# BANCA D'ITALIA

# **Temi di discussione**

del Servizio Studi

# Adverse Selection of Investment Projects and the Business Cycle

by Pietro Reichlin and Paolo Siconolfi



Number 326 - February 1998

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#### ADVERSE SELECTION OF INVESTMENT PROJECTS AND THE BUSINESS CYCLE (\*)

by Pietro Reichlin (\*\*) and Paolo Siconolfi (\*\*\*)

#### Abstract

In an economy where entrepreneurs with unequal "abilities" face alternative investment projects, which differ in degree of risk and productivity, we analyse the Nash equilibrium contracts arising from a banks-borrowers game in the context of asymmetric information. We show that, for a particular characterization of the game, one can determine the endogenous distribution of projects and the "type" of contracts (pooling or separating) as functions of the amount of loanable funds. We set this game in a general equilibrium aggregative economy with production, populated by overlapping generations of borrowers and lenders and show that for a range of the parameter values equilibria are characterized by persistent endogenous cycles.

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## **1** Introduction<sup>1</sup>

Many economists have argued that in economies with imperfect financial markets business cycle fluctuations are likely to be amplified. In particular, when lenders are not well informed about borrowers' investment projects, they tend to devise "second-best" contracts that may induce the borrowers to reveal some information. Typically, these contracts entail collateral requirements and credit rationing. As a consequence, real investment and consumption become highly dependent on the borrowers' balance sheet position, i.e., on the value of his net assets.

Various authors, including Bernanke and Gertler (1989), Greenwald and Stiglitz (1993) and Kiyotaki and Moore (1997), have noticed that the equilibrium quantity of lending and the default rates resulting from these second-best contracts may be highly sensitive to exogenous shocks. Thus the amplitude of the cycles is far greater than it would be with perfect financial markets and the effects of a shock to one sector may be more easily propagated to other sectors.

In general, this literature has focused on the role of imperfect financial markets in amplifying the propagation and the variability of the business cycle, assuming that the latter is originated by exogenous disturbances.

In this paper we set up a model in which informational asymmetries and second-best contracts in financial markets may be responsible for business cycle fluctuations that do not originate from any exogenous disturbance.

Our approach is different from that of the authors cited above, in whose models the main sources of business cycle fluctuations are borrowing limits, agency costs or collateral requirements. Instead, in our model a major role is played by the cyclical variability of the distribution of investment projects.

In our model entrepreneurs face different technologies to produce a single capital good <sup>1</sup>P. Reichlin acknowledges support from the HCM Program of the European Commission, contract CHRXCT94-0458. P. Siconolfi acknowledges the financial support of the GSB of Columbia University. We thank Gianni De Nicolò for many helpful conversations. and financial intermediaries are unable to observe both the borrowers' investment projects and their ability.

To simplify the analysis we assume the existence of only two types of investment projects, "good" and "bad", where the latter are dominated both in terms of risk and social productivity. The two projects have the same expected gross return, but the bad one is characterized by a lower probability of success (it is riskier) and it can only be operated by paying a fixed cost. As a result, the higher the proportion  $\lambda$  of agents undertaking the bad project, the higher is the loss of efficiency and real resources characterizing the associated equilibria.

Under some conditions, the equilibrium contracts allow for the existence of both types of project and their distribution is a function of the amount of loanable funds. In particular, loanable funds and bad projects are, at least in a critical region, positively correlated.

In our model, there is a sort of "cleansing effect of recessions", a phenomenon documented by Caballero and Hammour (1994) in a different context. During the cyclical upswing (when there is a large amount of loanable funds) competitive lenders devise contracts attracting a high proportion of bad projects; conversely, the opposite occurs during the downswing. The decline in the average quality of projects during the upswing, in turn, implies a loss of aggregate resources that may eventually lead to a recession.

The basic reason why cycles may be persistent in our model is that we find the existence of a fundamental discontinuity in the relation between loanable funds and the proportion of bad projects. This discontinuity is a consequence of a change in the characteristics of the second-best contracts that can emerge in financial markets and it may prevent the dynamics of the model from settling down to a stationary state.

In our model second-best contracts emerge in equilibrium since the relation between banks and firms is affected by moral hazard and adverse selection. In fact, the borrowers prepared to pay a higher interest rate are those who choose the riskier projects and this choice is affected by a costly action (where the cost of the action differs across borrowers).

To understand the banks-borrowers model, it may be useful to compare it to the Rothschild and Stiglitz (1976) insurance game (RS). There, the distribution of borrowers (insured) is exogenous and thus there is no moral hazard problem. In the RS model, Nash equilibrium contracts can only be separating and they exist only for a "large" proportion of high risk agents. As noted by Hellwig (1987), these two features of the model are a consequence of assuming that the game has two stages.

Following Hellwig (1987)) we choose a three stage game: lenders move first by offering loan contracts, borrowers apply for one of these contracts at the second stage and lenders accept or reject their applications at the last stage.

We define a "credit market equilibrium" as the set of Nash equilibrium contracts that satisfies the market-clearing conditions in the financial market. We prove that Nash equilibria always exist, that equilibrium contracts can be pooling and/or separating and that at a Nash equilibrium financial markets always clear.

Separating contracts can only be compatible with a Nash equilibrium when the interest rates are sufficiently high. Low rates induce a high proportion of borrowers to choose the safer projects and, as we know from the RS model, this makes separating contracts vulnerable to an upsetting deviation (by a pooling contract). The market-clearing condition in the credit market implies an inverse relation between loan rates and the amount of loanable funds. Thus, equilibrium separating contracts will only exist when the amount of loanable funds is small enough.

On the other hand, equilibrium pooling contracts are easily vulnerable to upsetting deviations (by separating contracts) when the size of the loan is small (i.e., when the amount of loanable funds is small). In fact, borrowers are always rationed with pooling contracts. Since technologies are linear and subject to a capacity constraint, credit rationing implies that the profits of the borrowers engaged on bad projects are increasing in the amount of the loan up to the capacity constraint. Thus, if the size of the loan is too small, a deviating bank can make higher profits by offering a contract at a slightly higher rate for a substantially bigger loan.

Since the amount of loanable funds is determined by the wage rate, we can say that separating contracts prevail when wages are low and pooling contracts prevail when wages are high.

However, there is a range of parameter values and wage levels for which separating and pooling contracts are both compatible with a Nash equilibrium. In order to eliminate this multiplicity problem, we introduce a natural selection criterion, i.e., the survival of the type of contract (separating or pooling) that corresponds to the highest rate on deposits (i.e., the highest bank revenues).

We then show that there is a unique switching value  $w^o$  for the amount of loanable funds w, such that the equilibrium contracts are separating (pooling) when  $w < w^o$  ( $w > w^o$ ), i.e., separating (pooling) contracts are a credit market equilibrium and they satisfy our selection criterion when  $w < w^o$  ( $w > w^o$ ).

Thus, when the amount of loanable funds crosses  $w^o$  from below, financial intermediaries replace separating with pooling contracts. This switch implies a sudden increase in the proportion of bad projects. In fact, the proportion of bad projects is increasing in interest payments (interest rate times loan) of the safe borrowers and the size of a loan granted to these borrowers falls short of the average size when contracts are separating and is just equal to the average size when contracts are pooling. Thus, interest payments deriving from a separating contract are always lower than those deriving from a pooling contract when the two contracts are generating the same rate on deposits, i.e., at the switching point  $w^o$ .

Embedding this framework in an overlapping generations model with two-period lived agents, we can describe the evolution of the distribution of projects along with output. In other words, contrary to previous contributions on the subject, we have a model in which the distribution of projects by degree of profitablity and risk is determined endogenously and is related to output.

The dynamics of this model are trivially monotonic when lenders are informed about borrowers' investment choices (and, a fortiori, under symmetric information) but turn out to be very rich under asymmetric information. We may have multiple steady states (with separating and/or pooling contracts) which are always associated with a lower income level than in the full information case. Moreover, due to the type of regime switch between separating and pooling contracts, steady states may not exist and, in this case, the equilibrium dynamics are characterized by "non-dying" and "non-exploding" cycles. Thus, our model is able to explain truly endogenous fluctuations, whereas most models of the business cycle based on asymmetric information and incentive constraints explain only the propagation mechanism and the amplification of the cycle initiated by exogenous perturbations.

## 2 The model

#### 2.1 Basic framework

We consider a two-good economy populated by two-period-lived overlapping generations of agents. Generations are identical and agents are decomposed into two sets, which we call the set of lenders and borrowers, each one distributed in the [0, 1] interval and having size one.

Lenders are endowed with one unit of labor when young, to be supplied to a set of competitive firms engaged in the production of a good y. Borrowers (or entrepreneurs) are endowed with the ability to run one of two storage technologies for producing a good z. All agents save and produce while young and consume in old age only.

The two goods are denoted by y and z. The former can be consumed or used as an input (capital) for the production of z. The latter can only be used as an input (materials) for the production of y. Capital and materials are fully depreciating in the production process.

Production of material z takes one period, it uses the good y as the only input and it is subject to random shocks. Entrepreneurs in this sector (borrowers) are risk-neutral, they can choose between two projects, a safe project L and a risky project H, and they differ in their ability to run them. We assume:

Assumption 1 Project j (j = H, L) transforms k units of good y invested at time t into a random output  $\tilde{z}_s^j(k)$  of good z at time t + 1, where:

$$ilde{z}^j_s(k) = \left\{egin{array}{cc} lpha^j(k-e^j(s)) & w.p. & p^j \ 0 & w.p & 1-p^j \end{array}
ight.$$

Assumption 2 All projects are subject to the capacity constraint  $k \leq 1$ .

Assumption 3  $\alpha^H > \alpha^L > 0$ ,  $p^L \alpha^L = p^H \alpha^H > 1$ .

Assumption 4  $\partial e^j(s)/\partial s \ge 0$  (j = H, L),  $e^H(s) > e^L(s)$  for all  $s \in [0, 1]$ .

Assumptions 3 and 4 are motivated by the fact that, in our model, we need technology L to dominate technology H, both in the sense that the latter has a higher fixed cost and that it is riskier (by first order dominance). For this reason, H-projects will be called "risky" or "bad". This dominance would be more robust with  $p^L \alpha^L \geq p^H \alpha^H$ , but the weaker assumption  $p^L \alpha^L = p^H \alpha^H$  greatly simplifies the algebra.

Production of y is instantaneous and it requires labor L and materials z as inputs. This technology is represented by a standard constant-returns-to-scale neoclassical production function:

$$y_t = F(z_t, L_t)$$

In order to guarantee the existence of non-trivial stationary states for the equilibrium dynamics of the model, we make the following assumption.

Assumption 5  $F_i(z,L) > 0$ ,  $F_{i,i}(z,L) < 0$ , (i = z,L) for all  $(z,L) \gg 0$ , F(0,L) = F(z,0) = 0,  $\lim_{z\to 0} F_z(z,L) = \infty$ ,  $\lim_{z\to 0} F_L(z,L)/z = \infty$ .

Under full employment and by the linear homogeneity of F(.) we can write:

$$y_t = F(z_t, 1) \equiv f(z_t)$$

and by the properties of F(.):

$$f(0) = 0, \ f'(z) > 0, \ f''(z) < 0$$

Letting all prices be measured in terms of the good y, perfect competition in the y sector insures that, at time t, the wage rate  $w_t$  and the price of the input z,  $q_t$ , are given by:

$$w_t = f(z_t) - z_t f'(z_t)$$
 (2.1)

$$q_t = f'(z_t) \tag{2.2}$$

Equations 2.1 and 2.2 also insure zero profit for all firms producing y.

#### 2.2 Financial intermediation

A finite number of financial intermediaries (banks) collect deposits from lenders and supply loans to entrepreneurs-borrowers.

In our economy the amount of deposits coincides with the wage rate w. Due to the presence of a capacity constraint, the total amount of loans that the borrowers can absorb cannot be greater than 1. In what follows we will always consider the case in which  $w_t \leq 1$  for all t. This is the interesting case, since it implies that the competitive economy solves a non-trivial problem of allocating scarce resources. If w > 1 there is excess supply of loanable funds and all firms are producing the good z at full capacity.

There are several ways of insuring  $w_t \leq 1$  for all t. The reader will see later on that the equilibrium sequence of wage rates is always bounded and thus a sufficiently small initial value  $w_0$  and/or a sufficiently small marginal productivity of labor for all z is enough to guarantee  $w_t \leq 1$ .

Banks behave competitively on both sides of the market in the following sense: they set contracts (specifying both quantities and prices) on the loan side and take the interest rate on deposits as given. Contracts are designed to maximize expected profits, and competition guarantees that the expected rate of return on loans equals the rate on deposits.

Contracts are denoted by:

$$c(i) = (R(i), B(i))$$

where R(i) is the interest factor on the loan and B(i) the amount lent by a bank *i*. Each borrower can apply for a single contract at most.

The set of borrowers applying for any contract is either empty or positive in measure. By the law of large numbers, banks' future revenues are deterministic and equal to the ex ante average revenue.

Since entrepreneurs have no endowment and the technology yields zero output in the bad state, c(i) is a standard debt contract entailing limited liability: the loan B(i) is repayed if and only if the investment project does not fail, otherwise the borrower pays nothing to the bank.

By risk neutrality, type-s borrower's expected profit with project j is:

$$\Pi^j_{s}(c) = (qar{lpha} - p^j R)B - qar{lpha} e^j(s)$$

where q is the relative price of z,  $\bar{\alpha} = p^L \alpha^L = p^H \alpha^H$  (for the moment the time subscript has been omitted from all variables).

It follows that:

$$\Pi_{s}^{H}(c) - \Pi_{s}^{L}(c) = (p^{L} - p^{H})RB - q\bar{\alpha}(e^{H}(s) - e^{L}(s))$$

In particular, since  $e^{j}(s)$  is small for any small s, entrepreneurs with high ability (small s) will choose to run the bad project. Thus, if the fixed cost were zero, all entrepreneurs would choose the bad project H. This is a consequence of limited liability. Then,  $\lambda < 1$  in equilibrium requires that  $p^{L}$  is not much higher than  $p^{H}$  and that  $e^{H}(.)$  is not too close to  $e^{L}(.)$  for all s. In order to get a simpler formalization of this type of conditions, we now specialize assumption 4 to the following simpler form:

Assumption 6  $e^L(s) = 0$  for all  $s \in [0, 1]$ ,  $e^H(s) = es$  with e > 0.

By assumption 6, we can ensure that both projects will survive in equilibrium by imposing lower bounds on the parameter e, as is made clear later on.

Before we characterize the competitive equilibria with asymmetric information, we give a brief account of the model under the assumption that banks have information about the borrowers' technology choice (known action).

### 2.3 The benchmark model

The model with "known action" is trivial, so we only give a quick description of the competitive equilibria and the welfare properties. Note that these properties, as well as the equilibria of the model, remain unchanged under the stronger assumption that financial intermediaries have complete information.

If banks know the technology choice of the borrowers, they can make contracts contingent on it. Since the type s of the borrower does not affect the banks revenues, these contracts are a pair:

$$c^{L} = (R^{L}, B^{L}), \ c^{H} = (R^{H}, B^{H})$$

Let r be the deposit rate. Nash equilibrium contracts are such that:

i) 
$$r = p^L R^L = p^H R^H$$
,  $R^L \le q \alpha^L$ ,  $R^H \le q \alpha^H$ ;  
ii)  $B^L = B^H = 1$  if  $R^j < q \alpha^j$ , and  $B^j \in [0, 1]$  if  $R^j = q \alpha^j$ ,  $j = H, L$ .

Since the total amount of loans must be equal to w < 1, the only Nash equilibrium compatible with market-clearing in the financial market is:

$$c^L = (q lpha^L, w), \ \ c^H = (q lpha^H, w)$$

Evidently, with this contract, all borrowers choose the *L*-project (recall that borrowers are assumed to choose the *L*-project when the profits deriving from the two technologies are equal) and nobody applies for the contract  $c^{H}$ .

Thus, the evolution of the equilibrium value of z is given by the following condition:

$$z_t = \bar{\alpha} w_{t-1}$$

From equation 2.1 and assumption 5, there is a continuous strictly increasing and differentiable function  $\Phi(.)$  such that  $z = \Phi(w)$  and:

$$\Phi(0)=0,\ \lim_{w
ightarrow\infty}\Phi(w)/w=+\infty,\ \lim_{w
ightarrow0}\Phi(w)/w$$

Thus, the dynamics of the equilibrium wage rate are described by:

$$\Phi(w_t) = ar{lpha} w_{t-1}$$

Now define  $w^* > 0$  from  $\Phi(w^*) = \bar{\alpha}w^*$  and assume:

#### Assumption 7 $w^* \in (0, 1)$

Under this assumption,  $w^*$  is the unique steady state for the dynamics of the equilibrium wage rate in the benchmark model for all initial conditions  $w_0 < 1$  and for all these initial conditions, the model produces a convergent sequence  $w_t \to w^*$ .

Corresponding to the stationary state  $w^*$ , there are unique stationary values of inputs and (average) consumption,  $k^*$ ,  $z^*$ ,  $c^*$ , such that:

$$z^* = ar{lpha} k^*, \; c^* + k^* = f(z^*), \; c^* = ar{lpha} f'(z^*) w^*$$

Now consider the Pareto optimal allocations. It is obvious that, within these allocations, the H-technology will never be activated. Hence, the Planner's resource constraints will simply read:

$$c_t + k_t \le f(z_t), \ z_t \le \bar{\alpha}k_{t-1}$$

where  $c_t$  is the time-t average consumption. It follows that a stationary Pareto optimal allocation implies a value z > 0 such that:

$$ar{lpha}f'(z)=ar{lpha}f'(ar{lpha}k)\geq 1$$

In the rest of the paper we assume that, given  $z^* > 0$ ,  $\bar{\alpha}$  is sufficiently large to guarantee that the competitive allocations of the benchmark model are Pareto optimal. In this way, we can single out a unique potential source of inefficiency in the model with uninformed lenders, i.e., the survival of the *H*-technology.

## 3 The banks-borrowers game

### 3.1 General structure

Now we assume that banks have neither ex-ante nor ex-post information about firms types and their technological choices. However, they can costlessly observe whether any project is successful. This implies that the game between banks and borrowers is characterized by moral hazard and adverse selection. The former arises since the action of the borrower (technological choice) affects the probability with which a project fails. Adverse selection arises since, for given technological choices, projects have two different degrees of risk (recall that  $p^L > p^H$  and  $\alpha^H > \alpha^L$ ).

Our model differs from most models of asymmetric information since the distribution of projects by the degree of risk has to be endogenously determined. However, the basic insights of the pure adverse selection model apply to our case also.

We know from the classic paper by Rothschild and Stiglitz (1976) that all Nash equilibria of a two stage game between borrowers and lenders display self-selection of borrowers according to the contracts offered by the lenders. In particular, the equilibrium contracts are always "separating" and they imply credit rationing for the borrowers having the safer projects.

However, equilibria fail to exist when the proportion of safe borrowers in the total population is sufficiently high. In this case a pooling contract can always upset any pair of separating contracts. Moreover, within the two-stage game, pooling contracts can always be upset by a separating pair attracting safe borrowers only.

The possible lack of a Nash equilibrium has led some authors to propose different solution concepts or different structures for the game (e.g., Wilson (1977), Riley (1979) and Hellwig (1986, 1987)).

In this paper we follow the approach proposed by Hellwig (1986, 1987) assuming that banks and lenders play a three stage game. Lenders move first by making contract offers. At the second stage borrowers can apply for at most one contract and select the project type. At the third stage, banks decide on applications after they observe the contract offers and the set of applications. We make the standard assumption that banks cannot cross-subsidize contracts.

Within the pure adverse selection model, Hellwig shows that a sequential Nash equilibrium will always exist in the three stage game. In particular, pooling contracts arise as equilibrium contracts whenever they Pareto dominate the separating pair.

In the next two sections we determine the symmetric Nash equilibria of the game between banks and borrowers with hidden action, for given wage rate  $w \leq 1$  and price q. To save notation, we omit the time subscript from all variables.

Given the structure of the game, we know that there may be only two sets of contracts as candidates for a symmetric Nash equilibrium: a pair of separating contracts and a pooling contract.

The pair  $(c^H, c^L)$ , with  $c^j = (R^j, B^j)$ ,  $c^H \neq c^L$ , j = H, L, is a separating contract if firms adopting the *j*-project apply for contract  $c^j$  and  $B^j > 0$  for all j = H, L. A pooling contract is  $c^p = (R^p, B^p)$ .

In fact, banks cannot make contracts contingent on the borrower's s-type and on the technology. Moreover, their revenues are clearly independent of the borrower's s-type, but not of his technological choice. Thus, banks can at most try to induce self-selection of borrowers in terms of the technology.

Later on we will impose a condition insuring that, in equilibrium, both projects will be implemented by the borrowers. Thus, in our model, there is always going to be a marginal type of borrower  $\lambda \in (0,1)$  who is indifferent between the *H* and the *L*-technology. In particular, if the equilibrium contracts are separating, the type  $\lambda$  is determined by the condition:

$$\Pi^H_\lambda(c^H) = \Pi^L_\lambda(c^L) \tag{3.3}$$

When the equilibrium contract is pooling:

$$\Pi^H_\lambda(c^p) = \Pi^L_\lambda(c^p) \tag{3.4}$$

Given assumptions 3 and 4, we will always have that, when  $\lambda \in (0, 1)$ , any type  $s < \lambda$  will adopt the *H*-technology and any type  $s \ge \lambda$  will adopt the *L*-technology.

The purpose of the next four sections is to characterize a credit market equilibrium. To this end we devide the discussion into the following steps.

1) In section 3.2 and 3.3 we determine when pooling and separating contracts may be compatible with a Nash equilibrium of the banks-borrowers game described above.

2) In section 3.4 we define (and show the existence of) market-clearing contracts as the Nash equilibrium contracts of the game which are verifying the equality between the supply of loanable funds w and the demand of loans arising in the Nash equilibrium. For some parameter values, separating and pooling Nash equilibrium contracts may both be market-clearing contracts for the same level of w.

3) Finally, in section 3.5 we determine the single type (pooling or separating) of marketclearing contract satisfying a selection criterion. We select the unique equilibrium contracts generating the highest rate on deposits. This type of contract, along with the supply of loanable funds, defines a credit market equilibrium. We show that there is a credit market equilibrium for all  $w \in (0, 1)$ .

From now on, to simplify the notation, the interest rates in any given loan contract will be normalized by  $q\alpha^{L}$ . In particular, for any contract  $c^{j} = (R^{j}, B^{j})$ , we define:

$$x^j = R^j/q\alpha^L$$

Moreover, it will be convenient to introduce the following definitions:

$$\mu=p^{H}/p^{L}, ~~\sigma=rac{1-\mu}{e}$$

#### **3.2** Pooling contracts

By definition, a pooling contract is a contract  $c^p = (x^p, B^p)$  accepted by all firms, whatever the technology (project) they choose.

When this contract does not induce specialization, equation 3.4 determines the proportion of borrowers adopting the *H*-technology as:

$$\lambda = \sigma x^p B^p \tag{3.5}$$

Since the entrepreneur of type s = 0 has no fixed cost, no pooling contract can implement technology L only. However, it is possible that  $c^p$  induce all firms to adopt technology H. In this case the contract will be called H-pooling.

Consider an equilibrium pooling contract  $c^p = (x^p, B^p)$ . Up to a normalization, banks profits are:

$$\Pi^B = [x^p(1 - (1 - \mu)\lambda) - r]B^p$$

The zero profit condition implies  $r = x^p(1 - (1 - \mu)\lambda)$ . Thus, a contract  $c^p = (x^p, B^p)$  is a Nash equilibrium when there is no deviating contract  $\hat{c} = (\hat{x}, \hat{B})$  generating higher revenue per unit of loan.

Since we are only interested in the case of no specialization, we now set a condition ruling out the existence of the H-pooling contract as a Nash equilibrium of the banks-borrowers game. In the light of the previous section, it is not surprising that this condition amounts to setting a lower bound on the fixed cost e. In fact, when technologies are very close to each other, pooling contracts tend to induce the adoption of bad projects. The next proposition establishes this claim precisely.

**Proposition 1** Let  $e \ge 1/2$ . Then, for all  $\mu = p^H/p^L \in (0,1)$  there cannot exist an H-pooling Nash equilibrium.

**Proof.** See the appendix.  $\Box$ 

From now on we will be assuming:

## Assumption 8 $e \ge 1/2$ .

Under the above assumption, it must be that:

$$B^p \leq 1, \ x^p \leq 1(R^p \leq q\alpha^L)$$

In the next proposition we characterize the set of pooling contracts  $c^p$  that are symmetric Nash equilibria of the banks-borrowers game.

For some parameter values, these conditions may involve a lower bound on the value of  $B^p$ .

**Proposition 2** There exist a continuous decreasing function  $x^p : [0,1] \rightarrow [0,1]$  and a value  $M(e,\mu)$  such that  $c^p$  is a Nash equilibrium of the banks-borrowers game if and only if  $x^p = x^p(B^p)$  and:

 $B^{p} \in [0,1]$  when  $e \geq \max(1/2, 2(1-\mu));$ 

 $B^{p} \in [M(e,\mu), 1]$  when  $1/2 \le e < 2(1-\mu)$  (i.e.,  $\mu < 3/4$ );

where  $M(e,\mu) \in (0,1)$  for all  $e \in [1/2, 2(1-\mu))$  when  $\mu < 3/4$ .

**Proof.** The proof of the proposition can be decomposed into two steps.

There are two possible ways of upsetting a pooling contract  $c^p$ : by a pooling and by a separating deviation. A deviation is separating if it attracts firms s undertaking the same project only. Otherwise it is called a pooling deviation.

Accordingly, conditions under which  $c^p$  cannot be upset by a pooling and a separating deviation are given in a first and in a second lemma of the proof respectively.

**Lemma 1** There is a continuous decreasing function  $x^p : [0,1] \to [0,1]$  with  $x^p(0) = 1$  such that a pooling contract  $c^p = (x^p, B^p)$  cannot be upset by a pooling deviation iff  $x^p = x^p(B^p)$ .

**Proof.** Let  $\hat{c} = (\hat{x}, \hat{B})$  be the pooling deviation. The deviating contract  $\hat{c}$ , in order to be pooling (i.e., attract all borrowers), needs to generate higher profits to the firms adopting

the L-project only. In fact, after the deviation, the only borrowers, if any, applying for the contract  $c^p$  are those adopting the H-project. Thus,  $c^p$  becomes unprofitable for the non deviating banks, which will reject all applications at the third stage of the game.

Now define the banks expected revenue for a unit of loan as:

$$V^{p}(x^{p}, B^{p}) = x^{p}[1 - (1 - \mu)\lambda] = x^{p}[1 - (1 - \mu)\sigma x^{p}B^{p}]$$

The contract  $\hat{c}$  will upset  $c^p$  if and only if:

(i) 
$$V^{p}(\hat{x}, \hat{B}) > V^{p}(x^{p}, B^{p})$$
  
(ii)  $(1 - \hat{x})\hat{B} \ge (1 - x^{p})B^{p}$   
(iii)  $\hat{B} \le 1$ 

Condition (i) says that the deviating bank makes higher profits, condition (ii) says that borrowers undertaking L-projects under the upsetting contract are not worse-off and condition (iii) guarantees that the upsetting contract satisfies the capacity constraint.

Clearly, we are in the best position to upset the existing contract when  $\hat{B}$  satisfies (ii) with equality, so that the upsetting loan will be chosen to be:

$$\hat{B}(\hat{x}) = \frac{1-x^p}{1-\hat{x}}B^p$$

Now substitute  $\hat{B}(\hat{x})$  in the left hand side of (i) and write:

$$V^p(\hat{x}, \hat{B}(\hat{x})) = \hat{x}Q(\hat{x})$$

where:

$$Q(\hat{x}) = 1 - (1 - \mu)\sigma \hat{x}\hat{B}(\hat{x})$$

is the repayment probability of the loan.

It is easy to verify that Q(.) and  $V^{p}(\hat{x}, B(\hat{x}))$  are respectively strictly decreasing and concave in  $\hat{x}$ . Hence, as in the standard monopoly case, the expected revenue  $V^{p}(.)$  is maximized when the elasticity:

$$\eta(\hat{x})=-Q'(\hat{x})\hat{x}/Q(\hat{x})=1$$

Evidently, the condition for the absence of a profitable deviation is that this maximizing condition is satisfied for  $\hat{x} = x^p$ .

One can easily verify that  $\eta(x^p) = 1$  requires a decreasing relation between  $x^p$  and  $B^p$ , which we define by:

$$x^p = x^p(B^p) \tag{3.6}$$

where  $x^{p}(.)$  is a continuous decreasing function in [0,1] such that  $x^{p}(0) = 1$ .

The proof of lemma 1 is sufficiently intuitive to explain the result. However, it may be worthy to stress the main point. In order to be a Nash equilibrium, a pooling contract must correspond to the maximum revenue that a deviating monopolistic bank would be able to make. The relation  $x^p = x^p(B^p)$  simply guarantees that the contract  $c^p$  satisfies this condition.

Now we turn to separating deviations. In this step we only need to show under what conditions  $c^p$  cannot be upset by a contract attracting *H*-projects only.

Let  $\hat{c} = (\hat{R}, \hat{B})$  be the upsetting contract. To save notation we will again normalize interest rates by  $q\alpha^{L}$ . Thus we write  $\hat{x}$  for  $\hat{R}/q\alpha^{L}$ .

It follows that the upsetting contract has to satisfy the following conditions:

(i')  $\mu \hat{x} > V^p(x^p, B^p) = x^p [1 - (1 - \mu)\lambda]$ (ii')  $\hat{B}(1 - \mu \hat{x}) \ge B^p (1 - \mu x^p)$ (iii')  $\hat{B}(1 - \hat{x}) \le B^p (1 - x^p)$ (iv')  $\hat{B} \le 1$  Condition (i') says that the deviating contract is more profitable for the bank, conditions (ii) and (iii) say that the deviating contract attracts H-type projects only and condition (iv) defines the capacity constraint.

It turns out that the existence of a capacity constraint  $(B \leq 1)$  is crucial for ruling out an upsetting separating deviation.

In fact, since  $V^p < 1$ , a deviating bank can pick  $\hat{c} = (\hat{x}, \hat{B})$  such that  $V^p < \mu \hat{x} < 1$  and  $\hat{B}$  is high enough to attract *H*-type borrowers. Evidently, this is a profitable deviation.

On the contrary, when  $\hat{B} \leq 1$  is imposed, the possibility of a profitable deviation attracting only *H*-technologies may be ruled out when  $c^p$  generates enough profits for the firms adopting the *H*-technologies, i.e., when:

$$B^p(1-\mu x^p)$$

is high enough. Since the above expression is increasing in  $B^p$ , any equilibrium pooling contract must satisfy the following lemma.

**Lemma 2** If  $\sigma \leq 1/2$ , a pooling contract  $c^p$  satisfying the condition  $x^p = x^p(B^p)$ , can never be upset by a separating deviation. If  $\sigma > 1/2$ , there is a value  $M(e, \mu) < 1$ , such that the same contract cannot be upset by a separating deviation iff:

$$B^p \ge M(e,\mu)$$

**Proof.** See the appendix.  $\Box$ 

Lemma 2 completes the proof of the proposition.  $\Box$ 

#### **3.3** Separating contracts

Separating contracts must satisfy two types of constraint: the participation and the incentive compatibility constraints. Given the linearity of the technology, the participation constraint

simply says that the interest rate on a loan contract cannot exceed the exogenous marginal product of capital for the technology for which the contract has been devised. Recalling our normalization, i.e.,  $x^j = R^j/q\alpha^L$  and the fact that  $p^L\alpha^L = p^H\alpha^H$  implies  $\alpha^L/\alpha^H = \mu$ , we get the following conditions:

(PC) 
$$\mu x^H \leq 1, \ x^L \leq 1;$$

(IC-1)  $\Pi_s^H(c^H) \ge \Pi_s^H(c^L)$ , for all  $s < \lambda$ ;

(IC-2) 
$$\Pi^L(c^H) \leq \Pi^L(c^L)$$
 for all  $s \geq \lambda$ ;

where, we recall,  $\lambda$  is such that:

$$\Pi_s^H(c^H) \ge \Pi^L(c^L) \text{ for all } s < \lambda; \ \Pi^L(c^L) \ge \Pi_s^H(c^H) \text{ for all } s \ge \lambda.$$

Competition among banks (on the deposit side) guarantees that in equilibrium they make zero profits. This imposes an extra condition on the set of separating contracts compatible with an equilibrium. In particular, since at the last stage of the game banks can reject applications on a specific contract, in a separating equilibrium the two contracts must generate the same revenue.

Then, zero profits imply  $p^H R^H = p^L R^L = r$ , i.e.:

$$\mu x^H = x^L \tag{3.7}$$

Clearly, no pair of separating contracts can be a Nash equilibrium if there is a deviating contract that makes some borrower better off and that does not violate the IC constraints. This implies that, at a Nash equilibrium, (IC-1) must be satisfied with equality and risky borrowers must produce at capacity. Formally:

$$(1 - \mu x^H)B^H = (1 - \mu x^L)B^L$$
(3.8)

$$B^H = 1 \tag{3.9}$$

From 3.7, 3.8 and 3.9 we get:

$$B^{L} = \frac{1 - x^{L}}{1 - \mu x^{L}} \tag{3.10}$$

Equation 3.10, defined for  $x^L \in [0, 1]$ , shows that  $B^L$  is a decreasing function of  $x^L$  ranging from 1 to zero.

From equations 3.3, 3.7 and 3.9 we get:

$$\lambda = \sigma x^L B^L$$

Using equation 3.10, the above can be written as:

$$\lambda = \phi(x^L) \equiv \sigma \frac{(1 - x^L)x^L}{1 - \mu x^L}$$
(3.11)

which is the proportion of agents undertaking the *H*-project. One can verify that, under assumption 8 it is always verified that  $\phi(x^L) < 1$ . Thus, there cannot be complete specialization on any one of the given technologies with separating contracts.

The function  $\phi$  is unimodal with  $\phi(0) = \phi(1) = 0$  and there is a value  $\theta \in (1/2, 1)$  such that:

$$\phi'(x^L) > 0 \Leftrightarrow x^L < \theta$$

The reason for the non-monotonicity of  $\phi(.)$  is easy to grasp. By equation 3.7 a rise of  $x^{L}$  implies a rise of the interest rates on loans for both projects. Since  $B^{H} = 1 > B^{L}$ , the profit from the *H*-technology falls more than the profit from the *L*-technology when  $x^{L}$  goes up and this would imply a fall in  $\lambda$ .

By the incentive compatibility constraint and equations 3.7 and 3.9, a rise of  $x^{L}$  implies a fall in  $B^{L}$  for any given value of  $\lambda$ . In fact, the profit from the *L*-project falls more than the profit from the *H*-project when  $x^{L}$  goes up, since:

$$d\Pi^L = -p^L B^L dx^L < d\Pi^H_s = -p^H B^L dx^L$$

for all s. In turn, a fall in  $B^L$ , by decreasing the profits from the L-project, has a positive effect on  $\lambda$ .

Therefore the final effect on  $\lambda$  of a rise in  $x^{L}$  is ambiguous.

The following proposition shows conditions under which a separating pair  $(c^{H}, c^{L})$  with  $\lambda < 1$  is a symmetric Nash equilibrium of the banks-borrowers game. By equations 3.7, 3.9 and 3.10, the separating contracts can be uniquely defined in terms of the variable  $x^{L}$ , and the proposition shows that we can always find a separating pair  $(c^{H}, c^{L})$  as a Nash equilibrium, provided that  $x^{L}$  is not too small.

**Proposition 3** A separating pair of contracts  $(c^H, c^L)$  is a Nash equilibrium of the banksborrowers game if and only if:

$$x^L \ge \gamma(e,\mu)$$

where  $\gamma(e,\mu) = 1$  for  $e \geq \max(1/2, 4(1-\mu))$  and it is in (0,1) otherwise.

**Proof.** See the appendix.  $\Box$ 

Here we give a sketch of the proof to convey some of the underlying intuition.

A separating pair  $(c^H, c^L)$  is a Nash equilibrium if there are no deviating contracts such that at least one set of borrowers are not worse off and the deviating bank is making higher revenues with these contracts. Since there are three stages in the game, the profitability of the deviation must be judged by taking into account that, as a response to the deviating contracts, non-deviating banks can reject the borrowers' applications.

There are two types of deviations in this game since the deviating bank can design a separating or a pooling contract. Since separating contracts satisfy both the IC constraints and  $B^{H} = 1$ , a simple argument shows that they are immune to separating deviations. Thus, in order for a deviating contract to upset the separating pair  $(c^{H}, c^{L})$ , it has to be a pooling contract  $\hat{c}^{p}$  satisfying:

- (a)  $[1 (1 \mu)\hat{\lambda}]\hat{x}^p > x^L;$
- (b)  $\Pi_{s}^{H}(\hat{c}^{p}) \geq \Pi_{s}^{H}(c^{H}), \, \Pi^{L}(\hat{c}^{p}) \geq \Pi^{L}(c^{L}).$

where  $\hat{\lambda}$  is the proportion of borrowers undertaking the bad project with the upsetting contract after the unprofitable contracts have been withdrawn.

Condition (a) basically requires the proportion of bad projects  $\lambda$  after the deviation to be small enough. Thus, a Nash equilibrium with separating contracts will only exist when this proportion is sufficiently high. This condition looks like the basic condition for the existence of separating contracts in the Rothschild and Stiglitz (1976) insurance game. However, in our framework, the proportion of risky projects is not an exogenous parameter but a function of the interest rates. In particular, by equation 3.4, any pooling deviation satisfies:

$$\hat{\lambda} = \sigma \hat{x}^p \hat{B}^p$$

The next step is to define the value  $\hat{B}^p$  for which banks profits are higher and condition (b) is not violated.

We can get a more immediate intuition of this proof when the capacity constraint is not binding. In this case the most profitable deviation is characterized by the highest loan compatible with conditions (b), i.e., by the loan  $\hat{B}^p(\hat{x}^p, x^L)$  at which  $\Pi^L(\hat{c}^p) = \Pi^L(c^L)$ .

Evidently, such  $\hat{c}^p$  is strictly preferred to  $c^H$  by any borrower adopting the H technology. Working out this condition, we obtain:

$$\hat{B}^p(\hat{x}^p,x^L)=rac{1-x^L}{1-\hat{x}^p}B^L$$

Under this specific contract we can derive that:

$$\hat{\lambda} = \lambda rac{\hat{x}^p (1 - x^L)}{x^L (1 - \hat{x}^p)}$$

Plugging this value into equation (a) and using these results, one can show that the condition for the non-existence of an upsetting deviation reduces to a simple relation between  $x^{L}$  and  $\lambda$ . In particular, when the capacity constraint is not binding we need  $\lambda \geq u(x^{L})$ , where:

$$u(x^L)=rac{1-x^L}{4(1-\mu)}$$

The reason why  $u(x^L)$  is decreasing can be derived from equation (a) and from the definition of  $\hat{\lambda}$ . In particular, when  $x^L$  goes up, we need a more profitable deviation in order to upset the separating contract and this requires a smaller value of  $\lambda$ .

Finally, using the fact that, with separating contracts,  $\lambda = \phi(x^L)$ , we can reduce the condition for the existence of a Nash equilibrium to a lower bound for  $x^L$ .

Figure 1 can illustrate the situation. The hump-shaped curve  $\phi(x^L)$  mapping the interest rate  $x^L$  onto the proportion of bad projects  $\lambda$  describes a relation to be satisfied by the potential equilibrium contracts (equation 3.11). The region above the decreasing line  $u(x^L)$ defines the combination of  $x^L$  and  $\lambda$  such that no profitable deviation from a separating contract exists and  $\gamma$  is the intersection between the two curves. Thus, a Nash equilibrium separating contract must be one such that  $x^L \geq \gamma$ , i.e., one for which the curve  $\lambda = \phi(x^L)$ and the region above the curve  $u(x^L)$  intersect.

#### **3.4** Market-clearing contracts

In the previous section we characterized the set of equilibrium separating and pooling contracts. In this section we impose that these contracts satisfy a market-clearing condition for each exogenously given level of saving (wage)  $w \in [0, 1]$ .

**Definition 1** For  $w \in (0,1)$ , a "market-clearing" (MC) contract is a Nash equilibrium contract (separating pair or pooling) such that the total amount of loans is equal to w.

The next proposition establishes the existence of MC contracts, either of the separating or of the pooling type (or both).

#### **Proposition 4** A market-clearing contract exists for all $w \in (0, 1]$ .

**Proof.** The proof has three steps. In the first step we will find a condition for the existence of pooling MC contracts, in the second step we will find a condition for the existence of separating MC contracts and in the final step we will examine these conditions at the same time.

Consider the market-clearing condition in the credit market when the contract is pooling. In this case the equality between demand and supply of loanable funds simply requires  $B^p = w$ . Then, we can state the following:

Lemma 3 If  $e \ge \max(1/2, 2(1-\mu))$  a pooling contract is an MC contract for all  $w \in (0, 1)$ . If  $1/2 \le e < 2(1-\mu)$  (i.e.,  $\mu \ge 3/4$ ), a pooling contract is an MC contract provided that  $1 \ge w \ge M(e, \mu) < 1$ .

**Proof.** The proof simply follows from proposition 2.  $\Box$ 

Now consider a pair of separating equilibrium contracts. Then, the proportion of high risk projects is  $\lambda^s = \phi(x^L)$ , where  $x^L = R^L/q\alpha^L$ ,  $B^H = 1$  and  $B^L$  is determined by equation 3.10, i.e.:

$$B^L = B^L(x) \equiv \frac{1 - x^L}{1 - \mu x^L}$$

Therefore, with separating contracts, the market-clearing condition in the credit market is:

$$\phi(x^{L}) + (1 - \phi(x^{L}))B^{L}(x^{L}) = w$$
(3.12)

Equation 3.12 defines implicitly  $x^{L}$  as a function of w, i.e.:

 $x^L = x^L(w)$ 

where  $x^{L}(.)$  is strictly decreasing in [0, 1],  $x^{L}(0) = 1$  and  $x^{L}(1) = 0$ .

By proposition 3,  $x^{L}$  is compatible with a Nash equilibrium only if it is greater then or equal to  $\gamma(e, \mu)$ . Then we have the following:

Lemma 4 If  $1/2 \le e < 4(1 - \mu)$ , (i.e.,  $\mu < 7/8$ ) there is a value  $L(e, \mu) \in (0, 1)$  such that a separating pair  $(c^H, c^L)$  is an MC contract for all  $w \in (0, L(e, \mu))$ .

**Proof.** Market clearing in the credit market with separating contracts requires  $x^{L}(w) \geq \gamma(e,\mu)$ , i.e.:

$$w \leq L(e,\mu)$$

for some value  $L(e,\mu) \in (0,1)$ . By proposition 3,  $x^L < 1$  in a Nash equilibrium only if  $1/2 \le e < 4(1-\mu)$ . Then, this assumption implies L(.) > 0 and the proposition follows.  $\Box$ 

The proof is completed by oserving that  $L(.) \ge M(.)$  for all  $\mu \in (0, 1)$  and  $e \ge 1/2$  (which is verified by assumption 8). Then, there is at least one market-clearing contract for each level of saving  $w \in (0, 1)$ .  $\Box$ .

#### 3.5 Credit market equilibrium

At this stage of the model separating and pooling market-clearing contracts may coexist for the same level of w. In particular, proposition 4 says that there is an open interval I in (0, 1)such that both a pooling and a separating pair of contracts are MC contracts associated with the same  $w \in I$ . In order to resolve this multiplicity problem, we impose the following selection criterion.

**Definition 2** A contract (separating pair or pooling) is a "credit market equilibrium" (CME) contract for some  $w \in (0, 1)$  if it is the MC contract that, among the MC contracts compatible with the same value of w, generates the highest deposit rate r.

For each  $w \in (0,1)$ , define  $r^s(w)$  and  $r^p(w)$  as the unique deposit rates arising from a separating and a pooling Nash equilibrium contract respectively. Then:

$$r^{s}(w) = q\bar{\alpha}x^{L}(w), \ r^{p}(w) = q\bar{\alpha}x^{p}(w)[1 - (1 - \mu)\lambda^{p}(w)]$$

where  $\lambda^{p}(w) = \sigma w x^{p}(w)$  is the equilibrium proportion of high risk projects with pooling contracts.

**Proposition 5** For  $\sigma < (3^{1/2} - 1)/2$  all CME contracts are pooling. Otherwise there exists a value  $w^{\circ} \in (0, 1)$  such that, for  $w \leq w^{\circ}$ , CME contracts are separating and, for  $w \geq w^{\circ}$ , CME contracts are pooling.

**Proof.** See the appendix  $\Box$ 

The above proposition says that:

A) when  $w \leq w^{o}$ , CME contracts are separating either because  $r^{s}(w) > r^{p}(w)$  or because  $w < M(e, \mu)$ , i.e., pooling contracts are not MC;

B) when  $w \ge w^{o}$ , CME contracts are pooling either because  $r^{p}(w) > r^{s}(w)$  or because  $w > L(e, \mu)$ , i.e., separating contracts are not MC.

Since we know that  $L(e,\mu) > M(e,\mu)$ , there may exist a value  $w' \in (M(.), L(.))$  such that  $r^s(w') = r^p(w')$ . In this case  $w^o = w'$ . Simple calculations show that L(.) > w' for all values of e and  $\mu$  in the relevant range. Thus  $w^o = \max\{w', M(e,\mu)\}$ . Figure 2 shows the case  $w' > M(e,\mu)$ .

In order to ensure the existence of both pooling and separating CME contracts, from now on it will be assumed:

Assumption 9  $e \le \frac{2(1-\mu)}{3^{1/2}-1}$ 

The above assumption implies  $e < 4(1 - \mu)$ , i.e., it implies the existence of equilibrium separating contracts for some w > 0.

### 3.6 Efficiency

A natural question to raise at this point is whether credit market equilibria are efficient.

For given  $w \in (0,1)$  (supply of loanable funds) and rw > 0 (lenders' consumption), a pair of contracts  $\omega = (\omega^H, \omega^L)$  with  $\omega^j = (\rho^j, \beta^j)$  (j = H, L) is feasible if it satisfies:

(f-1) 
$$\lambda(\omega)\beta^H + (1 - \lambda(\omega))\beta^L \le w;$$

(f-2) 
$$\lambda(\omega)p^{H}\rho^{H}\beta^{H} + (1-\lambda(\omega))p^{L}\rho^{L}\beta^{L} \ge rw;$$

the incentive compatibility constraints (IC-1), (IC-2), the participation constraints (PC) and the capacity constraints  $\beta^j \leq 1$  (j = H, L), where, as usual:

$$\lambda(\omega) = \min\{1, rac{(ar{lpha} - p^H 
ho^H)eta^H - (ar{lpha} - p^L 
ho^L)eta^L}{ear{lpha}}\}$$

It is straightforward to verify that  $\lambda$  can never be zero under the incentive compatibility constraints.

**Definition 3** For given  $w \in (0,1)$  and rw > 0, a feasible pair of contracts  $\omega^j = (\rho^j, \beta^j)$ (j = H, L) is efficient if there is no other feasible pair  $c = (c^H, c^L)$  with  $c^j = (R^j, B^j)$ (j = H, L) such that:

$$\max\{\Pi_s^H(c^H), \Pi_s^L(c^L)\} \ge \max\{\Pi_s^H(\omega^H), \Pi_s^L(\omega^L)\}, \ s \in [0, 1]$$
$$\lambda(c)p^H \rho^H \beta^H + (1 - \lambda(c))p^L \rho^L \beta^L \ge \lambda(\omega)p^H \rho^H \beta^H + (1 - \lambda(\omega))p^L \rho^L \beta^L$$

with at least one strict inequality.

This definition of efficiency takes into account the fact that the Planner cannot directly run the technology and has the same information as the lenders.

It is straightforward to show that all CME contracts are efficient. In fact, if there were a pair of contracts dominating the CME contract for given w and r, then this pair would define an upsetting deviation, contradicting the Nash property of the CME contracts.

## 3.7 The equilibrium share of bad projects

The proportions of borrowers undertaking the H-project with separating and pooling contracts respectively are given by:

$$\lambda^{s}(w)=\phi(x^{L}(w))=x^{L}(w)B^{L}(x^{L}(w)), \hspace{0.2cm} \lambda^{p}(w)=\sigma x^{p}(w)w$$

It is readily verified that  $x^{p}(w)w$  is increasing in w. Moreover, since the loan for L-projects with separating contracts is less than the average loan w ( $B^{H} = 1 > w > B^{L}$ ), then:

$$x^{L}(w) \le x^{p}(w) \Rightarrow \lambda^{s}(w) < \lambda^{p}(w)$$

Thus an increase in saving w will always bring about an increase in the proportion of "bad" projects when the contract is pooling, while it may or may not have this same effect when the contract is separating.

We now define the equilibrium share of the "bad" projects H as  $\lambda(w)$ . By proposition 5, we have:

$$\lambda(w) = \lambda^{s}(w)$$
 for  $w \leq w^{o}$ ;  $\lambda(w) = \lambda^{p}(w)$  for  $w > w^{o}$ .

From the previous section we know that  $w^o \ge w'$ , i.e., the wage rate at which deposit rates are equal under the two types of contract is higher than or equal to the switching point  $w^o$ . Hence:

$$x^{L}(w^{o}) \leq x^{p}(w^{o})[1 - (1 - \mu)\lambda^{p}] < x^{p}(w^{o})$$

and the following proposition can be established without proof:

**Proposition 6** The function  $\lambda(w)$  has a discontinuity in  $w^{\circ}$  where it makes a sudden jump to a higher value when w crosses  $w^{\circ}$  from the left, i.e.:

$$\lambda(w^o) = \lambda^s(w^o) < \lim_{w \to w^{o-}} \lambda^p(w)$$

## 4 Equilibrium dynamics

To determine the general equilibrium of the model we concentrate on the market for materials z. By Walras law, an equilibrium in this market implies an equilibrium in the entire economy.

Let  $b^j$  (j = H, L) be the amount of the loan used to finance project j under either one of the two types of equilibrium contract, i.e.,  $b^j = B^j$  (j = H, L) when contracts are separating and  $b^j = B^p = w$  for j = H, L when contracts are pooling.

To simplify the computation, assume that entrepreneurs are uniformly distributed in [0, 1]. Then the supply of materials z at time t + 1 is given by:

$$\Psi(w_t) = ar{lpha}[\lambda_t b_t^H + (1 - \lambda_t)b_t^L - \int_0^{\lambda_t} esds]$$
  
Since  $\lambda_t b_t^H + (1 - \lambda_t)b_t^L = w_t$ ,  $\lambda_t = \lambda(w_t)$  and  $\int_0^{\lambda_t} esds = rac{e}{2}\lambda_t^2$ , we have: $\Psi(w_t) = ar{lpha}[w_t - rac{e}{2}\lambda(w_t)^2]$ 

The dynamic structure of the model implies that market-clearing for the materials z can be written as:

$$z_t = \Psi(w_{t-1})$$

From equation 2.1 and assumption 5, there is a continuous strictly increasing and differentiable function  $\Phi(.)$  such that  $z = \Phi(w)$ . The dynamics of the equilibrium wage rate are thus described by the following equation:

$$\Phi(w_t) = \Psi(w_{t-1}) \tag{4.13}$$

A sequence  $\{w_t; t \ge 1\}$  satisfying equation 4.13 for all  $t \ge 2$  uniquely defines the equilibrium dynamics of the model for a given initial condition  $w_1$ .

Recall that, by assumptions 5 and 7, the benchmark model produces a unique positive asymptotically stable stationary wage rate  $w^* \in (0,1)$ . For this model, the corresponding map  $\Psi(w)$  is simply the linear function  $\bar{\alpha}w$ . When we have asymmetric information, the map  $\Psi(.)$  is nonlinear. In the previous section we showed that  $\lambda(w)$  has a discontinuity in the switching value  $w^{o}$ . The equilibrium share of bad projects  $\lambda$  jumps up (the value  $\Psi(w)$  falls) as w goes through  $w^{o}$  from the left. Additional properties of the map  $\Psi(.)$  can also be derived from the analysis of the equilibrium contracts, and they are summarized in the following proposition.

**Proposition 7** The map  $\Psi(w)$  has the following characteristics:

Ψ(0) = 0;
 Ψ(.) is increasing and piecewise continuous;
 Ψ(.) has a unique discontinuity at w° where Ψ(w°) > lim<sub>w→w°</sub>-Ψ(w);
 Ψ'(0) = ā;
 Ψ'(w) < ā for all w > w°;
 there is a value w<sup>m</sup> < 1 such that Φ(w) > Ψ(w) for all w > w<sup>m</sup>.

**Proof.** See the appendix.  $\Box$ 

From property 6 of proposition 7, it follows that the equilibrium sequence of wage rates  $\{w_t; t \ge 1\}$  is bounded. In fact,  $\Phi(w) > \Psi(w)$  for  $w > w^m$  implies that  $w_{t+1} < w_t$  whenever  $w_t > w^m$ . Other dynamic properties of the model can be easily derived from proposition 7.

#### Multiple steady states

A steady state of the dynamics generated by equation 4.13 is a value  $\bar{w} \ge 0$  such that:

$$\Phi(ar{w})=\Psi(ar{w})$$

Although the steady state of the benchmark model is always unique, we cannot exclude the existence of a multiplicity of steady states when there is asymmetric information. These may be associated with either pooling or separating contracts. By the properties of the maps  $\Phi(.)$  and  $\Psi(.)$ , at least one of these steady states is stable. Moreover, since:

$$\Psi(w) < \bar{\alpha}w$$

whenever  $\lambda > 0$ , it follows that  $\bar{w} < w^*$ . Thus, a first obvious claim that can be derived from the dynamics of the model is that asymmetric information produces poverty traps, in terms of wage and output levels.

If the stable steady state  $\bar{w} < w^o$  ( $\bar{w} > w^o$ ) then the economy converges asymptotically to a stationary equilibrium where contracts are separating (pooling). Clearly, the steady state associated with pooling contracts has a higher value, i.e., it implies a larger output and consumption level.

### Persistent cycles

Due to the discontinuity of the map  $\Psi(.)$  at  $w^{o}$ , there is an open set of parameter values such that steady states are non-existent in the model with asymmetric information. Note that this possibility may occur precisely because of the particular nature of the discontinuity, i.e., the fact that  $\lambda(w)$  jumps up at  $w^{o}$ .

Recall that  $\Psi(.)$  is piecewise continuous in [0, 1] and the discontinuity is unique. We can then partition the positive real line into the following two intervals:

$$I_1 = (0, w^o], \ \ I_2 = (w^o, \infty)$$

Since there is no  $\bar{w} > 0$  such that  $\Phi(\bar{w}) = \Psi(\bar{w})$ , we have:

$$w \in I_1 \Rightarrow \Phi(w) > \Psi(w), \ w \in I_2 \Rightarrow \Phi(w) < \Psi(w)$$

Thus, if  $w_1 \in I_1$ , the equilibrium sequence is initially monotonically increasing and there is a T > 0 such that  $w_1 \in I_1 \Rightarrow w_T \in I_2$ .

When  $w_t \in I_2$ , the equilibrium sequence becomes monotonically decreasing and there is a k > 0 such that  $w_T \in I_2 \Rightarrow w_{T+k} \in I_1$ .

Thus, we can state the following:

**Proposition 8** Assume that there is no  $\bar{w} \in (0,1)$  such that  $\Phi(\bar{w}) = \Psi(\bar{w})$ . Then the equilibrium dynamics of the wage rate is cyclical and it does not converge to any steady state.

Figure 3 illustrates the case of a periodic cycle.

## 5 Appendix

**Proof of proposition 1.** Assume that the *H*-pooling contract  $\bar{c} = (\bar{x}, \bar{B})$  is a Nash equilibrium of the game. Hence, it satisfies the following conditions:

(H1)  $\Pi_1^H(\bar{c}) \ge \Pi^L(\bar{c})$ (H2)  $\Pi_1^H(\bar{c}) \ge 0.$ 

The two inequalities state that all firms  $s \in [0, 1]$  make non zero profits with  $\bar{c}$ . It is clear that no *H*-pooling contract can survive a profitable deviation if it only attracts a subset of firms  $s \in [0, s']$  with s' < 1. In fact, in such a case the deviating contract  $\hat{c} = (\hat{x}, \hat{B})$  with  $\hat{x} = 1$  would upset, for a suitable choice of  $\hat{B}$ , the *H*-pooling contract.

Banks' revenue per unit of loan with the contract  $\bar{c}$  is equal to  $p^H \bar{R} = q \bar{\alpha} \mu \bar{x}$ , i.e., it is linearly increasing in  $\bar{x}$ . Thus, in seeking conditions under which  $\bar{c}$  is a Nash equilibrium, we can set  $\bar{B} = 1$ . In fact, by (H1), this value maximizes banks' revenue per unit of loan with  $\bar{c}$ , making the existence of the upsetting contract more problematic.

Now consider a family of deviating contracts  $\hat{c}$  satisfying:

$$\Pi^{L}(\hat{c}) > \Pi^{H}_{1}(\bar{c}), \ \Pi^{H}_{s}(\hat{c}) = \Pi^{H}_{s}(\bar{c})$$

The above family attracts L-projects only, thereby generating a revenue (per unit of loan) equal to  $q\bar{\alpha}\bar{x}$  for the deviating bank. Since a contract in this family attracts L-projects only, it does not modify the banks' revenue generated by the contract  $\bar{c}$ . Therefore, a deviation  $\hat{c}$ does not induce the non-deviating banks to reject applications for  $\bar{c}$  at the third stage of the game. One can easily show that, if  $\bar{c}$  survives this deviation, then it will survive any other deviation, i.e.,  $\bar{c}$  is a Nash equilibirum contract.

The deviating contract  $\hat{c}$  upsets  $\bar{c}$  if it generates higher profits for the banks than  $\bar{c}$ , i.e.,  $\hat{x} > \mu \bar{x}$ , where  $\bar{x} \leq (1 - e)/\mu$  by (H2). For the family of deviating contracts defined above, we have:

$$\hat{B} = rac{1-\muar{x}}{1-\mu\hat{x}}, \ \ (1-\hat{x})\hat{B} > (1-\muar{x}) - e$$

Using the above equations, the condition for the deviating contract to upset  $\bar{c}$  can be written as:

$$\Gamma(\hat{x}, \bar{x}) \equiv (1 - \mu \bar{x})(1 - \mu) \frac{\hat{x}}{1 - \mu \hat{x}} < e$$

Thus,  $\bar{c}$  is a Nash equilibrium under the condition that, for some  $\bar{x}$ ,  $\Gamma(\hat{x}, \bar{x}) > e$  for all  $\hat{x} \in (\mu \bar{x}, 1]$ . Since  $\Gamma(.)$  is increasing in  $\hat{x}$ , this condition reduces to:

$$\Gamma(\mu \bar{x}, \bar{x}) > e$$

for some  $\bar{x} \leq (1-e)/\mu$ . Trivial calculations show that this condition is verified only if  $e < (1-\mu)/(2-\mu)$ .

Hence,  $e \ge (1-\mu)/(2-\mu)$  implies that no *H*-pooling contract can be a Nash equilibrium contract of the game. In turn, this condition can only be verified for all  $\mu \in (0,1)$  when  $e \ge 1/2$ .  $\Box$ 

**Proof of lemma 2.** Here we consider the possibility that the pooling contract can be upset by a separating contract attracting *H*-projects only.

Inspection of conditions (i')-(iv') shows that we are in the best position to upset the existing contract if (ii') is satisfied with equality and  $\hat{B} = 1$ . These two conditions determine a unique value for the interest rate in the upsetting contract, such that:

$$\hat{x} = rac{1}{\mu} [1 - (1 - \mu x^p) B^p]$$

Substituting this value of  $\hat{x}$  into (i'), we directly find conditions under which deviating contracts cannot upset  $c^p$ . In particular, the expected revenue from the deviating contract defined in the left hand side of (i') becomes the function:

$$V^{H}(x^{p}, B^{p}) = 1 - (1 - \mu x^{p})B^{p}$$

Consider the set of pooling contracts characterized by lemma 1. Since the function  $x^{p}(.)$  defined by equation (15) is decreasing, we can find its inverse. Let  $B^{p}(.) = (x^{p})^{-1}(.)$  and write  $B^{p} = B^{p}(x^{p})$  to characterize the unique family of pooling contracts which cannot be upset by a pooling deviation.

Thus this family of pooling contracts is compatible with a Nash equilibrium if it is immune to separating deviations as well, i.e., if:

$$V^p(x^p, B^p(x^p)) \ge V^H(x^p, B^p(x^p))$$

It is verified that the above inequality will only hold under the condition:

$$x^p \le m \equiv \frac{1}{\mu + 2\sigma(1-\mu)}$$

Thus, when  $\mu + 2\sigma(1-\mu) \leq 1$ , i.e.,  $\sigma \leq 1/2$ , then  $m \geq 1$  and any pooling contract which cannot be upset by a pooling deviation, it cannot be upset by a separating deviation either.

On the other hand, when  $\sigma > 1/2$ , then m < 1 and  $c^p$  is immune to both a pooling and a separating deviation if and only if  $x^p \leq m$ . Now define  $M(e, \mu)$  from:

$$m = x^p(M(e,\mu))$$

It is verified that  $M(e, \mu) < 1$  for all values of  $e \ge 1/2$ . Recalling that  $x^p(.)$  is decreasing, we get the proposition.  $\Box$ .

**Proof of proposition 3.** First we show that separating deviations cannot be profitable. Let  $\hat{c}^H$  be the separating deviation attracting *H*-projects only. Since  $B^H = 1$ , the deviating bank will only attract borrowers with *H*-projects under the condition  $\hat{x}^H < x^H$ . However, this generates losses for the bank. Now let  $\hat{c}^L$  be a deviating contract attracting borrowers adopting the L-technology. Then:

$$(1-\hat{x}^L)\hat{B}^L \ge (1-x^L)B^L$$

In this case the deviating contract is incentive compatible only if  $\Pi_s^H(c^H) > \Pi_s^H(\hat{c}^L)$  and these two inequalities together imply that  $\hat{x}^L < x^L$ . Hence, this deviation is not profitable.

Thus, in order for a deviating contract to upset the separating pair  $(c^H, c^L)$ , it has to be a pooling contract  $\hat{c}^p$ . Then:

i)  $\hat{c}^p$  produces higher profits for the bank than the existing contracts when all applications with which banks make negative profits "after" the deviation are withdrawn;

ii) with  $\hat{c}^p$  a non-trivial fraction of borrowers undertaking both projects make at least the same profits than they used to make with the existing contracts.

Let  $\hat{\lambda}$  be the proportion of borrowers undertaking the bad project with the upsetting contract after the unprofitable contracts have been withdrawn. Then conditions (i) and (ii) above imply:

(a) 
$$[1 - (1 - \mu)\hat{\lambda}]\hat{x}^p > x^L;$$

(b)  $\Pi_{s}^{H}(\hat{c}^{p}) \geq \Pi_{s}^{H}(c^{H}), \ \Pi^{L}(\hat{c}^{p}) \geq \Pi^{L}(\hat{c}^{p}).$ 

By equation 3.4, any pooling deviation satisfies  $\hat{\lambda} = \sigma \hat{x}^p \hat{B}^p$ .

The next step is to define the value  $\hat{B}^p$  for which banks profits are higher and condition (b) is not violated. This value is clearly a function  $\hat{B}^p(\hat{x}^p, x^L)$  provided that the capacity constraint  $\hat{B}^p \leq 1$  is not binding. In particular, we need:

$$(1-\hat{x}^p)\hat{B}^p \ge (1-x^L)B^L$$

with  $\hat{B}^p \leq 1$ . Checking for a deviating contract that satisfies the above equation with equality is enough to guarantee the existence of a profitable deviation. Then, we will write:

$$\hat{B}^p = rac{1-x^L}{1-\hat{x}^p}B^L$$

for  $\hat{x}^p < m(x^L) = 1 - (1 - x^L)B^L$  (capacity constraint).

Under this specific contract we can derive that:

$$\hat{\lambda} = \lambda rac{\hat{x}^{m{p}}(1-x^L)}{x^L(1-\hat{x}^{m{p}})}$$

Plugging this value into equation (a) and using these results, one can show that the condition for the non-existence of an upsetting deviation reduces to a simple relation between  $x^{L}$  and  $\lambda$ . In particular, a profitable deviation will not exist if:

$$\hat{x}^{p}[1 - (1 - \mu)\lambda \frac{\hat{x}^{p}(1 - x^{L})}{x^{L}(1 - \hat{x}^{p})}] \le \hat{x}^{p}$$

for all  $\hat{x}^p \in (x^L, m(x^L))$ . Simple manipulations of the above expression show that a profitable deviation does not exist if and only if the polynomial in  $\hat{x}^p$  defined below:

$$P_{x^{L}}(\hat{x}^{p}) = A_{2}(\hat{x}^{p})^{2} - A_{1}\hat{x}^{p} + A_{0}$$

with  $A_2 = x^L + (1 - \mu)\lambda(1 - x^L)$ ,  $A_1 = x^L(1 + x^L)$ ,  $A_0 = (x^L)^2$  is greater than or equal to zero for all  $\hat{x}^p \in [x^L, m(x^L)]$ .

Observe that for all  $\hat{x}^p \in [x^L, 1]$ ,  $P_{x^L}(.)$  is strictly convex; it is positive for  $\hat{x}^p \leq x^L$  and decreasing for  $\hat{x}^p = x^L$ .

Let  $y^*(x^L) = \arg \min\{P_{x^L}(\hat{x}^p); \hat{x}^p \in [x^L, m(x^L)]\}$ . Clearly, either  $y^*(x^L) < m(x^L)$  and  $P_{x^L}$  is increasing in  $m(x^L)$ , or  $y^*(x^L) = m(x^L)$  and  $P_{x^L}$  is non increasing in  $m(x^L)$ .

Hence, for the existence of a separating equilibrium, we need to find conditions guaranteeing that either one of the following is verified:

(A) 
$$P'_{x^{L}}(m(x^{L})) > 0$$
 and  $P_{x^{L}}(y^{*}(x^{L})) \ge 0$ ;  
(B)  $P'_{x^{L}}(m(x^{L})) \le 0$  and  $P_{x^{L}}(m(x^{L})) \ge 0$ .

Now define  $y(x^L) = \arg \min\{P_{x^L}(\hat{x}^p); \hat{x}^p \in R\}$ . It is verified that:

i)  $P'_{x^L}(m(x^L)) > 0$  implies:

$$\lambda > \xi(x^L) \equiv \xi(x^L) = rac{1-(2-\mu)x^L}{2(1-\mu)(2-\mu-x^L)}$$

ii)  $P_{x^L}(y(x^L)) \ge 0$  implies:

$$\lambda \ge u(x^L) \equiv \frac{1 - x^L}{4(1 - \mu)}$$

iii)  $P_{x^L}(m(x^L)) \ge 0$  implies:

$$\lambda \ge v(x^L) \equiv [rac{1-x^L}{2-\mu-x^L}]^2$$

Thus, case (A) reduces to finding conditions under which:

$$\lambda > \xi(x^L), \ \lambda \ge u(x^L)$$

and case (B) reduces to finding conditions under which:

$$\xi(x^L) \ge \lambda \ge v(x^L)$$

Now recall that  $\lambda = \phi(x^L)$ . Then, the following facts can be readily verified:

- i. if  $\sigma > 1/4$ ,  $\phi(x^L) \ge u(x^L)$  iff  $x^L \in [\rho^1, 1]$ , where  $\rho = 1/[4\sigma(1-\mu) + \mu]$ , if  $\sigma \le 1/4$ ,  $\phi(x^L) \le u(x^L)$  for all  $x^L \in [0, 1]$ ;
- ii.  $\xi(\mu) = u(\mu) = v(\mu), \ \xi(x^L) > u(x^L) > v(x^L)$  for  $x^L \in (0, \mu), \ \xi(x^L) < v(x^L) < u(x^L)$  for  $x^L \in (\mu, 1);$
- iii.  $\phi(\mu) > \xi(\mu) = v(\mu) = u(\mu)$  iff  $\sigma > (1 + \mu)/4\mu$ ;
- iv. the curves  $\phi(x^L)$  and  $v(x^L)$  have a unique interesection  $\rho^2 \in (0, \rho^1)$  such that  $v(x^L) > \phi(x^L)$  for all  $x^L \in (0, \rho^2)$ .

From the above, it follows that case (B) cannot be verified under the assumption  $\sigma \leq (1 + \mu)/4\mu$ . Thus, only case (A) can hold.

Moreover, under the assumption  $\sigma \leq (1 + \mu)/4\mu$ , case (B) implies case (A). Finally, notice that  $\rho^1 \leq 1$  iff  $\sigma \geq 1/4$ .

These remarks lead to the following conclusions:

if  $\sigma \leq (1+\mu)/4\mu$ , the existence of a separating equilibrium requires  $x^L \in [\rho^1, 1]$  when  $\sigma > 1/4$ and  $x^L = 1$  when  $\sigma \leq 1/4$ ;

if  $\sigma > (1 + \mu)/4\mu$ , the existence of a separating equilibrium requires  $x^L \in [\rho^2, 1]$ .

By defining  $\gamma(e,\mu) = \rho^2$  when  $\sigma > (1+\mu)/4\mu$ ,

$$\gamma(e,\mu) = \begin{cases} 1 & \text{if } \sigma \le 1/4 \\ \rho^1 & \text{if } 1/4 < \sigma \le (1+\mu)/4\mu \\ \rho^2 & \text{if } \sigma > (1+\mu)/4\mu \end{cases}$$

**Proof of proposition 5.** To save notation, let  $z^j = r^j/q\bar{\alpha}$ , (j = s, p).

When contracts are separating, we have  $z^{s}(w) = x^{L}(w)$ . When contracts are pooling:

$$z^{\mathbf{p}} = x^{\mathbf{p}}(w)[1 - (1 - \mu)\lambda]$$

From the proof of proposition 2 we derived that:

$$(x^p)^{-1}(x^p) = rac{1-x^p}{\sigma(1-\mu)x^p(2-x^p)}$$

Then,  $(1-\mu)\lambda = (1-x^p)/(2-x^p)$  and we obtain:

$$z^p(w) = rac{x^p(w)}{2-x^p(w)}$$

Note that both  $z^{s}(w)$  and  $z^{p}(w)$  are decreasing functions of w. From now on it will be convenient to invert these functions in order to simplify the problem.

The inverse of  $z^{s}(.)$  is just equal to the inverse of  $x^{L}(.)$ , which we know from the proof of proposition 3. Thus, we define:

$$w^{s}(z) = (x^{L})^{-1}(z) = (1-z)rac{1-\mu z+\sigma(1-\mu)z^{2}}{(1-\mu z)^{2}}$$

as the unique saving level that is compatible with a given value z of the rate on deposits for a separating MC contract (provided the latter exists).

Similarly, from the definition of  $(x^p)^{-1}(.)$  and the relation between  $x^p$  and  $z^p$  found above, we can define:

$$w^p(z)=(1-z)rac{1+z}{4\sigma(1-\mu)z}$$

as the unique saving level which is compatible with a given value z of the rate on deposits for a pooling MC contract (provided the latter exists).

Now we study the two functions  $w^{s}(.)$  and  $w^{p}(.)$  to see whether they have one or more intersections.

Note that  $w^s(1) = w^p(1) = 0$ ,  $w^s(0) = 1$  and  $w^p(z)$  tends to  $+\infty$  for  $z \to 0$ . Thus, a sufficient condition for the two functions to have an intersection point is:

$$\lim_{z \to 1} \partial w^s / \partial z > \lim_{z \to 1} \partial w^p / \partial z$$

which is verified when  $2\sigma^2 + 2\sigma - 1 > 0$ , i.e.,  $\sigma > (3^{1/2} - 1)/2$ .

Now we show that  $w^{s}(.)$  and  $w^{p}(.)$  cannot have more than one intersection in (0, 1). In fact,  $w^{s}(z) = w^{p}(z)$  in (0, 1) implies:

$$rac{1-\mu z+\sigma(1-\mu)z^2}{(1-\mu z)^2}=rac{1+z}{4\sigma(1-\mu)z}$$

One can verify that the left hand side of the above equation is increasing and the right hand side is decreasing in z. Therefore, there cannot be more than one intersection point  $z^o$  between  $w^s(.)$  and  $w^p(.)$  in (0,1).

Now recall that there is an MC contract for all  $w \in (0,1)$  and that the MC contract can be separating for  $w \leq L(e,\mu)$  and it can be pooling for  $w \geq M(e,\mu)$ , with  $M(e,\mu) < L(e,\mu)$ . Hence, we get the proposition.  $\Box$ 

**Proof of proposition 7.** Property 1 in the proposition follows from the fact that the equilibrium contracts are separating for w small and that  $x^{L}(0) = 1$ , i.e.,  $\lambda^{s}(0) = 0$ .

For property 2 we need to show that:

$$e\lambda(w)\lambda'(w) < 1$$

for all  $w \in (0, 1)$ . When contracts are pooling, we have:

$$\lambda(w)=\phi(x^L(w))$$

where  $x^{L}(w)$  is implicitly defined by the condition:

$$\phi(x^L)=\psi(x^L,w)\equiv rac{w(1-\mu x^L)-(1-x^L)}{(1-\mu)x^L}$$

Then:

$$\partial\lambda^s(w)/\partial w = -\phi'rac{\psi_w}{\phi^L_x-\phi'}$$

Evidently,  $e\lambda(w)\lambda'(w) < 1$  whenever  $\phi' > 0$ . Then, let  $\phi' < 0$  and define  $\phi' = -\eta$ . Then:

$$e\lambda^s\partial\lambda^s/\partial w = e\lambda^srac{\eta}{\eta+\psi^L_x}\psi_w < e\lambda^s\psi_w = 1-x^L < 1$$

When contracts are pooling, we can derive the following expression for the derivative of  $\lambda$  with respect to w:

$$\partial\lambda^p/\partial w=\sigmarac{1}{1+\delta}$$

where:

$$\delta = rac{2(1-\mu)^2\lambda^2}{1-2(1-\mu)\lambda}$$

Thus:

$$e\lambda^p \partial \lambda^p / \partial w < e\lambda^p \sigma = (1-\mu)\lambda < 1$$

Property 3 in the proposition follows from the discussion in section 6.

Properties 4 and 5 follow from the observation that  $\lambda'(0) > 0$  and that  $\lambda(w)$  is increasing when contracts are pooling.

Property 6 follows from having assumed  $w^* < 1$ , i.e.,  $\Phi(w) > \bar{\alpha}w > \Psi(w)$  for all  $w > w^*$ .



Fig. l



Fig. 2



Fig. 3

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