## BANCA D'ITALA

## Temi di discussione <br> del Servizio Studi

A Model for<br>Contingent Claims Pricing on EMS Exchange Rates

by Antonio Roma



## BANCA D'ITALA

# Temi di discussione <br> del Servizio Studi 

A Model for<br>Contingent Claims Pricing on EMS Exchange Rates

by Antonio Roma

The purpose of the «Temi di discussione» series is to promote the circulation of working papers prepared within the Bank of Italy or presented in Bank seminars by outside economists with the aim of stimulating comments and suggestions.

The views expressed in the articles are those of the authors and do not involve the responsibility of the Bank.

## A MODEL FOR CONTINGENT CDAIMS PRICING ON EMS EXCHANGE RATES

The paper proposes a stochastic model for the bilateral exchange rate of currencies participating in the European Monetary System (EMS). A bivariate jump-diffusion process is employed to represent the exchange rate and the institutional constraints affecting it. A discrete time and state model is also provided. In the model, conditional mean reverting behaviour is posited for the exchange rate. Mean reverting behaviour can actually be empirically observed for EMS exchange rates, as opposed to the usual findings for freely floating exchange rates, and constitutes evidence in favour of the model. The proposed model may find application in asset pricing models, especially currency options pricing models.

## INTRODUCTION ${ }^{1}$

The European Monetary System (EMS) exchange rate mechanism influences heavily the behaviour of the participating currencies bilateral exchange rates. Modelling the resulting exchange rate process is essential in the context of asset pricing models that involve these exchange rates. Currency option pricing, for example, usually employs Black and Scholes type models [Garman and Kohlagen, 1983], where a continuous time and space stochastic process, a geometric Brownian Motion, is used for the representation of the exchange rate behaviour. Similar assumptions are made for the representations of exchange rates in equilibrium models of international capital markets [Solnik, 1974]. Although this may adequately describe the behaviour of freely floating exchange rates as those in terms of the US Dollar, a Brownian process does not describe accurately the behaviour of the exchange rates within the EMS. Due to the institutional enforcement of fluctuation bands subject to realignments, the EMS currencies bilateral exchange rates follow a complicated process, with discontinuous jumps alternating with "normal" periods within which the exchange rate may be considered to be following a continuous sample path process [see Figures 1-5].

In this paper, we model EMS exchange rates in view of potential analytical applications to asset pricing. Consequently, we represent

[^0]exchange rate behaviour as a Markov process, therefore preserving simple mathematical tractability ${ }^{2}$. Knowing that the process can be fitted by a more general ARMA model, for example, is not going to help directly in designing an appropriate currency option pricing model.

Initially, the aim of the analysis is to examine how the exchange rate process of EMS currencies is affected by the EMS constraint, conditional on a realignment not occurring. We would expect central bank intervention to maintain the exchange rates within the EMS bilateral limits, or the divergence thresholds in the case of ECU exchange rates. The resulting exchange rate behaviour is quite complex, consisting in a market dynamic on which central banks' intervention is at times superimposed. The actual timing and effect of the central banks, intervention is not easily representable. It is however argued that, at least in the proximity of the EMS barriers, these interventions will potentially be an important force affecting it. In particular, one expects some dependence of the distribution of exchange rate returns on the level of the exchange rate, or on the relative position of the exchange rate within the EMS band.

Few references exist in the literature on the analysis of the behaviour of constrained exchange rates. Farber, Roll and Solnick [1977] and Westerfield [1977] examine the distributional characteristics of exchange rate returns under the pre-1971 regime of fixed exchange rates as opposed to the subsequent freely floating regime. They found the distribution to be more leptokurtic under the fixed exchange regime than under floating exchange rates, and the volatility of returns to be higher in the second case. Also, considering EMS exchange rate

[^1]behaviour as a process within reflecting barriers, some implications of a model proposed by Cootner [1964] are relevant here. He postulates prices behaving as a random walk within reflecting barriers, in an attempt to account for the empirical leptokurtosis of stock prices. The sample moments of the distribution of continuously compounded returns computed for EMS bilateral exchange rates over the period 1979-1987 give results in accord with the above studies [Tables 1.1 and 1.2].

In a simplified context, we will assume central bank intervention as a simple linear function of the exchange rate displacement from a target value (as a first approximation the official central parity), and posit a stochastic process representation for EMS bilateral exchange rates compatible with this idea. Although reaction functions of the monetary authorities may not be so simple [see for example Neumann. 1984], the displacement from a target value is an important component in their specification. It seems that a process compatible with long term smooth functioning of EMS bands will have to be modelled as a stationary process, where some friction prevents the exchange rate from reaching limits, and where the central parity can be viewed as an equilibrium rate. We will therefore propose a stochastic process with similar characteristics for the modelling of the behaviour of EMS exchange rates between realignments, and we will contrast it with a random walk representation, as usually posited for freely-floating exchange rates. Moreover, we will model EMS realignments as jumps in the exchange rate regime which involve at the same time a jump in the exchange rate.

The plan of the paper is as follows: in section 1 a simple discrete time and state framework is introduced in order to provide some intuition on
the basic idea underlying the model. In section 2 a continuous time version is introduced, which offers some advantages in terms of analytical tractability and estimation. We treat the empirical plausibility of the proposed process in section 3.

## 1 - A MARKOV CHAIN MODEL

We illustrate in this section our model for the EMS exchange rates making use of a discrete time and state framework. Assume $2 \mathbf{2 a + 1}$ states constitute the state space of the EMS band, or values that the exchange rate may take. The logarithm of the exchange rate may be assumed to behave according to the Ehrenfest model [see Karlin and Taylor, 1975, or Cox and Miller, 1970], a state dependent random walk where the variable can move, between two points in time, only one state up or one state down with respective probability $\frac{n-i}{2 a} \cdot \frac{n+i}{2 a}$, where $i$ is the initial state, $\{-a \leq i \leq+a\}$. The random variable oscillates between the value in the highest state and the value in the lowest state, which represent reflecting barriers, and is pulled towards the mean value associated with the middle state 0 with a force proportional to the displacement from the mean state. The Ehrenfest model is a time homogeneous Markov chain with a binomial limiting distribution.

This process is fundamentally different from the usual random walk with identically and independently distributed (iid) increments assumed for exchange rates, which would have constant probabilities of an up or down move (apart from boundary behaviour at the barriers). A random walk with iid increments will in fact be our null hypothesis in the empirical
tests performed in the last section. In our model, the logarithm of the exchange rate between realignments follows an autoregressive process. This type of behaviour may be incompatible with market efficiency. However, when the possibility of a realignment is included, a much more plausible process results. Given the paucity of realignments, the modelling of a jump component can not be adequately tested empirically. As such we assume a simple model. Given $x$ the value of the logarithm of the exchange rate at time $t$, after a unit of time $T$ the state will either jump to $x+J_{x}$ with fixed probability $\lambda$, or, with probability (1- $\lambda$ ), it will undergo a normal transition, moving $\eta$ up or down with probabilities, $(1-\lambda) \frac{n-i}{2 a},(1-\lambda) \frac{n+i}{2 a}$. The jump $J_{x}$ can be $\frac{\rangle}{\bar{z}} 0$, and the expected change in logarithm ${ }^{3}$ over each unit time interval will be

$$
\begin{equation*}
E[\Delta x]=\lambda J_{x}-(1-\lambda) \frac{i \eta}{a}, \tag{1}
\end{equation*}
$$

where indicates the relative position within the band, and in represents the displacement from the mean.

By modelling the jump size as a simple function of the exchange rate displacement from the mean. it is possible to obtain expected returns that are $\frac{\rangle}{\bar{z}} 0$. Also, a more complex specification of the jump size as a function of other variables such as interest rate differentials, preferences, and time is possible.

Writing

$$
\begin{equation*}
E[\Delta x]=r_{d} \mathbf{r}_{\mathbf{f}}+\pi \tag{2}
\end{equation*}
$$

where $r_{d}$ is the domestic interest rate over the unit time period, $r_{f}$ the foreign rate and $\pi$ a risk premium, we could write, using (1) and (2) ${ }^{4}$,

[^2]\[

$$
\begin{equation*}
J_{x}=i \eta \frac{(1-\lambda)}{a \lambda}+\frac{r_{d}-r_{f}+\pi}{\lambda} \tag{3}
\end{equation*}
$$

\]

In what follows the exchange rate will be written as the central parity times a "one plus" percentage increment, or

$$
\mathbf{x}=\mathbf{P}+\mathbf{I}
$$

where $P$ is the logarithm of the central parity and $I=i \eta$. We assume that I follows an Ehrenfest process, and that $P$ follows a jump process. The size of the jump in $P$ is $J(i, a)=J_{x}+I$, where $J_{x}$ is given by expression (3), which assumes specification of $\pi, \lambda, \quad r_{d}, r_{f}$. As indicated notationally, J(i,a) depends on the position within the band, given by $i$ and the number of states into which the EMS band is subdivided, indexed by a. Each jump sets the central parity and simultaneously the exchange rate to a new level. As a special case, we may assume that when $a$ jump occurs, the new level $x+J_{x}=P+J(i, a)$, while, in general, after a jump I is restarted randomly around the new parity. Given a specific discretization of the state space within the EMS band, the feasible $x$ values will then be identified by three coordinates, $P$, $i$, and time. This follows from the fact that two different state variables are driving the process, a market-related stochastic movement around the central parity, and the variation of the parity. As a consequence, the behaviour of the exchange rate evolves in
with market equilibrium. This can be formally verified applying some fundamental theorems in Harrison and Kreps [1979] and Harrison and Pliska [1981]. Harrison and Kreps [1979] are able to show the existence of a one to one correspondence between the set of equivalent martingale measures under which discounted prices are martingales and the set of general equilibrium prices in an economy. It is easily verified that, under the hypothesis made, if the probability of a jump is excluded an equivalent martingale measure will not exist, and modelling a jump component is therefore a necessity.
a three-dimensional space. At each point in time, a matrix of $x$ values is generated, corresponding to each feasible central parity.

## 2 - A CONTINUOUS TINE MODEL

The Ehrenfest process, introduced in the first section to sketch our basic idea of the EMS exchange rate process, will converge, letting $\eta \rightarrow 0$, $\tau \rightarrow 0$, in the appropriate way, to a continuous time and state process, the Ornstein Uhlembeck process [see Karlin and Taylor, 1981], which may be represented by the Ito stochastic differential equation

$$
\mathrm{dx}=\alpha(\gamma-\mathrm{x}) \mathrm{dt}+\beta \mathrm{dZ}
$$

where $d Z$ is the increment to a standard Wiener process. The Ornstein Uhlembeck (henceforth $0 . U_{\text {.) }}$ process is a diffusion process with a central restoring tendency that makes it tend towards the long term mean $r$. This may model the tendency of EMS exchange rates to fluctuate around a central parity. This is again a simplification, since if any restoring force exists, pulling the exchange rate towards some level inside the EMS band, it may not have the characteristics of being continuous and proportional to the displacement from the equilibrium value, as posited by the O.U. model. Still this model may have some interesting features in relation to the problem at hand. It has an equilibrium distribution with constant variance, which may approximate the fact that, conditional
on no changes in parity occuring, the range of values that the exchange rate will take at different points in time in the future is fixed, it is easily tractable mathematically and takes account, although through a very simple specification, of the hypothized level dependency of the process. Its drawback is that it does not respect the EMS barriers, since, at a specific time, it has some positive probability to reach any level. However this probability is small, and for practical purposes may be disregarded ${ }^{5}$.

The O.U process has a gaussian transition density (which satisfies the appropriate forward and backward equation) with parameters:

$$
\begin{gathered}
E\left(x_{t} \mid x_{t-k}\right)=\gamma\left(1-e^{-\alpha k}\right)+x_{t-k} e^{-\alpha k} \\
\operatorname{VAR}\left(x_{t} \mid x_{t-k}\right)=\frac{\beta^{2}}{2 \alpha}\left(1-e^{-2 \alpha k}\right)
\end{gathered}
$$

and $\quad \operatorname{CORRELATION}\left(x_{t}, x_{t-k}\right)=e^{-\alpha k}$.

As $k \rightarrow \infty$, the process becomes stationary and has an equilibrium distribution with mean $r$ and variance $\frac{\beta^{2}}{2 \alpha}$.
As in section 1, a jump component will have to be included, to model realignments. This will result in a mixed jump-diffusion process.

A few continuous time mixed jump-diffusion models for security prices can be found in the literature.

[^3]In Press [1967] the following model was introduced:

$$
x(t)=x(0)+\sum_{k=1}^{N(t)} Y_{k}+Z(t)
$$

where $x(t)$ is the logarithm of the price at time $t, x(0)$ the known initial value at time $0, N(t)$ is a Poisson distributed random variable with mean $\lambda t, Y_{k}$ are the logarithms of jump sizes, which are assumed iid. random variables with common distribution ${ }^{6} N\left(. ; \mu, \sigma_{j}^{2}\right)$, and $Z(t)$ is a Wiener process the distribution of which is $N\left(. ; 0, \sigma^{2} t\right)$. The model describes the logarithm of the price evolving as a Wiener process on which, at exponentially distributed time intervals, jumps of stochastic size are added. The Wiener process $Z(t)$ is assumed independent of $Y_{k}$ and $N(t)$
$N(t)$. The term $\underset{\mathbf{k}=1}{\mathbf{N}_{\mathbf{k}}} \mathbf{Y}_{\mathbf{k}}$ represents a compound Poisson process.
Merton [1976] uses very similar dynamics: however, he does not assume zero drift for the Wiener process $Z(t)$ and, in the broader case, does not costrain the distribution of the jump size $Y_{k}$ to be normal.

Jarrow, Oldfield and Rogalski [1977] further generalize the model by allowing the random jump sizes $Y_{k}$ to be autocorrelated and by assuming a gamma distribution for the time interval between jumps, which includes the exponential distribution as a special case. Ahn and Thompson [1988] superimpose jumps on the square root process.

In the present case, the following model is proposed.
Let $x(t)$ be the logarithm of the exchange rate:

$$
x(t)=P(t)+I(t)
$$

where $I(t)$ is assumed to follow an $0 . U$. process,

$$
\mathrm{dI}=\alpha(\gamma-\mathrm{I}) \mathrm{dt}+\beta \mathrm{dz}
$$

$6_{\text {We define }} N\left(x: \mu, \sigma^{2}\right)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left[\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right]$

As mentioned above, the equilibrium distribution of $I(t)$ will be gaussian with mean $r$ and variance $\frac{\beta^{2}}{2 \alpha}$. If the equilibrium mean of the exchange rate is taken to be $\exp \{P\}, \gamma$ will be equal to 0 . Assume that a jump in $P$ may occur over each interval of time. Let the probability that at least one jump occurs in the interval of time $\Delta t$ be given by $\lambda \Delta t+0(\Delta t) ; 1-\lambda \Delta t-o(\Delta t)$ represents the probability that no jump occurs in the time interval $\Delta t$, while $o(\Delta t)$ represents the probability that more than one jump occurs in the interval of time $\Delta t$, and goes to zero faster than $\Delta t$. The occurrence of a jump is assumed independent of previous occurrences. Assume also that if a jump in $P$ occurs, I is restarted independently according to the equilibrium distribution $\mathrm{z}^{*}=N\left(. ; 0, \frac{\beta^{2}}{2 \alpha}\right)$. The expected change in $x$ per small unit time will be:

$$
\lambda \mathrm{J}-(\alpha+\lambda) \mathrm{I}
$$

where $J$ is the jump in $P$.
Setting the expression equal to $r_{d} r_{f}+\pi$, where $r_{d}$ and $r_{f}$ are the domestic and foreign instantaneous riskless rates, assumed constant as the risk premium $\pi$,

$$
J(I)=I \frac{\alpha+\lambda}{\lambda}+\frac{r_{d}{ }^{-r} f+\pi}{\lambda}
$$

The size of the jump in $P$ will be linear in $I$ and inversely related to $\lambda$. Note that the process for $I$ starts randomly according to its equilibrium distribution. This will imply that the distribution of $I(t)$, at any point in time "in the future", will still be the equilibrium distribution ${ }^{7} Z^{*}$. The value of $x(t)$ at any point in time, given an initial value of $P$, will be the sum of the initial value of $P$ plus a
${ }^{7}$ Cfr. Karlin and Taylor [1981] p. 220.
poisson number of jumps, $N(t)$, in $P$, plus the value of the increment $I$ :

$$
x(t)=P(0)+\sum_{k=1}^{N(t)} J_{k}+I(t)
$$

$\mathrm{J}_{\mathrm{k}}$ are iid gaussian random variables whose distribution is easily derived, since $J$ is a linear function of $I$ and the distribution of $I$ is $Z^{*}$. The distribution of the $J_{k}$ will be normal with

$$
\begin{aligned}
& E\left[J_{k}\right]=\frac{r_{d}-r_{f}+\pi}{\lambda} \\
& \operatorname{VAR}\left[J_{k}\right]=\left[\frac{\alpha+\lambda}{\lambda}\right]^{2} \frac{\beta^{2}}{2 \alpha}
\end{aligned}
$$

The process for $x(t)$ will be the sum of an independent $0 . U$. process in equilibrium and a compound Poisson process. The process for $\{x(t), P(t)\}$ will be Markov with mean and variance ${ }^{8}$ :

$$
\begin{gathered}
E[x(t)]=P(0)+\left(r_{d}-r_{f}+\pi\right) t \\
\operatorname{VAR}[x(t)]=\frac{\beta^{2}}{2 \alpha}\left[1+\left[\frac{\alpha+\lambda}{\lambda}\right]^{2} \lambda t\right]+\left[\frac{r_{d}-r_{f}+\pi}{\lambda}\right]^{2} \lambda t
\end{gathered}
$$

Also the probability distribution of $x(t)$ can be computed.

$$
\operatorname{Prob}[x(t) \leq x]=\sum_{n=0}^{\infty} \frac{(\lambda t)^{n} e^{-\lambda t}}{n!} F^{n}(x)
$$

where $F^{n}(x)=\operatorname{Prob}\left\{P(0)+J_{1}+J_{2}+\ldots \ldots+J_{n}+I \leq x\right\}$ is a normal distribution with mean $\frac{n\left(r_{d}-r_{f}+\pi\right)}{\lambda}+P(0)$ and variance $n\left[\frac{\alpha+\lambda}{\lambda}\right]^{2} \frac{\beta^{2}}{2 \alpha}+\frac{\beta^{2}}{2 \alpha}$.

Note that although the transition density of $x$ conditional on an initial

[^4]level of $P$ can easily be determined from the above result, the total process for the logarithm of the exchange rate is a bivariate process where $P$ is a stochastic variable as well. It is clear how each of the observed values of $x(t)$ may originate either from a transition within the same level of $P$, or may originate due to a shift in $P$. The additional problem faced in the estimation of the continuous time process is to detect whether each transition arises from a diffusion movement or from a jump in the mean of the process. Detecting jumps in the mean of the process may cause major difficulties. However, in our case, we may assume as a first approximation that the mean of the process, $P(t)$, changes only when a realignment occurs, resulting in a considerable simplification.

## 3 - A TENTATIVE ESTIMATION PROCEDURE

In this section we try to show that the $O$. $U$. representation is more plausible than Brownian motion for the behaviour of EMS exchange rates between realignments by fitting the two alternative processes.

In fitting a Markov process to a sample path of $n$ observations, $X_{0}, X_{1}, X_{2} \ldots X_{n}$, the fact can be used that the likelihood function of the sample path, L, is given by the product of the likelihood of subsequent changes ${ }^{9}$ :
$L\left(X_{0}, X_{1}, X_{2} \ldots X_{n}\right)=L\left(X_{n} / X_{n-1}\right) L\left(X_{n-1} / X_{n-2}\right) \ldots \ldots L\left(X_{2} / X_{1}\right) L\left(X_{1} / X_{0}\right)$.

[^5]In the case of the $0 . U$. process, due to the normality of the transition density, finding the parameters that maximize the likelihood of a sample path between jumps corresponds exactly to the ordinary least squares (OLS) fit of the linear regression

$$
x_{t}=\rho x_{t-1}+\mu+\epsilon_{t}
$$

where $\rho=e^{-\alpha}, \mu=\gamma(1-\rho)$, and the sum of the squared residuals yields an estimate of the variance. The null hypothesis is

$$
H_{0}: \rho=1, \mu=0,
$$

which implies that the continuous time process is a Brownian Motion. and the discrete realization a random walk with iid increments, against the alternative hypothesis

$$
\mathrm{H}_{1}: \rho<1 \text {, }
$$

which instead indicates that the continuous process is an $0 . \mathrm{U}$. process, and the discrete time process an autoregressive process of the first order (AR1). A further implication of $H_{0}$ is that the variance of the transition density will increase proportionally to time. Intuitively, this may not be compatible with the imposition of a fluctuation band. The alternative implies that the estimated variance parameter is $\frac{\beta^{2}}{2 \alpha}\left(1-e^{-2 \alpha}\right)$, and that the variance of the transition density becomes constant in the long run. There are some statistical complications in testing the hypothesis $\rho=1$. since the sample coefficent $\hat{\rho}$, under the null hypothesis, has an unusual distribution, and an alternative statistic for the ratio must be employed (see Dickey and Fuller [1981], and Fuller [1976]). The test is still quite simple, and references to this type of test for random walks, applied to US dollar exchange rates, may be found in the literature ${ }^{10}$. The estimation was

[^6]here implemented on the spot exchange rate in terms of Deutsche Mark of the French Franc, Belgian Franc. Italian Lira, Dutch Guilder and Danish Kroner. The data considered are the Wednesday exchange rates of the above currencies over the period March 13, 1979 - June 17, 1987 ${ }^{11}$. The data are obtained from the daily fixing of the Italian Lira, in Rome and Milan, and from the opening quotes in terms of US dollars of Baring Brothers in London. Data from the first source were used for the Lit/DM exchange rate and to obtain cross rates of the $\mathrm{FF} / \mathrm{DM}, \mathrm{BFr} / \mathrm{DM}$ and $\mathrm{DM} / \mathrm{DKr}$ exchange rate ${ }^{12}$. The $D F 1 / D M$ exchange rate was obtained crossing the Baring Brothers data, which were in the form of mid-points of bid and ask prices. Also data as published by the Financial Times were used for a cross check of the results, where possible.

In order to have a substantial number of observations, the test on the conditional process of the above EMS exchange rates was carried on considering only the longest periods between realignments; since in these periods the exchange rates considered fluctuated within a small band for some considerable time, no time trend is considered as an independent regression variable, even though in the case of the Lit/DM exchange rate it might have been more appropriate. The results of the OLS regression are presented in Tables 2.1 and 2.2. No conclusive statistical evidence can be found in favour of the O.U. model, since $t$ values which would lead to rejection of the null hypothesis are of ten

[^7]${ }^{12}$ The fixing of Lira exchange rates against EMS currencies is not perfectly synchronous. However, $15-30$ minutes at most elapse between the fixing of different exchange rates. Using standard asymptotic theory, an indication of the bias of the estimated coefficient $\hat{\rho}$ due to measurement error is given by the ratio of the exchange rate variance over 30 minutes to the exchange rate variance over a week, which is negligible.
associated with a possible misspecification of the model, as it appears from the residuals of the regressions; this on the one hand prevents statistical assessment, and on the other may raise questions about market efficiency. However, analysis of the residuals showed that large negatively correlated residuals of ten arose close to EMS realignment dates. This suggests that when the ex ante probability of a jump is considered, the exchange rate process may still be compatible with market efficiency. (A clear example of this type of equilibrium behaviour can be seen in the case of the $\mathrm{BFr} / \mathrm{DM}$ exchange rate behaviour in the six months preceding the October 1981 realignment, when it fluctuated very close to the EMS band with a strongly autoregressive sample path; see Table 2.1). Also, the choice of sample subperiods limits the number of observations available and the power of the test. When the regression was run on the deviations of the exchange rate from the central parity, it was never possible to reject the hypothesis that the mean of the process was equal to 0.

## 3.1 - A MORE PONERFUL TEST

Given that in this model we identify jumps with realignments, a more powerful test of the autoregressive tendency of the exchange rate becomes available, since we can easily estimate the parameters of the process using all available observations, including realignments. As a first approximation, we may assume that the mean of the process of the logarithm of EMS exchange rates, $P(t)$, changes only when a realignment occurs, and that between realignments it is equal to the logarithm of
the central parity. As a further simplification, we consider that, over each weekly interval, at most one jump may occur with fixed probability $\lambda$. Also, while in the general process described in section 2 we specify the size of the change in $P(t)$. $J$, as a function of a number of variables, in the present case $J$ is taken as given. Assuming that $x(t)$ follows an O.U. process around the mean $P(t)$, and knowing $P(t)$ at each point in time, it is possible to estimate the de-meaned process $I(t)$ which, conditional on no realignment, will itself follow an O.U. process with mean 0 .

In this simple version of the model, when a jump occurs, the value of the displacement of the variable $x$ from its new mean $P$, the increment $I(t)$, is fixed. This model will be called model A. Over each time interval, the distribution of $I(t)$ then is:

$$
\begin{array}{r}
\frac{1}{\sqrt{2} \pi \sigma_{2}^{2}} \exp \left[-\frac{1}{2 \sigma_{2}^{2}}[x(t)-\rho x(t-1)-(1-\rho) P(t)]^{2}\right] \\
\text { with probability (1- } \lambda) \\
\\
\text { (if a jump does not occur) }
\end{array}
$$

$\bar{I}(t) \quad$ with probability $\lambda$
(if a jump occurs)
where $\sigma_{2}^{2}=\frac{\beta^{2}}{2 \alpha}\left(1-\mathrm{e}^{-2 \alpha}\right)$.
This may be theoretically justified assuming, for example, that the exact value of the exchange rate after a realignment is set by monetary authorities. Under the present assumptions, we know from which of the two distributions each observation was drawn. Let $n$ be the number of
observations of the variable $x(t)$, denoted $x_{i}(i=1,2 \ldots n)$. The observations are separated into two groups: those which immediately follow a realignment are then indexed with a different subscript, $n_{j}$, which specifies that they are generated by the $\mathbf{j}$ th jump in $P(t)$. Since $P(t)$ is constant except for the occurrence of jumps, the notation $P_{j}$ ( $j=0,1,2 \ldots k$ ) will be used, where $k$ is the number of jumps observed.

In this case, the likelihood function of a sample path, $L$, takes the simple form:

$$
\begin{equation*}
L=\left[\prod_{j=0}^{k}\left[\prod_{i=n_{j}+1}^{n_{j+1}^{-1}} N_{i}\left(. ; P_{j}, \sigma_{2}^{2}\right)\right]\right] \lambda^{k}(1-\lambda)^{(n-k-1)} \tag{4}
\end{equation*}
$$

with $n_{j+1}{ }^{-1}>n_{j}+1 . N_{i}($.$) denotes the transition density of observation$ i.

To help understanding the unusual notation, consider the following example: of a sample path of $n=8$ observations, observations (1,2,3) relate to a first value of $P(t), P_{0}$, observations (4,5,6) relate to a second value of $P(t), P_{1}$, observations (7,8) are related to a third value of $P(t), P_{2}$.

$$
\frac{\cdots x_{1} x_{2} x_{3}}{P_{0}} \frac{x_{4} x_{5} x_{6}}{P_{1}} \frac{x_{7} x_{8}}{P_{2}}
$$

In this case, we have two jumps, and $x_{n_{1}}=x_{4}, x_{n_{2}}=x_{7}$. We define also $x_{n_{k+1}}=x_{n+1}$ and $x_{n_{0}}=x_{1}$. Assuming that the values $P_{j}$ are known (and recalling $I=x-P$ ), we can write the log-likelihood function, $\ell$, of a sample path as

$$
\begin{aligned}
& \ell=-n / 2 \ln 2 \pi \sigma_{2}^{2}-1 / 2 \sigma_{2}^{2} \sum_{j=0}^{k} \sum_{i=n}^{\sum_{j}+1}\left[I_{i}^{-1}-\rho I_{i-1}\right]^{2}+ \\
& +k \ln \lambda+(n-k-1) \ln (1-\lambda) \quad .
\end{aligned}
$$

and the maximum likelihood estimates of $\left\{\rho, \sigma_{2}^{2}\right\}$ are simply obtained running an OLS regression of the logarithm of the ratio of the exchange rate to the central parity on its lagged value, deleting the observations $\bar{I}(t)$ which represent jumps as dependent variables; the estimate of $\lambda$ separates out, since the jump process is independent, and can be done looking at the intervals of time between jumps only. The estimation results for model (A) are reported in Table 3.1.

Also, if the values of $P_{j}$ are not known (but only the time points at which they change are known), they can be estimated.

## 3.2 - STATISTICAL PROBLEMS

A statistical problem that arises in the estimation presented above is the assessment of the distribution of the sample coefficent $\hat{\rho}$ in the case in which the true coefficient is equal to 1 . In fact, when model
(A) is estimated, although the sample coefficient is obtained through an OLS regression, the presence of jumps will cause a difference with the distribution of the sample coefficient for the case $\rho=1$ derived by Dickey and Fuller. So attention will be concentrated on the distribution of the sample coefficent $\hat{\rho}$ under this model. Knowledge of this
distribution is essential in order to conclude, from Table 3.1, that the conditional process of the logarithm of the displacement of EMS exchange rates from the central parity may be appropriately represented by means of an O.U. process.

If model (A) is adopted, the maximum likelihood estimate of the coefficient $\hat{\rho}$ can be expressed as:

$$
\hat{\rho}=\frac{\sum_{j=1}^{k} \sum_{i=n_{j}+1_{i}}^{n_{j+1}^{1}} I_{i-1}}{\sum_{j=0}^{k} \sum_{i=n_{j}+1}^{n_{j+1}^{-1}} I_{i-1}^{2}}
$$

Since $I_{i}=\rho I_{i-1}+\epsilon_{i}$, defining
we can write

$$
\begin{equation*}
\hat{\rho}-\rho=\sum_{j=1}^{k}\left[\frac{C_{j}}{V_{j}} \frac{V_{j}}{\sum_{j=1}^{k} V_{j}}\right] \tag{5}
\end{equation*}
$$

If there were no jumps, i.e. if we were estimating using only observations relating a generic period $j$ between realignments, the expression would simply be:

$$
\hat{\rho}-\rho=\frac{C_{j}}{V_{j}}
$$

When $\rho=1$ and $\bar{I}(t)=0$, the distribution of $C_{j} N_{j}$ divided by the standard error of $\hat{\rho}$ (the t statistic) is the distribution derived by Dickey and Fuller, conditioned on the initial observation $=0$. The effect of a non-zero first observation vanishes asymptotically. Moreover, as opposed to the case when $|\rho|<1, V_{j}$ will not converge in probability to a constant [Dickey and Fuller 1979, pp.428-429]. As a consequence, determining the distribution of the sample coefficent $\hat{\rho}$ estimated from model (A) under the null hypothesis $\rho=1$ might be quite difficult.

In order to assess the significance of the regression results presented in Table 3.1, a Montecarlo simulation was run and the empirical distribution of the $t$ ratio under the null hypothesis $\rho=1$ was determined. The simulation was repeated 8000 times using parameters (standard deviation of the process, intensity of the jump process, standard deviation of the normal distribution according to which the process is restarted) very close to those estimated from the data. Table 3.2 shows the critical values of the empirical distribution and the values of the parameters of the process used. The Dickey and Fuller distribution is also reported.

On this basis, it seems from the results on Table 3.1 that some mean reverting tendency can be detected in the process for the logarithm of the EMS exchange rates considered. The result for the $\mathrm{BFr} / \mathrm{DM}$ exchange rate, which does not appear significant, can be explained by the fact that in this case it may not be appropriate to take the central parity as the mean of the process, since the exchange rate fluctuated for a considerable time around a quite different level (see earlier remark to

Table 2.1). As an indication, for the $\mathrm{BFr} / \mathrm{DM}$ exchange rate the regression was run again substituting for the central parity in the periods 79-81 and 83-86 the estimated mean of the process as in Table 2.1: the result was $\hat{\rho}=0.9775$ and $t=-1.704$.

In the case of the LIT/DM exchange rate some significant positive correlation of residuals is present, possibly due to the presence of a trend inside the wider band.

## 3.3 - ESTHATION WITH RANDOM RESTART AFTER A JUAP

In our model, as presented in section 2 , when a jump occurs, the central parity is moved to a new level, and the exchange rate restarts randomly around the new central parity. Assuming $\rho<1$, the simplifying assumption that the value of the exchange rate after a jump is deterministic, adopted in section 3.1 , may be removed, and the model fully estimated. Modelling the logarithm of the exchange rate, we now have the transition density:

$$
\begin{array}{r}
\frac{1}{\sqrt{2 \pi \sigma_{1}^{2}}} \exp \left[\begin{array}{r}
\left.-\frac{1}{2 \sigma_{1}^{2}}[x(t)-P(t)]^{2}\right] \text { with probability } \lambda \\
\text { (if a jump occurs) }
\end{array}\right. \\
\begin{array}{r}
\frac{1}{\sqrt{2 \pi \sigma_{2}^{2}}} \exp \left[-\frac{1}{2 \sigma_{2}^{2}}[x(t)-\rho x(t-1)-(1-\rho) P(t)]^{2}\right] \\
\text { with probability (1- } \lambda \text { ) } \\
\text { (if a jump does not occur) }
\end{array}
\end{array}
$$

where $\sigma_{2}^{2}=\sigma_{1}^{2}\left(1-\rho^{2}\right)$ and $\sigma_{1}^{2}$ is the variance of the stationary distribution
according to which $I(t)$ is restarted after a jump in $P(t)$. The behaviour of the variable $I(t)$ between realignments is an AR1 with a random starting observation [see Fuller 1976]. This model will be denoted model B.

The likelihood function of a sample path takes the form:

$$
L=\left[\prod_{j=0}^{k}\left[\prod_{i=n_{j}+1}^{n_{j+1}}(1-\lambda) N_{i}\left(. ; P_{j}, \sigma_{2}^{2}\right)\right]\right]\left[\prod_{j=1}^{k} \lambda N_{n_{j}}\left(. ; P_{j}, \sigma_{1}^{2}\right)\right] N_{n_{0}}\left(. ; P_{0}, \sigma_{1}^{2}\right)
$$

where $N_{i}($.$) denotes the normal likelihood function of anservation i$, conditional on no jump in the central parity occuring, and $N_{n}($.$) denotes$ the distribution if a jump in the parity occurs ${ }^{13}$.

The log-likelihood function can be written:

$$
\begin{aligned}
\ell & =(n-k-1) \ln (1-\lambda)+k \ln \lambda-n / 2 \ln \sigma_{2}^{2}-n / 2 \ln 2 \pi+ \\
& +(k+1) / 2 \ln \left(1-\rho^{2}\right)-1 / 2 \sigma_{2}^{2}\left[\left(1-\rho^{2}\right) \sum_{j=0}^{k}\left(x_{n}-P_{j}\right)^{2}+\right. \\
& \left.+\sum_{j=0}^{k} \sum_{i=n_{j}+1}^{n}\left[x_{i}-\rho x_{i-1}-(1-\rho) P_{j}\right]^{2}\right]
\end{aligned}
$$

Maximizing the function $\ell$ over the parameters $\left\{\lambda, p, \sigma_{2}^{2}\right\}$ leads to sample parameters $\left\{\hat{\lambda}, \hat{\rho}, \hat{\sigma}_{2}^{2}\right\}$ which satisfy:

$$
\begin{equation*}
\hat{\lambda}=\frac{k}{n-1} \tag{6}
\end{equation*}
$$

${ }^{13}$ It is assumed $N_{n_{0}}()=1.$.

$$
\begin{equation*}
\hat{\sigma}_{2}^{2}=\underline{\left(1-\hat{\rho}^{2}\right) \sum_{j=0}^{k} \hat{x}_{n_{j}}+\sum_{j=0}^{k} \sum_{i=n}^{n_{j+1}+1} \hat{x}_{i}^{2}} \tag{7}
\end{equation*}
$$

n

$$
\begin{equation*}
n \frac{\hat{\rho} \sum_{j=0}^{k} \hat{x}_{n_{j}^{2}}+\sum_{j=0}^{k} \sum_{i=n}^{n_{j+1}^{1}} \hat{\mathbf{x}}_{i} \bar{x}_{i-1}}{\left(1-\hat{\rho}^{2}\right) \sum_{j=0}^{k} \hat{x}_{n_{j}}+\sum_{j=0}^{k} \sum_{i=n_{j}+1}^{n_{j+1}^{1}} \hat{x}_{i}^{2}}-\frac{(k+1) \hat{\rho}}{\left(1-\hat{\rho}^{2}\right)}=0 \tag{8}
\end{equation*}
$$

where $\hat{x}_{i}=\left[x_{i}-\hat{\rho} x_{i-1}-(1-\rho) P_{j}\right], \hat{x}_{n_{j}}=\left[x_{n_{j}}-P_{j}\right], \bar{x}_{i-1}=\left[x_{i-1}-P_{j}\right]$

Although (8) results in a cubic equation and could be solved analytically for $\hat{\rho}$, it can also be easily solved numerically for $0<\hat{\rho}<1$. Table 4 presents the estimates of $\hat{\rho}$ and $\hat{\sigma}_{2}^{2}$ under model (B); the same table reports the results of the OLS estimation, using all available observations, of the alternative models:

$$
\begin{aligned}
& \text { C) } x(t)=\mu+\beta t+\rho x(t-1)+\epsilon_{t} \\
& \text { D) } x(t)=\beta P(t)+\rho x(t-1)+\epsilon_{t}
\end{aligned}
$$

with the assumption of normal iid errors.
Model (C) represents a process for the logarithm of the exchange rate which disregards the EMS institutional mechanism (which is the very aspect the present work tries to model), and it contains, as a special case, a random walk with drift. Model (D) differs from the simplified
jump models proposed in the way it treats observations which follow a realignment, and may present the problem of multicolinearity between the explanatory variables. Models (C) and (D) do not explicitly allow for a jump structure, which is the very aspect this paper tries to model. Although a statistical comparison of models B, C, and D is complex and beyond the scope of this paper, Table 3 provides some useful information.

We observe that, once the effect of the jump is eliminated by specifying a jump mechanism as in model (B), the estimated variance of the conditional process is much smaller. However, the DM/DFl exchange rate experienced very few realignments in the period considered (2 as opposed to 7-9 for the other four exchange rates) and the results for the jump model do not differ much from the others. We note also how the jump model generates higher values of the likelihood function when the data include many large discontinuities, again suggesting it as more appropriate model.

## CONCLUSION

We have presented a tractable stochastic process for the representation of the exchange rate dynamics of currencies linked by the European Monetary System. The model takes account of the institutional constraints imposed on these exchange rates and of the possibility of discontinuities in the exchange rate regime by means of a new type of bivariate Markov process. The proposed model incorporates a conditional mean-reverting behaviour around the value of EMS parities, while
preserving compatibility with market efficiency. We provide both a continuous time and discrete time version of the model.

We show that, in contrast to the empirical evidence for freely-floating exchange rates, some mean-reverting behaviour can be detected for EMS exchange rates. This is intuitively justified by the presence of an intervention mechanism which maintains these exchange rates in the proximity of a "mean value", the official central parity.

The proposed process may find application in asset pricing models, in particular for the pricing of currency options on these exchange rates.

Figure 1


* Weekly data. Source: Lira Fixing.

Figure 2
French Franc - Deutsche Mark Exchange Rate $12^{\text {th }}$ March 1979 through $19^{\text {th }}$ June $1987 *$


* Weekly data. Source: Lira Fixing.

Figure 3
Deutsche Mark - Danish Kroner Exchange Rate
$12^{\text {th }}$ Harch 1979 through $19^{\text {th }}$ Jume 1987 *


* Weekly data. Source: Lira Fixing.

Figure 4


* Weekly data. Source: Lira Fixing.


## Figure 5



* Weekly data. Source: Baring Brothers.

Table 1.1
Sample Moments - continuously compounded returns *

|  | MEAN $(x$ 100 $)$ | VARIANCE $(x$ 100) | SKEWNESS | KURTOSIS |
| :--- | :---: | :---: | :---: | :---: |
| BFr/DM | .033989 | .0008715 | -.39012 | 12.7311 |
| FF/DM | .041525 | .0008316 | -.15372 | 13.7286 |
| DM/DKr | -.042155 | .0010978 | -.48327 | 6.90292 |
| LIT/DM | .071343 | .0011595 | 1.37676 | 9.34972 |
| DM/DFl | -.007516 | .0005590 | -.31532 | 5.22560 |

Table 1.2
Sample Moments - first differences *

|  | MEAN $(x$ 100 $)$ | VARIANCE $(x \operatorname{100})$ | SKEWNESS | KURTOSIS |
| :--- | :--- | :--- | :--- | :--- |
| BFr/DM | .61550 | .2759 | -.30708 | 11.3421 |
| FF/DM | .114507 | .0057731 | .079530 | 10.7222 |
| DM/DKr | -.012469 | .0000969 | -.68258 | 5.30487 |
| LIT/DM | 41.5626 | 377.3351 | 1.51472 | 9.97046 |
| DM/DF1 | -.006798 | .0004582 | -.32763 | 5.30487 |

*     - March 1979 - June 1987; realignments excluded. Number of observations: $\mathrm{BFr} / \mathrm{DM}=424, \mathrm{FF} / \mathrm{DM}=425, \mathrm{DM} / \mathrm{DKr}=423, \mathrm{LIT} / \mathrm{DM}=423, \mathrm{DM} / \mathrm{DFl}=398$.

Table 2.1
Estimation of the $0 . \mathrm{U}$. model on the level of the exchange rate

| EXCHANGE RATE | PERIOD | $\begin{aligned} & \text { NUMBEF } \\ & \text { OF OBS } \end{aligned}$ | . $\boldsymbol{p}$ | t | H | CENTRAL PARITY | GAMMA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FF/DM | 9/79-10/81 | 106 | 0.9577 | -1.21 | 0.207 | 2.3556 | 2.3538 |
|  | 3/83-4/86 | 159 | 0.9508 | -2.34 | -2.75 | 3.0664 | 3.0596 |
| DM/DKR | 11/79-10/81 | 96 | 0.9352 | -1.69 | 0.004 | 0.3213 | 0.3206 |
|  | 3/83-4/86 | 159 | 0.9520 | -1.86 | -4.09 | 0.2753 | 0.2751 |
| BFR/DM | 9/79-10/81 | 106 | 0.9636 | -1.26 | 0.48 | 16.030 | 16.233 |
|  | 3/81-10/81 | 30 | 0.3244 | -3.81 | 0.19 | 16.030 | 16.357 |
|  | 3/83-4/86 | 159 | 0.9532 | -1.95 | -1.66 | 20.028 | 20.273 |
| LIT/DM | 9/79-3/81 | 78 | 0.9819 | -0.52 | 2.216 | 466.46 | 492.62 |
|  | 3/83-7/85 | 122 | 1.0073 | 0.530 | 2.175 | 626.043 | 558.15 |
| DM/DFL | 9/79-3/83 | 182 | 0.9371 | -2.39 | 1.56 | 0.9046 | 0.9087 |
|  | 3/83-11/86 | 190 | 0.8557 | -3.96 | -3.12 | 0.8875 | 0.8874 |

In Table $2.1 \hat{\rho}$ is the estimated regression coefficent. $t=\frac{\hat{\rho}-1}{\text { stand.error }(\hat{\rho})}$ : for critical values see Table 3.2. H is the Durbin(1970) statistic for serial correlation. GAMMA is the estimated mean of the process.
** - result relating to a smaller subperiod between realignments during which the exchange rate was close to an intervention limit

Table 2.2
Estimation of the $0 . U$. model on the logarithm of the exchange rate

| EXCHANGE RATE | PERIOD | NUMBE OF OB | . $\quad$ | t | H | CENTRAL PARITY | $\operatorname{EXP}\{r\}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FF/DM | 9/79-10/81 | 106 | 0.9582 | -1.20 | 0.256 | 2.3556 | 2.3539 |
|  | 3/83-4/86 | 159 | 0.9509 | -2.35 | -2.76 | 3.0664 | 3.0598 |
| DM/DKR | 11/79-10/81 | 96 | 0.9355 | -1.68 | 0.008 | 0.3213 | 0.3205 |
|  | 3/83-4/86 | 159 | 0.9527 | -1.85 | -4.05 | 0.2753 | 0.2751 |
| BFR/DM | 9/79-10/81 | 106 | 0.9636 | -1.26 | 0.48 | 16.030 | 16.237 |
|  | 3/83-4/86 | 159 | 0.9531 | -1.95 | -1.67 | 20.028 | 20.270 |
| LIT/DM | 9/79-3/81 | 78 | 0.9793 | -0.60 | 2.204 | 466.46 | 490.13 |
|  | 3/83-7/85 | 122 | 1.0064 | 0.471 | 2.218 | 626.043 | 560.36 |
| DM/DFL | 9/79-3/83 | 182 | 0.9370 | -2.39 | 1.56 | 0.9046 | 0.9086 |
|  | 3/83-11/86 | 190 | 0.8560 | -3.97 | -3.14 | 0.8875 | 0.8874 |

In Table $2.2 \hat{\rho}$ is the estimated regression coefficent.
$t=\frac{\hat{\rho}-1}{\text { stand.error }(\hat{\rho})}$ : for critical values see Table 3.2.
H is the Durbin(1970) statistic for serial correlation.
$\boldsymbol{\gamma}$ is the estimated mean of the process.

Table 3.1
Estimation of model A using the entire data set
FF/DM DM/DKR LIT/DM BFR/DM DM/DFL

| $\hat{\rho}$ | .95806 | .95704 | .98709 | .98981 | .95003 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{t}$ | $(\mathbf{- 3 . 2 8 )}$ | $(-3.12)$ | $(-1.56)$ | $(-0.93)$ | $(-3.07)$ |
| $\boldsymbol{\sigma}^{\mathbf{2}}$ | .8270 | 1.0915 | 1.2042 | .8362 | .5472 |
| D.W. | $\mathbf{2 . 0 6 6}$ | $\mathbf{2 . 0 8 0}$ | 1.636 | 2.032 | 1.977 |
| H | -0.7 | -0.86 | 3.807 | -0.338 | 0.242 |

In Table 3.1, $\hat{\rho}$ is the value of the estimated regression coefficent for model (A). where the first observation after each realignment is considered fixed: $\sigma^{2}$ is the value of the variance of the process estimated from the sum of the squared residuals, and it must be multiplied times $10^{-5}$; $t$ is the value of the $t$ statistic $(\hat{\rho}-1) / \sqrt{\left(\sigma^{2} / \Sigma x^{2}\right)}$; D.W. is the value of the Durbin Watson test, $H$ the value of the Durbin $H$ statistic.

## Table 3.2

EMPIRICAL CRITICAL VALUES - ONE SIDED TEST - SAMPLE SIZE $=500$

| probability <br> of a smaller <br> value | Dickey-Fuller <br> critical $t$ <br> value | "jump process" <br> $\sigma=.003$ <br> $\lambda=.018 \sigma_{j}=.03$ | "jump process" <br> $\sigma=0.03$ <br> $\lambda=.018 \sigma_{j}=.03$ |
| :--- | :--- | :---: | :--- |
| .025 | -2.23 | -2.21 | -2.11 |
| .05 | -1.95 | -1.89 | -1.81 |
| .1 | -1.62 | -1.55 | -1.47 |
|  |  |  |  |

In Table 3.2 the Dickey-Fuller distribution reported relates to the case of mean 0 and no time trend. In the case of the "jump process" $\sigma$ is the standard deviation of the process used in the simulation, $\lambda$ is the intensity of the jump process, $\sigma_{j}$ is the standard deviation of the normal distribution according to which the process is restarted after a jump.

Table 4
OOMPARISON OF MODELS B, C, D

FF/DM DM/DKR LIT/DM BFR/DM DM/DFL

| $\rho$ | $\begin{aligned} & .97475 \\ & (-7.16) \\ & .99084 \\ & (-1.45) \\ & .76416 \end{aligned}$ | $\begin{aligned} & .96399 \\ & (-6.83) \\ & .98761 \\ & (-1.82) \\ & .89610 \end{aligned}$ | $\begin{aligned} & .98804 \\ & (-2.27) \\ & .97750 \\ & (-2.23) \\ & .96684 \end{aligned}$ | $\begin{aligned} & .97598 \\ & (-7.42) \\ & .99588 \\ & (-0.75) \\ & .77490 \end{aligned}$ | $\begin{aligned} & .94665 \\ & (-3.51) \\ & .94864 \\ & (-3.30) \\ & .94347 \end{aligned}$ | (B) (C) (D) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\hat{\sigma}^{\mathbf{2}}$ | . 84061 | 1.0994 | 1.2053 | . 82758 | . 5456 | (B) |
|  | 2.9232 | 1.8262 | 2.1643 | 2.3602 | . 5712 | (C) |
|  | 2.1824 | 1.7055 | 2.15 | 1.8119 | . 5717 | (D) |
| $\boldsymbol{\ell}$ | 1869.2 | 1801.6 | 1776.8 | 1866.7 | 1844.8 | (B) |
|  | 1638.3 | 1739.6 | 1703 | 1684.4 | 1847 | (C) |
|  | 1701.2 | 1754.4 | 1704.5 | 1741.39 | 1846.8 | (D) |

In Table 4, $\hat{\rho}$ is the value of the estimated regression coefficent for model (C), model (D) and the jump model (B); $\hat{\sigma}^{2}$ is the value of the variance of the process estimated from the sum of the squared residuals, and it must be multiplied $10^{-5}$ times: $\hat{\ell}$ is the value of the maximized $\log$-likelihood function. In parentheses are reported the $t$ values for the null hypothesis $\rho=1$.

## REFERENCES

AHN, C.M. - THOMPSON, H.E. (1988), Jump Diffusion Processes and the Term Structure of Interest Rates, "Journal of Finance" 43, 1, pp. 155-174.

BALL, C.A. - TOROUS, W.N., (1983), A Simplified Jump Process for Common Stock Returns, "Journal of Financial and Quantitative Analysis", 18, March, pp. 53-65.
their Impact on Call Option 155-173.

COOTNER, P.H., (1964), Stock Prices: Random Versus Systematic Changes. in "The Random Character of Stock Market Prices", edited by P. Cootner, Cambridge (MA), MIT Press.

COX, D.R. - MILLER H.D., (1965), The Theory of Stochastic Processes, New York, Methuen; reprinted by Chapman \& Hall, London, 1972.

DICKEY, D.A. - FULLER, W.A. (1979), Distribution of the Estimators for Autoregressive Time Series With a Unit Root. "Journal of the American Statistical Association". 74. No. 366, June, pp. 427-431.
—— (1981), Likelihood Ratio Statistics for Autoregressive Time Series With a Unit Root, "Econometrica", 49, N.4. pp. 1057-1072.

DOUKAS, J. - RAHMAN, A., (1987), Unit Root Tests: Evidence from the Foreign Exchange Futures Market, "Journal of Financial and Quantitative Analysis", 22, No. 1, March, pp. 101-108.

EVANS, G.B.A. - SAVIN, N.E., (1981), Testing for Unit Roots: 1, "Econometrica", 49, No. 3, pp. 735-779.
, (1984), Testing for Unit Roots: 2,
"Econometrica". 52, No. 5,pp. 1241-1269.
FARBER, A., ROLL, R., SOLNIK, B.H. . (1977), An Empirical Study of Risk under Fixed and Flexible Exchange, "Carnegie-Rochester Conference on Policy", Vol. 5 pp.235-265.

FULLER, W.A., (1976), Introduction to Statistical Time Series, London, John Wiley \& Sons.

GARMAN, M.B., - KOHLHAGEN, S.W. (1983), Foreign Currency Option Values, "Journal of International Money and Finance", 2, pp. 231-237.

HARRISON, J. - KREPS, D., (1979), Martingales and Arbitrage in Multiperiod Securities Markets, "Journal of Economic Theory", 20, No. 3, June, pp. 381-408.

-     - PLISKA, S. (1981). Martingales and Stochastic Integrals in the Theory of Continuous Trading, "Stochastic Processes and their Applications". 11, pp. 215-260.

KARLIN, S. - TAYLOR, H.M. (1975), A First Course in Stochastic Processes, New York, Academic Press.

Processes, New York, Academic Press.
LO, A.W., (1986), Statistical Tests of Contingent Claims Asset Pricing Models - A New Methodology, "Journal of Financial Economics", 17, 1, pp. 143-173.

MEESE, R.A. - SINGLETON, K.J. (1982), On Unit Roots and the Empirical Modelling of Exchange Rates, "Journal of Finance". 37, 4. pp. 1029-1035.

MERTON, R. (1976), Option Pricing When the Underlying Stock Returns are Discontinuous, "Journal of Financial Economics". 3, Jan-March, pp. 125-144.

NEUMANN, M.J.M. . (1984), Intervention in the Mark/Dollar Market: the Authorities. Reaction Function, "Journal of International Money and Finance", 2, pp. 223-239.

OLDFIELD, G.S., ROGALSKI, R.J., JARROW, R.A., (1977), An Autoregressive Jump Process for Common Stock Returns. "Journal of Financial Economics", 5. Dic, pp. 389-418.

PRESS J., (1967), A Compound Events Model for Security Prices, "Journal of Business". 40, July, pp. 317-335.

SHILLER, R.J. - PERRON. P. (1985), Testing the Random Walk Hypothesis: Power Versus Frequency of Observations, "Economics Letters", 18, No.4, pp. 381-386.

SOLNIK, B.H., (1974), An Equilibrium Model of the International Capital Market, "Journal of Economic Theory", 8, 4, July, pp. 500-524.

WESTERFIELD. J.M. (1977), An Examination of Foreign Exchange Risk under Fixed and Floating Rate Regimes, "Journal of International Economics", 2, May, pp. 181-200.

## RECENTLY PUBLISHED «TEMI» (*)

n. 108 - Modello mensile del mercato monetario, (ottobre 1988).
n. 109 - Ilmercato unico europeo e l' armonizzazione dell'IVA e delle accise, di C. A. Bollino V. Ceriani - R. Violi (dicembre 1988).
n. 110 - Ilmercato dei contratti a premio in Italia, di E. BARONE - D. CUOCO (dicembre 1988).
n. 111 - Delegated screening and reputation in a theory of financial intermediaries, by D. TERlizZese (dicembre 1988).
n. 112 - Procedure di destagionalizzazione dei depositi bancari mensili in Italia, di A. CIVIDINI - C. COTTARELLI (gennaio 1989).
ก. 113 - Intermediazione finanziaria non bancaria e gruppi bancari plurifunzionali: le esigenze di regolamentazione prudenziale, (febbraio 1989).
n. 114 - La tassazione delle rendite finanziarie nella CEE alla luce della liberalizzazione valutaria (febbraio 1989).
n. 115 - Il ruolo delle esportazioni nel processo di crescita e di aggiustamento dei PVS, di L. Bini Smagh - D. Porciani - L. Tornetta (marzo 1989).
n. 116 - LDCs' repayment problems: a probit analysis, by F. Di Mauro - F. Mazzola (maggio 1989).
n. 117 - Mercato interbancario e gestione degli attivi bancari: tendenze recenti e linee di sviluppo, by G. Ferri - P. Mardllo Reedtz (giugno 1989).
n. $118-L a$ valutazione dei titoli con opzione di rimborso anticipato: un'applicazione del modello di Cox, Ingersoll e Ross ai CTO, di E. BARONE - D. CUOCO (giugno 1989).
n. 119 - Cooperation in managing the dollar (1985-87): interventions in foreign exchange markets and interest rates, by E. GAIOTTI - P. GIUCCA - S. MICOSSI (giugno 1989).
n. 120 - The US current account imbalance and the dollar: the issue of the exchange rate pass-through, by C. MASTROPASQUA - S. VONA (giugno 1989).
n. 121 - On incentive-compatible sharing contracts, by D. TERLIZZESE (giugno 1989).
n. 122 - The adjustment of the US current account imbalance: the role of international policy coordination, by G. Gomel - G. Marchese - J. C. Martinez Oliva ((luglio 1989).
n. 123 - Disoccupazione e dualismo territoriale, di G. Bodo - P. SESTITO (agosto 1989).
n. 124 - Redditi da lavoro dipendente: un'analisi in termini di capitale umano, di L. CANNARI G. Pellegrini - P. Sestito (settembre 1989).
n. 125 - On the estimation of stochastic differential equations: the continuous-time maximum-likelihood approach, by R. CESARI (settembre 1989).
n. 126 - La misurazione dell' efficienza nei modelli di "frontiera", di M. Gresti (settembre 1989).
n. 127 - Do intergenerational transfers offset capital market imperfections? Evidence from a cross-section of Italian households, by L. Guiso - T. JAPPELLI (settembre 1989).
n. 128 - La struttura dei rendimenti per scadenza secondo il modello di Cox, Ingersoll e Ross: una verifica empirica, di E. BARONE - D. Cuoco - E. ZAUTZIK (ottobre 1989).
n. 129 - Il controllo delle variabili monetarie e creditizie: un'analisi con ilmodello monetario della Banca d'Italia, di I. ANGELONi - A. CIVIDINI (novembre 1989).
n. 130 - L'attività in titoli delle aziende di credito: un' analisi di portafoglio, di G. FERRI C. MONTICELLI (dicembre 1989).
n. 131 - Are asymmetric exchange controls effective? by F.PAPADIA - S. ROSSI (gennaio 1990).
n. 132 - Misurazione dell' offerta di lavoro e tasso di disoccupazione, di P. SESTITO (marzo 1990).
n. 133 - Progressing towards EuropeanMonetary Unification:SelectedIssues and Proposals, by L. Bini Smaghi (aprile 1990).
n. 134 - Il valore informativo delle variabili finanziarie: un'analisi con il modello econometrico trimestrale della Banca d'Italia, di I. ANGELONI e A. CIVidini (aprile 1990).

[^8]
[^0]:    ${ }^{1}$ The present paper is a revised version of a manuscript presented at the $15^{\text {th }}$ Annual Meeting of the European Finance Association, September 1-3 1988, Istanbul, Turkey.
    The fundamental contribution of Clifford Ball, especially on section 2, is gratefully acknowledged. I am also indebted to Richard Brealey. Ian Cooper, and Stephen Schaefer for many comments and suggestions. Ignazio Angeloni and an anonimous referee commented helpfully on an earlier draft.

[^1]:    ${ }^{2}$ For a general treatment of Markov processes see, among others, [Cox and Miller, 1965] and [Karlin and Taylor, 1975].

[^2]:    ${ }^{3}$ This defines continuously compounded expected return; in the limit, continuously compounded returns and percentage returns will tend to coincide.
    ${ }^{4}$ Due to the presence of reflecting barriers, under the hypothesis of constant $r_{d}, r_{f}, \pi$, the Ehrenfest process alone is not even compatible

[^3]:    5 The need to model reflecting barriers does not arise when modelling ECU exchange rates in terms of EMS currencies, which however fluctuate inside divergence thresholds.

[^4]:    $8_{\text {Cfr. Karlin }}$ and Taylor [1981] p.429. We assume that I(0) starts according to $Z^{*}$.

[^5]:    9 See Lo [1986] pag. 165. This follows since the Markov property implies:
    $P\left[x_{t}=a / x_{t-1}=b, x_{t-2}=c, \ldots \ldots x_{t-n}=i, t\right]=P\left[x_{t}=a / x_{t-1}=b, t\right]$ that is, the distribution at a point in time in the future depends at most on the present state and time.

[^6]:    ${ }^{10}$ See Meese and Singleton [1982] and Doukas and Rahman [1987].

[^7]:    ${ }^{11}$ Except for the Dutch Guilder series, which ends in November 1986.

[^8]:    ${ }^{(*)}$ Requests for copies should be sent to: Banca d'Italia - Servizio Studi - Djvisione Biblioteca e Pubblicazioni - Via Nazionale, 91-00184, Rome.

