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of financial intermediaries**

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Summary

Why do people lend to banks rather than to final borrowers? This paper provides a rationale for the preference to indirect lending (writing a deposit contract with a bank) over direct financing of individual entrepreneurs based on the presence of ex-ante asymmetric information. Ex-ante asymmetric information might create a "lemon" problem and therefore produce a need for screening, which in turn makes delegation profitable. The intermediary is then characterized as an agent performing a delegated screening role. However, delegation gives rise to a standard agency problem. This is solved, in a sequential context, by the incentives provided by reputation.

1. Introduction¹

People are often unwilling to lend to entrepreneurs who need to finance risky projects. Yet the same people willingly lend to banks that then use the money borrowed to finance the same entrepreneurs.

Why should somebody refuse to lend to a second person but agree on lending to a third "person" that, in turn, will lend to the second one?

This apparently strange behaviour, far from being evidence of irrationality, points to the crucial role of financial intermediaries.

Indeed, there are a number of conditions under which the direct transfer of funds generated by individual savers to entrepreneurs who need to finance real investment might be vastly inefficient. Financial intermediaries are then to be interpreted as a more efficient, alternative way to channel savings into investments.

In this paper we provide a simple reason for the preference of indirect over direct lending on the basis of a specific form of asymmetry in information: namely, that entrepreneurs know the return distribution of the projects better than do potential investors (ex-ante asymmetric information).

Given this asymmetry, the entrepreneurs have an incentive to misrepresent the risk of the projects they want to finance, offering the investors a contract that might be a "lemon".

The distortion induced by the "lemon" problem might be more or less severe according to the actual distribution of project risk, with higher proportions of risky projects increasing the severity of the problem. One may theoretically conceive of circumstances under which no investment at all would take place,

1. The author thanks G. Galli, C. Giannini, M. Messori and D. Pyle for very useful comments. The present version of the paper has greatly benefited from detailed comments by G. Ferri.

due to the risk of financing a "lemon". And even in less extreme cases the exchange of financial assets that would result could be highly inefficient, with many potential investors discouraged and many relatively safe investments not financed.

As an alternative, the asymmetry could be removed through a screening process, with each single investor collecting information on the project to be financed. However, this involves a massive duplication of effort in information gathering; it is this element that opens the stage for a financial intermediary that avoids duplication, playing a delegated screening role. Yet delegation only shifts the "lemon" problem a step away, for now the intermediary has an incentive of its own to misrepresent the riskiness of the projects in its portfolio.

Clearly, if each agent had to pay the information cost of determining the riskiness of the intermediary's assets there would be no reason for the intermediary to exist in the first place. Also, any state-dependent contract with the intermediary would engender the same distortions of direct investment, so that again there would be no reason to contract with the intermediary. Investors will give their funds to the intermediary only if it offers them a "cheating-free", state-independent deposit contract.

Ruling out perfect diversification, however, a riskless deposit contract is costly for the intermediary in terms of forgone expected return. In other words, a deposit contract is not generally self-enforcing, since the intermediary would prefer to keep a lower level of reserves than that required to guarantee the promised repayment in every state². A solution based on the incentives provided by reputation, formalizing the problem as a repeated game, is analyzed.

Following Kreps and Wilson (1982) and Milgrom and Roberts

2. In general, the intermediary can guarantee a riskless deposit only if its reserves are equal to the difference between the promised return and the lowest possible return on the investment. Clearly if the latter were zero only a 100% reserve would guarantee the deposit (aside from interest payment). In what follows it is assumed that the lowest possible return on the risky investment is strictly greater than zero.

(1982), it can be shown that there exists a sequential equilibrium in which the optimal strategy for the savers is (often) to deposit and the optimal strategy for the intermediary is (often) to keep enough reserves to guarantee in all states the promised return on deposits.

This sequential equilibrium captures the idea that in a long-term credit relationship previous performance can substitute for the actual evaluation of the riskiness of the bank's assets, which is the ultimate determinant of the riskiness of the bank's liabilities.

Reputation solves the agency problem posed by delegated screening and therefore makes it possible for the intermediary actually to perform its role. But reputation is costly, for the intermediary is often forced to choose an amount of reserves larger than that resulting from a static optimization. Following Diamond (1984) we shall call the utility loss due to this constraint the delegation cost. Because of the presence of a delegation cost, we are not in general guaranteed that one of the agents would in fact choose to act as intermediary.

It can be shown, however, that the delegation cost is decreasing with diversification.

This implies that there exists a level of diversification such that acting as intermediary dominates direct investment.

In Section 2.1 the set-up will be introduced. In Section 2.2, problems of adverse selection or incentive compatibility will be shown to arise under asymmetric information. In Section 3 it will be argued that one of the agents could be delegated to screen the risky projects, avoiding duplication in information gathering. This however would give rise to incentive compatibility problems akin to those arising between entrepreneurs and savers. In Section 4 a state independent (deposit) contract that solves these problems will be shown to be supported by "reputation" in a repeated game, albeit one that is only informally related to the original set-up. In Section 5 the relationship between delegation cost and diversification will be analyzed. Section 6 gives references to the literature and a few concluding observations.

2. Contracting without an intermediary

2.1 The set-up

Suppose there are n types of risk-neutral entrepreneurs, indexed by $i = 1 \dots n$. Each type is endowed with a private investment opportunity, or project, that requires an initial investment of $(\bar{W}-c)$ and yields, at the end of the period, a return represented by a random variable X_i , with distribution function $F_i(\cdot)$.

Let the support of all random variables be the same, $[X_1 \dots X_m]$. The projects are totally ordered by the criterion of mean preserving spread and project i is riskier than project j if $i > j$.

There is also a large number of risk-averse savers (also referred to as investors), each endowed with initial wealth \bar{W} . Both sets of agents are interested in maximizing the expected utility of final wealth.

We assume that ex-ante asymmetric information prevails, so that:

(AI) entrepreneurs know their type whereas savers cannot distinguish among projects, unless they pay a fixed cost c . In this case they know the type of each project.

2.2 The "lemon" problem

It can be shown that if we had assumed complete information rather than (AI) the standard debt contract (SDC) would be the

optimal revenue sharing contract in the set-up considered³. Under (AI), however, there is no reason to believe that this result would continue to hold. Indeed we want to show that if the sharing contract is constrained to be an SDC there will be incentives for entrepreneurs to misrepresent the riskiness of their projects.

If the sharing contract is an SDC, we can associate with project i the value \bar{R}_i , which is the fixed repayment in case of solvency characterizing the optimal contract under complete information.

As usual in contract problems, characterized by utility taking rather than price taking behaviour, the solution depends on the utility reservation level of one of the agents which is taken as a constraint by the other.

The extremes of the contract curve for an SDC correspond to the two polar cases:

(a) the utility of the entrepreneur is maximized subject to the constraint that the utility of the saver is equal to the utility of his initial wealth \bar{W} .

(b) the utility of the saver is maximized subject to the constraint that the utility of the entrepreneur is equal to the leisure forgone when the entrepreneur embarks upon his project⁴, say \bar{U} .

In both cases we have the following result:

Provided that the probability of insolvency in the optimal contract is strictly positive, the optimal fixed repayment \bar{R} is increasing with riskiness.

3. This result is crucially dependent on the assumed asymmetry in the attitude towards risk. It might be useful to recall that an SDC is characterized by a fixed repayment to the lender unless the borrower is declared bankrupt. In this case the SDC requires that the lender recovers as much as possible from the proceeds of the borrower's investment.

4. If we had allowed the entrepreneur to have some initial wealth his reservation utility level would be given by the maximum utility he can achieve without external financing.

Proof: see Appendix A.

The implication of this result is that since the utility of the entrepreneur is decreasing with \bar{R} , and since $\bar{R}_j > \bar{R}_1 \quad \forall j > 1$, each entrepreneur will pretend to be of type 1 irrespectively of his real type.

Under (a), it then immediately follows that no SDC would be written, because of the danger of a "lemon". Indeed, if we let the savers have a prior probability distribution over the set of projects, $\pi_1 \dots \pi_n$, where π_i is the probability that a project is of type i , we find that the expected utility from accepting the contract is

$$\sum_{i=1}^n \pi_i V(\bar{R}_1, F_i) < U(\bar{W}) \quad \text{whenever } \pi_1 < 1 \quad \text{since}$$

$$V(\bar{R}_1, F_i) < U(\bar{W}) \quad \forall i > 1$$

Under (b) the consequences of the misrepresentation of riskiness might be less dramatic. The equation that defines the optimal \bar{R} is now

$$(i) \quad \int_{\bar{R}_i}^X (X - \bar{R}_i) dF_i(X) = \bar{U} \quad \text{where } \bar{U} \text{ is exogenous.}$$

We can assume that $V(\bar{R}_i, F_i) \geq U(\bar{W}) \quad \forall i$, since $V(\cdot)$ is maximized subject to the constraint given by (i). But no general conclusion can be reached concerning the sign of $V(\bar{R}_1, F_1) - U(\bar{W})$ when $i > 1$, i.e. when misrepresentation does take place. Indeed, note that if there exists a j , say j_0 , such that⁵

5. To be sure, we could slightly modify the original set-up introducing among the possible projects a true "lemon", i.e. a project which would not be financed even if its true type were known with certainty. This would guarantee that there exists an i such that $\bar{V}(R_1, F_i) < \bar{U}(W)$.

$V(\bar{R}_1, F_{j_0}) \leq U(\bar{W})$, then $V(\bar{R}_1, F_j) < U(\bar{W}) \forall j > j_0$.

Under this assumption the expected utility from accepting the contract would then be smaller than $U(\bar{W})$ if the probability weights $\pi_{j_0} \dots \pi_n$ were large enough.

The inefficiency implied by (AI) shows most dramatically in situations where no SDC would be written. However, the inefficiency is present even when the contract is in fact viable. On the one hand, all the surplus from misrepresenting the riskiness is extracted by the riskiest projects, and this, in a dynamic perspective, could drive the safest ones out of business; on the other hand, possible differences in the utility functions and in the subjective probabilities π_i could make it unprofitable for some savers to accept the sub-optimal contract, thus reducing the flow of resources which could finance profitable investment.

More generally, the inefficiency of writing an SDC under (AI) raises the issue of whether a different contractual form could satisfy an appropriately defined incentive compatibility constraint. We examined this problem elsewhere (Terlizzese (1988)), characterizing the form of the optimal incentive compatible sharing contract and showing that, in general, that contract would allow the investor to classify the entrepreneurs into as many risk classes as there are types. What is of present interest, however, is the general conclusion that incentive compatibility is costly to achieve. In the case examined we show that revelation of type riskiness is paid in terms of sub-optimal risk sharing. More generally, given (AI) there will be a welfare loss associated with any contract, even with the optimal one.

3. Contracting with an intermediary

One possible alternative is to pay the fixed cost of information c , remove (AI) and then write an optimal sharing contract freed from incentive compatibility constraints.

This solution is preferred if the utility of investing $\bar{W}-c$

in a first-best way is greater than that of investing $\bar{W}-c$ in a second-best way and storing c (we assume that there exists a riskless storing technology). This is not likely to be the case if c is large with respect to \bar{W} . However two savers pooling their wealth would be in a better position since the information cost could be shared between the two. Equivalently, a saver with initial wealth equal to $2\bar{W}$ could invest in two projects reducing to $c/2$ the per-project information cost. It is then clear that for large enough initial wealth the optimal contract under symmetric information, achieved by paying the cost c , dominates the sub-optimal contract under (AI). In other words, under (AI) a risky financial investment is characterized by scale economies.

This in turn suggests the possibility of exploiting such scale economies by delegating to one of the agents - the intermediary - the task of assessing the riskiness of the projects, thus avoiding the duplication of costly information gathering.

To be sure, scale economies have often been invoked to justify the role and nature of intermediaries. Here, however, scale economies are not simply assumed: they result endogenously from assumption (AI). More interestingly, we are forced to consider whether the solution of delegating the screening of the projects to one of the agents is a viable one - a step that, as Diamond (1984) correctly points out, is often forgotten when ad hoc scale economies are assumed.

In principle a large enough intermediary can offer a contract to individual savers that dominates the second-best direct investment under (AI), since the per-project cost of screening is monotonically decreasing with size.

The delegated screening role nonetheless poses a standard agency problem since the riskiness of the projects, which is the key determinant of the riskiness of the contract between the intermediary and the savers, is the private knowledge of the intermediary. The intermediary thus has an incentive to misrepresent project riskiness in order to obtain a more favourable contract with savers. In other words, delegation merely

shifts the problem of incentive compatibility to the level of relationship between intermediary and savers.

Indeed, suppose that the savers write a state-dependent contract with the intermediary based on the "average riskiness" of project types. The intermediary would then find profitable to actively select entrepreneurs riskier than the average, since the higher risk would be borne by savers.

Any state-dependent contract with the intermediary would present the same problems of inefficiency as individual contracts written under (AI). Moreover, the problem cannot be solved by removing (AI), i.e. paying the cost of information, because in this case there would be no reason to give the money to the intermediary in the first place. As a result, the intermediary can hope to receive savers' wealth only by offering them a state-independent contract, i.e. a deposit contract.

Ruling out perfect diversification, in order for the intermediary to guarantee a fixed repayment in all states, it must be endowed with initial wealth, net of the payment of the fixed cost of information, sufficiently high to cover the difference between the promised payment and the lowest possible return on the project⁶.

The trouble with this solution is that money kept as reserve has a lower expected return than money invested in risky projects. The optimal strategy of the intermediary, therefore, is to offer riskless deposit contracts, collect the funds and then invest in the risky projects the fraction of total available capital (deposits + intermediary initial wealth) which maximizes its own expected utility. This fraction in general will be larger than is needed to guarantee the offered deposit contract in all states. In other words, the intermediary has, ex-post, an incentive to

6. The existence of a collateral would reduce the amount of reserves that the bank would need to hold, and would therefore alleviate the problem. In what follows it is implicitly assumed that the collateral is not enough, i.e. that, even by liquidating the collateral, if the intermediary did not keep enough reserves, it would not be able to comply with its commitment in the event of a "bad" state occurring.

"cheat" by taking a position more risky than would be necessary to comply with its ex-ante commitments.

A possible solution to this incentive problem would be to assume that the bank's choice of reserves is observable.

A second solution relies on diversification. High degrees of diversification, while would not reduce the incentive to "cheat", would decrease the probability of the events in which misrepresentation of riskiness is harmful, i.e. the events in which the bank cannot comply with its ex-ante commitment with depositors. Provided that that probability is small enough the expected loss due to bank cheating would be offset by the saving in information costs and a deposit contract would be viable.

While there is something to both these solutions to the agency problem, we think that the informational requirements they impose on savers are somewhat too high. This is particularly disturbing since the theory we are proposing is essentially based on the role of delegated information gathering when information gathering is costly. It would sound a bit odd to assume that the savers do not know the riskiness of intermediary projects and then let them free to observe the reserves chosen by the bank or its degree of diversification. For this reason we want to explore a different solution based on the incentives provided by reputation.

The informational content of reputation is in principle limited to the observation of past performance and this seems to represent more closely what actual savers do in fact know about their banks. We will not manage, however, to keep up with such a low informational requirement. This would probably require the analysis of a dynamic game in which the choice of bank reserves and the degree of diversification are explicitly modelled and the size of the bank is one of the state variables summarizing previous choices. In the following we shall only analyze a repeated game (which we call the credit game), neglecting the dynamic role of stocks (wealth of the bank, reserves). As a result we shall assume that the probability of a bank cheat resulting in depositors losses is given and known. This can only be justified as an intermediate step towards a more thorough understanding of

reputation phenomena in credit markets.

4. The credit game

The analysis of the credit game follows closely the seminal work by Kreps and Wilson (1982) and Milgrom and Roberts (1982), as will become apparent in the sequel. The payoff and the probability structure of the game are chosen ad hoc and only loosely relate to the set-up assumed up to now. This is not to say that the game is inconsistent with the previous analysis, but only that we do not carefully explore the rigorous derivation of one from the other. To establish some convenient notation, let us refer to the intermediary as agent B (bank) and to the depositors as agent DE. DE can choose whether to deposit his money with the bank (choice D) or not to deposit (ND). If DE plays D, B can choose whether to play "risky" (R), i.e. keep reserves lower than is needed to guarantee the promised return with certainty, or to play "safe" (S), keeping enough reserves. Suppose that the structure of the payoff is the following:

		Payoff		
		B	DE	
(I) if DE plays D	{	B plays R	a	b-rq
		B plays S	1	b
if DE plays ND		0	0	

where $b > 0$, $a > 1$, $r = b-f$, q is the probability that, given that the bank plays R, it is not able to repay DE, $f < 0$ is the expected utility of DE in this case, b and r are chosen so that $b-rq < 0$. Note that if $q < \frac{b}{r}$, the expected loss due to bank "cheating" (i.e. playing risky when it promised a safe deposit contract) is so small that the savers are willing to take the risk. By

assumption we rule out this case.

The only Nash equilibrium of this game corresponds to the last row of the payoff table, where DE does not deposit and the bank is therefore not active. In other words, the gain from delegated monitoring cannot be appropriated by the bank, whose role therefore fails to be explained.

It nevertheless seems appealing to conjecture that repetitions of the credit game would give rise to a reputation phenomenon. The bank would play S in the early stages in order to convince later agents to deposit, and this is known to DE, who will therefore deposit. To put it differently, the bank would play safe in order to acquire a reputation for safety.

It is well known (the point was first made by Selten (1978)) that when the number of repetitions is finite, the argument is not watertight. Indeed, the so called Selten paradox can easily be adapted to this case. In the last stage there is no more reputation to be gained from playing safe; therefore the equilibrium of the game will be the Nash equilibrium (N.E.) described before, regardless of what the reputation was in the penultimate stage. But now consider the second-last stage. Since nothing that could happen in that stage could modify the outcome in the last stage, once again there is no point in gaining reputation (playing S), and the game would again settle on the N.E. The induction carries on backwards to the first stage, and in the only (perfect) equilibrium the bank would never be active.

On the other hand, if the horizon is infinite, we run into well known problems of indeterminacy: almost anything can be an equilibrium, supported by the appropriate trigger strategy.

Yet the existence of long-term relationships seems to be a salient feature of credit markets and reputation is indeed a matter of major concern for the banking system. We therefore feel it fruitful to pursue this line of argument, trying to solve the Selten paradox by mimicking the solution provided by Kreps and Wilson.

We introduce an incompleteness in information, so that the depositors do not know the true bank payoff, being uncertain

between (I) and the following:

		Payoff
		B
(II)	DE plays D	$\left\{ \begin{array}{l} \text{B plays R} \\ \text{B plays S} \end{array} \right.$
		$\left. \begin{array}{l} 1 \\ a \end{array} \right\}$
	DE plays ND	0

We will refer to the bank with payoff (I) as the risky bank, B^R , and to the bank with payoff (II) as the safe bank, B^S .

Obviously B^S would always play S if DE plays D.

As usual, we will index time in a backward fashion, beginning with stage N (when there are N repetitions of the game to be played) and finishing with stage 1.

We envisage the game as being played by N large cohorts of identical depositors⁷ assuming that each depositor is informed on whether the bank complied with its commitments in previous stages of the game. To be precise, we assume that each cohort is composed by a continuum of depositors, uniformly distributed in [0,1]. Therefore the bank payoffs should be interpreted as integrals over [0,1] of per-depositor payoff.

Suppose that each depositor at stage N assigns a positive - possibly very small - probability $(1-\delta)$ to the bank being of the safe type. We can interpret this probability assessment as the reputation for safety of the bank.

A sequential equilibrium for the repeated game (see Kreps and Wilson (1982a) for general definitions and properties of the sequential equilibrium) is:

7. The assumption that at each stage of the game new depositors enter is restrictive, since it rules out the possibility of the depositors wealth to vary over time. As with the problem raised by the wealth of the bank, already mentioned, this suggests the opportunity of using a dynamic rather than repeated game. We leave this for future work.

- (i) a strategy for each player and
- (ii) for each stage $n=N, N-1, \dots, 1$, a function p_n which maps histories of moves up to stage n into $(0,1)$ (call this function a probability revision function)

such that:

- (a) starting from every point in the game where it is the bank's turn to move, the bank strategy is the best response to the depositor strategy
- (b) for each n , the depositor strategy is the best response to the bank strategy, given that the bank is of the risky type B^R with probability p_n
- (c) the game starts with $p_N = \delta$
- (d) each p_n is computed from p_{n+1} and the knowledge of the bank strategy, using Bayes' theorem whenever it applies.

Let us provisionally agree on the following intuitive requirement for the probability revision function: if at some stage the depositor observes that the bank is not able to repay, he will be certain that the bank is of the risky type for all subsequent stages of the game.

Therefore if a risky bank gives itself away, revealing that it is indeed of the risky type, its payoff for all later stages will be zero. Suppose now that for some integer h

$$(1) \quad a(1-q)^h > 1$$

Then the risky bank plays risky from stage h until the end of the game. Indeed, compare the strategy of playing R always with the strategy of playing S at stage h and reverting to play R for all

subsequent stages.

If the bank plays R its current payoff is a but, with probability q , its type is revealed and its payoff will be zero up to the end of the game. Thus the bank will be allowed to play in the next stage only with probability $(1-q)$. At that time playing R would again yield a together with the probability $(1-q)$ of continuing the game. The expected payoff of the first strategy (always R) is then

$$(2) \quad a + (1-q)a + (1-q)^2a + \dots + (1-q)^{h-1}a$$

Following the same argument the expected payoff of the second strategy (S now and R for the rest of the game) is

$$(3) \quad 1 + a + (1-q)a + \dots + (1-q)^{h-1}a$$

Therefore if (1) is satisfied the first strategy is better than the second. But the argument can be repeated at stage $h-1$ and again, under (1), we conclude that playing R always is better than playing S at that stage and reverting to R the next one. This clearly leads to the initially proposed interpretation of (1).

Let us now denote by h the largest integer such that (1) is satisfied.

We consider the following probability revision function and strategies.

Probability revision:

(i) if at stage $n+1$ no deposit occurs then $p_n = p_{n+1}$

(ii) if at stage $n+1$ either DE plays D and the bank fails to repay or $p_{n+1} = 1$, then $p_n = 1$

For stage $h+j$, $j=1, \dots, N-h$

(iii) if at stage $h+j+1$ DE plays D, the bank is able to repay and $P_{h+j+1} < 1$, then $P_{h+j} = \min(P_{h+j+1}, \beta\mu_j)$, where μ_j follows $\mu_1 = 1$, $\mu_j = \mu_{j-1}(1-q\beta) + q$, $\beta = \frac{b}{rq}$

For stages $n = h, h-1, \dots, 1$

(iv) if at stage $n+1$ DE plays D, the bank is able to repay and $P_{n+1} < 1$, then $P_n = \frac{(1-q) P_{n+1}}{1-q P_{n+1}}$

Bank strategy:

if the bank is of the safe type, it plays S if DE plays D

if the bank is of the risky type, and DE plays D;

At stage $h+j$, $j=2, \dots, N-h$

play S if $P_{h+j} < \beta\mu_{j-1}$

play R with probability $\frac{P_{h+j} - \beta\mu_{j-1}}{P_{h+j}q(1-\beta\mu_{j-1})}$ if $P_{h+j} \geq \beta\mu_{j-1}$

At stages $h+1, h, \dots, 1$

play R with certainty

Depositor strategy:

At stage $h+j$, $j=2 \dots N-h$

play D if $P_{h+j} < \beta\mu_j$

play ND if $P_{h+j} > \beta\mu_j$

randomize playing D with probability $\frac{a-1}{a-1+q}$, if $P_{h+j} = \beta\mu_j$

At stage $h+1$

the same strategy if $P_{h+1} \neq \beta\mu_1 = \beta$

randomize, playing D with probability $\frac{a(1-q)^{h-1}}{a[(1-q)^h - (1-q)^{h+1}]}$
if $p_{h+1} = \beta$

At stage $n = h, h-1 \dots 1$
play D if $p_n \leq \beta$
play ND if $p_n > \beta$

We then have the result:

The probability revision function and the strategies proposed above constitute a sequential equilibrium for the credit game

Proof: see Appendix B.

Remark 1

The credit game we have analysed is somewhat more complex than the entry game discussed by Kreps and Wilson.

In the entry game the monopolist's choice of his best response to entries in the stage game (i.e. share the market) would reveal with certainty that he is not of the "strong" type.

In the credit game, even if the risky bank acts in accordance with its best short term response to D (i.e. R), it is still possible that the depositor might not find out its true type. In the language of game theory the credit game is characterized not only by incomplete information but also by imperfect information.

Remark 2

The sequential equilibrium concept has recently been questioned by Kohlberg and Mertens (1986). Their critique points to the possibility of unsatisfactory sequential equilibria. Sequential equilibrium would not be a strong enough concept to rule out all strategically unstable

equilibria. In spite of this critique we find that the sequential equilibrium concept is a useful one, precisely because it allows an explicit, albeit not formal, analysis of the "plausibility" of beliefs. It is this analysis which would eventually eliminate strategically unstable equilibria (see Kreps and Wilson 1982a, section 8). Moreover, we do not claim that the equilibrium found for the credit game is unique. On the other hand we do claim that the beliefs which are involved in our equilibrium are "plausible", and that the equilibrium itself is strategically stable in the sense of Kohlberg and Mertens.

An intuitive explanation of the optimal strategies and probability revision function is difficult because of the simultaneous nature of the problem: at each stage the strategies are optimal given the probabilities assigned to bank types, which in turn are consistent, via Bayes theorem, with the optimal strategies. However, it is perhaps useful to give a very loose description of a likely development of the game along the sequential equilibrium path.

Provided that the horizon is long enough, the game starts with $p_N < \beta\mu_N$. DE plays D since he knows that even B^R would play S for sure in order to gain reputation. However p_N will not be updated, precisely because both banks would play S and no information is obtained by observing that the repayment occurs. At some later stage n , when $\beta\mu_n \leq p_N$, B^R would start playing R with positive probability. Knowing this, DE would update p_N according to Bayes theorem, and the probability would follow the dynamics of $\beta\mu_n$. This would continue until stage h , with DE and B^R both randomizing. From stage h until the end of the game B^R plays R for sure whereas DE is indifferent between D and ND. In this case we have assumed that DE chooses to play D.

The existence of the sequential equilibrium of the credit game shows that a solution to the agency problem posed by delegated screening can be provided by the incentives of reputation and therefore, that the informational problems of

evaluating the riskiness of the projects can be efficiently approached through financial intermediation. Indeed, the information costs of screening the projects are minimized (avoiding duplications) and the problem of screening the intermediary is substituted by the simpler problem of taking records of its past performance.

It should be noted, however, that the same mechanism could operate directly between entrepreneurs and savers. We did not consider this possibility, implicitly assuming that the time horizon of individual entrepreneurs is too short (e.g. they only live one period) or that they have not enough initial wealth. One of the implications of our theory is that an entrepreneur who is believed to have a sufficiently long time horizon and enough initial wealth (in U.S. terms a AAA rated entrepreneur) could issue liabilities ex-ante evaluated as riskless. A second implication is that bank loans should be demanded by low wealth, short horizon entrepreneurs.

Both implications find some supporting evidence, though the theory cannot explain why high wealth, long horizon entrepreneurs also demand bank loans. One possible explanation, not inconsistent with our theory, is that a loan, which we know implies a screening process, could be a reputation-enhancing signal.

A second caveat in interpreting to sequential equilibrium existency result concerns the absence of a central bank in our setting.

The "private production" of reputation that emerges in the credit game should not be overstated. Indeed, we have considered a single bank. Turning to a banking system is likely to make things much worse, since reputation is an asset with tremendous externalities. The stability of an unregulated banking system therefore deserves much more work. We simply note that the emphasis that our analysis puts on reputation appears to be consistent with the stringent "moral requisites" imposed by regulatory authorities on potential entrants into the banking business. Moreover, required capital ratios, reserve requirements and supervisory controls could all be interpreted as signals

directed to sustain reputation.

5. Diversification and delegation cost

In deriving the equilibrium of the credit game we assumed that the probability of the bank not being able to face its commitments when it plays R (i.e. q) is exogenously given and common knowledge.

This, as we pointed out earlier, is rather unsatisfactory since q is itself the result of bank decisions concerning both the level of reserves and the degree of diversification and these should be properly endogenized. This will not be done here so as to keep the exposition within reasonable bounds of complexity and length. The hope is that the insight gained with a simpler approach would survive a more thorough analysis. We shall however performe a sort of sensitivity experiment, assuming that the bank can in fact signal in a credible way any chosen q in a given interval, and exploring the consequences of varying q on the equilibrium found for the credit game.

We interpret this as a sensitivity experiment concerning either the degree of diversification or the amount of reserves (or both), since greater diversification would reduce the probability weight on the right tail of the bank assets distribution and larger reserves would reduce the number of project realizations that would engender insolvency problems. In both cases the result would be a smaller q .

The results of such an experiment are relevant for the analysis of what Diamond (1984) called the delegation cost. Indeed we showed how reputation can solve the agency problem raised by delegating to the bank the screening of the projects. However, in order to gain reputation the bank is forced to choose, on a number of occasions, a level of reserves larger than the level that it would have chosen otherwise. We define the delegation cost as the utility loss due to this constraint. With an abuse of terminology, let us call optimal the level of reserves chosen in a static (one

shot) game. It is clear that the delegation cost varies monotonically with the expected number of occasions on which reserves greater than optimal are chosen. We can then establish the result:

Provided that a is chosen large enough the delegation cost is a non decreasing function of q .

Proof: see Appendix C.

Note that this result is not immediately obvious. The smaller is q , the safer is the contract with the bank. This implies that the depositors are more inclined to accept that the bank plays R since the associated expected loss is small. On the other hand, the smaller is q the less informative is a good outcome in concluding whether the bank is of the risky type. What is proved is that the first effect dominates if a is large, i.e. if the payoff of playing R for the risky bank is large. Intuitively, this is because a larger payoff would increase the probability of playing R for the risky bank and would thus restore the informational content of observing a deposit repayment.

The result is interesting because it suggests that the costs of solving the agency problem posed by delegated screening can be made small by a well diversified intermediary.

6. Conclusions

In this paper we attempted to provide a rationale for the preference to indirect lending (writing a deposit contract with a bank) over direct financing of individual entrepreneurs. Our explanation is based on the presence of ex-ante asymmetric information. We have shown that ex-ante asymmetric information might create a lemon problem and therefore produce a need for screening, which in turn makes delegation profitable. The intermediary is then characterized as an agent performing a

delegated screening role. This is, however, not enough, since delegation gives rise to a standard agency problem. We have shown that this can be solved, in a sequential context, by the incentives provided by reputation.

The contents and the approach of this paper are related to two somewhat different lines of research. On the one hand there are alternative explanations for a possible preference to indirect lending. On the other hand there is ongoing research on the consequences of asymmetric information (both ex-ante and ex-post) on credit markets.

A different rationale for indirect lending has been recently analyzed by Diamond and Dybvig (1983). They have analyzed a situation characterized by uncertainty concerning intertemporal preferences. This enables them to make precise the claim that funds generated by agents who care for "liquidity" might not be channelled to agents who need to finance illiquid investments. They have shown that the efficient allocation of resources requires the development of an insurance market for the risk of "liquidity shocks" and that well diversified banks can provide such insurance. Moreover, they have also shown the multiplicity of equilibria resulting from this solution with the related possibility of bank runs.

Ex-ante asymmetric information in the analysis of financial intermediation is stressed in Leland and Pyle (1977) and in Boyd and Prescott (1986).

However, Leland and Pyle do not really address the question of what sort of service is provided by the intermediaries. In Boyd and Prescott intermediaries are seen as a large coalition of agents, and no attention is paid to the form of the contract written between participants in the coalition.

An analysis which focuses on information reliability and reputation, albeit in a single period framework, and which is therefore similar in spirit to our paper, can be found in Ramakrishnan and Thakor (1984).

A second line of research centers on the asymmetry of information concerning the realized outcome of a given project

(ex-post asymmetric information). Gale and Hellwig (1985) and Diamond (1984) have shown that in this situation a standard debt contract is the optimal incentive compatible contract among risk neutral agents. The bankruptcy clause in the SDC, which is required to meet the incentive compatibility constraint, imposes however a dead-weight loss on the contract. This can be avoided if the outcome is observed (i.e. if there is monitoring), but given that monitoring is costly some form of delegation appears desirable. The dead-weight loss has then to be imposed on the contract between depositors and the intermediary (the agent to whom the monitoring is delegated), but its expected value can be driven close to zero through diversification.

It must be noted that when the intermediary is rationalized as providing delegated monitoring services, the contracts that it writes with the entrepreneurs will not be a state-independent debt contracts, as Diamond explicitly shows. Indeed the SDC is the solution to the incentive compatibility problems in direct financing and the intermediary will improve upon direct financing only if it allows more efficient contracts to be written. This is in fact the case if the intermediary observes the state and enforces a first best state dependent contract.

In our analysis, on the contrary, the intermediary exists precisely because it allows individual savers to escape the dilemma between being cheated and paying information costs. Since the bank has private information on the projects, any offer of a state-dependent contract to depositors would bring back the dilemma and would thus nullify the rationale for the intermediary's existence.

In so far as ex-ante asymmetric information is considered relevant, therefore, it appears that no bank should find it possible to have its depositors agree on state-dependent deposit contracts.

This point seems to be overlooked by proponents of liberalization in banking, who advocate state-dependent bank liabilities.

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Appendix A

The proof will be given with reference to case (a). It can be easily adapted to cover case (b).

Recall that, by definition, if project j is a mean preserving spread of project i ,

$$(i) \quad \int_{X_1}^Y (F_j(X) - F_i(X))dX \geq 0 \quad \forall Y$$

(ii) $F_j - F_i$ changes sign exactly once

and

$$(iii) \quad E_j(X) = E_i(X)$$

where the suffix for the $E(\cdot)$ operator refers to the appropriate distribution function.

Let us define, for any given \bar{R}_i

$$V(\bar{R}_i, F_i) = \int_{X_1}^{\bar{R}_i} U(X)dF_i(X) + \int_{\bar{R}_i}^X U(\bar{R}_i)dF_i(X)$$

Under (a), \bar{R}_i is defined by the solution of the following equation:

$$(iv) \quad V(\bar{R}_i, F_i) = U(\bar{W})$$

Suppose now that the level of \bar{R} (strictly greater than X_1) associated with projects i and j is the same, so that

$$(v) \quad V(\bar{R}, F_i) = V(\bar{R}, F_j)$$

Integrating by parts and simplifying⁸, we get:

8. We are adopting the convention that $F(h) = \Pr(X \leq h)$ and we assume that each point in the support has positive probability.

$$\int_{X_1}^{\bar{R}} U(X) dF_i(X) + \int_{\bar{R}}^X U(\bar{R}) dF_i(X) = U(\bar{R}) - U(X_1)F_i(X_1) - \int_{X_1}^{\bar{R}} U'(X)F_i(X) dX$$

Therefore (v) can be written as

$$U(X_1) [F_j(X_1) - F_i(X_1)] + \int_{X_1}^{\bar{R}} U'(X) [F_j(X) - F_i(X)] dX = 0$$

However, the first term is strictly positive because of conditions (i) - (iii). As for the second term, because of conditions (i) and (ii), together with the concavity of $U(\cdot)$, we can conclude that it is non negative.

Hence (v) cannot be true and we have that

$$V(\bar{R}, F_i) > V(\bar{R}, F_j) \quad \text{whenever } j > i.$$

Since $\frac{\partial V}{\partial \bar{R}} = [1 - F(\bar{R})] U'(\bar{R}) > 0$ we conclude that

for equation (iv) to be satisfied, $\bar{R}_j > \bar{R}_i$ whenever $j > i$, which is the desired result.

Appendix B⁹

(1) We will first verify the Bayesian consistency of the proposed probability revision function, given the strategies. If no deposit occurs at stage n+1 nothing is learned about the type of bank and $p_n = p_{n+1}$.

If $p_{n+1} = 1$ the depositor is sure at stage n+1 that B is of the risky type. This is common knowledge, and therefore there is no point in B^R playing safe to gain reputation. Again, this is common knowledge, hence DE does not deposit and $p_n = p_{n+1} = 1$.

If DE deposited and F occurs DE knows that the bank must have played risky, and since only B^R plays R it follows $p_n = 1$.

If DE plays D and $\sim F$ occurs, Bayes' theorem applies:

$$(1.1) \quad p_n = \Pr(B^R/\sim F) = \Pr(B^R \cap \sim F)/\Pr(\sim F) = \Pr(\sim F/B^R) \cdot \Pr(B^R)/\Pr(\sim F) =$$

$$\{[\Pr(\sim F/R, B^R) \cdot \Pr(R/B^R) + \Pr(\sim F/S, B^R) \cdot \Pr(S/B^R)] \cdot \Pr(B^R)\}/\Pr(\sim F) =$$

$$\{[(1-q) \cdot \Pr(R/B^R) + \Pr(S/B^R)] \cdot p_{n+1}\}/\Pr(\sim F)$$

$$\text{where } \Pr(\sim F) = [(1-q)\Pr(R/B^R) + \Pr(S/B^R)]p_{n+1} + (1-p_{n+1})$$

Note that both the occurrence of $\sim F$ and the choice of R (or S) are relative to stage n + 1.

At stage $n \leq h+1$ B^R is supposed to play R, hence $\Pr(R/B^R) = 1$. Therefore, at stage $n \leq h$ (1.1) specializes to:

$$p_n = \frac{(1-q) \cdot p_{n+1}}{(1-q)p_{n+1} + 1 - p_{n+1}} = \frac{(1-q) p_{n+1}}{1 - q p_{n+1}}$$

At stage $h+j$, $j=2 \dots N-h$ if $p_{h+j} < \beta \mu_{j-1}$ B^R is supposed to play S, hence nothing is learned by the occurrence of $\sim F$. Therefore $p_{h+j-1} = p_{h+j}$.

9. A detailed, constructive proof is available from the author on request.

If $P_{h+j} \in [\beta\mu_{j-1}, 1)$ B^R is supposed to play R with probability

$$\Pr(R/B^R) = \frac{P_{h+j} - \beta\mu_{j-1}}{P_{h+j}q(1-\beta\mu_{j-1})}$$

and (1.1) specializes to

$$P_{h+j-1} = \left\{ \left[(1-q) \frac{P_{h+j} - \beta\mu_{j-1}}{P_{h+j}q(1-\beta\mu_{j-1})} + 1 - \frac{P_{h+j} - \beta\mu_{j-1}}{P_{h+j}q(1-\beta\mu_{j-1})} \right] P_{h+j} \right\} / \Pr(\sim F)$$

A little algebra then shows that

$$P_{h+j-1} = \beta\mu_{j-1}$$

We have then confirmed the proposed probability revision function.

(2) Let us now verify that, given the probability revision function and the strategies of the others, each agent is playing optimally.

Depositor strategy

The expected return from deposit is

$$\begin{aligned} & f \cdot \Pr(F/R, B^R) \cdot \Pr(R/B^R) \cdot \Pr(B^R) + b[1 - \Pr(F/R, B^R)] \cdot \Pr(R/B^R) \cdot \Pr(B^R) + \\ & + b[\Pr(S/B^R) \cdot \Pr(B^R) + \Pr(S/B^S) \cdot \Pr(B^S)] = \end{aligned}$$

$$(2.1) \quad (b-rq) \cdot \Pr(R/B^R) \cdot \Pr(B^R) + b[\Pr(S/B^R) \cdot \Pr(B^R) + \Pr(B^S)]$$

At stage $h+j$, $j-2 \dots N-h$ this specializes to

$$(b-rq) \left[\frac{P_{h+j} - \beta\mu_{j-1}}{P_{h+j}q(1-\beta\mu_{j-1})} P_{h+j} \right] + b \left[\left(1 - \frac{P_{h+j} - \beta\mu_{j-1}}{P_{h+j}q(1-\beta\mu_{j-1})} \right) P_{h+j} + (1-P_{h+j}) \right] =$$

$$b - rq \left[\frac{P_{h+j} - \beta\mu_{j-1}}{q(1-\beta\mu_{j-1})} \right] = b - rq\lambda$$

If $p_{h+j} = \beta\mu_j$, $\lambda = \frac{\beta(\mu_j - \mu_{j-1})}{q(1 - \beta\mu_{j-1})} = \beta = \frac{b}{rq}$. Hence $b - rq\lambda = 0$

If $p_{h+j} > \beta\mu_j$, $\lambda > \beta$. Hence $b - rq\lambda < 0$

If $p_{h+j} < \beta\mu_j$, $\lambda < \beta$. Hence $b - rq\lambda > 0$

At stage $n = 1, 2 \dots h+1$

(2.1) Specializes to

$$(b-rq) p_n + b(1-p_n) = b - rqp_n \begin{matrix} > \\ < \end{matrix} 0 \text{ according to } p_n \begin{matrix} < \\ > \end{matrix} \beta.$$

We have then confirmed the proposed strategy for DE.

Bank strategy

B^S : in the short run playing S is better than playing R. In the long run playing S results in more deposits, hence it is better. It is therefore rational for B^S always to play S.

B^R : It can be verified by induction that given the strategy of DE and given the probability revision function, the following is the maximum expected utility at stage n given that at that stage the depositor probability evaluation that the bank is of the risky type is p_n .

If $n < h + 1$

$$V_n(p_n) = \begin{cases} a \sum_{t=0}^{n-1} (1-q)^t & \text{if } p_n \leq \beta \\ 0 & \text{if } p_n > \beta \end{cases}$$

If $n = h + 1$

$$V_{h+1}(p_{h+1}) = \begin{cases} a \sum_{t=0}^h (1-q)^t & \text{if } p_{h+1} < \beta \\ \frac{a-1}{q} & \text{if } p_{h+1} = \beta \\ 0 & \text{if } p_{h+1} > \beta \end{cases}$$

If $n \geq h + 2$, write $n = h+j$, $j = 2, \dots, N-h$

$$V_{h+j}(p_{h+j}) = \begin{cases} (j-1) + a \sum_{t=0}^h (1-q)^t & \text{if } p_{h+1} < \beta \\ (j-s) + \frac{a-1}{q} & \beta\mu_s \leq p_{h+1} < \beta\mu_{s+1} \quad s = 1 \dots j-1 \\ \frac{a-1}{q} & \text{if } p_{h+1} = \beta\mu_j \\ 0 & \text{if } p_{h+1} > \beta\mu_j \end{cases}$$

Suppose now that at stage $n+1$ DE plays D and B^R plays R. B^R receives a now, with probability q it receives 0 for the rest of the game ($V_n(1) = 0$), with probability $(1-q)$ it receives $V_n(p_n)$. If B^R plays safe it receives 1 now and $V_n(p_n)$ for the rest of the game.

Hence B^R will play risky if

$a + (1-q)V_n(p_n) > 1 + V_n(p_n)$ i.e. if

$$V_n(p_n) < \frac{a-1}{q}$$

At stage $h + j$, $j = 2, \dots, N - h$

if $p_{h+j+1} < \beta\mu_j$ and $\sim F$ occurs, $p_{h+j} = p_{h+j+1} < \beta\mu_j$. Hence

$$V_{h+j}(p_{h+j}) \geq 1 + \frac{a-1}{q} \text{ and } B^R \text{ plays } S$$

if $p_{h+j+1} \geq \beta\mu_j$ and $\sim F$ occurs, $p_{h+j} = \beta\mu_j$. Hence

$$V_{h+j}(p_{h+j}) = \frac{a-1}{q} \text{ and } B^R \text{ is indifferent}$$

At stage $n \leq h+1$ we find that if $p_{n+1} \leq \beta$ and $\sim F$ occurs,

$p_n < p_{n+1} < \beta$. Hence

$$V_n(p_n) = a \sum_{t=0}^{n-1} (1-q)^t < \frac{a-1}{q} \text{ because of the definition of } h$$

if $p_{n+1} > \beta$ DE does not deposit, $p_n = p_{n+1} > \beta$,

$$V_n(p_n) = 0 < \frac{a-1}{q}. \text{ Therefore } B^R \text{ always plays } R.$$

This verifies the proposed strategy for the bank.

Appendix C

First of all note that if q is such that $b \geq rq$, then reputation becomes irrelevant and DE choose to play D even if it is sure that the bank is of the risky type. In this case, then, B^R will trivially choose its optimal level of reserves at each stage and the delegation cost will be zero.

Let us therefore consider the case $q > b/r$, and let us always choose b and r in such a way that, when q , decreases, this condition is still satisfied.

Let us define (i) $h(q)$ as the largest integer such that $(1-q)^{h(q)} \geq 1/a$
(ii) $n(q)$ as the smallest integer such that $\beta \mu_{n(q)} \geq \delta$

$h(q) + 1$ is the maximum number of moves where R is played for sure.

$n(q)$ is the minimum number of periods in excess of $h(q)$ which guarantees that, for given initial reputation $(1-\delta) > 0$, DE will start playing D (possibly randomizing). Hence $h(q) + n(q)$ is the minimal horizon for the credit game. Also, in all moves $h(q)+n(q), \dots, 1$ B^R will play R with positive probability.

Since when B^R plays R it chooses the optimal level of reserves, to evaluate the expected number of moves where the optimal level of reserves is chosen it is crucial to evaluate the behaviour of $n(q) + h(q)$ when q varies.

It is immediate to verify that $h(q)$ is non decreasing and that $n(q)$ is non increasing with q . They are, however, non continuous functions, and we cannot hope that their sum is a monotone function. We can, however, characterize the "quasi monotonicity" of $s(q) = h(q) + n(q)$ showing that $s(q)$ is "close" to a continuous monotonous function $\sigma(q)$ ¹⁰.

More precisely, let us define

10. I would like to thank Renato Serafini for suggesting this characterization.

(iii) $\eta(q)$ such that $(1-q)^{\eta(q)} = \frac{1}{a}$

(iv) $v(q)$ such that $\left(\frac{r-b}{r}\right)^{v(q)-1} = (1-\delta)\left(\frac{rq}{rq-b}\right)$

which are readily verified to be the continuous versions of (i) and (ii) above¹¹.

Comparing (i) with (iii) we see that $h(q) \leq \eta(q)$.

Also, comparing, (ii) with (iv) we can conclude that $n(q) \geq v(q)$.

From $h(q) \leq \eta(q)$ we derive $h(q) = \text{int}(\eta(q)) = \eta(q) - \text{dec}(\eta(q))$ where $\text{int}(\cdot)$ is the integer part of a real number and $\text{dec}(\cdot)$ is its decimal part.

Similarly, from $n(q) \geq v(q)$ we derive $n(q) \leq \text{int}(v(q)) + 1 = v(q) + 1 - \text{dec}(v(q))$

We now define

$$\begin{aligned}\sigma(q) &= \eta(q) + v(q) \\ \sigma^+(q) &= \eta(q) + v(q) + 1 \\ \sigma^-(q) &= \eta(q) + v(q) - 1\end{aligned}$$

Hence

$$\sigma^-(q) \leq s(q) \leq \sigma^+(q)$$

This implies that we can use the continuous function $\sigma(q)$ to characterize the behaviour of the non continuous, non monotonous function $s(q)$.

Simple algebra would show that

$$\frac{d\sigma}{dq} < 0 \quad \text{if} \quad \frac{\lg(a)}{\lg(r)-\lg(r-b)} > \frac{b(1-q) [\lg(1-q)]^2}{q(rq-b)}$$

11. Let us recall that

$$\mu_n = - \left(\frac{rq-b}{b}\right) \left(\frac{r-b}{r}\right)^{n-1} + \frac{rq}{b}$$

Hence $\beta\mu_n = \frac{b}{rq} \mu_n = - \left(\frac{rq-b}{rq}\right) \left(\frac{r-b}{r}\right)^{n-1} + 1$

This condition imposes a lower bound for the values of q such that $\sigma(q)$ is a decreasing function. This lower bound can be made smaller than b/r (where the problem becomes trivial) by appropriately choosing a . Therefore, if a is large enough $\sigma(q)$ is decreasing over the entire relevant range. Loosely speaking, this implies that $s(q)$ is also decreasing. The expected number of moves in which the optimal level of reserves is (possibly) chosen, which is obviously increasing with $s(q)$, also depends:

- (i) on the probability of playing R at each stage $h(q) + 2, \dots, s(q)$
- (ii) on the probability of continuing the game, which in turn depends on $(1-q)$ and on the probability of DE randomizing.

One immediately verifies that all the probabilities mentioned under (i) - (ii) are decreasing with q . Hence the expected number of moves where no delegation cost is imposed increases as q decreases which is the desired result¹².

12. Note that for $q = b/r$ the delegation cost is trivially zero. However, we cannot conclude that delegation cost can be made arbitrarily small since we cannot appeal to continuity and only the weaker statement in the text is possible.

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