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Summary

When entrepreneurs know the riskiness of the projects they want to finance better than potential investors, the transfer of funds between surplus and deficit units in the economy will in general be inefficient.

This paper explores the possibility of dealing with ex-ante asymmetric information in financial problems through the offer of an appropriate set of incentive compatible (i.c.) sharing contracts, i.e. contracts specifying how to share the proceeds of a risky project, such that, when the entrepreneur selects one out of the offered set, he thereby reveals his riskiness.

No collateral requirements are employed to achieve incentive compatibility, which is obtained by appropriately choosing the expected value and the concavity of the contracts in the optimal set. Characterization and welfare properties of the set of optimal i.c. sharing contracts are provided, showing the existence of a tradeoff between surplus extraction and risk-sharing.

The paper also provides a new insight on the relationship between ex-ante asymmetric information and credit rationing. Stiglitz and Weiss showed the former to be responsible for the latter. We show that equilibrium credit rationing will be avoided if optimal i.c. sharing contracts are used and an appropriate notion of competitive equilibrium is employed (Riley, 1979). Thus credit rationing appears rather as the consequence of a "legal restriction", whereby the contracts are constrained to be standard debt contracts.

1. Introduction[1]

Credit markets are often characterized by (ex-ante) asymmetric information, with entrepreneurs knowing better than potential investors the riskiness of the projects they want to finance. The possibility of adverse selection might then undermine the efficiency of credit contracts.

The problems posed by (ex-ante) asymmetric information can be approached by recognizing that the choice of a credit contract from a range of available ones might reveal something about the entrepreneur's private information. The information revealed by the choice of the contract then becomes one of the key elements in the design and the assessment of different contractual forms.

Indeed, let us define the general class of sharing contracts as the set of all rules that specify how to divide each of the possible outcomes of a risky project between the entrepreneur who managed the project and the investor who financed it.[2]

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1. The author thanks L. Guiso, M. Messori and I. Visco for discussing a previous version of the paper.

2. A standard debt contract is a special form of a sharing contract, which specifies that the entrepreneur pays a constant amount in all non bankrupt states and gets nothing in bankrupt ones.

An equity contract, neglecting ownership complications, is also an example of a sharing contract. Clearly equity is a much more flexible sharing contract than debt, since the payment to the investor can be different in each state.

It might be that a set of sharing contracts can be devised in such a way as to provide the entrepreneur with the appropriate incentives to reveal truthfully any (pay-off relevant) differential information he might possess. In fact the problem can be approached imposing the weaker requirement that no entrepreneur would be better-off misrepresenting the riskiness of his project (in short, his riskiness). When this is the case we shall call the resulting sharing contracts incentive-compatible (ICSC henceforth). ICSC, thus, are contracts specifying how to share the proceeds of a risky project such that, when the entrepreneur selects one out of the offered set, he thereby reveals his riskiness.

In this paper we explore a setting characterized by risk neutral entrepreneurs, risk averse investors, contracts constrained by limited liability, increasing riskiness of projects represented by mean preserving spreads. In this setting we show that, under an appropriate regularity condition, the problem posed by ex-ante asymmetric information can be solved and we provide a simple characterization of the set of optimal ICSC. In section 4 we show that in the set of optimal ICSC, the riskier is the entrepreneur, the lower is the expected value of the share that goes to the investor; on the other hand, the riskier is the entrepreneur, the more concave[3] is the appropriate contract. This means that in the set of optimal ICSC there is a trade-off between larger expected value and improved risk sharing.

In section 5 some of the welfare properties of the optimal ICSC are also analyzed. All surplus is only extracted from the safest entrepreneurs. Optimal risk sharing is only achieved with the riskiest entrepreneurs.

The intuition behind these results should be clear. As usual in this sort of problems incentive compatibility is obtained introducing some distortions with respect to a first best (full-information) contract. In the setting of the paper, the

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3. A precise meaning will be given to this expression in section 4.

first best contract would be a standard debt contract (SDC henceforth), which achieves optimal risk sharing, with larger expected value (to the investor) the greater is the riskiness of the entrepreneurs. In the set of optimal ICSC some of the surplus that could be extracted from high risk types is forgone in order to make the contract meant for lower risks unattractive. But low risk types should not find attractive contracts meant for high risks either. This is achieved by making the latter the more concave the riskier is the type, so that proportionately more has to be paid when the project yields outcomes that correspond to the "center" of the distribution, where the probability weight is larger for low risks. In order to obtain the required ranking in the concavity, the projects meant for low risk types are distorted as compared to an SDC. Precisely, they are less concave.

The strategy of proof underlying this argument does not generalize easily to projects with more than three outcomes, since the concavity would then become a local rather than a global feature of the contract. The economics behind it appears however to be robust. The marginal utility of the entrepreneurs is state-dependent: in fact, given risk neutrality, it coincides with the probabilities of the outcomes of their project. Entrepreneurs of two different types thus have different state-dependent marginal utility, even though they are both risk-neutral. Separation can be obtained by making the marginal state-dependent cost of different contracts to match the appropriate state-dependent marginal utility.

One interesting implication of the existence of a set of optimal ICSC that in general allow the investor to sort the entrepreneurs into risk classes according to their true types is explored in section 6. In a seminal paper Stiglitz and Weiss (1981) showed that the informational failures of the kind considered in this paper can lead to credit rationing when an increase of the interest rate yields unfavourable change in the composition of the loan applicants.[4]

The possibility of sorting, however, avoids adverse selection effects. We conjecture, then, that no rationing is required in equilibrium. To verify this we need an appropriate notion of equilibrium and we use the reactive equilibrium proposed by Riley (1979). We show that the reactive equilibrium is given by the solution of a sharing problem supplemented by the appropriate "zero profit" condition. In equilibrium, no pooling contract will survive competitive entry and therefore no rationing will occur.[5]

We interpret this result as showing that the well known Stiglitz and Weiss proof of the optimality of credit rationing is somewhat of an artifact of a "legal restriction" whereby the contracts are constrained to be SDCs.

The optimality of credit rationing was already questioned. Bester (1987), Besanko and Thakor (1987) showed that including collateral requirements and loan quantity in the contract design avoid rationing.

Stiglitz and Weiss (1987) replied, however, that in a model modified to allow for differences in wealth an increase in collateral could also have adverse selection effects if, for example, the entrepreneurs willing to put up more collateral (i.e. the wealthier) were in that position because, in previous periods, they "gambled and won". Thus raising collaterals could lead to higher proportions of risk-lovers in the pool of the loan applicants.

Our result is more robust in that it hinges merely on the

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4. In the paper by Stiglitz and Weiss credit rationing is shown to result both from ex-ante asymmetric information concerning the riskiness of the entrepreneurs (adverse selection) and from ex-ante asymmetric information concerning the project choosen by the entrepreneurs (moral hazard).

In our set up the entrepreneurs will be endowed with an idiosincratic project of fixed size. Therefore no moral hazard problems could possibly arise and we shall focus on the adverse selection issue.

5. This of course is subject to the qualification spelled out in the previous note.

appropriate choice of a state dependent sharing schedule.[6]

This paper bears relationships with two different strands in the literature. The first aims at deriving the optimal financial contract when information is imperfect. Notable examples are Gale and Hellwig (1985) and Diamond (1984). In both papers the imperfection considered is ex-post asymmetric information, i.e. a situation in which only the entrepreneur can observe the outcomes of the project.

The second examines the robustness of the Stiglitz and Weiss rationing result by endogenizing various aspects of the financial contract (collateral, amount of the loan...), keeping however fixed the basic feature of the standard debt contract: a state-independent, fixed repayment except when bankruptcy occurs. Examples of this literature are Bester (1987) and Besanko and Thakor (1987). The informational imperfection that is considered in those papers is, as in Stiglitz and Weiss, ex-ante asymmetric information. This paper, to some extent, bridges a gap between these two strands. On the one hand it parallels the analysis of endogenously determined contractual forms covering the interesting case of ex-ante asymmetric information. On the other hand it takes up the validity of the rationing result allowing for a contract different from a standard debt contract to be employed.

2. The set-up

Suppose there are n types of risk neutral entrepreneurs, indexed by $i=1...n$. Each type is endowed with a private investment opportunity, or project, which requires an initial investment of $\bar{W}$ and yields, at the end of the period, a (gross) return represented by a random variable x^i.

6. As noted by Gale and Hellwig (1985), in the set-up chosen by Stiglitz and Weiss no state dependent sharing schedule is possible since there is only one state in which sharing does in fact occur.

Let $[x_1, x_2, x_3]$ be the common finite support of all random variables, and denote with $p^i = [p_1^i, p_2^i, p_3^i]$ the probability distribution of project i.

The projects are totally ordered according to the criterion of mean preserving spread, with project i being riskier than project j if i > j.

In general, with a three outcomes project, a mean preserving spread requires $p_s^i > p_s^j$, s = 1,3, $p_2^i < p_2^j$, for i > j.

For expositional purposes, it will sometimes be convenient to imagine that the riskiness varies continuously.[7] This can be expressed, with no loss of generality, as follows:

$p_s^i = p_s(\theta_i) = p_s + \theta_i h_s$, s = 1,2,3 where $\theta_i \in [0,1]\ \forall i$,
$p = [p_1, p_2, p_3]$ is a probability distribution and
$q = [q_1, q_2, q_3]$, $q_s = p_s + h_s$ is a mean preserving spread of p[8]. Therefore we have

$$\frac{dp_s(\theta)}{d\theta} > 0 \qquad s = 1,3 \qquad \text{and} \qquad \frac{dp_2(\theta)}{d\theta} < 0$$

Clearly riskier types will be characterized by a larger θ.

There is a risk-averse investor with strictly concave utility function u(·), endowed with initial wealth $\bar{W}$, who has to decide whether to finance (one of) the projects. Each agent is interested in maximizing the expected utility of final wealth.

We assume that ex-ante asymmetric information prevails, so that:

(AI) entrepreneurs know their type (i.e. their θ) whereas the investor cannot distinguish among projects.

A sharing contract, designed for type i, is a function

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7. This will be implicitely used in the definition of the utility curves, section 4.

8. This of course requires $\sum_s h_s = 0$, $\sum_s h_s x_s = 0$.

$R^i(x_s) = [R^i_1, R^i_2, R^i_3]$ that specifies for each realization of the project the amount that goes to the investor. The entrepreneur who chooses the contract R^i clearly gets $x_s - R^i_s$.

Under (AI) there is no guarantee that each type i would choose the contract R^i.

To be specific, we imagine the timing of the contracting problem to be as follows:

(a) the investor announces a sharing policy, i.e. the vector of all sharing contracts $R = [R^1..R^n]$, one for each type;

(b) he then meets only one entrepreneur, who chooses one of the sharing contracts;

(c) the project is financed and the revenue is shared according to the chosen contract.

We are then assuming that once the investor announced a sharing policy, he cannot use the information extracted from the entrepreneur choice either to modify the contract or to wait for a different entrepreneur.[9]

Clearly, each entrepreneur will choose the contract that maximizes his expected utility, even if the optimal contract from his point of view is not the one that was originally meant for his type. The problem of the investor is then to design a sharing policy that maximizes his expected utility taking into account the rational behaviour of the entrepreneurs. This means that it must be optimal for type i to choose contract R^i, which in turn requires that there should be no other contract R^j that yields to entrepreneur i a larger expected utility (evaluated according to p^i) and that the expected utility of contract R^i should be no less than the entrepreneur's reservation level $\bar{w}$, exogenously given.

If we let $\pi = [\pi_1..\pi_n]$ to be the investor probability distribution over the set of types, we can formally state the sharing problem as follows

9. This is equivalent to say that any entrepreneur who were turned down would not be replaced. With this assumption, the number of entrepreneurs that the investor meets in each period can be fixed to one without loss of generality.

$$
(P1) \begin{cases}
\max\limits_{R} \sum\limits_{i=1}^{n} \left[\sum\limits_{s=1}^{3} u(R_s^i) p_s^i \right] \pi_i \\
\text{s.t. (VP)} \quad \sum\limits_{s} [x_s - R_s^i] p_s^i \geq \bar{w} \quad \forall \; i \\
\text{(IC)} \quad \sum\limits_{s} [x_s - R_s^i] p_s^i \geq \sum\limits_{s} [x_s - R_s^j] p_s^i \quad \forall \; i,j, \; i \neq j \\
\text{(LL)} \quad x_s - R_s^i \geq 0 \quad \forall \; i,s \\
\text{(N)} \quad R_s^i \geq 0 \quad \forall \; i,s
\end{cases}
$$

Constraints (VP) (voluntary participation) assures that each entrepreneur is at least indifferent between choosing the contract designed for him and not undertaking the project.

Constraints (IC) are the incentive-compatibility constraints that guarantee that each entrepreneur i has no advantage in pretending to be of a different type j.

Contraints (LL) are the limited liability constraints and state that the entrepreneur cannot pay more than the outcome of the project. Contraints (N) are the usual non negativity constraints.

Given that $\Sigma \, x_s \, p_s^i = \Sigma \, x_s \, p_s^j = \bar{x} \; \forall j,i$ we can notationally simplify (P1) writing the constraints as follows

$$\text{(VP)} \quad \Sigma \, R_s^i p_s^i \leq k \qquad \forall i$$

$$\text{(IC)} \quad \Sigma \, R_s^i p_s^i \leq \Sigma R_s^j p_s^i \qquad \forall i,j, \; i \neq j$$

where $k = \bar{w} - \bar{x}$.

<u>Remark 1</u>. We assumed that each project has only three possible outcomes, corresponding to low, medium and high return. This is the minimum number of outcomes required in order for the con-

tracting problem not to be trivial and to retain the ordering of the projects according to the mean preserving spread criterion.

The assumption is not harmless, however. With three outcomes any sharing contract is either convex or linear or concave. This will be heavily used in almost all the proofs, which cannot thus be generalized to projects with more than three outcomes. A more general assumption on the support of the projects would have required more restrictive assumptions on the probability distributions of outcomes in order to obtain determinate results.

Remark 2. The assumed asymmetry in risk attitued is different from the universal risk neutrality assumed by Stiglitz and Weiss (1981) in the first part of their paper. However universal risk neutrality would make any analysis of the form of the optimal contract indeterminate. At least one risk-averse agent is needed. We choose to have the investor risk-averse and the entrepreneur risk neutral since this assumption, together with limited liability, yields the SDC as the optimal full-information sharing contract. It provides, therefore, a useful benchmark.

3. The full-information problem

As a benchmark we first briefly analyze the full-information problem, i.e. (P1) without constraints (IC), whose only role is to ensure that the type declared by the entrepreneur is the true one. If we temporarily assume that the investor can observe the entrepreneur's type we can clearly neglect constraints (IC).

It can be easily shown that an SDC is the sharing contract that solves the full-information problem. Formally, the solution to the full-information problem, denoted by R_D^i, is given by

$$R_D^i(x_s) = \begin{cases} \bar{R}^i & \forall\, x_s > \bar{R}^i \\ x_s & \text{otherwise} \end{cases}$$

where R_D^i satisfies constraints (VP) with equality. In particular note that in all states where the limited liability constraint is not binding, the SDC achieves optimal risk sharing as represented by

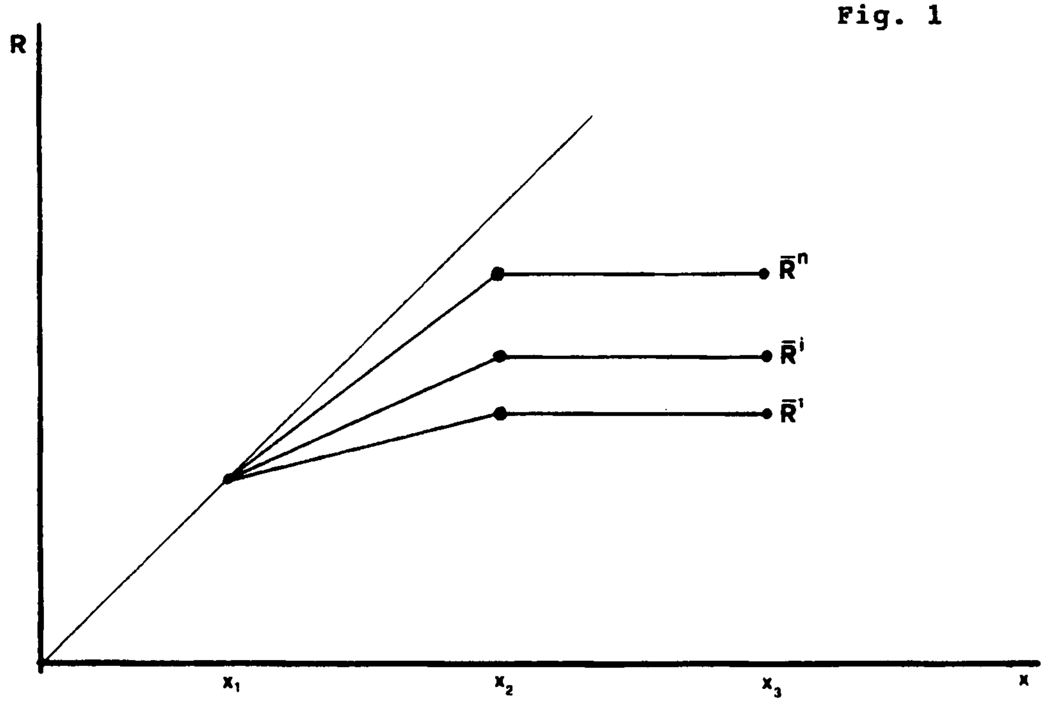
(1) $u'(\bar{R}^i) = \phi_i \quad \forall \; x_s > \bar{R}^i$, where ϕ_i are the Lagrange multipliers associated with constraints (VP).

In words, equation (1) says that, under an SDC, the entrepreneur provides the investor with full insurance against risk whenever this is possible. When the limited liability constraint is binding the investor gets all outcome of the project. We shall call bankruptcy this situation.

It can be shown that $\bar{R}^i$ is a non decreasing function of riskiness (i.e. of i). Given that the utility of the entrepreneur is a decreasing function of $\bar{R}$, this at once implies that the optimal SDC is not incentive-compatible.

Note, for further reference, that an SDC is a concave function of x_s. If constraint (LL) is binding in the worst state, we can represent the set of optimal full-information contracts as in figure 1

Fig. 1



It might be conjectured that a solution to (P1) could be given by R_D^1 for all types. This is clearly incentive compatible, since the same contract is offered to all entrepreneurs. It also achieves optimal risk sharing. However this solution implies that no attempt at obtaining the maximum surplus from each entrepreneur is made.

Indeed, we shall see that the solution to (P1) strikes a balance between risk sharing and surplus extraction. In particular R_D^1 would never be a solution to (P1) (that is, if there are at least two types).

4. The optimal incentive compatible contract

Problem (P1) is a very complex one, due to the large number (n(n-1)) of (IC) constraints and to the fact that each control variable appears both on the LHS and on the RHS of some of the (IC) constraints. It is therefore extremely difficult to obtain a characterization of the optimal sharing policy, since the standard technique of proving by contradiction whether a constraint bites largely fails in this case.

The usual approach to the solution of problems involving (IC) constraints is to reduce the number of constraints substituting the global characterization of incentive compatibility given by (IC) with a local one, in which only "adjacent" (IC) constraints are considered.

$$\text{(AIC)} \quad \sum_s R_s^i p_s^i \le \sum_s R_s^{i+h} p_s^i \qquad h = -1,+1$$

We shall refer to (P2) as to (P1) with (IC) substituted by (AIC).

Remark 3. This corresponds to consider only the first and second order conditions of the entrepreneur's maximization problem that is defined by the (IC) constraints in the continuous version of (P1):

$$\theta \in \arg\max_{\hat{\theta}} \Sigma R_s(\hat{\theta})p_s(\theta) \quad \forall \theta$$

However, as Matthews and Moore (1987) point out, this approach is not legitimate unless one can show that the solution to the relaxed problem obtained by substituting (AIC) to (IC) are the same of the solution to the original problem. In fact Mirrles (1986) gives an example where the first order conditions are not necessary nor sufficient for the general problem.

Following Matthews and Moore let us define the utility curve as a function $v(\hat{R},\cdot) : [0,1] \times R^3_+ \rightarrow R^1$ that gives the utility of a given contract $\hat{R}$ when θ varies, i.e. gives the utility obtained by different types from the same contract. Define also what they call the single crossing property:

A given set of contracts $\{R^1..R^S\}$ satisfies the single crossing property if:

(SCP) no two utility curves cross more than once and they actually cross at any point of tangency.

The relevance of (SCP) lies in the following

Lemma 1: under (SCP), (AIC) can be substituted to (IC) without loss of generality.

Proof: see Matthews and Moore (1987).

Matthews and Moore explore conditions under which (SCP) holds (at least for the set of optimal contracts).

In our set-up it is particularly easy to show that this is the case. We need a few definitions, which will be extensively used in the following.

Let λ be the solution to the following equation:

$$x_2 = \lambda x_1 + (1-\lambda)x_3.$$

We shall say that a contract R^i is convex, linear or concave according to δ_i being respectively less, equal or greater than zero in the following equation:

$$R_2^i = \lambda R_1^i + (1-\lambda)R_3^i + \delta_i.$$

Moreover, we shall restrict the attention to the set of contracts where the (LL) constraint is binding at least in the worst state. Formally, we shall consider contracts belonging to

$$BB = \{R^i \in R_+^3 \mid R_1^i = x_1\}.$$

We shall later discuss the significance of restricting the analysis to contracts in BB, the set of contracts in which bankruptcy is possible, and present the condition required for the set of optimal contracts to be a subset of BB (see lemma 5).

It is also useful to explicitely record the following straightforward implication of the definition of mean preserving spread[10]:

$$(MPS) \begin{cases} p_1^i + \lambda p_2^i = p_1^j + \lambda p_2^j \\ p_3^i + (1-\lambda)p_2^i = p_3^j + (1-\lambda)p_2^j \end{cases} \quad \forall\ j,i$$

We can now easily prove:

Lemma 2: for all set of distinct contracts in BB, (SCP) holds.

10. (MPS) does follow from the definition of mean preserving spread if we consider only random variables with three possible realizations. For more general random variables (MPS) does not hold.

Proof: the explicit expression of the utility curve is

$$v(R^i,\cdot) = \sum_s p_s(\theta)R_s^i = [p_1(\cdot)+\lambda p_2(\cdot)]x_1 + [p_3(\cdot)+(1-\lambda)p_2(\cdot)]R_3^i+p_2(\cdot)\delta_i$$

Define $\Delta(\theta) = v(R^i,\theta) - v(R^j,\theta) = [p_3+(1-\lambda)p_2](R_3^i-R_3^j) + p_2(\delta_i-\delta_j)$.

If there is no θ such that $\Delta(\theta) = 0$ (SCP) holds trivially since the utility curves never cross.

Let θ_0 be such that $\Delta(\theta_0) = 0$.

Therefore

$$[p_3(\theta_0)+(1-\lambda)p_2(\theta_0)](R_3^i-R_3^j) = p_2(\theta_0)(\delta_j-\delta_i)$$

Consider the possible cases

(i) $\delta_j = \delta_i$. This implies $R^i \equiv R^j$, i.e. the contracts are not distinct.

(ii) $\delta_j > \delta_i$. Because of (MPS) the LHS does not vary with θ. The RHS, remembering that $p_2(\cdot)$ is decreasing in θ, decreases. Therefore $\Delta(\theta) \gtrless 0$ according to $\theta \gtrless \theta_0$.

(iii) $\delta_j < \delta_i$. As in (ii), we conclude $\Delta(\theta) \gtrless 0$ according to $\theta \lessgtr \theta_0$. This completes the proof.

As a straightforward implication of lemmas 1 and 2 we then have:

Proposition 1: if the set of optimal contracts is a subset of BB, then (P1) and (P2) have the same set of solutions.

Although proposition 1 allows us to reduce the number of (IC) constraints to 2(n-1) it is still difficult to characterize the optimal contracts.[11]

We shall therefore adopt a non-standard technique to solve (P1), in the spirit of Matthews and Moore (1987).

Let us consider the "upward" incentive constraints

$$\text{(UIC)} \quad \sum_s R_s^i p_s^i \leq \sum_s R_s^j p_s^i \qquad j < i$$

We shall refer to (P3) as to (P1) with (IC) replaced by (UIC). (UIC) constraints formalize the idea that a contract meant for type j should not be preferred by riskier types to the contract meant for them. In other words (UIC) constraints protect against the possibility that each type pretends to be less risky than in fact it is. What we need to verify is that in providing the incentives for (UIC) to be satisfied we do not also provide incentives for some type to pretend to be riskier than it is.

In the course of proving lemma 2 we implicitely obtained some results on concavity and convexity of contracts in BB which is useful to extend a little and explicitely record.

First, we need a definition:

Given two contracts R^i and R^j in BB that have the same expected value according to some p, we say that R^i is more concave than R^j if $\delta_i > \delta_j$.

The reader can easily convince himself that the definition is in accord with the intuitive meaning of "more concave" by

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11. The reason for this difficulty can be seen from the first order conditions to (P2). Assuming interior solutions, we have:

$$u'(R_s^i) = \frac{1}{\pi_i} [\gamma_i + \mu_{i-1,i} + \mu_{i+1,i} - (\mu_{i,i-1} \frac{p_s^{i-1}}{p_s^i} + \mu_{i,i+1} \frac{p_s^{i+1}}{p_s^i}) + \eta_{is}],$$

where γ_i, μ_{ij}, $\eta_{is} p_s^i$ are Lagrange multipliers associated respectively to constraints (VP), (IC), (LL). The difficulty arises from the term in the inner brackets.

Indeed, p_s^{i-1}/p_s^i is a concave function of s whereas p_s^{i+1}/p_s^i is a convex one. Short of determining the magnitude of μ_{ij}, j=i-1, i+1, we are not able to establish whether R^i should be a concave or convex function.

drawing two contracts that satisfy the definition.

We then have:

Lemma 3:

(a) for all linear contracts, $\Sigma\, p_s^j\, R_s^i$ is a constant $\forall$ j

for all concave contracts $\Sigma\, p_s^{i+h} R_s^i < \Sigma\, p_s^i R_s^i \quad \forall h \geq 1$

for all convex contracts $\Sigma\, p_s^{i+h} R_s^i > \Sigma\, p_s^i R_s^i \quad \forall h \geq 1$

(b) Let R^i be more concave than R^j (suppose for definiteness they have the same expected value according to p^i).

Then $\Sigma\, R_s^j\, p^{i+h} > \Sigma\, R_s^i\, p_s^{i+h}$

$\forall h \geq 1$

$\Sigma\, R_s^i\, p_s^{i-h} > \Sigma\, R_s^j\, p_s^{i-h}$

Proof: part (a) of the lemma is clearly a special case of results proved in Rotschild and Stiglitz. It is a straightforward implication of definitions given and of (MPS).

As for part (b), let us recall that by definition

$$R_2^i = \lambda x_1 + (1-\lambda) R_3^i + \delta_i, \quad R_2^j = \lambda x_1 + (1-\lambda) R_3^j + \delta_j, \quad \delta_i > \delta_j.$$

We then have, because of (MPS):

$\Sigma\, R_s^i (p_s^i - p_s^{i+h}) = (p_2^i - p_2^{i+h})\delta_i > (p_2^i - p_2^{i+h})\delta_j = \Sigma\, R_s^j (p_s^i - p_s^{i+h})$ since p_2^i is decreasing in i. The opposite inequality holds if h < 0. Since $\Sigma\, R_s^i p_s^i = \Sigma\, R_s^j p_s^i$, (b) follows.

The inequalities in (a) mean that each linear contract is evaluated in the same way by all types, irrespective of their riskiness. The value of a concave contract is higher the riskier is the type which makes the evaluation. The opposite holds for a convex contract.

The interpretation of the inequalities in (b) is the following:

If contract R^i is more concave than contract R^j and type i is indifferent between the two contracts, then R^i is strictly preferred to R^j by all types more risky than i and R^j is strictly preferred to R^i by all types less risky than i.

Remark 4. As these relationships will be frequently used in the following, it is perhaps useful to spell out some of their implications for the choice of the optimal contracts.

Let us remember that for each contract R^i we have the following constraints

$$\sum_s R_s^i p_s^i \leq k$$

$$\sum_s R_s^i p_s^i \leq \sum R_s^j p_s^i \qquad j < i$$

Moreover contract R^i appears in all the constraints relative to types j > i, i.e.

$$\sum_s R_s^j p_s^j \leq \sum R_s^i p_s^j \qquad j > i$$

Suppose now that we consider a new contract $\hat{R}^i$ that has the same expected value of R^i from the point of view of type i but is less concave. Substituting $\hat{R}^i$ for R^i would not modify any of the constraints in which R^i appeared on the LHS. Also, because of (b), $\hat{R}^i$ would satisfy all constraints in which R^i appeared on the RHS. Hence, $\hat{R}^i$ is a new feasible contract with the same expected value (for type i) and less concave. Therefore it is in general possible to rank $\hat{R}^i$ and R^i according to the investor expected utility. This is clearly a very helpful device in the design of an optimal contracts set.

For a set of contracts $\{R^1..R^{i+1}\}$ in BB define now the following "linear-concave" characterization

$$(LC)_i \begin{cases} \sum_s R_s^h p_s^h = \sum_s R_s^{h-1} p_s^h \quad h = 2, \ldots i+1 \\ \text{if } R^h \text{ is linear, } R^{h-j} \equiv R^h, \ j = 1.. \ h-1 \\ \text{if } R^h \text{ is concave, } R^{h+j} \text{ is at least as concave as } R^h, \ j=1..i-h+1 \end{cases}$$

We shall refer to $(LC)_{n-1}$ simply as to (LC). If the contracts satisfy (LC) this means that each type is indifferent[12] between the contract meant for him and the contract meant for the next less risky type. Moreover, (LC) establishes an ordering in the concavity of the contracts.

Also define the monotone characterization

$$(M) : x_1 \leq R_2^i \leq R_3^i \qquad \forall i .$$

We are now ready to state and prove the important

Lemma 4. Restricting the contracts to BB,

(a) the set of optimal contracts satisfies (LC)

(b) the set of optimal contracts satisfies (M)

(c) (P1) and (P3) have the same set of solutions.

The proof is a little involved and it is perhaps useful to give the basic intuition of the way we proceed.

The chief difficulty in problems with incentive compatibility constraints is that when we define a contract, say R^i, we implicitely impose some constraints on the design of other contracts R^j, $j \neq i$. In other words, the choice of R^i has a direct pay-off in terms of (expected) utility obtained by the investor on R^i and an indirect pay-off in terms of the possibly more stringent constraints imposed on other contracts. Clearly an optimal contract balances these two aspects of the pay-off. The advantage of considering (UIC) is that we only need to worry about the

12. Here and in the following we make the standard assumption that if the entrepreneur is indifferent between lying and telling the truth he choses the latter.

constraints imposed on contracts meant for riskier types. Using the implications of lemma 3 recorded in Remark 4 we then show that characterization (LC) must hold. But the ordering according to the concavity of the contracts implies, again because of lemma 3, that also the "downward" incentive compatibility constraints are satisfied.

Proof. Let us first consider (b).

From F.O.Cs to (P3) we have

$$(1) \qquad u'(R_s^i) = \frac{1}{\pi_i} [\gamma_i + \sum_{j<i} \mu_{ji} - \sum_{j>i} \mu_{ij} \frac{p_s^j}{p_s^i} + \eta_{is}],$$

assuming interior solutions, where γ, μ, $\eta \cdot p$ are Lagrange multipliers corresponding to (VP), (UIC) and (LL). Neglecting η_{is}, from (1) and the ordering of p according to mean preserving spread we would deduce that $R_1^i > R_2^i$, $R_3^i > R_2^i$, provided not all μ_{ij}, $j > i$ are zero.

The first of the two inequalities need not be satisfied for contracts in BB, where $\eta_{i1}>0$ and $R_1^i = x_1$.

Consider the second. It is satisfied if $\eta_{is} > 0$, $s = 2,3$, and if $\eta_{i2}=0$, $\eta_{i3}>0$ since $x_2 < x_3$.

Suppose $\eta_{i2} > 0$, $\eta_{i3} = 0$ and $R_3^i < R_2^i$. We show that this cannot be optimal.

Consider a new contract $\hat{R}^i = [x_1, \bar{R}^i, \bar{R}^i]$ where

$$\bar{R}^i = R_2^i \frac{p_2^i}{p_2^i+p_3^i} + R_3^i \frac{p_3^i}{p_2^i+p_3^i}.$$

Clearly $\Sigma p_s^i \hat{R}_s^i = \Sigma p_s^i R_s^i$. It can also be immediately verified that R^i is more concave than $\hat{R}^i$. Therefore, from the second part of lemma 3 we conclude that $\hat{R}^i$ would not violate any of the (UIC) constraints. Moreover, $\Sigma p_s^i\, u(R_s^i) < \Sigma\, p_s^i\, u(\hat{R}_s^i)$ because of the concavity of the $u(\cdot)$. Hence $R_3^i \geq R_2^i$ in any optimal contract. This, together with (a), gives (b).

We now prove (a) by induction on i. This is done in a number of steps.

Step 1. At least one of γ_i, $\mu_{j,i}$, $j<i$ is greater than zero $\forall i$.

Consider the structure of the constraints in (P3). Each contract i has an upper bound given by min $[k, \sum_s R_s^j p_s^i (j<i)]$. Contract i also appears as a possible upper bound of all contracts j, j>i. If the expected value of contract i were smaller than its upper bound, then R_s^i, s = 2,3 could be increased. This would possibly made slacker constraints on contracts j>i and would increase the utility of investor.

Step 2. $\Sigma R_s^1 p_s^1 = k$.

This is a straightforward consequence of step 1.

Step 3. R^1 cannot be convex.

Because of lemma 3, if R^1 were convex $\Sigma R_s^1 p_s^j > \Sigma R_s^1 p_s^1 = k \quad \forall j>1$. Therefore $\Sigma R_s^1 p_s^1$ would never be a binding upper bound on any of the other contracts. Consider now a new contract $\hat{R}^1$ with the same expected value of R^1 according to p^1 but linear. Since $\Sigma \hat{R}_s^1 p_s^j = k$ $\forall j$ none of the constraints on j>1 would become tighter because of $\hat{R}^1$ and the utility of the investor would increase. This is because a concave utility is an increasing function of a mean preserving reduction of the difference $R_3^i - R_2^i$. Therefore a convex R^1 would be dominated by a linear R^1.

Step 3 implies that the optimal R^1 is either linear or concave.

Step 4. It is possible that the optimal R^1 contract be concave.

If R^1 is concave, $\Sigma R^1 p^j < k$ because of lemma 3. This implies that R^1 becomes a binding upper bound on the choice of contract R^2 and possibly on other contracts R^j, j > 2. This means that the choice of a concave R^1 involves a loss in utility due to a lower expected value on those contracts. There is however a potentially offsetting increase in utility deriving by improved risk sharing ($R_3^1 - R_2^1$ is reduced as compared with a linear contract). Whether the balance between the increase and the decrease in utility is positive or negative depends on the probability weights π_i and on the degree of risk aversion of the $u(\cdot)$.

Step 5. The optimal R^2 is either linear or at least as concave as R^1.

(5.1) Suppose R^1 is linear. Then, because of step 1, $\Sigma R_s^2 p_s^2 = k = \Sigma R_s^1 p_s^1$. Paralleling the argument used in step 3 we conclude that R^2 is either linear or concave. If it is concave it is trivially more concave than R^1 which is linear.

(5.2) Suppose R^1 is concave. Because of lemma 3 $\Sigma R_s^1 p_s^2 < k$. This implies, because of step 1, that $\Sigma R_s^1 p_s^2 = \Sigma R_s^2 p_s^2$. If R^2 were convex or linear, $\Sigma R_s^2 p_s^j > k > \Sigma R_s^1 p_s^2 > \Sigma R_s^1 p_s^j \; \forall j > 2$. Hence R^2 would never be a binding upper bound and, according to the argument in step 3, would be dominated by a concave R^2. Moreover, from the second part of lemma 3 it follows that if R^1 were more concave than R^2, $\Sigma R_s^2 p_s^j$ $(j>2)$ would never be binding and there would be an utility gain in moving towards a more concave R^2.

A straightforward implication of steps 2-5 is that

Step 6. $(LC)_1$ holds for the optimal contracts.

Step 7. Suppose $(LC)_{i-1}$ holds. Then $(LC)_i$ holds.

Since $(LC)_{i-1}$ holds, because of lemma 3 $\Sigma R_s^i p_s^{i+1} \leq \Sigma R_s^j p_s^{i+1} \leq k$ $j<i$. Therefore $\Sigma R_s^i p_s^{i+1}$ is the most binding upper bound on R^{i+1} and, because of step 1, $\Sigma R_s^i p_s^{i+1} = \Sigma R_s^{i+1} p_s^{i+1} \leq k$ (equality only if R^i is linear).

Suppose R^i is linear. Then, according to the same argument used in step 2, R^{i+1} is either linear or concave.

Suppose R^i is concave. Paralleling the argument used in step (5.2) we conclude that R^{i+1} is at least as concave as R^i.

By induction, we then conclude

Step 8. (LC) holds for the set of optimal contracts.

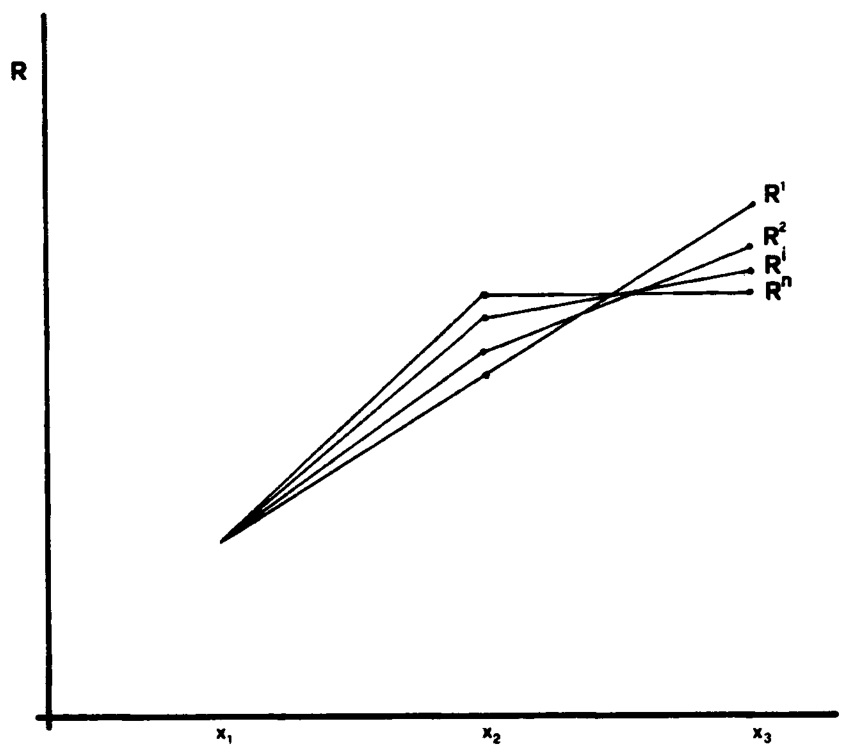
In order to prove (b), because of proposition 1 we only need to prove that any solution to (P3) satisfies the "adjacent downward" incentive constraints:[13]

(ADIC) $\Sigma\, R_s^i p_s^i \leq \Sigma\, R_s^{i+1} p_s^i \;\forall i.$

Indeed, because of (a), $\Sigma\, R_s^{i+1} p_s^{i+1} = \Sigma\, R_s^i p_s^{i+1}$ and R^{i+1} is at least as concave as R^i. Therefore the second part of lemma 3 implies $\Sigma\, R_s^i p_s^i \leq \Sigma\, R_s^{i+1} p_s^i$, i.e. (ADIC) holds. This completes the proof.

Lemma 4 provides a simple characterization of the set of optimal ICSC. Optimal contracts are more concave the riskier is the type for which the contract is meant ("relative concavity"); any type is indifferent between choosing its own contract and pretending to be the next less risky type ("adjacent indifference"). A typical set of optimal ICSC is represented in figure 2.

Fig. 2



13. Alternatively, we could verify that all "downward" incentive constraints are satisfied, without relying on proposition 1.

To understand the rationale for these two features of the optimal contracts note first the following:

a) under the assumption that (LL) is binding at least in the worst state we know that the optimal full-information contract is a concave function of x (see section 3);

b) we also know (Rotschild and Stiglitz (1970)), that a mean preserving spread of x reduces the expected value of a concave function of x.

The design of the optimal ICSC exploits these two facts. For given expected value of a contract, because of (a) we are interested in making the contract concave. But the concavity of the contract, because of (b)[14] implies that a less risky type would obtain from that contract a lower expected value than the type for which the contract is meant. This essentially means, given "adjacent indifference", that we need not worry about "downward" incentive compatibility.

It is also interesting to note the trade-off between risk sharing and expected value in the design of the optimal contracts. Optimal risk sharing is obtained with an SDC and we can say, loosely speaking, that the shape of an SDC gives the maximum degree of concavity desired by the investor (this is indeed guaranted by (M)). Considering a linear contract R^i, better risk sharing means greater concavity (i.e. a shape closer to an SDC). But greater concavity means that the expected value of that contract according to a riskier distribution p^{i+1} is correspondingly lower. Since the expected value of the contract is a cost for the entrepreneur, the greater concavity of R^i means that larger surplus must be left to type i+1. Clearly the optimal contract minimizes that extra-surplus, and this gives "adjacent indifference".

We now turn to consider how restrictive is to constrain the optimal contracts to belong to BB.

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14. Let us remember that the entrepreneur gets $\bar{x}-\Sigma p_s R_s$, so that a concave R means a convex share for the entrepreneur.

From F.O.Cs to (P3) it is apparent that the optimal R^n is an SDC. If R^n were not in BB, then the whole problem would become trivial. Indeed, if $R^n \notin B$ then $R^n_s = \bar{R}^n \ \forall s$. This means that in the optimal contract type n provides the investor with full insurance. However, because of (LC), $\Sigma R^1_s p^1_s \geq \Sigma R^2_s p^2_s \geq .. \geq \Sigma R^n_s p^n_s$.

Therefore all entrepreneurs, whose utility is decreasing with $\Sigma R_s p_s$, would pretend to be of type n, since $\Sigma R^n_s p^n_s = \bar{R}^n \cdot \Sigma p^n_s = \bar{R}^n \cdot \Sigma p^j_s = \bar{R}^n \ \forall j$.

As a result the optimal contract must be a riskless repayment irrespective of types, and the asymmetry of information becomes totally immaterial. In fact it is precisely the possibility of bankruptcy that, in spite of the assumed risk neutrality of the entrepreneurs (the party with private information), makes the problem of adverse selection relevant.

Remark 5. A standard claim in the principal-agent literature (Levinthal (1988)) is that incentive problems can be trivially solved if the agent is risk-neutral. In this case the agent would bear all the risk and the optimal contract would take the form of a fixed payment to the principal, with the agent receiving the residual outcome.

However, if there is a strictly positive probability of bankruptcy (i.e. if there is even a small chance that the agent cannot pay what required by the contract) that trivial solution would not work and incentive problems would be relevant in spite of the agent risk-neutrality.

Therefore the assumption that $\bar{w}$ is such that in the optimal contract the probability of bankruptcy for the riskiest type is strictly positive is not really restrictive. It is rather a condition for the problem to be of interest. It will therefore be implicitely assumed in the following.

But we know that $R^{n-j}_3 \geq R^n_3$, $j = 1 .. n - 1$ because of (LC). Since $R^n_3 = R^n_2 > x_1$ we have that $R^{n-j}_3 > x_1$, $j = 1 .. n - 1$. Now, if the distributions p^i were symmetrical (i.e. if $p^i_1 = p^i_3 \ \forall \ i$) from first order conditions we would immediately conclude that, if $\eta_{n-j,1}$ were zero, $R^{n-j}_1 = R^{n-j}_3$ $j=1 .. n-1$. But $R^{n-j}_3 > x_1$ and we have that $\eta_{n-j,1}$ cannot be zero.

The same conclusion holds by continuity provided that the probability distributions p^i are such that p_1^i is not too small compared to p_3^i, for all i. This requirement is made precise in the following

Lemma 5. If the probability distributions of the projects outcomes satisfy

$$\text{(A)} \quad \frac{p_3^i}{p_1^i} \leq \frac{1-\lambda}{\lambda} \quad \forall i$$

then the set of optimal contracts solving (P1) is a subset of BB.

Proof. Suppose not, i.e. suppose that for one of the optimal contracts, say R^i,

$$R_2^i = \lambda R_1^i + (1-\lambda)R_3^i + \delta_i \qquad \text{with } R_1^i < x_1$$

Consider now a different contract $\hat{R}^i$ defined as follows

$$\hat{R}_1^i = x_1, \ \hat{R}_2^i = \lambda x_1 + (1-\lambda)\hat{R}_3^i + \delta_i, \ \hat{R}_3^i = R_3^i - (x_1 - R_1^i)\ \frac{p_1^i + \lambda p_2^i}{p_3^i + (1-\lambda)p_2^i}$$

It is immediate to verify that because of (MPS)[15]

$$\Sigma\ \hat{R}_s^i p_s^j = \Sigma\ R_s^i p_s^j \ \forall j$$

Therefore $\hat{R}^i$ satisfies all constraints satisfied by R^i.

Moreover $\hat{R}_1^i > R_1^i$, $\hat{R}_3^i < R_3^i$ and

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15. Indeed

$$\Sigma\ \hat{R}_s^i p_s^i = x_1(p_1^i + \lambda p_2^i) + (p_3^i + (1-\lambda)p_2^i)[R_3^i - (x_1 - R_1^i)\ \frac{p_1^i + \lambda p_2^i}{p_3^i + (1-\lambda)p_2^i}] + p_2^i \delta_i =$$

$$= R_1^i(p_1^i + \lambda p_2^i) + R_3^i(p_3^i + (1-\lambda)p_2^i) + p_2^i \delta_i = \Sigma\ R_s^i p_s^i$$

Moreover $\Sigma\ \hat{R}_s^i p_s^j$ varies with j only because of the term $p_2^j \delta_i$ which is unchanged with respect to $\Sigma\ R_s^i p_s^j$.

$$\hat{R}_2^i - R_2^i = \lambda(x_1-R_1^i) - (1-\lambda)\ (x_1-R_1^i)\ (\frac{p_1^i+\lambda p_2^i}{p_3^i+(1-\lambda)p_2^i}) =$$

$$(x_1-R_1^i)\ [\lambda-(1-\lambda)\ \frac{p_1^i+\lambda p_2^i}{p_3^i+(1-\lambda)p_2^i}] \leq 0 \quad \text{if}$$

$$[\lambda-(1-\lambda)\ \frac{p_1^i+\lambda p_2^i}{p_3^i+(1-\lambda)p_2^i}] \leq 0 \quad \text{which is true because of (A).}$$

Since in lemma 4, step 1 we proved that in any optimal contract $R_2^i \leq R_3^i$ we have that $\hat{R}_2^i \leq R_2^i \leq R_3^i$.

Therefore the new contract $\hat{R}^i$ has the same expected value of R^i but has a smaller interval of variation and this means that the utility of the investor is increased by shifting from contract R^i to contract $\hat{R}^i$ belonging to BB. This contradicts the optimality of R^i and concludes the proof.

Remark 6. Using (MPS) it can be easily verified that we only need to check that condition (A) is satisfied by any one of the p^i.

As a straightforward implication of lemmas 4 and 5 we have

Proposition 2: Under the assumption (A) (P1) and (P3) have the same solution set. Moreover the set of optimal contracts is a subset of BB and is characterized by (LC) and (M).

Remark 7. The characterization of the set of optimal ICSC allows us to operationally solve the sharing problem using dynamic programming techniques. This is shown in the Appendix.

5. Welfare properties of the optimal contracts

The characterization of the set of optimal ICSC given in proposition 2 allows us to derive very simply some of the welfare properties of the contracts.

We have

Proposition 3. In the set of optimal ICSC

(a) all surplus is extracted from type 1;

(b) if the contract R^1 is concave, then all riskier types 2, ..n strictly prefer the contract to their reservation choice, i.e. they obtain part of the surplus;

(b1) if R^1 is linear, let R^{i_0} be the riskiest linear contract; then all types i_0+2, ..n obtain part of the surplus;

(c) optimal risk sharing is only achieved with the riskiest type.

Proof. Part (a) was proved in the course of proving lemma 4. Because of (LC), $\Sigma R^2_s p^2_s = \Sigma R^1_s p^2_s < \Sigma R^1_s p^1_s = k$ if R^1 is concave Similarly $\Sigma R^i_s p^i_s = \Sigma R^{i-1}_s p^i_s < \Sigma R^{i-1}_s p^{i-1}_s < k \ \forall\ i > 2$ since R^i is concave. Hence (b) holds.

(b1) is obvious given (b)

(c) is a straightforward consequence of the corollary to proposition 2 proved in the Appendix.

Let us remember that under full information the set of optimal contracts is given by R^i_D, i=1, ..n, where R^i_D is an SDC and (VP) is binding for each i.

Comparing proposition 3 with the welfare properties of the set of optimal contracts under full information we see that only riskier types gain from the informational asymmetry and only the investor loses. The safest (or the i_0+1 safest in case R^{i_0} is the riskiest linear contract) obtain the same expected return they would obtain under full information. All riskier types extract a surplus and strictly prefer the optimal ICSC to an SDC with (VP) binding.

The expected utility obtained by the investor from contract R^i is lower than the expected utility obtained from contract R^i_D, for $\Sigma R^i_{Ds} p^i_s = k \geq \Sigma R^i_s p^i_s$, with strict inequality for all $i \geq i_0 + 2$ if i_0 is the riskiest linear contract, and the risk sharing is improved by R^i_D, for all $i < n$. For the safest and the riskiest types, the SDC only improves on risk sharing or expected value, respectively. Therefore all the welfare loss involved by the informational failure is borne by the investor.

It might seem at first sight, however, that there is here no dead-weight loss, i.e. no net welfare loss, since the investor loses but the entrepreneur unambiguously gains. The following simple argument shows that gains and losses do not calcel out. Consider the allocation of welfare that is guaranteed to the entrepreneurs by the optimal ICSC, i.e. consider for each type the surplus over the reservation level $\bar{w}$ that is obtained accepting the optimal contract meant for him. If incentive compatibility problems were not present we could appropriately define a set of SDCs in such a way as to replicate the same welfare allocation as far as entrepreneurs are concerned and yet guarantee strictly higher an expected utility for the investor. Therefore the difference between that higher level of utility and the level obtained with optimal ICSC provides a measure of the dead-weight loss due to asymmetric information.

Note that the effect of asymmetric information is twofold. On one hand it involves a move along the contract curve in a direction more favourable to the entrepreneurs, i.e. a "redistribution" of welfare. On the other hand it distorts the contracts causing a net welfare loss.

6. Optimal ICSC and credit rationing

One of the implications of proposition 2 is that the appropriate design of a sharing contract allows the investors to sort the entrepreneurs into risk classes according to their type (riskiness).

As Stiglitz and Weiss (1981) showed that credit rationing arises as a rational response to the impossibility of sorting into risk classes entrepreneurs with private information[16], the pos-

16. In the present paper we do not consider the possibility of credit rationing arising from moral hazard. In fact, we assume that entrepreneurs cannot choose their projects precisely to avoid moral hazard. Similarly, our analysis is static and disregards the incentive role of credit rationing in repeated credit relationships (see Stiglitz and Weiss (1983)).

sibility of sorting leads quite naturally to ask whether equilibrium credit rationing could be avoided. The answer to this question, however, requires to supplement the analysis of the optimal sharing contract with an appropriate equilibrium condition.

6.1 Reactive equilibrium

It is well known, since Rotschild and Stiglitz, that there are problems with the notion of competitive equilibrium in presence of asymmetric information[17]. In particular, we need to stipulate how agents are supposed to behave in the face of the information made available by the contract. The solution originally proposed by Rotschild and Stiglitz, as well as the most obvious Nash solution, have been shown to be crucially sensitive to the assumption of a discrete number of types; in fact Riley (1979) has shown that with a continuum of types neither the equilibrium proposed by Rotschild and Stiglitz nor the Nash equilibrium exist. As an alternative, Riley proposed a partially strategic notion of equilibrium.

A set of contracts constitute a (Riley's) Reactive Equilibrium (RRE) if, for any additional contract that generates an expected gain to the agent making the offer, there is another contract which yields a gain to a second agent and losses to the first. Moreover, no further addition to the set of contracts generates losses to the second agent.

More formally, let us define a set of sharing contracts to be informationally consistent (INC) if there is a contract R^i for each i such that

(i) $R^i \succsim_i R^j \quad \forall i,j$

(ii) $E_i[u(R^i)] = \bar{u} \ \forall i$, where $\succsim_i$ denotes the preference ordering

17. More precisely, as F. Hahn (1985) noted, the relevant problem is the market dependent nature of information.

of a type-i entrepreneur, $E_i(\cdot)$ denotes expectation taken according to p^i and $\bar{u} = u(\bar{W})$.

Condition (i) is simply the incentive compatibility requirement.

Condition (ii) states that the investor is offering to each type a contract that, when accepted by that type, barely "breaks-even" in utility terms. It must be stressed that there are two different requirements embodied in (ii). The first is that the investor is not being fooled, since he computes the expected utility of the contract meant for type i using the appropriate probability distribution p^i. The second requirement is that the investor obtains from the contract precisely what he gives ($\bar{W}$) and therefore he does not extract any surplus.

Because of the second requirement, we shall refer to condition (ii), with little abuse of terminology, as to the zero profit condition (ZP).

Riley showed that the Pareto optimal set of INC sharing contracts constitute the unique RRE. The Pareto-optimal INC contracts can be found by solving

$$\text{(P4)} \quad \begin{cases} \min\limits_{[R]} \sum\limits_i \left[\sum\limits_s p_s^i R_s^j \right] \pi_i \\ \text{s.t.} \\ \quad \text{(IC)} \ \sum\limits_s p_s^i R_s^i \leq \sum\limits_s p_s^i R_s^j \quad \forall i, j \\ \quad \text{(ZP)} \ \sum\limits_s p_s^i \, u(R_s^i) = u(\bar{W}) \quad \forall i \\ \quad \text{(LL)} \ x_s - R_s^i \geq 0 \quad \forall i, s \\ \quad \text{(N)} \ R_s^i \geq 0 \quad \forall i, s \end{cases}$$

We shall assume, for the time being, that the entrepreneurs (VP) constraints (see (P1)) are satisfied. The possibility of (VP) binding will be taken up at the end of the section.

Replicating arguments developed in section 4, we could easily show that (P4) can be simplified by substituting (IC) with (AIC). A more difficult task is to show that a solution to (P4) can be found solving the problem that obtains when we substitute (IC) with (UIC). We shall refer to this problem as to (P5).

As we did in section 4, we shall restrict our attention to contracts belonging to BB. Only at a later stage this restriction will be shown to be required to rule out a trivial solution to the incentive problem posed by asymmetric information[18]. The main difference between the sort of problem solved in section 4 and (P4) (or (P5)) is the presence of (ZP) constraints. It is therefore useful to explore preliminarly the structure of such constraints.

Let us define, for each i and for contracts belonging to BB, the set of contracts satisfying the corresponding constraint (ZP), i.e. define

$$V_i = \{(x_1,R_2,R_3)/p_1^i u(x_1) + p_2^i u(R_2) + p_3^i u(R_3) = \bar{u}\}.$$

Under strict concavity of $u(\cdot)$ the projection of the set V_i on the (R_2,R_3) plane is represented by a decreasing, strictly convex line. We shall refer to that line as v_i.

The slope of v_i, easily obtained from the implicit function theorem, is given by

$$\frac{dR_3}{dR_2} = - \frac{p_2^i}{p_3^i}\ \frac{u_2'}{u_3'} \qquad \text{where } u_s' = u'(R_s)\ , \qquad s = 2,3$$

Clearly, the slope of v_i is smaller than $-(p_2^i/p_3^i)$ for all points above the 45° line, whereas it is larger for all points below. The point $R_2 = R_3 = \bar{R}^i$, located at the intersection between v_i and the 45° line, represents the SDC ε V_i, i.e. $\bar{R}^i$ is the solution of $p_1^i u(x_1) + (p_2^i+p_3^i)u(R) = \bar{u}$. With abuse of notation we shall refer to the contract $[x_1,\bar{R}^i,\bar{R}^i]$ as $\bar{R}^i$. Contract $\bar{R}^i$ has the following important property: the expected value of the contracts belonging to V_i (evaluated according to p^i) is minimized by $\bar{R}^i$, i.e.

$$\min_{R\ \varepsilon\ V_i} \Sigma p_s^i R_s^i = p_1^i x_1 + (p_2^i+p_3^i)\bar{R}^i$$

18. Alternatively, one might think that $x_1=0$.

Equivalently, the straight line with slope $-(p_2^i/p_3^i)$ passing through $\bar{R}^i$ is tangent to v_i (see fig. 3)

It can be easily shown that $\bar{R}^i<\bar{R}^j$ if $i<j$. Also, consider the contract $\overset{o}{R}=[x_1,\overset{o}{R}_2,\overset{o}{R}_3]$ that solves the following system

(1.1) $u(R_2) = \lambda u(x_1) + (1-\lambda)\ u(R_3)$

(1.2) $p_1^i u(x_1) + p_2^i u(R_2) + p_3^i u(R_3) = \bar{u}$

Substituting (1.1) into (1.2) we get:

(2) $(p_1+\lambda p_2)\ u(x_1) + (p_3+(1-\lambda)p_2)\ u(R_3) = \bar{u}$

where we can drop the superscript i because of MPS.

Solving (2) for R_3, substituting into (1.1) and solving for R_2 yields the desired contract. By construction, $\overset{o}{R} \varepsilon V_i\ \forall i$. Therefore all v_i cross in the point $(\overset{o}{R}_2,\overset{o}{R}_3)$.

Also note that the smaller is i (i.e. the safer is the type) the steeper is the slope of v_i at $(\overset{o}{R}_2,\overset{o}{R}_3)$ because, by assumptions, $-(p_2^i/p_3^i) < -(p_2^j/p_3^j)$ whenever $j>i$.

Using the strict convexity of the v_i, we then conclude that if a contract R is such that $R_2<\overset{o}{R}_2$ and $R_3>\overset{o}{R}_3$ then

$\sum_s p_s^i u(R_s) < \sum_s p_s^j u(R_s)$ if $j>i$;

if a contract R is such that $R_2>\overset{o}{R}_2$ and $R_3<\overset{o}{R}_3$ then

$\sum_s p_s^i u(R_s) > \sum_s p_s^j u(R_s)$ if $j>i$.

The geometric counterpart of these properties is that, for $i>j$, v_i lies to the left of v_j NW of $(\overset{o}{R}_2,\overset{o}{R}_3)$ and v_i lies to the right of v_j SE of $(\overset{o}{R}_2,\overset{o}{R}_3)$ (see fig. 3)

Remark 8. $\overset{o}{R}$ is a convex contract. Indeed, for $u(R_2) = \lambda u(x_1)+(1-\lambda)u(R_3)$ it must be the case that $R_2 = \lambda x_1+(1-\lambda)R_3-\delta$, $\delta>0$.

We see that the convex contract $\overset{o}{R}$ provides a separation between the set of contracts whose expected utility is increased and the set of contracts whose expected utility is decreased by an increase in riskiness of the original project.

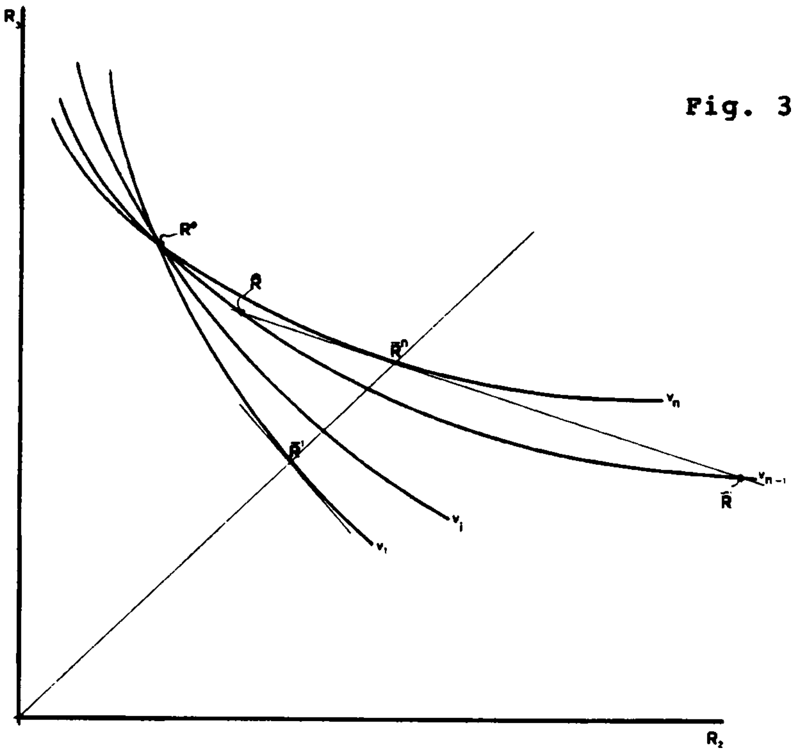


Fig. 3

A similar role was played in lemma 3 by a linear contract, separating the sets of contracts whose expected <u>value</u> is respectively increased or decreased by an increase in riskiness.

There is one further property of the set of contracts satisfying (ZP) that is useful to record explicitly.

Consider the set of contracts (belonging to BB) that have the same expected value of $\bar{R}^n$ according to p^n, i.e. the contracts lying on the line

$$\sum_s p_s^n R_s = p_1^n x_1 + (p_2^n + p_3^n)\bar{R}^n$$

This is a straight line, in the (R_2, R_3) plane, with slope $-(p_2^n/p_3^n)$ passing through $\bar{R}^n$. It crosses v_{n-1} in two points, say $(\hat{R}_2, \hat{R}_3)$ and $(\tilde{R}_2, \tilde{R}_3)$, respectively above and below the 45° line (see fig. 3). Let us define the two contracts $\hat{R} = [x_1, \hat{R}_2, \hat{R}_3]$ and $\tilde{R} = [x_1, \tilde{R}_2, \tilde{R}_3]$. We then have:

<u>Lemma 6</u>

$$\sum p_s^{n-1} \hat{R}_s < \sum p_s^{n-1} \tilde{R}_s$$

<u>Proof</u>. Define $R_2^{n-1} = \bar{R}^{n-1} + \delta_2$, $R_3^{n-1} = \bar{R}^{n-1} + \delta_3$.

This allows us to represent the v_{n-1} line in the (δ_2, δ_3) plane.

In particular, since $\bar{R}^{n-1}$ minimizes $\Sigma p_s^{n-1} R_s^{n-1}$, all points that are candidates for v_{n-1} must lie strictly above the line $\delta_3 = - \dfrac{p_2^{n-1}}{p_3^{n-1}} \delta_2$; since $u(\cdot)$ is increasing we must clearly exclude all points belonging to the first quadrant (see the shaded area in fig. 4).

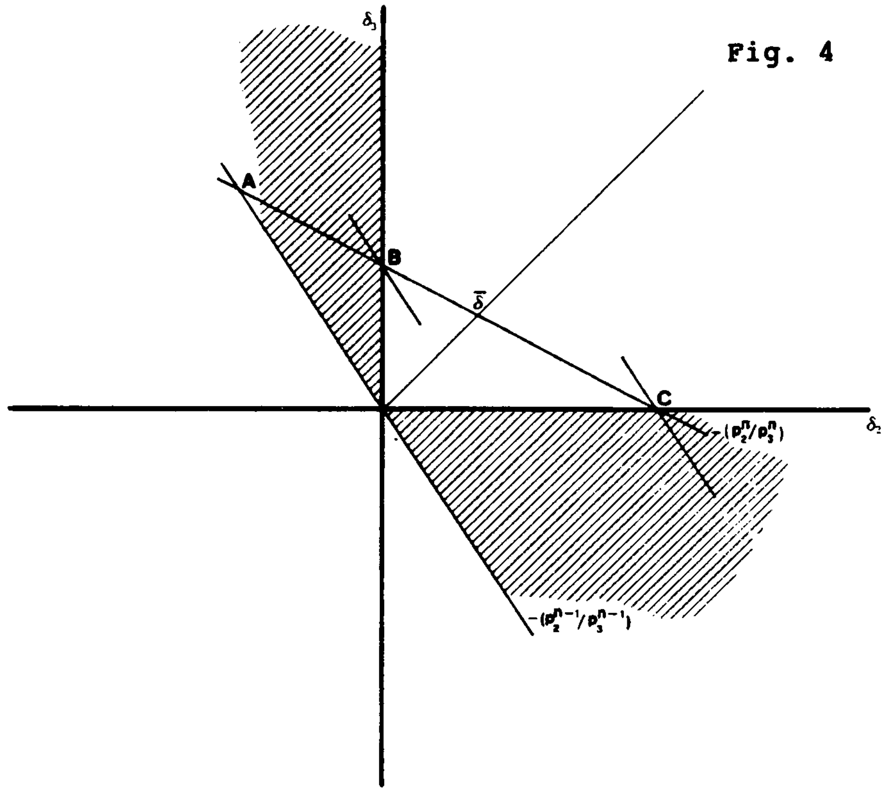


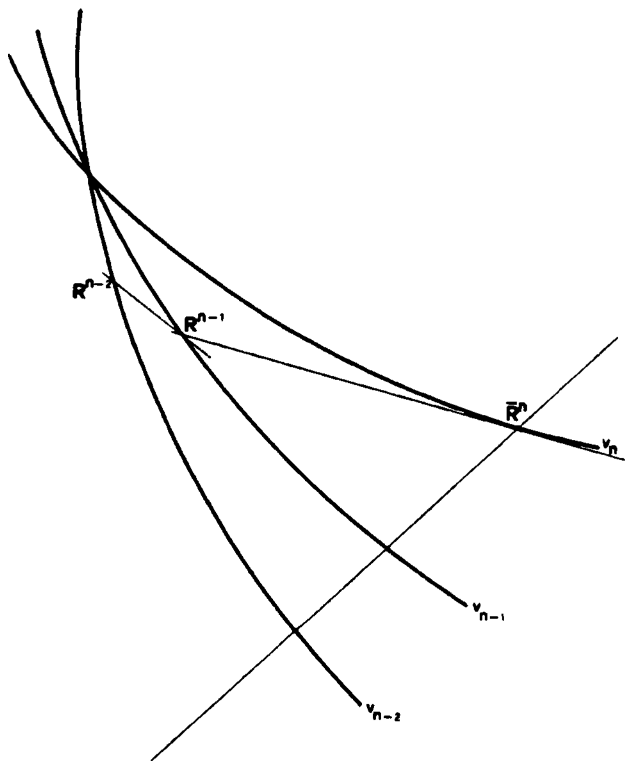
Fig. 4

Since $\bar{R}^n = \bar{R}^{n-1} + \bar{\delta}$, we can represent $\bar{R}^n$ by the point $(\bar{\delta}, \bar{\delta})$. The points representing $\hat{R}$ and $\tilde{R}$ are to be found somewhere in the shaded area along the line with slope $-(p_2^n/p_3^n)$ passing through $(\bar{\delta}, \bar{\delta})$. More precisely $\hat{R}$ can be represented by some point on the segment AB and $\tilde{R}$ by some point on the line starting from C. To prove our lemma it is enough to prove that $\Sigma R_s^B p_s^{n-1} < \Sigma R_s^C p_s^{n-1}$, where R^B and R^C are the contracts corresponding to the points B and C. A little use of analytic geometry suffices to show that this is indeed the case as long as $- \dfrac{p_2^n}{p_3^n} > - \dfrac{p_2^{n-1}}{p_3^{n-1}}$. But this follows from the assumed ranking of projects riskiness, and the lemma is proved.

Lemma 6 proves that among the contracts meant for type n-1 that both satisfy ZP (i.e. contracts belonging to V_{n-1}) and are not preferred by type n, the contract with $R_2 < R_3$ is preferred by type n-1 to the contract with $R_2 > R_3$. This result will be used in deriving the solution to problem (P4)

It is useful, before presenting the formal proof, to provide a geometric interpretation of the solution. This can be obtained through the following construction (see fig. 5).

Fig. 5



We start from the SDC meant for the type n ($\bar{R}^n$); we draw the tangent to v_n through $\bar{R}^n$ (slope $-(p_2^n/p_3^n)$), and we consider the contract R^{n-1}, determined by the intersection between the v_{n-1} and the tangent in the half plane above the 45° line. We then draw a straight line with slope $-(p_2^{n-1}/p_3^{n-1})$ through R^{n-1}, and we determine R^{n-2} at the intersection between this line and v_{n-2} (again in the half plane above the 45° line). We continue in this way until we define a contract for each type. It can easily be checked that the contracts so determined satisfy IC. We shall prove that this set of contracts indeed maximizes the average expected utility of entrepreneurs, i.e. it solves (P4). It is worth noting that the set of optimal contracts does not depend on the distribution of types. This is perhaps the main difference with the optimal ICSC found without imposing "zero profit" conditions. Also note that whereas proposition 2 merely allowed us to characterize the set of optimal ICSC, imposing the "zero profit" conditions yields an explicit solution of the sharing problem.

Lemma 7. Restricting the optimal contracts to belong to BB, we have that

(a) the solution to (P5) is recursively given by

$$R_2^{n-j} = R_2^{n-j+1} - \delta_j, \quad R_3^{n-j} = R_3^{n-j+1} + \delta_j \frac{p_2^{n-j+1}}{p_3^{n-j+1}}, \quad j = 1, .. n-1$$

where δ_j is the positive solution of

$$p_1^{n-j}u(x_1) + p_2^{n-j}u(R_2^{n-j+1}-\delta_j) + p_3^{n-j}u(R_3^{n-j+1}+\delta_j\frac{p_2^{n-j+1}}{p_3^{n-j+1}}) = \bar{u}$$

and $R_2^n = R_3^n = \bar{R}^n$

(b) (P4) and (P5) have the same solutions.

Proof. We first prove part (a) of the lemma in a number of steps.

Step 1. The optimal R^n is an SDC, namely $\bar{R}^n = [x_1, \bar{R}^n, \bar{R}^n]$.

From F.O.C.s we have, considering interior solutions[19]

$$\pi_n p_s^n + \lambda_n u'(R_s^n)p_s^n + p_s^n \sum_{j<n} \mu_{nj} = 0 \qquad s = 2,3$$

where λ and μ are Lagrange multipliers associated to (ZP) and (UIC) constraints.

Equivalently

19. Note that we are assuming that none of the constraints $R_2^n \leq x_2$, $R_3^n \leq x_3$ is binding. This amounts to impose an upper bound on the investor reservation level $\bar{u}$. It can be easily shown that if $R_3^n = x_3$ then $R_2^n = x_2$.

$u'(R_s^n) = k_n$, where k_n is a constant independent of s. Hence $R_2^n = R_3^n$. This condition, together with the constraint that the contract $\varepsilon\ V_n$, completely determine R^n. Indeed, the optimal R^n is the unique solution of

$p_1^n u(x_1) + (p_2^n + p_3^n)\ u(R) = \bar{u}$, i.e. precisely what we defined as $\bar{R}^n$.

Step 2. In the set of optimal contracts, no other contract is an SDC. This follows immediately from observing that for any other SDC, say $\bar{R}^j$, satisfying the constraint $\bar{R}^j\ \varepsilon\ V_j$, we have

$\bar{R}^j < \bar{R}^n \quad \forall j < n$. Therefore $\Sigma p_s^n\ \bar{R}^j < \Sigma p_s^n\ \bar{R}^n$, violating IC.

Step 3. $\Sigma R_s^{n-1} p_s^n = \Sigma R_s^n\ p_s^n$ (i.e. AUIC is binding on R^n).
Because of step 2, it must be

$$\Sigma R_s^{n-1} p_s^{n-1} > \Sigma \bar{R}_s^{n-1} p_s^{n-1}.$$

Hence we can decrease the expected value of R^{n-1} and still satisfy $R^{n-1}\ \varepsilon\ V_{n-1}$. Suppose now that the claim were not true, i.e. suppose $\Sigma R_s^{n-1} p_s^n > \Sigma R_s^n p_s^n$; we could then reduce R^{n-1} (along the v_{n-1}) without violating any of the constraints in which R^{n-1} appears on the left. This would clearly increase the objective function, leading to a contradiction. Thus the claim is true.

Step 4. $R_2^{n-1} = \bar{R}^n - \delta$, $\quad R_3^{n-1} = \bar{R}^n + \delta\ \dfrac{p_2^n}{p_3^n}$ where δ is the positive solution of

(4.1) $p_1^{n-1} u(x_1) + p_2^{n-1} u(R_2^{n-1}) + p_3^{n-1} u(R_3^{n-1}) = \bar{u}$.

Because of step 3 and because the optimal R^{n-1} must be on v_{n-1}, we have that the optimal R^{n-1} must be a solution of 4.1. Equation (4.1) has only two possible solutions, one with $\delta>0$ and one with $\delta<0$. In lemma 6 we showed that the solution with $\delta<0$ yields a contract with a larger expected value (according to type n-1). Therefore it is dominated by the solution with $\delta>0$.

Step 5. $R_2^{n-2} = R_2^{n-1} - \delta$, $R_3^{n-2} = R_3^{n-1} + \delta\ \dfrac{p_2^{n-1}}{p_3^{n-1}}$, where δ is the positive solution of

(5.1) $p_1^{n-2}u(x_1) + p_2^{n-2}u(R_2^{n-2}) + p_3^{n-2}u(R_3{}^{n-2}) = \bar{u}$.

Because of step 2 we know that $\Sigma R_s^{n-2} p_s^{n-2} > \Sigma \bar{R}_s^{n-2} p_s^{n-2}$, and, as in step 3, we can easily prove that it must be the case that

(5.2) $\Sigma R_s^{n-2} p_s^{n-1} = \Sigma R_s^{n-1} p_s^{n-1}$.

As in step 4 this, together with $R^{n-2} \varepsilon V_{n-2}$, leaves only two possibilities. Suppose that the negative solution for δ yields the optimal R^{n-2}, i.e. suppose

(5.3) $R_2^{n-2} > R_2^{n-1}$ and $R_3^{n-2} < R_3^{n-1}$.

In order to prove that this cannot be optimal there are three cases to consider. Indeed, we proved that R^{n-1} is less concave than $\bar{R}^n$. Moreover (5.2) and (5.3) are equivalent to say that R^{n-2} is more concave than R^{n-1}. However it might be the case that R^{n-1} is a convex contract on its own[20]. Then we have the possible cases

(i) R^{n-1} concave, R^{n-2} concave

(ii) R^{n-1} convex, R^{n-2} concave

(iii) R^{n-1} convex, R^{n-2} convex.

Let us consider (i). Because of previous steps we can write

(5.4) $\Sigma \bar{R}_s^n p_s^n = \Sigma R_s^{n-1} p_s^n < \Sigma R_s^{n-1} p_s^{n-1} = \Sigma R_s^{n-2} p_s^{n-1} > \Sigma R_s^{n-2} p_s^n$,

where the inequalities follow from concavity and lemma 3. Write (5.4) in abstract algebraic terms as

$a = b < c = d > e$. Clearly, if we can prove that

(5.5) $c - b < a - e$ we then conclude

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20. Remember that $(\overset{o}{R}_2, \overset{o}{R}_3)$, defined at pag 36, is convex. We are going to prove that any optimal contract is SE of $(\overset{o}{R}_2, \overset{o}{R}_3)$, but this does not rule out convex contracts.

a > e , i.e. that

$\Sigma \bar{R}_s^n p_s^n > \Sigma R_s^{n-2} p_s^n$, thus violating incentive compatibility.

The condition (5.5), written explicitly, is

$$R_2^{n-1}(p_2^{n-1}-p_2^n) + R_3^{n-1}(p_3^{n-1}-p_3^n) < R_2^{n-2}(p_2^{n-1}-p_2^n) + R_3^{n-2}(p_3^{n-1}-p_3^n),$$

which is indeed true if (5.3) is true.

Case (ii) is very simple, since the first inequality is reversed and we get at once the desired result that $\Sigma \bar{R}_s^n p_s^n > \Sigma R_s^{n-2} p_s^n$.

Case (iii) is essentially the same as case (i) and is left to the reader. We conclude that (5.3) cannot be optimal and that the optimal R^{n-2} corresponds to the posivite solution to (5.1).

<u>Step 6</u>. It is clear that we can replicate the argument in step 5 for each type n-3, .. 1. This proves part (a).

<u>Step 7</u>. We only need to verify that the ADIC is satisfied by the proposed solution. From part (a) we know that the optimal set of contracts is characterized by a (strict) ranking of concavity similar to (LC). In fact, we cannot exclude convex contract but we have that each contract is strictly less concave than the contract meant for the next more risky type. Formally, the following characterization of relative concavity is satisfied by the set of optimal contracts

$$\text{(RC)} \quad \begin{cases} \sum_s R_s^h p_s^h = \sum_s R^{h-1} p_s^h & h = 2, \ldots n \\ R^i \text{ is more concave that } R^{i-1} \end{cases}$$

Given (RC) we easily conclude, as in lemma 4, step 8, that ADIC holds. This complete the proof.

We now take up the restrictive clause that contracts must belong to BB. The argument is very similar to that presented in section 4. Consequently we shall be somewhat sketchy. Preliminarly, it is useful to extend some of the definitions previously given to cover the case of contracts not belonging to BB. Let us define the set

$$V_j(R_1) = \{ (R_1,R_2,R_3) \,/\, p_1^j u(R_1) + p_2^j u(R_2) + p_3^j u(R_3) = \bar{u} \}$$

and the curve $v_j(R_1)$, which is the locus of the projection of $V_j(R_1)$ in the (R_2,R_3) space. Clearly our former definitions are special cases of the latter, namely

$$V_j \equiv V_j(x_1) \text{ and } v_j \equiv v_j(x_1).$$

All properties that belong to V_j and v_j are also shared by $V_j(R_1)$ and $v_j(R_1)$. In particular, the minimal contract in $V_j(R_1)$ is

$$\bar{R}^j(R_1) = [R_1,\ \bar{R}^j(R_1),\ \bar{R}^j(R_1)] \quad \text{where } \bar{R}^j(R_1) \text{ solves}$$

$$\min_{R \,\varepsilon\, V_j(R_1)} \; p_1^j R_1 + p_2^j R_2 + p_3^j R_3.$$

Equivalently, the slope of the $v_j(R_1)$ is equal to $-\dfrac{p_2^j}{p_3^j}$ at the point $[\bar{R}^j(R_1),\ \bar{R}^j(R_1)]$.

Also, let us define $g^j(R_1) = p_1^j R_1 + (p_2^j+p_3^j)\ \bar{R}^j(R_1)$.

In words, $g^j(R_1)$ is the minimal value (according to p^j) of the contracts belonging to $V_j(R_1)$.

It can be easily shown that, because of the concavity of the $u(\cdot)$, $g^j(\cdot)$ is a continuous, decreasing function in the range $0<R_1\leq x_1$.

The same is clearly true of the function

$$g_i^j(R_1) = p_1^i R_1 + (p_1^i+p_2^i)\ \bar{R}^j(R_1), \quad \forall i.$$

The useful result, used in the proof of lemma 7, according to which type j prefers the SDC meant for type j-1, for all j, can be then rephrased, using previous definitions, as follows

$g^j(x_1) > g_j^{j-1}(x_1)$

We are now ready to state and prove the following

Lemma 8

Assume that the investors' utility reservation level prevents riskless contracts, i.e. assume

(NRC) $u(x_1) < u(\bar{W})$.

Moreover assume that the probability distribution of projects outcomes satisfies

$$\text{(A)} \quad \frac{p_3^i}{p_1^i} \leq \frac{1-\lambda}{\lambda} \quad \forall i$$

then the set of contracts solving (P5) is a subset of BB.

Proof. As shown in the proof of lemma 7, the optimal contract meant for type n has the form $R_1 = R_2 = R_3$ unless some of the (LL) constraints is binding. Under assumption (NRC) it cannot be that $R_1 < x_1$ for (ZP) would be violated. Hence the contract meant for type n solving (P5) is indeed the contract $\bar{R}^n = [x_1, \bar{R}^n, \bar{R}^n]$ or, using the definition previously given $\bar{R}^n = \bar{R}^n(x_1)$.

Let us now consider the contract meant for type n-1. We shall reserve the notation R^{n-1} for the contract defined in lemma 7. Assume that the optimal contract meant for type n-1 is $R \in V_{n-1}(R_1)$ for some $R_1 < x_1$.

First of all we show that AUIC must be binding for such a contract, i.e. we show that

$$\Sigma R_s p_s^n = \Sigma \bar{R}_s^n p_s^n.$$

Suppose not, i.e. suppose

(1) $\Sigma\, R_s p_s^n < \Sigma\, \bar{R}_s^n p_s^n$. There are two cases to consider

(i) $R \equiv \bar{R}^{n-1}(R_1)$. Hence (1) reads

$\Sigma\, \bar{R}_s^{n-1}(R_1) p_s^n > \Sigma\, \bar{R}_s^n p_s^n$ or, equivalently

$g_n^{n-1}(R_1) > g^n(x_1)$. Since

$g_n^{n-1}(x_1) < g^n(x_1)$ and $g_n^{n-1}(\cdot)$ is a continuous, decreasing function there exists a value $\hat{R}_1$, $\hat{R}_1 > R_1$, such that

$g^{n-1}(\hat{R}_1) = g^n(x_1)$.

Note that, since $g^{n-1}(\cdot)$ is decreasing

$g^{n-1}(\hat{R}_1) < g^{n-1}(R_1)$.

Therefore the contract $\bar{R}^{n-1}(\hat{R}_1) = [\hat{R}_1, \bar{R}^{n-1}(\hat{R}_1), \bar{R}^{n-1}(\hat{R}_1)]$ strictly dominates the orginal $\bar{R}^{n-1}(R_1)$, satisfies by construction (ZP) and is such that AUIC is binding.

(ii) It can be easily checked that, given any contract $R \in V_{n-1}(R_1)$, $R \neq \bar{R}^{n-1}(R_1)$, such that AUIC is not binding, a new contract $\hat{R}$ can be defined that dominates R such that either AUIC is binding or $\hat{R} \equiv \bar{R}^{n-1}(R_1)$, with AUIC possibly not binding.

Because of (i) and (ii) we then conclude that for the optimal contract meant for type (n-1) AUIC must be binding.

(iii) Suppose now that the optimal $\hat{R}^{n-1}$ does not belong to BB. We know that AUIC must be binding, i.e. we know that

(2) $\sum_s \hat{R}_s^{n-1} p_s^n = \sum_s \bar{R}_s^n p_s^n$

Consider the (only) contract $\tilde{R}^{n-1}$ with $\tilde{R}_1^{n-1} = \hat{R}_1^{n-1}$ that satisfies (2) and is such that

(3) $\sum_s \tilde{R}_s^{n-1} p_s^{n-1} = \Sigma\, R_s^{n-1} p_s^{n-1}$.

It is just a matter of simple algebra to check that

$$\tilde{R}_2^{n-1} = R_2^{n-1} + (x_1 - \hat{R}_1^{n-1})\gamma_1 > R_2^{n-1} \quad \text{with } \gamma_1 = \frac{p_1^{n-1}p_3^n - p_3^{n-1}p_1^n}{p_2^{n-1}p_3^n - p_3^{n-1}p_2^n} > 0$$

$$\tilde{R}_3^{n-1} = R_3^{n-1} + (x_1 - \hat{R}_1^{n-1})\gamma_2 > R_3^{n-1} \quad \text{if} \quad \gamma_2 = \frac{p_2^{n-1}p_1^n - p_2^n p_1^{n-1}}{p_2^{n-1}p_3^n - p_3^{n-1}p_2^n} > 0$$

which is true if (A) is satisfied. But this means that $\Sigma\, p_s^{n-1}u(\tilde{R}^{n-1}) > \Sigma\, p_s^{n-1}u(R_s^{n-1}) = \bar{u}$. Therefore $\hat{R}^{n-1}$, in order to satisfy ZP, must be such that

$$\sum_s \hat{R}_s^{n-1}p_s^{n-1} > \Sigma\, R_s^{n-1}p_s^{n-1}.$$

This proves that R^{n-1} is the optimal contract meant for type n-1.

(iv) Given R^{n-1}, steps (i)-(ii) can be easily replicated to show that for any optimal contract meant for type n-2 AUIC must be binding.

As in step (iii), then, we can show that R^{n-2} dominates any contract not belonging to BB. The same procedure can clearly be replicated for all contracts to obtain the desired result.

A straightforward consequence of lemmas 7 and 8 is the following:

<u>Proposition 4</u>

Under the assumptions (NRC) and (A) the set of contracts solving (P4) is determined as in lemma 7, part (a).

As we mentioned earlier, the solution to (P4) is the Pareto dominant member of the family of informationally consistent (INC) contracts, which, as Riley showed, is the unique RRE.

The fact that the contracts $[R^1, R^2 \ldots R^{n-1}, \bar{R}^n]$ constitute indeed an RRE can be easily checked.

Let us consider the simple case of 2 types (a similar argument can be easily extended to any number of types). In fig. 6 we show the solution to (P4), (R^1,R^2). Suppose now an entrant offers a pooling contract R^P. R^P is clearly preferred by both types to their original contracts. R^P is a feasible contract if it lies on or to the left of the indifference curve relative to the pool of entrepreneurs,

$\bar{V} = \{(x_1, R_2, R_3)/\pi_1 \; \Sigma p_s^1 u(R_s) + \pi_2 \; \Sigma p_s^2 u(R_s) = \bar{u}\}$, with corresponding $\bar{v}$.

We assume that π_1 and π_2 are such that the pooling contract R^P is in fact viable (Riley (1979) showed that with a continuum of types there is always unraveling of the competitive equilibrium at the lower end point of the types distribution).

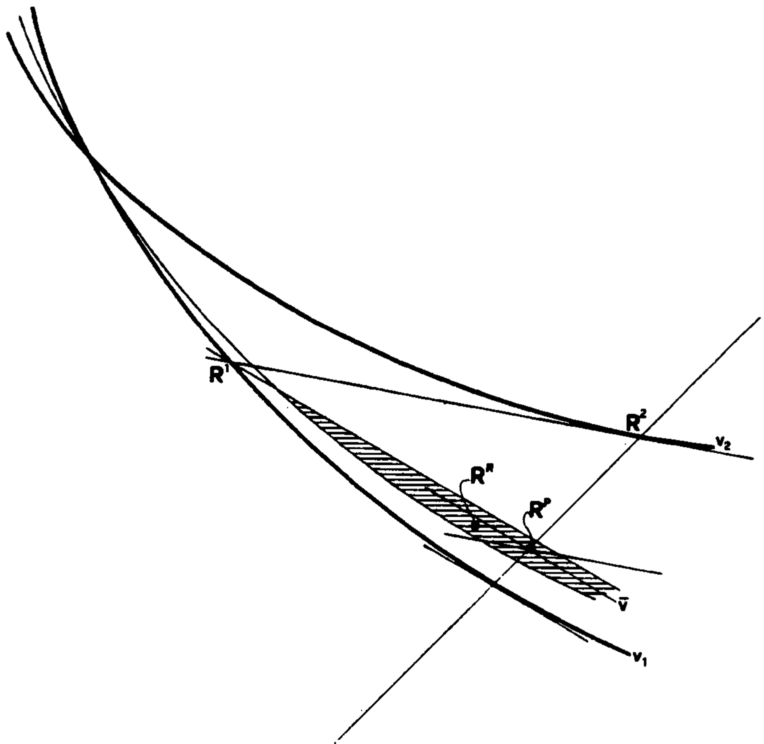

Fig. 6

To show that the original (R^1, R^2) constitute an RRE consider a reactive offer R^R. It is preferred by type 1 to R^P, whereas type 2 strictly prefers R^P. Hence when R^R is offered the entrant offering R^P would only lure the most risky types and would therefore suffer a loss[21] (since R^P is to the left of v_2). On the

21. Note that a similar reactive offer could be devised for each of the feasible pooling contracts that might possibly disturb the original equilibrium (all the points in the shaded area are candidate pooling contracts).

other hand given the possible reactions by other agents, the worst outcome for the reacting agent is to lose all his customers, and this is not worse than his outcome if he did not respond ro R^P.

This therefore confirms that the proposed set of contracts constitute an RRE.

Remark 9. In order to be compared with results presented in section 5, the welfare properties of the Pareto dominant INC contracts are here briefly recorded.

The contract with the most risky type is the same as the contract obtained by that type under full information. The burden of providing incentives to correctly reveal projects riskiness is beared by all the other entrepreneurs, for the investors break-even any way.

The important point to notice is that the solution to (P4) is not constrained Pareto efficient. There are contracts (in the shaded area of fig. 6) that would be preferred by all agents. However those contracts would not survive (reactive) competition. Thus, as Riley points out, the assumption of asymmetric information implies a major departure from the standard competitive model.

6.2 Incentive compatibility and credit rationing

We can now take up explicitly the rationing issue.

What is shown before is that no pooling contract will survive the notion of competition embodied in the RRE.

Thus, an intuitive consequence of accepting the idea of "reactive competition" is that no credit rationing will be found in an RRE. Indeed, credit rationing can be a feature of optimizing behaviour[22] in the face of asymmetric information when the same contract is offered to entrepreneurs belonging to different risk classes, i.e. when there is pooling. If we rule out pooling we rule out credit rationing.

22. See note 16.

Although intuitive, the point is important and it is worth while to be precise. However, it is not quite clear what is the most appropriate way to do so.

A very simple route is to interpret the "zero profit" constraint as to implying an unlimited availability of identical investors, who are indifferent (at the RRE) between partecipating to the market or staying out.

As a result all entrepreneurs will find their financing needs fulfilled at the terms implied by the RRE; the market, in other words, would be cleared and no rationing would show-up.

An interpretation which is perhaps more interesting is the following.

Suppose that each investor is in fact an intermediary that borrows from depositors at a riskless rate r, guaranteed by some form of (government funded) deposit insurance[23].

Our original "zero profit" constraint should now read[24]

$$(ZP') \quad \sum_s p_s^i u(\max[R_s^i-(1+r)\bar{W},\ 0]) = u(0) \quad \forall$$

with the proviso that $u(0) > -\infty$, where the expression max[.,.] formalizes the existence of a deposit insurance.

Given r, we can proceed as before to obtain a set of curves v_i in the space (R_2, R_3) that is analogous to the set drawn, for example, in fig. 3. Given these curves we obtain, duplicating previous arguments, the optimal set of contract corresponding to the given r. The whole procedure can clearly be repeated starting from a different r. We then have that the optimal set of contracts is parametrized by r, as we shall make explicit by writing $R(r) = [R^1(r), \ldots, R^n(r)]$. The larger is r, the larger is, coeteris

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23. We assume the existence of deposit insurance to avoid confronting with the problem of the form of the contract between the intermediary and the depositors.

24. We shall assume that each intermediary can at most obtain the amount $\bar{W}$ as deposits. Alternatively the (ZP) constraints should see that for each type bundle the investor obtains no surplus. This would complicate our analysis without modifying its qualitative conclusions.

paribus, the expected value of each contract needed to guarantee to the investor the same utility reservation level. Graphically, this corresponds to shift each v_i curve to the right.

This implies that the entrepreneurs' expected utility is a decreasing function of r. There will be values of r so high that some of the entrepreneurs will receive from the contract an utility lower than their reservation level. Therefore they would not accept the contract. Let us define, for each r, the set of types that would accept the offered contract, i.e.

$$F(r) = \{i / \Sigma\, p_s^i\, R_s^i(r) \leq k\},$$

where we remember that $\bar{k} = \bar{w} - x$ and w is the entrepreneur's reservation level. Because of what we already said, $F(r) \subseteq F(r')$ if $r' < r$.

Also, let us define the demand for funds as

$$D(r) = N\bar{W} \cdot \sum_{i \varepsilon F(r)} \pi_i$$

where N is the total number of entrepreneurs[25].

Finally, we suppose that there is an upward sloping supply of funds, S(r).

Suppose now that D(r) > S(r), i.e. suppose that, with the funds available at the current riskless rate, not all investors can in fact meet the demand from entrepreneurs accepting the offered contracts.

Thus, there will be investors who bid up r, attracting more deposits at a rate $\hat{r} > r$ and offering a new set of contracts solving (P4) parametrized by $r^1 \geq \hat{r}$. These contracts will be incentive compatible and they will earn the investors an expected utility at least as big as their reservation level.

Moreover the investors offering $\hat{r}$ to depositors would attract all deposits, and all investors will eventually adapt the contracts they offer to the new riskless rate. Competition between

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25. Since we assumed a finite number of types, D(r) is a step function. With a continuum of types D(r) should be accordingly redefined.

investors will then force the offered contracts to be parametrized by $\hat{r}$. At the new values of the contracts offered, there might be less entrepreneurs demanding funds (i.e. $F(\hat{r}) \subset F(r)$); alternatively, it might be the case that, even when the new, more costly contracts are offered, all entrepreneurs are better off accepting the contract than refusing it (i.e. $F(\hat{r}) = F(r)$). In any case, we have

$$D(\hat{r}) \leq D(r) \quad \text{and} \quad S(\hat{r}) > S(r).$$

If it is still the case that $D(\hat{r}) > S(\hat{r})$, the whole process will start again. In this way the riskless rate is driven up to the market clearing level.

A similar argument can be made if $D(r) < S(r)$.

We therefore conclude that, accepting a reactive notion of competition, no equilibrium credit rationing will ever be found in our simple model. Clearly this conclusion might fail to hold in more complicated models, with moral hazard or repeated credit relationships taken into account.

A similar result is obtained by Besanko and Thakor (1987) and by Bester (1987). In those papers, the element that allows the investor to sort out the entrepreneurs is the choice of collateral instead of the state structure of repayment schedule. The latter, as in Stiglitz and Weiss, is assumed to correspond to an SDC.

Stiglitz and Weiss (1987), however, have shown that under plausible circumstances the increase of collateral requirements might also have adverse selection effects and therefore their original motivation for credit rationing would persist. We believe that our result is more robust in that it does not requires the use of a new instrument to generate a sorting contract.

7. Conclusions

The optimal design of contractual arrangements to solve financial problems is a topic of great theoretical interest that, in a financial environment which underwent deep changes and is expected to show even deeper ones, is likely to have interesting policy implications as well.

Recent papers, for example Diamond (1984), Gale and Hellwig (1985), have explored the field under the assumption that financial relationships are plagued by ex-post asymmetric information; i.e. a situation in which entrepreneurs can privately observe the results of their projects whereas the investors cannot. In this setting, and assuming universal risk neutrality, it is shown that the SDC is indeed the optimal one since the incentive-compatibility problem of revealing truthfully the outcome to be shared can be solved by a fixed repayment subject to a bankruptcy clause.

This is a reassuring result, to be contrasted with the well known conclusion that an SDC provides inefficient risk sharing among risk-averse agents,[26] since SDCs are a very widespread contractual form among private agents and, most importantly, since legal restrictions often forbid the banks from writing all but SDCs with their borrowers.

The traditional rationale for such a restriction is that a bank writing debt rather than equity contracts on its assets side is somewhat more likely to face its fixed committments on its liabilities side and is prevented from behaving collusively with the borrower to the detriment of depositors.

These arguments are not entirely convincing, and we are not aware of their rigorous proof. Thus the theoretical result showing the optimality of SDCs is not without practical interest.

This paper follows the same lead of Diamond and Gale and Hellwig papers, considering however the implication of a different informational imperfection: ex-ante asymmetric information. We have assumed a particular asymmetry in risk sharing attitueds that

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26. It must be noted, however, that the optimality of the SDC is obtained assuming universal risk neutrality, i.e. a situation in which risk sharing is immaterial and therefore the drawback of the SDC (in terms of risk sharing) is irrelevant. Relaxing that assumption (Gale and Hellwig (1985), section 4) no longer yields the SDC as the solution of the problem.

is most favourable to SDCs, since it generates the SDC as the optimal one in the full-information problem. Yet, considering ex-ante asymmetric information we have shown that the incentive-compatibility problem would require a contract with state dependent repayments.

In other words, we have shown that restricting to SDCs the choice of financing contracts has the cost of precluding the efficient diffusion of information.

It is not clear whether that cost might tilt the balance against the choice of SDCs. What we believe is clear is that that cost need to be taken into account.

A more integrated analysis of the pros and cons is, in our opinion, badly needed.

A further contribution of this paper is to the long standing quarrel concerning the rationality of credit rationing. We confirm that the issue is sensitive as to whether the financial contract is exogenously given or endogenously determined to cope with the adverse selection problem. We pursue the latter possibility and, exploiting the differences in the state-dependent marginal value of the same contract for different entrepreneurs, we achieve incentive compatibility without resorting to collateral requirements.

Incentive compatibility, together with a notion of competitive equilibrium appropriate to a situation in which the information available is affected by the agent actions (Riley (1979)), allows us to produce a separating equilibrium without rationing.[27]

We stress that this result is obtained without resorting to collateral. It appears thus to be immune to the reply (Stiglitz and Weiss (1987)) that the endogenous determination of collateral to avoid adverse selection could be self-defeating, because of a positive correlation between the willingness to put up more

27. It must be reminded that our analysis does not deals with moral hazard nor with long-term credit relationships. Therefore we have nothing to say about the possibility of rationing arising because of these problems.

collateral and the riskiness of the borrower.

The result that rationing can be avoided if SDCs are substituted by appropriate state-dependent contracts can be taken as a further instance of the informational costs associated with restricting to SDCs the choice of financing contracts.

Appendix

The (LC) characterization of the optimal contract suggests the possibility of using dynamic programming techniques to solve (P3). In fact, given (LC) the problem can be conveniently approached by backward induction.

Because of the characterization given in proposition 2 we can define the admissible region of the contracts as follows

$$AR = \{R^i \in R^3_+ \mid R^i \in BB,\ \Sigma R^i_s p^i_s \leq k,\ R^i \text{ is linear or concave}\}$$

For each contract $R^{i-1} \in AR$ define also

$$AR(R^{i-1}) = \{R^i \in R^3_+ \mid R^i \in AR,\ \Sigma R^i_s p^i_s = \Sigma p^i_s R^{i-1}_s\}$$

The requirement that the optimal contract R^i belongs to the set $AR(R^{i-1})$ for each R^{i-1} belonging to AR reflects the fact that for the optimal contracts (AUIC) is binding.

A more convenient representation of that requirement is the following:

for each $R^{i-1} \in AR$, $R^i \in AR(R^{i-1})$ if

$$R^i_1 = x_1,\ R^i_2 = R^{i-1}_2 + \delta_i \qquad R^i_3 = R^{i-1}_3 - \delta_i \frac{p^i_2}{p^i_3},\quad \delta_i \geq 0$$

Moreover, because of characterization (M) we require

$$(1) \qquad \delta_i \leq (R^{i-1}_3 - R^{i-1}_2) \frac{p^i_3}{p^i_3 + p^i_2} \quad \forall i$$

$$\text{with} \quad R^0_3 = \frac{k - x_1(\lambda p^1_2 + p^1_1)}{p^1_3 + (1-\lambda) p^1_2} \qquad \text{and} \qquad R^0_2 = \lambda x_1 + (1-\lambda) R^0_3$$

This is because if $\delta_i = (R^{i-1}_3 - R^{i-1}_2) \dfrac{p^i_3}{p^i_3 + p^i_2}$ then $R^i_2 = R^i_3$.

Let us now consider contract R^n. We know that it must be an SDC and it must belong to BB.

Given R^{n-1}, this implies that

$$R_1^n = x_1, \quad R_2^n = R_3^n = R_2^{n-1} \frac{p_2^n}{p_2^n+p_3^n} + R_3^{n-1} \frac{p_3^n}{p_2^n+p_3^n} = \bar{\bar{R}}^n$$

This is equivalent to

$$\delta_n^* = (R_3^{n-1}-R_2^{n-1}) \frac{p_3^n}{p_3^n+p_2^n}$$

Define, for each $R^{n-1} \varepsilon$ AR

$$V(R^{n-1}) = \pi_n(p_2^n+p_3^n)\ u(R_2^{n-1}+\delta_n^*).$$

Here and thereafter we shall neglect the terms involving only x_1, which is a constant.

The Bellman equation that is associated with the choice of contract R^{n-1} is then given, for each R^{n-2} ε AR, by

$$V(R^{n-2}) = \max_{\delta_{n-1}} \{\pi_{n-1}[p_2^{n-1}u(R_2^{n-2}+\delta_{n-1}) + p_3^{n-1}u(R_3^{n-2}-\delta_{n-1} \frac{p_2^{n-1}}{p_3^{n-1}})]+V(R^{n-1})\}$$
$$\text{s.t. } (1)$$

From the F.O.Cs we have then

$$(2)\quad \pi_{n-1}p_2^{n-1}[u'(R_2^{n-2}+\delta_{n-1})-u'(R_3^{n-2}-\delta_{n-1} \frac{p_2^{n-1}}{p_3^{n-1}})] +$$

$$+ \pi_n[u'(R_2^{n-1}+\delta_n^*)\cdot(p_2^n+p_3^n)\gamma] \leq 0 \ (= 0 \text{ if } \delta_{n-1}>0)$$

where $\gamma = \frac{1}{p_2^n+p_3^n} (p_2^n - \frac{p_3^n\ p_2^{n-1}}{p_3^{n-1}}) < 0$ and the expression of $R_2^{n-1}+\delta_n^*$ in terms of R^{n-2} and δ_{n-1} is given by

$$R_2^{n-1} + \delta_n^* = \frac{p_2^n}{p_2^n+p_3^n}(R_2^{n-2}+\delta_{n-1}) + \frac{p_3^n}{p_2^n+p_3^n}(R_3^{n-2}-\delta_{n-1}\cdot\frac{p_2^{n-1}}{p_3^{n-1}})$$

The first expression in (2) is positive as long as (1) is slack. The second is clearly negative.

If (1) is binding, the first is zero and (2) can be satisfied only if either π_n is zero or $R_2^{n-2} = R_3^{n-2}$, so that (1) and the non negativity constraint on δ_{n-1} together imply $\delta_{n-1} = 0$.

Therefore R^{n-1} is an SDC if either R^{n-2} is an SDC or no riskier type exists.

For each R^{n-2} define $\delta(R^{n-2})$ as the function that solves (2). Clearly

$$V(R^{n-2}) = \pi_{n-1}\{p_2^{n-1}u(R_2^{n-2}+\delta(R^{n-2})) + p_3^{n-1}u(R_3^{n-2}-\delta(R^{n-2})\frac{p_2^{n-1}}{p_3^{n-1}})\} +$$

$$+ \pi_n(p_2^n+p_3^n)u(R_2^{n-1}+\delta_n^*).$$

It is convenient, for given R^{n-3}, to redefine $\delta(\cdot)$ as $\delta(\delta_{n-2})$, where δ_{n-2} is defined by

$$R_2^{n-2} = R_2^{n-3} + \delta_{n-2}.$$

The Bellman equation relative to R^{n-2} now becomes, for each $R^{n-3} \in AR$

$$V(R^{n-3}) = \max_{\delta_{n-2}} \{\pi_{n-2}\{p_2^{n-2}u(R_2^{n-3}+\delta_{n-2}) + p_3^{n-2}u(R_3^{n-3}-\delta_{n-2}\frac{p_2^{n-2}}{p_3^{n-2}})\} +$$

$$+ V(R^{n-2})\}$$

subject to (1).

From the F.O.Cs we have

$$(3)\ \pi_{n-2}p_2^{n-2}\ [u'(R_2^{n-2}) - u'(R_3^{n-2})] + \pi_{n-1}\ \cdot$$

$$\{p_2^{n-1}u'(R_2^{n-2}+\delta(\delta_{n-2}))(1+\delta'(\cdot)) - p_3^{n-1}u'(R_3^{n-2}-\delta(\cdot)\frac{p_2^{n-1}}{p_3^{n-1}})(\frac{p_2^{n-2}}{p_3^{n-2}} + \delta'(\cdot)\frac{p_2^{n-1}}{p_3^{n-1}})\}$$

$$+\pi_n\{(p_2^n+p_3^n)u'(\bar{R}^n)[\frac{p_2^n}{p_2^n+p_3^n}(1+\delta'(\cdot)) - \frac{p_3^n}{p_2^n+p_3^n}(\frac{p_2^{n-2}}{p_3^{n-2}} + \delta'(\cdot)\frac{p_2^{n-1}}{p_3^{n-1}})]\} \leq 0$$

$$(= 0 \text{ if } \delta_{n-2} > 0)$$

Using (2) it is immediate to verify that (3) cannot be satisfied unless either π_n and π_{n-1} are zero or $R_2^{n-3} = R_3^{n-3}$.

Therefore R^{n-2} is an SDC if either R^{n-3} is an SDC or no riskier type exists.

This conclusion can be generalized by backward induction and we can state the following corollary to proposition 2:

Corollary

In the set of optimal ICSC, as long as there are at least two types, the only SDC is the contract written with the riskiest type.

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