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REAL INTEREST RATES, SOVEREIGN RISK AND OPTIMAL DEBT MANAGEMENT

by Francesco Drudi (*) and Raffaela Giordano (**)

Abstract

The role of movements in real rates in explaining the relationship between long and short-term interest rates is explored using a model of optimal government debt management. The government's incentives to resort in the future to inflation and ex-post debt taxation in order to reduce the real value of its nominal liabilities have an impact on term premia and hence on the short-long spread. Inflation risk and default risk are perceived to be higher the larger the stock of outstanding debt. A policy of lengthening the maturity of debt may reduce the risk of default, while indexed bonds may increase it.

Contents

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1. Introduction [1]

In the past decade economists have devoted considerable attention to the apparent existence of inflation premia embedded in rates of return on government debt. The presence of government's inflationary incentives, related to different potential sources, has been at the heart of the analysis of the problem of time inconsistency in monetary and fiscal policies. Calvo and Guidotti (1990a, 1990b) and Missale and Blanchard (1994), among others, have investigated the government's incentives to use opportunistic inflation to reduce the ex-post real value of its nominal obligations, and explored the role of debt maturity in managing inflation. Fewer works have been addressed to the analysis of default on government debt, either in the form of simple repudiation of debt obligations or some ex-post tax on debt repayments. When investors perceive the possibility of partial (or complete) default, they require compensation for such risk, which is incorporated in nominal interest rates on government debt. A pioneering work in this area is the paper by Calvo (1988). Giavazzi and Pagano (1990) and Alesina, Prati and Tabellini (1990) consider the possibility of financial crises in a context of multiplicity of equilibria. Drudi and Prati (1993) analyze the problem with incomplete information. Alesina, De Broeck, Prati and Tabellini (1992) document some evidence of risk premia on government debt in OECD countries.

The risk of default on government debt is not immediately

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intuitive. It is a widespread opinion that the government would always prefer to inflate away its debt instead of defaulting on it explicitely (by simply not repaying or restructuring it). After all, even though default is a non-distortionary lump-sum tax, it also has very high costs in terms of loss of reputation for defaulting governments, income redistribution and risk of bankruptcies in the financial sector. However, in the case of Italy several indicators seem to suggest that a default premium sometimes arose on bonds issued by the Treasury, as it could be detected in the spread between swap and bond rates or in the differential between bonds issued by the Italian Treasury and those issued by other governments or supernational agencies. The high and time-varying spread between the yields on floaters issued by the Treasury and short-term rates might be a further indication of the existence of default premia.

In this paper we investigate the emergence of default risk premia under the hypothesis that the government has the option to reduce its liabilities also by means of inflation. In our model the government weighs an exogenous cost of default, which is proportional to the amount of repudiated debt, against the costs arising from inflation and income taxation. Given this setup, we focus on the relationship between default risk and the real interest rate level, and analyze the consequences for optimal debt management.

The microeconomic foundations of the cost associated with debt repudiation have been discussed in a number of papers on this topic. Defaulting governments lose reputation and may find it difficult to borrow in the future when necessary (Grossman and van Huyck, 1988; Chari and Kehoe, 1990). Default redistributes income away from debt-holders (Alesina, 1988; Drudi and Prati, 1993; Eichengreen, 1990; Tabellini, 1991).

Default may lead to financial disruption in the banking sector, if financial institutions hold significant amounts of government debt in their portfolios (Alesina, 1988; Spaventa, 1988). Defaulting governments bear transaction costs associated with legal actions which may be undertaken when repudiation is open (Calvo, 1988).

We distinguish situations of explicit default on debt from those of financial crisis. The former occur when the government does not fully repay its debt obligations. The latter originate when investors refuse to buy government debt.

In this model, default is triggered only when the burden of debt reaches a critical level. Before that point, the government prefers to refrain from defaulting and resort to inflation or income taxation. This feature allows us to rationalize the casual observation that default occurs very rarely (among the major economies, no government has ever defaulted since the Second World War). In our setup, default is precipitated by an increase in interest rates. With stochastic and auto-correlated interest rates, risk premia appear once the level of rates is so high that a further increase in them would induce the government to default. In the extreme case when rates jump so much to force a future default in any state of the economy, a financial crisis emerges.[2]

The introduction of real indexation of government debt, in our setting, reduces the inflationary incentive and increases the risk of default. Similarly, high costs of inflation make equilibria with risk of default more likely

[2] Similar results might apply as a consequence of shocks to government expenditure.

to occur. Therefore, the combination of large stocks of public debt, high real interest rates and anti-inflationary governments, or central banks, generally makes real indexation less desirable.

Finally, we investigate the choice of optimal debt maturity. We assume that the government is able to issue short-term and long-term bonds, and that the release of information about real rates is after the repayment of short-term debt. Therefore, default risk affects long-term bonds only. Uncertainty about future rates assigns a role to debt maturity even in the presence of time-consistent government policies. As in the existing literature, the government's incentives to inflate away its nominal obligations influence the optimal maturity structure. In our context, the possibility of emergence of default risk attaches an additional role to debt maturity. To this extent, a large share of long-term in total debt represents a hedge against a steep rise in interest rates. However, when the stock of debt is extremely large, the compensation for default risk required by the public and embedded in long-term rates becomes unsustainable, so that only short-term debt can be issued.

The paper is organized as follows.

Section 2 presents the basic model.

Section 3 analyzes the situation where full precommitment on both inflation and debt repudiation policies on the part of the government is allowed. The equilibrium which results is efficient and involves, in the absence of uncertainty about future realizations of the real rate, perfect tax smoothing. In the presence of stochastic real rates, the maturity structure may act as a hedge against such a risk.

Moreover, since neither inflation risk nor default risk is perceived by the public, long-term rates do not exhibit any term premium.

In Section 4, we consider the case of no risk of bankrupcy: the government is assumed to be able to precommit only its defaulting strategy but not its inflation policy. In the time-consistent equilibrium the government finds it optimal to reduce the amount of nominal obligations outstanding in period 2 by increasing tax revenues in period 1 relative to what is optimal under full precommitment. Optimal maturity can be either shorter or longer than in the case of full precommitment, since here better tax smoothing is achieved at the expense of higher inflation biases.

Section 5 addresses the case of no precommitment, in which the term premium may also account for a positive risk of default in the second period. Such a risk is shown to be increasing with the size of the debt, the level of real rates and the cost of inflation for the government. It decreases as the maturity lengthens. For this reason, optimal maturity is, in general, longer than in the case of partial precommitment.

Finally, Section 6 considers the case of indexed debt. Here the absence of inflationary means to reduce the real value of nominal debt obligations makes the emergence of default risk more likely.

Section 7 contains the results of some numerical simulations.

Section 8 concludes.

2. The basic model

2.1 The economy

Agents set nominal rates of return on government debt according to the following no arbitrage conditions:

$$(1)\quad R_{01} = E_0\left[\frac{r_1\Pi_1}{1-\Theta_1}\right],$$

$$(2)\quad R_{02} = E_0\left[\frac{r_1 r_2\Pi_1\Pi_2}{1-\Theta_2}\right],$$

$$(3)\quad R_{12} = E_1\left[\frac{r_2\Pi_2}{1-\Theta_2}\right],$$

where R_{ij} denotes the nominal interest factor (i.e., 1 plus the corresponding interest rate) between periods i and j, Π_t denotes the inflation factor in period t (i.e., $P_t = P_0\Pi_1\Pi_2,\ldots,\Pi_t$, where P is the price level), and Θ_t denotes the tax rate on debt maturing in period t. E_t is the expectation operator given the information set available to agents in period t.[3]

Real interest rate factors, r_t, are exogenous with respect to monetary policy and follow a random walk

$$(4)\quad r_t = r_{t-1} + \epsilon_t,$$

where ϵ_t can take values ϵ and $-\epsilon$ with probability 1/2. Agents observe the current-period real interest rate, and

[3] Equations (1), (2) and (3) are first-order conditions of a simple intertemporal optimization problem, in which risk neutral agents choose a path for consumption and saving, given that the nominal rate of return on saving between periods i and j is R_{ij} and their time preference discount rate equals the real interest rate.

formulate expectations about the future-period real interest rate when setting nominal returns on long-term government debt.

2.2 The government

The government is assumed to have a three-period horizon. Government expenditure occurs at three dates, period 0, period 1 and period 2. In period 0 the government does not levy any tax and the debt issued is equal to the expenditure, g_0. Debt issued in period 0 can be both short-term debt, maturing in period 1, and long-term debt, maturing in period 2. In period 1 the government finances a constant (exogenous) flow of expenditure, g_1, and repays the maturing debt by levying distortionary taxes on labor income, by using the revenue from inflation, or by issuing new short-term nominal debt (maturing in period 2). In addition, it may tax government debt ex-post. Finally, in period 2 the government finances current expenditure, g_2, and repays the debt issued in periods 0 and 1 by resorting to conventional taxation, to the inflation tax, or by taxing both short-term and long-term outstanding debt.[4] Therefore, the government's budget constraints in the three periods are:

$$(5) \quad g_0 = D_{01} + D_{02}$$

$$(6) \quad g_1 + \frac{D_{01}R_{01}}{\Pi_1}(1 - \Theta_1) = \tau_1 + D_{12}$$

[4] The assumption of a government having a three-period horizon is not essential in deriving the results of the paper. For the results to hold, the only requirement is that government budgets balance at some point of time (i.e. no Ponzi schemes are allowed). Further, the result, to be described later in this paper, that debt repudiation will occur, if at all, only in the last period depends crucially on the assumption about the timing of release of information, which makes it worthwhile deferring default until it is certain that the fiscal regime is no longer sustainable.

$$(7) \quad g_2 + \left(\frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2}\right)(1-\Theta_2) = \tau_2,$$

where D_{ij} denotes the value, at time i, of nominal public debt issued in period i with maturity in period j, and τ_t and Θ_t are respectively the tax rates on labor income (normalized to one) and government debt in period t.

In period 0 the only decision faced by the government is choosing the maturity structure of the initial stock of debt necessary to finance the given public expenditure g_0. In periods 1 and 2 the government chooses a sequence of taxes on income, inflation and default (τ_1, τ_2, Π_1, Π_2, Θ_1, Θ_2,). In addition, in period 1 the goverment decides the amount of maturing debt to be rolled over to period 2.

The government's optimal choice of instruments responds to the objective of minimizing the value of the following intertemporal cost function:

$$\begin{aligned}(8) \quad L^g \;=\; & \frac{1}{2}\tau_1^2 + \frac{\gamma}{2}(\Pi_1-1)^2 + r_1^{-1}E_0\left[\frac{1}{2}\tau_2^2 + \frac{\gamma}{2}(\Pi_2-1)^2\right] + \\ & \alpha\left\{\Theta_1\frac{D_{01}R_{01}}{\Pi_1} + r_1^{-1}E_0\left[\Theta_2\left(\frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2}\right)\right]\right\},\end{aligned}$$

where the time preference discount factor is equal to the real interest rate factor. α stands for the cost per unit of repudiated debt.

The government's policy affects welfare in three ways: non-linearly via the distortionary costs of taxation and inflation, and linearly via the cost of defaulting on government debt.[5]

We characterize default as any action that reduces the value of government debt obligations and that was not contractually specified. The cost associated with such an action may have different interpretations. It could be thought of as transaction costs connected with legal actions, or political costs that have to be incurred because of the contract-breaking behavior of the government. Alternatively, default costs may be brought out by considerations of redistribution or risk of bankruptcies in the financial sector. Such costs arise if and only if default is unanticipated. They should therefore be modeled as a function of the amount of debt unexpectedly repudiated. However, results qualitatively analogous to those contained in this paper are obtained by specifying cost functions associated with surprise default only.

The timing of the game is the following. In period 0 all agents in the economy observe the real interest rate r_1. Given the amount of public expenditure to be financed, g_0, the government issues short-term and long-term debt, D_{01} and D_{02}. Private agents set nominal returns on government debt according to equations (1) and (2). In period 1 the goverment chooses inflation Π_1, taxes on income and debt τ_1 and Θ_1, and the amount of new short-term debt to be issued

[5] Assuming quadratic costs of taxation and inflation is quite standard: they could be thought of as originated by an underlying production technology, increasing and concave in labor and money. Debt repudiation is observed very rarely. The choice of linear default costs allows us easily to generate emergence of default only after high inflation costs have been suffered. Notice that this feature can be reproduced alternatively by specifying a lump-sum cost of default (which occurs whenever repudiation is open, independently of the amount of defaulted debt). Such specification is more appropriate if political costs or costs linked to loss of reputation are assumed to be the main concern of defaulting governments.

D_{12}. Then, r_2 realizes. Given this new information, private agents set R_{12} according to (3). Finally, in period 2 the government sets Π_2, τ_2 and Θ_2.

In the sections which follow we consider four cases. First, we investigate the trivial case in which the government can fully precommit its action both in terms of inflation and in terms of default. In the second case the government is assumed to be able to precommit its defaulting policy: a regime in which debt repudiation is an option not available to the government. The third case addresses the situation where no precommitment is allowed: the government is free to rely on both inflation and ex-post debt taxation. Finally, we analyze the case in which government debt is totally indexed, that is, a regime where inflation can be precommitted.

3. The case of full precommitment

In this section we briefly analyze the first-best case where complete precommitment on the part of the government is possible and, therefore, policies choosen in period 0 are credible. This case serves as a benchmark against which to evaluate the outcomes that arise under more realistic setups. The intertemporal government budget constraint is given by

$$(9)\quad g_2 + D_{02}r_1^2 + (D_{01}r_1 + g_1 - \tau_1)r_2 = \tau_2.$$

Equation (9) is obtained by combining equations (6) and (7), taking into account that under full precommitment inflation and default are perfectly anticipated.

The problem of the government in period 1 is to minimize loss function (9), subject to budget constraint (9).[6]

The first-best choice of taxes on income and debt and inflation rates implies

$$\tau_1 = \frac{(D_{01}r_1 + g_1)(r_1 + \epsilon^2/r_1) + D_{02}r_1^2 + g_2}{1 + r_1 + \epsilon^2/r_1},$$

$$E_1[\tau_2] = \frac{(D_{01}r_1 + g_1)r_1 + (D_{02}r_1^2 + g_2)(1 + \epsilon^2/r_1)}{1 + r_1 + \epsilon^2/r_1},$$

$$\Theta_1 = \Theta_2 = 0,$$

$$\Pi_1 = \Pi_2 = 1.$$

Under a full precommitment regime, optimal debt taxation and inflation are zero. In the absence of uncertainty about future realizations of the interest rate (i.e. $\epsilon = 0$), it would be optimal to achieve perfect smoothing of taxes over time. With stochastic real rates, instead,

$$\tau_1 \gtrless E_1[\tau_2]$$

depending on whether

$$D_{01}r_1 + g_1 \gtrless D_{02}r_1^2 + g_2,$$

(or, equivalently, $D_{12} \gtrless 0$) since the uncertainty about r_2 makes the optimal amount of short-term debt maturing

[6] Since period 2 real interest rate is not observed at time 1, the government optimizes taking expectations over the outcomes associated with the two possible realizations of r_2.

in period 1 rolled over to period 2 too small compared with what would be necessary to achieve perfect tax smoothing.

In period 0 the decision faced by the government is choosing the maturity structure of the given initial stock of public debt g_0. Since the presence of uncertainty introduces a distortion by providing an incentive in period 1 not to smooth taxes completely over time, debt maturity plays a role in such a context. As a matter of fact, maturity will be optimal if the government in period 1 is able to achieve perfect tax smoothing by selecting $D_{12} = 0$, so that the risk associated with the uncertain realization of r_2 is eliminated. This implies

$$D_{01}r_1 + g_1 = D_{02}r_1^2 + g_2,$$

if $|g_1 - g_2| < g_0 r_1^2$.[7]

Since inflation risk, as well as default risk, is perceived to be null, term premia are systematically equal to zero under this setup (i.e. $R_{01} = r_1$ and $R_{02} = r_1^2$).[8]

4. Partial precommitment: the choice between income taxation and inflation

We consider now a case of partial precommitment where the government is able to make commitments regarding ex-post debt taxation only.

[7] If $|g_1 - g_2| > g_0 r_1^2$, the optimal share of long-term in total debt is at a corner with $D_{02}/g_0 = 1$, if $g_1 - g_2 > 0$, and $D_{02}/g_0 = 0$, if $g_1 - g_2 < 0$.

[8] The implication of zero slope of the yield curve crucially hinges on the hypotheses of total absence of uncertainty and agents' risk neutrality.

With partial precommitment time inconsistency of government behavior may arise because of the presence of nominal debt, which provides an incentive to the government to resort in the future to inflation in order to reduce the real value of nominal debt obligations.

The problem of the government consists in choosing a sequence of inflation and income taxes $(\Pi_1, \Pi_2, \tau_1, \tau_2)$ to minimize loss function (9) subject to budget constraints (5), (6) and (7), where now nominal returns on government debt reflect the total absence of uncertainty about default.

In order to characterize time consistent policies we solve the government's problem starting from period 2.

4.1 The last period problem

The government in period 2 faces budget constraint (7), where the only non predetermined variables are Π_2 and τ_2. Given the objective of minimizing the value of the cost function

$$\frac{1}{2}\tau_2^2 + \frac{\gamma}{2}(\Pi_2 - 1)^2,$$

subject to equation (7), the optimal choice of period 2 inflation is given by

$$(10)\ \gamma(\Pi_2 - 1)\Pi_2 = \tau_2 \left(\frac{D_{02}R_{02}}{\Pi_1 \Pi_2} + \frac{D_{12}R_{12}}{\Pi_2} \right).$$

Unlike the previous case, here the absence of precommitment of period 2 variables leaves the government free to resort to inflation in order to reduce the real value of its obligations (recall that in period 2 the nominal value of outstanding debt, as well as the nominal interest rate factors, R_{02} and R_{12}, is a predetermined variable). Equation (10) implies that the marginal cost of inflation equals the cost reduction from the tax cut that is induced by the associated larger inflation tax (including the fall in the real value of debt obligations maturing in period 2). The gains resulting from reducing the real value of government obligations are, however, at least on average, an illusion. The market perceives the future incentive to inflate on the part of the government and, at the time the nominal debt is being issued, nominal interest rates reflect future inflation. The only possibility of error comes from the uncertainty about the realization of the period 2 real interest rate, which the public faces when setting nominal returns on long-term government debt. This implies that for government bonds issued in period 0, we have in equilibrium

(11) $R_{01} = r_1 \Pi_1$,

(12) $R_{02} = r_1 \Pi_1 \bar{R}_2$,

where $\bar{R}_2$ is the average equilibrium nominal interest rate in period 2. Furthermore, the equilibrium interest rate for debt issued in period 1 satisfies

(13) $R_{12} = r_2 \Pi_2$.

Since market interest rates reflect actual equilibrium inflation, we have, combining (10) with (7), (12) and (13),

$$(14)\ \gamma(\Pi_2 - 1)\Pi_2 = \left(\frac{D_{02}r_1\bar{R}_2}{\Pi_2} + D_{12}r_2\right)^2 + g_2\left(\frac{D_{02}r_1\bar{R}_2}{\Pi_2} + D_{12}r_2\right).$$

Equation (14) shows that Π_2 and τ_2 are positively linked at optimum. Furthermore, Π_2 increases with r_1 and decreases with γ. Higher realizations of period 2 real interest rate (i.e. $\epsilon > 0$) are associated with higher values of Π_2 if and only if $D_{12} > 0$.[9]

4.2 The intermediate period problem

In period 1 the government formulates its time consistent policy taking into account that in equilibrium Π_2 will be chosen according to equation (14), which shows that Π_2 is a function of the stock of nominal debt maturing in period 2. However, while setting policy variables in period 1, the government is not able to observe the realization of the real interest rate in period 2. It therefore has to form expectations over two possible outcomes.

Let $\tau_{2,s}$ and $\Pi_{2,s}$ denote income tax and inflation in the last period under the two possible realizations of the real rate ($s = l, h$ respectively when $r_2 = r_1 - \epsilon$ and $r_2 = r_1 + \epsilon$). Let us also define $r_{2,l} \equiv r_1 - \epsilon$ and $r_{2,h} \equiv r_1 + \epsilon$. Then, the problem of the government in period 1 is to minimize expected social loss

$$(15)\ \frac{1}{2}\tau_1^2 + \frac{1}{2r_1}\left[\frac{1}{2}\tau_{2,l}^2 + \frac{\gamma}{2}(\Pi_{2,l} - 1)^2 + \frac{1}{2}\tau_{2,h}^2 + \frac{\gamma}{2}(\Pi_{2,h} - 1)^2\right],$$

[9] In fact, a higher r_2 makes the government budget constraint tighter or looser depending on whether the government borrows or lends in period 1.

subject to budget constraints (6) and (7), where the perfect-foresight condition (13) holds, and the incentive compatibility constraint, equation (14) (of course, (6), (7) and (14) apply for both $r_{2,l}$ and $r_{2,h}$). This problem involves the choice of τ_1, Π_1, D_{12}, $\tau_{2,s}$ and $\Pi_{2,s}$ $(s = l, h)$.

The first-order conditions for optimization with respect to Π_1, D_{12} and $\Pi_{2,s}$ imply respectively

$$(16)\ \gamma(\Pi_1 - 1)\Pi_1 = \tau_1 \frac{D_{01}R_{01}}{\Pi_1} + r_1^{-1}E_1\left[\tau_2 \frac{D_{02}R_{02}}{\Pi_1\Pi_2}\right],$$

$$(17)\ \tau_1 = r_1^{-1}E_1\left[\tau_2 r_2 + 2\lambda r_1(2\tau_2 - g_2)r_2\right],$$

$$(18)\ \gamma(\Pi_{2,s} - 1)\Pi_{2,s} = \tau_{2,s}\frac{D_{02}R_{02}}{\Pi_1\Pi_{2,s}} + \Lambda,\ s = l, h$$

where

$$\begin{aligned}\Lambda \ = \ & 2r_1\left[\lambda_s(2\gamma\Pi_{2,s}^2 - \gamma\Pi_{2,s} + (2\tau_{2,s} - g_2)D_{02}r_1r_{2,-s}\Pi_{2,-s}(2\Pi_{2,s})^{-1}) - \right. \\ & \left. \lambda_{-s}(2\tau_{2,-s} - g_2)\, D_{02}r_1r_{2,s}\Pi_{2,s}(2\Pi_{2,-s})^{-1}\right]\end{aligned}$$

is a term associated with the incentive compatibility constraint (14) (λ_s is the Lagrange multiplier and $-s$ stands for the state alternative to s).

Equations (16) and (18) say that the marginal cost of inflation, at optimum, equals the marginal benefit from the tax reduction associated with the larger inflation tax and the fall in the real value of debt obligations. Equation (17) implies that, compared with the first-best optimum, even in the absence of uncertainty it is no longer optimal to completely smooth out taxes over time. In particular, the time consistent equilibrium exhibits higher (lower) taxes in period 1 relative to period 2 whenever $\lambda > 0$ ($\lambda < 0$). Further analysis of the first-order conditions shows that the intertemporal distribution of taxes is directly related to whether the government borrows or lends in period 1. In fact, solving (18) for λ_s and using (14), we obtain $\tau_1 \geq E_1[\tau_2]$ (i.e. $\lambda_s \geq 0$, $s = l, h$) if and only if $D_{12} \geq 0$. This inequality indicates that the only time inconsistency problem that matters to the government in period 1 for altering the intertemporal distribution of taxes is the one concerning D_{12}. In particular, if $D_{12} > 0$, the government in period 2 is provided with an additional (to D_{02}) incentive to increase Π_2. As a result, the government finds it optimal to lower the amount of nominal obligations left in period 2 by reducing its borrowing in period 1. This can be done by raising tax revenues in period 1. This phenomenon is what has been recognized in the literature as debt aversion (Calvo and Guidotti, 1990). The opposite reasoning applies if $D_{12} < 0$, since a negative D_{12} provides an incentive to deflate.

4.3 The first period problem

Equations (14), (16), (17) and (18), together with budget constraints (6) and (7), characterize the time consistent policy for the government in period 1 as a function of the maturity structure of initial debt and the exogenous variables of the model.

The optimal maturity structure for the initial stock of government debt is the one which minimizes the expected discounted sum of periods 1 and 2 social losses, subject to budget constraints (6) and (7) and incentive-compatibility constraints (14), (16), (17) and (18).

To investigate how optimal debt maturity and term-spread depend on exogenous variables, we numerically simulate the model. The results are summarized in Table 1.

Table 1

OPTIMAL DEBT MATURITY AND TERM SPREAD UNDER PARTIAL PRECOMMITMENT
(in parentheses, optimal debt maturity under full precommitment)

	$g_0 = 60\%$		$g_0 = 20\%$		$g_0 = 120\%$	
	$g_1 = 40\%$ $g_2 = 40\%$	$g_1 = 40\%$ $g_2 = 0\%$	$g_1 = 40\%$ $g_2 = 40\%$	$g_1 = 40\%$ $g_2 = 0\%$	$g_1 = 40\%$ $g_2 = 40\%$	$g_1 = 40\%$ $g_2 = 0\%$
D_{02}/g_0	35% (50%)	50% (80%)	35% (50%)	100% (100%)	30% (50%)	40% (65%)
Spread	−0.0264	−0.0075	−0.0067	0.0039	−0.0682	−0.0401
Π_1	1.1034	1.0771	1.0260	1.0160	1.2635	1.2209
Π_2^e	1.0536	1.0628	1.0133	1.0235	1.1367	1.1456

In column 1 we report our benchmark case, which is characterized by the following parameter values: the initial stock of debt, g_0, is assumed to be equal to 60% of gross national product (GNP), government expenditure in both periods, g_1 and g_2, equals 40% of GNP, and the inflation cost, γ, equals 4. The real interest factor, r_1, and the shock, ϵ, are assumed to be equal to 1.05 and 0.05, respectively. We also report the results obtained for a case of low debt

($g_0 = 20\%$) and a case of high debt ($g_0 = 120\%$). For each size of debt we analyze both situations where government expenditures (or deficits) increase and decrease over time. We report here only a case of decreasing pattern (i.e. $g_1 = 40\%$, $g_2 = 0\%$). Opposite conclusions have to be drawn when deficits exhibit an increasing pattern.

The simulations show that optimal debt maturity in the absence of precommitment on inflation can be either shorter or longer than optimal maturity under full precommitment. Under both regimes, optimal maturity lengthens in the presence of decreasing patterns of government expenditure (or deficit) and shortens with increasing deficits, as a more balanced distribution of government liabilities across periods improves tax smoothing. The absence of precommitment on inflation in general dampens these effects, since better tax smoothing is achieved at the expense of larger inflationary biases. Changes in equal proportion in g_1 and g_2 are shown to affect optimal maturity only slightly; the corresponding results are not reported in the table. The effects of changes in g_0 on optimal maturity are negligible in the presence of a balanced distribution of government expenditures over time (i.e $g_1 = g_2$). In contrast, with downward-sloping (upward-sloping) time profiles of deficits, optimal maturity tends to shorten (lengthen) as g_0 increases.

The results of changes in g_1 and g_2 on the long-short spread, measured by the difference $R_{02}^{1/2} - R_{01}$, are strictly associated with optimal maturity: the spread systematically narrows as the maturity shortens. In particular, the term-spread is positive whenever the base for Π_2 at optimum (i.e. $D_{02}R_{02} + D_{12}R_{12}$) exceeds the base for Π_1 (i.e. $g_0 r_1$). This is in general the case for low-enough levels of initial debt g_0 and high-enough values of g_1 relative to g_2, that is, for steep-enough downward-sloping time profile of government expenditures (or deficits). For the same reason, the term-spread decreases as g_0 increases.

Finally, the simulations suggest that optimal maturity tends to lengthen with increases in the volatility of the real rate process as well as in the cost of inflation. Higher values of ϵ and γ are associated with larger term-spreads. As a matter of fact, optimal maturity and term-spread in the benchmark case rise respectively to 37% and -0.0262 when $\epsilon = 0.1$, and to 38% and -0.0093 when $\gamma = 12$.

5. No precommitment: the case of bankruptcy risk

When no precommitment is allowed, investors require compensation for both inflation and default risk, which is embedded in nominal returns on government debt.

In order to find the subgame perfect equilibrium of the game, we solve the government's problem by backward induction.

5.1 The last period problem

In the last period, the government chooses Π_2, τ_2 and Θ_2 to minimize the value of the cost function

$$(19)\ \frac{1}{2}\tau_2^2 + \frac{\gamma}{2}(\Pi_2 - 1)^2 + \alpha\Theta_2\left(\frac{D_{02}R_{02}}{\Pi_1\Pi_2} + \frac{D_{12}R_{12}}{\Pi_2}\right),$$

subject to its period 2 budget constraint, equation (7).

The government's optimal policy in the last period is characterized by the following first order conditions:

$$
\begin{aligned}
(20) \qquad & \tau_2 - \lambda \geq 0 \\
(21) \qquad \gamma(\Pi_2 - 1)\Pi_2 - (\tau_2 - g_2)\left(\lambda + \frac{\alpha\Theta_2}{1-\Theta_2}\right) & \geq 0 \\
(22) \qquad & \alpha - \lambda \geq 0,
\end{aligned}
$$

($= 0$ if $\tau_2 > 0$, $\Pi_2 > 0$, $\Theta_2 > 0$, respectively), where λ is the Lagrange multiplier associated with the budget constraint.

Two possible situations may arise from the solution to this system of equations. Solving (20), (21) and (22), all with equality sign, together with budget constraint (7), gives

(23) $\tau_2 = \alpha$,

(24) $\gamma(\Pi_2 - 1)\Pi_2(1 - \Theta_2) = \alpha(\tau_2 - g_2)$,

(25) $\Theta_2 = 1 - \dfrac{(\tau_2 - g_2)\Pi_2}{D_{02}R_{02}\Pi_1^{-1} + D_{12}R_{12}}$.

τ_2 and Π_2 can be interpreted as the maximum amount of income and inflation taxes that the government is willing to levy in period 2. Therefore, the government's problem has an interior solution and $\Theta_2 > 0$ whenever the total stock of debt maturing in period 2, $D_{02}R_{02}\Pi_1^{-1} + D_{12}R_{12}$, is larger than the maximum amount of income taxes (net of current expenditure) and inflation that the government is ready to levy, namely $(\tau_2 - g_2)\Pi_2$.

If instead $D_{02}R_{02}\Pi_1^{-1}+D_{12}R_{12} < (\tau_2-g_2)\Pi_2$, then the government's problem has a corner solution and $\Theta_2 = 0$, since the maximum amount of income and inflation taxes exceeds the maturing stock of debt. Under such circumstance, optimal inflation and income tax rates are given by the solution to the following system of equations

(26) $\gamma(\Pi_2 - 1)\Pi_2 = \tau_2(\tau_2 - g_2)$,

(27) $\dfrac{D_{02}R_{02}}{\Pi_1\Pi_2} + \dfrac{D_{12}R_{12}}{\Pi_2} + g_2 = \tau_2$,

which solved for Π_2 gives back the incentive compatibility constraint under partial precommitment, equation (10).

5.2 The intermediate period problem

In period 1 the government chooses inflation Π_1, taxes on income and debt, τ_1 and Θ_1, and the amount of maturing debt to be rolled over to period 2, D_{12}, to minimize (9), subject to its period 1 and period 2 budget constraints, equations (6) and (7), and the incentive compatibility constraints in period 2 (either (26) and (27) or (23), (24) and (25), depending on whether the problem in period 2 admits a corner or an interior solution). Recalling that in period 1 the government does not observe r_2, its objective is given by the following expected social loss

(28)
$$\frac{1}{2}\tau_1^2 + \frac{\gamma}{2}(\Pi_1 - 1)^2 + \alpha\Theta_1\frac{D_{01}R_{01}}{\Pi_1} +$$
$$\frac{1}{2r_1}\left[\frac{1}{2}\tau_{2,l}^2 + \frac{\gamma}{2}(\Pi_{2,l} - 1)^2 + \frac{1}{2}\tau_{2,h}^2 + \frac{\gamma}{2}(\Pi_{2,h} - 1)^2\right] +$$

$$\frac{\alpha}{2}\left[\Theta_{2,l}\left(\frac{D_{02}R_{02}}{\Pi_1\Pi_{2,l}}+D_{12}r_{2,l}\right)+\Theta_{2,h}\left(\frac{D_{02}R_{02}}{\Pi_1\Pi_{2,h}}+D_{12}r_{2,h}\right)\right],$$

where $\Theta_{2,s}$ is the ex-post tax on debt maturing in period 2 when $r_{2,s}$ realizes $(s=l,h)$. Obviously, expected loss (29) is minimized taking into account that budget and incentive compatibility constraints must hold for both $r_{2,l}$ and $r_{2,h}$.

The first-order condition with respect to τ_1 gives

(29) $\tau_1 \leq \alpha,$

which implies that α is the maximum amount of income taxes that the government is ready to levy in period 1. As in period 2, the government's problem can have either an interior solution with $\Theta_1 > 0$ or a corner solution with $\Theta_1 = 0$. In period 1 the government also chooses inflation tax Π_1 and how much debt D_{12} to roll over to the last period. Since in equilibrium $R_{01} = r_1\Pi_1$, from (29) it follows that, in the absence of default, D_{12} must be at least equal to $D_{01}r_1 + g_1 - \alpha$.

In the following sections we consider only equilibria in which the government finds it optimal to borrow in period 1 (i.e. $D_{12} > 0$). The analysis of the case, empirically less important, with optimal lending in period 1 can be provided by the authors upon request.

In order to investigate all situations which in principle may arise, we need to introduce some notation.

Let Π_1^p, τ_1^p, D_{12}^p, $\tau_{2,s}^p$ and $\Pi_{2,s}^p$ represent the solution to the problem under partial precommitment, equations (14), (16), (17), (18) ($s = l, h$). By investigating first-order conditions (14), (16), (17), (18) it immediately appears that if $D_{12}^p > 0$, then $\tau_1^p > \tau_{2,h}^p > \tau_{2,l}^p$.

Three possibilities may thus occur.

5.2.1 Equilibrium with no risk of default

In this section we investigate the conditions under which it is never optimal for the government to repudiate its debt.

Let $\hat{\tau}_{2,h}$ and $\hat{\Pi}_{2,h}$ denote the solutions to the incentive compatibility constraints, equations (26) and (27), when $D_{12} = D_{01}r_1 + g_1 - \alpha$ and $r_2 = r_1 + \epsilon$. That is, $\hat{\tau}_{2,h}$ and $\hat{\Pi}_{2,h}$ are income and inflation taxes that the government would optimally implement in period 2 if it observes a high realization of the real interest rate and the amount of debt rolled over from the previous period is the minimum compatible with optimal income taxation in period 1 (i.e. $\tau_1 = \alpha$). Similarly, let $\hat{\tau}_{2,l}$ and $\hat{\Pi}_{2,l}$ be the equilibrium value of τ_2 and Π_2 when $D_{12} = D_{01}r_1 + g_1 - \alpha$ and $r_2 = r_1 - \epsilon$.

Proposition I: Assume $D_{12}^p > 0$. Then, the unique perfect equilibrium of the game never involves default if and only if

$$\hat{\tau}_{2,h} \leq \alpha.$$

In this equilibrium the government chooses τ_1^p, Π_1^p, $\tau_{2,s}^p$ and $\Pi_{2,s}^p$ ($s = l, h$), if $\tau_1^p \leq \alpha$. That is, the equilibrium is at a corner and coincides with the solution to the unconstrained system of first-order conditions under partial precommitment.[10] If instead $\tau_1^p > \alpha$, then optimization implies $\tau_1 = \alpha$; $\tau_{2,s}$ and $\Pi_{2,s}$ $(s = l, h)$ will be determined according to equations (26) and (27), where $D_{12} = D_{01}r_1 + g_1 - \alpha$ is imposed (i.e. $\tau_{2,s} = \hat{\tau}_{2,s}$ and $\Pi_{2,s} = \hat{\Pi}_{2,s}$).

The public sets nominal returns on government debt according to equations (1), (2) and (3), assuming zero risk of default (i.e. $\Theta_1 = \Theta_2 = 0$).

(Proof is in the appendix).

Proposition I provides the necessary and sufficient condition for the game to admit an equilibrium with no risk of default in the presence of borrowing by the government in period 1. A more compact but equivalent condition is that for both realizations of r_2

$$(30) \quad \frac{D_{02}R_{02}}{\Pi_1 \hat{\Pi}_2} + \left(\frac{D_{01}R_{01}}{\Pi_1} + g_1 - \alpha\right) r_2 + g_2 \leq \alpha,$$

that is, the intertemporal budget constraint can be always satisfied by relying on inflation and income taxation only.

5.2.2 Equilibrium with default risk

Suppose now that under a high realization of r_2, even by choosing $D_{12} = D_{01}r_1 + g_1 - \alpha$ (i.e. as small as possible) the real value of the total stock of outstanding debt in

[10] Notice that $\tau_1^p \leq \alpha$ implies $\tau_{2,s}^p \leq \alpha$ $(s = l, h)$. Furthermore, from incentive compatibility constraints (26) and (27) it follows that $\tau_{2,s}^p \leq \alpha$ implies $\Pi_{2,s}^p \leq \Pi_{2,s}^i$, where $\Pi_{2,s}^i$ is given by the solution to (24) and (25) with $r_2 = r_1 - \epsilon$ for $s = l$ and $r_2 = r_1 + \epsilon$ for $s = h$.

period 2 is larger than the maximum amount of income taxes that the government is willing to levy. That is,

$$(31)\ \frac{D_{02}R_{02}}{\Pi_1\hat{\Pi}_2}+\left(\frac{D_{01}R_{01}}{\Pi_1}+g_1-\alpha\right)r_2+g_2>\alpha,$$

if and only if $r_2 = r_{2,h}$ (i.e., $\hat{\tau}_{2,h} > \alpha$ and $\hat{\tau}_{2,l} < \alpha$).[11] Then, the government has to resort to ex-post debt taxation if $r_2 = r_{2,h}$ occurs.

The problem of the government thus consists in choosing the optimal defaulting policy, that is, how much to tax short-term debt maturing in period 2 as opposed (or in addition) to taxing debt maturing in period 1. In making this choice the government faces the following trade-off. If it taxes away, fully or partially, its short-term debt maturing in period 1, the associated cost will be proportional to the amount of defaulted debt, $\Theta_1 D_{01}R_{01}\Pi_1^{-1}$. It may alternatively decide to roll over that amount to period 2. Then, depending on the realization of the real interest rate, total outstanding debt may prove unsustainable. If, and only if, this is the case, the government has to resort to default and bear the cost of taxing the stock of debt rolled over from period 1. This reasoning provides the intuition for the following

Lemma: Assume that condition (31) holds. Then the optimal strategy for the government is to tax away long-term debt maturing in period 2 as much as necessary, and never repudiate short-term debt maturing in period 1.

(Proof is in the appendix).

[11] If both $\hat{\tau}_{2,h}$ and $\hat{\tau}_{2,l}$ were greater than α no sustainable equilibrium with long-term debt would ever be attained.

The result stated in the lemma relies on the fact that repudiation in period 2 occurs with probability 1/2. Therefore, before having observed the realization of the real interest rate in period 2, it is less costly to roll over short-term debt which cannot be repaid by current taxation rather than to default on it.

The following proposition provides the conditions under which an equilibrium with partial default on long-term debt exists.

Proposition II: Let condition (31) hold. Then the game admits a unique perfect equilibrium which implies risk of partial default if and only if $\exists$ a pair $(\Theta_2^*, \Pi_2^*), \Theta_2^* \in [0,1)$, which solves incentive compatibility constraints in period 2, equations (23), (24) and (25) with $D_{12} = D_{01}r_1 + g_1 - \alpha$, such that

$$(32)\ (1-\Theta_2^*)\frac{D_{02}R_{02}}{\Pi_1\Pi_2^*} + \left(\frac{D_{01}R_{01}}{\Pi_1} + g_1 - \alpha\right) r_2 + g_2 \leq \alpha$$

is satisfied both when $r_2 = r_{2,l}$ and $r_2 = r_{2,h}$, and holds with equality for $r_2 = r_{2,h}$.

In this equilibrium the government never repudiates short-term debt maturing in period 1 (i.e. $\Theta_1 = 0$).

Optimal ex-post tax on long-term debt is zero when $r_2 = r_{2,l}$ (i.e. $\Theta_{2,l}^* = 0$). Furthermore, the government chooses $\tau_1 = \alpha$, $\tau_{2,h} = \alpha$, $\Pi_{2,h} = \Pi_{2,h}^*$, $\Theta_{2,h} = \Theta_{2,h}^*$, and $\tau_{2,l}$ and $\Pi_{2,l}$ according to equations (26) and (27), where $D_{12} = D_{01}r_1 + g_1 - \alpha$ is imposed.

The public sets nominal returns on government debt according to equations (1), (2) and (3), where $\Theta_1 = 0$, $\Theta_{2,l} = 0$ and $\Theta_{2,h} = \Theta^*_{2,h}$.

(Proof is in the appendix).

5.2.3 Financial crisis

The cases addressed in propositions I and II do not exhaust all the possible situations which in principle may arise.

In particular, two cases of financial crisis may be conceived. If the maximum amount of income taxes that the government is willing to levy is not large enough to satisfy the government's intertemporal budget constraint in any state of the economy (i.e., for any realization of period 2 real interest rate), then a case of financial crisis emerges. Under such a circumstance, the government is not able to issue any debt in period 0, since default is expected to be total with probability one (i.e. $\Theta_1 = \Theta_{2,l} = \Theta_{2,h} = 1$).

Suppose instead that the fiscal regime becomes unsustainable when a high realization of the real rate in period 2 occurs (i.e., condition (31) is met for $r_2 = r_{2,h}$), but condition (32) in proposition II, necessary and sufficient for having an equilibrium with risk of default, is not satisfied (i.e. $\Theta^*_{2,h} \geq 1$). Then, a different case arises, characterized by the emergence of a risk of financial crisis. In this situation, the government is able to issue debt in period 0, since Θ_1 and $\Theta_{2,l}$ are expected to be 0. However, it might not be able to borrow in period 1, since, if $r_2 = r_{2,h}$ realizes, optimal default on outstanding debt will be total (i.e. $\Theta^*_{2,h} = 1$), and the return required by the public for holding government debt will become infinity.[12]

5.3 The first period problem

The existence of the option for the government to rely on default has important implications for the choice of the maturity structure.

As a matter of fact, under a no precommitment regime high realizations of the real rate may generate a risk of default which is larger the shorter the maturity (since a larger stock of debt has to be rolled over at the new interest rate). In particular, default risk may emerge only in the presence of short maturities. If this is the case, the government will find it optimal to increase the fraction of long-term in total debt up to the level which is compatible with an equilibrium with no risk of default. Therefore, optimal maturity under no precommitment is in general longer than under partial precommitment for all parameter values such that equilibria with risk of default are attained. Optimal maturity tends to lengthen with increases in the real interest rate. Neverthless, there may be cases in which real rates are so high that no equilibria with long-term debt is achieved. Under such a circumstance, the only sustainable equilibrium may be one involving risk of financial crisis. In this equilibrium optimal maturity drops to zero since government is able to issue short-term debt only.

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[12] Notice that this equilibrium relies on the assumption that the cost of financial crisis is not excessively high. In particular, such cost must be low enough to make the government prefer to risk financial crisis with probability 1/2 rather than to default on short-term debt.

Table 2 illustrates this intuition. The parameter values characterize a case of large government indebtness and downward-sloping time profile of current expenditures (i.e. $g_0 = 120\%$, $g_1 = 40\%$, $g_2 = 0\%$; $\alpha = 0.885$, $\gamma = 4$, $\epsilon = 0.05$).

Table 2

OPTIMAL DEBT MATURITY AND TERM SPREAD UNDER NO PRECOMMITMENT

	Real interest rate				
	5%	6%	7%	8%	8.5%
D_{02}/g_0	40%	45%	50%	60%	0%
Spread	−0.0401	−0.0391	−0.0378	−0.0365	-

In the presence of low levels of the real interest rate, the model admits equilibria with no risk of default. Optimal maturity is constant and coincides with optimal maturity under partial precommitment. As real rates increase, a default risk emerges at the maturity which was optimal in the presence of lower rates, and the government optimally increases the share of long-term in total debt. At rates equal to 8.5%, no equilibrium with long-term debt exists (i.e. condition (32) cannot be satisfied for any $D_{02} > 0$), and only short-term debt can be issued under the risk of financial crisis.

Notice that optimal maturity will exhibit a similar evolution if government debt (instead of real rates) increases:

when the size of debt is small, no default premia are required and optimal maturity is determined only by hedging and inflation considerations. With larger stocks of debt, the government lengthens the maturity in order to avoid the emergence of default premia. If government indebtedness were to reach extremely high levels, compensation for default risk embedded in long-term rates would be unbearable, and only short-term debt could be issued.

The term-spread increases (i.e. it reduces in absolute value) as real rates increase.

6. The case of indexed debt

In the presence of real indexation of government debt, the incentive of the government to reduce the real value of its nominal obligations through inflationary means disappears. Hence, equilibria with zero inflation are always achieved. The problem of the government consists in choosing a path of taxes on income and debt $\tau_1, \tau_2, \Theta_1, \Theta_2$ to minimize loss function (9), subject to its budget constraints, equations (6) and (7), where nominal rates now reflect the total absence of inflation risk.

The government's optimal policy in the last period is characterized by one of the following two situations. If $D_{02}R_{02}+D_{12}R_{12}+g_2 \leq \alpha$, then the government's problem has a corner solution and $\Theta_2 = 0$, since the maximum amount of income tax (net of current expenditure) that the government is ready to levy in period 2 exceeds the stock of maturing debt. If instead $D_{02}R_{02}+D_{12}R_{12}+g_2 > \alpha$, then the solution is at an interior with $\tau_2 = \alpha$ and $\Theta_2 = 1-(\alpha-g_2)(D_{02}R_{02}+D_{12}R_{12})^{-1}$.

In period 1 the government chooses taxes on income and debt, τ_1 and Θ_1, and the amount of maturing debt to roll over to period 2, D_{12}, to minimize (9), subject to its period 1 and period 2 budget constraints and the incentive compatibility constraints in period 2.

The first-order conditions with respect to τ_1 and D_{12} give, respectively

$$\tau_1 \leq \alpha,$$
$$\tau_1 = r_1^{-1} E[\tau_2 r_2],$$

which imply the same tax smoothing behavior as under full precommitment, subject to the constraint that the marginal cost of income taxation cannot exceed the marginal cost of default.

In the following section, some numerical simulations illustrate how the presence of indexed debt affects the results.

7. Numerical simulations

To investigate how the risk of default depends on exogenous variables like g_0 ,γ, ϵ, and on the presence of indexed debt, we numerically simulate the model.

Table 3 reports the minimum levels of the real interest rate compatible with equilibria incorporating risk of default for different parameter values, with and without real indexation of government debt. In parentheses we report the corresponding default premia, measured approximatively by the difference

$[R_{02}(\Pi_1\Pi_2^e)^{-1}]^{1/2} - R_{01}\Pi_1^{-1}$, where $\Pi_2^e = (\Pi_{2,l} + \Pi_{2,h})/2$. The benchmark case is characterized by the same parameter values as in Table 2.

The table shows that the risk of default increases with the size of debt and the cost of inflation. This finding would suggest that the combination of very high levels of public debt and anti-inflationary governments, or central banks, makes default premia more likely to arise.[13] More volatile real interest rate processes increase the risk of default.

Table 3

LOWEST REAL RATE LEVELS COMPATIBLE WITH RISK OF DEFAULT
(in parentheses, the corresponding default premium x 100)

Benchmark		$g_0 = 130\%$		$\gamma=12$		$\epsilon = 0.1$	
no ind.	ind.	no ind.	ind.	no ind.	ind.	no ind.	ind.
8.37% (0.011)	8.26% (0.002)	3.15% (0.003)	3.01% (0.003)	8.36 (0.006)	8.26 (0.002)	8.30% (0.010)	8.09% (0.005)

If government debt is totally indexed, the real value of nominal obligations cannot be lowered by means of inflation taxes and repudiation is more likely to occur at lower interest rate levels. The higher the variance of the real rate shock,

[13] The effect of the cost of inflation on the defaut risk is small, however. This is due to the fact that in equilibrium inflation is anticipated (at least on average). Therefore, the conditions under which the government is not able to satisfy its budget constraint with conventional taxes and has to rely on default are independent of the size of inflation that it optimally implements.

the greater the impact of debt indexation on the risk of default. Furthermore, the simulations show that government losses tend to decrease as the fraction of indexed debt increases, since the presence of real indexation of government debt eliminates the distortion associated with the inflationary incentive. Nonetheless, whenever fiscal sustainability is doubtful and the inflationary bias is not excessively large, having real indexation of government debt may be suboptimal (i.e. in the presence of equilibria with default risk and very high values of γ, government losses turn out to be greater with real indexation than without it).

Comparing the default premia illustrated in the table is not very informative, since they refer to different real interest rate levels. Notice, however, that the presence of debt indexation, together with a high cost of inflation and a high volatility of the interest rate process, has the effect of increasing the default premium. In particular, for a real rate of 8.37%, the default premium which in the benchmark case equals 0.00011, goes up to 0.00079 in the presence of indexed debt, to 0.00022 when $\gamma = 12$, and to 0.00139 when $\epsilon = 0.1$.

8. Concluding remarks

In this paper we construct a model of optimal debt management where the inability of the government to precommit its policies generates time inconsistency of government behavior. The potential for time inconsistency exists because the presence of nominal debt provides an incentive for the government to resort in the future to inflation, in order to reduce the real value of nominal debt obligations. In addition, the government may resort to ex-post debt taxation if the fiscal regime becomes unsustainable.

We assume that future realizations of the real interest rate are uncertain. The cost of government borrowing depends on such realizations and so, consequently, does the incentive of the government to inflate away the stock of outstanding debt. Moreover, the presence of large stocks of debt associated to high levels of the real rate makes the sustainability of the fiscal regime doubtful and leads the market to perceive a positive risk of default.

We show that different patterns of the term spread may be generated by different types of precommitment which the policymaker is able to enter into.

Whenever precommitment is imperfect (i.e. the government cannot commit either its inflationary or defaulting policy, or both), the model is able to generate overreactions of long-term rates to movements in short interest rates.

In particular, if, on the one hand, the government is able to fully precommit its action in terms of both inflation and default, then the term structure reflects the total absence of risk (i.e. long-term interest rates move together with short-term interest rates). On the other hand, if the government can only precommit its defaulting strategy, an inflation risk is perceived by the market and embedded in nominal rates. Long-term rates react more or less than short-term rates to changes in such risk depending on the time profile of government expenditures (or deficits). Finally, the paper examines the case in which there is no precommitment. This situation introduces a discontinuity in the long-short spread behavior, since when real rates reach the maximum level compatible with fiscal sustainability a positive default risk adds to the original term premium. Such risk increases with the size of the debt, the cost of inflation for the government and the volatility of the real rate process.

When government debt is totally indexed, inflation risk disappears but default risk becomes more likely. As a consequence, the optimal fraction of indexed debt tends to be lower than one, and in general it depends on the sustainability of the fiscal regime and the cost of inflation for the government.

We show that also optimal debt maturity is heavily dependent on the precommitment regime we consider. Under full precommitment, since future realizations of the real interest rate are uncertain, an appropriate choice of debt maturity can serve as a hedge against such a risk. Optimal maturity helps improve tax smoothing by balancing government liabilities across periods. Lack of precommitment on inflation may have the effect of either shortening or lengthening the optimal maturity, depending on the distribution of government expenditures over periods. Finally, since default risk increases as the maturity structure of the debt shortens, optimal maturity under bankruptcy risk is in general longer than in the case in which debt repudiation policies can be precommitted.

APPENDIX

Proof of Proposition I

Recall that first-order conditions (14), (16), (17), (18) imply, when $D_{12} > 0$, $\tau_1^p > \tau_{2,h}^p > \tau_{2,l}^p$ (debt aversion).

If: Suppose $\hat{\tau}_{2,h} \leq \alpha$. The solution to the government's problem under no precommitment gives $\tau_t \leq \alpha$, $t = 1, 2$. Two situations may thus occur. If $\tau_1^p \leq \alpha$, then debt aversion implies $\tau_{2,s}^p \leq \alpha$ $(s = l, h)$. Therefore, the equilibrium under bankruptcy risk is at a corner and coincides with the solution to the partial precommitment problem. From incentive compatibility constraints (23), (24) and (25) it follows that $\Theta_{2,s} = 0$ $(s = l, h)$: no default occurs in any state of the economy. If instead $\tau_1^p > \alpha$, then optimality in the presence of the option of debt repudiation requires $\tau_1 = \alpha$. Further, since $\hat{\tau}_{2,h} \leq \alpha$ by assumption, the government in period 1 will always find it optimal to set $D_{12} = D_{01}r_1 + g_1 - \alpha$ and $\Theta_1 = 0$. By doing so, it avoids bearing the cost of default in both periods. Given $D_{12} = D_{01}r_1 + g_1 - \alpha$, the solution to the government's problem in period 2 is again at a corner, with $\tau_{2,s} = \hat{\tau}_{2,s}$, $\Pi_{2,s} = \hat{\Pi}_{2,s}$ and $\Theta_{2,s} = 0$ $(s = l, h)$.

Only if: Suppose $\hat{\tau}_{2,h} > \alpha$. Equilibrium in period 1 implies $\tau_1 \leq \alpha$ and hence $D_{12} \geq D_{01}r_1 + g_1 - \alpha$. Therefore, in the absence of default, the budget constraint in period 2 implies $\tau_{2,s} \geq \hat{\tau}_{2,s}$, $s = l, h$. But, since $\hat{\tau}_{2,h} > \alpha$ by assumption, from incentive compatibility constraints (23), (24) and (25) it follows that $\tau_{2,h} = \tau_{2,h}^i = \alpha$, $\Pi_2 = \Pi_{2,h}^i$ and $\Theta_{2,h} > 0$: the solution in period 2 is at an interior and default occurs, at least in the presence of a high realization of the real rate.

Proof of Lemma

When condition (31) holds, the problem faced by the government in period 1 is to minimize the cost function (29), given $\tau_1 = \alpha$ and $\tau_{2,h} = \alpha$. By substituting budget constraints (6) and (7), where condition (13) holds, for $\Theta_1 D_{01} R_{01} \Pi_1^{-1}$ and $\Theta_{2,h}(D_{02}R_{02}(\Pi_1\Pi_2)^{-1}+D_{12}r_{2,h})$ into (29) ($\Theta_{2,l} = 0$, by assumption), and differentiating with respect to D_{12} gives $-\alpha+\alpha/2$. The first term in this expression is the marginal benefit (negative cost) for the government if it does not default today, whereas the second term is the expected cost of defaulting tomorrow rather than today. Since the first term is greater than the second term, it is always optimal to choose D_{12} as large as possible, which implies Θ_1 as small as possible.

Proof of Proposition II

If: Let $(\Theta_{2,s}^*, \Pi_{2,s}^*)$, $\Theta_{2,s}^* \in [0,1)$, solve incentive compatibility constraints in period 2, equations (23), (24) and (25) when $D_{12} = D_{01}r_1 + g_1 - \alpha$ and $r_2 = r_{2,s}$, $s = l, h$. If condition (31) holds, then, because of debt aversion, it must be the case $\tau_1^p > \alpha$. Therefore, first-order condition in period 1, equation (29), implies $\tau_1 = \alpha$; $\Theta_1 = 0$ and $D_{12} = D_{01}r_1 + g_1 - \alpha$, because of the lemma. The government in period 2 faces budget constraint (7), where $D_{12} = D_{01}r_1 + g_1 - \alpha$ and $R_{12} = r_2\Pi_2(1-\Theta_2)$, because of perfect foresight. Now, if $r_2 = r_{2,l}$ realizes, then condition (31) implies that the equilibrium is at a corner, with $\Theta_{2,l} = 0$, and $\tau_{2,l}$ and $\Pi_{2,l}$ given by incentive compatibility constraints (26) and (27). Condition (32) is satisfied with $\Theta_{2,l}^* = 0$ and the inequality sign. If instead $r_2 = r_{2,h}$, then the solution is interior: $\tau_{2,h} = \alpha$, $\Theta_{2,h} = \Theta_{2,h}^*$, $\Pi_{2,h} = \Pi_{2,h}^*$, and condition (32) holds with equality.

Only if: Suppose that condition (32) does not hold, for at least one realization of r_2. Then, the intertemporal budget constraint cannot be satisfied for any $\Theta_2^* \in [0,1)$.

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