BANCA D'ITALIA

Temi di discussione

del Servizio Studi

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DEBT STABILIZATION UNDER FISCAL REGIME UNCERTAINTY

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Abstract

Should debt stabilization be rapid or gradual? A sudden fiscal tightening is preferable, when the benefits of lower interest payments outweigh the costs of an imperfect tax smoothing. We characterize the optimal debt stabilization plan in a theoretical framework, where the interest rates on government debt depend endogenously on the government's ability to signal the sustainability of the fiscal regime. The main finding is that a lasting surplus net of interest payments is a sufficient condition for resolving the uncertainty and eliminating the risk premia. The model also allows to discuss whether the fiscal pre-requisites of the Maastricht Treaty would exclude from the EMU countries with non-sustainable fiscal regimes.

Contents

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1. Introduction[1]

Should debt stabilization be achieved with a gradual or sudden fiscal tightening? An intuitive answer to this question is the following. When the sustainability of the fiscal regime is not in doubt and no risk premium is paid on the debt, a gradual tightening will certainly be preferred, since it allows to smooth the stabilization costs over time. Instead, under fiscal regime uncertainty, the government may choose to bear the costs of a sudden fiscal tightening, that would eliminate the uncertainty and the risk premium. In this paper, we investigate in a theoretical setting the conditions under which the optimal policy of the government leads to an early resolution of uncertainty.

A number of papers has recently addressed the question of the timing of stabilizations. Fernandez and Rodrik (1991) show that the difficulty of identifying ex-ante individual gainers and losers of a reform can generate a bias towards a status quo. Alesina and Drazen (1991) and Drazen and Grilli (1990) argue that a "war of attrition" between different social groups determines the timing of the stabilizations: the first paper focuses on the causes of delayed stabilizations, while the second shows that the economic crises may have positive

1. We are grateful to the participants in seminars at the Bank of Italy, IGIER, Universidad Carlos III de Madrid, Universitat Pompeu Fabra de Barcelona and University of Rome and to one anonymous refeeree for several helpful comments. Angela Gattulli and Paola Caprari provided valuable editorial assistance. The views expressed in the paper are not necessarily those of the Bank of Italy or of CEPR.

welfare effects[2].

The approach of this paper is different from those cited above. We consider a game of incomplete information, where the public does not know the welfare objective of the government in power, as in the reputation models of monetary policy[3]. We say that a fiscal regime is non-sustainable if the government in power prefers to levy an ex-post surprise inflation - or explicit - tax on the stock of debt[4].

The cost of a tax on government debt, differently from the rest of the literature[5], is endogenized as a redistributive cost. In fact, we assume that debt is more concentrated than income, as in most real world economies, and that the tax system is such that repaying the stock of debt with income taxes will redistribute income from the low debt to the high debt group. In this context, the costs of default are modelled as a function of the redistributive goals of the government, so that only governments sufficiently in favor of

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2. A related paper is Bertola and Drazen (1993), that derives a nonlinear relationship between private consumption and government spending in a model where the latter is expected to fall when it reaches specific "target points".

3. Persson and Tabellini (1990, Chap. 3-4) review this literature; see Cukierman and Liviatan (1991) and Vickers (1986) for an approach similar to the one of this paper.

4. It is easy to show that in our model this definition of non-sustainability is equivalent to the one based on the violation of the trasversality condition. In fact, in Section 3 we show that, whenever the expected discounted present value of taxation is smaller than the stock of debt, the public will expect an ex-post surprise tax on the stock of debt and it will not buy any debt.

5. Stylized models with exogenous default costs, aimed at studying the possible existence of multiple equilibria and debt management policies, are in Calvo (1988) and in Alesina, Prati and Tabellini (1990). Debt management issues in models with exogenous inflation costs are also the focus of the papers by Calvo and Guidotti (1990) and Blanchard and Missale (1991).

the high debt group will have costs of default large enough to rule it out[6]. Note that the costs of default are incurred only if the tax on government debt is unexpected, since otherwise it would not cause any wealth redistribution.

Given the assumptions on the tax system and the wealth distribution, a government willing to fully repay its debt can be considered as "right wing", since the net effect of debt repayment by means of income taxes amounts to a redistribution from the low wealth group to the high wealth group. Alternative distributions of government bonds (e.g., held mostly by low income agents while high income agents prefer other assets) or different (progressive) income tax systems may reverse the redistributive effects of debt repayment and make a "left-wing" government willing to repay the debt.

In the initial period of the model, the public is uncertain about the preferences of the government in power and associates a positive probability to the event that it will levy ex-post a surprise tax on the stock of debt outstanding. This policy characterizes government N, whose fiscal regime is always non-sustainable (i.e., it violates the transversality condition that the discounted present value of taxes must be larger than the outstanding debt). If government N were known to be in power with certainty, nobody would buy its debt. Government S's preferences are, instead, such that it would never levy a surprise tax on the stock of debt, as long as the risk premium is not too large. In fact, if government S were in power with certainty, its fiscal regime would be sustainable (i.e., the discounted present value of taxes would be larger than the outstanding debt).

The model is designed to address the following questions: when will each type signal its nature? when will the

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6. Models with redistributive costs of default but no uncertainty are in Aghion and Bolton (1990) and Tabellini (1991).

interest rate on government bonds incorporate a risk premium? what is the maximum risk premium that government S is willing to pay? how does the optimal policy of the government depend on the initial distribution of wealth?

In the presence of a deficit net of interest payments, the public will not always be able to tell the two governments apart: both separating equilibria (where government N reveals its nature at an early stage by taxing away part of the debt) and pooling equilibria (where government N initially mimicks the policy of type S) are possible depending on the preferences of the governments, the total expenditure to be financed and the initial reputation. Instead, when a lasting surplus net of interest payments is achieved, the uncertainty will always be resolved. In this case, government N does not have any incentive to mimick government S in order to issue debt, since it can finance the current expenditure with the current taxes.

This result has interesting implications for the time path of interest rates. It is evident that, as long as the economy is in a pooling equilibrium, no risk premium will be paid on government debt, since government N mimicks government S's policy and does not tax government debt. However, the period before a surplus net of interest payments is achieved, the public anticipates that in the following period the uncertainty will be resolved and requires a risk premium.

The problems of countries which are currently trying to repay large stocks of debt and the historical experience confirm the two findings above. For example, in France in the 1920s, the risk premium on government debt and the refinancing difficulties of the Treasury increased when the government was about to reach a surplus net of interest payments[7].

The model also predicts that, when the initial

7. See Prati (1991a).

reputation is "bad" or the total expenditure is large, the risk premium becomes "too high" and the only possible equilibrium is a financial crisis where no debt is sold. This happens because the risk premia and the interest payments are so large that the public anticipates both type of government would levy a surprise tax on the stock of debt.

The theoretical framework allows to test whether the fiscal pre-requisites of the Maastricht Treaty are sufficient to exclude the countries subject to the risk of default from the EMU[8]. In particular, we investigate whether, given the optimal debt stabilization policies of the two types, the fiscal rules of the Maastricht Treaty would allow in the monetary union only type S governments. Our model predicts that the deficit pre-requisite (not larger than 3 per cent of GDP) should separate type S from type N governments. In fact, in high debt countries, the 3 per cent rule implies huge primary surpluses, which solve the uncertainty on the nature of the government. The debt pre-requisite (less than 60 per cent of GDP) may instead be "too tight", since it excludes from the monetary union governments that have accumulated large stocks of debt as a result of the "bad" reputation and the high interest rates, while it neglects whether they have signalled or not to run sustainable fiscal policies. This problem may be serious since it is exactly in the instances of "bad" reputation that government S has strong incentives to signal early.

On the basis of our theoretical model, we conclude that the deficit criterion, though not necessary, is sufficient to select governments running sustainable fiscal policies; the

8. See, among others, Giovannini and Spaventa (1991). The default may be explicit or via inflation; it is feared, in fact, that a large stock of debt in one or more countries partecipating in the union could force the European Central Bank to a less strict monetary policy in response to a liquidity crisis and to avoid a default of high debt governments. Buiter, Corsetti and Roubini (1993) discuss critically the relevance of such argument.

debt criterion, instead, is certainly not necessary nor sufficient. As a consequence, the joint application of the two criteria may leave out of the union governments that have signalled the sustainability of the fiscal regime, but whose initial reputation was bad. These indications should be taken with caution, for they refer to only one possible rationale for the Maastricht rules (the exclusion of default-risk countries)[9].

The paper is organized as follows. Section 2 describes the setup of the model (the economy, the two types of government, the private learning process and the equilibrium definition). Section 3 discusses the equilibria of the model in the two cases where the government expenditure in the period before last is relatively low (surplus net of interest payments in the last two periods) or high (deficit net of interest payments in the last two periods). Section 4 summarizes the features of the equilibria of the latter case, in which both pooling and separating equilibria exist. Section 5 debates the rationale for the Maastricht Treaty rules.

2. The model

2.1 The economy

The population is divided in two groups (R and P), formed by an equal number of risk neutral atomistic agents

9. Another motive of concern is that the pooling of high debt countries with low debt ones could lead after the monetary union and in a non-ricardian world to an increase in the level of interest rates for low debt countries ; see for example Gros and Thygesen (1992).

living for 3 periods[10]. They differ only for the period zero endowment, which is larger in the case of type R, $e_R > e_P$. For simplicity, we assume that the discount factor is equal to 1, so that individual h (h=R,P) maximizes the expected utility function:

$$U_h = E_0 \{ c_{h,0} + c_{h,1} + c_{h,2} \} \qquad h = R,P \tag{1}$$

In the model, the preferences of the government, unknown to the public in period zero are the only source of uncertainty.

In period zero, the agents do not work and allocate their endowment between consumption and government debt:

$$c_{h,0} = e_h - d_{h,1} \geq 0 \tag{2}$$

In order to rule out negative consumption, we assume that the total debt is smaller than the total endowment and that each type buys bonds proportionally to its initial endowment[11]. Define:

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10. The game studied in this paper is part of a general class of dynamic games with a large player (the government) and a large number of small players. The play of the large players is observed and is therefore part of the public history of the game, while the individual plays of the small players are not observed, so that only their aggregate play is part of the public history of the game. Similar games in an infinite horizon model are in Chari and Kehoe (1990), Stokey (1992) and Celentani and Pesendorfer (1992).

11. The conclusions of the paper hold for any other arbitrary distribution of government debt, which satisfies the non-negativity constraint on consumption and that gives type R a larger stock of debt than the one of type P.

$$\phi = \frac{e_R}{E} = \frac{d_{R,t}}{D_t} > 1$$

where

$E = \frac{1}{2}(e_R+e_P)$ = average per-capita initial endowment;

$D_t = \frac{1}{2}(d_{R,t}+d_{P,t})$ = average per-capita debt in period t;

so that

$$c_{R,0} = \phi \; (E - D_1) \tag{3}$$

$$c_{P,0} = (2 - \phi) \; (E - D_1) \tag{4}$$

In periods 1 and 2, the two groups have an identical labor income, which is reduced by labor taxes and distortionary costs of taxation[12]:

$$y(\tau_t) = y - \tau_t - h(\tau_t) \tag{5}$$

where

y = labor income;

τ_t = taxes levied on each individual in period t;

$h(\tau_t) = \frac{h}{2}\tau_t^2$ = distortionary costs of taxation.

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12. This form of distortionary costs is often used in the literature and it is not particularly ad hoc. Prati (1991b, Appendix 1) shows that by modelling explicitly the labor supply decision of the agents one would get first order conditions analogous to those derived under equation (5).

In period t (=1,2), the government may decide to levy an <u>ex-post</u> tax on the stock of debt plus interest payments, $\theta_t d_{h,t} R_t$. If the distribution of debt is unchanged in period 1, the agents' budget constraints can be written as:

$$c_{R,1} = y(\tau_1) + \phi \, [\, D_1 R_1 (1-\theta_1) - D_2 \,] \tag{6}$$

$$c_{R,1} = y(\tau_1) + (2 - \phi) \, [\, D_2 R_2 (1-\theta_1) - D_2] \tag{7}$$

$$c_{R,2} = y(\tau_2) + \phi \, [D_2 R_2 (1-\theta_2)] \tag{8}$$

$$c_{R,2} = y(\tau_2) + (\, 2 - \phi \,) \, [D_2 R_2 (1-\theta_2)] \tag{9}$$

where

R_t = interest factor on government debt maturing at t.

The following no arbitrage condition is the solution of the agents' maximization problem[13]:

$$R_t \, (\, 1 - \theta_t^e \,) = 1 \qquad t = 1,2 \tag{10}$$

where

θ_t^e = expected tax rate on debt maturing in period t, given the information set of agents in period t-1.

The assumptions on the distribution of income and wealth imply that in this model, as in most real world

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13. The first-order condition of the private sector would be identical, if we considered a small open economy with perfect capital mobility and a risk and tax free international asset. Alternatively, we could have assumed a risk-free domestic technology with a rate of return equal to 1.

economies, income is less concentrated than wealth. The additional assumption that the tax system cannot discriminate between the high-debt group R and the low-debt group P, implies that a policy of debt rapayment by means of income taxes redistributes income from group P to group R. A straightforward implication of these assumptions is that only a government sufficiently in favor of the wealthy group R will decide not to tax government debt. The effect of alternative hypotheses on the distribution of government debt and on the tax system is discussed in the conclusions.

2.2 The government

Government expenditure occurs at two dates, period 0 and period 1. In period 0, the government does not levy any tax and the debt issued is equal to the expenditure. In periods 1 and 2, the government levies distortionary taxes on labor income and may tax ex-post government debt. Therefore, the government's budget constraints are:

$$g_0 = D_1 \tag{11}$$

$$D_1 R_1 (1 - \theta_1) + g_1 = \tau_1 + D_2 \tag{12}$$

$$D_2 R_2 (1 - \theta_2) = \tau_2 \tag{13}$$

We assume that within each period the government chooses debt and taxes before the public sets the interest rate. The timing of the game is then the following[14]:

14. The results of the paper would be analogous, if the government's and the public's actions were simultaneous within each period but the algebra would be more complicated. An opposite timing, where the public moves first, would instead cause a multiplicity of equilibria (see Prati 1991 a, Chap. 3).

<u>Period 0</u>

D_1

$R_1 = R_1(D_1)$

<u>Period 1</u>

$D_2, \ \tau_1, \ \theta_1$

$R_2 = R_2(D_2)$

<u>Period 2</u>

$\tau_2, \ \theta_2$

In period zero, the public does not know which government is in power, but it knows that the social welfare function maximized by <u>government S</u> is characterized by a larger weight on the utility of type R than the one maximized by <u>government N</u>:

$$W_i = \omega_i U_R + (1 - \omega_i) U_P \qquad i=S,N \qquad \omega_N<\omega_S<1 \qquad (14)$$

Government i chooses a sequence of taxes on income and debt $(\tau_1,\tau_2,\theta_1,\theta_2)$, that maximizes W_i subject to the individuals' and the government's budget constraints (equations (3)-(9) and (11)-(13)) and to the no-arbitrage conditions (equation (10)).

In order to have an interesting problem, it is not enough to assume $\omega_S > \omega_N$, but we need to impose an additional restriction on preferences:

<u>Assumption 1:</u> $\frac{1}{2} < \omega_N < \frac{1}{2} + \frac{h(g_0 + g_1)}{4(\phi - 1)} < \omega_S$

The right-hand side inequality requires government S to have preferences sufficiently in favor of type R individuals. This requirement implies that, if government S were in power <u>with certainty</u>, it would not tax government debt and it would raise the whole revenue with income taxes (see the discussion of Lemma 1 below).

The intermediate inequality requires government N to give a small enough weight to the utility of type R individuals. As shown in Appendix 1, this assumption implies that, even if government N were in power with certainty and could issue debt at the risk free rate, it would levy a positive tax on government debt (i.e., either $\theta_1 > 0$ or $\theta_2 > 0$).

The assumption $\omega_i > 1/2$, which implies that the preferences of both governments are biased in favor of the type R individuals, does not affect the results in any crucial way. On the contrary, it allows us to solve a general form of the model in which the preferences of the two governments are relatively similar (i.e., they can both be labeled "right-wing"), though government N is not sufficiently in favor of type R individuals to rule out a tax on government debt[15].

2.3 The information structure

We investigate Sequentially Rational Perfect Bayesian Equilibria, since in the initial period there is incomplete information on the government in power and we assume that neither government can commit its policies in advance. In period 0, there is a prior probability, p_0, that government N is in power. When the first period taxation on labour income and debt ($\tau_1 = \hat{\tau}_1$ and $\theta_1 = \hat{\theta}_1$) is observed, the probability is revised according to Bayes' rule:

15. If the preferences were more polarized than in Assumption 1 (i.e., $\omega_N<1/2$), the solution of the model would be similar to the one discussed in the rest of the paper, but less complicated. The unconstrained optimal policy of government N would be to levy a negative or zero tax on labor income. However, since negative taxes are not allowed, income taxes will be set to zero and the entire stock of debt will be taxed away.

$$p_1 = \frac{p_0\ Prob(\theta_1 = \hat{\theta}_1, \tau_1 = \hat{\tau}_1 : \omega = \omega_N)}{p_0 Prob(\theta_1 = \hat{\theta}_1, \tau_1 = \hat{\tau}_1 : \omega = \omega_N) + (1 - p_0)\ Prob(\theta_1 = \hat{\theta}_1, \tau_1 = \hat{\tau}_1 : \omega = \omega_S)} \tag{15}$$

Furthermore, since Bayes' rule does not apply off the equilibrium paths, we assume that, if $\theta_1 > (\hat{\theta}_1 : \omega = \omega_S) = 0$ or $\tau_1 < (\hat{\tau}_1 : \omega = \omega_S)$, the public infers that government N is in power and sets $p_1 = 1$.

2.4 The government's problem

The problem of government i can be written as:

$$\begin{aligned} Min_{\{\theta_1, \theta_2, D_2\}}\ & \frac{1}{2}[D_1 R_1 (1 - \theta_1) + g_1 - D_2]^2 + \frac{1}{2}[D_2 R_2 (1 - \theta_2)]^2 \\ & + \hat{\alpha}_i D_1 [1 - R_1 (1 - \theta_1)] + \hat{\alpha}_i D_2 [1 - R_2 (1 - \theta_2)] \end{aligned} \tag{16}$$

subject to $R_t\ (1 - \theta_t^e) = 1$

where $\hat{\alpha}_i = \dfrac{\omega_i \phi + (1 - \omega_i)(2 - \phi) - 1}{h} > 0.$

The above formulation is derived by plugging the individuals' utility functions in the welfare function, susbtituting from the budget constraints and dropping constants.

The government's policy affects welfare in two ways: non-linearly via the distortionary costs of taxation (first and second elements of equation (16)) and linearly via the redistribution brought about by a surprise tax on government debt. In fact, the last two elements of equation (16) are greater than zero and can be interpreted as a redistributive cost of debt taxation only when $R_t\ (1 - \theta_t) < 1$ (i.e. the net of tax return on government debt is smaller than expected). The latter cost is larger the larger is the parameter $\hat{\alpha}_i$, which increases with ω_i. The intuition is that in this model the only cost of a tax on government debt is the redistributive cost, which is evidently larger for government S that favors the R group than for government N[16].

In order to find the optimal policy of the government that satisfies the sequential rationality condition, we solve the government's problem backwards.

The last period problem

The optimal policy of government i in the last period is characterized by the following first order condition, taken with respect to θ_2:

$$D_2 R_2 (1 - \theta_2) \leq \hat{\alpha}_i \tag{17}$$

The comparison of (13) and (17) indicates that $\hat{\alpha}_i$ can be interpreted as the maximum amount of income taxes that government i is ready to levy in period 2. When (17) holds with the equality sign, the government's problem has an interior solution and $\theta_2>0$, since the maturing stock of debt D_2R_2 is larger than the maximum amount of income taxes that government i is ready to levy. When (17) holds with the inequality sign, government i is at a corner and $\theta_2=0$, since the maximum amount of taxes is larger than the maturing stock of debt.

Intermediate period problem

In period 1, the first order condition of the problem of government i with respect to θ_1 is:

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16. If we considered the case of a 'left-wing' government, $\omega_N \leq 1/2$ (so that $\hat{\alpha}_N \leq 0$), a positive surprise tax on government debt would increase welfare.

$$D_1 R_1 (1 - \theta_1) + g_1 \leq \hat{\alpha}_i + D_2 \qquad (18)$$

As in period 2, the government problem can have either an interior solution with $\theta_1>0$ or a corner solution with $\theta_1=0$.

In period 1, the government chooses also how much debt D_2 roll over to the last period. Under the assumed timing, the first order condition of this problem is:

$$D_1R_1(1 - \theta_1) + g_1 = \hat{\alpha}_i + D_2 +$$
$$+ [D_2R_2 (1 - \theta_2) - \hat{\alpha}_i] [R_2(1 - \theta_2) + D_2\frac{\partial R_2(1 - \theta_2)}{\partial D_2}]$$

The derivative $\partial R_2(1 - \theta_2)/\partial D_2$ enters the first order condition for D_2 because, under the assumed timing, the government sets D_2 before the public chooses R_2. As we shall see, the values of this derivative depend on whether the uncertainty is resolved in period 1 (separating equilibrium) or in period 2 (pooling equilibrium).

3. The equilibria

3.1 Solution strategy

The solution of the model depends crucially on whether the government's problem has an interior or a corner solution with respect to θ_t. A few preliminary results will simplify the search for equilibria.

First, note that the definition of $\hat{\alpha}_i$ allows to rewrite Assumption 1 as follows:

<u>Assumption 1':</u> $2\hat{\alpha}_N < g_0 + g_1 < 2\hat{\alpha}_S$

Appendix 1 shows that, given the left-hand side inequality, government N will be at an interior solution either in period 1 or in period 2. Assumption 1' implies that government N is not willing to levy enough income taxes to finance government expenditure as well as interest payments.

Suppose government N is at an interior solution with respect to θ_t, can also government S be at an interior solution with $\theta_t>0$? Assumption 1' rules out this possibility when $R_t=1$, since in this case there are no interest payments and government expenditure is smaller than the maximum amount of income taxes government S is ready to levy. However, it is evident that a sufficiently large R_t can force also government S to choose $\theta_t>0$. In this case, Lemma 1 below states that there cannot be an equilibrium.

<u>Lemma 1:</u> when the problem of both governments has an interior solution in the last period with $\theta_t>0$, there is no equilibrium.

<u>Proof (by contradiction):</u> suppose that the problem of government S has an interior solution in period 2 with $\theta_2^S>0$. In this case $D_2R_2>\hat{\alpha}_S>\hat{\alpha}_N \rightarrow \theta_2^N>\theta_2^S>0$, $D_2R_2>\hat{\alpha}_i$ and $D_2R_2(1-\theta_2^i)=\hat{\alpha}_i$, which is possible only if (recall equation (10)) $R_2=\frac{1}{1-\theta_2^e}<\frac{1}{1-\theta_2^i}$,

so that $\theta_2^i>\theta_2^e$ and $\theta_2^N>\theta_2^S>\theta_2^e = (1-p)\theta_2^S+p\theta_2^N$. But the latter inequality never holds as long as $0\leq p\leq 1$. The proof can easily be repeated for period 1. Q.E.D.

A crucial implication of Lemma 1 is that in equilibrium we will never observe both governments choosing $\theta_t>0$. The intuition is that nobody will buy government debt as long as he expects that both governments will tax it by surprise. We take advantage of this result for designing the following <u>solution strategy</u>:

1) we assume that government S's problem has a corner solution with respect to θ_t, so that government S is characterized by $\theta_t=0$;
2) we derive the optimal policy of the two governments in equilibrium under the step 1 assumption;
3) we find the range of parameters in which government S's optimal policy has an ex-post corner solution, so that the step 1 assumption is consistent with the optimal policy of government S in equilibrium;
4) we take 3) into account when we define the range of parameters where the equilibrium exists.

Step 3 of the solution strategy is equivalent to impose the transversality condition that the expected discounted present value of taxation must be not smaller than the stock of debt. As an example, consider the condition for a corner solution of government S's problem in the last period:

$$D_2 \, R_2 < \hat{\alpha}_S$$

Under the assumption that government S is at a corner (step 1) and that government N is at an interior solution in period 2 (Assumption 1'), it is easy to verify that the inequality above is equivalent to the following one:

$$D_2 < (1-p)\,\hat{\alpha}_S + p\hat{\alpha}_N$$

where the right hand side is equal to the maximum expected level of taxes in period 2.

According to the above solution strategy and given Assumption 1, type N is characterized by $\theta_t^N > 0$ and type S by $\theta_t^S = 0$. In this context, two equilibria may prevail: a separating equilibrium or a pooling equilibrium. In the first one, the N government chooses $\theta_1^N > 0$ and reveals itself in period 1. In the second one, type N imitates type S in period 1 ($\theta_1^N = \theta_1^S = 0$ and $\tau_1^N = \tau_1^S$) and reveals its type only in the last period, choosing $\theta_2^N > 0$.

We distinguish between a case with low and one with high first period government expenditure, g_1. The discriminating level is $2\hat{\alpha}_N$. In the case $g_1 < 2\hat{\alpha}_N$, the taxes government N is ready to levy in period 1 and 2 are larger than the government expenditure in period 1 (overall <u>surplus net of interest payments</u> in the last two periods). Instead, when $g_1 > 2\hat{\alpha}_N$, the expenditure in period 1 is larger than the taxes government N is ready to levy in the last two periods (overall <u>deficit net of interest payments</u> in the last two periods). In the first case, the taxes of government N will be at the optimum level ($2\hat{\alpha}_N$) even if it reveals its type in period 1. In the second case, if government N reveals itself in period 1, it will be unable to issue more than $D_2 = \hat{\alpha}_N$ and it will have to tax in period 1 in excess of its optimal level ($\tau_1 = g_1 - \hat{\alpha}_N > \hat{\alpha}_N$). Therefore, one would expect that the incentives for government N to mimick government S are larger in the latter case. This result is derived formally in the rest of the paper.

In the characterization of equilibria we will make extensive use of two parameters. The first is a non-sustainability parameter, given by:

$$\psi = \frac{g_0 + g_1}{2\hat{\alpha}_N} - 1 \qquad \psi > 0$$

The value of ψ increases with the debt not repaid by type N ($g_0 + g_1 - 2\hat{\alpha}_N$).

The second is a polarization parameter, related to the preferences of the two governments:

$$\gamma = \frac{\hat{\alpha}_N}{\hat{\alpha}_S} = \frac{\omega_N - \frac{1}{2}}{\omega_S - \frac{1}{2}} \qquad 0 < \gamma < 1$$

When γ is close to 1, the two governments are very similar and, in an interior optimum, type S taxes labor income only a little more than type N. As γ goes to zero, the redistributive preferences of the two governments become very different and type S taxes labor income much more than type N; as we shall see, the smaller is γ, the larger is ceteris paribus the sustainable stock of debt.

3.2 Case I ($g_1 < 2\hat{\alpha}_N$, surplus net of interest payments of type N in the last two periods)

3.2.1 Equilibrium definition

Separating equilibrium

In a separating equilibrium, $R_2 = 1$, since the uncertainty is resolved in period 1 and there is no risk premium on the debt rolled over. In fact, in a separating equilibrium, $Prob(\theta_1 = 0, \tau_1 = \tau_1^{S,S} : \omega = \omega_N) = 0$ and, by Bayes' rule, $p_1 = 0$, if $\theta_1 = 0$ and $\tau_1 = \tau_1^{S,S}$ are observed, and $p_1 = 1$, otherwise. As a consequence, $\theta_2^{N,S} = \theta_2^{S,S} = 0$, $R_2(1 - \theta_2) = 1$ and $\partial R_2(1 - \theta_2)/\partial D_2 = 0$.

The policy of type N in period 1, $\theta_1^{N,S}$ and $D_2^{N,S}$, is derived considering jointly the first order conditions for θ_1 and D_2, with $R_2\ (1 - \theta_2) = 1$ and $\partial R_2(1-\theta_2)/\partial D_2 = 0$; note that

$\theta_1^{N,S}<1$, since we are considering the case where $g_1 < 2\hat{\alpha}_N$. The equilibrium values for the taxation in periods 1 and 2 follow from the budget constraint of the government. The expression for the interest factor in period 1 follows from the arbitrage condition, with $\theta_1^e = \theta_1^N p_0$. Similarly, from the first order condition for D_2, the policy of type S is $D_2^{S,S} = (D_1 R_1 + g_1)/2$, which amounts to a perfect tax smoothing. After substitution of the equilibrium value for R_1 we get the following characterization of a separating equilibrium (subscripts t=1,2 refer to the period, the first superscript to the type and the second to the equilibrium):

<u>type N</u>

$$\theta_1^{N,S} = \frac{D_1 R_1 + g_1 - 2\hat{\alpha}_N}{D_1 R_1} = \frac{D_1 + g_1 - 2\hat{\alpha}_N}{D_1 + (g_1 - 2\hat{\alpha}_N)\ p_0} \tag{19}$$

$$\tau_1^{N,S} = \tau_2^{N,S} = D_2^{N,S} = \hat{\alpha}_N \tag{20}$$

<u>type S</u>

$$\tau_1^{S,S} = \tau_2^{S,S} = D_2^{S,S} = \frac{D_1 + g_1 - 2\hat{\alpha}_N\, p_0}{2\ (1 - p_0)} \tag{21}$$

<u>the economy</u>

$$R_1 = \frac{D_1 + (g_1 - 2\hat{\alpha}_N)\ p_0}{D_1\ (1 - p_0)} > 1 \tag{22}$$

$$R_2 = 1 \tag{23}$$

Figure 1 shows how the optimal policy of type S varies with ψ

Fig. 1a

Separating equilibrium $(g_1 < 2\hat{\alpha}_N)$: $t_1^s = \frac{\tau_1^s}{\hat{\alpha}_N} = \frac{\tau_2^s}{\hat{\alpha}_N} = \frac{D_2^s}{\hat{\alpha}_N}$

$(\gamma = \frac{1}{2})$

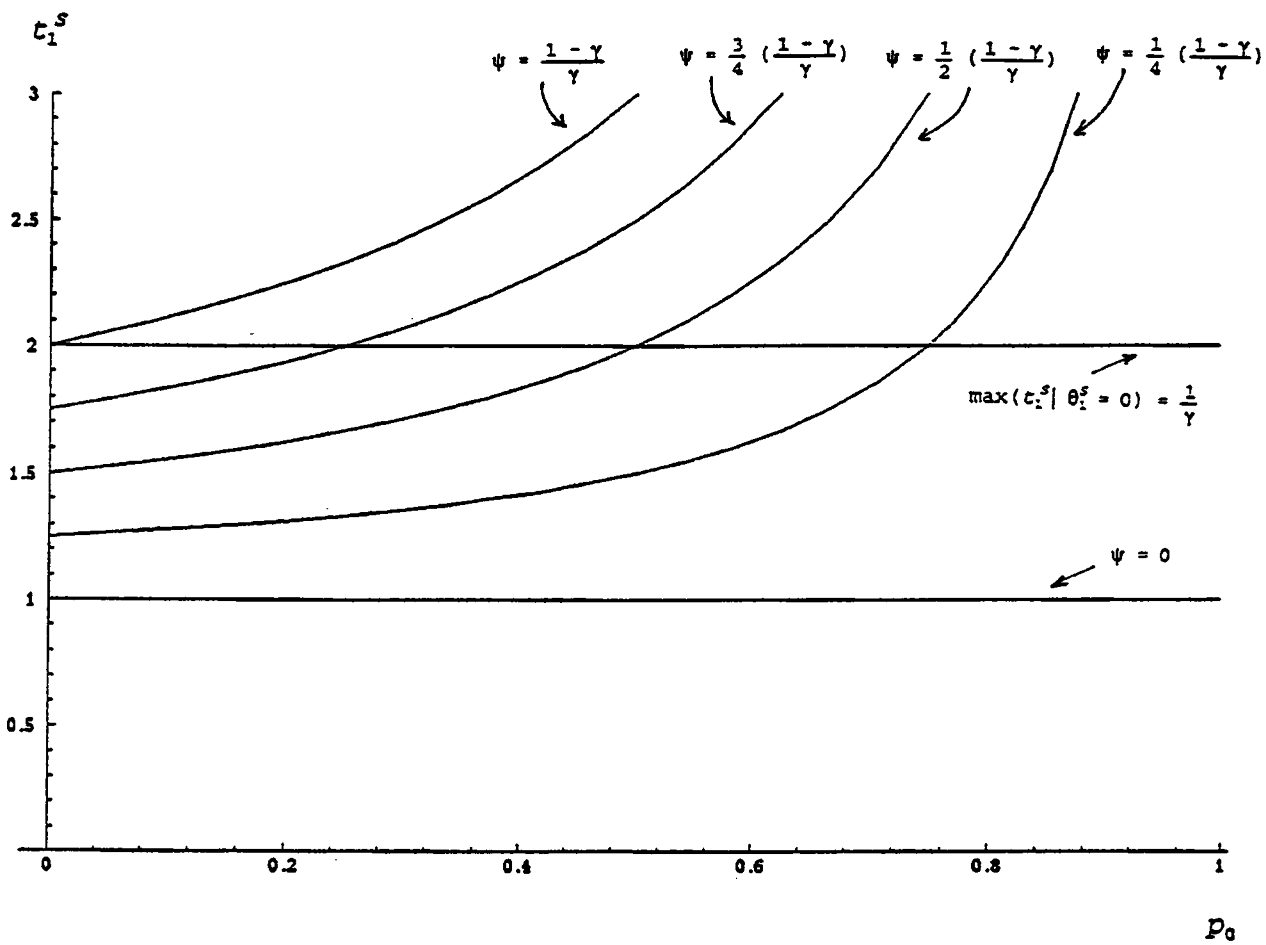
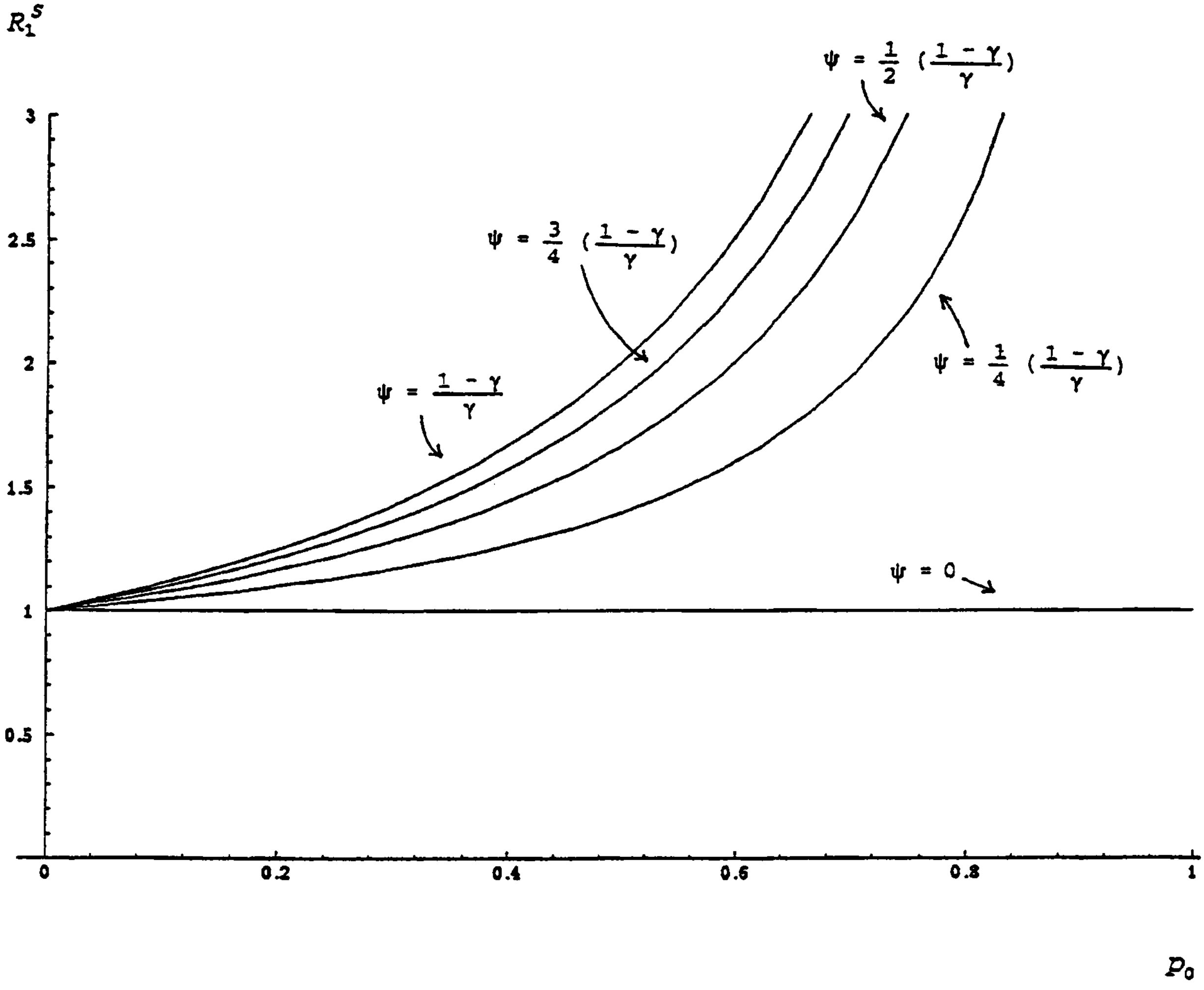


Fig. 1b

Separating equilibrium $(g_1 < 2\hat{\alpha}_N)$: R_1^S

$$(\gamma = \frac{1}{2},\ g_0 = g_1)$$



and p_0. Higher taxes result from a larger non-sustainability parameter ψ (larger government expenditure or lower ω_N) or a higher probability that type N is in power (higher p_0). Note that, as p_0 goes to 1, taxes of type S go to infinity. However, there is no equilibrium with an infinite level of taxes and a positive stock of debt, since $\hat{\alpha}_S$ is finite (even $\omega_S=1 \rightarrow \hat{\alpha}_S=(\phi-1)/h<\infty$) and, when the taxes reach this level, also type S would prefer to tax government debt. In other words, an infinite level of taxes is not consistent with $\theta_1^S=0$: when the interest factor R_1 is so large that $\tau_1^{S,S}>\hat{\alpha}_S$, type S is no longer at a corner solution ($\theta_1^S>0$) and the condition in Lemma 1 for the existence of an equilibrium is not satisfied. This happens for a sufficiently high p_0 or a large enough government expenditure (the maximum level of taxes consistent with$\theta_1^S=0$ is shown in figure 1 by the horizontal line at $1/\gamma=\hat{\alpha}_S/\hat{\alpha}_N$).

Note that all variables in the figures are standardized with respect to $\hat{\alpha}_N$. We also draw the graphs for$\gamma=1/2$ ($\rightarrow \hat{\alpha}_S = 2\hat{\alpha}_N \rightarrow \omega_S = 2\omega_N - 1/2 > \omega_N > 1/2$, i.e. the maximium level of taxes of types S is twice the maximum level of taxes of type N); different γs do not affect the shape of the curves but only their vertical position.

Figure 1b shows that the interest factor R_1 grows with government expenditure (ψ) and with p_0 and it goes to infinity as p_0 goes to 1[17].

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17. Figure 1b is drawn for a particular distribution of government expenditure over time ($g_0 = g_1$), since the interest factor R_1 varies with the distribution of government expenditure. In fact, a smaller $g_0 = D_1$ makes type N choose a larger $\theta_1^N>0$ in order to receive the same revenue from the tax on government debt; as a result the risk premium will be larger (R_1 will increase). Note, instead, that figure 1a is not affected by the distribution of government expenditure, since $D_1R_1 + g_1$

Pooling equilibrium

In a pooling equilibrium, government N replicates the choice of equilibrium values in period 1 of government S. As a consequence $Prob(\theta_1 = 0, \tau_1 = \tau_1^{S,P} : \omega = \omega_N) = 1$, so that Bayes' rule implies $p_1 = p_0$. In period 2, given the choice of taxation, $\theta_2^{N,P}$ follows from the budget constraint. The derivation of R_2 is by straightforward substitution in the arbitrage condition.

For $D_2^{S,P}$, consider the derivative of $R_2(1-\theta_2)$ with respect to D_2: $\frac{\partial R_2(1-\theta_2)}{\partial D_2} = \frac{\partial R_2}{\partial D_2} = \frac{\hat{\alpha}_N p_0}{D_2^2(1-p_0)}$

After the substitution of the equilibrium values of R_1 and R_2 and of the derivative in the first order condition for $D_2^{S,P}$, evaluated at $\theta_1 = \theta_2 = 0$ and $\hat{\alpha}_i = \hat{\alpha}_S$, we get the following characterization of the pooling equilibrium:

type N:

$$\tau_1^{N,P} = \tau_1^{S,P} = D_1 + g_1 - D_1^{S,P} \tag{24}$$

$$D_2^{N,P} = D_2^{S,P} \tag{25}$$

$$\theta_1^{N,P} = 0 \tag{26}$$

$$\tau_2^{N,P} = \hat{\alpha}_N \tag{27}$$

---

does not depend on it; this feature is due to the fact that the amount of surprise taxation on debt chosen by type N is fixed and equal to the amount necessary to satisfy the budget constraint given $\tau_t^N = \hat{\alpha}_N$.

$$\theta_2^{N,P} = \frac{D_2^{S,P} - \hat{\alpha}_N}{D_2^{S,P} - \hat{\alpha}_N p_0} < 1 \tag{28}$$

type S:

$$D_2^{S,P} = \frac{(D_1 + g_1)\ (1 - p_0)^2 + \hat{\alpha}_N p_0 + \hat{\alpha}_S p_0 (1 - p_0)}{1 + (1 - p_0)^2} > \hat{\alpha}_N \tag{29}$$

$$\tau_1^{S,P} = \frac{(D_1 + g_1) - \hat{\alpha}_N p_0 - \hat{\alpha}_S p_0 (1 - p_0)}{1 + (1 - p_0)^2} \tag{30}$$

$$\tau_2^{S,P} = D_2^{S,P} R_2 = \frac{(D_1 + g_1)\ (1 - p_0) - \hat{\alpha}_N p_0\ (1 - p_0) + \hat{\alpha}_S p_0}{1 + (1 - p_0)^2} > \hat{\alpha}_N \tag{31}$$

the economy:

$$R_1 = 1 \tag{32}$$

$$R_2 = \frac{D_2^{S,P} - \hat{\alpha}_N p_0}{D_2^{S,P}\ (1 - p_0)} > 1 \tag{33}$$

$$p_1 = p_0 \tag{34}$$

The behavior of the equilibrium values for different levels of the parameters ψ and p_0 are shown in figure 2.

One feature of government S's optimal strategy in the pooling equilibrium is worth noting. If ψ is small enough, $\psi < (1 - \gamma)/2\gamma$, as p_0 grows, type S will first increase and then decrease the stock of debt $D_2^{S,P}$ rolled over to the last period (see fig. 2a). This policy may look counterintuitive,

Fig. 2a

Pooling equilibrium $d_2^P = \frac{D_2^P}{\hat{\alpha}_N}$

$(\gamma = \frac{1}{2})$

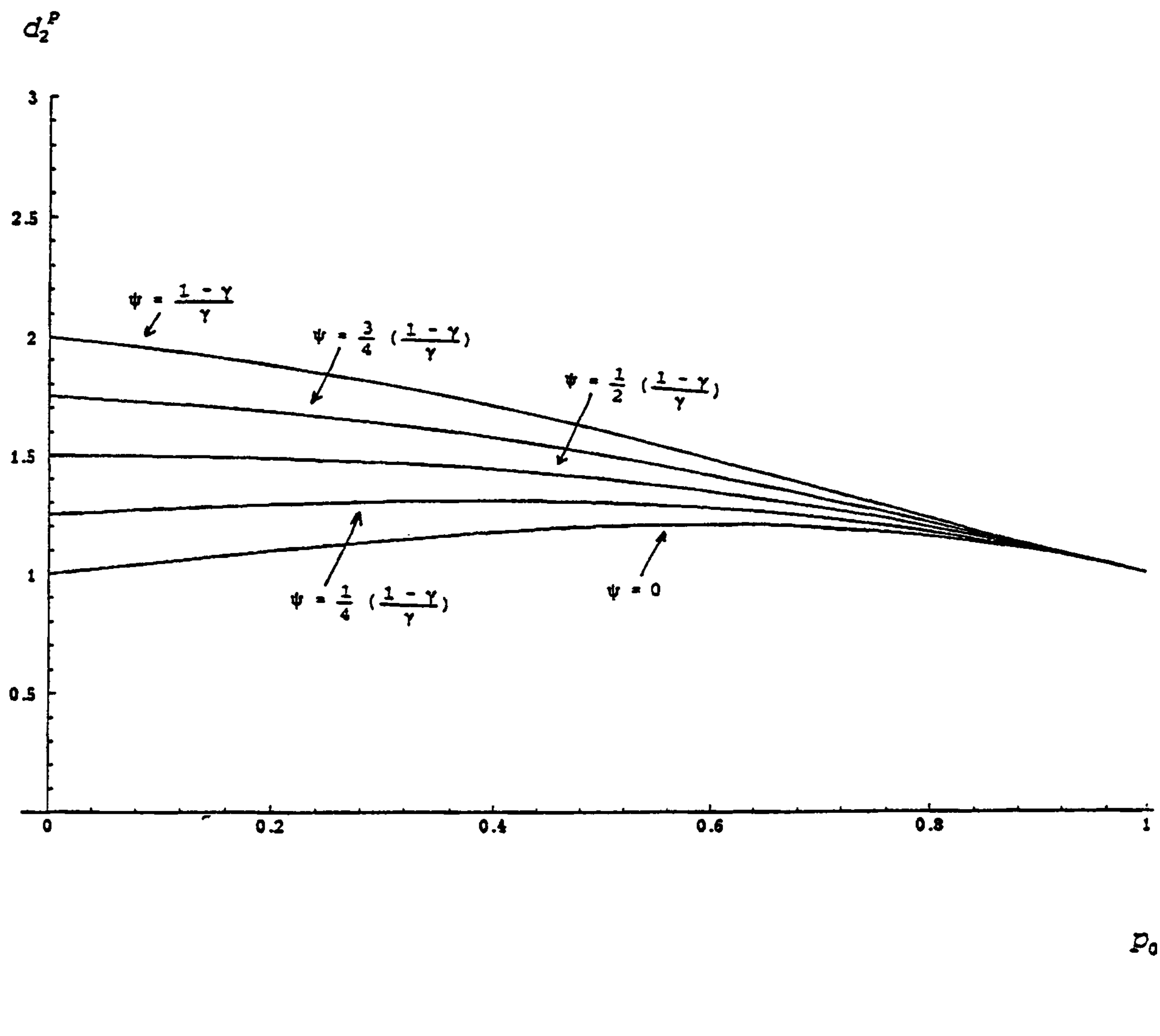


Fig. 2b

Pooling equilibrium $t_1^P = \frac{\tau_1^P}{a_N}$

$(\gamma = \frac{1}{2})$

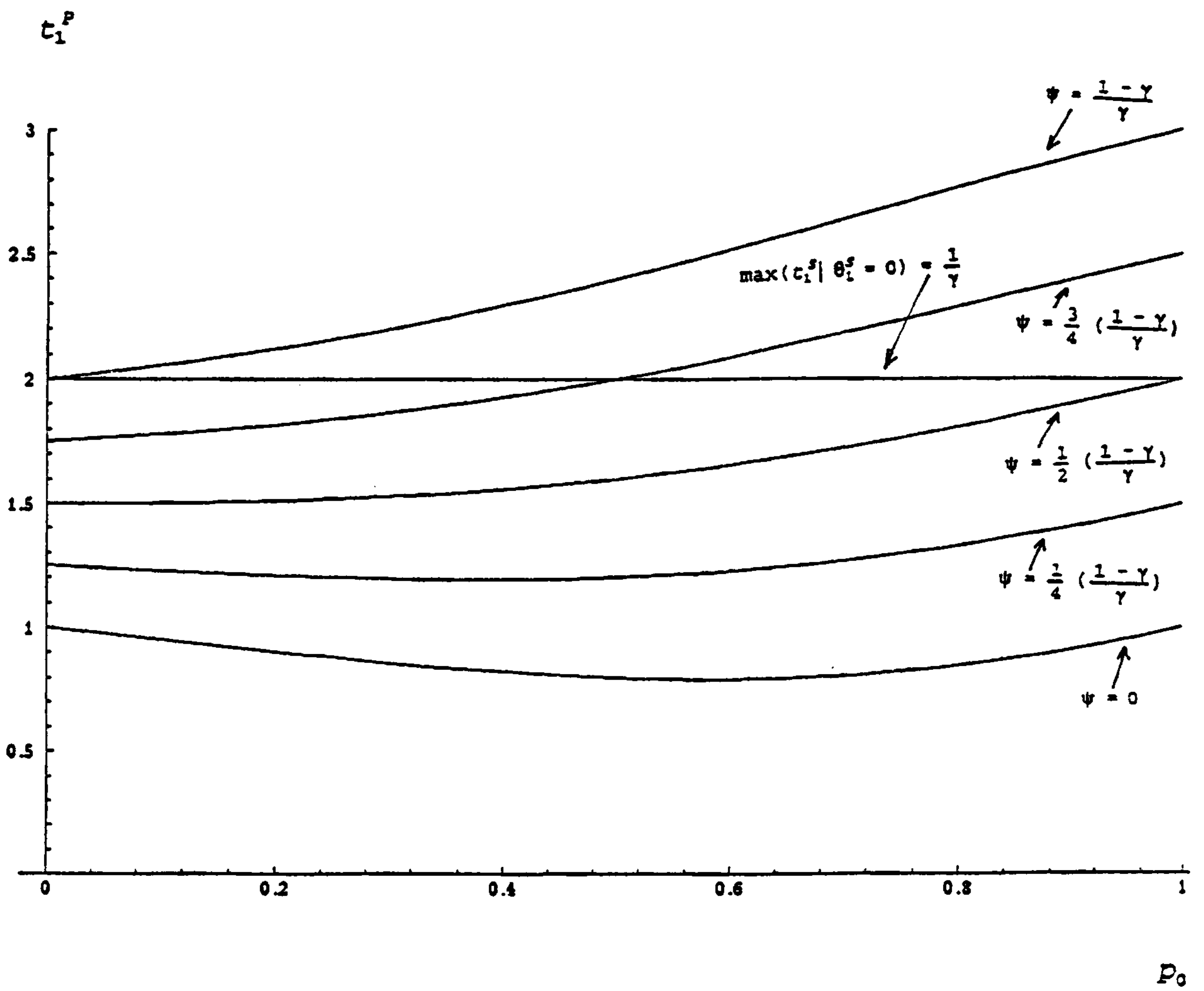


Fig. 2c

Pooling equilibrium R_2^p

$(\gamma = \frac{1}{2})$

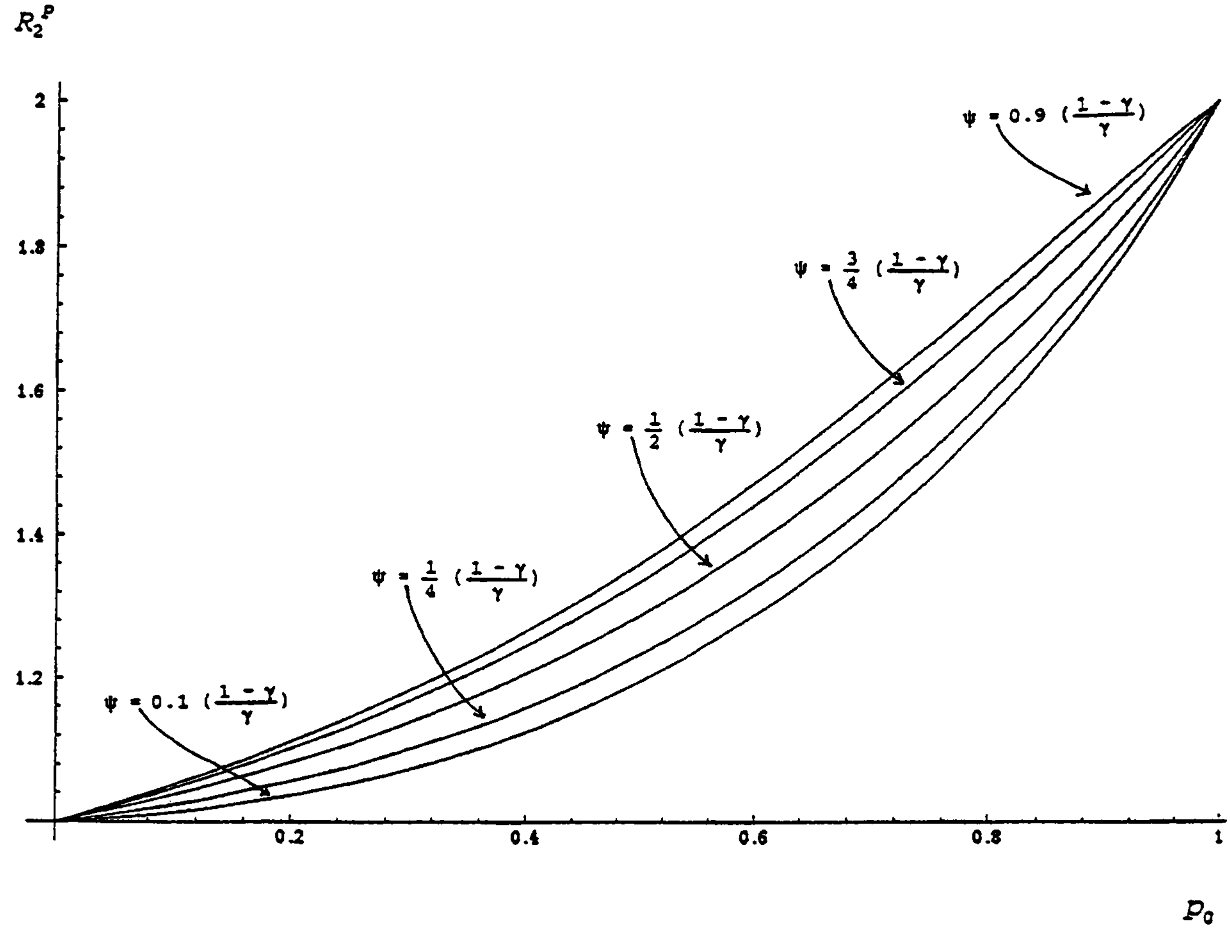
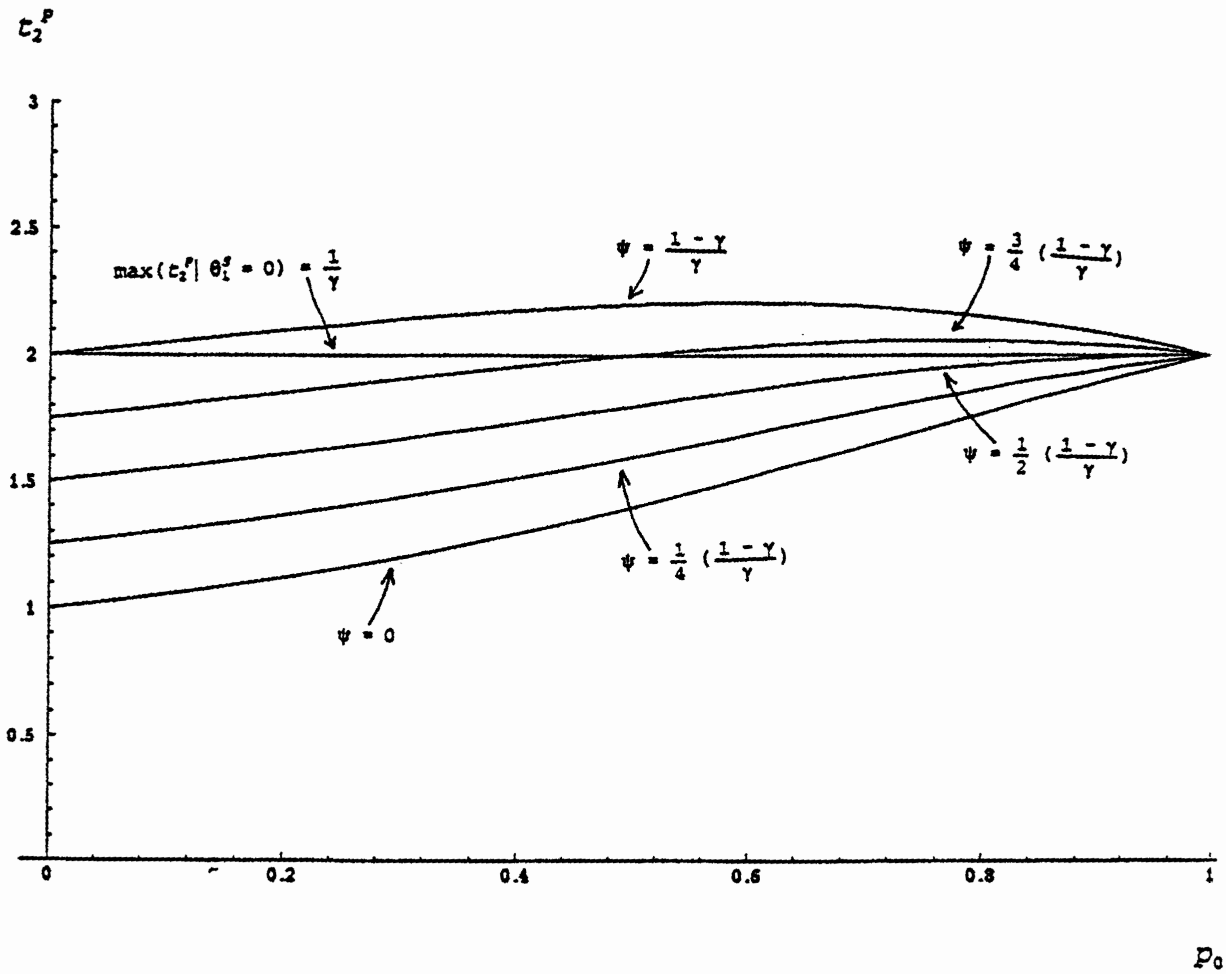


Fig. 2d

Pooling equilibrium $t_2^P = \frac{\tau_2^P}{\hat{a}_N}$

$(\gamma = \frac{1}{2})$



since a larger p_0 leads to a larger R_2 and to more taxes to be levied over the two periods; but, in this model, the government trades off the distortionary costs against the redistributive costs of taxation. Government S knows that tomorrow it will not tax government debt ($\theta_2^S = 0$) and that the larger is R_2, the larger will be the interest rate net of taxes and the 'surprise redistribution' towards the "rich" group. Therefore, as long as the marginal redistributive benefit is larger than the marginal cost of taxation, government S will roll over a larger stock of debt as R_2 increases, using strategically the stock of debt to increase the redistribution towards the "rich". However, government S chooses this policy only when risk premia and government expenditure are not too large (p_0 and ψ small enough)[18].

The behavior of the other choice variables is straightforward. $\tau_1^{S,P}$ varies as $D_2^{S,P}$ with the opposite sign (fig. 2b). The equilibrium value of R_2 always increases with ψ and p_0 (fig. 2c) at a rate that more than compensates any reduction of $D_2^{S,P}$, so that $\tau_2^{S,P}$ always grows (fig. 2d).

3.2.2 Equilibrium existence

In the case of low levels of expenditure in period 1, we can state the following proposition:

Proposition 1: When $g_1 < 2\hat{\alpha}_N$, no pooling equilibria exist; a

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18. A right-wing government accumulates strategically government debt also in Alesina and Tabellini (1990), because it does not know whether it will be in power in the future and it wants to constrain the expenditure policy of future governments with different preferences. Here a right-wing government (type S) accumulates debt, because it knows it will be in power in the future and wants to take advantage of the risk premium to increase the size of wealth redistribution.

necessary and sufficient condition for the existence of a separating equilibrium is $\psi < \frac{(1-\gamma)(1-p_0)}{\gamma}$.

We discuss the intuition behind Proposition 1; a formal proof is in Appendix 2. Figure 3 shows the range of parameters where the separating equilibrium exists.

When $g_1 < 2\hat{\alpha}_N$, pooling equilibria do not exist because type N always deviates from the pooling equilibrium policy once period 1 is reached. The reason is that type N will be at an interior optimum, even if it reveals its type in period 1: the taxes levied in the last two periods will be optimal ($\tau_t = \hat{\alpha}_N$), as well as the amount of surprise taxation on government debt[19]. On the other hand, any pooling equilibrium requires type N to levy taxes in period 1 equal to those of type S ($\tau_1^{N,P} = \tau_1^{S,P}$) and different from $\hat{\alpha}_N$, so that type N incurs higher taxation costs than in the interior optimum. Since the public takes government's incentive into account, it anticipates that no pooling is possible in period 1 and demands a risk premium on the debt issued in period 0 ($R_1 > 1$).

The first requirement for the existence of a separating equilibrium is that type S's problem must have a corner solution with respect to θ_t ($\theta_t = 0$, see Lemma 1), otherwise no debt would be issued in period 0 and the concept of separating equilibrium itself would be meaningless; this leads to the inequality in Proposition 1. In fact, a large government expenditure relative to the type N's redistributive preferences(i.e. ψ too high), or a high probability p_0 of type

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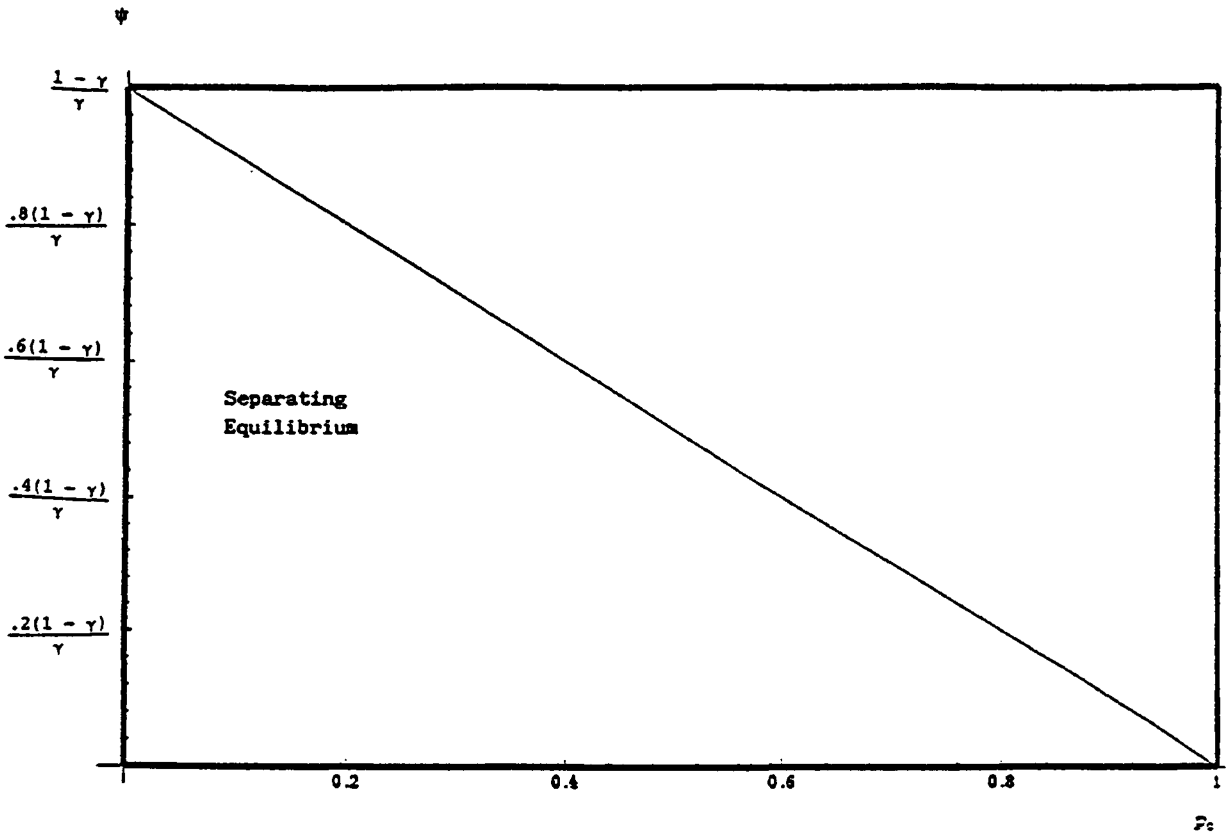
19. The redistributive costs of the tax on government debt are the same when the tax is levied in period 1 or in period 2, because they are linear and depend only on the size of the surprise taxation (so that they are unaffected by the level of the interest factor R_t).

Fig. 3

Case I $(g_1 < 2\hat{\alpha}_N)$ Existence separating equilibrium



N being in power would force type S to choose a positive tax on government debt in period 1, in order to avoid the high taxation costs. As discussed earlier (Proposition 1), the public would anticipate that both governments would attempt a surprise taxation and would not buy any debt in period 0. This instance can interestingly be interpreted as a financial crisis, where "bad" reputation can make the government unable to issue any debt.

If the condition for government S to be at a corner is satisfied, then the separating equilibrium policies of the two governments will be sequentially rational. In fact government N does not deviate in period 1 from the separating equilibrium since it can implement its optimal policy at no cost. Government S does not deviate either, since mimicking government N would require government S to choose $\theta_1 > 0$ and this, by Lemma 1, is in contrast with a positive stock of debt. Therefore, a separating equilibrium with a positive stock of debt exists only for the range of parameters satisfying the inequality in Proposition 1.

3.3 Case II ($g_1 > 2\hat{\alpha}_N$, deficit net of interest payments of type N in the last two periods)

3.3.1 Equilibrium definition

In order to characterize the equilibria in case II, we introduce a parameter, w, related to the expenditure distribution over time defined as:

$$w = \frac{D_1}{2\hat{\alpha}_N \psi} = \frac{g_0}{g_0 + g_1 - 2\hat{\alpha}_N}$$

In case II, w varies between 0 and 1. It is 0 when the expenditure in period zero is equal to 0, so that the indebtedness of the government is due only to the expenditure in period 1 ($g_1 > 2\hat{\alpha}_N$); it is equal to 1 when the second period expenditure is minimal for case II, i.e. equal to the "sustainable" expenditure of type N, $g_1 = 2\hat{\alpha}_N$. Case I corresponds instead to the case where w>1 ($g_1 < 2\hat{\alpha}_N$).

Pooling equilibrium

The equilibrium values for the pooling equilibrium are equal to case I.

Separating equilibrium.

The solution of type N's problem is straightforward. The type S's policy is derived by simply substituting the value of the equilibrium interest factor in the first order condition for D_2 and recalling that in a separating equilibrium $R_2(1 - \theta_2) = 1$ and $\partial[R_2(1 - \theta_2)]/\partial D_2 = 0$:

type N:

$$\theta_1^{N,S} = 1 \tag{35}$$

$$D_2^{N,S} = \hat{\alpha}_N = \tau_2^{N,S} \tag{36}$$

$$\tau_1^{N,S} = g_1 - \hat{\alpha}_N > \hat{\alpha}_N \tag{37}$$

type S:

$$\tau_1^{S,S} = \tau_2^{S,S} = D_2^{S,S} = \frac{D_1 + g_1(1 - p_0)}{2(1 - p_0)} > \hat{\alpha}_N \tag{38}$$

the economy:

$$R_1 = \frac{1}{1 - p_0} > 1 \tag{39}$$

The crucial difference between case I and case II is that type N in the separating equilibrium of case II is no longer at an interior solution. In fact the unconstrained optimal policy of government N would require $\tau_t = \hat{\alpha}_N$ and $\theta_1^{N,S} > 1$, which is clearly unfeasible since no government can tax more than fully the debt outstanding; therefore, government N's optimal policy is characterized by a corner solution for $\theta_1^{N,S}$ ($\theta_1^{N,S} = 1$) so that government N cannot follow the optimal unconstrained tax policy and chooses $\tau_1^{N,S} = g_1 - \hat{\alpha}_N > \hat{\alpha}_N$[20]. As we shall see, the fact that government N cannot achieve the unconstrained optimum in the separating equilibrium makes pooling equilibrium possible when government S's taxes in period 1 are not too large.

3.3.2 Equilibrium existence

In case II both pooling and separating equilibria may prevail; we shall discuss separately the conditions for their existence.

20. Type S's policy is different from case I: taxes of type S are no longer invariant to the distribution of expenditure over time (the larger is g_0 relatively to g_1 the larger are type S's taxes). In case I D_1 R_1 is invariant to g_0, because type N's interior solution implies that the total revenue from the surprise tax on government debt is constant, so that $\theta_1^{N,S}$ and correspondingly R_1 falls as D_1 increases. On the contrary, in case II, D_1 R_1 grows with D_1 because type N takes the opportunity of a higher D_1 to get a higher revenue from the tax on government debt; R_1 will also increase.

Existence of the pooling equilibrium

The following proposition gives the conditon for the existence of a pooling equilibrium in case II:

Proposition 2: when $g_1 > 2\hat{\alpha}_N$, a necessary and sufficient condition for the existence of a pooling equilibrium is $\underline{\psi}^P < \psi < \overline{\psi}^P$, where

$$\underline{\psi}^P = \frac{(1-\gamma)p_0(1-p_0)}{\gamma[1+w-(1-w)(1-p_0)^2]}$$

$$\overline{\psi}^P = \begin{cases} \overline{\psi}_1^P, & \text{for } 0 < p_0 < Max\left[1-\sqrt{\dfrac{w}{(1-w)}},\ 0\right] \\ Min\left[\overline{\psi}_1^P, \overline{\psi}_2^P\right], & \text{for } Max\left[1-\sqrt{\dfrac{w}{(1-w)}},\ 0\right] < p_0 < 1 \end{cases}$$

where

$$\overline{\psi}_1^P = \frac{(1-\gamma)\ (2-p_0)}{2\gamma}$$

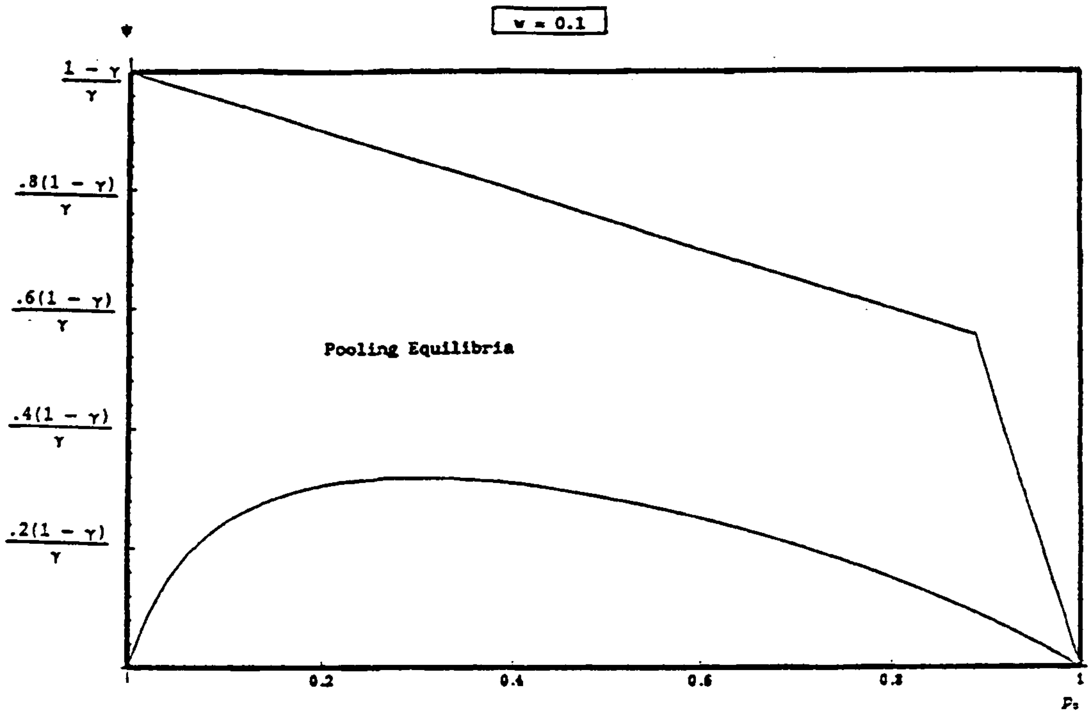
$$\overline{\psi}_2^P = \frac{(1-\gamma)\ p_0\ (1-p_0)}{2\gamma[w-(1-w)(1-p_0)^2]}$$

We discuss here the intuition behind Proposition 2; a formal proof is in Appendix 3. Figure 4 illustrates Proposition 2 by showing how the range of parameters where pooling equilibria exist shrinks as w goes to 1.

Lemma 1 requires government S's problem to have a corner solution with respect to θ_t ($\theta_t^{S,P} = 0$). This requirement

Fig. 4a

Case II: Existence of pooling equilibria



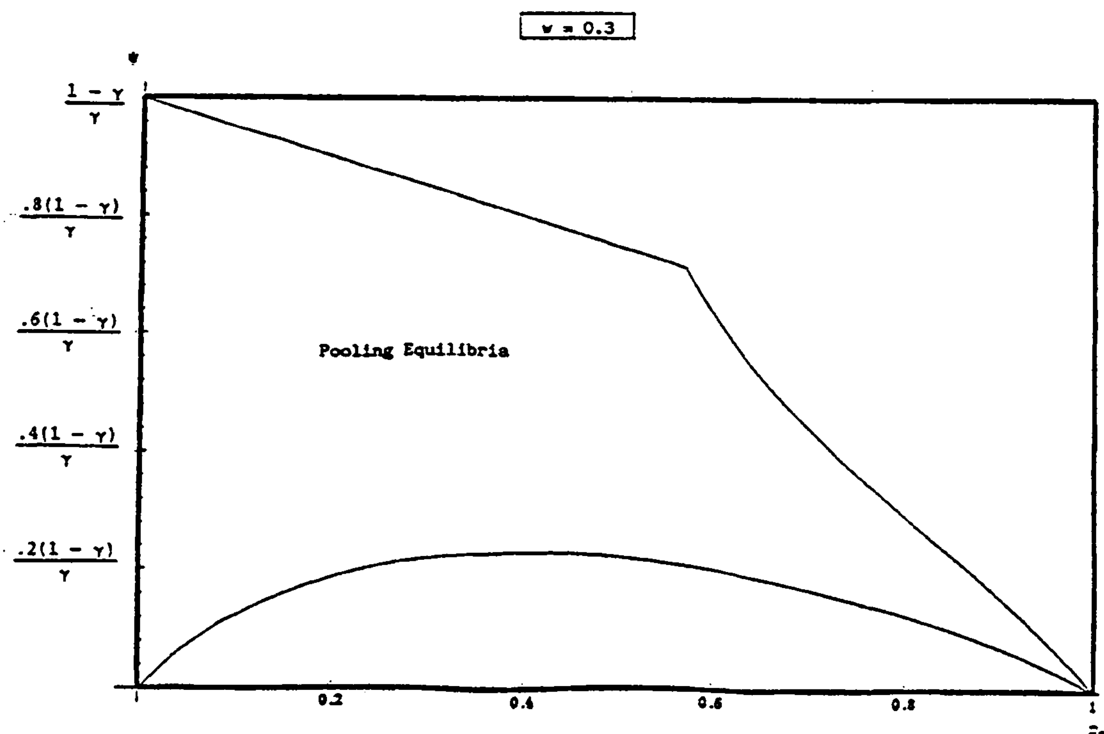
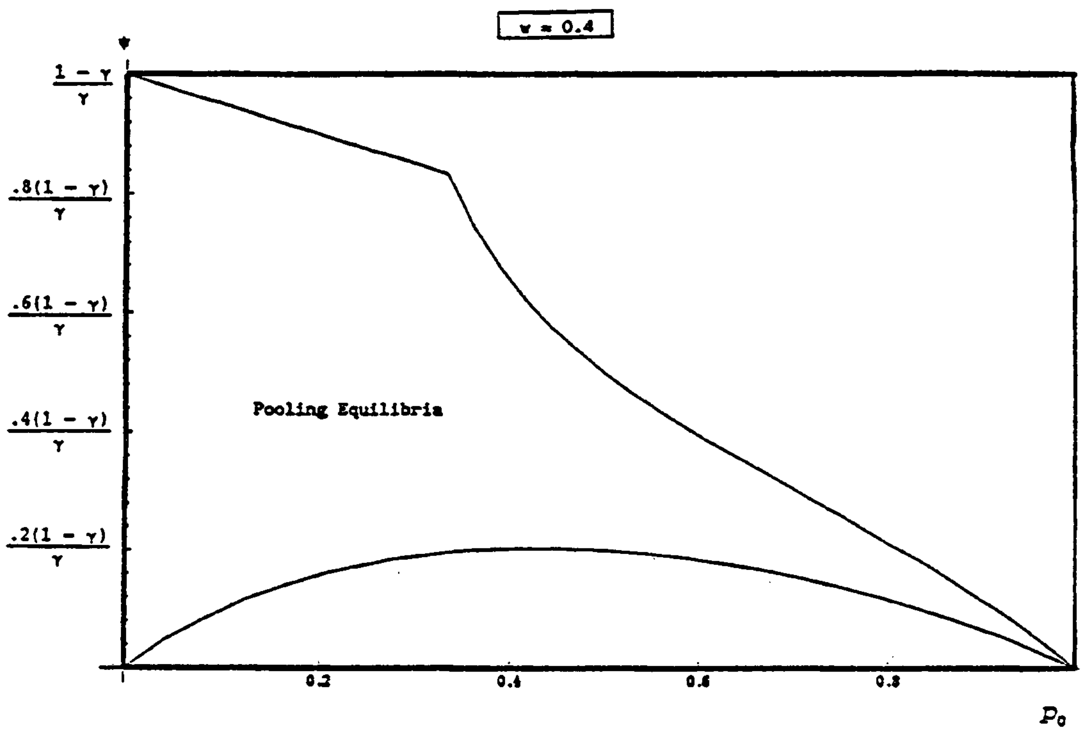


Fig. 4b

Case II: Existence of pooling equilibria



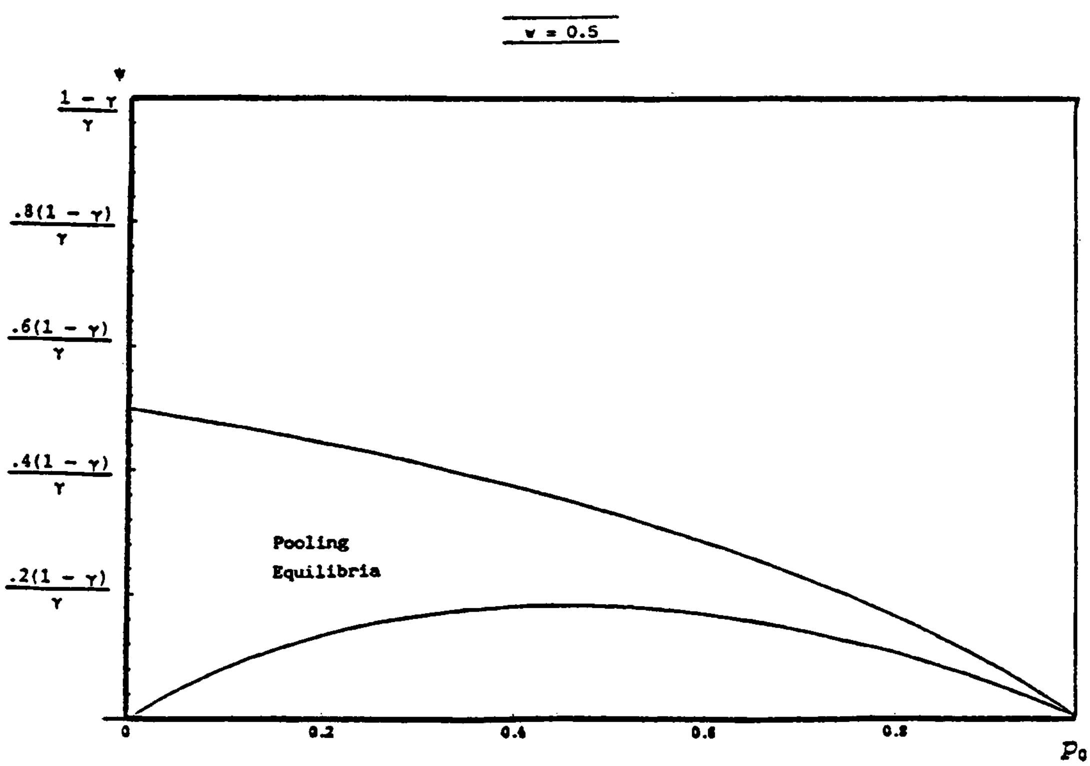
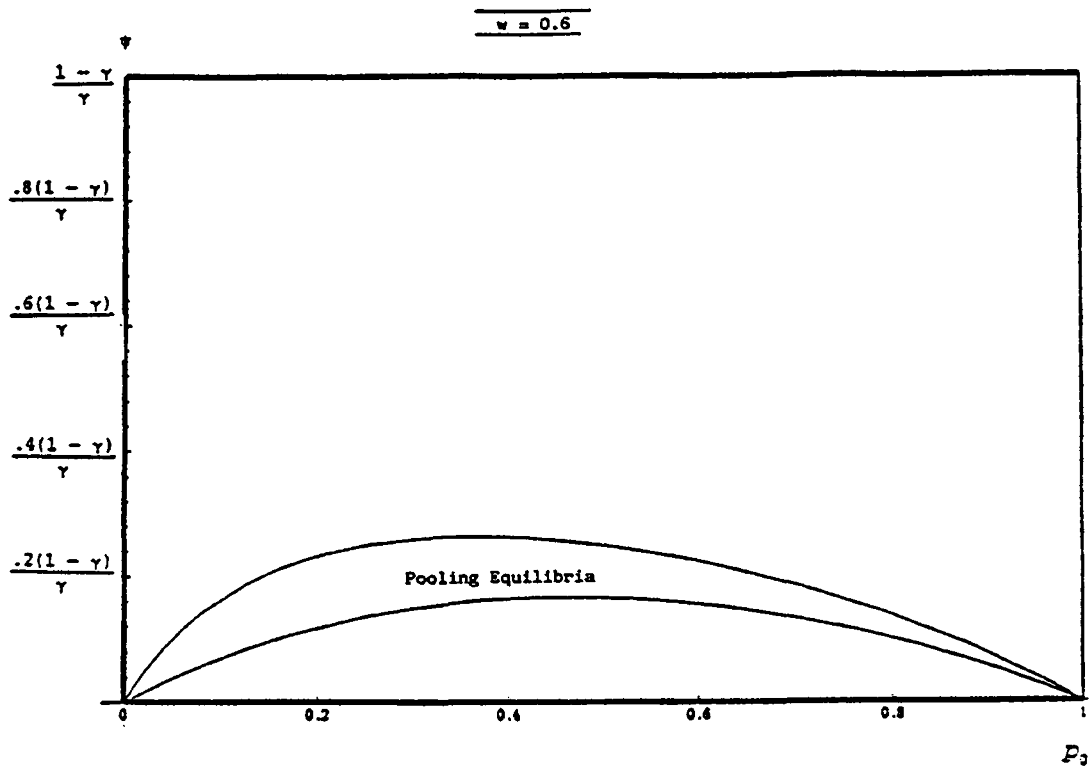


Fig. 4c

Case II: Existence of pooling equilibria



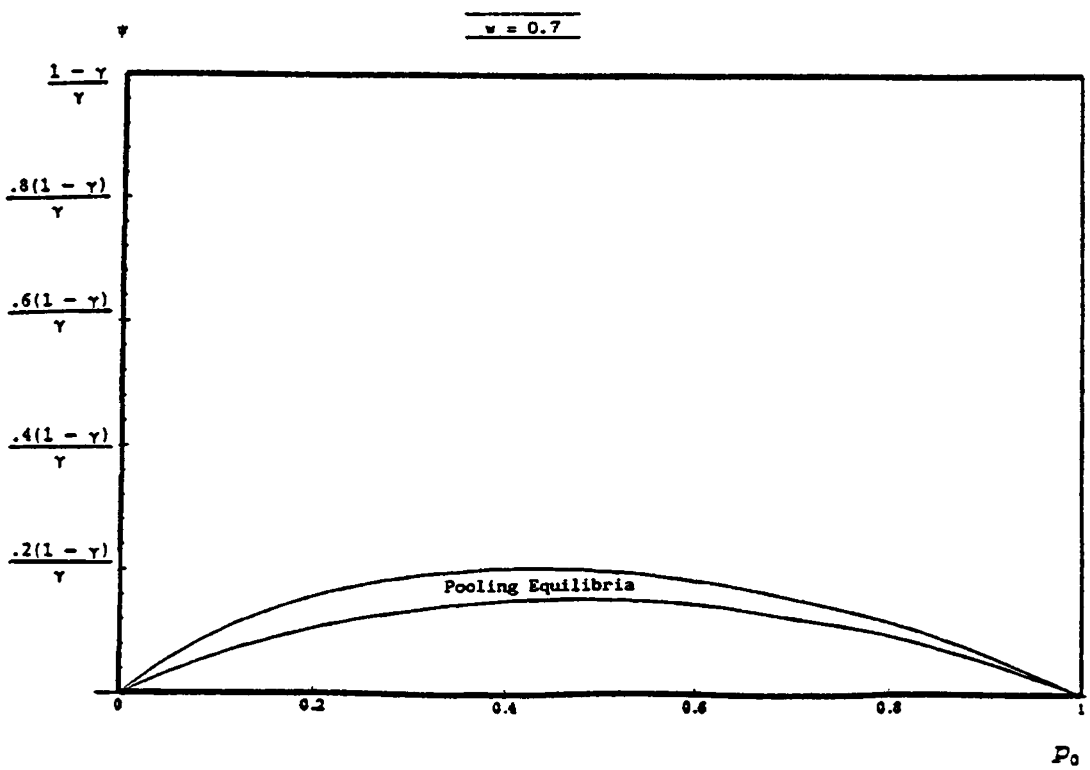
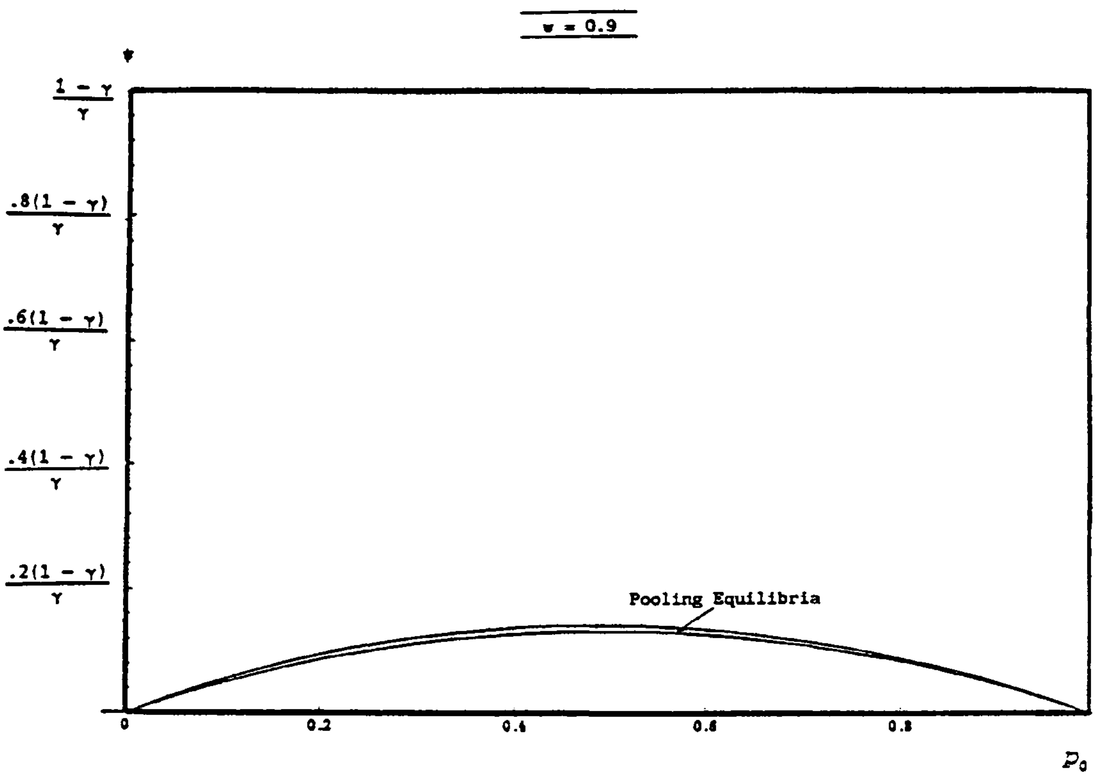


Fig. 4d

Case II: Existence of pooling equilibria



is satisifed when ψ is smaller than Ψ_1^P. Note that the range of parameters for which the inequality holds is larger than in the separating equilibrium of case I (Ψ_1^P > $[(1-\gamma)(1-p_0)]/\gamma$, see Proposition 1). The reason is that in a pooling equilibrium the risk premium is paid on the debt issued in period 1, while in a separating equilibrium the risk premium is paid on the debt issued in period zero; given that the debt issued in period 1 is smaller, the same prior probability p_0 corresponds to less interest payments. As a result, in a pooling equilibrium higher p_0 than in a separating equilibrium are necessary to increase interest payments enough to make government S tax government debt and cause a financial crisis.

Proposition 2 also states that ψ must be smaller than Ψ_2^P and greater than $\underline{\psi}^P$ in order to have a separating equilibrium, because otherwise government N would deviate in period 1 and the pooling equilibrium strategy would not be sequentially rational. In fact, if ψ is too high, the pooling equilibrium taxes in period 1 are larger than those type N would levy if it revealed its type, $\tau_1^P > g_1 - \hat{\alpha}_N$, so that type N would not mimick type S. Similarly, if ψ is too low, τ_1^P is enough smaller than the optimal level of taxes of type N, $\hat{\alpha}_N$, to make it prefer to separate[21].

It is important to notice that the upper bound Ψ_2^P gets smaller as w gets larger since the shift of expenditure from period 1 to period 0 reduces $g_1 - \hat{\alpha}_N$ and type N's incentive to

21. In a pooling equilibrium, type S may choose a very low τ_1^P when either ψ or p_0 are small, because the goal of redistributing towards the "rich" leads to an increase of D_2^P and a corresponding reduction of τ_1^P (see section 3.2.1).

mimick. In the limit, when w = 1, $\underline{\Psi}^P = \overline{\Psi}^P$ and there is no range of parameters where a pooling equilibrium exists.

No other conditions are considered in Proposition 2 because government S never deviates from a pooling equilibrium. The result reflects the redistributive goal of type S: if type S deviates from a pooling equilibrium in period 1 and increases taxes enough to separate from type N, it will not pay a risk premium in period 0 nor in period 1; as a result, it will not take advantage of the risk premium to redistribute towards the "rich". It turns out that, in the range of parameters where type S's problem has a corner solution with respect to θ_t, $(\psi < \overline{\Psi}_1^P)$, type S prefers a positive risk premium to a zero risk premium and never deviates from the pooling equilibrium strategy.

Existence of the separating equilibrium

For a separating equilibrium we can state the following proposition:

Proposition 3: when $g_1 > 2\hat{\alpha}_N$, a necessary and sufficient condition for the existence of a separating equilibrium is $p_0 > \overline{p} = Max\ [\frac{1-2w}{1-w}, 0]$

and $\underline{\psi}^S < \psi < \overline{\Psi}^S$,

where:

$$\overline{\Psi}^S = \frac{(1-\gamma)(1-p_0)}{\gamma[1-p_0(1-w)]}$$

$$\underline{\psi}^S = Min\ \{\ \underline{\psi}_1^S\ ,\ Max[\underline{\psi}_2^S, \underline{\psi}_3^S]\ \}$$

$$\underline{\psi}_1^S = \frac{(1-\gamma)(1-p_0)\ \left[\ (1-p_0) + \sqrt{\dfrac{1+(1-p_0)^2}{2}}\ \right]}{\gamma\ (2-p_0)\ \left[(1-p_0) + w\,p_0\right]}$$

$$\underline{\psi}_2^S = \frac{(1-\gamma)\ p_0\ (1-p_0)^2}{2\gamma\ \left[\ w - (1-w)(1-p_0)^3\ \right]}$$

$$\underline{\psi}_3^S = \frac{2\ (1-\gamma)\ w\,p_0}{\gamma\ \left\{2\ \left[\ (1-w)^2(1-p_0)^2 + \dfrac{w^2}{(1-p_0)^2}\ \right] - \left[(1-p_0) + w\,p_0\right]^2\right\}}$$

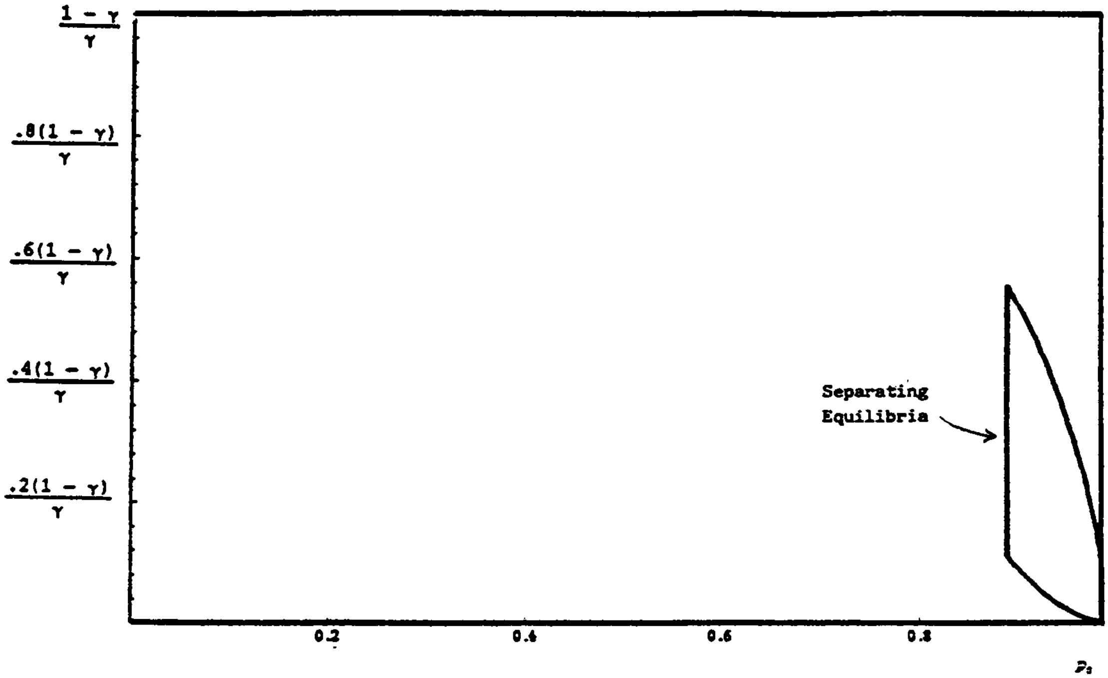
Figure 5 illustrates Proposition 3 by showing how the range of parameters where separating equilibria exist grows as w goes to 1. The intuition behind Proposition 3 is the following (a formal proof is in Appendix 4): the condition $\psi<\overline{\psi}^S$ guarantees that type S's problem has a corner solution with respect to θ_t ($\theta_t^{S,S}=0$). The range of parameters satisfying the condition is large when w is close to zero and shrinks as it grows; when w=1, the range is identical to the one of case I (see Proposition 1). When w is close to zero, financial crises are then possible only for extreme values of the parameters, because little debt is issued in period 0 ($D_1=g_0$ is small) and only high ψ and p_0 would make R_1 high enough to force type S to tax government debt.

In a separating equilibrium, government N will signal its nature in period 1 only if it is not better off mimicking type S's policy. This is the case only if $\tau_1^{S,S} > g_1 - \hat{\alpha}_N$, which amounts to require $p_0 > \overline{p}$. The larger is w, the smaller is g_1 and the easier is to satisfy the inequality: as w grows, $\overline{p}$ falls and the condition becomes not binding for w > 1/2. Instead, when w is small, the debt issued in period 0 is small and only a large p_0 causes a risk premium large enough to increase interest payments and type S's separating equilibrium taxes above the critical level, i.e. $\tau_1^{S,S} > g_1 - \hat{\alpha}_N$.

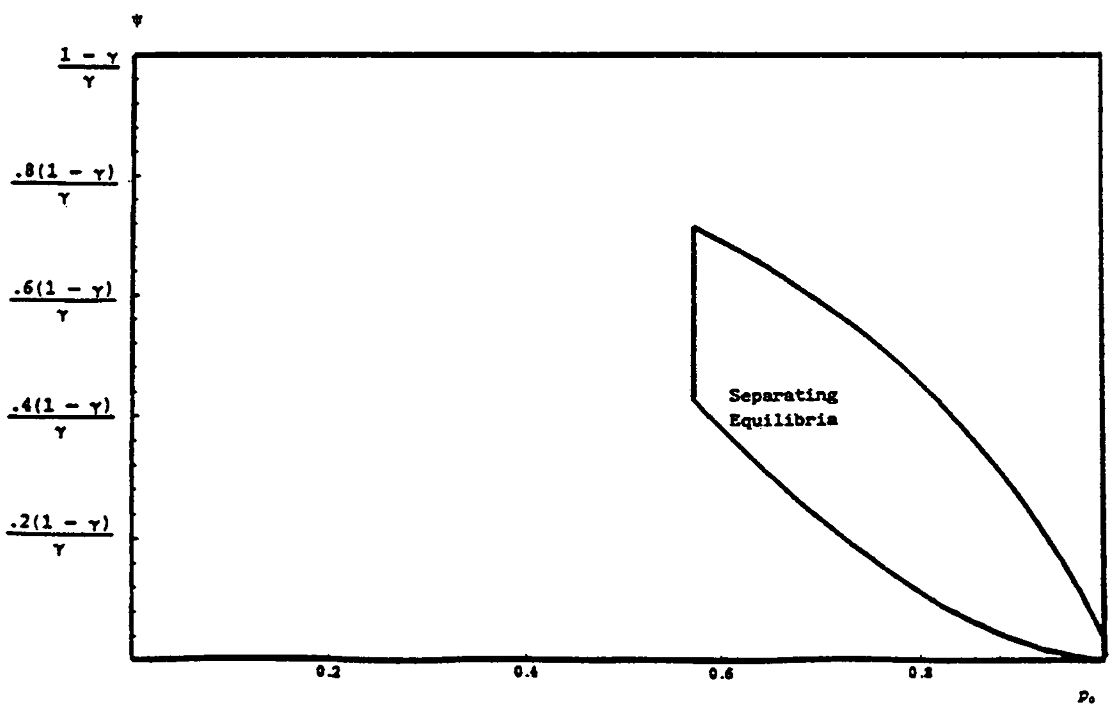
Fig. 5a

Case II: Existence of separating equilibria

$\psi = 0.1$

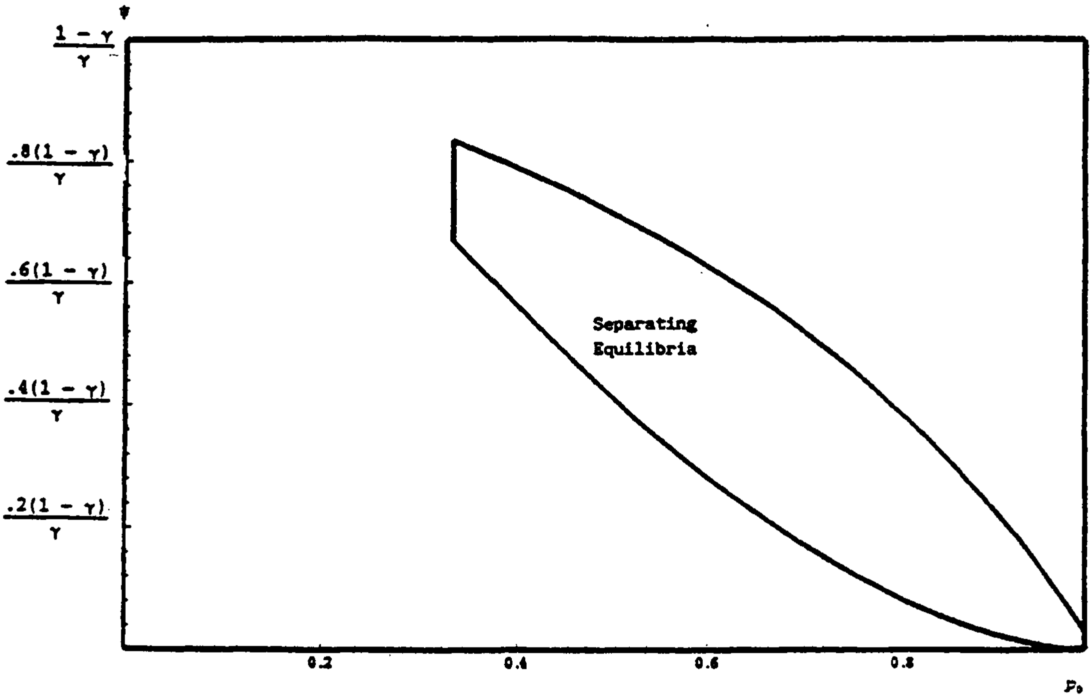


$\psi = 0.3$



Case II: Existence of separating equilibria

w = 0.4



w = 0.5

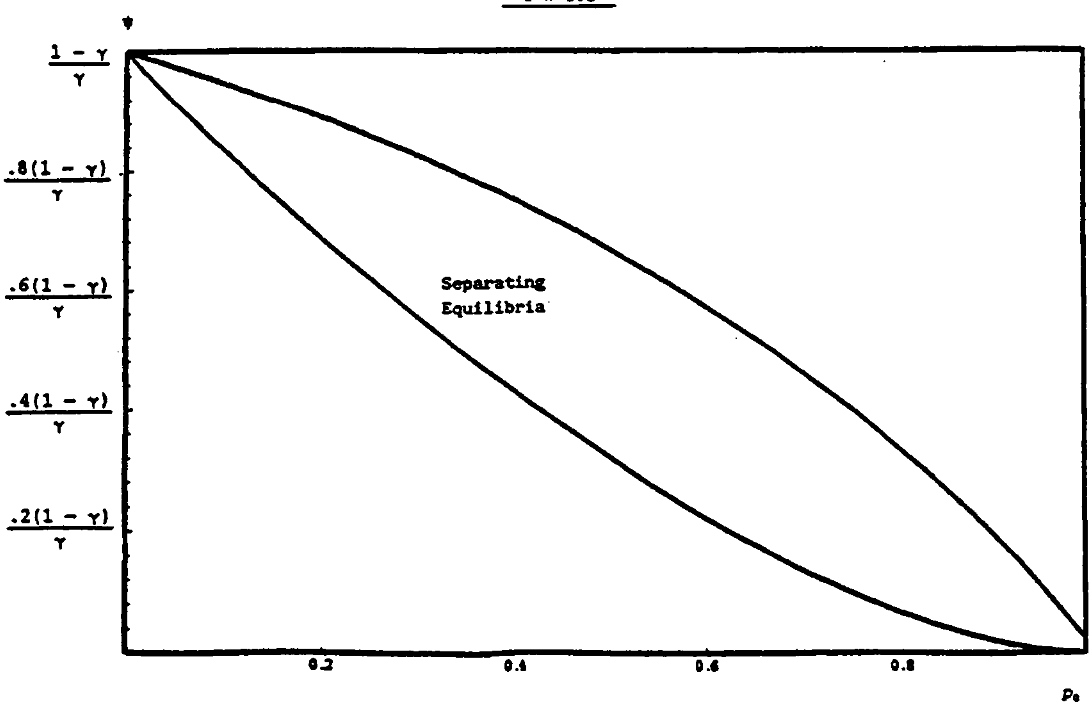
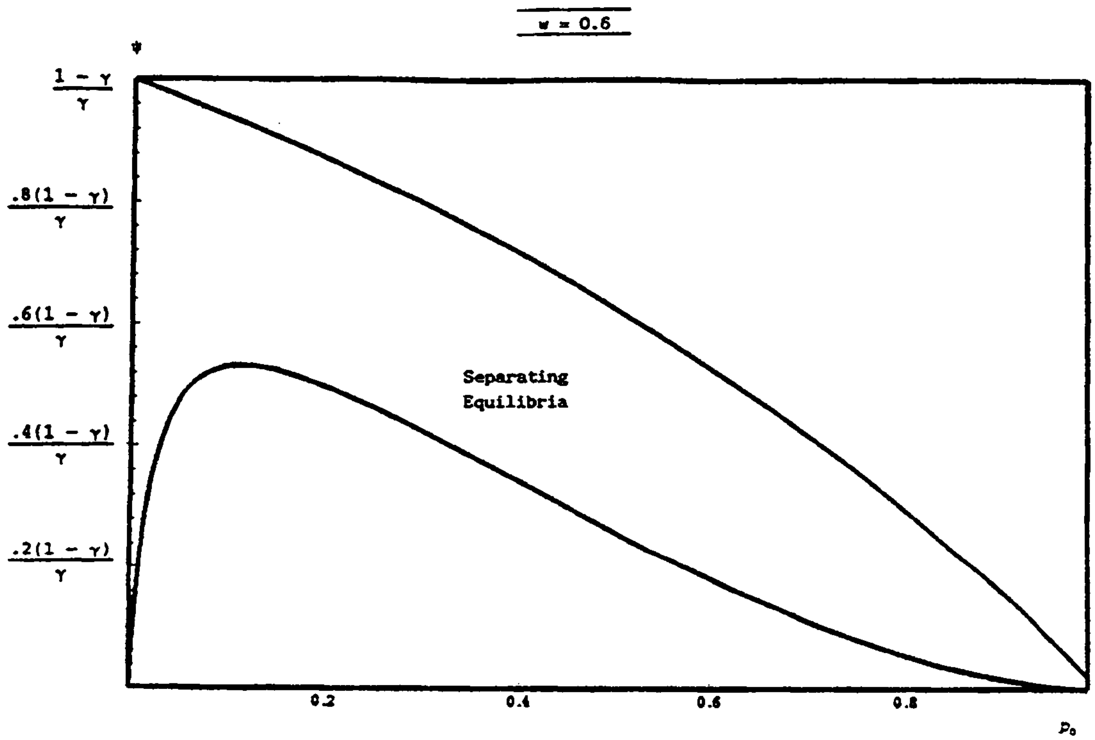


Fig. 5c

Case II: Existence of separating equilibria



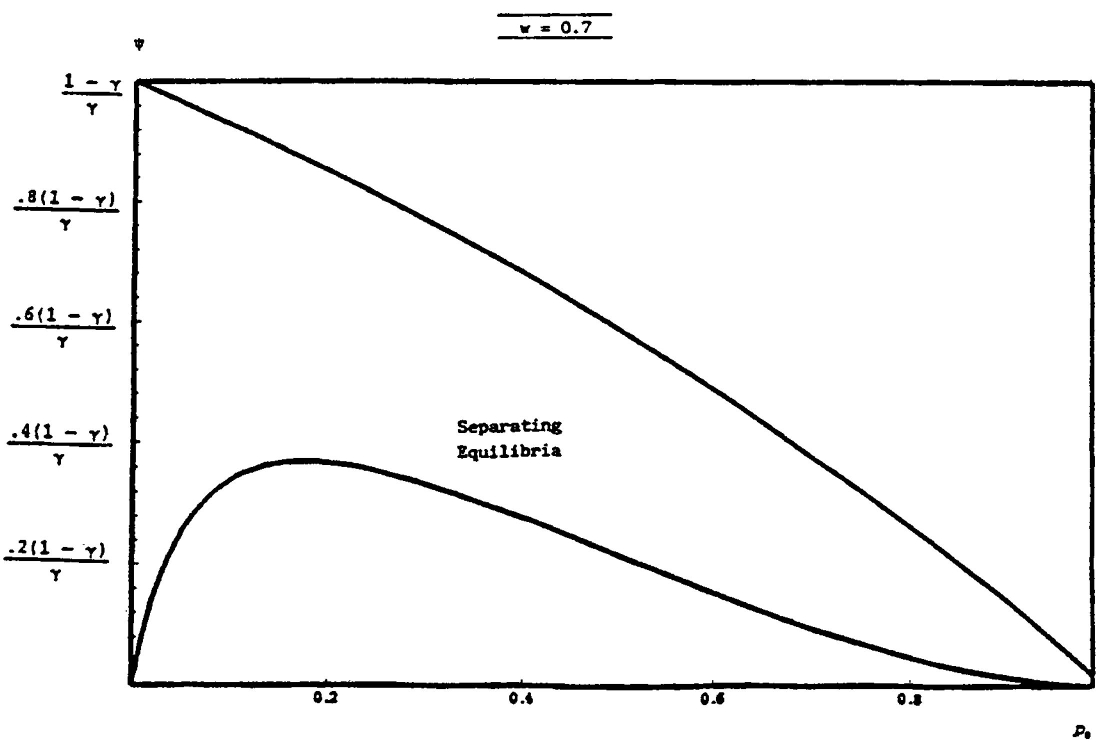
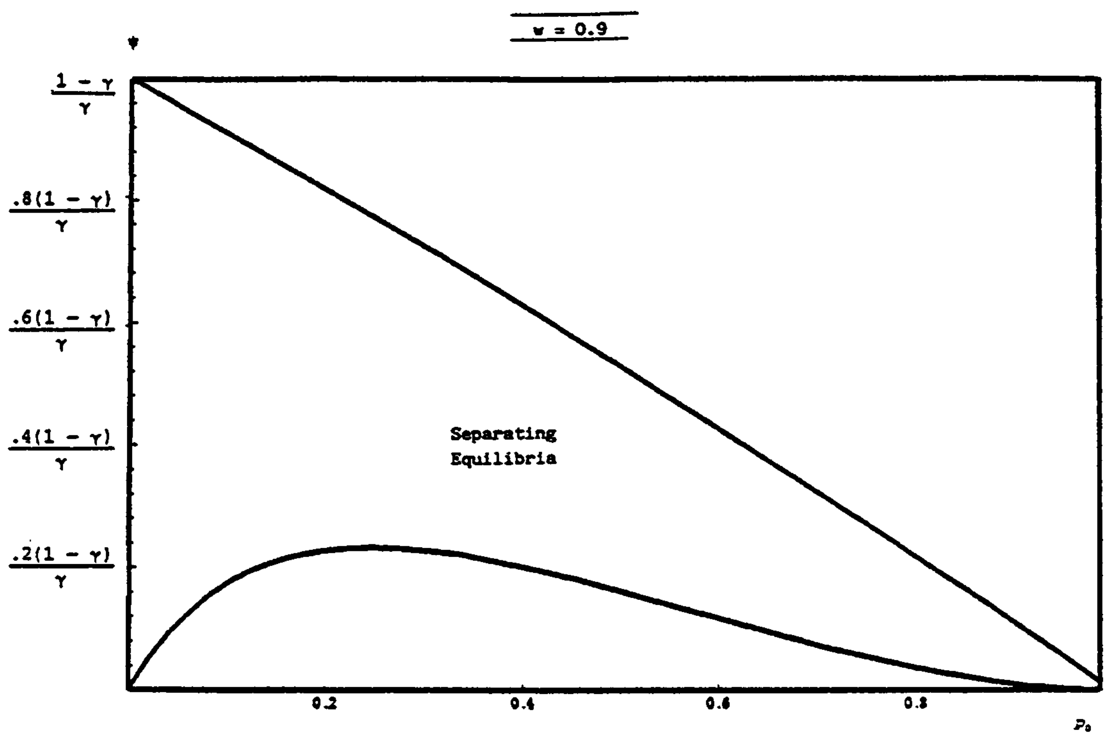


Fig. 5d

Case II: Existence of separating equilibria



In the deviation from the separating equilibrium, type S would choose a level of taxes in period 1 smaller than $g_1 - \hat{\alpha}_N$, so that it could be mimicked by type N. The rationale would be to pay a risk premium also on the debt issued in period 1 (in a separating equilibrium a risk premium is already paid on the debt issued in period 0) in order to increase the redistribution of wealth towards the "rich". Type S will weight the redistributive benefits of the deviation policy against the costs of the additional taxation. It turns out (the details are in Appendix 4) that for low enough ψ, type S prefers to deviate and the separating equilibrium policy is not sequentially rational. Separating equilibria could then exist only for $\psi > \underline{\psi}^S$. The same intuition accounts for the enlargement of the separating equilibrium range as w goes to 1, shown in Figure 5.

4. Equilibium summary (Case II)

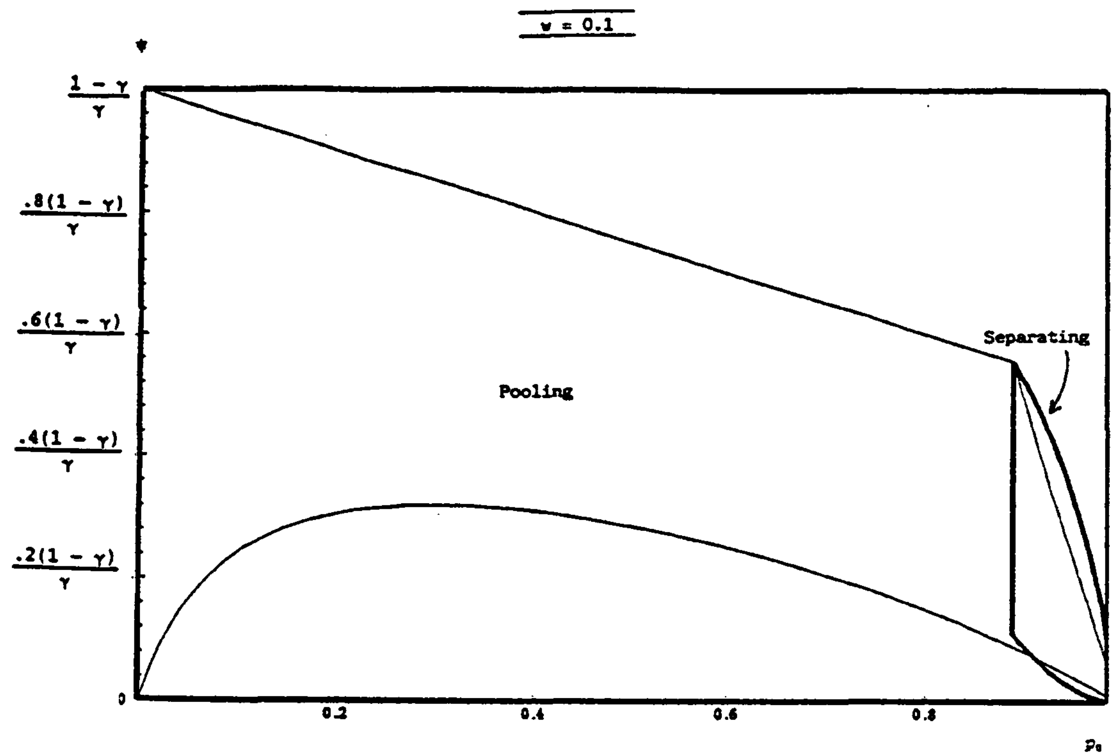
Figure 6 combines in the same graph the range of parameters where the pooling and the separating equilibria exist (fig. 4 and 5).

When $w < 0.5$, the <u>separating equilibria</u> exist only for relatively high values of p_0 (the prior probability of type N being in power). In fact, when $w < 0.5$, the expenditure g_0 is small and so is the debt issued in period 0; only if p_0 is large, the interest payments in period 1 will be large enough to make type S choose a level of taxes in period 1, that will not be mimicked by type N. When $w > 0.5$, this problem does not arise and the separating equilibria exist also for small values of p_0.

When $w < 0.5$, the <u>pooling equilibria</u> prevail, since the expenditure g_1 in period 1 is large and so are the incentives of type N to mimick type S (if type N does not mimick,

Fig. 6a

Comparison of pooling and separating equilibria



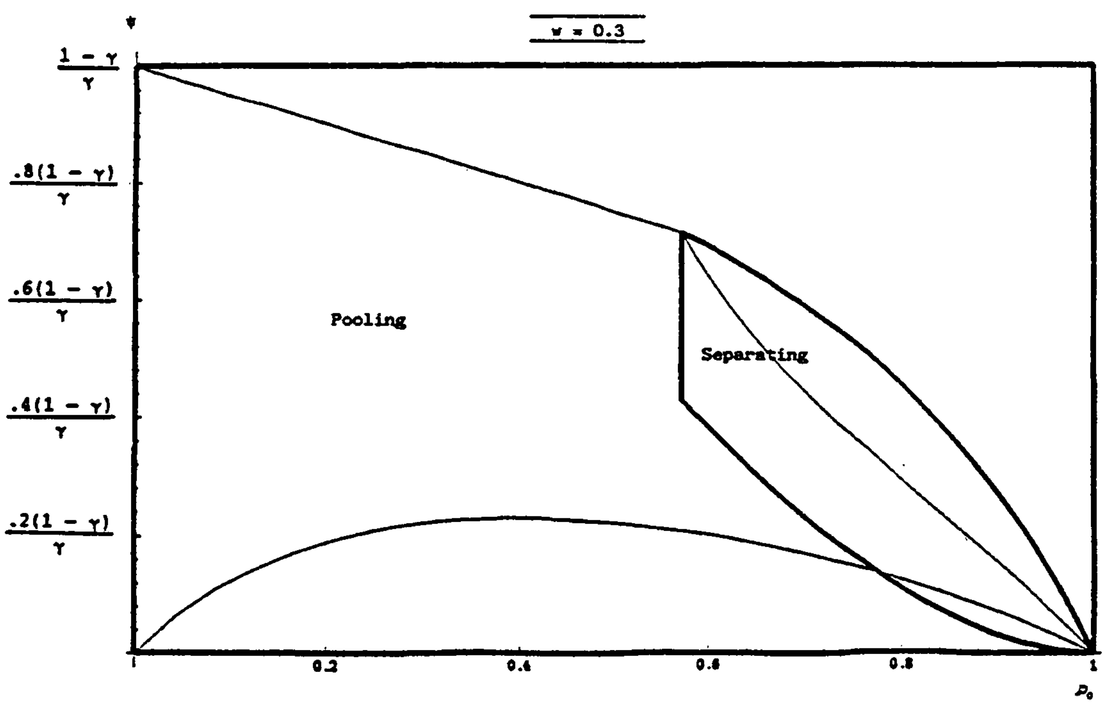


Fig. 6b

Comparison of pooling and separating equilibria

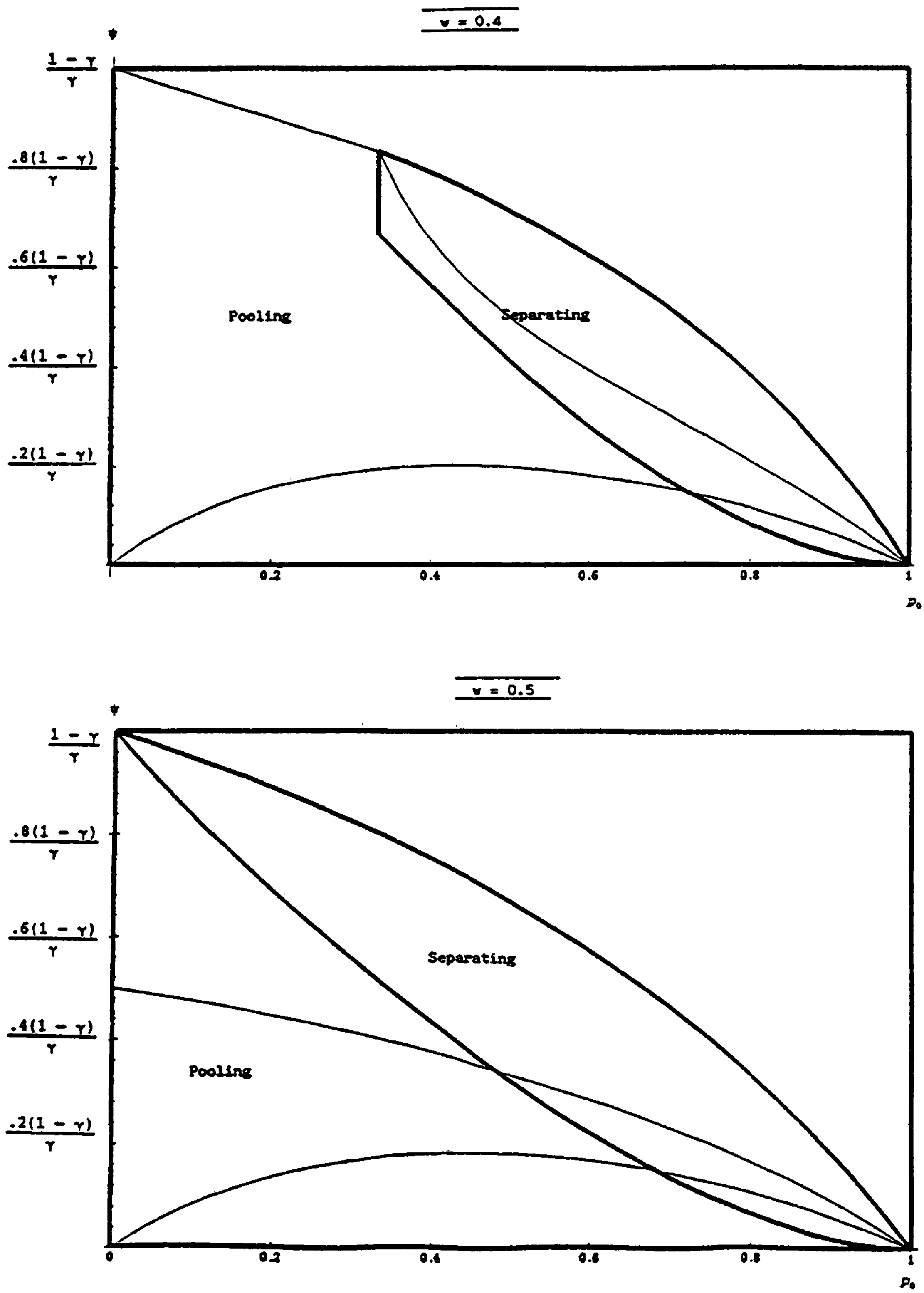
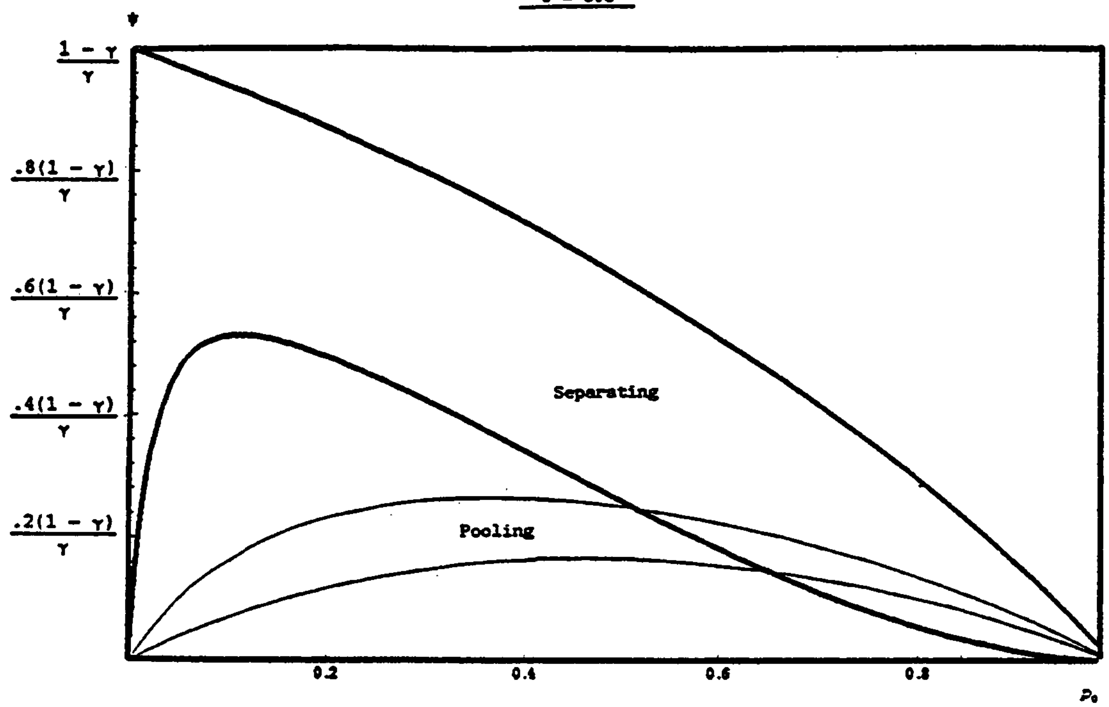


Fig. 6c

Comparison of pooling and separating equilibria

$\psi = 0.6$



$\psi = 0.7$

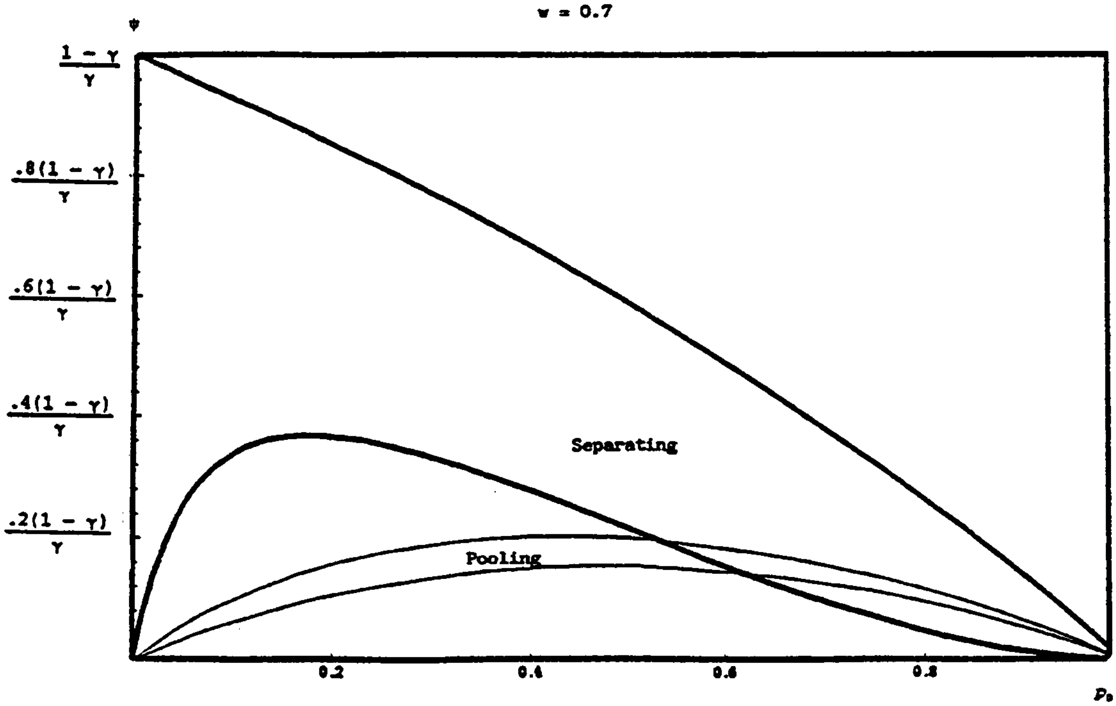
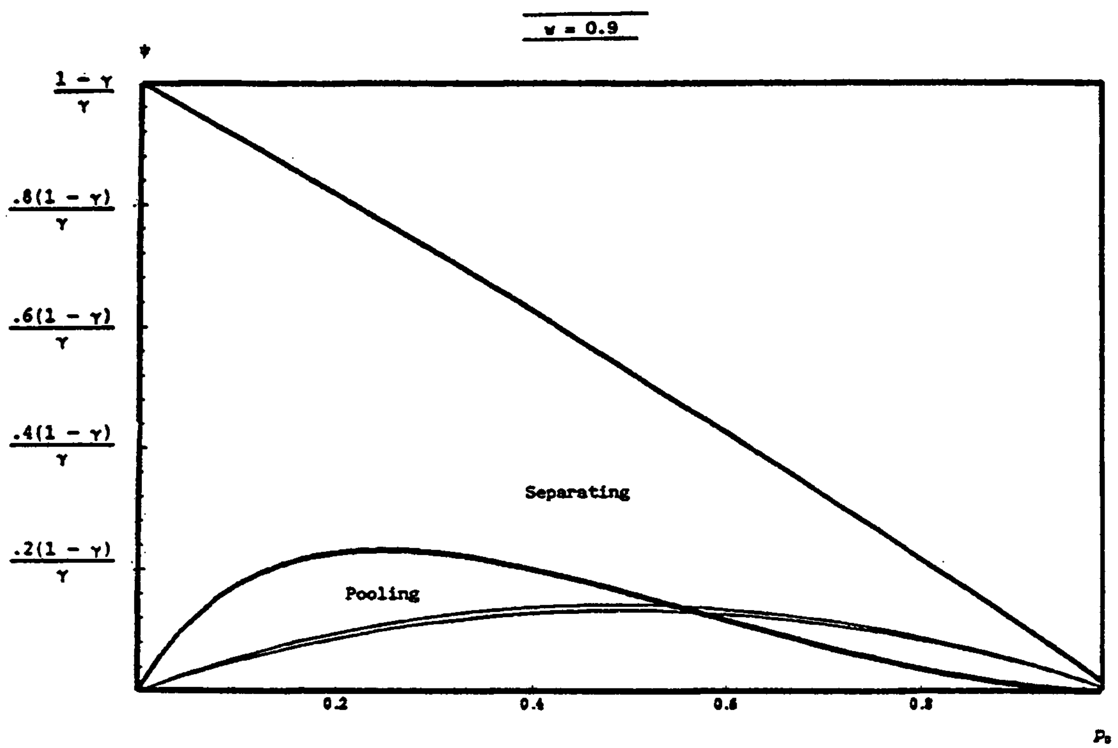


Fig. 6d

Comparison of pooling and separating equilibria



everybody will know that it is in power and it will be unable to finance g_1 by issuing debt). As w becomes larger than 0.5, the incentives to mimick get smaller and the range of pooling equilibria shrinks.

When p_0 is relatively large, multiple equilibria are possible: the smaller is w, the larger is the multiple equilibrium range.

No equilibria with a positive stock of debt exist when the total expenditure (measured by the parameter ψ on the vertical axis) and p_0 are high (upper right-hand side of all graphs); in this range, no debt can be sold ("financial crisis"), since the interest payments are high and the public knows that both types would attempt a surprise ex-post tax on the stock of debt. No equilibria exist also when ψ is low, unless p_0 is either very low or very high (bottom of the box); in this area, neither the pooling equilibrium holds, because type N in period 1 would consider type S's taxes too low and deviate from it, nor the separating équilibrium holds, because type S would "break" it by choosing a level of taxes in period 1 so low that type N mimicks. In both cases, type S chooses a low level of taxes in period 1, in order to increase the stock of debt rolled over to the last period and in turn the redistribution of wealth towards the "rich". Finally, when w > 0.5, no equilibria exist for an intermediate range of ψ and small p_0, since type S deviates from the separating equilibrium, in order to increase the redistribution of wealth, and type N deviates from the pooling equilibrium, because period 1 taxes of type S are too large.

5. The fiscal pre-requisites of the Maastricht Treaty

We consider now whether the Maastricht Treaty pre-requisites prevent countries subject to default risk from entering the monetary union. In order to perform the conceptual experiment we are interested in, consider the end of period 1 as the time by which the fiscal pre-requisites of the Maastricht Treaty should be met.

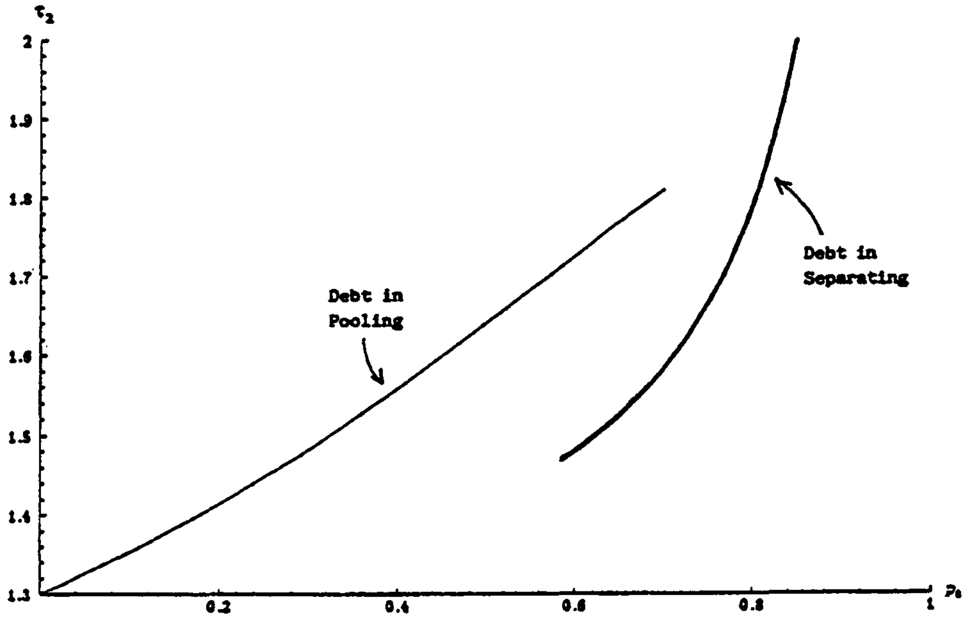
The deficit pre-requisite (less than 3 per cent of GDP) should certainly be able to identify type S governments. In fact, deficits smaller than 3 per cent of GDP imply in most high debt countries huge primary surpluses, that we showed to correspond to an early resolution of uncertainty (case I). Defaulting governments (type N) would then be identified before the monetary union deadline.

Figure 7 helps us discussing the debt pre-requisite (less than 60 per cent of GDP) by showing - for w=.4 and different values of ψ - the debt maturing in period 2 when type S is in power, $\tau_2^{S,i}$ (where i=S or N depending on whether the separating or the pooling equilibrium exists). The figure shows that the maturing debt increases with the prior probability p_0, because of the growing interest payments; as a consequence, the Maastricht debt threshold, that can be thought as a horizontal line in Figure 7, will tend to leave out of the monetary union the governments with a "bad" reputation. However, these governments are those that in equilibrium signal to be type S (high levels of p_0 correspond to separating equilibria), while the governments with a "good" reputation and a low level of debt do not signal and may actually be type N (low levels of p_0 correspond to pooling equilibria). In this example, the debt pre-requisite <u>alone</u> has perverse implications, since it leaves out of the union the type S governments and it lets in the type N governments. The debt prerequisite has, instead, the desired effect in the range where multiple equilibria exist (intermediate values of p_0), since in that range it favors the separating equilibria.

The policy implications of the model discussed in the paper should be considered with caution. First, the exclusion of default-risk countries may not be the only rationale for the

Fig. 7a

Debt maturing in period 2 (ψ = .3, γ = .5 and w = .4)



Debt maturing in period 2 (ψ = .5, γ = .5 and w = .4)

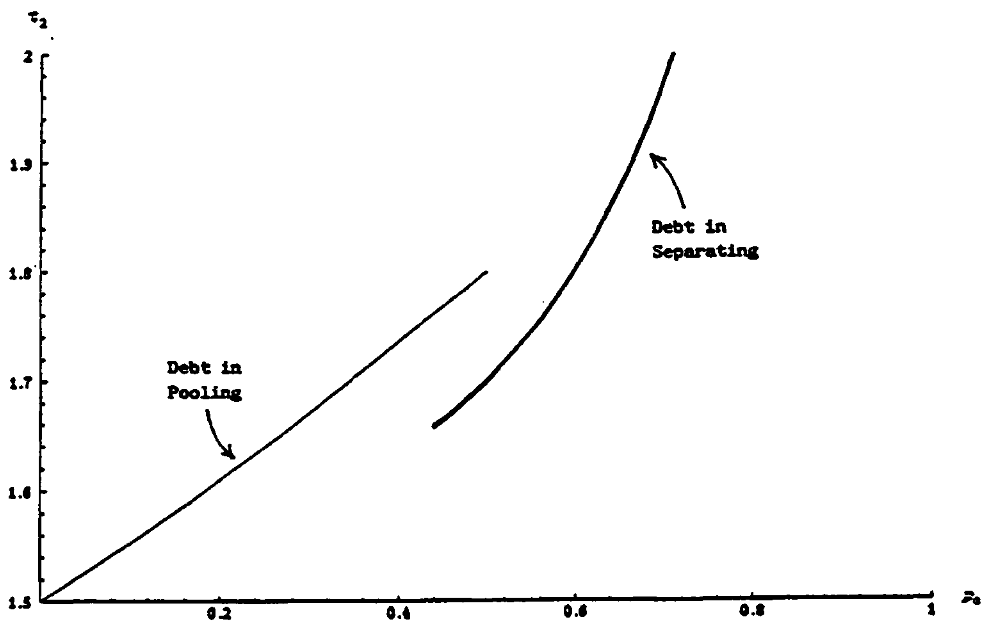
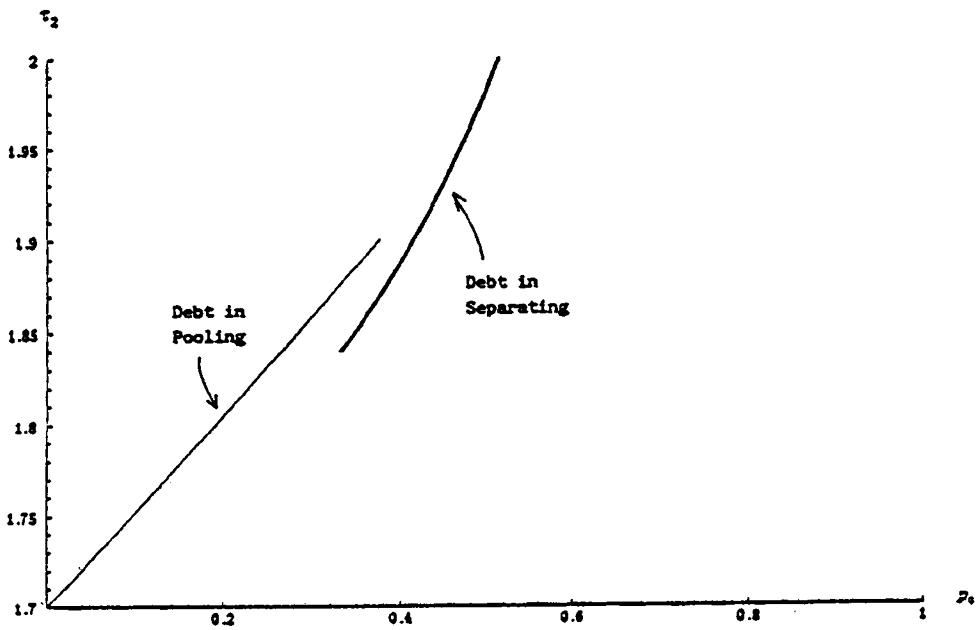


Fig. 7b

Debt maturing in period 2 ($\psi = .7$, $\gamma = .5$ and $w = .4$)



Maastricht rules, as mentioned in the Introduction. Secondly, the model assumes that the same government is in power in all periods; this assumption implies that, once the government in power signals its type, no risk premia are paid on government debt; this may not be the case in the real world, where frequent government changes could make impossible a complete resolution of uncertainty. Finally, we assume an exogenous process for the government expenditure and study the optimal policy to finance it; but, if the expenditure path were determined endogenously (the fiscal pre-requisites of EMU could be met by cutting the expenditure as well as by increasing the taxes), the policy implications could be different; we plan to extend our research in this direction.

Appendix 1

Lemma A1: a necessary and sufficient condition for type N to choose either $\theta_1 > 0$ or $\theta_2 > 0$ is $D_1+g_1>2\hat{\alpha}_N$.

Proof:

Note that if type N chooses $\theta_1 = \theta_2 = 0$ at $R_1 = R_2 = 1$, then it will do so for any R_1 and R_2 greater than 1. From the first order conditions, evaluated at $R_1 = R_2 = 1$ we have:

$$(1 - \theta_1) D_1 = \frac{(2 - \theta_2)}{(1 - \theta_2)} \hat{\alpha}_N - g_1$$

For necessity, assume $\theta_1 = 0$. Then:

$$\theta_2 = \frac{D_1 + g_1 - 2\hat{\alpha}_N}{D_1 + g_1 - \hat{\alpha}_N}$$

which is > 0 only if $D_1 + g_1 > 2\hat{\alpha}_N$.

Assume $\theta_2 = 0$. Then:

$$\theta_1 = \frac{D_1 + g_1 - 2\hat{\alpha}_N}{D_1}$$

which is > 0 only if $D_1 + g_1 > 2\hat{\alpha}_N$.

For sufficiency, consider

$$D_1 + g_1 - 2\hat{\alpha}_N = \theta_1 D_1 + \theta_2 D_1 (1 - \theta_1) - \theta_2 (\hat{\alpha}_N - g_1)$$

so that:

$$D_1 + g_1 - 2\hat{\alpha}_N > 0 \rightarrow \theta_1 D_1(1 - \theta_2) + \theta_2(D_1 + g_1 - \hat{\alpha}_N) > 0$$

which requires either $\theta_1 > 0$ or $\theta_2 > 0$.

Appendix 2

Proof of Proposition 1

The proof is organized in four steps[22]: a) type S is at a corner (i.e. he prefers to choose $\theta_1 = \theta_2 = 0$) in a separating equilibrium in the required parameter range; b) type N never deviates from a separating equilibrium; c) type S never deviates from a separating equilibrium; d) type N always deviates from a pooling equilibrium.

a) To prove the first step we formulate the following:

Lemma A2: when $g_1 < 2\hat{\alpha}_N$, a necessary and sufficient condition for type S to choose $\theta_1 = \theta_2 = 0$ in a separating equilibrium is $\psi < \frac{(1-\gamma)(1-p_0)}{\gamma}$

Proof: for the S type to be at a corner, the following condition has to be satisfied:

$$\tau_1^{S,S} = \tau_2^{S,S} = D_2^{S,S} < \hat{\alpha}_S$$

$$\rightarrow \psi < \frac{(1-\gamma)(1-p_0)}{\gamma}$$

b) type N deviates from a separating equilibrium iff he gets a higher welfare from mimicking the S type, i.e iff $W_N^S > W_N^{SD}$, where W_i^j indicates the welfare of type i in the equilibrium (or deviation from it) j:

22. In order to impose sequential rationality and check the off-equilibrium strategies, since Bayes' rule does not apply outside the equilibrium, we assume that, if $\theta_1 > 0$ or $\tau_1 < \tau_1^{S,i}$, $i = S, P$ is observed, then the bondholders infer that they face a type N government.

$$-2\hat{\alpha}_N^2-2\hat{\alpha}_N(2\hat{\alpha}_N-g_1) < -(\tau_1^{S,S})^2-\hat{\alpha}_N^2+2\hat{\alpha}_N D_1 R_1+2\hat{\alpha}_N D_2^{S,D}(R_2^{S,D}(1-\theta_2^{S,D})-1)$$

$$\rightarrow \qquad (\tau_1^{S,S} - \hat{\alpha}_N)^2 < 0$$

which never occurs.

c) In the separating, if type S mimicked type N, it would choose $\tau_1^{S,S} = \tau_1^{N,S} = \hat{\alpha}_N$ and $\theta_1^{S,S} = \theta_1^{N,S} > 0$. But the latter is not compatible with a positive stock of debt (see Proposition 1). As a result, type S never deviates from a separating[23].

d) In the deviation from the pooling equilibrium, type N plays $\tau_1^{N,P} = \hat{\alpha}_N$ instead of $\tau_1^{N,P} = \tau_1^{S,P}$. Type N deviates iff the welfare from staying in the pooling is lower than the welfare in the deviation, i.e. iff $W_N^P < W_N^{PD}$:

$$-(\tau_1^{S,P})^2 -\hat{\alpha}_N^2 +2\hat{\alpha}_N D_1 +2\hat{\alpha}_N (-D_1-g_1+\tau_1^{S,P}+\hat{\alpha}_N) < -2\hat{\alpha}_N^2 +2\hat{\alpha}_N(2\hat{\alpha}_N -g_1)$$

$$\rightarrow \qquad -(\tau_1^{S,P} - \hat{\alpha}_N)^2 < 0$$

which always occurs.

---

23. The N type is actually indifferent, in a separating equilibrium, between choosing θ_1 or $\theta_2 > 0$. We assume that the type N, when indifferent, chooses $\theta_1 > 0$, ruling out the deviation of the S type.

Appendix 3

Proof of Proposition 2

To prove the proposition we need to check three conditions: a) type S is at a corner (i.e. he prefers to choose $\theta_1 = \theta_2 = 0$) in a pooling equilibrium in the relevant parameter range; b) type N does not deviate from a pooling equilibrium in the relevant parameter range; c) type S does not deviate from a pooling equilibrium in the relevant parameter range.

a) For the pooling equilibrium, we can state the following lemma, which provides the value of the parameter $\overline{\Psi}_1^P$:

Lemma A3: for any level of g_1, a necessary and sufficient condition for type S to choose $\theta_1 = \theta_2 = 0$ in a pooling equilibrium is $\psi < \frac{(1-\gamma)(2-p_0)}{2\gamma} = \overline{\Psi}_1^P$.

Proof: For necessity, plug the expression for $D_2^{S,P}$ in the corner condition for θ_1 or θ_2. For sufficiency:

$$\psi < \frac{(1-\gamma)}{2\gamma}(2 - p_0) \rightarrow \frac{D_1 + g_1 - 2\hat{\alpha}_N}{\hat{\alpha}_N} < \frac{\hat{\alpha}_S - \hat{\alpha}_N}{\hat{\alpha}_N} - \frac{(\hat{\alpha}_S - \hat{\alpha}_N)}{\hat{\alpha}_N}\frac{p_0}{2}$$

$$\rightarrow \quad D_1 + g_1 - \hat{\alpha}_S \leq \hat{\alpha}_N - (\hat{\alpha}_S - \hat{\alpha}_N)\frac{p_0}{2}$$

Note that, if $D_2^{S,P}$ is greater than the RHS of the last inequality, then the corner condition for θ_1 is satisfied. This is always the case, since:

$D_2^{S,P} > \hat{\alpha}_N$ and $\hat{\alpha}_S > \hat{\alpha}_N$[24].

Similarly, it can be shown that $\psi < \Psi_1^P$ implies that the corner condition for θ_2 is always satisfied.

b) In the deviation from the pooling, type N reveals its type in period 1 (the first superscript refers to the type, the second to the equilibrium and to the deviation):

$$\theta_1 = 1 \quad , \quad \tau_1^{N,PD} = g_1 - \hat{\alpha}_N \quad , \quad D_2^{N,PD} = \hat{\alpha}_N$$

where $D_2^{N,PD}$ is chosen for the continuation of the game. Furthermore, $R_2^{N,PD} = 1$. Type N prefers the pooling iff $W_N^P > W_N^{PD}$, which implies:

$$-(\tau_1^P)^2 - \hat{\alpha}_N^2 + 2\hat{\alpha}_N D_1 + 2\hat{\alpha}_N(\hat{\alpha}_N - D_2^P) > -(g_1 - \hat{\alpha}_N)^2 - \hat{\alpha}_N^2$$

Therefore, the deviation is ruled out iff:

$$3\hat{\alpha}_N - g_1 < \tau_1^P < g_1 - \hat{\alpha}_N$$

In terms of the fiscal and reputation parameters, the LHS inequality becomes:

24.
$$\frac{D_2^{S,P}}{\hat{\alpha}_N} = \frac{2\gamma(1+\psi)(1-p_0)^2 + \gamma p_0 + p_0(1-p_0)}{\gamma[1+(1-p_0)^2]} > 1$$

$$\rightarrow \quad \psi > -\frac{1-\gamma}{2\gamma}\frac{p_0}{1-p_0}$$

which is always satisfied since $\gamma<1$ and $\psi>0$.

$$\underline{\psi}^P = \frac{(1-\gamma)p_0(1-p_0)}{\gamma[1+w-(1-w)(1-p_0)^2]} < \psi$$

and the RHS inequality

$$2\gamma\ [w-(1-w)\ (1-p_0)^2]\ \psi\ <\ (1-\gamma)\ p_0\ (1-p_0)$$

which always holds in the range $0 < p_0 < 1 - \sqrt{\frac{w}{1-w}}$ and can otherwise be rewritten as

$$\psi < \frac{(1-\gamma)p_0(1-p_0)}{2\gamma[w-(1-w)(1-p_0)^2]} = \overline{\Psi}_2^P$$

In summary, the range of parameters where type N never deviates from a pooling equilibrium is:

$$\underline{\psi}^P < \psi \qquad for \quad 0 < p_0 < 1-\sqrt{\frac{w}{1-w}}$$

$$\underline{\psi}^P < \psi < \overline{\Psi}_2^P \qquad for \quad 1-\sqrt{\frac{w}{1-w}} < p_0 < 1$$

c) In the deviation from the pooling equilibrium, type S picks a level of taxation large enough to make type N indifferent between mimicking and revealing its nature ($\tau_1^{S,PD}=g_1-\hat{\alpha}_N$). The deviation is excluded iff $W_S^P > W_S^{PD}$, which occurs when:

$$-(\tau_1^P)^2-(\tau_2^P)^2+2\hat{\alpha}_S(\tau_2^P-D_2^P) > -(g_1-\hat{\alpha}_N)^2-(D_1+\hat{\alpha}_N)^2$$

Notice that W_S^{PD} depends only on the distortionary costs of taxation. In fact, the debt repayment policy in the deviation does not imply any wealth redistribution since $R_1 = R_2 = 1$. However, the distortionary costs of taxation in the deviation are not perfectly smoothed, so that the welfare of type S in the deviation is always smaller than the one from a policy of perfect tax smoothing. Therefore, in order to show that type S does not to deviate from the pooling equilibrium, it is sufficient to show that W_S^P is greater than the welfare from a policy of perfect tax smoothing:

$$- (\tau_1^P)^2 - (\tau_2^P)^2 + 2\hat{\alpha}_S(\tau_2^P - D_2^P) > - (\frac{D_1 + g_1}{2})^2 - (\frac{D_1 + g_1}{2})^2$$

After some manipulations, the above inequality can be rewritten in terms of the parameters ψ and p_0:

$$(2 - p_0)\psi^2 - \frac{2\,(1 - \gamma)(1 - p_0)}{\gamma}\,\psi - \frac{p}{2}\,(\frac{1 - \gamma}{\gamma})^2 < 0$$

If one takes into account that ψ must be positive, then the negative root of the quadratic expression in ψ can be neglected and the inequality above is equivalent to:

$$\psi < \frac{1 - \gamma}{\gamma\,(2 - p_0)}\,[\,(1 - p_0) + \sqrt{\frac{1 + (1 - p_0)^2}{2}}\,] = \psi^+$$

where ψ^+ is the positive root of the quadratic expression. The last inequality identifies a range of parameters that encompasses the one where type S chooses $\theta_1 = \theta_2 = 0$ ($\psi^+ > \Psi_1^P$, as it can be easily verified). We conclude that type S never deviates from a pooling equilibrium in the range of parameters where pooling equilibria with a positive stock of debt exist.

Appendix 4

Proof of Proposition 3

To prove the proposition we need to check three conditions: a) type S is at a corner (i.e. he prefers to choose $\theta_1 = \theta_2 = 0$) in a separating equilibrium in the relevant parameter range; b) type N does not deviate from a separating equilibrium in the relevant parameter range; c) type S does not deviate from a separating equilibrium in the relevant parameter range.

a) In order to meet the first condition, we formulate the following:

Lemma A4: When $g_1 > 2\hat{\alpha}_N$, a necessary and sufficient conditon for type S to choose $\theta_1 = \theta_2 = 0$ in a separating equilibrium is $\psi < \frac{(1-\gamma)(1-p_0)}{\gamma[1-p_0(1-w)]} = \bar{\Psi}^S$.

Proof: For necessity, plug the expression for $D_2^{S,S}$ in the corner condition for θ_1 or θ_2:

$$\tau_1^{S,S} = \tau_2^{S,S} = D_2^{S,S} < \hat{\alpha}_S \quad \rightarrow \quad \psi < \frac{(1-\gamma)}{\gamma}\frac{(1-p_0)}{1-p_0(1-w)}$$

The reverse causation holds as well.

b) In the deviation from a separating equilibrium, type N mimicks type S's optimal policy in the separating and, as a result, the interest factor in the continuation of the game is greater than 1:

$$D_2^{N,SD} = D_2^{S,S} = D_1R_1 + g_1 - \tau_1^{S,S}$$

$$\tau_2^{N,SD} = D_2^{N,SD}R_2^{N,SD}(1 - \theta_2^{N,SD}) = \hat{\alpha}_N$$

No deviation of type N requires $W_N^S > W_N^{SD}$:

$$-(g_1 - \hat{\alpha}_N)^2 - \hat{\alpha}_N^2 > -(\tau_1^{S,S})^2 - \hat{\alpha}_N^2 + 2\hat{\alpha}_N D_1R_1 + 2\hat{\alpha}_N(\hat{\alpha}_N - D_2^{N,SD})$$

$$\rightarrow \quad \tau_1^{S,S} > g_1 - \hat{\alpha}_N$$

which implies that type N does not deviate from a separating equilibrium when the separating equilibrium taxes of type S are large enough. The inequality above can be rewritten in terms of the parameter p_0 as:

$$p_0 > \frac{1-2w}{1-w}$$

which is always satisfied when $1/2 < w < 1$. We have then as in Proposition 3:

$$p_0 > \bar{p} = Max[\frac{1-2w}{1-w} , 0]$$

c) In the deviation from the separating, type S shifts to a level of taxation such to be mimicked by type N, in order to pay a risk premium also between period 1 and period 2. This policy is preferred to the separating equilibrium policy as long as the benefits from the larger wealth redistribution are larger than the costs from the higher taxes.

The optimal deviation policy is the one that maximizes the welfare of type S in period 1 and period 2, given the actions taken by the government and the public in period 0 and under the constraint that taxes in the deviation, $\tau_1^{S,SD}$, are low enough to be mimicked by type N. First, we let type S reoptimize in period 1 without imposing the above constraint and we identify a range of parameters where the unconstrained optimal deviation dominates the separating equilibrium policy ($\psi < \underline{\psi}_1^S$). Second, we consider whether this policy is feasible, i.e. it satisfies the constraint of being mimicked by type N and we identify the range of parameters where this is the case ($\psi < \underline{\psi}_2^S$); the range where the unconstrained deviation is preferred and feasible is then $\psi < Min[\underline{\psi}_1^S , \underline{\psi}_2^S]$. Finally, we identify the range of parameters where the corner solution of the reoptimization problem ("constrained deviation") is preferred to the separating equilibrium ($\psi < \underline{\psi}_3^S$); as a result, the range of parameters where type S deviate becomes larger, $\psi < Min[\underline{\psi}_1^S , Max[\underline{\psi}_2^S , \underline{\psi}_3^S]]$.

c1) The unconstrained optimal deviation policy is analogous to the one of a pooling equilibrium, where $D_1R_1+g_1$ replaces D_1+g_1:

$$\tau_1^{BEST} = \tau_1^{S,P} + \frac{D_1 p_0}{(1 - p_0)[1 + (1 - p_0)^2]}$$

Type S prefers the unconstrained deviation to the separating equilibrium if:

$$- 2(t_1^{S,S})^2 < - (t_1^{BEST})^2 - d_2^{BEST}R_2^{BEST} + \frac{2}{\gamma} d_2^{BEST}(R_2^{BEST} - 1)$$

where

$$t_1^{S,S} = \frac{\tau_1^{S,S}}{\hat{\alpha}_N}$$

$$t_1^{BEST} = \frac{\tau_1^{BEST}}{\hat{\alpha}_N} = \frac{\tau_1^{P}}{\hat{\alpha}_N} + \frac{2wp_0\psi}{(1-p_0)[1+(1-p_0)^2]}$$

$$R_2^{BEST} = \frac{D_2^{BEST} - \hat{\alpha}_N p_0}{D_2^{BEST}(1 - p_0)}$$

$$d_2^{BEST} = \frac{D_2^{BEST}}{\hat{\alpha}_N} = \frac{2w\psi}{1 - p_0} + 2(1 + (1 - w)\psi) - t_1^{BEST}$$

The above inequality can be rewritten in terms of the parameters ψ and p_0 as:

$$p_0(2 - p_0)[(1-p_0)+wp_0]^2 \,.\, \psi^2 - \frac{2(1-\gamma)p_0(1-p_0)^2[(1-p_0)+wp_0]}{\gamma} \,.\, \psi - \frac{1}{2}\left(\frac{1-\gamma}{\gamma}\right)^2 p_0^2(1-p_0)^2 < 0$$

One of the roots of the quadratic expression in ψ is always negative and can be neglected, so that the positive root defines the parameter range where the unconstrained deviation is preferred by type S to the separating equilibrium:

$$\psi < \frac{(1-\gamma)(1-p_0)\left[(1-p_0) + \sqrt{\frac{1+(1-p_0)^2}{2}}\right]}{\gamma(2-p_0)[(1-p_0) + w\,p_0]} = \psi^{+} = \psi^{S}_{-1}$$

c2) The feasibility of τ_1^{BEST} implies:

$$3\hat{\alpha}_N - g_1 < \tau_1^{BEST} < g_1 - \hat{\alpha}_N$$

$$1 - 2(1 - w)\psi < t_1^P + \frac{2wp_0}{(1 - p_0)(1 + (1-p_0)^2)}\psi < 1 + 2(1 - w)\psi$$

The LHS inequality simplifies to

$$\psi > \frac{(1 - \gamma)p_0(1 - p_0)^2}{2\gamma\{[w + (1 - w)(1 - p_0)[1 + (1 - p_0)]\}} = \psi_{-2L}$$

which is neglected in Proposition 4 for the reasons discussed below at point c3).
The RHS inequality can be rewritten as:

$$2[\frac{w}{1-p_0}-(1-w)(1-p_0)^2]\ \psi \quad < \quad (\frac{1-\gamma}{\gamma})p_0(1-p_0)$$

which is always satisfied in the range $1-(\frac{w}{1-w})^{\frac{1}{3}}<p_0<1$ and

simplifies otherwise to:

$$\psi < \frac{(1 - \gamma)p_0(1 - p_0)^2}{[2\gamma(w - (1 - w)(1 - p_0)^3]} = \psi^S_{-2}$$

In summary, the feasibility of τ_1^{BEST} requires:

$$\underline{\psi}_{2L} < \psi \,, \qquad for \;\; 0<p_0<1-\left(\frac{w}{1-w}\right)^{\frac{1}{3}}$$

$$\underline{\psi}_{2L} < \psi < \underline{\psi}_2^S \,, \qquad for \;\; 1-\left(\frac{w}{1-w}\right)^{\frac{1}{3}}<p_0<1$$

c3) When the feasibility condition at point c2) is not satisfied by the unconstrained deviation, the only possible deviation is at $\tau_1 = g_1 - \hat{\alpha}_N$. This deviation will be preferred to the separating equilibrium policy iff:

$$-2\,(t_1{}^S)^2 < -\left(\frac{g_1 - \hat{\alpha}_N}{\hat{\alpha}_N}\right)^2 - (d_2^{CD}\,R_2^{CD})^2 + \frac{2}{\gamma}\,(d_2^{CD}\,R_2^{CD} - d_2^{CD})$$

where the superscript CD refers to the constrained deviation. The above inequality simplifies to:

$$\left\{ 2\,[(1-w)^2(1-p_0)^2 + \frac{w^2}{(1-p_0)^2} - [1-p_0) + w\,p_0]^2 \right\} \psi^2 - $$
$$- 2\left[\frac{(1-\gamma)}{\gamma}\,w\,p_0\right]\,\psi < 0$$

One of the two solutions is ruled out, being for $\psi = 0$. The other defines the range of parameters:

$$\psi < \frac{2\,(1-\gamma)\,wp_0}{\gamma\{\,2\,[(1-w)^2(1-p_0)^2 + \frac{w^2}{(1-p_0)^2} - [(1-p_0)+w\,p_0]^2\,\}} = \underline{\psi}_3^S$$

Note that $\underline{\psi}_3^S > \underline{\psi}_{2L}$, so that, in the range $\psi < \underline{\psi}_{2L}$ where the optimal unconstrained deviation is not feasible, type S deviates in any case because its welfare in the constrained deviation is larger than in the separating equilibrium. As a result, $\underline{\psi}_{2L}$ can be neglected in Proposition 3.

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Printing Office of the Banca d'Italia
Rome, September 1993