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ASYMMETRIES AND NONLINEARITIES IN ECONOMIC ACTIVITY

by Fabio Fornari (*) and Antonio Mele (**)

Abstract

This paper analyses the cyclical component of the industrial productions of three countries. A GARCH framework is employed to model the conditional variances of the cycles which are found to react asymmetrically to shocks of opposite sign; further, they present, in one case out of three, long-memory features. The ability of GARCH models at capturing all the heteroscedasticity of the data is tested against the null of deterministic chaos.

Contents

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1. Introduction[1]

The analysis of the statistical properties of gross domestic product's (GDP) cyclical component has recently received new attention. Many economists, among which Keynes (1936) and Hicks (1950), have initially suggested that cycles are asymmetric, in that their reaction to past positive and negative GDP's rates of change (i.e. contractions or expansions) is different. More recently, on the contrary, Neftci (1984) has provided novel support for the existence of a symmetric behaviour, while Beaudry and Koop (1993) have again evidenced asymmetry. More importantly for our analysis, French and Sichel (1993) employed a recently developed statistical technique, EGARCH (exponential generalised autoregressive heteroscedasticity) and concluded that the conditional variance of the US GDP reacts asymmetrically to positive and negative innovations and reveals quite persistent, indicating that the effects of a shock are prolonged over time.

In this paper the presence of nonlinear dependence and asymmetry is investigated in three industrial production series by means of asymmetric conditionally heteroscedastic models; further, tests are performed to understand whether the whole nonlinear dependence can be successfully captured by autoregressive conditional heteroscedastic models or if this hypothesis has to be rejected against the alternative of

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1 The authors wish to thank Giuseppe Schlitzer (Banca d'Italia), whose comments remarkably improved both structure and content of this paper, as well as Enrico Giovannini (ISTAT) and an anonimous referee for his careful review. As usual, errors remain authors' responsibility.

deterministic chaos, which has been increasingly put forward as an alternative explanation in many recent theoretical contributions.

2. Asymmetry in the business cycle

In two recent papers, Diebold and Rudebusch (1990, 1991) have investigated about the correlation between economic expansions and the successive contractions, finding no evidence of causal links between the length of such phases. Zellner (1990), however, found that a relation existed between the length of an expansion and the successive contraction, or viceversa, based on pre-War data; further, such a relation was found to be negative (i.e. a long expansion leads to a short contraction). Diebold (1993) regressed the length of post-War expansions on the corresponding length of the subsequent contractions; he found the negative relation to hold.

Such findings, however, refer to a concept which we shall call length-asymmetry; another concept, which represents the focus of the paper is the depth-asymmetry, i.e. contractions being more violent than expansions or viceversa. This concept was initially evidenced by Mitchell (1927), who found that the distances between trough to peak and from peak to trough in a business cycle indicator, were not equal. The two concepts, depth and length asymmetry were cast together by Keynes (1936): he argued that economic contractions are more violent but also last for shorter periods than expansions.

An explanation for the presence of asymmetry in economic variables came from Burns and Mitchell (1946). They tried to create a time scale, possibly common to all business cycles, supposing that typical cycles could be divided in nine stages; to generate asymmetries, they hypothesised that the dynamics of such a time measure were different in each of the stages. Their analysis has been recently revived by Stock (1987): he sought to find a nonlinear transformation which allows one to move from economic to calendar time; the latter is then supposed to diffuse according to a different speed, the so-called time deformation.

Many have been the attempts to detect and model the asymmetric features of economic time series, ranging from the use of the skewness (DeLong and Summers, 1986) to the threshold autoregressive process (TAR) of Tiao and Tsay (1991). Aim of the paper is to individuate and model depth-asymmetry (to this concept we will refer with the term asymmetry) by means of conditionally heteroscedastic models; the presence of length-asymmetry will be acknowledged throughout the above mentioned papers.

3. Nonlinearities and asymmetry

3.1 The Information of the variance

An increasing number of papers have been dealing with GARCH (generalised autoregressive conditional heteroscedastic) models since Engle's (1982) seminal paper. GARCH is a simple and appealing scheme to handle nonlinearity; it models the conditional variance of a stationary series, h_t, as a restricted ARMA model. Though this structure has been found to capture most of time series' nonlinear dynamics (see Bollerslev, Chou and Kroner, 1992), nonetheless the assumption of a GARCH structure for the variance should be tested against the alternative of more general forms of nonlinear dependence. Deterministic chaos is another appealing possibility; deterministic models, in fact, are able to generate chaotic dynamics under specific conditions, and such an hypothesis has revealed hardly rejectable in an increasing number of applications involving financial and macroeconomic series.

All the investigations for chaotic dynamics have been based upon a recently developed test called BDS (Brock et al., 1988), distributed as a normal variate under the null of identical independent distribution. It has high power to detect the presence of serial dependence, whose form is supposed to be as general as possible, even deterministic chaos, and is based on the so-called correlation dimension integral; intuitively, the latter tests whether data can be cast on a much lower dimension space than that in which they are spanned. When applied to the rates of change of the industrial production indices filtered with a moving average process of the first order (so to eliminate serial correlation), this test rejects the null of i.i.d. distribution, thus enabling one to consider the presence of a GARCH structure in the data.

As already mentioned, French and Sichel (1993) employed the Exponential GARCH model of Nelson (1991) to estimate the variance of the US GDP; they had three main results, namely:

- the conditional variance of real GNP increases after negative shocks but not after positive ones;
- asymmetry is present mainly in the cyclically sensitive sectors of the economy;
- shocks to the conditional variance of GDP persist for a long time period.

We will further investigate about the first and the third finding, focusing later on the plausibility of the GARCH assumption. The model employed in this paper (SCGARCH) is a generalisation of the GARCH introduced by Bollerslev (1986) and is simpler to estimate than the EGARCH, needing just a minor modification to the standard GARCH likelihood. A GARCH structure assumes that the conditional variance h_t of a stationary series (e_t) evolves through time in a fashion resembling a restricted ARMA model; for the simplest model, the GARCH(1,1), Bollerslev proposed the following structure, in a regression context:

$$r_t = a + b \cdot x_t + e_t, \qquad \text{with } e_t | F_{t-1} \sim N(0, h_t) \tag{1}$$

$$h_t = a_0 + a_1 \cdot e^2{}_{t-1} + b_1 \cdot h_{t-1} \tag{2}$$

with r_t and x_t being respectively the endogenous and k exogenous variables, $a_0 > 0$, a_1 and $b_1 \geq 0$, and F_{t-1} the information set dated t-1.

The sign-conditional GARCH model (SCGARCH) developed by Fornari and Mele (1994) is obtained when equation (2) is replaced with:

(3) $$h_t = a_0 + a_1 \cdot e^2{}_{t-1} + b_1 \cdot h_{t-1} + \delta \cdot s_{t-1}$$

where s_{t-1} is a dummy variable, equal to one in the case of a positive variation of the previous level of the index (I_t) and minus one in the opposite case; the remaining parameters have the same restrictions as in (2) and, further, $|a_0|>|\delta|$. Model (3) allows the variance to react more strongly to past negative values of the lagged first difference of the industrial production index (than to positive ones), as far as $\delta<0$. Further, instead of a structural specification for I_t, as implicit in (1), we made use of a time series approach, being more interested in the specification of the conditional variance, rather than the conditional mean. Thus, we replaced (1) with (1'), where the logarithmic rate of change of I_t, r_t, is modeled as a moving average process of the first order, i.e.

(1') $$r_t = \phi \cdot e_{t-1} + e_t$$

Before estimating the SCGARCH model, it is possible to have an idea about the strength of the asymmetry by running three types of tests recently developed by Engle and Ng (1993). Suppose to estimate a GARCH(p,q) model for a stationary series, r_t, and to calculate the standardised residuals, z_t, as

$$z_t = r_t/h_t^{0.5}$$

where h_t represents the conditional variance. If the latter is influenced in a different way by past shocks of different sign, a test for omitted variables should reveal a lack of fit. What Engle and Ng propose is to generate three series: the first one z_1, is a dummy variable which equals one when the previous change of the industrial production index is negative, and is zero in the remaining cases. The second variable, z_2, is the product of z_1 times the first difference of the (filtered) index ($z_t \cdot r_t$); the third, z_3, is obtained as the product of ($1-z_1$) by r_t.

The test is obtained as the Student's-t of the coefficients which multiply, respectively, z_1, z_2 and z_3. Should the first be significant, there would be evidence of a different impact of negative errors on the conditional variance. When the second coefficient is significant, negative errors have a greater impact on the variance; the opposite holds when the third test is significant.

The test can also be carried out preliminarly, without estimating the GARCH model; in this case it represents a test for asymmetry in variance (rather than a test for the adequacy of a symmetric GARCH model) and is run by first standardising the filtered first differences of the original series with their sample standard deviation, and then squaring the series thus obtained; the latter is regressed on the one period lagged values of z_1, z_2 and z_3.

Before coming to the analysis of the results, it is worth noting that, unlike French and Sichel (1993, henceforth FS), our scheme, reported in (1') - (3), provides us with a direct tool to test whether variances are higher during contractions than during expansions. In fact, the variable which regulates the asymmetry in our conditional variance specification is not the sign of the previous forecast error of the mean equation (1') (in which the first difference of the industrial production index is modeled as a moving average process of the first order) but the sign of the previous first difference of the production index itself. In the former case, in fact, the sign could be positive or negative irrespective of the specific phase of the business cycle; temporary falls (rises) of the index occur in periods of expansion (recession) as well; in our setup, on the contrary, the sign of the dummy variable is expected to be mostly negative in recessions and positive in expansions. Thus, based on δ being negative, we can conclude that variances are higher in the negative phase of the cycle; however, we will also make use of a further practical test to provide more evidence for our conclusions.

3.2. Data and results

The series that we employed to test for the presence of asymmetric responses to cycles' expansions or contractions are seasonally adjusted industrial productions for the United States, Great Britain and Italy, observed monthly from January 1957 to June 1993, obtained from the International Monetary Fund Financial Statistics. We decided to work on industrial production, rather than on GDP, for two main reasons: first, in order to achieve efficiency in the estimation phase we needed observations at a higher frequency than the quarterly one at which GDP is typically available; second, French and Sichel (1993) show that asymmetry is present only in the strongly cyclical sectors (i.e. industry). Thus GDP would not be a candidate to evidence asymmetry, since the share of services in it is by far larger than industry's.

Each of the three industrial production series is integrated of order one, as evidenced by a Dickey and Fuller's (1979) (DF) test, thus differencing will produce stationary series (r_t), made up of cycle and noise (seasonality had been already eliminated). Before estimating the SCGARCH models, the presence of conditional heteroscedasticity in the cycles has been ascertained with a more specific test than the BDS. Specifically, we used Engle's (1982) TR^2, based on the regression of the squared filtered[2] logarithmic rates of change of the industrial production indices, $r_{i,t}$ (with $i = 1, 2, 3$), on their p previous values, which is distributed according to a chi-square with p degrees of freedom under the null of homoscedastic conditional variance; its sample values, evaluated up to the fifth lag were 114.7, 117.3 and 111.7 for Italy, the UK and the US respectively, highly significant at any level of confidence, which support the presence of time-varying second moments.

To evaluate the presence of asymmetry in variance, the tests proposed by Engle and Ng (1993) have been applied, preliminarly, to the three filtered rates of change of the

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2 With a moving average process of the first order.

indices, standardised with their sample standard deviation (results are commented but not reported). As far as Italy is concerned, there is evidence of positive and negative changes of the industrial production index having different impact of the level of volatility; the tests based on z_2 and z_3 would support a greater effect of positive changes, though such indications build on a standardisation (the unconditional standard deviation) which is not the most efficient. With concern for the UK and the US the tests point at a greater effect of negative changes of the indices on the level of the conditional volatility.

The estimated coefficients of the three SCGARCH models are reported in Table 1, together with their Student's-t values, degree of persistence in variance and model's logarithm of the likelihood function.

Table 1

SCGARCH MODELS*

	r	a_0	a_1	b_1	δ	Pers.	Log of Likel.
US	-0.434	1.58E-5	0.106	0.753	-8.93E-6	0.859	1843.2
t-Stud.	-9.53	9.52	7.20	27.54	-3.74		
UK	0.111	1.23-4	0.373	0.163	-3.81E-5	0.536	1673.5
t-Stud.	3.78	10.31	12.21	2.83	-4.12		
IT	0.200	4.43E-4	0.306	0.02	-1.55E-4	0.335	1434.7
t-Stud.	3.71	16.83	6.91	1.85	-5.84		

* Persist. refers to the degree of persistence of the conditional variances, as measured by the sum $a_1 + b_1$; Log of Likel. indicates the logarithm of the likelihood function.

Asymmetry appears to be a common feature of the series, since the parameter δ is always negative and significant. The asymmetric reaction function of the conditional variance to

Fig. 1

RESPONSE FUNCTION OF ITALIAN BUSINESS CYCLE'S VARIANCE TO SHOCKS OF OPPOSITE SIGN

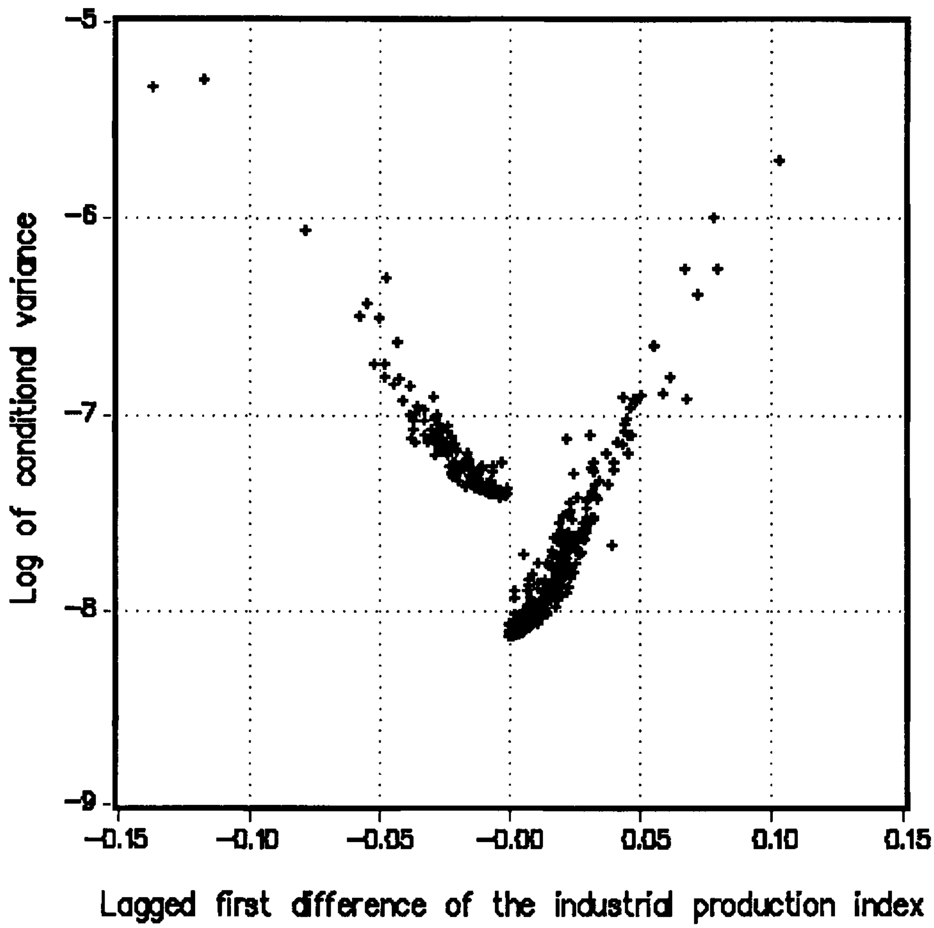
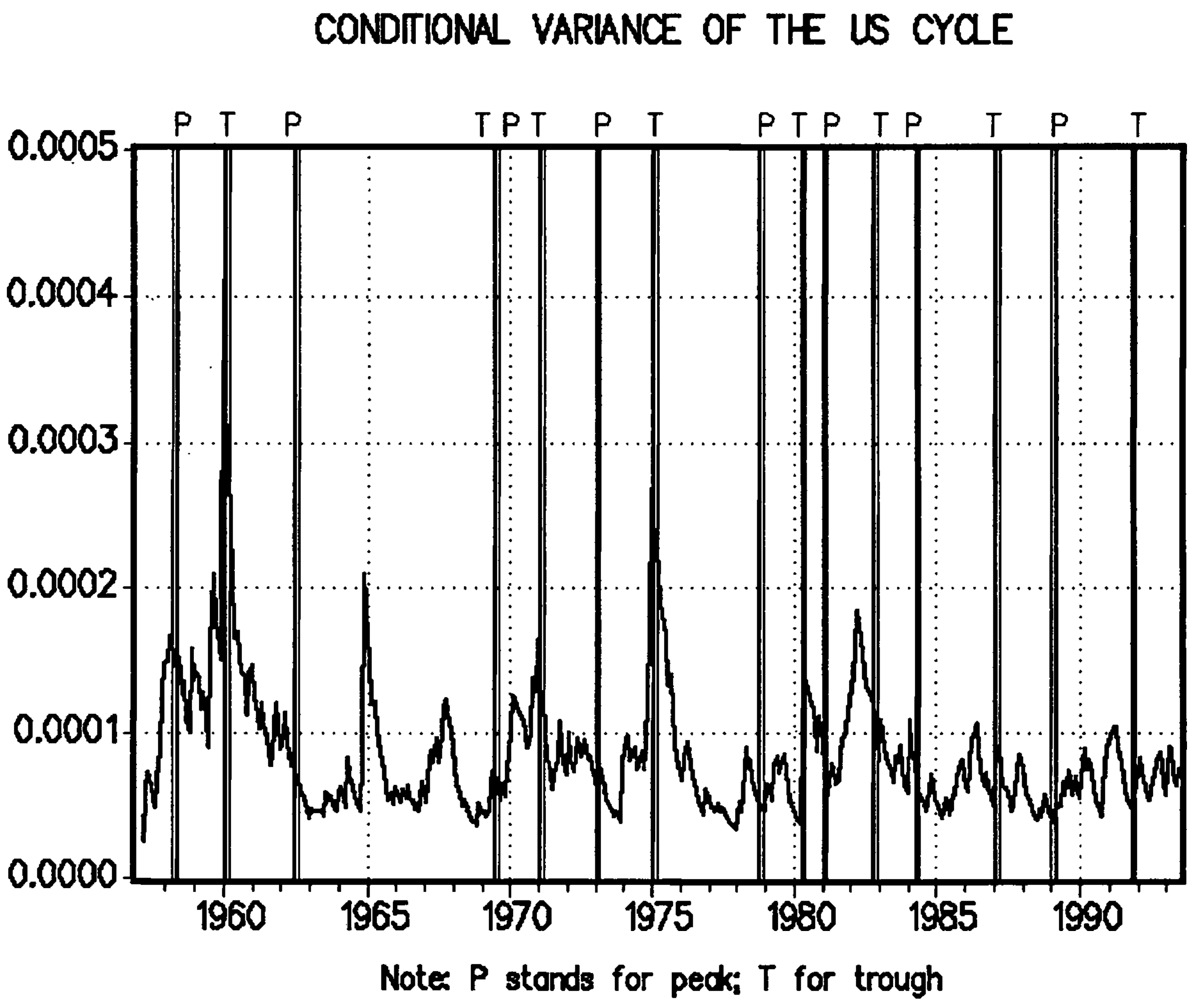


Fig. 2


CONDITIONAL VARIANCE OF THE US CYCLE
P T P T P T P T P T P T P T P T
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
1960 1965 1970 1975 1980 1985 1990
Note: P stands for peak; T for trough

previous innovations is reported in Figure 1 with concern for Italy, while Figure 2 shows the estimated conditional volatility of the US industrial production, together with the dates at which peaks and troughs have occurred, according to the NBER classification of business cycles; as clearly evidenced, highest variances occur near the troughs.

However, in order, to strengthen out results we have employed a further test, as illustrated by FS. It is based on the regression of the conditional variance, h_t, on a constant and a variable, called maxgap, which equals zero if the current industrial production index (I_t) is above the preceding value and $(\log(I_{max}) - \log(I_t))$ if the current index is below the previous highest value (I_{max}); a positive coefficient for maxgap would signal highest variances at troughs. Thus, we ran the following regression

$$h_t = \alpha + \beta \cdot maxgap_t + \varepsilon_t$$

for each of the three countries, expecting β to be positive in the case of highest conditional variances at troughs.

With concern for the US and the UK β equalled 0.001 and 2.38E-4 respectively, statistically significant at any level of confidence; for Italy it was 0.0051 with a Student's-t exceeding 5, thus strengthening the conclusions derived from the analysis of δ, based on the SCGARCH estimation.

Our methodology, which makes use of the sign of the last change in the production index as a conditioning variable for h_t, instead of the sign of the last forecast error made for r_{t-1} (i.e. e_{t-1} in (1)) gives direct information about the behaviour of the conditional variance in periods of expansion and recession. However, we lack information about the existence of an overall tendency of h_t to react sharper to negative forecast errors (e_t) than to positive ones. To examine such an opportunity we estimated the following GARCH(1,1) model, incorporating both signs, i.e.

$$r_t = \phi \cdot e_{t-1} + e_t$$
$$e_t \mid F_{t-1} \sim N(0,h_t)$$
$$u_t = e_t \;/\; |e_t|$$
$$s_t = r_t \;/\; |r_t|$$
$$h_t = \alpha_0 + \alpha_1 \cdot e^2{}_{t-1} + \beta_1 \cdot h_{t-1} + \delta \cdot s_{t-1} + \gamma \cdot u_{t-1}$$

where s_t and u_t record the sign of r_t and e_t, respectively. In the phase of estimation u_t has been modified, by setting it at zero when both u_{t-1} and s_{t-1} equalled minus one; this has been done in order to let the main role be played by the sign of the industrial production index change, consistently with the focus of our paper. In all the three cases γ was found to be negative but not significant, with Student's-t ranging from -1.23 (US) to -0.72 (UK); thus, no evidence has been found of a general tendency of the conditional variances to overreact to negative shocks, in addition to the finding of such a behaviour following drops of the economic activity.

As far as the reaction of the conditional variance to the available information set is concerned, an interesting feature regards the values of the persistence, defined as a_1+b_1 in (3); when this sum equals one the conditional variance is generated by a random walk process, so that shocks affect variances for any future time interval (Bollerslev and Engle, 1986). As regards the three industrial productions, there is no evidence of integration in variance, even if high persistence is observed in the case of Italy (where the effects of a shock last about a year) and to a lesser extent for the US (where the same effects last five months; Figure 2); this feature drops further in the case of the UK (Table 1). The finding of absence of integration in variance implies that the effect of a shocks is eventually absorbed.

The diagnostic checking of the estimated models has been carried out mainly by comparing the coefficients of <u>kurtosis</u> of the original series, standardised with their sample standard deviation, in one case, and with the SCGARCH conditional standard deviation, in another. Further, a Pagan and Sabau's (1988) (PS) test has been performed under the null of GARCH conditional variance accounting for all data's heteroscedasticity. As far as the former control is concerned, the <u>kurtoses</u> fell from 7.27, 9.97 and 6.63 to 5.00, 5.11 and 5.00 for Italy, the UK and the US respectively. Such values are higher than 3, the standard normal's <u>kurtosis</u>, revealing the limited ability of GARCH models to capture the heteroscedastic features of the data. This feature is not as clearly evidenced by the second test (PS), based on the regression of the squared industrial production's logarithmic rates of changes on a constant and the conditional variance. Under the null of GARCH capturing most of data's heteroscedasticity, the constant and the slope are expected to be zero and one respectively. In the three cases the constants did not deviate significantly from nil while the slopes were 1.22, 1.15 and 1.43 respectively for Italy, the UK and the US; however the null that they equal unity cannot be rejected in the three cases, as the corresponding tests (roughly normally distributed) were 1.48, 1.08 and 1.9 respectively. Thus, apart for the U.S. where the test is at the limit of the rejection area, there are no strong evidences of lack of fit; nonetheless, we will try to further improve the current specification in the following section.[3]

Finally, an additional test has been carried out in order to evidence whether the asimmetry found in the conditional variance could originate from neglected asymmetries in the mean equation. In order to include such a consideration we estimated again the mean equation (1') by means of a sort of threshold model, i.e.

$$r_t = \phi \cdot e_{t-1} + \beta \cdot s_{t-1} + e_t$$

where s_{t-1} is the same dummy variable as in the SCGARCH specification (3); if its coefficient (β) is negative, the first differences of the (logarithmic) industrial production index will be higher following negative values than after positive values of the same size. However, β turned out to be significant for none out of the three cases, thus revealing

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3 Apart from the occurrence of misspecification of the model, which will be addressed in the next Section, the slight lack of fit for the US could also originate from a wrong distributional assumption. In many applications (though concerning high frequency data) it is common to model the variance of the residuals of (1') as being conditionally Student's-t or G.E. (General Error) distributed, rather than normal.

no interaction between the first two moments of the conditional distributions of the logarithmic rates of change of the industrial production, in terms of asymmetric behaviour.

4. Long memory and deterministic chaos

4.1 Long memory

To test whether the SCGARCH models can be additionally improved, so to capture more of the nonlinearity of the data, two further investigations have to be carried out. First, we have to make sure that the mapping between cycle's values and their conditional variance is appropriate, i.e. that the power at which the industrial productions' rates of change are raised, so to maximise their autocorrelation function, equals two (which is the implicit assumption in GARCH models); further, standardising the (filtered) cycles with their conditional standard deviations and testing whether the series thus constructed are independently and identically distributed, *via* the BDS, provides an important tool to evidence different sources of nonlinearity.

As far as the first issue is concerned, fractionally differenced or long-memory series have been a widely analysed topic in the traditional ARIMA modeling; let us simply recall here that fractional integration implies that a series x_t has to be differenced d times, with d a non integer scalar, to achieve stationarity, i.e. $(1-L)^d \cdot x_t = e_t$, where e_t is white noise. Such a feature has been recently discovered also in conditional variances and various attempts have been made to model it (see for example Ding, Engle and Granger, 1993; Bollerslev and Mikkelsen, 1993; Harvey, 1993). We will make use of the Power ARCH (P-ARCH) model developed in the first of the three above mentioned papers, according to which the standard deviations of the cycles should be raised to a power, say d, to be estimated from the data, in order to maximise their autocorrelation function. The structure of the Power ARCH(1,1) model is:

(4) $$r_t = e_t + r \cdot e_{t-1}, \text{ with } e_t | F_{t-1} \sim N(0,h_t)$$

(5) $$\sigma_t^d = a_0 + a_1 \cdot (|e_{t-1}| - c \cdot e_{t-1})^d + b_1 \cdot \sigma^d{}_{t-1}$$

with $a_0 > 0$, $a_1 \geq 0$, $b_1 \geq 0$, $-1 \leq c \leq 1$, $d > 0$ and $\sigma_t = h_t^{0.5}$.

To bring about economic justification for the empirical observation of long memory in the conditional variance, one can recall the explanation for long memory in the first moment of the industrial production given by Haubrick and Lo (1989). In their framework, this feature is derived by aggregating the generating processes of the single industrial productions, independently defined for each of N production activities. We briefly outline here their model in order to draw the lines along which long memory can be supposed to exist also in the second moments of the rates of change of the industrial production.

Agents are assumed to maximise a utility function of the form:

(6) $$U(c_t) = C'_t \cdot I - 0.5 \cdot C'_t \cdot B \cdot C_t$$

where C_t expresses consumption of each of the N goods in the economy at time t, I is a vector of ones and B is a diagonal matrix (with b_i as elements). Budget constraints are defined as:

$$C_t + S_t \cdot I = Y_t$$

with S an N·N matrix denoting the quantity of good j invested in process i at time t. Output is determined according to

$$Y_t = A \cdot S_{t-1} + e_t$$

with A a matrix of input-output parameters a_{ij}, which is also assumed to be symmetric (denote $a_{ii} = a_i$). The solution to (6) obtained *via* dynamic programming can be shown to be:

(7) $$Y_{i,t+1} = c_i \cdot Y_{i,t} + K_i + e_{i,t+1}$$

with c_i function of both a_i and b_i.

According to this scheme, output follows an AR(1) process so that higher output today will bring about higher output in the future.[4]

At this point, the one-step ahead forecast errors of (1'), i.e. $e_{i,t+1}$ should be autocorrelated when raised to the

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4 Haubrick and Lo make then use of Granger's (1988) aggregation results to show how the sum of N independent AR(1) processes yields an ARMA(N,N-1) (they also derive a test for long memory, based on the Hurst's (1951) Range to Standard Deviation Ratio Statistics). Of course, when N is large enough, such a process can be well approximated by a fractional ARIMA model.

second power, owing to the nature of the generating process of Y_t. In fact, as noted earlier, high output values are followed by high output values, hence producing the well-known clustering of observations (which had already been noted by Mandelbrot, 1963, with concern for financial variables), which is typically captured by GARCH models. Thus, variances should be autocorrelated and an ARMA structure can be supposed to reasonably approximate their evolution over time. As shown in Pantula (1986) ARMA and GARCH models are in a direct relation, so that the aggregation of GARCH(1,1) models will yield a GARCH(p,q) with high values of p and q; in analogy to what observed for the first moments, the latter could be summarised by a fractionally integrated GARCH. Of course such an explanation rests on the feasibility of aggregating sectoral variances obtaining analogous results to those yielded by the aggregation of the sectoral indices.

To analyse this issue from an empirical point of view, we first estimated the autocorrelation functions of the absolute values of the logarithmic rates of change of the industrial production indices raised to the following set of powers: 0.5, 0.8, 1, 1.2, 1.5 and 2 (with concern for Italy they are shown in Figure 3). None of the powers which were employed gave more information than the second, though additional spikes existed for some of them. Power ARCH (P-ARCH) models were anyway estimated to determine whether d could significantly diverge from two, and the coefficients are reported in Table 2; note that P-ARCH models are asymmetric since the conditional variance explicitly depends upon the lagged values of the cycle. Figure 4 reports the asymmetric response function of the Italian variance, which looks sharper than the corresponding SCGARCH-based function of Figure 1.

Table 2

POWER ARCH MODELS*

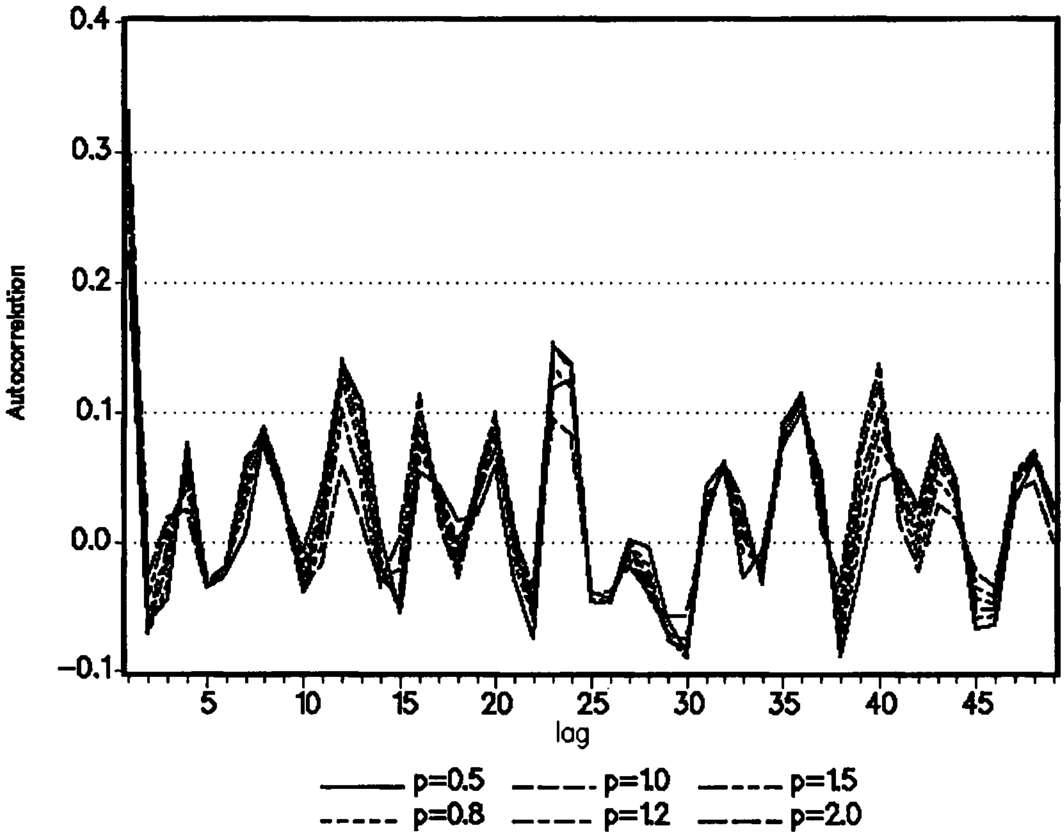
	r	a_0	a_1	b_1	c	d	Pers.	Log of Likel.
US	-0.377	3.17E-6	0.0026	0.562	0.512	1.13	0.565	1851.8
t-Stud.	7.09	14.57	22.95	55.19	3.32	123.6		
UK	-0.184	4.75E-11	3.3E-4	0.468	0.292	1.89	0.469	1608.8
t-Stud.	4.08	15.72	38.32	19.92	4.42	557.7		
IT	0.257	0.00311	0.018	0.67	1.000	0.636	0.164	1424.0
t-Stud.	8.71	62.85	62.85	45.89	Restr.	11.75		

* Persist. refers to the degree of persistence of the conditional variances, as measured by the sum $a_1 + b_1$; Log of Likel. indicates the logarithm of the likelihood function.

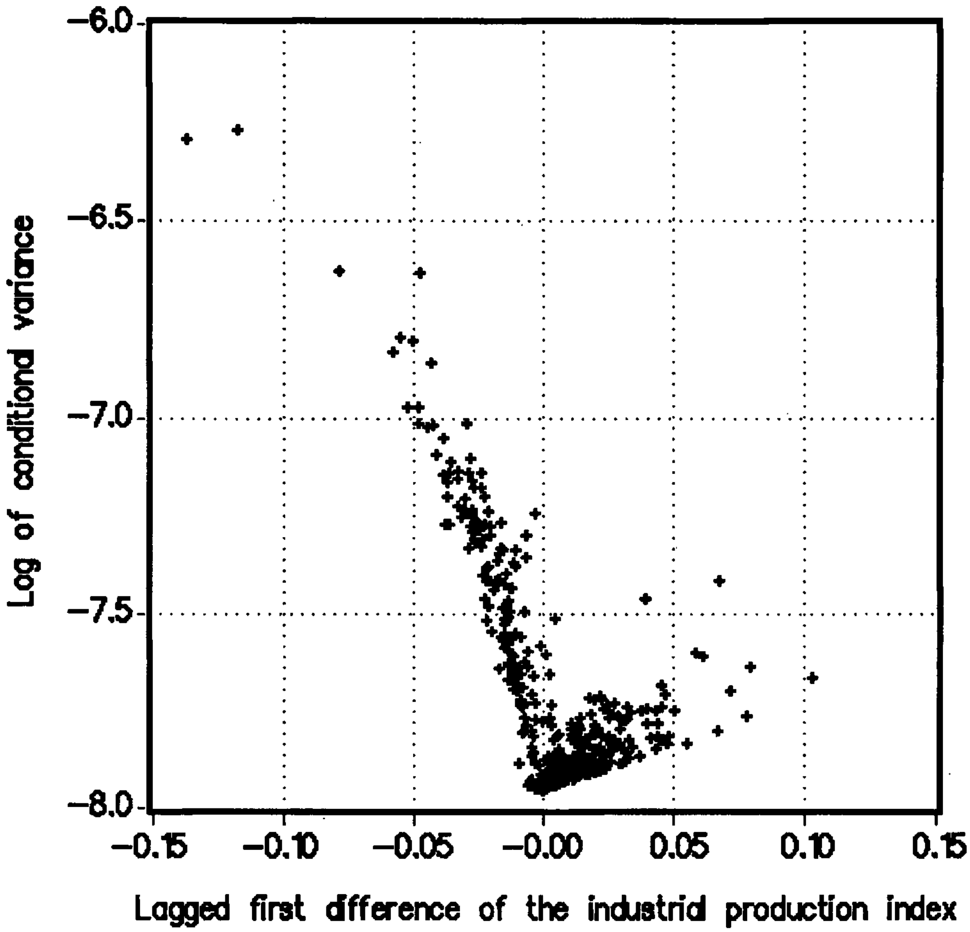
The logarithm of the likelihood of the P-ARCH models is lower than the corresponding value of the SCGARCH for the UK and Italy (a likelihood ratio test would reject the null of better fit of the Power ARCH at any level of confidence), but significantly higher for the United States. It is interesting to examine the values of the power transformations which maximise the likelihood (and the cycle's autocorrelation) function in the three cases: 0.64, 1.90 and 1.13 for Italy, the UK and the US respectively. Thus the squared mapping assumption between cycle and conditional variance would be plausible only for the UK; for the remaining two countries, an asymmetric GARCH model for absolute cyclical values would be preferable; however, the likelihood of the model for Italy does not support this choice. Also, though the <u>kurtosis</u> of the US cycles standardised with the Power ARCH standard deviation is lower

Fig. 3

AUTOCORRELATION FUNCTION OF THE ITALIAN CYCLE FOR VARIOUS POWER TRANSFORMATIONS



POWER ARCH BASED RESPONSE FUNCTION OF ITALIAN CYCLE'S VARIANCE



than the <u>kurtosis</u> obtained after standardising with the SCGARCH standard deviation, this is not true for Italy and the UK.

4.2 Deterministic chaos

Coming now to the second issue, should the cyclical values standardised with the SCGARCH standard deviation depart from an i.i.d. distribution, the alternative of deterministic chaos might be considered. As will be showed shortly, the GARCH model can reveal a useful tool to test this assumption.

As expected, the hypothesis of i.i.d. distribution for the cycles standardised with the SCGARCH standard deviation is (again) rejected by the BDS test.[5] This implies the

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5 Results are not reported, but are available on request from the authors. As a referee pointed out, the hints of misspecification for the GARCH(1,1) coming from the residuals' excess kurtoses and, with concern for the US, from the Pagan and Sabau's test could bias the standardisation upon which the results of the BDS are based. On this respect, we tried different specifications for the conditional variance, though none of them significantly improved, in terms of likelihood, over the simpler SCGARCH(1,1). Further, as Nelson and Foster (1994) have shown, GARCH models are consistent estimators of the true underlying model for the conditional variance and perform better than misspecified structural models. Though such considerations hold as far as the sampling frequency drops to zero, they nonetheless evidence that the estimated conditional variance is the best (linear) forecast that we can make and that its bias (i.e. the difference over the true, but unknown, conditional variance) vanishes asymptotically, and at an exponentially growing deterministic rate.

existence of a small embedding dimension which contains the data and reveals that conditional heteroscedasticity is not the only source of nonlinearity. Nonetheless, in the situation at hand, GARCH models represent a useful tool to test for deterministic chaos. It is worth recalling here that the existence of chaotic dynamics is not a new concept; they were already known to Poincaré at the beginning of the century and extensively applied to physics by Lorenz (1963); the most surprising features of even small systems of nonlinear equations is to generate dynamics which, though deterministic, look completely random, proving a simpler alternative to stochastic linear models.

To understand the usefulness of GARCH models in detecting chaotic dynamics, we will largely draw from Grandmont (1985), who developed a deterministic model able to generate chaotic fluctuations, and Reichlin (1986). For sake of simplicity, we will here only roughly report the main characteristics of their models.

The formal setup is based on an overlapping generation model, composed by two representative agents, young and old, who live for two periods, have perfect foresight about future prices and quantities and maximise an additive utility function of two arguments, consumption and leisure (the difference between labour endowment and labour in each of the two periods of their life). Utility functions can be of two different forms: constant relative risk aversion (CRRA) and constant absolute risk aversion (CARA); of course, risk-aversion has quite a strange meaning in a perfect foresight model, but has been nonetheless used in order to indicate different types of utility functions. It can be shown that chaos may arise because of time-variations in the curvature of agents' indirect utility function; this is produced by young agents' uncertainty about the price level prevailing in period two (although everything in the model is deterministic) which has uncertain final outcome, since both income and substitution effects work simultaneously and in opposite directions. Chaotic dynamics evolve around two attractors; one of them (the axes' origin (0,0)) is unstable while the other attracts the system when the parameters which define the utility function lie within a given range. However, what we are most willing to recall is that the curvature of agents' utility function (which induces departures from the attractor) is the Arrow-Pratt absolute risk aversion coefficient, i.e. $-(V''(a_t))/V'(a_t) = k$, where V' and V'' indicate the first and second derivative of the indirect utility function, respectively. This framework naturally nests in Engle, Lilien and Robins' (1987) GARCH in mean model; the latter allows for feedback effects between conditional mean and variance of a series and, to our aims, can be cast as follows (note also that the conditional variance as of time t is perfectly predictable at time t-1, so that the only uncertainty comes from future returns, i.e. future prices):

$$r_t = e_t + r \cdot e_{t-1} + \tau \cdot h_t, \text{ with } e_t|F_{t-1} \sim N(0,h_t)$$
$$h_t = a_0 + a_1 \cdot e^2{}_{t-1} + b_1 \cdot h_{t-1} + \delta \cdot s_{t-1}$$

In this model, τ has the same meaning of the Arrow-Pratt risk aversion coefficient, measuring the curvature of the utility function, which depends upon the conditional mean and standard deviation (the link between the two formulations can be established by means of a consumption CAPM assumption). Of course we are here assuming the existence of a representative economic agent who holds all the shares of the only existing firm, whose activity accounts for all the industrial production of the economy (however, though far from reality, such assumptions are quite common in theoretical macroeconomic formulations). Thus, if the model is estimated recursively and if τ is reasonably constant, a GARCH structure would be the main source of nonlinearity. In this case τ would not vary through time due to the high correlation between r_t and h_t. On the contrary, if τ is found to be time-varying, then chaotic effects are likely to occur on the ground of Grandmont's analysis.

Table 3 reports the estimated coefficient of the SCGARCH in mean models. The parameter τ, which represents the degree of risk aversion is always significant and negative and supports the presence of a relationship between conditional first and second moments of the cycle series.

However, concern of the present analysis is not just the overall significance of the relation but, rather, its stability, given that changes in the curvature of the utility function generate chaotic effects. Thus, we estimated the SCGARCH in mean models recursively, increasing the samples by 24 months per time, then focusing on the trend of τ in the three series, which is reported in Figures 5, 6 and 7 for Italy, the UK and the US. At the same time we also estimated τ on rolling samples, on a fixed window of 24 months, thus controlling the coherence of the results, which are again reported in Figures 5, 6 and 7. Constancy over time is clearly not supportable when such figures are inspected; as far as Italy is concerned, the parameter based on the recursive estimation ranges between -20 and -0.1 (between -52 and 10 under the rolling estimation), with even wider ranges for the remaining countries.[6]

Table 3

SCGARCH IN MEAN MODELS*

	r	a_0	a_1	b_1	δ	τ	Pers.	Log of Likel.
US	-0.369	1.40E-5	0.098	0.765	-7.1E-6	-32.11	0.863	1851.9
t-Stud.	-6.45	10.75	7.48	33.07	-3.43	-4.25		
UK	0.15	1.86E-4	0.368	0.211	-2.9E-6	-7.69	0.579	1641.9
t-Stud.	2.68	10.95	11.68	4.15	-3.22	-2.80		
IT	0.28	3.48E-4	0.229	0.172	-1.1E-4	-5.99	0.401	1441.5
t-Stud.	4.62	18.62	6.14	4.55	-4.48	-4.08		

* Persist. refers to the degree of persistence of the conditional variances, as measured by the sum $a_1 + b_1$; Log of Likel. indicates the logarithm of the likelihood function.

Such an evidence indicates that the amount of nonlinearity which remains even after the data have been standardised with the GARCH standard deviation may be attributed to the presence of chaotic effects, originating from a switching curvature (or trade off between income and substitution effect) of economic agents' utility function.

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6 Though results will not be reported, it has to be noted that (apart from the observed time variability of τ) a test of equality between couples of point estimates of τ leads, in most cases, to reject such an hypothesis against the alternative of significant difference.

Fig. 5

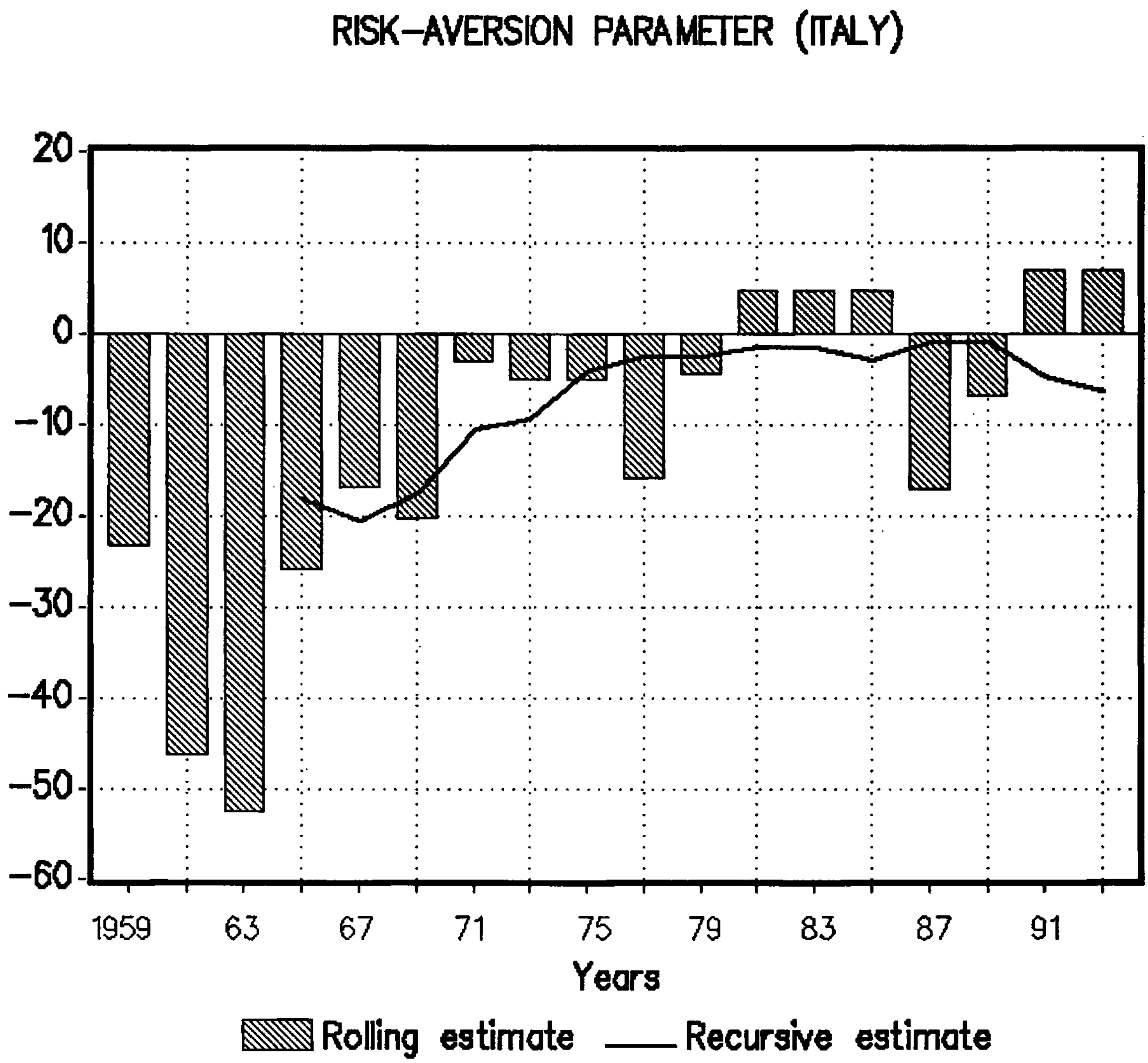
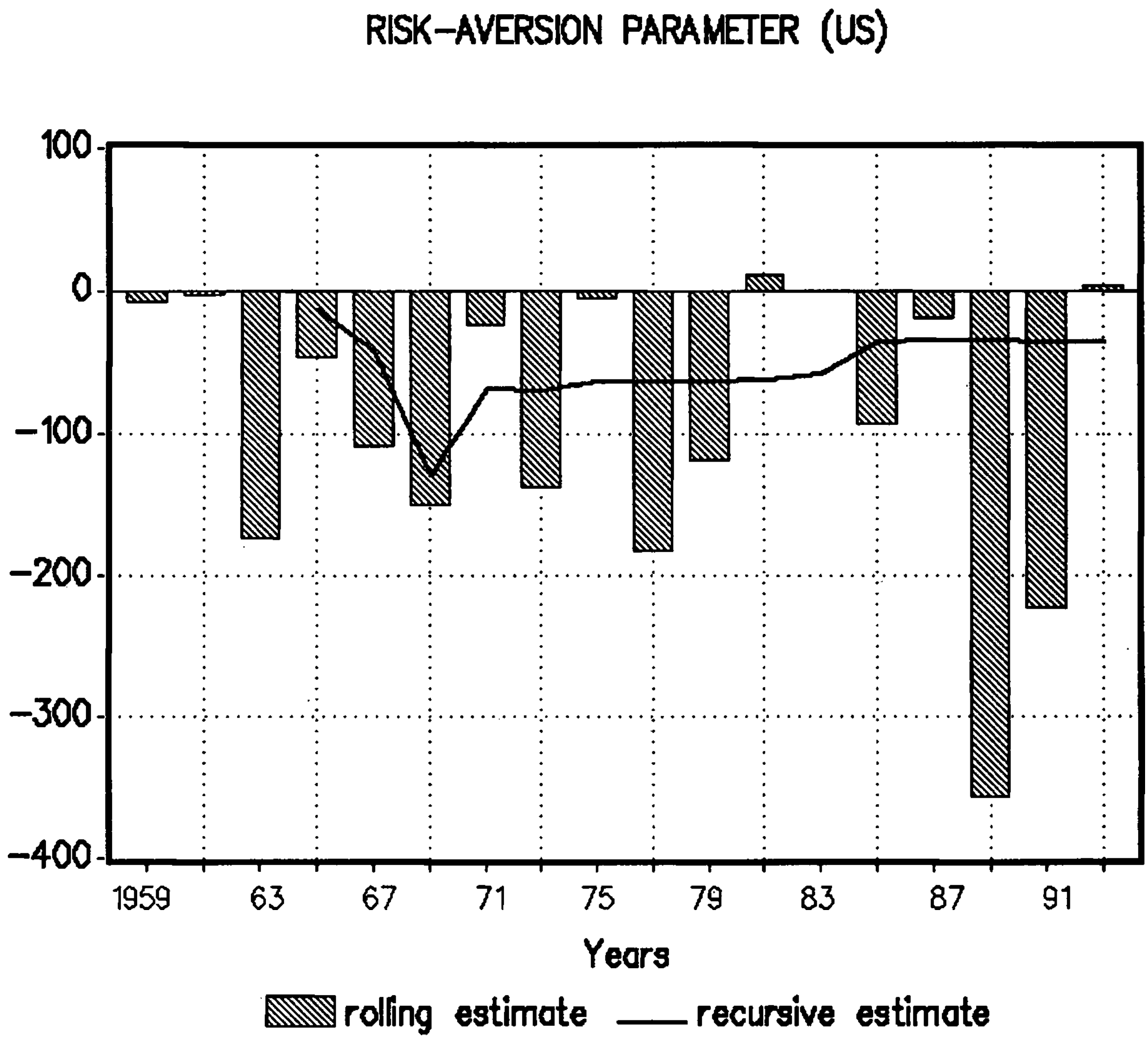
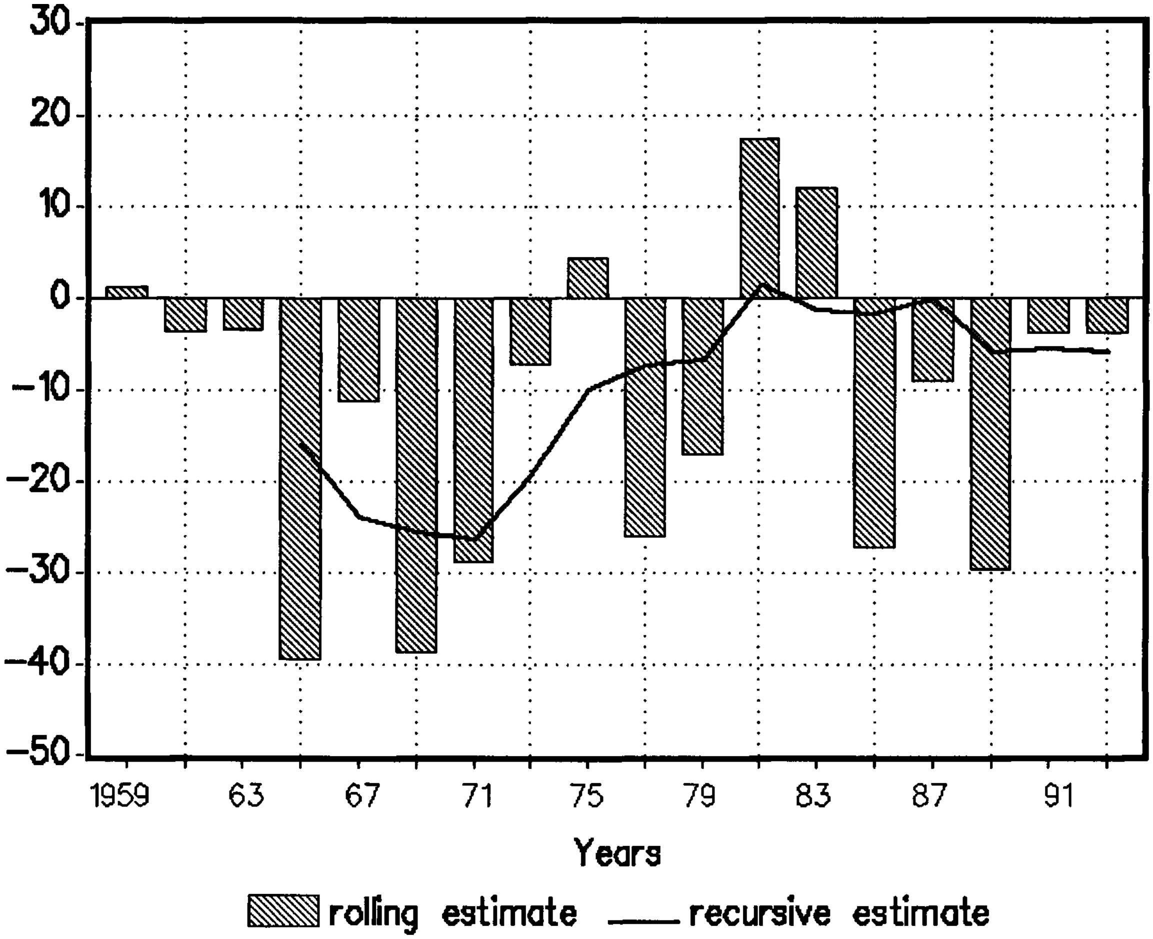

RISK-AVERSION PARAMETER (ITALY)
20
10
0
-10
-20
-30
-40
-50
-60
1959
63
67
71
75
79
83
87
91
Years
Rolling estimate
Recursive estimate

Fig. 6



RISK-AVERSION PARAMETER (US)
100
0
-100
-200
-300
-400
1959
63
67
71
75
79
83
87
91
Years
rolling estimate
recursive estimate

RISK-AVERSION PARAMETER (UK)



5. Conclusions

This paper has produced novel international evidence regarding the asymmetry of real economic variables to contractions or expansions.

Asymmetric behaviour of monthly conditional variances of the logarithmic rates of change of the industrial production indices (the cycles) has been found for the US, the UK and Italy, in a period ranging from January 1957 to June 1993. The extent to which shocks to the three economies persist over time is significantly different across countries; however, shocks eventually fade away. Though evidence of variances with long memory features has been found for the three series, employing a model which makes use of a fractional power, the likelihood of the latter is above the corresponding value obtained with standard GARCH models (i.e. assuming a squared mapping between cycle values and conditional variance) only for one series (the US) out of three.

The SCGARCH models are not able to capture all the nonlinearity of the data. Since a significant and negative relation exists between conditional means and variances, whose instability is the main source of chaotic dynamics according to a business cycle model developed by Grandmont, the SCGARCH in mean model was employed to test for this occurrence. The estimates of the risk-aversion coefficients for the three countries are found to vary considerably when the models are estimated recursively and on rolling samples, pointing at the presence of possible chaotic effects, which had already been signalled by the BDS test.

It is our opinion that the conclusions of previous studies, mainly focused on the US economy, can be extended also to the real economy of other industrialised countries. Nonetheless, the results in this paper concerning long-memory features and chaotic behaviour suggest further investigation on other macroeconomic variables, closely related to industrial production. Analogous results would be hint of a transmission of such phenomena throughout the economy, and not just a particular (and independent) feature of a single set of series.

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