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Debt Restructuring with Multiple Creditors and the Role of Exchange Offers

by Enrica Detragiache and Paolo G. Garella



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DEBT RESTRUCTURING WITH MULTIPLE CREDITORS AND THE ROLE OF EXCHANGE OFFERS

by Enrica Detragiache (*) and Paolo G. Garella (**)

Abstract

Exploiting the analogy with the private provision of a public good, this paper studies debt restructuring with an arbitrary number of creditors using mechanism design. Creditors differ in the value they expect to receive in bankruptcy, and this value is private information. As with public goods, too little debt forgiveness is granted in equilibrium relative to the first best. Creditors are more willing to make concessions under common values than under pure private values, an opposite phenomenon to the "winners' curse" in auctions. Exchange offers are an optimal restructuring scheme for the debtor, because they allow creditors to contribute to debt forgiveness at different levels.

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1. Introduction

An important issue in the theory of corporate finance is the nature of the costs of financial distress. While there are no costs of financial distress if debt contracts can be efficiently renegotiated (Haugen and Senbet, 1978), in practice debt renegotiation is often difficult. Granting sufficient debt forgiveness to an insolvent firm requires a high degree of coordination among creditors, and coordination is difficult to achieve when creditors are many. The concern about losses of firm value in financial distress has led a number of countries to introduce special bankruptcy statutes which, by weakening creditors' contractual rights, make it easier for indebted firms to obtain debt forgiveness (Mitchell, 1990). Various authors have identified the existence of coordination problems among multiple creditors as the main economic rationale for such statutes (see for instance Jackson, 1986, and Webb, 1991)¹.

In practice, many firms voluntarily avoid restructuring under court supervision, and obtain that creditors grant some debt forgiveness even if their number is large. In recent years, exchange offers have emerged as the most popular mechanism for restructuring distressed bonds out of court in the U.S.² In an exchange offer, bondholders are offered to trade their old claims for a new security, such as a bond with lower face value or a mix of debt and equity; bondholders are free to exchange any fraction of their portfolio. The success of the offer is usually conditional on a minimum tendering requirement announced before tendering begins.

In this paper we develop a model of debt renegotiation with an arbitrary number of privately informed creditors designed to answer a normative and a positive question: the

¹Debt renegotiation may also be inefficient when the indebted firm has private information about its going concern value. On this subject, see Giammarino (1989), Webb (1987), and Detragiache (1995).

²For empirical evidence on exchange offers and corporate debt workouts, see Weiss (1990), Gilson *et al.* (1990), Asquith *et al.* (1994), Altman (1993).

normative question is whether it is possible, given creditors' pre-bankruptcy contractual rights, to devise a debt renegotiation mechanism that is *ex post* efficient, i.e. that ensures that no going concern value is lost. In other words, we ask whether the common view that out-of-court renegotiation tends to yield too little debt forgiveness is borne out, once a fairly general model of renegotiation is analyzed. The positive question is why exchange offers are so widespread a device for debt renegotiation.

To construct an analytical framework for this inquiry, we exploit the analogy between debt renegotiation and the classic economic problem of financing the production of a public good through private contributions. If the indebted firm is viable, debt renegotiation can benefit the creditors (it can keep the firm in business, creating more value to be divided among claimholders). However, the benefit is non-excludable, because creditors who refuse to write down their claims cannot be prevented from receiving a higher repayment rate if the firm remains in business. So the increase in the value of the firm due to debt renegotiation is a public good for the creditors. Using mechanism design, Laffont and Maskin (1979), Rob (1989), and Mailath and Postlewaite (1990) study whether the production of a public good can be efficient when it must be financed through voluntary contributions from consumers, and consumers are privately informed about their willingness to pay. Their results can be directly applied to debt renegotiation.

In our model, debt forgiveness can increase the value of the firm because it improves investment incentives (Myers, 1977)³. There is an arbitrary but finite number of creditors. Creditors differ in the utility that they expect to receive in case of bankruptcy. Creditor heterogeneity stems from differences in preferences and/or differences in the information used to forecast the bankruptcy value of the firm. In the language of auction theory, if creditors differ only because of different preferences the model is one of private values. If creditors have also different information, the model is one of common values, and the

³When a company has a large debt outstanding, creditors appropriate some of the returns from new investment, so shareholder-oriented management invests too little.

expected bankruptcy payoff of a creditor is affected by the private information that other creditors may reveal through their actions⁴.

As with public goods, it can be easily shown that all renegotiation mechanisms yield less debt forgiveness than would be *ex post* efficient. The reason is the standard free-rider problem: inducing creditors to reveal their true willingness to contribute to debt forgiveness is costly. Since creditor participation is voluntary, some of the cost must be borne by the shareholders of the indebted firm. Unless the going concern value of the firm is substantial, the costs of debt restructuring exceed the benefits, and renegotiation fails. This result depends crucially on the assumption that creditors have private information: because creditors are non-atomistic in our model, with full information a renegotiation plan could be devised to make each creditor pivotal. Such a plan could implement the *ex post* efficient outcome, as in the takeover model of Bagnoli and Lipman (1988). On the other hand, if it is plausible to assume that creditors are privately informed about their 'willingness to pay', then our results support the argument that financial distress is costly, and that a bankruptcy statute that facilitates debt forgiveness can be welfare-improving.

We also find that creditors can be more easily coordinated when they have common values. The intuition is the following: each creditor knows that a restructuring plan succeeds only if other creditors are also willing to make concessions. But if other creditors are willing to write down their claims, then they must have information that the bankruptcy value of the firm is low. Such knowledge makes creditors more willing to offer debt forgiveness. This is the opposite of the well-known 'winners' curse' in auctions: with common values, bidders bid less aggressively because winning conveys unfavorable information about the true value of the object (see, for instance, Milgrom, 1987).

In the second part of the analysis it is shown that exchange offers allow the debtor to maximize expected profits from renegotiation. Since it is natural to assume that the

⁴The work on public goods cited above is restricted to the pure private values case.

renegotiation scheme is chosen by the debtor, this result explains why exchange offers are so popular. In contrast with other restructuring mechanisms, exchange offers allow the debtor to engage in a form of ‘price discrimination’: by choosing to exchange a different fraction of their loan portfolio, creditors contribute to debt forgiveness at different levels, and the debtor can take advantage of their different ‘willingness to pay’. Predictably, creditors who expect a high payoff in bankruptcy choose to tender a small fraction of their claims, and vice versa. The probability of success of the offer is increasing in the going concern value of the firm.

Exchange offers are also studied by Gertner and Scharfstein (1991). These authors model bondholders as homogeneous and atomistic, and find that exchange offers succeed only if the new claims have a higher priority status (or shorter maturity) than the old ones. Otherwise, all bondholders hold out, and the offer fails no matter how large the welfare gain from debt forgiveness. In practice, however, successful debt restructurings involving exchanges of debt of similar maturity and priority have been observed empirically (Asquith *et al.*, 1994). Our results indicate that, if creditors are not atomistic, so that their decisions have a non-negligible effect on the probability of success of renegotiation, then exchange offers involving lower priority debt succeed with positive probability, although less often than it would be socially efficient.

The paper is organized as follows: section 2 contains the basic model and the main results about the efficiency of debt renegotiation. Section 3 shows the optimality of exchange offers. In section 4 some extensions and open issues are discussed, and section 5 concludes.

2. A Model of Debt Restructuring

2. 1. The Basic Set-up

A firm has a project that requires an investment of I in the current period, and yields future cash-flows with present discounted value of $C + I$ (with $C > 0$). The project is lost if the firm files for bankruptcy. The firm has a total debt outstanding of D and no cash in hand, so it needs external funds to finance both investment and debt service. If the firm is allowed to issue new claims senior to existing debt, then financing should be forthcoming, because $C > 0$. However, if outstanding debt has protective covenants forbidding the issuance of senior debt, then a new loan is profitable if and only if

$$(1) \quad (C + I) - D - I = C - D > 0.$$

If this inequality fails, then the firm must either file for bankruptcy or try to obtain debt forgiveness from creditors. This is just a version of the underinvestment problem identified by Myers (1977). Since we want to focus on debt renegotiation, we will assume that inequality (1) fails, i.e. that $C - D < 0$, and that the issuance of higher priority debt is prohibited by existing covenants⁵.

The debt outstanding is in the hands of n creditors, who are risk-neutral and have equal priority. For simplicity, creditors hold identical shares $d = \frac{D}{n}$ of total debt outstanding. Creditors do not trade debt amongst themselves (see section 4 below for a discussion of trading). Existing debt contracts are incomplete in that they lack provisions specifying how renegotiation should occur. The only provision is that contracts cannot be altered without the consensus of both the debtor and the creditor⁶.

⁵The possibility of renegotiating the seniority covenant instead of renegotiating the face value of the debt is discussed in section 4 below.

⁶Our analysis does not address the issue of how to design optimal renegotiation provisions to include in debt contracts. Rather, we take contractual incompleteness as given, and study

Before debt restructuring begins, each creditor i privately learns his expected payoff if the firm files for bankruptcy x_i ⁷. x_i , which will be referred to as the 'type' of creditor i , is the realization of the random variable $X_i : [\underline{x}, \bar{x}] \rightarrow [0, 1]$. The distribution function of X_i is $F(x_i)$, and the density is $f(x_i)$. $x \equiv (x_1, \dots, x_n)$ denotes the vector of creditor types, while $I_k \equiv \{x \mid \underline{x} \leq x_j \leq \bar{x} \text{ for } j = 1, 2, \dots, k\}$ (with $k = 1, 2, \dots, n$) is the set of all possible types of k creditors. Creditor types are assumed to be identically and independently distributed⁸. In the rest of the paper, the following definitions will be used:

$$f(x) \equiv \prod_{j=1}^n f(x_j)$$

$$x_{-i} \equiv (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$$

$$f(x_{-i}) \equiv \prod_{j \neq i} f(x_j).$$

Different values of x_i across creditors may arise because creditors have different preferences: for instance, tax considerations and regulatory constraints may make bankruptcy more or less attractive to different investors⁹. In this case, the model would be one of pure private values. If creditors have also different information about the outcome of bankruptcy, then the model is one of common values. In this case, the private information received by creditor i can help other creditors improve their forecast of their bankruptcy payoff. Following Myerson (1981), if creditor i knew the payoff expected by other

what outcomes can be achieved through renegotiation.

⁷The debtor may also have private information about the bankruptcy value of the firm. See section 4 below for a discussion of this possibility.

⁸On mechanism design when private information is correlated see Crémer and McLean (1988).

⁹For instance, debt forgiveness may be more attractive to bank creditors who are poorly capitalized and/or already have large tax credits from loss carry-forwards, because it can often be designed to avoid explicit debt write-offs.

creditors, his revised expected payoff in bankruptcy would be

$$(2) \quad l_i(x_i, x_{-i}) \equiv x_i + \sum_{j \neq i} e_i(x_j),$$

where the revision functions $e_i: [\underline{x}, \bar{x}] \rightarrow \mathbb{R}$ are non-decreasing, and satisfy

$$\int_{\underline{x}}^{\bar{x}} e_i(x_j) f(x_j) dx_j = 0$$

for all $j \neq i$. This property of the revision functions implies

$$\int_{I_{n-1}} l_i(x_i, x_{-i}) f(x_{-i}) dx_{-i} = x_i,$$

so we will continue to refer to x_i as the expected bankruptcy value of creditor i . The common values model can be easily given a pure private values interpretation by letting the revision functions be identically equal to zero.

Definition. *The model is one of pure private values if $e_i(x_j) = 0$ for all possible values of i, j , and x_j . This implies $l_i(x_i, x_{-i}) = x_i \quad \forall x_{-i} \in I_{n-1}$.*

To simplify the notation, define

$$e_i(x_{-i}) \equiv \sum_{j \neq i} e_i(x_j).$$

For the sake of realism, it is assumed that the payoff that creditors expect to receive in bankruptcy does not exceed the full face value of the debt:

Assumption 1. $l_i(x_i, x_{-i}) \leq d$ for all $x_i \in [\underline{x}, \bar{x}]$, $x_{-i} \in I_{n-1}$, and $i = 1, \dots, n$.

The next definition makes explicit the notion of efficient debt renegotiation in the context of this model.

Definition. *Debt restructuring satisfies ex post efficiency if and only if the new investment project is undertaken (and the firm remains in business) with probability one when*

$$C \geq \sum_{i=1}^n l_i(x_i, x_{-i}),$$

and the project is undertaken with probability zero (and the firm goes bankrupt) otherwise.

2. 2. The Revelation Game

To answer the normative question, of whether *ex post* efficiency can be obtained through debt renegotiation, we adopt a mechanism design approach similar to that used by Laffont and Maskin (1979), Rob (1989), and Mailath and Postlewaite (1990) to study the private provision of a public good. Specifically, we ask whether there exists a set of rules (a mechanism) for renegotiating the debt that yields an *ex post* efficient equilibrium outcome. We limit our investigation to a special class of mechanisms called revelation mechanisms, because by the Revelation Principle (Myerson, 1981) all equilibrium outcomes of all possible mechanisms can be obtained as truth-telling equilibrium outcomes of a revelation mechanism.

The structure of the revelation mechanism is as follows: creditors confidentially and simultaneously report their private information (their type x_i) to a fictitious mediator. Based on the reports, the mediator instructs the debtor to invest (remain in business) with probability p , and not to invest (file for bankruptcy) with probability $(1 - p)$. The mediator also instructs the firm to make a vector of payments $\omega \equiv (\omega_1, \omega_2, \dots, \omega_n)$ to each

of the creditors if it remains in business (ω_i is the payment to creditor i). Creditors are asked to accept the payment, and to cancel all remaining outstanding debt. Of course, if the payment is less than d , the full face value of the debt, creditors are effectively asked to provide some debt forgiveness. The payment may consist of cash or of a claim on the future cash-flow of the firm.

In a truth-telling equilibrium of the revelation game, it must be a best response for each creditor to report his true type, and all the creditors and the debtor must be willing to follow the recommendation of the mediator. Let $\hat{x} \equiv (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$ be the vector of reported types. Let $\omega: I_n \rightarrow \mathbb{R}^n$ be the payment vector recommended by the mediator as a function of the reported types, and let $p: I_n \rightarrow [0, 1]$ be the probability of investment recommended by the mediator as a function of the reported types. Accordingly, if the mediator chooses a mechanism (ω, p) , then $\omega_j(\hat{x}_i, x_{-i})$ is the payment to creditor j and $p(\hat{x}_i, x_{-i})$ is the probability of investment when creditor i reports $\hat{x}_i$ and all the other creditors report their true types.

In an equilibrium in which all other creditors report their true type, the expected utility to creditor i from a mechanism (ω, p) is:

$$\int_{I_{n-1}} \{ p(\hat{x}_i, x_{-i}) \omega_i(\hat{x}_i, x_{-i}) + [1 - p(\hat{x}_i, x_{-i})] l_i(x_i, x_{-i}) \} f(x_{-i}) dx_{-i}.$$

Recalling the definition of $l_i(x_i, x_{-i})$ and the properties of the revision functions, the expression above can be rewritten as

$$x_i + \int_{I_{n-1}} p(\hat{x}_i, x_{-i}) [\omega_i(\hat{x}_i, x_{-i}) - l_i(x_i, x_{-i})] f(x_{-i}) dx_{-i}.$$

Since creditors get their expected bankruptcy payoff x_i if the mechanism is not implemented, the change in expected utility from the implementation of the mechanism is:

$$(3) \quad U(x_i, \hat{x}_i, \omega_i, p) \equiv \int_{I_{n-1}} p(\hat{x}_i, x_{-i}) \left[\omega_i(\hat{x}_i, x_{-i}) - l_i(x_i, x_{-i}) \right] f(x_{-i}) dx_{-i}.$$

In equilibrium, each creditor must be at least as well off as without debt renegotiation, hence the following participation constraints must hold:

$$(4) \quad U(x_i, x_i, \omega_i, p) \geq 0$$

for all $x_i \in [\underline{x}, \bar{x}]$ and all i ¹⁰. In a truth-telling equilibrium, the debtor's expected utility from a mechanism (ω, p) is

$$(5) \quad V(x, p, \omega) \equiv \int_{I_n} p(x) \left[C - \sum_{i=1}^n \omega_i(x) \right] f(x) dx.$$

Accordingly, the participation constraint for the debtor is

$$(6) \quad V(x, p, \omega) \geq 0.$$

An alternative specification of the debtor participation constraint is discussed in section 4. For truthful reporting to be an equilibrium, the mechanism must satisfy the following incentive-compatibility constraints:

$$(7) \quad U(x_i, x_i, \omega_i, p) \geq U(x_i, x_j, \omega_i, p)$$

for all $x_i, x_j \in [\underline{x}, \bar{x}]$, and for all i and j . Using standard techniques (see Myerson, 1981), it can be shown that the following are necessary and sufficient conditions for

¹⁰Much of the work on public goods does not impose voluntary participation constraints on the beneficiaries of the goods, so the results are not directly applicable here (see, for instance, d'Aspremont and Gérard-Varet, 1979).

incentive-compatibility:

$$(8) \quad P(x_i) \equiv \int_{I_{n-1}} p(x_i, x_{-i}) f(x_{-i}) dx_{-i} \text{ must be decreasing in } x_i \text{ for all } x_i \in [\underline{x}, \bar{x}];$$

$$(9) \quad U(x_i, x_i, \omega_i, p) = U(\bar{x}, \bar{x}, \omega_i, p) + \int_{I_{n-1}} \int_{x_i}^{\bar{x}} p(u, x_{-i}) du f(x_{-i}) dx_{-i} \text{ for all } x_i.$$

The first condition states that the higher the expected bankruptcy payoff of a creditor the smaller the probability, as expected by that creditor, that the firm will stay in business. This condition is quite natural, since creditors with high expected bankruptcy payoffs are less eager to see the firm continue. The second condition (derived in Appendix 1) states that the mechanism must give each creditor the expected change in utility of the highest type $\bar{x}$ plus a mark-up. Since by the participation constraints $U(\bar{x}, \bar{x}, \omega_i, p) \geq 0$, any incentive-compatible renegotiation plan that satisfies the participation constraints gives all creditor types (except type $\bar{x}$) more than their reservation utility: the second term on the RHS of (9) represents the informational rent captured by a creditor of type x_i .

The informational rent is decreasing in x_i . To understand why this must be the case, note that, for incentive-compatibility to hold, creditors' change in utility must be constant in $\hat{x}_i$, the reported type. But then the only difference in equilibrium utility between a type x_i and a type $x_j > x_i$ is that x_i has a smaller 'opportunity cost' of letting the firm remain in business (see equation (3)). Hence, the increase in utility from participating in the renegotiation mechanism must be larger for creditors with low x_i .

Consider now an incentive-compatible mechanism in which $U(\bar{x}, \bar{x}, \omega_i, p) = 0$. Among all incentive-compatible mechanisms, this is the most favorable to the debtor, because creditors' utility is kept at a minimum. In the Appendix it is shown that the amount the debtor expects to pay to creditor i according to such a mechanism is equal to the expected bankruptcy payoff of creditor i plus an additional term, which is just a

reformulation of the informational rent on the RHS of equation (9):

$$(10) \quad \int_{I_n} p(x) \omega_i(x_i, x_{-i}) f(x) dx = \int_{I_n} p(x) \left[l_i(x_i, x_{-i}) + \frac{F(x_i)}{f(x_i)} \right] f(x) dx.$$

Hence, an incentive-compatible mechanism must pay each creditor more than their reservation utility to elicit truthful information revelation. Substituting in (5), the debtor's expected utility becomes

$$(11) \quad \int_{I_n} p(x) \left[C - \sum_{i=1}^n \left[l_i(x_i, x_{-i}) + \frac{F(x_i)}{f(x_i)} \right] \right] f(x) dx.$$

PROPOSITION 1. *A mechanism (ω, p) is incentive-compatible and satisfies the participation constraints (4) and (6) if and only if*

$$(11') \quad \int_{I_n} p(x) \left[C - \sum_{i=1}^n \left[l_i(x_i, x_{-i}) + \frac{F(x_i)}{f(x_i)} \right] \right] f(x) dx \geq 0.$$

This result is essentially the same as Laffont and Maskin (1979), Rob (1989), and Mailath and Postlewaite (1990). Inspection of (11') shows that debt renegotiation can succeed even if there is a multiplicity of creditors, because $p(x)$ can be non-zero for some values of x without violating (11')¹¹. However, successful renegotiation is too rare compared with what is required by *ex post* efficiency: if $p(x) = 1$ whenever $C \geq \sum_{i=1}^n l_i(x_i, x_{-i})$ and $p(x) = 0$ otherwise, condition (11) is violated unless the going concern value C happens to be very large. Hence, with multiple, privately informed creditors financial distress is costly, because firms with a positive going concern surplus may go bankrupt.

¹¹Mailath and Postlewaite (1990) show that, with pure private values, as $n \rightarrow \infty$ the probability that (8) be satisfied goes to zero, so inefficiency becomes the rule as in Gertner and Scharfstein (1991).

The inefficiency arises because of asymmetric information: with full information, the debtor could just offer a plan that gives creditors their expected bankruptcy payoff $l_i(x_i, x_{-i})$. Creditors would accept, and the debtor would invest whenever it is efficient to do so. With asymmetric information, to implement this mechanism creditors would have to report their private information truthfully, but would have no incentive to do so, as reporting a higher expected bankruptcy payoff would make them better off. To avoid this type of free riding, an incentive-compatible mechanism must be devised. As shown above, this implies that the debtor must give up rents to the creditors. Since obtaining debt forgiveness is more costly than under full information, it may not be profitable for the debtor even if the investment project is socially efficient.

Next we will show that if creditor heterogeneity is due to differences of information and not just to differences in preferences, then achieving coordination is easier, and the debtor is better off.

PROPOSITION 2. *For any incentive-compatible investment rule $p(x)$, the debtor's expected profits are higher under common values than under pure private values.*

Proof. see Appendix.

The intuition behind this result is the following: each creditor knows that debt restructuring will succeed only if other debtholders have a low expected bankruptcy payoff. With common values, this means that the expected bankruptcy value of the firm conditional on a successful restructuring is smaller than the unconditional expected bankruptcy value. Hence, with common values a creditor is more willing to contribute to debt forgiveness. This is the opposite of the 'winners' curse' in auctions (Milgrom, 1987): in an auction with common values, obtaining the object reveals to the bidder that his estimate of the value was too high, so bidding behavior is more conservative. Proposition 2

also implies that under common values condition (11') of Proposition 1 is more likely to be satisfied for any given investment rule $p(x)$. Hence, debt restructuring is less likely to be inefficient when creditor heterogeneity is due, at least in part, to differences in information.

3. An Interpretation of Exchange Offers

In the U.S. bonds are often restructured outside of bankruptcy through exchange offers. In an exchange offer, the debtor proposes to exchange existing debt for a new security with a lower interest rate or a lower face value than the old one. Sometimes the new claim is a composite of debt and other securities. Usually, the success of the offer is contingent on a minimum tendering requirement. There are two features that characterize exchange offers as a renegotiation mechanism: first, creditors choose which fraction of their holdings to exchange, effectively choosing how much to contribute to debt forgiveness; second, the offer specifies in advance that the transaction is void unless a sufficient amount of securities are tendered. In this section, we will show that, when there is a multiplicity of privately informed creditors, it is optimal for an indebted firm to restructure its debt through an exchange offer. To show this result, we will first derive the equilibrium outcome of a revelation mechanism that maximizes the debtor's expected profits. This mechanism will be referred to as the 'optimal' revelation mechanism, although it is optimal only from the point of view of the debtor¹². Because of the Revelation Principle, this mechanism yields the debtor the maximum payoff from any renegotiation mechanism. Then, we construct an exchange offer game, and show that (under some additional parameter restrictions) there exists a Bayesian–Nash equilibrium of that game in which all players get exactly the same payoffs as in the equilibrium of the optimal mechanism.

¹²Because the debtor uses his monopoly power to extract rents from the creditors, the outcome of this mechanism leads to larger *ex post* inefficiencies than what asymmetric information alone would imply.

Hence, choosing the exchange offer as a renegotiation scheme maximizes the debtor's expected profits.

3. 1. An Optimal Revelation Mechanism

In section 2 it was shown that the maximum expected profits that the debtor can obtain from an incentive-compatible revelation mechanism that does not violate the creditors' participation constraints are given by equation (10). Choosing $p(x)$ to maximize (10) pointwise:

$$(12) \quad \hat{p}(x) \equiv \begin{cases} 1 & \text{if } C \geq \sum_{i=1}^n \left[l_i(x_i, x_{-i}) + \frac{F(x^i)}{f(x^i)} \right], \\ 0 & \text{otherwise.} \end{cases}$$

$\hat{p}(x)$ and the condition that $U(\bar{x}, \bar{x}, \omega, p) = 0$ fully determine the expected profits of the debtor. Through (9), the expected utility of each creditors can also be computed without derive the payment function $\omega(x)$. To prove the optimality of exchange offers in the next section, however, it is necessary to derive an explicit solution for $\omega(x)$. To this end, rewrite the investment rule $\hat{p}(x)$ as follows:

$$(12') \quad \hat{p}(x) = \begin{cases} 1 & \text{if } C \geq A(x_i) + Q(x_{-i}) \\ 0 & \text{otherwise,} \end{cases}$$

where

$$(13) \quad A(x_i) = x_i + \frac{F(x_i)}{f(x_i)} + \sum_{j \neq i} e_j(x_i)$$

and

$$(14) \quad Q(x_{-i}) = \sum_{j=1}^n \sum_{k \neq j, i} e_j(x_k) + \sum_{j \neq i} \left[x_j + \frac{F(x_j)}{f(x_j)} \right].$$

Following Rob (1989), let us assume that the hazard rate of $F(x)$ is monotonic:

Assumption 2. $\frac{F(x_i)}{f(x_i)}$ is strictly increasing for all $x_i \in [\underline{x}, \bar{x}]$ and all $i = 1, \dots, N$.

This property holds for several commonly used distribution functions. Under this assumption, $A(x_i)$ is strictly increasing (equation (13)). Hence, for each vector x_{-i} there exists a unique value of x_i that solves

$$(15) \quad C = A(x_i) + Q(x_{-i}).$$

Let $\tilde{x} \equiv \tilde{x}(x_{-i})$ denote this value. Given that the other creditors report x_{-i} , $\tilde{x}$ is the largest type that creditor i can report to the mediator without forcing bankruptcy. Hence, $\tilde{x}(x_{-i})$ is the ‘pivotal’ type of creditor i according to the investment rule $\hat{p}(x)$. Note that, if the other creditors report low types, the term $Q(x_{-i})$ is small, and the ‘pivotal’ type for creditor i is large. Hence, $\tilde{x}(x_{-i})$ measures the extent of the opportunities for holding out (Rob, 1989).

Lemma 1. *In the optimal mechanism, the payment to creditor i is given by*

$$(16) \quad \hat{\omega}_i(x_i, x_{-i}) = \hat{\omega}_i(x_{-i}) = \tilde{x}(x_{-i}) + e_i(x_{-i}).$$

Proof. see Appendix.

According to Lemma 1, creditor i receives a payment that is independent of the type that

he reports, and may be increasing or decreasing in the types reported by the other creditors. The function $\bar{x}(x_{-i})$ is decreasing in each component of x_{-i} , as remarked above. On the other hand, $e_i(x_{-i})$ is increasing in each component of the vector x_{-i} , because, if other creditors report high bankruptcy payoffs, then creditor i revises his expected bankruptcy payoff upwards, and is less willing to contribute to debt forgiveness. If the 'holding out effect' dominates the 'revision of expectations' effect, then the payment from the optimal mechanism is decreasing in each component of x_{-i} . In the case of pure private values there is no revision of expectation effect, and $\hat{\omega}_i(x_i, x_{-i})$ is decreasing in the type of the other creditors.

3. 2. An Exchange Offer Game

Consider the following debt renegotiation mechanism. The debtor offers creditors a new security with face value b in exchange for their old claims (in equilibrium, $b < d$, the face value of the old debt). The offer states that exchange will be valid, and that the debtor will remain in business, only if the amounts tendered by the creditors satisfy a certain condition specified in the offer. Otherwise, the exchange will be void, the creditors' original claims will remain in force, and the debtor will file for bankruptcy. After the offer is announced, creditors simultaneously exchange a fraction $h_i \in [0, 1]$ of their holdings. Let $h \equiv (h_1, h_2, \dots, h_n)$ denote the vector of amounts tendered. After creditors tender, if the offer succeeds the debtor undertakes the investment project, and if the offer fails the debtor goes bankrupt. Note that no new decision is made at this stage of the game: the debtor just follows the rules stated in the offer¹³.

This renegotiation mechanism, while it resembles an exchange offer, is not a revelation mechanism, since there is no fictitious mediator to receive reports and to recommend actions based on those reports. In the revelation game creditors choose the

¹³On the issue of *ex post* renegotiation of the mechanism, see section 4 below.

type that they report and whether or not to participate. In the exchange offer game the only action chosen by the creditors is the amount tendered h_i . Also, the investment rule chosen by the debtor is a function of the amounts tendered and not of the reported types. To show that there is an equilibrium of the exchange offer game that yields the same payoffs as the optimal revelation mechanism, it will be shown that the amounts that creditors tender in the exchange offer are monotonic functions of the type. Hence, the debtor can infer the type of each creditor by observing the amounts tendered, and he can then follow the same investment rule of the optimal mechanism. To prove these results, it is necessary to impose an additional assumption on the parameter values.:

Assumption 3. The parameters $\underline{x}$, $\bar{x}$, C , and n , and the functions $e_i(x_j)$, $F(x_i)$, and $f(x_i)$ are such that for all x_{-i} and $x'_{-i} \in I_{n-1}$, if $Q(x_{-i}) > Q(x'_{-i})$, then $\hat{\omega}_i(x_{-i}) < \hat{\omega}_i(x'_{-i})$ (where $Q(x_{-i})$ is defined in (16)).

This assumption states that the payment assigned to creditor i by the optimal revelation mechanism (see Lemma 1) must be smaller the 'larger' is the vector of other creditor types, where 'larger' means to yield a larger $Q(x_{-i})$. This assumption is satisfied if the 'revision of expectation' effect on $\hat{\omega}_i(x_i, x_{-i})$ is small relative to the 'holding out' effect.

PROPOSITION 3. *Under Assumptions 1–3, there is an equilibrium of the exchange offer game that yields the debtor and the creditors the same expected utility as the optimal revelation mechanism. In this equilibrium, creditors with lower expected bankruptcy payoff tender a larger fraction of their portfolios, making a greater contribution to debt forgiveness.*

Sketch of the proof (see Appendix 3 for the complete proof). The first step is to derive the amount that a creditor of type x_i must tender to obtain the same expected utility as in the equilibrium of the optimal revelation mechanism. Let $\hat{H}(x_i)$ denote this amount. Then, it

is shown that, if the face value of the new security b is chosen appropriately, $\hat{H}(x_i) \in [0, 1]$ for all $x_i \in [\underline{x}, \bar{x}]$. The third step is to show that the functions $\hat{H}(x_i)$ are monotonically decreasing, i.e. that creditors who expect to receive a higher payoff in bankruptcy must tender a smaller share of their portfolio, and therefore contribute less to debt forgiveness. Assumption 3 is needed for this result. Once it is established that $\hat{H}(x_i)$ is monotonic, the function can be inverted to yield

$$(17) \quad x_i = H^{-1}(h_i) \equiv G(h_i).$$

Using this relationship, the investment rule of the revelation mechanism $\hat{p}(x)$ can be expressed as an investment rule for the exchange offer, i.e. as a mapping from the set of amounts tendered to the unit interval. Accordingly, in the equilibrium of the exchange offer game the debtor announces that the exchange offer is valid if and only if the amounts tendered satisfy:

$$(18) \quad C \geq \sum_{i=1}^N \left\{ l_i[G(h_i), G(h_{-i})] + \frac{F[G(h_i)]}{f[G(h_i)]} \right\},$$

(where $h_{-i} \equiv (h_1, \dots, h_{i-1}, h_{i+1}, \dots, h_n)$). Since the debtor is committed to invest whenever the offer succeeds, this announcement results in the investment rule $\hat{p}(G(h))$. If creditors of type x_i tender $H(x_i)$, then $\hat{p}(G(h)) = \hat{p}(x)$, and in the equilibrium of the exchange offer the firm remains in business for the same realizations of the vector of types x as in the equilibrium of the optimal revelation mechanism. Since the investment rule $\hat{p}(x)$ maximizes the debtor's expected profits, if creditors respond by tendering $\hat{H}(x)$, $\hat{p}(G(h))$ is optimal for the debtor. On the other hand, given the investment rule $\hat{p}(G(h_i))$, by construction tendering $\hat{H}(x_i)$ gives a creditor of type x_i the same expected utility as the revelation mechanism. Deviating from $\hat{H}(x_i)$ means imitating the action of another type.

By incentive-compatibility, this is suboptimal, and $\hat{H}(x_i)$ is a best response. •

Exchange offers are optimal for the debtor because they allow creditors who are optimistic about the outcome of bankruptcy to contribute less to debt forgiveness, and vice versa. By exploiting the creditors' different 'willingness to pay' for the benefit of staying out of bankruptcy, the debtor effectively engages in price discrimination¹⁴.

(18) is the condition for the exchange offer to succeed. Generally, this condition is more complex than a minimum tendering requirement, so the equilibrium of the exchange offer game of Proposition 3 does not exactly match real world exchange offers. However, condition (18) can be expressed as a minimum tendering requirement for particular parameterizations of the model. Specifically, if there exists a monotonically increasing function $R(\cdot)$ such that

$$(19) \quad R \left[\sum_{i=1}^N h_i \right] = \sum_{i=1}^N \left\{ l_i[G(h_i), G(h_{-i})] + \frac{F[G(h_i)]}{f[G(h_i)]} \right\},$$

then the exchange offer succeeds if and only if

$$\sum_{i=1}^N h_i \geq R^{-1}(C),$$

and $R^{-1}(C)$ is the minimum tendering requirement. The function $R(\cdot)$ exists if, for instance, the distribution function $F(x_i)$ is of the class $k(x_i)^\alpha$, with α and k constant, and if the revision functions are linear.

¹⁴This result is related to Cornelli (1992). She shows that, for a monopolist facing a fixed cost of production, the optimal selling procedure is to offer customers a menu of possible prices. As in our case, high valuation consumers are willing to pay a price above the minimum to increase the probability of the good being produced.

4. Extensions and Open Issues

4. 1. Trading Distressed Debt

In our analysis of debt renegotiation, we have ignored the possibility that creditors trade amongst themselves or with other potential investors. In practice, however, distressed debt can be traded. The type of debt most highly marketable consists of bonds, because underwriters usually commit to make a secondary market in the paper that they issue. Secondary markets for corporate bonds in general, and distressed corporate bonds in particular, are usually quite illiquid with large bid–ask spreads¹⁵. The largest share of distressed debt consists of claims for which there is no organized secondary market, such as bank loans, or trade credits¹⁶. The opportunities to trade this type of debt are likely to be even more limited than in the case of bonds. Hence, while trading in distressed debt takes place, it is unlikely to result in the perfectly competitive allocation.

Let us now consider how introducing limited trading opportunities would change our analysis of debt renegotiation. First of all, we have interpreted $l_i(x_i, x_{-i})$, the payoff that creditors expect to receive in the absence of (or in case of failure of) renegotiation, as the expected bankruptcy payoff. If debt can be traded, $l_i(x_i, x_{-i})$ should be reinterpreted as the maximum of two quantities: the value that creditor i expects to receive in bankruptcy, and the price that he expects to receive if he sells the debt. What complicates the analysis is that, in principle, the resale price may be affected by the information that is revealed during renegotiation. If creditors are aware of this effect, then their strategies in renegotiation may change¹⁷. However, as long as secondary market trading opportunities

¹⁵Ramaswami and Moeller (1990) report extreme buy and sell spreads in the distressed bond market (p. 22). Also Altman (1991) reports that distressed bonds are often illiquid.

¹⁶For instance, in the sample of 103 financially distressed firms studied by Altman (1993), the ratio of non–traded to traded debt is 4 to 1.

¹⁷A similar problem arises in Treasury bill auctions (Bikhchandani and Huang, 1989). In many countries, Treasury bills are auctioned off to primary dealers, who then resell the securities to the public at large. Bidding strategies in the primary auction, if observable to

are limited, neglecting this effect should not seriously distort our analysis of renegotiation.

A second issue is whether the assumption that debt is held by heterogeneous investors is consistent with the existence of limited trading opportunities *ex ante*. In our model, creditors of different type receive a different expected utility in equilibrium, so identical debt claims have a different value to different creditors. If debt was traded in a perfectly competitive market before renegotiation, then all gains from trade would be exhausted, creditor heterogeneity would be eliminated, and our analysis of debt renegotiation would be irrelevant. However, with trading frictions, such as asymmetric information, this is not necessarily the case. In a dealer's market, for instance, a large bid–ask spread can be a sign that potential gains from trade remain unexploited. As remarked above, spreads tend to be large in distressed debt markets, suggesting that the assumption of creditor heterogeneity is not completely off the mark. On this subject, in his empirical study of distressed LDC debt in 1986–87, James (1990) finds evidence supporting the hypothesis that creditor banks had heterogeneous valuations in spite of the existence of a secondary market for the debt.

Nonetheless, a complete theory of debt renegotiation should introduce limited trading opportunities explicitly into the analysis. Our results on debt renegotiation should be seen as a first step in a direction in which more advance is to be expected.

4. 2. Debtor's Private Information about the Bankruptcy Value of the Firm

The debtor may also receive private information (let's call it x_0) about the value of the firm in bankruptcy. In this case, the analysis of section 2 should be modified by extending the vector of types x to include also x_0 , and by imposing an additional incentive–compatibility constraint in the revelation game. However, since the debtor's payoff is identically zero in bankruptcy, his expected payoff does not depend on x_0 directly,

the public, reveal information that is reflected in the secondary market price.

and for incentive-compatibility to hold the functions ω and p must be constant in x_0 : because the debtor's expected profits are not a function of the bankruptcy value of the firm, the private information that the debtor may have concerning that value cannot be credibly communicated. This is true even if the debtor has better information than the creditors, or if he knows the true bankruptcy value of the firm. Hence, neglecting the debtor's information is without loss of generality. On the other hand, if the debtor's private information is about the going concern value of the firm, then this conclusion no longer holds, as we discuss below.

4. 3. Asymmetric Information about the Continuation Value of the Firm

In the model presented here the value of the firm as a going concern is known to all parties, while creditors have private information as to the value that they expect to receive in bankruptcy. Alternatively, it could be assumed that the going concern value of the firm is private information. In this case the renegotiation problem would change as follows: first, the type of security offered in the exchange offer (cash, risky debt, a mix of debt and equity) would dictate how creditors' expected utility depends on creditors' private information. For instance, creditors' expected payoff is independent of the going concern value of the firm if the security is riskless debt, but it is linear in the estimated going concern value if the security is equity. Second, the debtor's expected profits would be a function of the going concern value, so the debtor's private information would become important. The choice of the restructuring plan (including the choice of security) would reveal some of the debtor's private information.

Information revelation through the choice of the new security in debt workouts has been studied in models in which the debtor knows the true continuation value of the firm and creditors behave like a coalition¹⁸. However, the general case, in which all parties have

¹⁸See Brown et al. (1994) for an empirical study of information revelation through choice of security, and Detragiache (1995) for a theoretical study of how the ability to choose the

potentially relevant private information and there are coordination failures among creditors, remains to be investigated.

4. 4. *Ex Ante* versus *Ex Post* Budget Balance

In the literature on public goods the participation constraint imposed on the debtor (equation (6)) is often referred to as '*ex ante* budget balance'. This constraint requires the mechanism to make the debtor better off *ex ante*, i. e. before the debtor knows the payments that he will have to make to the creditors. An alternative form of the participation constraint is '*ex post* budget balance'. In this case, the continuation value of the firm must exceed the total amount paid out to creditors whenever the mechanism requires the firm to stay in business with positive probability:

$$(20) \quad p(x) [C - \sum_{i=1}^n \omega_i(x)] \geq 0 \quad \forall x \in I_n.$$

Which of the constraints is the most appropriate depends on how the financing for the new investment project is arranged. If no financing arrangement is made before debt renegotiation takes place, then for realizations of x such that $C < \sum_{i=1}^n \omega_i(x)$ no investor would be willing to finance the new project, and *ex post* budget balance would be a more appropriate constraint.

Alternatively, financing for the new project may be arranged before renegotiation. The debtor may have obtained a commitment by a lender to finance the project conditional on debt renegotiation to succeed. Pre-arranged financing of this type is commonly used by firms that restructure debt through exchange offers. In this case, *ex ante* budget balance is the appropriate constraint because, whenever *ex ante* budget balance holds, a risk-neutral investor should be willing to supply pre-arranged financing. In any case, the distinction

mix of securities affects renegotiation efficiency.

between the two types of constraint is not very important for the results: since agents are risk-neutral and there is a continuum of possible types, for any mechanism that satisfies the creditors' participation constraints and incentive-compatibility constraints there exists a payment scheme that preserves *ex post* budget balance whenever *ex ante* budget balance holds (see Mailath and Postlewaite, 1990).

4. 5. Renegotiation with Majority Voting

In the analysis it is assumed that debt can be restructured only with the unanimous approval of all creditors. In terms of the model, the participation constraints of all creditors have to be satisfied with probability one. Sometimes, however, a restructuring plan may be subject to majority voting. In the U.S., for instance, if a firm is in Chapter 11 a restructuring plan can be approved by just a 2/3 majority of each class of claimants. We claim that also under majority voting exchange offers remain useful as tools for price discrimination. Consider the following two-part mechanism: the debtor proposes a plan that includes a flat minimum debt forgiveness rate, as well as an offer to exchange the old debt for another asset with an even lower face value. Creditors vote on the plan. If the majority is in favor, all debt is scaled down, and the exchange offer takes place. If the minimum tendering requirement is met, then the firm continues, otherwise bankruptcy follows. This mechanism certainly allows shareholders to replicate the outcome of a plan that gives all debtholders the same payoff rate: it is sufficient to set the flat rate of debt forgiveness high enough to ensure that continuation is profitable even if nobody participates to the exchange offer. On the other hand, shareholders may grant themselves a higher expected profit by asking for a lower flat rate (thereby increasing the probability that the plan will be approved), and extracting further debt forgiveness from low bankruptcy value creditors through the exchange offer. As in the model of section 3.2, some types of creditor should be willing to contribute more than the flat rate to increase the probability that the firm will remain in business.

4. 6. Renegotiation of the Seniority Covenant

As an alternative to debt forgiveness, the indebted firm could ask creditors to waive the strict seniority covenant attached to existing debt contracts, and allow the debtor to finance the new investment project by issuing a senior loan¹⁹. Depending on the original contractual agreements, such a waiver may need unanimous consent of the creditors, or a majority vote may be sufficient. A waiver of the seniority covenant, while it allows the debtor to finance the new project, does not result in any debt forgiveness. Hence, if the return from the project is non-stochastic, renegotiating the seniority covenant yields shareholders of the indebted firm a payoff of zero (see (1)). Clearly, the debtor is better off renegotiating using an exchange offer even if the covenant can be renegotiated by majority vote. On the other hand, if the return from the investment project is stochastic, there may be large enough realizations of the returns to leave room for a positive payoff for equity even if no debt forgiveness is obtained. If the covenant waiver requires only a majority vote, then asking for the waiver will increase the probability of remaining in business. Hence, even though it leaves the firm with a larger debt outstanding, and does not exploit creditors' different 'willingness to pay', shareholders may be better off than with debt forgiveness.

4. 7. Renegotiation of the Mechanism *Ex Post*

Depending on the particular mechanism, the *ex post* inefficiencies highlighted in section 2 may create incentives to renegotiate the outcome of the game; if renegotiation is anticipated, then the nature of the equilibrium may change dramatically (see Myerson, 1991, Ch. 10). This is a well-known problem in the static mechanism design literature. Ausubel and Deneckere (1989, 1993) have shown that, in the case of bilateral trading

¹⁹We are indebted to a referee for suggesting this point.

problems, the solution to the static mechanism design problem can be sustained as a sequential equilibrium of a multiperiod game. If arguments similar to those of Ausubel and Deneckere can be extended also to mechanism design problems with many agents, then our results are valid. This is an important topic for future research.

5. Concluding Remarks

The analogy between debt forgiveness and the private provision of a public good indicates that, with an arbitrary number of privately informed creditors, the free rider problem causes out-of-court debt restructuring to succeed only if efficiency gains are large enough. In general, firms are likely to go bankrupt too often for *ex post* efficiency to obtain, so financial distress is costly.

An important question is whether inefficiencies of this type justify the existence of bankruptcy statutes which, by weakening the pre-bankruptcy rights of creditors, make restructuring more likely (such as Chapter 11 in the U.S.). This question is quite complex: for instance, *ex ante* firms may be better off when they have the option of issuing a security that is hard to renegotiate *ex post*, as in the models of Rajan (1992), and Detragiache (1994). Also, court intervention generates deadweight costs of its own, and it is not clear whether the party who makes the filing decision will make the socially efficient choice between liquidation and reorganization. Critics of Chapter 11 often point out that managers of distressed firms always file for reorganization, in the hope that a 'miracle' will restore the firm to solvency before a final decision is made (see, for instance, Bradley and Rosenzweig, 1992). Such perverse incentives may be reduced by improving the design of reorganization laws, but they may be hard to eliminate altogether.

Our analysis also shows that, under some conditions, exchange offers are an optimal way to renegotiate debt for a firm facing a multiplicity of creditors whose willingness to contribute to debt forgiveness is private information. In an exchange offers, by choosing

which fraction of the portfolio to tender, creditors choose how much debt forgiveness to supply. This helps reducing the impact of the free rider problem. We find that exchange offers could benefit the debtor also when debt is renegotiated through a majority vote, as it is the case in Chapter 11.

Since creditor heterogeneity is a barrier to efficient debt renegotiation, and trade among creditors should reduce heterogeneity, the growth of increasingly liquid secondary markets for distressed securities should make financial distress less costly. While in Europe no such market has yet emerged (The Economist, 1994), in the U.S. an informal market for non-performing LDC loans has existed since 1986, and the market for distressed corporate bonds has grown in recent years (Altman, 1993). Specialized intermediaries ('vulture funds'), who buy large quantities of a distressed issue, have emerged. By reducing the number of creditors involved in restructuring, vulture funds can facilitate out-of-court debt restructuring.

Related to debt renegotiation is the issue of covenant renegotiation. Berlin and Mester (1992) suggest that private debt usually carries more stringent covenants than public debt because covenant renegotiation is easier when it involves only a small number of creditors. Although we have not studied the issue of an optimal mechanism to renegotiate covenants, our results suggest that if a mechanism could be devised to give heterogeneous creditors different payoffs, then renegotiating covenants with multiple creditors would be easier, and perhaps it would become possible to attach more restrictive covenants to public debt as well.

To focus on the free rider problem among creditors our model has abstracted from several important features of real world debt restructuring. For instance, we have assumed that all debt is homogeneous, while in practice firms issue several classes of debt claims (bank debt, senior bonds, subordinated bonds, commercial paper). Different categories of creditors may also have different information: for instance, banks who have a long-term relationship with the debtor are likely to be better informed than public creditors, and in

fact banks appear to play a special role in debt restructuring. Brown *et al.* (1993) and James (1993) are empirical studies of this subject. Another issue that we have neglected is the possibility of partial liquidation. Distressed firms frequently engage in asset sales (Asquith *et al.*, 1994). Brown *et al.* (1994) present evidence that asset sales by distressed firms reflect pressure from short-term creditors, and that, in contrast with asset sales by healthy firms, they benefit creditors and hurt shareholders. Finally, debt restructuring can also lead to a reallocation of control powers within the firm (Gilson, 1990). In this case, the resolution of financial distress has potential effects on agency problems between management and shareholders, and the going concern value of the firm depends on how control rights are redistributed in the workout. A complete theory of financial distress will need to encompass all these aspects. Empirical studies of debt workouts and bankruptcy, which have grown dramatically in recent years, will certainly provide important insights for the development of such a theory.

Appendix 1

Derivation of equation (8). For the incentive-compatibility constraints to hold, it must be

$$x_i = \operatorname{argmax}_{\hat{x}_i} U(x_i, \hat{x}_i, \omega_i, p).$$

Let $\hat{U}(x_i, x_i, p, \omega)$ be the maximum utility of creditor i from the mechanism (p, ω) . Then, by the envelope theorem

$$\frac{d \hat{U}}{d x_i} = - \int_{I_{n-1}} p(x_i, x_{-i}) f(x_{-i}) dx_{-i} \leq 0.$$

Hence, the equilibrium utility from the mechanism must be decreasing in the creditor type.

Reintegrating this expression,

$$\hat{U}(x_i, x_i, p, \omega) = U(\bar{x}, \bar{x}, \omega, p) + \int_{I_{n-1}} \left[\int_{x_i}^{\bar{x}} \frac{d \hat{U}}{d u} du \right] f(x_{-i}) dx_{-i} =$$

$$U(\bar{x}, \bar{x}, \omega, p) + \int_{I_{n-1}} \int_{x_i}^{\bar{x}} p(u, x_{-i}) du f(x_{-i}) dx_{-i}.$$

Derivation of equation (10). From (3) and (9)

$$(A1) \quad \int_{I_{n-1}} p(x_i, x_{-i}) \omega_i(x_i, x_{-i}) f(x_{-i}) dx_{-i} =$$

$$\int_{I_{n-1}} p(x_i, x_{-i}) l_i(x_i, x_{-i}) f(x_{-i}) dx_{-i} + \int_{I_{n-1}} \int_{x_i}^{\bar{x}} p(u, x_{-i}) du f(x_{-i}) dx_{-i}.$$

The second term on the RHS of this equation is the informational rent earned by creditor i .

Taking expectations over all possible values of x_i :

$$(A2) \quad \int_{I_n} p(x) \omega_i(x_i, x_{-i}) f(x) dx =$$

$$\int_{I_n} p(x_i, x_{-i}) l_i(x_i, x_{-i}) f(x) dx + \int_{I_{n-1}} \left[\int_{\underline{x}}^{\bar{x}} \int_{x_i}^{\bar{x}} p(u, x_{-i}) du f(x_i) dx_i \right] f(x_{-i}) dx_{-i}.$$

Integrating by parts the term in brackets:

$$(A2) \quad \int_{\underline{x}}^{\bar{x}} \int_{x_i}^{\bar{x}} p(u, x_{-i}) du f(x_i) dx_i = \int_{\underline{x}}^{\bar{x}} p(x_i, x_{-i}) F(x_i) dx_i,$$

So the expected informational rent can be written as

$$\int_{I_n} p(x_i, x_{-i}) [F(x_i)/f(x_i)] f(x) dx.$$

Substituting in (A2) yields (10).

Appendix 2

Proof of Proposition 2. If $p(x)$ is incentive-compatible, the debtor's expected profits are given by equation (10). Under pure private values, the revision functions $e_i(x_{-i})$ are identically zero, so it is sufficient to show that

$$\int_{I_n} p(x) \left[\sum_{i=1}^n e_i(x_{-i}) \right] f(x) dx < 0.$$

Using the definitions of $e_i(x_{-i})$ and of $P(x_i)$, this inequality can be rewritten as

$$\sum_{i=1}^n \sum_{j \neq i} \left[\int_{\underline{x}}^{\bar{x}} P(x_j) e_i(x_j) f(x_j) dx_j \right] < 0.$$

Since by definition $\int_{\underline{x}}^{\bar{x}} e_i(x_j) f(x_j) dx_j = 0$ for all i and j , and $e_i(x_j)$ is increasing, there exists an $\hat{x}_i$ such that $e_i(x_j) \leq 0$ as $x_j \leq \hat{x}_i$, and we can write

$$\int_{\underline{x}}^{\hat{x}_i} P(\hat{x}_i) e_i(x_j) f(x_j) dx_j + \int_{\hat{x}_i}^{\bar{x}} P(\hat{x}_i) e_i(x_j) f(x_j) dx_j = 0.$$

$P(x_j)$ is decreasing because of incentive-compatibility, hence

$$\int_{\underline{x}}^{\hat{x}_i} P(x_i) e_i(x_j) f(x_j) dx_j < \int_{\underline{x}}^{\hat{x}_i} P(\hat{x}_i) e_i(x_j) f(x_j) dx_j,$$

and

$$\int_{\hat{x}_i}^{\bar{x}} P(\hat{x}_i) e_i(x_j) f(x_j) dx_j > \int_{\hat{x}_i}^{\bar{x}} P(x_i) e_i(x_j) f(x_j) dx_j.$$

But then

$$\int_{\underline{x}}^{\bar{x}} P(x_j) e_i(x_j) f(x_j) dx_j < P(\hat{x}_i) \int_{\underline{x}}^{\bar{x}} e_i(x_j) f(x_j) dx_j = 0.$$

Proof of Lemma 1. From the definition of $\tilde{x}(x_{-i})$ it follows that the investment rule of the optimal mechanism can be rewritten as

$$p(x) = \begin{cases} 1 & \text{for } x_i \leq \tilde{x}(x_{-i}) \\ 0 & \text{otherwise.} \end{cases}$$

Hence, from equation (7) the change in utility to creditor i from the optimal mechanism is

$$(A3) \quad U(x_i, x_i, \hat{p}, \hat{\omega}) = \int_{I_{n-1}} \int_{x_i}^{\bar{x}} \hat{p}(u, x_{-i}) du f(x_{-i}) dx_{-i} = \\ \int_{I_{n-1}} \max [\tilde{x}(x_{-i}) - x_i, 0] f(x_{-i}) dx_{-i}.$$

From equations (A3) and (3),

$$(A4) \quad \int_{I_{n-1}} \hat{p}(x_i, x_{-i}) \omega_i(x_i, x_{-i}) f(x_{-i}) dx_{-i} = \\ \int_{I_{n-1}} \left[x_i + e_i(x_{-i}) + \max [\tilde{x}(x_{-i}) - x_i, 0] \right] f(x_{-i}) dx_{-i}.$$

Since $\tilde{x}(x_{-i}) - x_i < 0$ when $\hat{p}(x_i, x_{-i}) = 0$, and $\tilde{x}(x_{-i}) - x_i > 0$ when $\hat{p}(x_i, x_{-i}) = 1$, a solution to (A4) is

$$\hat{\omega}_i(x_i, x_{-i}) = \tilde{x}(x_{-i}) + e_i(x_{-i}) \bullet$$

Proof of Proposition 3. Define the set $\Gamma(x_i) \equiv \{ x_{-i} \mid C \geq A(x_i) + Q(x_{-i}) \}$. This set contains all the vectors x_{-i} such that $\hat{p}(x_i, x_{-i}) = 1$. If x_i is large, then the set $\Gamma(x_i)$ may be empty, but to keep the exposition simple this possibility is neglected in the rest of the proof. The proof for the general case is in the working paper version of this article. Let $f(x_{-i} \mid \Gamma(x_i))$ denote the density of x_{-i} conditional on $x_i \in \Gamma(x_i)$. If the exchange offer is expected to succeed with probability $\hat{p}(x)$, then the expected change in utility to creditor i from tendering an amount h_i is

$$U(x_i, h_i, p) = \int_{I_{n-1}} \hat{p}(x_i, x_{-i}) [h_i b + (1 - h_i) d - l_i(x_i, x_{-i})] f(x_{-i}) dx_{-i}.$$

For the exchange offer to yield the same change in expected utility as the revelation mechanism, the amount tendered h_i must satisfy

$$\int_{I_{n-1}} \hat{p}(x_i, x_{-i}) [h_i b + (1 - h_i) d] f(x_{-i}) dx_{-i} = \int_{I_{n-1}} p(x_i, x_{-i}) \hat{\omega}(x_{-i}) f(x_{-i}) dx_{-i},$$

hence

$$(A5) \quad [h_i b + (1 - h_i) d] = \int_{\Gamma(x_i)} \hat{\omega}_i(x_{-i}) f(x_{-i} \mid \Gamma(x_i)) dx_{-i} \equiv W(x_i).$$

Lemma 3. *Under Assumptions 1–3 $W(x_i)$ is strictly increasing.*

Proof. As x_i increases, the set $\Gamma(x_i)$ loses some elements. Specifically, only ‘smaller’ vectors x_{-i} (vectors x_{-i} that yield a small $Q(x_{-i})$) remain in the set. By Assumption 3, the payments $\hat{\omega}_i$ associated with these vectors are larger, so $W(x_i)$ is increasing.

For a creditor i of type x_i to obtain the same expected utility as in the revelation game, in the exchange offer he must tender

$$(A6) \quad H(x_i) = \frac{d - W(x_i)}{d - b}.$$

Since $W(x_i)$ is monotonically increasing by Lemma 3, $H(x_i)$ is monotonically decreasing: creditors with a large expected bankruptcy payoff exchange a smaller fraction of their portfolio. For $H(x_i)$ be feasible, it must be the case that $H(x_i) \in [0, 1]$ for all $x_i \in [\underline{x}, \bar{x}]$. From (A6) this is equivalent to $W(x_i) \in [b, d]$ for all $x_i \in [\underline{x}, \bar{x}]$. Since $W(x_i)$ is increasing, to ensure that $W(x_i) \geq b$ it is sufficient that the face value of the new asset be $b = W(\underline{x})$. To see that $W(x_i) \leq d$, recall that in the optimal revelation mechanism the utility of the highest bankruptcy payoff type is equal to zero, hence

$$\int_{I_{n-1}} p(\bar{x}, x_{-i}) \hat{w}(x_{-i}) f(x_{-i}) dx_{-i} = \int_{I_{n-1}} \hat{p}(\bar{x}, x_{-i}) l_i(\bar{x}, x_{-i}) f(x_{-i}) dx_{-i}.$$

Hence,

$$W(\bar{x}) = \int_{\Gamma(\bar{x})} l_i(\bar{x}, x_{-i}) f(x_{-i} \mid \Gamma(x_i)) dx_{-i}.$$

Since $l_i(\bar{x}, x_{-i}) \leq d$ by Assumption 1, it follows that also $W(\bar{x}) \leq d$. Hence, if $b = W(\underline{x})$, then $H(x_i) \in [0, 1]$.

Since $H(x_i)$ is monotonic, it can be inverted, and for all $x_i \in [\underline{x}, \bar{x}]$ one can write

$$x_i = H^{-1}(x_i) \equiv G(h_i).$$

Consider the following strategy for the debtor: offer a new asset with face value $b = W(\underline{x})$,

and announce that the exchange offer will succeed if and only if the amounts tendered satisfy the following condition:

$$(A7) \quad C \geq \sum_{i=1}^n \left\{ l_i[G(h_i), G(h_{-i})] + \frac{F[G(h_i)]}{f[G(h_i)]} \right\}$$

The announced rule for the success of the exchange offer yields the same probability of investment as the optimal revelation mechanism whenever creditors tender $H(x_i)$. By the incentive-compatibility of the revelation mechanism, it is a best response for creditors with bankruptcy payoff x_i to tender $H(x_i)$. Finally, the debtor's strategy maximizes his expected profit by construction. •

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