BANCA D'ITALIA

Temi di discussione

del Servizio Studi

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The views expressed in the articles are those of the authors and do not involve the responsibility of the Bank.

Summary

The paper examines the effects of exogenous exchange rate movements in a model of international duopoly with differentiated products where prices are the choice variables. Alternative schemes of strategic interaction between firms are considered. If each duopolist makes "consistent" conjectures about the way the rival reacts to its price changes, the effects of a nominal depreciation of its currency on domestic and foreign prices and the real exchange rate may differ substantially from those found in the case of Cournot competition. Numerical simulations suggest that the differences between the Cournot and the consistent conjectures equilibrium (C.C.E.) depend crucially on the degree of substitution between foreign and domestic goods. In particular, if goods are close substitutes and wages are not affected by exchange rate variations, the C.C.E. implies a negligible effect on the real exchange rate, while the Cournot equilibrium predicts a significant real depreciation.

1. Introduction [1]

The effects of exchange rate movements on the pricing behaviour of firms operating in international markets have recently become an important issue in economic analysis, owing to the large exchange rate fluctuations of the last decade and to the need to formulate a satisfactory theoretical explanation of the observed persistent deviation of relative prices from Purchasing Power Parity.

The pricing strategies of firms selling in more than one market have been analysed in several different contexts. In turn, the many models proposed have variously emphasised the role played by market structure and organization, the degree of substitutability between domestic and foreign goods, uncertainty about future exchange rate levels, adjustment costs and imperfect information. [2]

One of the purposes of this paper is to identify the main channels through which the exchange rate affects pricing strategies in a static context with symmetric and complete information; for this reason particular emphasis is given to the cost structure and the functional forms of consumers' demand curves. The paper also investigates how the effects of exogenous movements in the exchange rate vary with alternative schemes of strategic interaction among firms.

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1. I wish to thank D.Terlizzese and two anonimous referees for helpful comments and remarks on a previous version of this paper. My special acknowledgement goes to J. Smith for carefully reviewing the English and to M. Chiurato for valuable editorial help. The responsability of any errors is of course my own.

2. For a summary of the "pricing to market" literature see Dornbusch (1987b) and Krugman (1989).

The model of international duopoly I present is very similar in spirit to those proposed by Brander (1981), Brander-Krugman (1983) and Dornbusch (1987a and 1987b). The set up can be summarised as follows: there are two firms (one domestic and one foreign) selling differentiated goods that are produced with an identical technology involving labour and one intermediate input, say energy.

In order to keep the structure of the model relatively simple, the domestic firm is assumed to sell its products only on the domestic market, while the foreign one sells its products on both markets. Accordingly, the analysis focusses on the competition between the duopolists on the domestic market.

This assumption obviously introduces an asymmetry between the duopolists and has some important implications for the results of the paper. The case presented here, though admittedly not the most general one, is nonetheless interesting in many respects, especially because consideration is given to all the channels through which prices are affected by movements of the exchange rate.

The price of energy is set in foreign currency and domestic money wages are indexed to an aggregate price index including domestic and foreign prices. Movements in the exchange rate thus have a direct and an indirect impact on domestic costs.

It is also assumed that prices rather than quantities are the choice variables; in the model presented in Section 2 each firm chooses its own optimal strategy, taking its rival's strategy as given; the solution thus represents a Cournot-Nash equilibrium. An analysis follows of how the two firms' optimal prices are affected by exogenous movements in the exchange rate. Strictly speaking, the Cournot equilibrium is defined for homogeneous goods models where quantities are the strategic variables; however, the term is used here in a broader sense to identify a non-cooperative equilibrium with no strategic interaction between the duopolists. (3)

In Section 3 the assumption of Cournot-Nash behaviour is relaxed and consideration is given to a scheme of strategic interaction whereby each firm forms "conjectures" about its rival's reaction function. The model is also solved by imposing the further economic condition that conjectures are consistent in equilibrium, in the sense proposed by Bresnahan (1981 and 1983), Kamien-Schwartz (1983) and Dixit (1986).

3. In the oligopoly literature, price setting models are often referred to as Bertrand models (Benassy (1989), Okuguchi (1987), Bresnaham (1981)). Friedman (1977a, 1977b, 1983), however, also defines the price model equilibrium as a Cournot type equilibrium, in the sense that each firm is maximising its own profits with respect to its price, taking other prices as fixed.

2. The Cournot-Nash duopoly model

The model consists of two firms, one foreign (F) and one domestic (H), each producing a good with labour and energy. F is assumed to sell its goods on the domestic and foreign markets, while products of H are sold only on the domestic market. Although this assumption obviously introduces an asymmetry between the duopolists, it is fairly standard in the literature because it makes it possible to analyse the competition on a single market separately. (4)

To get simple analytical results, the same Cobb - Douglas production function with constant returns to scale is used for the two firms. It is also assumed that the price of energy is set in the foreign currency. (5) Thus, the cost functions of the two firms are given by:

1a) $C^* = K\ w^{*a} \left(pm^*\right)^{(1-a)} X^*$ Foreign firm

1b) $C = Kw^a \left(pm^*\ e\right)^{(1-a)} x$ Home firm

where C, C^* : total costs in the domestic and foreign currencies

K : constant term

w, w^* : money wage rates in the domestic and foreign

4. See Yamawaki (1986). This assumption is often made only implicitly.

5. More realistically, it is possible to imagine that the price of energy is set in a third currency and then analyse the effects of movements in the different cross-rates. This would complicate the algebra a little, but would not change the basic implications of the model.

		currencies
pm^*	:	price of energy in the foreign currency
e	:	exchange rate (domestic currency units per unit of the foreign currency)
x	:	domestic output
x^*	:	foreign output sold on the domestic and foreign markets.

Domestic money wages (w) are linked to a price index (P) including both domestic and imported consumer goods:

$$w = \tilde{w} \; P^{\mu} \qquad 0 \leq \mu \leq 1$$

$$P = p^{\alpha} \, (p^* \, e)^{(1-\alpha)} \qquad 0 \leq \alpha \leq 1$$

where p, p^* are the prices of domestic and foreign consumer goods and α, μ, $\tilde{w}$ are exogenous.

For the sake of simplicity, foreign money wages (w^*) are taken as fixed in the foreign currency. The analysis would be more complicated, without substantially altering the results, if foreign money wages were indexed to some aggregate foreign price index, as long as they were not affected by movements in the exchange rate, which is plausible here since domestic goods are not sold on the foreign market. Under the assumption of constant returns to scale, the foreign firm can therefore derive independent optimal strategies on the two markets. (6)

On the domestic market the duopolists are faced with the following symmetric linear demand functions:

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6. See Krugman (1981), Brander-Krugman (1983), Yamawaki-Audretsh (1989) and Okuguchi (1989) for a formal approach to this problem.

2a) $x^* = h - \beta p^* e + \gamma p$

2b) $x = h - \beta p + \gamma p^* e$

where $h, \beta, \gamma > 0$ and $\beta > \gamma$.

2a) represents the demand for firm F's product and 2b) the demand for firm H's product.(7)

The two profit functions, in the domestic and foreign currencies, are given by:

3a) $F : \Pi^* = (p^* - Kw^{*a} (pm^*)^{(1-a)}) (h - \beta p^* e + \gamma p)$

3b) $H : \Pi = (p - Kw^a (pm^* e)^{(1-a)}) (h - \beta p + \gamma p^* e)$

Strictly speaking, foreign profits also depend on the price and output on the foreign market. However, as pointed out above, the assumption of constant return to scale allows firm F's maximization process to be analysed separately on the two markets.

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7. These demand functions can be derived from the following quadratic utility function in x^* and x (Dixit (1979), Singh-Vives (1984)):

$U = b (x + x^*) - 1/2 (cx^2 + 2dxx^* + cx^{*2})$

where $b, c, d > 0$ and $c > d$.

Then, it can be easily shown that :

$h = b (c - d) / (c^2 - d^2)$
$\beta = c / (c^2 - d^2)$
$\gamma = d / (c^2 - d^2)$

The two goods are perfect substitutes when d is equal to c. This particular case cannot be analysed in this context since the two demand functions are not defined; however, I will consider the case of high substitutability which implies that c approaches d and β approaches γ.

Since the goods of F and H are imperfect substitutes, prices rather than quantities are likely to be the strategic variables (Friedman (1983)); accordingly, each firm sets its own price in order to maximize its profits, taking as given the other firm's pricing strategy. In this model, however, the indirect effect of prices on domestic costs means that the problem is not identical. For the foreign firm the first order conditions are given by:

4a) $\Pi^*_{p*} = x^* - p^* e \quad \beta + \quad Kw^{*a}(pm^*)^{(1-a)} \quad e\beta = 0$

and solving for p^*

5a) $$p^* = \left(\frac{\varepsilon^*_1}{\varepsilon^*_1 - 1} \right) J^*$$

where $\varepsilon_1^* = \beta e p^* / x^*$ is the own price elasticity of demand for F.

$J^* = K\, w^{*a}\, (pm^*)^{(1-a)}$ is the constant marginal cost of F.

5a) represents the optimal pricing strategy if the second order conditions are also satisfied. This will be discussed below. It is worth noting that, in this context, ε_1^* is not constant, but depends on domestic and foreign prices and on the exchange rate. [8] Let:

8. Dornbusch (1987a), Aizenman (1989) and Colombo (1988 and 1989) consider demand functions with constant elasticity of substitution between domestic and foreign goods similar to those found in Dixit-Stiglitz (1977).

$$M^* = \frac{\varepsilon_1^*}{\varepsilon_1^* - 1} = M^*(p^* e, p)$$

be the mark-up function for the foreign firm. The partial derivatives of M^* with respect to foreign prices (M_1^*) and domestic prices (M_2^*) are respectively negative and positive [9]. Note that for log-linear demand functions $M_1^* = M_2^* = 0$ and the mark-up is independent on the rival's price and the exchange rate.

For the domestic firm the first order conditions are given by:

$$\text{4b)}\quad \Pi_p = x - p\beta + Kw^a(pm^*e)^{(1-a)}\beta - xK(pm^*e)^{(1-a)}\frac{\partial w^a}{\partial p} = 0$$

$$1-\varepsilon_1+J\varepsilon_1 p^{(a\alpha\mu-1)}(ep^*)^{(1-\alpha)\mu a} - J(ep^*)^{(1-\alpha)\mu a}\alpha\mu a p^{(\alpha\mu a-1)} = 0$$

where $J = K(pm^*e)^{(1-a)}\tilde{w}^a$ and $\varepsilon_1 = \beta p / x$

and solving for p:

9. In fact:

$$\frac{\partial M^*}{\partial (p^* e)} = \frac{-\beta(h + \gamma p)}{(2\beta p^* e - h - \gamma p)^2} < 0 \text{ always}$$

$$\frac{\partial M^*}{\partial p} = \frac{\gamma\beta p^* e}{(2\beta p^* e - h - \gamma p)^2} > 0 \text{ always}$$

$$\text{5b)} \quad p = M^{\sigma} \left[J\,(ep^{*})^{(1-\alpha)\mu a} \right]^{\sigma}$$

$$\text{where } M = \frac{\varepsilon_1 - \alpha\,\mu a}{\varepsilon_1 - 1} \quad \text{and } \sigma = \frac{1}{1 - \alpha\mu a}$$

With $\mu \neq 0$ and $\varepsilon_1 = \varepsilon_1^{*}$, M is always less than M^{*}. [10] This means that with indexed money wages the domestic firm cannot exploit all its duopolistic power on the market since, when setting its optimal pricing strategy, the domestic firm knows that its behaviour will affect money wages and therefore its own costs. It thus faces a trade-off between revenues and costs that the foreign firm does not face. [11] Obviously this result depends crucially on the assumption of constant foreign marginal costs. If foreign wages were also indexed it would not be possible to determine a precise relationship between the two mark-ups.

Equation 5b) clarifies the ways in which the exchange rate can affect domestic pricing behaviour. Three main channels can be distinguished: [12]

a) the <u>first</u> produces a <u>direct</u> effect through the mark-up

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10. In general, with $\varepsilon^{*} \neq \varepsilon_1$, M^{*} is larger than M if $\varepsilon_1^{*} < (\varepsilon_1 - \alpha\mu a)/(1-\alpha\mu a)$. 1

11. It is easy to check that $M_1 < 0$ and $M_2 > 0$ still hold.

12. There is another channel that works when the assumption of constant marginal cost (constant return to scale) is relaxed. In some "pricing to market" models this is actually the only channel (Aizenman (1989) and Colombo (1989)). Increasing and decreasing returns to scale are not easy to deal with when money wages are indexed to foreign and domestic prices. Moreover they can also cause problems in assessing the conditions of stability of the Cournot equilibrium with more than one market (Okuguchi (1989)).

function (M) that, in general, depends on the exchange rate and on the rival's price;

b) the <u>second</u> also produces a <u>direct</u> effect on costs whenever there is some intermediate input which has to be paid for in foreign currency;

c) the <u>third</u> may produce an <u>indirect</u> effect on money wages or, in general, any indexed costs.

In general, the implications of a "pricing to market" model will differ considerably, substantially depending on which effects are working. For instance, models with log-linear demand functions actually consider effects b) and c); if money wages are also fixed, or are indexed only to domestic prices, then only channel b) may operate.

A solution of 5a) and 5b) represents a Cournot-Nash equilibrium if and only if the second order conditions are satisfied; that is:

$$\Pi^*_{p^*p^*} < 0 \qquad \Pi_{p\,p} < 0$$

It is easy to check that they are always met for the foreign firm while for Π_{pp} to be negative the direct effect of a price increase on marginal revenues must not be fully offset by the effect on marginal costs. In the following, it is assumed that the second order conditions are satisfied for both firms, so that an equilibrium ($\bar{p}$, $\bar{p}^*$) exists at positive prices and quantities.

By approximating the two reaction functions 5a) and 5b) around the equilibrium we obtain: [13]

13. Since the foreign reaction function is linear, the model can also be solved by directly substituting 5a) into 5b). However, I have preferred to follow here the same methodology used in Section 3 to solve the model with "conjectural variations".

$$6a) \quad dp^* = \gamma_0 dp + \gamma_1 de + \gamma_2 dw^* + \gamma_3 dpm^* \qquad \text{F.R.F.}$$

$$6b) \quad dp = \beta_0 dp^* + \beta_1 de + \beta_2 d\tilde{w} + \beta_3 dpm^* \qquad \text{H.R.F.}$$

where
$$\gamma_0 = \eta_2^* / (1 + \eta_1^*)(p^* / p)$$

$$\gamma_1 = - \eta_1^* / (1 + \eta_1^*) \quad (p^* / e)$$

$$\gamma_2 = a / (1 + \eta_1^*)(p^* / w^*)$$

$$\gamma_3 = (1 - a) / (1 + \eta_1^*) \quad (p^* / pm^*)$$

$$\beta_0 = \sigma ((1-\alpha)\mu a + \eta_2) / (1+\sigma\eta_1)(p/p^*)$$

$$\beta_1 = \sigma ((1-\alpha)\mu a + (1-a)+\eta_2) / (1+\sigma\eta_1)(p/e)$$

$$\beta_2 = \sigma a / (1+\sigma\eta_1)(p/\tilde{w})$$

$$\beta_3 = \sigma (1-a) / (1+\sigma\eta_1)(p/pm^*)$$

and

$$\eta_1^* = - M_1^* J^* e \ , \quad \eta_2^* = M_2^* J^* (p/p^*)$$

$$\eta_1 = - M_1 (p/M) , \quad \eta_2 = M_2 (p^* e/M)$$

are the positive domestic and foreign mark-up elasticities with respect to foreign and domestic prices, respectively.

It is easy to check that the system given by 6a) and 6b) is stable since $\gamma_0\beta_0$ is always positive and less than one, which implies that the domestic firm's reaction function is steeper than the foreign firm's, as in Figure 1. (14)

We can now analyse the effects of an exchange rate variation. Assuming that the domestic currency depreciates (de > 0), the two reaction functions will shift downward:

$$\left.\frac{dp^*}{de}\right|_{\substack{p = \bar{p} \\ \text{F.R.F.}}} = \gamma_1 = \left(- \eta_1^* / (1+\eta_1^*)\right) \left(p^* / e\right)$$

$$\left.\frac{dp}{de}\right|_{\substack{p^* = \bar{p}^* \\ \text{H.R.F.}}} = \beta_1 = \sigma((1-\alpha)\, \mu a + (1-a)+\eta_2)/(1+\sigma\eta_1)(p/e)$$

The F.R.F. is affected by exchange rate movements <u>only</u> if the mark-up function depends on relative prices; hence, with particular classes of demand functions the F.R.F. would be horizontal and would not shift after a change in the

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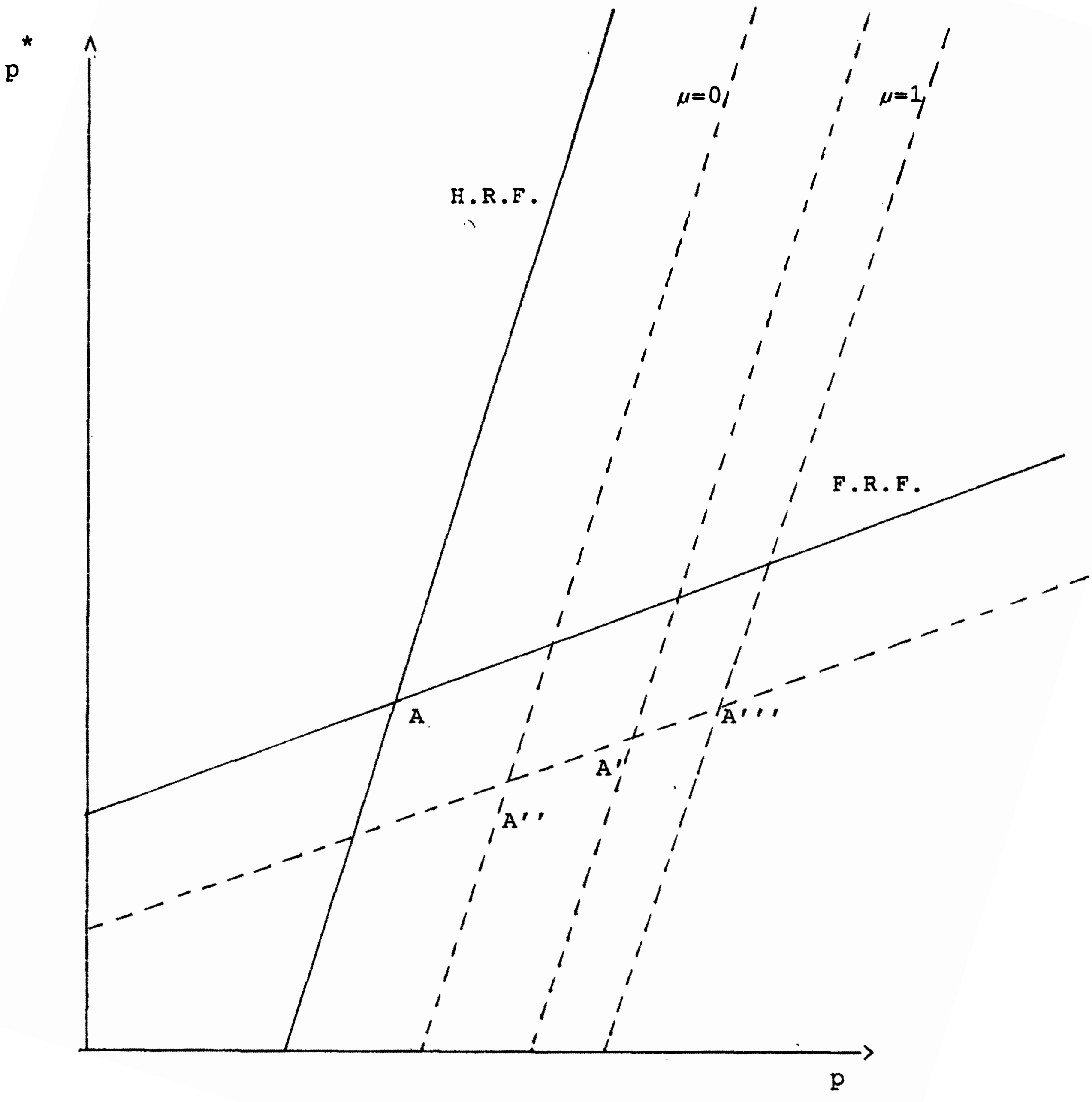
14. More generally, the stability conditions can be set as:

$$\Pi_{pp}\, \Pi^*_{p^*p^*} - \Pi^*_{p^*p}\, \Pi_{pp^*} > 0$$

which implies that the own effects of prices on marginal profits exceed the cross effects. General conditions for stability of price adjusting oligopoly can be found in Quandt (1967), Okuguchi (1969) and Friedman (1977b).

Fig. 1



exchange rate. By contrast, the shift of the H.R.F. depends on several other factors: the production technology, the share of domestic goods in the aggregate price index and the degree of wage indexation. Since the outward shift of the F.R.F. is larger than that of the H.R.F. (15) at the new equilibrium (A') the domestic price level is higher and the foreign price level is lower after the home currency depreciation, as shown in Figure 1.

At this stage it may be useful to summarize the comparative static exercise results as follows. Defining the real exchange rate as the ratio between domestic and foreign prices (in domestic currency), $\lambda = p/(p^{*}e)$, the real exchange rate elasticity with respect to the nominal exchange rate ($\eta_{\lambda,e}$) is given by:

$$\eta_{\lambda,e} = \eta_{p,e} - \eta_{p*,e} - 1$$

$$= \Omega / \Delta$$

where $\Omega = \sigma\,((1-\alpha)\mu a + (1-a)(1+\eta^{*}_{1}-\eta^{*}_{2}) + \eta_{2} - \eta_{1}) - 1$

and $\Delta = (1+\eta^{*}_{1})(1+\sigma\eta_{1}) - \eta^{*}_{2}\,\sigma\,((1-\alpha)\mu a + \eta_{2})$

which is always less than one in absolute value. (16) Thus, in general, the pass-through from the nominal to the real exchange rate is less than complete and depends in a rather

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15. In fact:

$\eta^{*}_{1} > \eta^{*}_{2}$ and $\eta_{1} > \eta_{2}$ and this assures that $(-\gamma_1/\gamma_0) > \beta_1$

16. $\eta_{\lambda,e} = -1$ when labour is the only productive input, money wages are not indexed and the demand functions exhibit constant elasticity. This result can be found in Dornbusch (1987a).

complex way on all the parameters of the model. Let's consider two special cases. First, if domestic money wages are not indexed, $\eta_{\lambda,e}$ simplifies to: (17)

$$\eta_{\lambda,e} = - a / (1 + \eta_1^* + \eta_2^*)$$

which is proportional to the share of labour in the productive process and equal to zero if energy is the only input. If the demand functions exhibit constant elasticity, $\eta_{\lambda,e}$ reduces to: (18)

$$\eta_{\lambda,e} = a\ (\mu -1)/\ (1 - \alpha\mu a)$$

and the domestic and foreign price elasticities are given by:

$$\eta_{p,e} = ((1-\alpha)\mu a + (1-a))/\ (1 - \alpha\mu a)$$

$$\eta_{p*,e} = 0$$

It is worth noting that the foreign price is not affected by exchange rate movements, while $\eta_{p,e}$ is always positive and reaches a maximum of 1 with full indexation. In this case, the pass-through from the exchange rate to domestic prices is complete and the real exchange rate remains constant.

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17. It is easy to check that in this case $\eta_1 = \eta_1^*$ and $\eta_2 = \eta_2^*$.

18. In this case $\eta_1^* = \eta_2^* = \eta_1 = \eta_2 = 0$.

3. The duopoly model with "conjectural variations"

In the previous section it was shown that with constant returns to scale the exchange rate can affect pricing behaviour only through the cost structures and demand elasticities. It appears worth investigating, at this stage, whether these results depend on the particular strategic interdependence between the two firms: in the Cournot-Nash equilibrium, in fact, each duopolist acts as if its own strategy has no effect on its rival's. This seems a strong assumption since in oligopolistic markets firms are likely to believe that the prices (or quantities) they choose will affect rivals prices (or quantities) and will therefore take this into account in choosing optimally. Although this idea has very much to do with expectations and dynamic phenomena, it can be included in a static framework by introducing "conjectural variations", that is the degree to which a change in one firm's policy variables induces expected changes in those of the other firm.

In one of his contributions, Dornbusch (1987a) seems to suggest that trying to model strategic interactions in a standard Cournot framework of international trade would imply an interdependence of pricing decisions, even in the case of constant marginal costs and demand elasticities.

Conjectural variations have been extensively used in the static oligopolistic framework [19] and have been specified in slightly different ways depending on the particular

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19. For a survey see Kamien-Schwartz (1983) and Martin (1989).

features of the models.[20] Conjectural variations are introduced in the model presented in Section 2 in the following way; let

$$k = (\partial p^*/\partial p) \,/\, (p^*/p) \qquad 0 \leq k < 1$$

$$k^* = (\partial p/\partial p^*) \,/\, (p/p^*) \qquad 0 \leq k^* < 1$$

represent, respectively, the <u>conjectured elasticities of response</u> of the foreign and domestic prices with respect to changes in the rival's price.[21] At first I will assume that k and k^* are constant, so that they do not depend on any relevant variable of the model.[22] In this context the first order conditions are modified as follows:

$$\text{7a)} \quad \Pi^*_{p^*} = x^* - p^* \left[\beta e - \gamma k^* (p/p^*) \right] + J^* \left[\beta e - \gamma k^* (p/p^*) \right] = 0$$

$$\text{7b)} \quad \Pi_p = x - p \left(\beta - \gamma e k (p^*/p) \right) + \left(\beta - \gamma e k (p^*/p) \right) \cdot$$

$$Jp^{\alpha\mu a} (p^* e)^{(1-\alpha)\mu a} - Jx (p^* e)^{(1-\alpha)\mu a} \alpha\mu a p^{\alpha\mu a-1}$$

$$- Jx p^{\alpha\mu a} (1-\alpha)\mu a (p^* e)^{(1-\alpha)\mu a-1} k ((p^* e)/p) = 0$$

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20. Martin (1989) discusses the different ways of modelling conjectural variations and the implications and relationship between them.

21. Note that the implicit assumption in the Cournot equilibrium is $k = k^* = 0$.

22. This assumption will be relaxed later with the introduction of "consistent" conjectures.

Solving for p^* and p we obtain:

$$8a) \quad p^* = \left(\frac{\varepsilon_1^* - \varepsilon_2^* k^*}{\varepsilon_1^* - \varepsilon_2^* k^* - 1} \right) J^* = \tilde{M}^* J^*$$

$$\text{where} \quad \varepsilon_2^* = \gamma \, p \, / \, x^*$$

$$8b) \quad p = \left(\frac{\varepsilon_1 - \varepsilon_2 k - a\mu(k+\alpha(1-k))}{\varepsilon_1 - \varepsilon_2 k - 1} \right)^{\sigma} \left(J(p^* e)^{(1-\alpha)\mu a} \right)^{\sigma}$$

$$= \tilde{M}^{\sigma} \left(J \, (p^* e)^{(1-\alpha)\mu a} \right)^{\sigma}$$

$$\text{where} \quad \varepsilon_2 = \gamma \, p^* e/x$$

It is easy to check that the second order conditions are always met for the foreign firm (i.e. $\Pi^*_{p^*p^*} < 0$). For the domestic firm, we have also to assume that the indirect effects of a price change on marginal costs do not offset the direct effects on marginal revenues. At equilibrium the slopes of the two reaction functions are given by:

$$9a)\ dp^*/dp = \tilde{\eta}_2^* / (1 + \tilde{\eta}_1^*)\ (p^*/p) \qquad \text{F.R.F.}$$

$$9b)\ dp/dp^* = \sigma(\tilde{\eta}_2 + (1-\alpha)\mu a)/(1+\sigma\tilde{\eta}_1)(p/p^*) \qquad \text{H.R.F.}$$

where $\tilde{\eta}_1^*$, $\tilde{\eta}_2^*$ are the foreign mark-up elasticities with respect to (p^*e) and p, $\tilde{\eta}_1$, $\tilde{\eta}_2$ are the domestic mark-up elasticities with respect to p and (p^*e); they are all positive and depend in a rather complex way on k^* and k.

The two reaction functions are still upward sloping in the p^*, p space. However, conjectures about rival strategies imply that the foreign reaction function is steeper and that the domestic reaction function is less steep than in the Cournot case, since the domestic and foreign mark-up elasticities depend positively on the degree of strategic interaction.

At this stage some points need to be emphasized; first, strategic interaction between duopolists implies higher equilibrium prices, since $\tilde{M}$ and $\tilde{M}^*$ are respectively higher than M and M^* ; second, the exchange rate continues to affect prices only through the cost structures and demand elasticities. (23) Thus, strategic interaction does not seem to be either a necessary or a sufficient condition for the exchange rate to impact on pricing decisions. One could

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23. Since:

$$dp^*/de = - \tilde{\eta}_1^*/(1 + \tilde{\eta}_1^*)\ (p^*/e)$$

$$dp/de = \sigma((1 - \alpha)\ \mu a + (1 - a) + \tilde{\eta}_2)\ /\ (1+\sigma\tilde{\eta}_1)\ (p/e)$$

the F.R.F. and H.R.F. shifts depend on k and k^* <u>only</u> if mark-up functions are not constant.

object that conjectures by firms are likely to be functions of prices and of the exchange rate. Indeed the literature provides analytical examples of variable conjectural variations; (24) however, all of them adopt very "ad hoc" specification, whose economic interpretation is hard to be found. In a static context, in which each duopolist forms independent conjectures about the <u>elasticity of response</u> of his rival's strategic variable, it seems plausible to assume that such conjectures are not functions of prices and exchange rate <u>levels</u>.

Since the model is highly non-linear, the implications of strategic interaction are better analysed by solving the system given by equations 2a), 2b), 8a) and 8b) numerically for different values of k, k^* and increasing values of μ, given a particular parametric configuration. The results in terms of the relevant elasticities are summarized in Table 1. (25)

For any value of k and k^* domestic price elasticities are increasing with the degree of wage indexation, while foreign price elasticities are decreasing in μ. Moreover, full indexation implies a constant real exchange rate for any k and k^*. However, this result is attained with different domestic and foreign price elasticities; in particular the pass-through of the exchange rate onto domestic prices is a decreasing function of the degree of strategic interaction. It is worth noting that with low values of μ, the impact on the real exchange rate crucially depends on k and k^*: the

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24. See, for example, Bresnahan (1981), Kamien-Schwartz (1983) and Dixit (1986).

25. I have chosen $a = 0.7$, $\alpha = 0.5$ and e, $\tilde{w}$, w^*, pm^* and K equal to one. The parameters of the demand functions are: $h = 0.5$, $\beta = 50.1$ and $\gamma = 49.9$. This implies that the two goods are nearly perfect substitutes.

TABLE 1

DOMESTIC AND FOREIGN PRICE ELASTICITIES WITH RESPECT TO THE NOMINAL EXCHANGE RATE

μ	COURNOT EQUILIBRIUM $k = k^* = 0$			$k = k^* = 0.3$			$k = k^* = 0.5$			$k = k^* = 0.9$		
	$\eta_{p,e}$	$\eta_{p^*,e}$	$\eta_{\lambda,e}$	$\eta_{p,e}$	$\eta_{p^*,e}$	$\eta_{\lambda,e}$	$\eta_{p,e}$	$\eta_{p^*,e}$	$\eta_{\lambda,e}$	$\eta_{p,e}$	$\eta_{p^*,e}$	$\eta_{\lambda,e}$
0.0	0.53	- 0.24	- 0.23	0.55	- 0.27	- 0.18	0.57	- 0.29	- 0.14	0.58	- 0.39	- 0.03
0.1	0.55	- 0.23	- 0.22	0.57	- 0.26	- 0.17	0.59	- 0.28	- 0.13	0.59	- 0.38	- 0.03
0.2	0.58	- 0.21	- 0.21	0.60	- 0.24	- 0.16	0.61	- 0.27	- 0.13	0.60	- 0.37	- 0.03
0.3	0.61	- 0.20	- 0.19	0.62	- 0.23	- 0.15	0.63	- 0.25	- 0.12	0.62	- 0.35	- 0.03
0.4	0.65	- 0.18	- 0.17	0.66	- 0.21	- 0.14	0.66	- 0.23	- 0.11	0.64	- 0.34	- 0.03
0.5	0.69	- 0.16	- 0.15	0.69	- 0.19	- 0.12	0.69	- 0.21	- 0.10	0.66	- 0.32	- 0.02
0.6	0.73	- 0.14	- 0.13	0.73	- 0.16	- 0.11	0.73	- 0.19	- 0.08	0.68	- 0.30	- 0.02
0.7	0.78	- 0.11	- 0.11	0.78	- 0.14	- 0.09	0.77	- 0.16	- 0.07	0.71	- 0.27	- 0.02
0.8	0.84	- 0.09	- 0.08	0.83	- 0.10	- 0.06	0.83	- 0.12	- 0.05	0.75	- 0.23	- 0.02
0.9	0.91	- 0.05	- 0.04	0.90	- 0.07	- 0.04	0.89	- 0.08	- 0.03	0.81	- 0.18	- 0.01
1.0	0.99	- 0.01	0.00	0.98	- 0.02	0.00	0.97	- 0.03	0.00	0.87	- 0.12	0.00

The parametric configuration is:

$h = 0.50$	$a = 0.7$	$pm^* = 1$
$\beta = 50.1$	$e = 1$	$K = 1$
$\gamma = 49.9$	$w^* = 1$	
$\alpha = 0.5$	$\tilde{w} = 1$	

gain in competitiveness is maximum in the Cournot case and it tends to zero as k and k^* approach one.

Thus, strategic interaction significantly alters the effects of nominal exchange rate variations on relative prices, especially when wage indexation is less than complete.

Conjectural variation models have been criticised on several grounds. First, while they are clearly concerned with dynamic phenomena, such models are not dynamic; in fact, firms are assumed to maximise current-period profits rather than profits discounted over some time horizon (Friedman, 1983). Secondly, the dependence of market equilibrium on the conjectural variations dilutes the predictive power of the theory (Kamien-Schwartz (1983) and Dixit (1986)). [(26)] Thirdly, the expectations about rivals's behaviour described by conjectural variations may not be correct (Friedman(1983), Kamien-Schwartz (1983), Bresnahan (1981 and 1983)).

While little progress has been made so far in analysing dynamic models, fruitful efforts have been made to overcome the other criticisms. In particular, several attempts have been made to narrow the possible market equilibria by imposing the additional constraint that the conjectural variations be "consistent". The basic idea is that if conjectural variations are consistent, then the firm's beliefs about its rivals' rate of response will coincide with the actual rates of response, at least in equilibrium [(27)]. As Bresnahan (1981) points out, the essential feature of

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26. This is due to the fact that k and k^* are independent and are allowed to assume any value between 0 and 1.

27. "The precise sense in which conjectures are to be consistent is this; the <u>conjectural variation</u> and the <u>reaction function</u> will be equated", (Bresnahan (1981), pag.934).

Cournot equilibrium is that each firm's beliefs about the level of all other firms's action turn out to be confirmed. However, firms are right in their beliefs for the wrong reason, since rivals' reaction functions do not have zero slopes, although conjectures do. Consistent conjectures imply that firms will be correct in equilibrium not only about the levels, but also about the slopes of the functions according to which rivals are reacting.[28]

Thus, consistency of conjectures makes the way firms react to one another endogenous by requiring that it be correct.[29] Imposing consistent conjectural variations in the previous model implies solving the system of non-linear equations given by 2a), 2b), 8a), 8b) and

10a) $k = \tilde{\eta}_2^{*}/(1 + \tilde{\eta}_1^{*})$

10b) $k^{*} = (\tilde{\eta}_2 + (1-\alpha)\mu a)/(1-\alpha\mu a + \tilde{\eta}_1)$

10a) equates the elasticity of the foreign firm's reaction function to the elasticity of response "conjectured" by the domestic firm and, similarly, 10b) implies that, at the equilibrium, the elasticity conjectured by the foreign firm is equal to the actual elasticity of the domestic reaction function.

Also in this case the model has been solved numerically for values of μ ranging from zero to unity; moreover, in

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28. According to Basar (1986), this is a first order consistent conjectural variations equilibrium. He shows that with non-linear conjectured response functions the equilibrium may not be completely defined unless higher orders of consistency are imposed.

29. Bresnahan (1981) adds : "One might view this as a kind of rational expectations oligopoly theory" (pag.935).

order to assess the importance of the degree of substitution between foreign and domestic goods, five different values of γ have been chosen, such that β/γ ranges from 4 (low substitution) to 1.005 (high substitution).[30]

Figure 2 shows how the equilibrium values of k and k^* are affected by the degree of wage indexation and product substitutability : k and k^* are always increasing in μ; k^* increases faster than k because the slope of the domestic reaction function depends directly on wage indexation. It is worth noting that when goods are very close substitutes, the consistent conjectures equilibrium (C.C.E.) implies a fairly high degree of strategic interaction even if money wages are not indexed.

If we compare the Cournot and C.C.E. outcomes in terms of domestic and foreign prices levels (Figg. 3 and 4), the role played by the degree of sustitution becomes even more evident.[31] When substitution is high, the Cournot equilibrium implies that both domestic and foreign prices are very close to the competitive level (i.e.: prices equal to marginal costs), while under C.C.E. prices are above marginal costs and are significantly affected by wage indexation.

Thus, when substitution is low the model suggests that wage indexation matters a lot in determining prices and profit levels and the Cournot assumption of no strategic interaction between duopolists is plausible. On the contrary, with high product substitutability the role of strategic interaction should be emphasised with respect to wage indexation.

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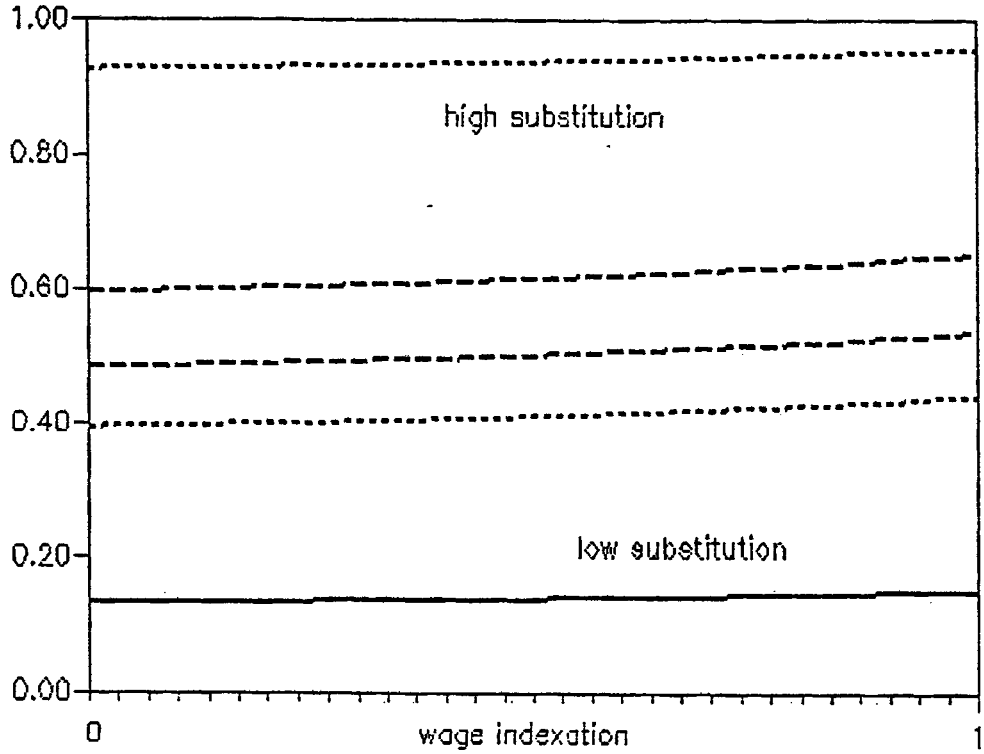
30. The remaining parameters are those of the previous exercise (see Table 1).

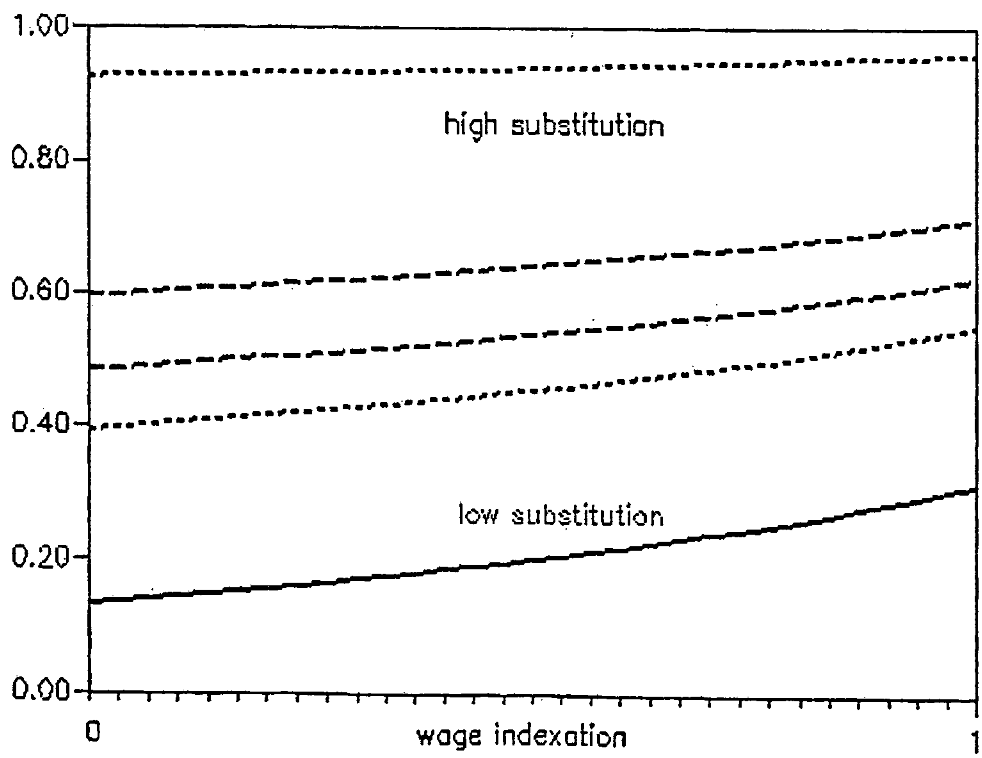
31. The Cournot solutions were found by simulating the previous model and imposing the constraint $k = k^* = 0$.

Fig. 2

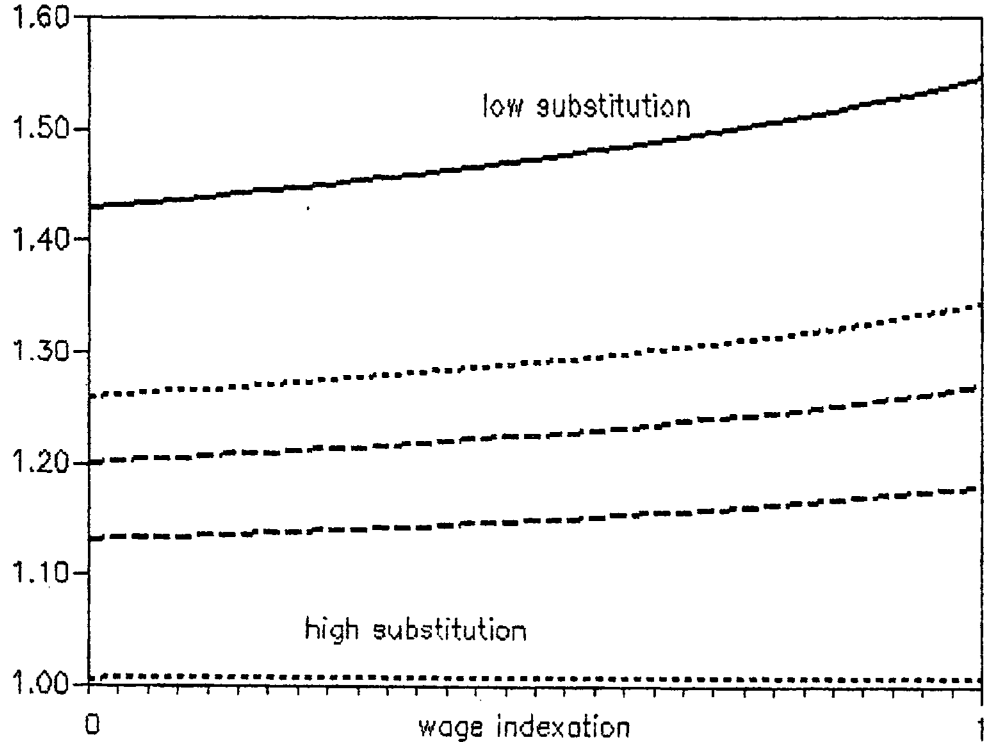
CONSISTENT CONJECTURES EQUILIBRIUM : K



CONSISTENT CONJECTURES EQUILIBRIUM : K*



COURNOT EQUILIBRIUM : DOMESTIC PRICE LEVELS



CONSISTENT CONJECTURES EQUILIBRIUM : DOMESTIC PRICE LEVELS

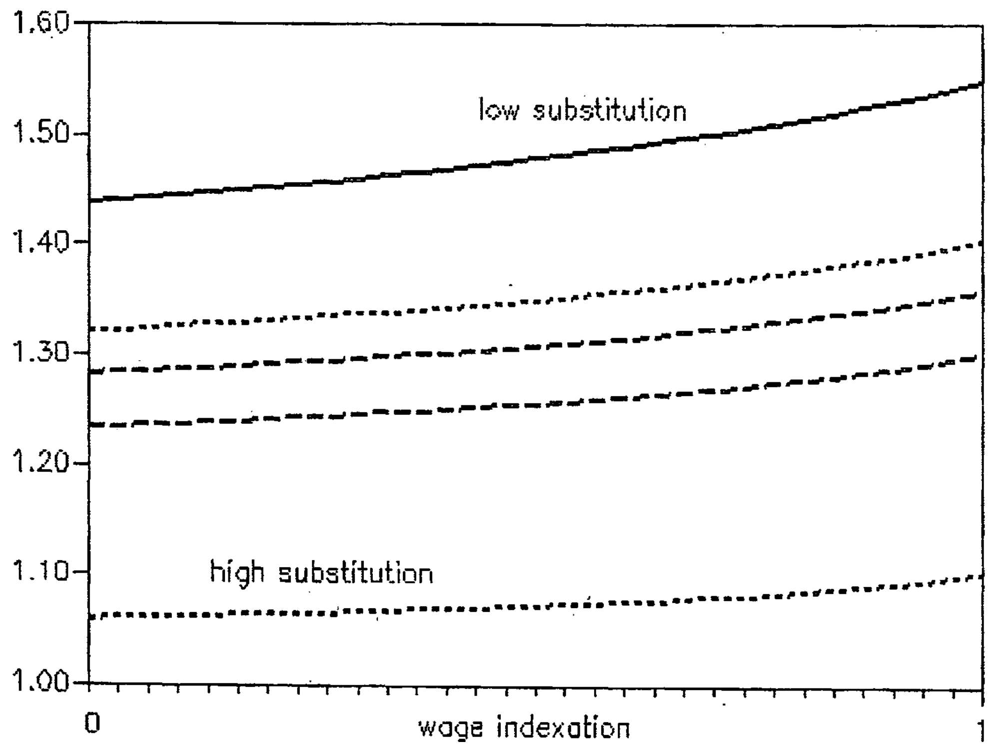
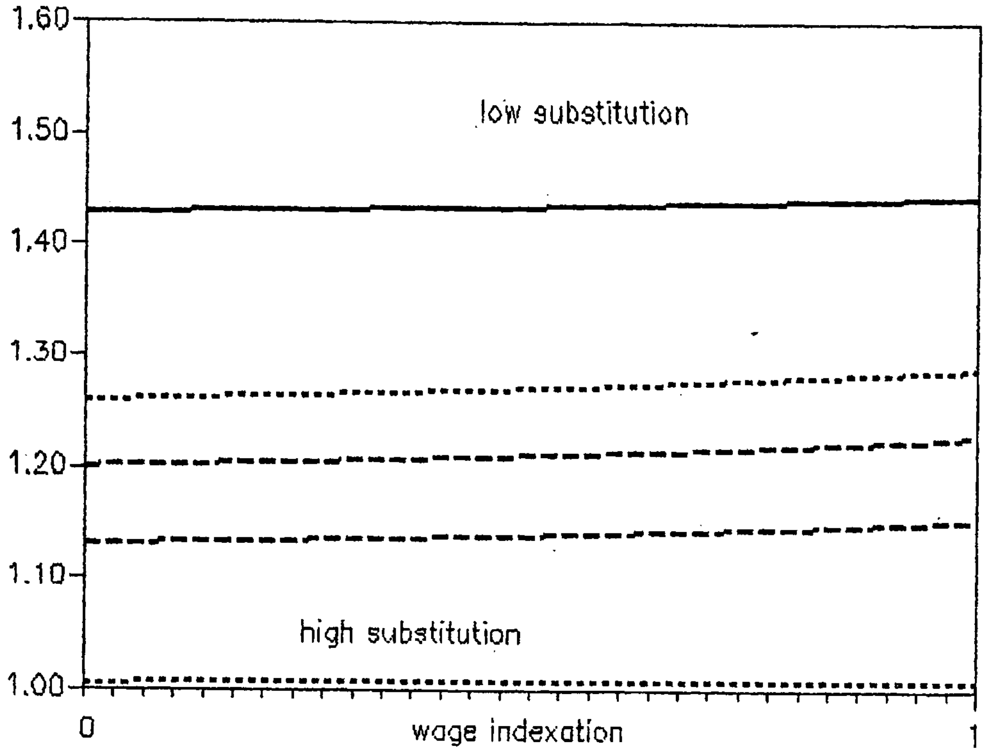
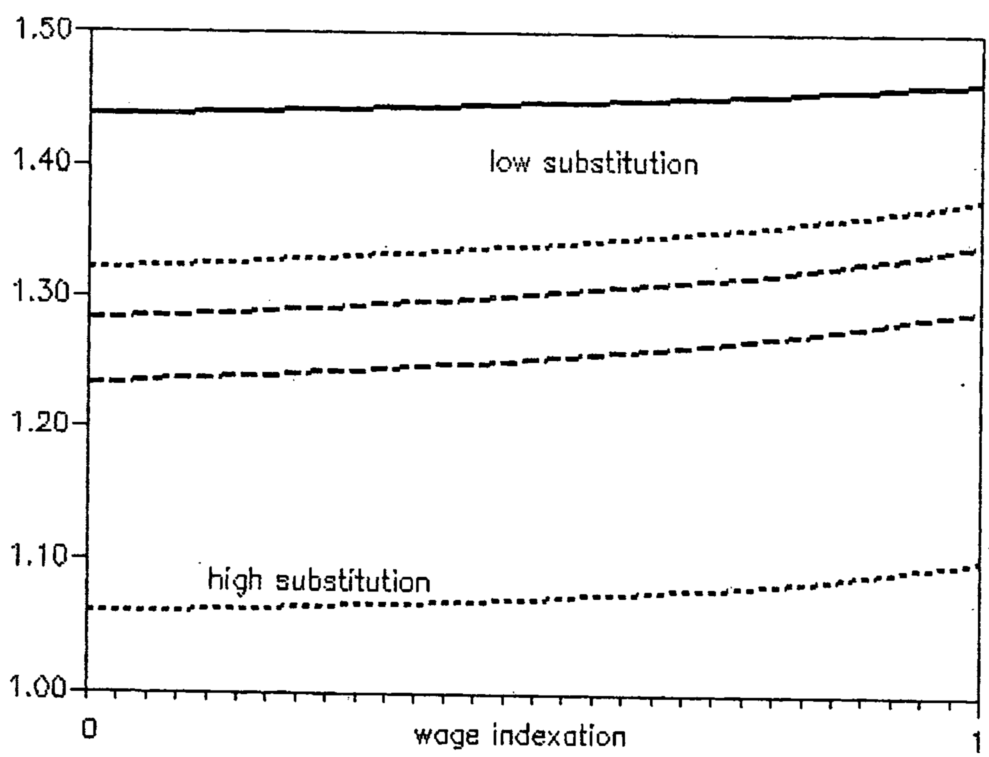


Fig. 4

COURNOT EQUILIBRIUM : FOREIGN PRICE LEVELS

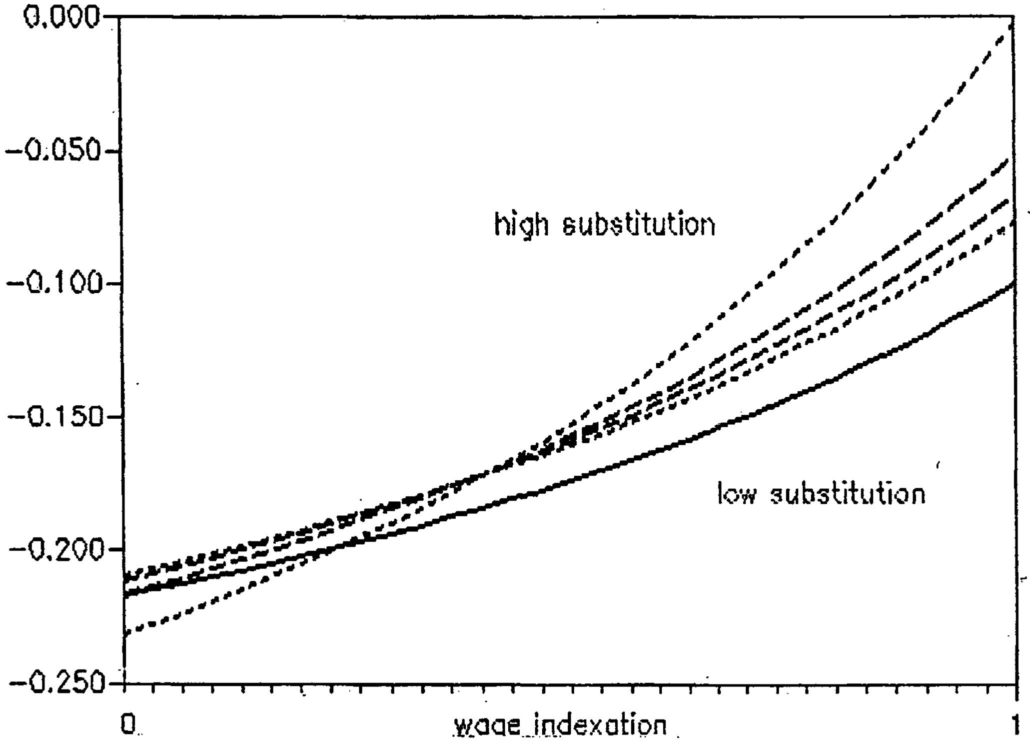


CONSISTENT CONJECTURES EQUILIBRIUM : FOREIGN PRICE LEVELS

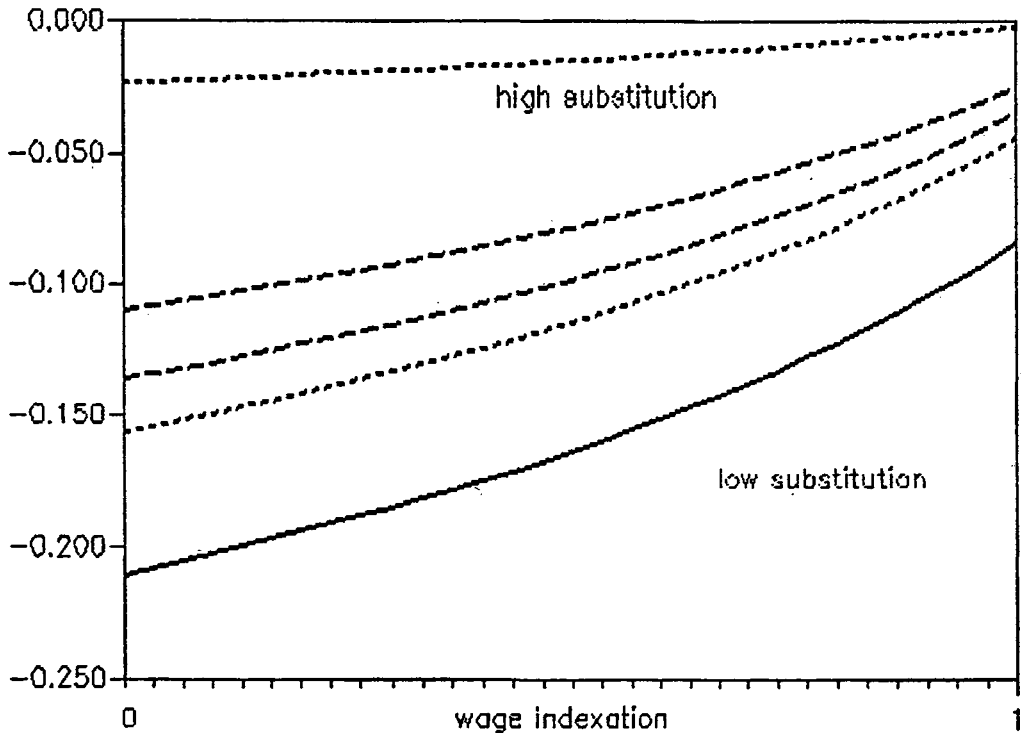


These remarks also apply if we consider the effects of exogenous movements in the nominal exchange rate (Fig. 5): with low substitution the predictions of both models are very similar and depend crucially on the degree of indexation. On the contrary, when substitution is high, C.C.E. implies nearly no effect on the real exchange rate while, in the Cournot case, this result is attained only with full indexation.

COURNOT EQUILIBRIUM : REAL EXCHANGE RATE ELASTICITIES
WITH RESPECT TO THE NOMINAL EXCHANGE RATE



CONSISTENT CONJECTURES EQUILIBRIUM:REAL EXCHANGE RATE ELASTICITIES
WITH RESPECT TO THE NOMINAL EXCHANGE RATE



4. Conclusion

This paper examines the effects of exogenous exchange rate movements in a model of international duopoly under alternative schemes of strategic interaction in a context where prices are the choice variables and technologies exhibit constant returns to scale.

If both duopolists act as Cournot-Nash players and domestic money wages are not indexed to foreign prices, the slopes of the two reaction functions will depend only on mark-up elasticities. In this case an exogenous depreciation will imply lower foreign prices and higher domestic prices; the real exchange rate will depreciate by an amount proportional to the share of labour in the productive process. Indexation of domestic money wages would amplify the effects of exchange rate movements on domestic prices and might imply full pass-through and a constant real exchange rate when the foreign and domestic products are very close substitutes.

If we consider a scheme of strategic interaction, according to which each duopolist makes conjectures about the elasticity of response of its rival's price to its own price changes, the previous results are significantly altered.

First, the equilibrium domestic and foreign prices are always higher than in the Cournot case. Secondly, the effects of exchange rate movements on relative prices depend crucially on the degree of strategic interaction. In particular, when goods are very close substitutes and each duopolist believes that his action will induce a similar change in the rival's strategy, a nominal depreciation of the domestic currency will have almost no impact on the real exchange rate, even when domestic money wages are not indexed.

The paper also considers the more general case of "consistent" conjectures, which implies that, in equilibrium, duopolists are correct not only about the rival's price level, as in the Cournot equilibrium, but also about the first derivative of the rival's reaction function. Conjectures are thus endogenously determined and depend on all the variables and the parameters of the model. This scheme of strategic competition is quite attractive, since it implies a better understanding by firms of the consequences of their own actions; it can therefore be used to discriminate between different models. In the case analysed in the paper, it has been found that the level of "consistent" strategic interaction is essentially determined by the degree of product substitutability. In particular, if substitution is low, the reaction functions tend to be very flat. Thus, the Cournot assumption of zero conjectures appears to be a fairly plausible simplification. On the contrary, with high substitutability, the C.C.E. predictions are very different from those of the Cournot model: domestic and foreign prices and profits are significantly higher, while a nominal depreciation has nearly no effect on the real exchange rate whatever the degree of domestic wage indexation.

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