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1. Introduction

The question of whether aggregation is necessarily "bad" has been addressed extensively in the econometric literature.[1] Despite the diversity of the empirical results obtained, the main conclusion which seems to have emerged is that, for predicting macro-variables, the gain from disaggregation may be slight.

However, the prediction criterion proposed by Grunfeld and Griliches (1960) which is commonly employed for the choice between the aggregate and disaggregate linear specifications is only appropriate when all the disaggregate equations have the same specification and the sample size is large. In the case of small samples the use of this criterion of the sums of squared residuals will be valid only if in addition to a uniform specification of micro-relations the disturbances of the disaggregate equations are also contemporaneously uncorrelated. This is clearly a rather restrictive framework, especially in view of the fact that the gain from disaggregation is likely to be materialised only when the micro-equations are not too poorly specified, and for this to be possible different specifications for the micro-relations may be required.

In this paper the Grunfeld-Griliches (GG) prediction criterion is generalised to allow for non-zero contemporaneous covariances between the disaggregate equations and the possibility of different linear parametric restrictions on the equations of the disaggregate model. In addition, for the case of a known error covariance matrix, an exact statistical test of the hypothesis of perfect aggregation is developed which, unlike the test proposed by Zellner (1962) in the context of the seemingly unrelated regressions model, does not

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[1] The main references are Grunfeld and Griliches (1960), Boot and De Wit (1960), Orcutt, Watts and Edwards (1968), Edwards and Orcutt (1969), Lovell (1973), Sasaki (1978), Barker (1970) and Winters (1980).

necessitate the requirement that all coefficients across the equations of the disaggregated model are the same. The new test allows for the possibility of valid aggregation either through coefficient equality or through the invariance of the composition of the regressors across the micro-units over time. In the case where the error covariance matrix of the disaggregate model is unknown, an asymptotic justification for the test will be provided by allowing the degree of disaggregation to increase while keeping the sample size fixed. A proof of this rather novel asymptotic justification of the test will be given for the special case where the disturbances of the micro-equations are contemporaneously uncorrelated.

The choice criterion and the test of perfect aggregation developed in the paper are then applied to two alternative specifications of employment functions for the UK economy disaggregated by 39 industries, and for the manufacturing sector disaggregated by 23 industries. As far as the choice criterion is concerned, the empirical results show that for the economy as a whole the disaggregate model is preferable to the aggregate specification, while the reverse is true for the manufacturing industries taken as a group. The choice of the aggregate model in the case of the manufacturing industries should not, however, be taken to mean that there are no aggregation problems at this level. In fact the application of the test of perfect aggregation to the employment functions provides strong evidence in favour of rejecting the hypothesis of perfect aggregation both for the economy as a whole, and for the manufacturing sector. The better performance of the aggregate specification in the case of the manufacturing industries should be interpreted as providing an important indicator of the misspecification of the disaggregate equations.

The plan of the paper is as follows. Section 2 sets out the basic econometric framework. Section 3 examines the small sample bias of the GG prediction criterion. Section 4 generalises the basic model so that different specifications for the micro-equations is possible, and derives a goodness-of-fit criterion for discrimination between aggregate and disaggregate models that does not suffer from the small sample problem. Section 5 considers alternative methods of testing for the errors of aggregation, and develops a new test of the hypothesis of perfect aggregation. Section 6 deals with the problem of misspecification of the disaggregate model and the implications that this has for the use of the proposed choice criterion. Section 7 contains the application of the econometric methods developed in the paper to the UK employment functions.

2. The basic econometric framework

We start with the micro-model analysed by Theil (1954), and subsequently by Grunfeld and Griliches (1960), and others, and suppose that the n observations on the m micro-units $\{y_{it},\ i = 1,2,\ \ldots,\ m;\ \ t = 1,2,\ \ldots,\ n\}$ are generated according to the following linear specifications

$$y_{it} = \sum_{j=1}^{k} \beta_{ij}\, x_{i,jt} + u_{it}, \qquad \begin{array}{l} i = 1,2,\ \ldots,\ m \\ t = 1,2,\ \ldots,\ n \end{array}$$

or in matrix notation

$$(2.1) \qquad H_d:\ \underset{n\times 1}{\underset{\sim}{y}_i} = \underset{n\times k}{X_i}\ \underset{k\times 1}{\underset{\sim}{\beta}_i} + \underset{n\times 1}{\underset{\sim}{u}_i}, \qquad i = 1,2,\ \ldots,\ m.$$

In the above specification it is assumed that the variations in dependent variables of all micro-units can be explained by means of linear combination of the same set of k explanatory variables. (This assumption will be relaxed in the next section.) Writing (2.1) as a System of Seemingly Unrelated Equations (SURE), following Zellner (1962) we have

$$(2.2) \quad \underset{\sim}{y} = X \underset{\sim}{\beta} + \underset{\sim}{u}$$

where $\underset{\sim}{y} = (\underset{\sim}{y}_1', \underset{\sim}{y}_2', \ldots, \underset{\sim}{y}_m')'$, $\underset{\sim}{\beta} = (\underset{\sim}{\beta}_1', \underset{\sim}{\beta}_2', \ldots, \underset{\sim}{\beta}_m')'$, $\underset{\sim}{u} = (\underset{\sim}{u}_1', \underset{\sim}{u}_2', \ldots, \underset{\sim}{u}_m')'$, and X is an $mn \times mk$ block-diagonal matrix of full column rank with matrix X_i as its ith block. We also make the following assumption:

Assumption 1: The $mn \times 1$ disturbance vector $\underset{\sim}{u}$ is distributed independently of X, has mean zero and the variance matrix $\Omega = \Sigma \otimes I_n$, where $\Sigma = (\sigma_{ij})$, and I_n is the identity matrix of order n.

The problem of aggregation can arise when an investigator interested in the behaviour of the macro-variable

$$(2.3) \quad \bar{\underset{\sim}{y}} = \sum_{i=1}^{m} \underset{\sim}{y}_i$$

considers the single macro-equation

$$(2.4) \quad H_a: \underset{n\times 1}{\bar{\underset{\sim}{y}}} = \underset{n\times k}{\bar{X}} \underset{k\times 1}{\underset{\sim}{b}} + \underset{n\times 1}{\bar{\underset{\sim}{u}}_a},$$

where

$$(2.5) \quad \bar{X} = \sum_{i=1}^{m} X_i$$

instead of the m micro-equations in (2.1). In this paper we are interested primarily in the prediction problem discussed by Grunfeld and Griliches, where the focus of the analysis is whether to predict $\bar{\underset{\sim}{y}}$ using the macro-equation (2.4), or the micro-equations (2.1). The problem of 'aggregation bias', defined by the deviation of the macro-parameters from the average of the corresponding micro-parameters and already discussed in detail by Theil (1954) will not be addressed here.

3. The small sample bias of the Grunfeld-Griliches criterion

The GG prediction (or more accurately the in-sample goodness-of-fit) criterion for the discrimination between the disaggregate model, H_d defined by (2.1), and the aggregate model, H_a defined by (2.5) can be written as

Choose H_d if $\bar{\underset{\sim}{e}}_d'\bar{\underset{\sim}{e}}_d < \bar{\underset{\sim}{e}}_a'\bar{\underset{\sim}{e}}_a$, otherwise choose H_a,

where $\bar{\underset{\sim}{e}}_d$ and $\bar{\underset{\sim}{e}}_a$ are the estimates of the errors in predicting $\bar{\underset{\sim}{y}}$ under H_d and H_a respectively. The estimates employed by GG for $\bar{\underset{\sim}{e}}_d$ and $\bar{\underset{\sim}{e}}_a$ are based on the Ordinary Least Squares (OLS) method and are given by

$$(3.1) \qquad \bar{\underset{\sim}{e}}_a = \bar{M}\,\bar{\underset{\sim}{y}}, \qquad \bar{M} = I - \bar{X}(\bar{X}'\bar{X})^{-1}\bar{X}'$$

and

$$(3.2) \qquad \bar{\underset{\sim}{e}}_d = \sum_{i=1}^{m} M_i\,\underset{\sim}{y}_i, \quad M_i = I - X_i(X_i'X_i)^{-1}X_i'$$

It is important to note that in general $\bar{\underset{\sim}{e}}_d$ is not an efficient estimate of $\bar{\underset{\sim}{u}}_d = \bar{y} - \sum_{i=1}^{m} X_i\,\beta_i$, unless the disturbances of the micro-equations are contemporaneously uncorrelated (i.e. $\sigma_{ij} = 0$, for $i \neq j$), or X_i can be written as exact linear functions of $\bar{X}$. The problem of efficient estimation of $\bar{\underset{\sim}{u}}_d$

and the consequence that this has for the GG criterion will be discussed later. For the moment we assume that the GG criterion, as specified above, is applied even in the case where the micro-equation disturbances are contemporaneously correlated, and investigate the small sample bias that such a procedure entails.

Like the justification offered for Theil's $\bar{R}^2$ criterion, the rationale behind the use of the GG criterion must lie in the fact that if the micro-equations are correctly specified, then 'on average' the prediction of $\bar{y}$ from the macro-equation should not be any better than that obtained from the micro-equations. That is we should have

$$(3.3) \qquad E_d(\bar{e}_d'\bar{e}_d) \leq E_d(\bar{e}_a'\bar{e}_a),$$

where $E_d(.)$ represents the mathematical expectations operator under H_d. However, using (3.1) and (3.2) it is easily seen that[1]

$$(3.4) \qquad E_d(\bar{e}_d'\bar{e}_d|X) = (n - k)\bar{\sigma}^2 - 2\sum_{i>j}^{m} \sigma_{ij}\{k - Tr(A_iA_j)\}$$

$$(3.5) \qquad E_d(\bar{e}_a'\bar{e}_a|X) = (n - k)\bar{\sigma}^2 + \xi'\bar{M}\xi$$

where

$$(3.6) \qquad \xi \doteq \sum_{i=1}^{m} X_i\beta_i - \bar{X}b$$

$$(3.7) \qquad A_i = X_i(X_i'X_i)^{-1}X_i', \qquad i = 1,2, \ldots, m$$

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[1] The notation Tr(A) stands for the trace of matrix A.

and

$$(3.8) \quad \bar{\sigma}^2 = \sum_{i,j} \sigma_{ij} = \mathrm{Var}\left(\sum_{i=1}^{m} u_{it}\right).$$

Given the above results it therefore follows that in small samples the inequality relationship defined by (3.3) may not be satisfied even if the micro-equations are correctly specified.(1)

Now using familiar results from the literature on canonical correlations we have

$$k - \mathrm{Tr}(A_i A_j) = \sum_{s=1}^{k} (1 - \rho_{s,ij}^2) \geq 0$$

where $\rho_{s,ij}$ is the sth canonical correlation coefficient between the explanatory variables of the ith and the jth micro-equations. This result has two interesting implications for the use of the GG criterion. Firstly, unless all the canonical correlations of X_i, X_j for all i and j are equal to unity (a condition satisfied only if X_i can be written as exact linear functions of X_j, for all i and j), the use of the GG criterion is justified in small samples only in the special case where the micro-disturbances

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(1) Notice that the unconditional expectations of $\bar{\underset{\sim}{e}}_d'\bar{\underset{\sim}{e}}_d$ and $\bar{\underset{\sim}{e}}_a'\bar{\underset{\sim}{e}}_a$ can be obtained from (3.4) and (3.5) in the following manner

$$E_d(\bar{\underset{\sim}{e}}_d'\bar{\underset{\sim}{e}}_d) = (n - k)\bar{\sigma}^2 - 2 \sum_{i>j} \sigma_{ij}\{k - E[\mathrm{Tr}(A_i A_j)]\},$$

$$E_d(\bar{\underset{\sim}{e}}_a'\bar{\underset{\sim}{e}}_a) = (n - k)\bar{\sigma}^2 + E(\underset{\sim}{\xi}' \bar{M} \underset{\sim}{\xi}).$$

are all contemporaneously uncorrelated. Secondly, the direction of the bias involved in the use of the GG criterion depends on the signs of the contemporaneous correlation of the micro-disturbances. The GG criterion favours the disaggregate model whenever $\sigma_{ij} > 0$, and vice versa.

The small sample bias in the use of the GG criterion will not disappear even when $\bar{u}_d$ is estimated efficiently by the SURE method. Consider the simple case where Σ is known. The SURE estimate of $\bar{u}_d$, which we denote by $\bar{e}_s$, will be

$$\bar{e}_s = S\, e_s,$$

$$= S[y - X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y],$$

where S stands for the $n \times nm$ summation matrix

(3.9) $$S = [I_n \vdots I_n \vdots \cdots \vdots I_n],$$

with the property that $Sy = \bar{y}$, etc. Under H_d we have

$$\bar{e}_s = S(I - A)u,$$

in which

(3.10) $$A = X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}.$$

Hence

$$E_d(\bar{e}_s'\bar{e}_s|X) = n\,\bar{\sigma}^2 - \mathrm{Tr}\{(X'\Omega^{-1}X)^{-1}X'S'SX\}.$$

Again leaving the case where X_i are exact linear functions of $\bar{X}$ to one side, the inequality

$$E_d(\bar{e}_s'\bar{e}_s|X) < E_d(\bar{e}_a'\bar{e}_a|X)$$

holds only in the special case where $\sigma_{ij} = 0$, for $i \neq j$.

4. A generalised goodness-of-fit criterion for discrimination between aggregate and disaggregate models

From the results of the previous section it is now a straightforward matter to derive a choice criterion for discrimination between the disaggregate and the aggregate models that does not suffer from the small sample bias of the GG criterion. But it is first important to extend the econometric framework of section 2, so that different specifications for the micro-equations can be considered. Such a generalisation is particularly important when the primary purpose of the disaggregation is to achieve a better explanation of the macro-variables. Accordingly, we consider the following generalisation of the disaggregate model

$$\tilde{H}_d: \begin{cases} \underset{n\times 1}{\underset{\sim}{y}_i} = \underset{n\times k_i}{X_i}\, \underset{k_i\times 1}{\underset{\sim}{\beta}_i} + \underset{n\times 1}{\underset{\sim}{u}_i}, & i = 1,2,\ldots, m \\[2ex] \underset{q_i\times k_i}{R_i}\, \underset{k_i\times 1}{\underset{\sim}{\beta}_i} = \underset{q_i\times 1}{\underset{\sim}{r}_i}, & \text{with Rank } (R_i) = q_i < k_i, \end{cases}$$

where R_i and $\underset{\sim}{r}_i$ are the $q_i \times k_i$ and $k_i \times 1$ matrices of known constants. For the aggregate model we now specify that

$$\tilde{H}_a; \begin{cases} \underset{n\times 1}{\underset{\sim}{\bar{y}}} = \underset{n\times k}{Z}\, \underset{k\times 1}{\underset{\sim}{\gamma}} + \underset{n\times 1}{\underset{\sim}{\bar{u}}_a}, \\[2ex] \underset{q\times k}{R}\, \underset{k\times 1}{\underset{\sim}{\gamma}} = \underset{q\times 1}{\underset{\sim}{r}}, & \text{with Rank } (R) = q < k, \end{cases}$$

where R and $\underset{\sim}{r}$ are the $q \times k$ and $k \times 1$ matrices of known constants.

In the above formulation each micro-equation is subject to its own linear, possibly non-homogeneous parametric restrictions; thus allowing for a wide range of specifications across the micro-equations. The specification of the macro-equation is also generalised so that the investigator is not confined to using only columns of $\bar{X} = \Sigma X_i$ as the explanatory variables in the aggregate model. This, for example, allows for the inclusion of distributional variables in the aggregate model for the purpose of capturing the changes in the composition of the macro-variables over time.

The restricted least squares estimates of the parameters of the disaggregate model are

$$(4.1) \qquad \hat{\underset{\sim}{\beta}}_i = D_i X_i' \underset{\sim}{y}_i + \underset{\sim}{g}_i,$$

where

$$(4.2) \qquad G_i = I_n - X_i D_i X_i' = I_n - Q_i,$$

$$(4.3) \qquad D_i = (X_i'X_i)^{-1} - (X_i'X_i)^{-1}(R_i'C_i^{-1}R_i)(X_i'X_i)^{-1},$$

$$(4.4) \qquad C_i = R_i(X_i'X_i)^{-1}R_i',$$

$$(4.5) \qquad \underset{\sim}{g}_i = (X_i'X_i)^{-1}R_i'C_i^{-1}\underset{\sim}{r}_i.$$

Similarly, for the aggregate model we have

$$(4.6) \qquad \hat{\underset{\sim}{\gamma}} = DZ'\bar{\underset{\sim}{y}} + \underset{\sim}{g},$$

$$(4.7) \qquad G = I_n - ZDZ' = I_n - Q,$$

$$(4.8) \qquad D = (Z'Z)^{-1} - (Z'Z)^{-1}(R'C^{-1}R)(Z'Z)^{-1},$$

$$(4.9) \qquad C = R(Z'Z)^{-1}R',$$

$$(4.10) \qquad \underset{\sim}{g} = (Z'Z)^{-1}R'C^{-1}\underset{\sim}{r}.$$

Consider the following 'adjusted' goodness-of-fit criteria for the aggregate and the disaggregate models

$$(4.11) \qquad s_a^2 = \bar{e}_a'\bar{e}_a/(n - \tilde{k}),$$

and

$$(4.12) \qquad s_d^2 = \sum_{i,j=1}^{m} \hat{\sigma}_{ij},$$

where

$$(4.13) \qquad \hat{\sigma}_{ij} = \{n - \tilde{k}_i - \tilde{k}_j + Tr(Q_i'Q_j)\}^{-1} e_i'e_j,$$

with $\bar{e}_a = \bar{y} - Z\hat{\gamma}$, $e_i = y_i - X_i\hat{\beta}_i$, $\tilde{k}_i = k_i - q_i$, and $\tilde{k} = k - q$. The matrix Q_i is already defined by (4.2). Under $\tilde{H}_d$, it can be shown that

$$e_i = G_i u_i,$$

and

$$\bar{e}_a = G(\bar{u} + \xi) + Z(Z'Z)^{-1}R'C^{-1}(R\gamma - r),$$

where $\xi = \sum_{i=1}^{m} X_i\beta_i - Z\gamma$. Substituting these results in (4.11) and (4.12) and taking expectations conditional on X and Z yields

$$(4.14) \qquad E_d(s_d^2|X) = \bar{\sigma}^2,$$

$$(4.15) \qquad E_d(s_a^2|X, Z) = \bar{\sigma}^2 + (n - \tilde{k})^{-1}x'x,$$

where $x = G\xi + Z(Z'Z)^{-1}R'C^{-1}(R\gamma - r)$. From these results it now readily follows that (unconditionally)

$$E_d(s_d^2) \leq E_d(s_a^2),$$

which provides the basis, in small samples, for the following choice criterion for the discrimination between the aggregate and the disaggregate models

Choose $\tilde{H}_d$ if $s_d^2 < s_a^2$, otherwise choose $\tilde{H}_a$.

Unlike the GG criterion, the above model selection criterion 'on average' will result in the choice of the disaggregate model in small samples, assuming of course that the disaggregate model is correctly specified. The implications for the above choice criterion when the disaggregate model is subject to errors of specification will be discussed below. Here, for comparison purposes it is worth considering the following decomposition of the s_d^2 criterion.

$$(4.16) \qquad (n - \tilde{k})s_d^2 = \bar{\underset{\sim}{e}}_d'\bar{\underset{\sim}{e}}_d + \sum_{i=1}^{m} (\tilde{k}_i - \tilde{k})\hat{\sigma}_{ii} + 2\sum_{i>j}^{m} \left(\frac{\phi_{ij}}{1 - \phi_{ij}}\right)\underset{\sim}{e}_i'\underset{\sim}{e}_j$$

where $\bar{\underset{\sim}{e}}_d = \sum_{i=1}^{m} \underset{\sim}{e}_i$, and

$$\phi_{ij} = \frac{\tilde{k}_i + \tilde{k}_j - \tilde{k} - Tr(Q_i'Q_j)}{n - \tilde{k}}$$

The GG prediction criterion focuses on the first term on the right-hand side of (4.16) and ignores the asymptotically negligible second and third terms. The second term represents the contribution to the s_d^2-criterion arising out of the possible differences in the number of estimated coefficients between the aggregate and the disaggregate models. The third term in (4.16) captures the effect of the contemporaneous correlation amongst the disturbances of the micro-equations.

The above choice criterion is also applicable to situations where the disaggregate and the aggregate equations are themselves parts of a system of simultaneous equations. A proof in the case where the disaggregate and the aggregate equations are estimated by the Instrumental Variable (or the Two Stage Least Squares) method is given in Appendix A.

5. Tests of aggregation

In studying the aggregation problem our emphasis so far has been on the model selection procedures. An alternative approach would be to employ classical hypothesis testing procedures and develop a statistical test of the conditions necessary for valid aggregation. In the context of the basic disaggregate model (2.1), the necessary condition for perfect aggregation is given by $\underset{\sim}{\xi} = 0$, where $\underset{\sim}{\xi}$ is defined in (3.6). The hypothesis of 'perfect aggregation' can now be written as

$$H_\xi: \quad \underset{\sim}{\xi} = \sum_{i=1}^{m} X_i \underset{\sim}{\beta}_i - \bar{X}\underset{\sim}{b} = 0.$$

Under H_ξ, it readily follows from (4.14) and (4.15) that $E_d(s_d^2) = E_d(s_a^2) = \bar{\sigma}^2$, and we should not expect to gain from disaggregation. In this sense H_ξ is the appropriate hypothesis of perfect aggregation.[1]

Before developing a test of H_ξ, it is important to note that the condition $\underset{\sim}{\xi} = 0$ is satisfied under a variety of circumstances. Two important cases under which this condition are fulfilled are

(i) Micro-homogeneity hypothesis $H_\beta: \quad \underset{\sim}{\beta}_1 = \underset{\sim}{\beta}_2 = \dots = \underset{\sim}{\beta}_m$

and

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[1] The hypothesis H_ξ when the number of regressors is one is equivalent to the n-covariance condition discussed in Lancaster (1966).

(ii) Compositional stability hypothesis H_x: $X_i = \bar{X} C_i$, $i = 1,2, \ldots, m$

where C_i are $k \times k$ non-singular matrices of fixed constants, such that $\sum_{i=1}^{m} C_i = I_k$. This hypothesis states that the composition of the regressors across micro-units remain fixed over time.(1) Under H_x, the macro-coefficient vector $\underset{\sim}{b}$, is defined in terms of the micro-coefficients through the identity $\underset{\sim}{b} = \sum_{i=1}^{m} C_i \underset{\sim}{\beta}_i$. The condition $\underset{\sim}{\xi} = 0$ will also be met under the mixed hypothesis

$$H_{\beta x}: \quad \underset{\sim}{\beta}_1 = \underset{\sim}{\beta}_2 = \ldots = \underset{\sim}{\beta}_s, \qquad s < m$$

$$X_i = \bar{X} C_i, \qquad i = s+1, s+2, \ldots, m$$

where in this case $\bar{X} = \sum_{i=s+1}^{m} X_i$, and $\sum_{i=s+1}^{m} C_i = I_k$.

The test proposed by Zellner (1962) for aggregation bias is a test of the micro-homogeneity hypothesis, H_β, and is not necessarily relevant as a test of H_ξ: $\underset{\sim}{\xi} = 0$. The Zellner test can therefore be unduly restrictive. Rejection of H_β does not necessarily imply that the perfect aggregation hypothesis H_ξ should also be rejected. What is needed is a direct test of $\underset{\sim}{\xi} = 0$.

5.1 A test of perfect aggregation: case of known Σ

We first develop a test of H_ξ in the case where Σ, the covariance matrix of the micro-disturbances, is known. An efficient estimate of $\underset{\sim}{\xi}$ in this case can be written as

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(1) This condition for valid aggregation has been discussed in the econometric literature by Klein (1953), Wold and Juréen (1953).

(5.1) $\tilde{\xi} = SX\tilde{\beta} - \bar{X}\hat{b}$,

where $\tilde{\beta}$ and $\hat{b}$ are the SURE and the OLS estimates of the parameters of the disaggregate and the aggregate equations respectively, and S is the summation matrix defined by (3.9). Substituting

$$\tilde{\beta} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y,$$

$$\hat{b} = (\bar{X}'\bar{X})^{-1}\bar{X}'\bar{y},$$

in (5.1) now yields

$$\tilde{\xi} = Hy,$$

where

$$H = SA - \bar{A}S,$$

with the matrix A already defined by (3.10), and

(5.2) $\bar{A} = I - \bar{M} = \bar{X}(\bar{X}'\bar{X})^{-1}\bar{X}'$.

On the null hypothesis that $\xi = 0$, we have

$$\tilde{\xi} = Hu.$$

Therefore, under the assumption that u is normally distributed with zero means and a known non-singular variance matrix $\Omega = \Sigma \otimes I_n$

(5.3) $\tilde{\xi}'(H\Omega H')^{-1}\tilde{\xi} \sim \chi^2_n$.

A necessary condition for $H\Omega H'$ to have a full rank can be obtained in the following manner. Since, by assumption Ω is a non-singular matrix then Rank $(H\Omega H')$ = Rank (H). But,

$$\text{Rank }(H) \leq \text{Rank }(SA) + \text{Rank }(\bar{A}S),$$

$$\text{Rank }(\bar{A}S) = \text{Rank }(\bar{A}) = k,$$

$$\text{Rank }(A) = \text{Tr}(A) = mk, \quad \text{Rank }(S) = n$$

and

$$\text{Rank }(SA) \leq \text{Min}(n, mk).$$

Consequently,

$$\text{Rank }(H) \leq k + \text{Min}(n, mk),$$

and for matrix H to have the full rank equal to n, it is necessary that

$$k + \text{Min}(n, mk) \geq n,$$

or

$$(5.4) \qquad k(m + 1) \geq n.$$

In situations where the number of micro-equations is relatively large, the computational burden of obtaining the SURE estimates $\tilde{\beta}$ in (5.1) can be considerable. One possibility would be to construct a test of H_ξ based on the OLS estimates of β instead of the SURE estimates. The estimate of ξ based on the OLS estimates is given by

$$\hat{\xi} = \sum_{i=1}^{m} X_i\hat{\beta}_i - \bar{X}\hat{b},$$

$$= \bar{e}_a - \bar{e}_d,$$

where $\bar{e}_a$ and $\bar{e}_d$ are already defined by (3.1) and (3.2) respectively. Under H_d, and on the assumption that the hypothesis of perfect aggregation H_{ξ} holds, we have[1]

$$(5.5) \qquad \hat{\underset{\sim}{\xi}} = \sum_{i=1}^{m} (A_i - \bar{A})\underset{\sim}{u}_i = \sum_{i=1}^{m} H_i \underset{\sim}{u}_i .$$

Now assuming that $\underset{\sim}{u}_i$ are normally distributed, then conditional on X_i we have

$$\hat{\underset{\sim}{\xi}} | X_i \sim N(0, \psi)$$

where

$$(5.6) \qquad \psi = \sum_{i,j=1}^{m} \sigma_{ij} H_i H_j .$$

Therefore, assuming that ψ is a non-singular matrix,[2] we arrive at the result

$$(5.7) \qquad (\bar{\underset{\sim}{e}}_a - \bar{\underset{\sim}{e}}_d)' \psi^{-1} (\bar{\underset{\sim}{e}}_a - \bar{\underset{\sim}{e}}_d) \sim \chi_n^2$$

which is the OLS counterpart of (5.3).

5.2 Case of unknown Σ

When $\Sigma = (\sigma_{ij})$ is unknown, it is still possible to obtain an 'approximate' test of the perfect aggregation hypothesis by replacing σ_{ij} in (5.3) or (5.6) with their SURE or the OLS estimates. Here, we focus on the latter and consider testing H_{ξ} by means of the statistic

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(1) The matrices A_i and $\bar{A}$ are already defined in (3.7) and (5.2) respectively.

(2) Notice that a necessary condition for ψ to be invertible is given by (5.4)

(5.8) $$\underline{a}_m = (\bar{e}_a - \bar{e}_d)'\hat{\psi}^{-1}(\bar{e}_a - \bar{e}_d)$$

where

(5.9) $$\hat{\psi} = \sum_{i,j=1}^{m} \hat{\sigma}_{ij} H_i H_j$$

(5.10) $$\hat{\sigma}_{ij} = \{n - 2k + tr(A_i A_j)\}^{-1} e_i' e_j$$

We shall refer to a test of H_ξ based on (5.8) as the perfect aggregation test, or the $\underline{a}$-test for short.

The exact distribution of the $\underline{a}_m$ statistic under H_ξ is no longer a χ^2_n, and unfortunately does not lend itself to a simple derivation either. But it is possible to approximate the distribution of $\underline{a}_m$ by means of a 'suitable' limiting distribution. The usual asymptotic theory where the limiting distribution is obtained by letting n, the sample size, tend to infinity is not, however, applicable here. The relevant asymptotic framework for testing the hypothesis of perfect aggregation is to allow the level of disaggregation, m, to increase without a bound, while keeping the sample size fixed. In such an asymptotic framework in order to ensure that the macro-variables $\bar{u}$ and $\bar{X}$ have finite limiting distributions as $m \to \infty$, the following assumptions will be needed.

<u>Assumption 2</u>: the $n \times k$ matrix of observations on the explanatory variables of the ith micro-equation in the disaggregate model (2.1) which is composed of m micro-equations satisfy the condition

$$X_i(m) = m^{-1} \Delta_i + O_p(m^{-3/2})$$

where Δ_i is an $n \times k$ matrix such that $m^{-1} \sum_{i=1}^{m} \Delta_i$ tends to the finite limit $\bar{X}$, as $m \to \infty$,[1] and elements of $P_i = \Delta_i(\Delta_i'\Delta_i)^{-1}\Delta_i'$ are bounded in absolute value by matrix P. Notationally we write $|P_i| < P < \infty$.

<u>Assumption 3</u>: the variance-covariance of the disturbances of the ith and the jth micro-equations in the disaggregate model (2.1) which is composed of m micro-equations satisfy the condition

$$\sigma_{ij}(m) = m^{-1}\ \tau_{ij} + 0\ (m^{-3/2})$$

such that $m^{-1} \sum_{i,j=1}^{m} \tau_{ij}$ tends to the finite limit $\bar{\sigma}^2 = V(u_t)$ as $m \to \infty$, and

$$|\tau_{ij}| < \tau^2 < \infty.$$

<u>Assumption 4</u>: the variance matrix ψ defined by

$$\psi = \sum_{i,j=1}^{m} \sigma_{ij}\ H_i H_j$$

has a full rank.[2]

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(1) In the case where $X_i(m)$ are stochastic, we need to assume that in the limit $m^{-1} \sum_{i=1}^{m} \Delta_i$ has the same distribution as $\bar{X}$.

(2) Recall that the necessary condition for this assumption to hold is given by (5.4) and is clearly met as $m \to \infty$, but n is kept fixed.

When assumptions 1-4, hold, it seems reasonable to suppose that the distribution of $\underline{a}_m$, on the null hypothesis of perfect aggregation will tend towards a χ_n^2 as $m \to \infty$. Although, at this stage we are not able to present a proof of this proposition in its present form, we can nevertheless offer the following less general theorem.

Theorem: consider the disaggregate model (2.1) and suppose that the standardised micro-disturbances $u_{it}/\sqrt{\sigma_{ii}}$ are identically distributed, independently both across time periods and across equations, with zero means, unit variances and finite third order moments. Then conditional on X, and under assumptions 2-4, the statistic

$$\underline{a}_m = (\bar{e}_a - \bar{e}_d)'\left(\sum_{i=1}^{m} \sigma_{ii} H_i^2\right)^{-1}(\bar{e}_a - \bar{e}_d)$$

will be asymptotically distributed as a χ_n^2 variate on the null hypothesis of perfect aggregation (i.e. $\xi = 0$), as $m \to \infty$.

It is worth noting that the above theorem is applicable even when micro-equations contain lagged dependent variables or macro-variables and other common variables such as an intercept term or a time trend. In the case of these variables assumption 2 will be met so long as the common variables are included in the micro-equations in an 'average' form (i.e. averaged over the number of micro-units). For example, if the specification of the micro-equations under consideration contains a time trend, the trend variable should be entered in the micro-equations as $m^{-1}t$, $t = 1,2, \ldots, n$. This procedure also ensures that the coefficients of macro-variables in micro-equations remain finite as $m \to \infty$.

6. Disaggregation and specification error

The model selection criterion and the aggregation tests developed in this paper are based on the assumption that the disaggregate model is correctly specified. In reality, however, both the disaggregate and the aggregate models may suffer from errors of specification, with the latter also being subject to the additional problem of aggregation error. In such a circumstance the issue of whether disaggregation is useful for the study of macro-phenomena and the extent of the gain that may be expected from disaggregation depends very much on the relative importance of the two types of errors of specification and aggregation. In this section the implications that errors of specification may have for the use of our proposed choice criterion will be examined.

Let the correctly specified disaggregate model be

$$(6.1)\quad \underset{n\times 1}{\underset{\sim}{y}_i} = \underset{n\times k}{X_i}\underset{k\times 1}{\underset{\sim}{\beta}_i} + \underset{n\times s}{W_i}\underset{s\times 1}{\underset{\sim}{\gamma}_i} + \underset{n\times 1}{\underset{\sim}{u}_i}, \qquad i = 1,2, \ldots, m$$

which in a stacked form can also be written as

$$(6.2)\quad \underset{\sim}{y} = X\underset{\sim}{\beta} + W\underset{\sim}{\gamma} + \underset{\sim}{u},$$

where $\underset{\sim}{\gamma} = (\underset{\sim}{\gamma}_1', \underset{\sim}{\gamma}_2', \ldots, \underset{\sim}{\gamma}_m')$, and W is an $mn \times ms$ block-diagonal matrix with W_i on its ith block. The other notations are as in relation (2.2). Suppose now that a researcher misspecifies this model by omitting the variables in W, and continues to employ the model selection criterion based on s_a^2 s_d^2, defined by (4.11) and (4.12) respectively. Clearly, the result $E_d(s_d^2) \leq E_d(s_a^2)$, which provided the rationale for the choice criterion, need no longer hold.

Stacking the OLS residuals $\underset{\sim}{e}_i = M_i\underset{\sim}{y}_i$ in the vector $\underset{\sim}{e} = (\underset{\sim}{e}_1', \underset{\sim}{e}_2', \ldots, \underset{\sim}{e}_m')'$, s_d^2 can also be written as

$$s_d^2 = \underset{\sim}{e}' \, N \, \underset{\sim}{e}$$

in which $N = (\bar{N} \otimes I_n)$, and $\bar{N}$ is an $m \times m$ matrix with a typical element equal to $[Tr(M_iM_j)]^{-1}$. Now under the correctly specified model (6.2),

$$\underset{\sim}{e} = M\underset{\sim}{y}, \qquad M = I - X(X'X)^{-1}X'$$

$$= MW\gamma + Mu.$$

Hence

$$(6.3) \qquad E_d(s_d^2|X, W) = \bar{\sigma}^2 + \underset{\sim}{\gamma}'W'MNMW\underset{\sim}{\gamma}.$$

Since in general N may not be a positive semi-definite matrix, without further information about the nature of the specification error, it will not be possible to say whether misspecification leads to an upward or a downward bias in the application of the choice criterion. Expanding (6.3) in terms of the misspecification of the individual micro-equations we have

$$(6.3)' \qquad E_d(s_d^2|X, W) = \bar{\sigma}^2 + (n - k)^{-1} \sum_{i=1}^{m} \underset{\sim}{d}_i'\underset{\sim}{d}_i$$

$$+ 2 \sum_{i>j}^{m} \{\underset{\sim}{d}_i'\underset{\sim}{d}_j/Tr(M_iM_j)\}$$

where $\underset{\sim}{d}_i = M_iW_i\underset{\sim}{\gamma}_i$, and $Tr(M_iM_j) = n - 2k + Tr(A_iA_j)$.

The direction of the bias resulting from misspecification clearly depends on the sign of the cross-equation terms $\underset{\sim}{d}_i'\underset{\sim}{d}_j$, $i \neq j$, and their quantitative importance relative to the equation-specific terms $\underset{\sim}{d}_i'\underset{\sim}{d}_i$. In practice, however, it is reasonable to expect that $E_d(s_d^2) > \bar{\sigma}^2$.

Now turning to the s_a^2 criterion, under (6.1) we obtain

$$(6.4) \qquad E_d(s_a^2|X, W) = \bar{\sigma}^2 + (n-k)^{-1}\ \tilde{\xi}'\ \bar{M}\ \tilde{\xi} \geq \bar{\sigma}^2$$

where

$$(6.5) \qquad \tilde{\xi} = \sum_{i=1}^{m} X_i \tilde{\beta}_i + \sum_{i=1}^{m} W_i \tilde{\gamma}_i = \tilde{\xi}_a + \tilde{\xi}_s .$$

Comparing (6.3) and (6.4) it is clear that in general it is not possible to say whether $E_d(s_a^2)$ exceeds $E_d(s_d^2)$. The result depends on the relative importance of the specification error and the aggregation error for the explanation of the macro-variable $\bar{y}$. In their work, Grunfeld and Griliches (1960), consider a special case of some interest where there are micro-specification errors that cancel out in the aggregate. In the context of model (6.1) this can arise either when there are, for example, errors of measurement in the micro-variables that cancel out exactly in the aggregate[1] (i.e. $\tilde{\xi}_s = \sum_{i=1}^{m} W_i \tilde{\gamma}_i = 0$), or when the micro-specification errors involve omission of macro-variables already included in the aggregate model,[2] (i.e. $\bar{M}\tilde{\xi}_s = 0$). In such a case, using (6.4), we have

$$E_d(s_a^2|X, W) = \bar{\sigma}^2 + (n-k)^{-1}\ \tilde{\xi}_a'\ \bar{M}\ \tilde{\xi}_a,$$

and only aggregation errors ($\tilde{\xi}_a \neq 0$) cause the expectations of s_a^2 to exceed the true error variance of the aggregate model. However, even in this special case it is not possible

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[1] The problem of measurement errors in a disaggregate model with $m = k = 2$ is discussed by Aigner and Goldfeld (1974).

[2] It is beyond the scope of the present paper to go into the reasons for the importance of macro-variables in the explanation of micro-behaviour. In general they may arise because individual micro-behavioural relations are not independent but are influenced or constrained by outcomes (or expectations of outcome) of the market as a whole.

to say whether it is better to use the aggregate model. The answer still depends on the relative importance of the micro-specification errors in the disaggregate model and the aggregation error in the aggregate model for the explanation and prediction of macro-behaviour. The issue of whether one should choose the aggregate or the disaggregate model cannot be resolved by a priori reasoning alone and has to be settled with respect to particular problems and in the context of specific models.

7. Applications: employment demand functions in the UK

In this section the methods described in the preceding sections will be illustrated by estimating employment demand functions for the UK economy disaggregated into 39 industries. We adopt the following log-linear approximation to the employment function

$$(7.1) \qquad LEH_{it} = \beta_{i1}/m + \beta_{i2}(T/m) + \beta_{i3}LY_{it} + \beta_{i4}LEH_{i,t-1} + \beta_{i5}LW_{it} + \beta_{i6}(SLY_t/m) + u_{it},$$

$$i = 1,2, \ldots, m,$$
$$t = 1,2, \ldots, n,$$

where

LEH_{it} = log of man-hours employed in industry i at time t,

T = time trend,

LY_{it} = log of industry i output at time t,

LW_{it} = log of average wage rate per man-hours employed in industry i at time t,

SLY_t = $\sum_{i=1}^{m} LY_{it}$.

This specification can be justified theoretically when employment decisions are made at the industry level by cost minimising firms with identical production functions and the same given demand and factor price expectations. In this framework the lagged employment variable enters the equation because of the adjustment costs involved in the hiring and firing of workers. The time trend is there to capture the effect of the neutral technical progress on the labour productivity, and the level of aggregate output (SLY_t) is a proxy measure intended to capture changes in demand expectations arising from the perceived interdependence of the demand in economy by the firms in the industry.[(1)] Apart from this last variable, the employment function (7.1) is a simplified version of the equations estimated by Peterson (1986), as a part of the Cambridge Multisectoral Dynamic Model (MDM) of the UK economy. Details of the data used are provided in Appendix C.

For the aggregate employment function we estimated

$$(7.2) \qquad SLEH_t = b_1 + b_2T + b_3SLY_t + b_4SLEH_{t-1} + b_5SLW_t + u_t, \qquad t = 1,2, \ldots, n$$

where

$$SLEH_t = \sum_{i=1}^{m} LEH_{it}, \quad \text{and} \quad SLW_t = \sum_{i=1}^{m} LW_{it}.$$

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(1) Notice that, as required by assumption 3, the intercept term, the time trend, and the aggregate output variable that are common to all the micro-equations are specified in the 'average' form. Clearly this has no effect on the overall fit of the equations for a fixed level of disaggregation.

Here we are assuming that the purpose of the study is to explain $SLEH_t$, which is the sum of the logarithms of industry employment. This is clearly different from the more usual practice of specifying aggregate employment functions in terms of the logarithm of the sum of industry employment. For our purposes the specification (7.2) has the advantage that it fits directly within the theoretical framework of the paper, and as is pointed out, for example, by Lovell (1973), it also satisfies the Klein-Nataf consistency conditions. A theoretical analysis of the alternative methods of aggregating micro-specifications such as (7.1), and an econometric investigation of the relative merits of such aggregation methods is beyond the scope of the present paper.

7.1 Results for the economy as a whole

The estimates of the unrestricted version of the industry demand functions (7.1) for the 39 industries in MDM over the sample period 1955-81 are set out in Table 1. The estimates of the standard errors of the regression coefficients are given in the brackets. The Table also includes the equations' standard errors (SE), the values of the log-likelihood function (LLF), the Lagrange Multiplier statistic for testing against first order residual autocorrelation (LM1), and the Box-Pierce statistic for testing against general autocorrelation of up to the fourth order (BoxP). In addition, the Tables relating to restricted versions of the equations include an asymptotic chi-squared test of the validity of the imposed restrictions (RST).

The estimate of the corresponding aggregate specification is

$$(7.3)\qquad \underset{}{SLEH_t} = \underset{(43.51)}{-83.54} - \underset{(0.05)}{0.11549}\,T + \underset{(0.05)}{0.4501}\,SLY_t$$

$$+ \underset{(0.10)}{0.6075}\,SLEH_{t-1} - \underset{(0.05)}{0.3315}\,SLW_t + \hat{u}_t$$

$\bar{R}^2 = 0.989$, $\hat{\sigma} = 0.4207$, LLF = -12.166, LM1 = 0.0639

BoxP(4) = 1.7979.

Overall, the industry estimates are well determined and the majority of estimated coefficients are of the right signs. The results provide further evidence in support of the view that both the demand and the product wage variables are significant determinants of changes in employment, although, as is already stressed by Peterson (1986), in the case of most industries changes in demand have been historically more important than changes in product wages in the explanation of employment changes. We also estimated a restricted version of the industry employment functions by imposing zero restrictions on the coefficients of those variables that were clearly insignificant. The results for this 'restricted' specification are summarised in Table 2.

In order to compare the in-sample 'predictive' performance of the aggregate and the disaggregate models, we computed the s_d^2 criterion [as defined by (4.13)] for the unrestricted and the restricted versions of the disaggregate model. These were 0.1384 and 0.1164 respectively, thus providing evidence in favour of the restricted version of the disaggregate model. The value of the goodness-of-fit criterion for the aggregate equation (7.3) was equal to 0.1769. Therefore, on the basis of the proposed choice criterion the restricted as well as the unrestricted versions of the disaggregate model are preferable over the aggregate equation. The computation of the statistic for the test of perfect aggregation defined by (5.7) also provided additional support in favour of the disaggregate model. In the case of the unrestricted version the value of this test statistic was equal to 178.70, which is approximately distributed as a χ^2_{27}, thus firmly rejecting the hypothesis of perfect aggregation.

7.2 Results for the manufacturing industries

Having rejected the aggregate employment function in favour of the disaggregate model, the question of what the appropriate level of disaggregation should be naturally arises. One possibility would be to repeat the above analysis for all possible levels of disaggregation. Here in the way of illustration we only consider the problem in the case of the manufacturing industries. This industry grouping is composed of the industries labelled 5 to 27 inclusive in Table 1. For these industries the disaggregate results for the unrestricted and the restricted specifications are given in Tables 3 and 4 respectively. We also obtained the following estimate of employment demand for the manufacturing sector as a whole

$$(7.4) \qquad SLEH_t = \underset{(18.21)}{-95.26} + \underset{(0.05)}{0.06522}\, T + \underset{(0.03)}{0.4893}\, SLY_t + \underset{(0.07)}{0.7091}\, SLEH_{t-1} - \underset{(0.05)}{0.4403}\, SLW_t + \hat{u}_t$$

$\bar{R}^2 = 0.996$, $\hat{\sigma} = 0.1979$, LLF = 8.19, LM1 = 0.003, BoxP(4) = 0.3862.

In this application the values of the goodness-of-fit criterion (s_d^2) for the unrestricted and the restricted models were 0.0483 and 0.0444 respectively, indicating that the restricted version of the disaggregate model has a better in-sample predictive performance. We also obtained the goodness-of-fit criterion for the aggregate estimate (7.4) and obtained the value of 0.0392. Hence, on the basis of the choice criterion, for the manufacturing industries the aggregate model is preferable to either of the disaggregate models. This, of course, does not mean that the aggregate model is not subject to the aggregation error problem. In fact the application of the test of perfect aggregation to this example resulted in the value of 174.7 for the $\underline{a}_m$ statistic which is well in excess of the 5% critical value of the χ^2 distribution with 27 (= n) degrees of freedom. The better performance of the aggregate model should be interpreted as an important indication that the disaggregate employment functions are misspecified. This suggests the need for a much more detailed analysis of employment demand at the industry level, which may involve experimenting with a different choice of functional forms across industries, or searching for new industry-specific explanatory variables, or even compiling a more reliable set of micro-data. But, in the absence of further industry-specific analysis of employment, the aggregate specification of employment function, at least for the manufacturing industries, seems to be providing a better explanation of the aggregate employment variable than our disaggregate specifications.

Table 1

Disaggregate Employment functions
Whole Economy (Unrestricted)

Part 1

		INPT	TIME	LY	LEH1	LW	SLY	BOXP/RST	SE/LM1	LLF
1	Agriculture etc.	200.3148	0.2888	-0.0106	0.7142	-0.1436	-0.4006	2.3520	0.0180	73.5783
		(63.6571)	(0.1603)	(0.1811)	(0.0859)	(0.0898)	(0.0919)	(0.0000)	0.0697	
2	Coal Mining	45.1631	-0.1641	0.0172	0.7538	-0.3358	-0.1413	4.2805	0.0304	59.4369
		(97.7052)	(0.1444)	(0.0886)	(0.1175)	(0.0758)	(0.1887)	(0.0000)	0.0587	
3	Mining nes	213.3453	0.1264	0.6856	-0.0289	-0.6985	-0.8028	3.6686	0.0426	50.3048
		(122.0582)	(0.2339)	(0.1853)	(0.1307)	(0.0785)	(0.4247)	(0.0000)	0.0495	
4	Petroleum & Nat. gas	343.4249	0.3466	0.1882	0.7611	-0.0783	-1.1346	0.6105	0.2082	7.4574
		(440.8264)	(1.5109)	(0.0941)	(0.1585)	(0.0522)	(1.3558)	(0.0000)	0.0061	
5	Food Manufacturing	-153.9159	-0.5356	0.5251	0.6661	-0.1385	0.1873	2.1233	0.0155	77.5890
		(83.5237)	(0.1980)	(0.1710)	(0.1931)	(0.0678)	(0.0777)	(0.0000)	0.0001	
6	Drink	67.7753	-0.3466	0.3127	0.6419	-0.0356	-0.1624	2.3595	0.0266	62.9689
		(79.0321)	(0.2229)	(0.3315)	(0.1939)	(0.0588)	(0.3315)	(0.0000)	0.0126	
7	Tobacco	35.2325	-0.7767	-0.0946	0.5021	0.0298	0.4397	1.3190	0.0379	53.4412
		(110.7756)	(0.2822)	(0.2606)	(0.2785)	(0.0436)	(0.1545)	(0.0000)	0.1917	
8	Coal Products	-189.0199	-0.9537	0.6683	0.0198	-0.3994	0.5694	4.1313	0.0428	50.1853
		(69.1786)	(0.3650)	(0.1746)	(0.1948)	(0.0516)	(0.2658)	(0.0000)	0.0089	
9	Petroleum Products	-80.6781	-0.6654	0.1845	0.5606	-0.2810	0.2286	3.4307	0.0584	41.7955
		(125.6271)	(0.2493)	(0.2338)	(0.1666)	(0.1265)	(0.5138)	(0.0000)	0.0034	
10	Chemicals	-67.7700	-0.0083	0.0814	0.6875	-0.2657	0.2932	4.3614	0.0161	76.6404
		(70.5894)	(0.0978)	(0.1268)	(0.1256)	(0.0635)	(0.1901)	(0.0000)	0.0290	
11	Iron & Steel	-193.8070	-0.4080	0.2738	0.6855	-0.3419	0.4305	1.6580	0.0297	60.0266
		(59.2294)	(0.2716)	(0.0867)	(0.1053)	(0.1023)	(0.2023)	(0.0000)	0.0221	
12	Non-ferrous Metals	75.7069	-0.1883	0.5031	0.4055	-0.0997	-0.1721	9.3375	0.0225	67.5629
		(36.3757)	(0.1621)	(0.0941)	(0.0888)	(0.0371)	(0.1455)	(0.0000)	0.2802	
13	Mech. Engineering	-173.5971	0.4307	0.4943	0.5566	-0.7183	-0.0049	5.0589	0.0206	69.8457
		(84.8910)	(0.2590)	(0.1008)	(0.1018)	(0.1959)	(0.1476)	(0.0000)	0.1341	
14	Instr. Engineering	-60.7478	-0.1419	0.2596	0.5481	-0.3763	0.1496	3.0554	0.0235	66.3774
		(52.2311)	(0.1693)	(0.1377)	(0.1351)	(0.1252)	(0.1831)	(0.0000)	0.0433	
15	Elect. Engineering	-161.8519	0.1455	0.5517	0.6741	-0.5943	-0.1604	1.0417	0.0182	73.2768
		(42.2856)	(0.2093)	(0.1044)	(0.0610)	(0.1296)	(0.1107)	(0.0000)	0.0458	
16	Ship Building	-52.5748	-0.0632	0.2482	0.9403	0.0357	0.0460	2.2198	0.0290	60.6603
		(68.7806)	(0.1875)	(0.0915)	(0.1229)	(0.0771)	(0.1587)	(0.0000)	0.0651	
17	Motor Vehicles	-80.4445	-0.5104	0.3540	0.2526	-0.2730	0.5787	0.8899	0.0212	69.1531
		(45.5710)	(0.1434)	(0.0414)	(0.0976)	(0.0684)	(0.1277)	(0.0000)	0.0292	
18	Aerospace Equipment	279.8769	-0.4246	0.1201	0.3245	-0.0366	-0.2777	3.2530	0.0295	60.2566
		(99.2985)	(0.1452)	(0.0685)	(0.1494)	(0.0921)	(0.1368)	(0.0000)	0.0087	
19	Other Vehicles	-130.8584	-0.2149	0.3656	0.7052	-0.1870	0.2579	1.3417	0.0315	58.4228
		(87.2358)	(0.2020)	(0.0788)	(0.1090)	(0.0636)	(0.2139)	(0.0000)	0.0885	
20	Metal Goods nes	-67.4355	-0.0204	0.3476	0.6622	-0.2473	0.0620	5.1918	0.0175	74.2838
		(59.1631)	(0.1379)	(0.0695)	(0.0820)	(0.1046)	(0.1361)	(0.0000)	0.0981	
21	Textiles	-191.8663	0.1407	0.3933	0.5738	-0.4794	0.3188	1.5252	0.0192	71.8470
		(42.3205)	(0.4404)	(0.1257)	(0.1235)	(0.0675)	(0.2543)	(0.0000)	0.0200	
22	Leather, Clothing et	-16.7573	-0.3231	0.3659	0.3654	-0.3769	0.1333	4.1750	0.0129	82.6064
		(24.7463)	(0.1598)	(0.0666)	(0.1003)	(0.0716)	(0.0828)	(0.0000)	0.0000	
23	Bricks	-204.2333	-0.2881	0.3679	0.6358	-0.4899	0.3084	2.5290	0.0169	75.2218
		(58.5311)	(0.1464)	(0.1136)	(0.0818)	(0.0788)	(0.1986)	(0.0000)	0.0418	

Table 1
(contd.)

Part 2

		INPT	TIME	LY	LEH1	LW	SLY	BOXP/RST	SE/LM1	LLF
24	Timber & Furniture	113.0376	-0.3327	0.2940	0.2198	-0.1579	0.1333	1.5096	0.0135	81.2999
		(32.2402)(	0.1102)(	0.0572)(	0.0952)(	0.0512)(	0.1164)(	0.0000)	0.0044	
25	Paper & Board	-21.6897	0.1365	0.5950	0.2959	-0.4017	-0.0513	4.9944	0.0186	72.6984
		(34.8470)(	0.1292)(	0.0912)(	0.0918)(	0.0514)(	0.1233)(	0.0000)	0.3797	
26	Printing & Publishin	85.1112	-0.0126	0.3070	0.6212	-0.1569	-0.2561	3.8617	0.0212	69.1203
		(37.2213)(	0.1206)(	0.1219)(	0.1162)(	0.1310)(	0.1369)(	0.0000)	0.3187	
27	Other Manufacturing	23.6801	-0.9817	0.4609	0.5188	0.1019	0.1139	4.6006	0.0201	70.6345
		(84.0324)(	0.1986)(	0.1515)(	0.1529)(	0.1201)(	0.2632)(	0.0000)	0.2353	
28	Construction	-33.7043	0.0313	0.6277	0.2501	-0.5884	-0.1008	13.3864	0.0226	67.4270
		(55.6259)(	0.1309)(	0.1121)(	0.1089)(	0.1033)(	0.1501)(	0.0000)	0.5395	
29	Gas	-101.6012	-0.4864	0.0514	0.6667	-0.1705	0.4874	3.6843	0.0247	65.0320
		(63.7289)(	0.1606)(	0.0386)(	0.1250)(	0.0398)(	0.1083)(	0.0000)	0.0125	
30	Electricity	126.1077	-0.1620	0.3578	0.5614	-0.1589	-0.4152	2.0996	0.0239	65.9219
		(51.4807)(	0.1948)(	0.1437)(	0.1409)(	0.0680)(	0.1725)(	0.0000)	0.0426	
31	Water	-5.7136	0.5776	1.3500	0.2529	-0.4686	-0.7631	3.8952	0.0440	49.4163
		(78.8126)(	0.2350)(	0.4401)(	0.1625)(	0.1197)(	0.2748)(	0.0000)	0.2023	
32	Rail	28.1213	0.0990	0.3602	0.8104	-0.0146	-0.2058	7.9427	0.0260	63.6575
		(111.1621)(	0.1493)(	0.1327)(	0.1281)(	0.1269)(	0.2103)(	0.0000)	0.0065	
33	Road	-24.7397	-0.2445	0.1006	0.8193	-0.0657	0.1465	3.4204	0.0166	75.7122
		(75.7105)(	0.1067)(	0.0709)(	0.1377)(	0.0630)(	0.1103)(	0.0000)	0.3575	
34	Other Transport	-10.0589	0.0076	0.2376	0.6483	-0.2290	0.0187	6.0183	0.0175	74.3242
		(80.3592)(	0.0979)(	0.1250)(	0.1354)(	0.0554)(	0.1022)(	0.0000)	0.1119	
35	Communication	-30.8580	-0.6136	0.4681	0.6165	-0.1043	0.0105	8.8701	0.0204	70.1672
		(65.3423)(	0.2309)(	0.1823)(	0.1418)(	0.0999)(	0.1017)(	0.0000)	0.6084	
36	Distribution	63.0362	0.2154	0.2596	0.5825	-0.2831	-0.1501	0.2939	0.0153	77.8874
		(49.1404)(	0.1347)(	0.1713)(	0.1123)(	0.0938)(	0.1236)(	0.0000)	0.0009	
37	Business Services	176.0958	0.7588	-0.1097	0.5283	-0.1323	0.0760	6.9777	0.0157	77.2283
		(99.5864)(	0.2800)(	0.1412)(	0.2181)(	0.0531)(	0.0939)(	0.0000)	0.0527	
38	Professional Service	205.8510	0.6391	0.0241	0.3858	-0.1719	-0.0496	3.4485	0.0198	70.9905
		(100.7069)(	0.1935)(	0.0610)(	0.1817)(	0.0733)(	0.1091)(	0.0000)	0.0018	
39	Misc. Services	14.9571	0.1365	0.4158	0.6851	-0.1410	-0.2158	4.1185	0.0175	74.2990
		(90.1482)(	0.1018)(	0.1276)(	0.1383)(	0.0755)(	0.1106)(	0.0000)	0.0350	

Key to Tables

Standard errors in brackets

BOXP is Box-Pierce residual correlogram test $\chi^2(4)$

RST is χ^2 test of imposed restrictions (where appropriate)

SE is equation standard error

LM1 is L.M. test for AR(1) errors $\chi^2(1)$

LLF is maximised value of the log-likelihood function.

Table 2

Disaggregate Employment functions
Whole Economy (Restricted)

Part 1

		INPT	TIME	LY	LEH1	LW	SLY	BOXP/RST	SE/LM1	LLF
1	Agriculture etc.	197.9147	0.2852	0.0000	0.7117	-0.1465	-0.4039	2.3066	0.0176	73.5761
		(48.6127)	(0.1475)	(0.0000)	(0.0752)	(0.0754)	(0.0731)	(0.0034)	0.0695	
2	Coal Mining	-24.5354	-0.1903	0.0000	0.8207	-0.3602	0.0000	4.6718	0.0294	59.0720
		(24.1090)	(0.1361)	(0.0000)	(0.0684)	(0.0683)	(0.0000)	(0.5755)	0.0327	
3	Mining nes	161.3095	0.0000	0.6130	0.0000	-0.6931	-0.6011	3.9337	0.0410	50.1177
		(47.0511)	(0.0000)	(0.1074)	(0.0000)	(0.0745)	(0.1410)	(0.2930)	0.0712	
4	Petroleum & Nat. gas	-13.3603	0.0000	0.1178	0.8651	-0.0771	0.0000	0.3482	0.2041	6.7572
		(17.8877)	(0.0000)	(0.0534)	(0.1052)	(0.0518)	(0.0000)	(1.1179)	0.0204	
5	Food Manufacturing	-153.9159	-0.5356	0.5251	0.6661	-0.1385	0.1873	2.1233	0.0155	77.5890
		(83.5237)	(0.1980)	(0.1710)	(0.1931)	(0.0678)	(0.0777)	(0.0000)	0.0001	
6	Drink	50.0819	-0.2822	0.1572	0.6890	-0.0264	0.0000	2.4202	0.0262	62.8155
		(70.2962)	(0.1801)	(0.0955)	(0.1685)	(0.0557)	(0.0000)	(0.2400)	0.0137	
7	Tobacco	5.4802	-0.7074	0.0000	0.5251	0.0000	0.4132	1.1646	0.0366	53.1344
		(70.0305)	(0.2289)	(0.0000)	(0.2745)	(0.0000)	(0.1455)	(0.4826)	0.1634	
8	Coal Products	-185.0692	-0.9517	0.6794	0.0000	-0.4019	0.5622	4.0992	0.0418	50.1787
		(57.2100)	(0.3645)	(0.1360)	(0.0000)	(0.0454)	(0.2560)	(0.0103)	0.0084	
9	Petroleum Products	-35.8462	-0.5963	0.2717	0.5235	-0.3106	0.0000	3.3607	0.0573	41.6689
		(74.9952)	(0.1950)	(0.1274)	(0.1441)	(0.1077)	(0.0000)	(0.1979)	0.0006	
10	Chemicals	-62.6492	0.0000	0.0000	0.6726	-0.2532	0.3969	3.5299	0.0155	76.3051
		(67.8694)	(0.0000)	(0.0000)	(0.1238)	(0.0465)	(0.0906)	(0.5281)	0.0260	
11	Iron & Steel	-193.8070	-0.4080	0.2738	0.6855	-0.3419	0.4305	1.6580	0.0297	60.0266
		(59.2294)	(0.2716)	(0.0867)	(0.1053)	(0.1023)	(0.2023)	(0.0000)	0.0221	
12	Non-ferrous Metals	75.7069	-0.1883	0.5031	0.4055	-0.0997	-0.1721	9.3375	0.0225	67.5629
		(36.3757)	(0.1621)	(0.0941)	(0.0888)	(0.0371)	(0.1455)	(0.0000)	0.2802	
13	Mech. Engineering	-174.7656	0.4283	0.4920	0.5574	-0.7188	0.0000	5.0700	0.0202	69.8450
		(77.2136)	(0.2484)	(0.0724)	(0.0990)	(0.1953)	(0.0000)	(0.0011)	0.1335	
14	Instr. Engineering	-55.2525	0.0000	0.3398	0.5397	-0.4622	0.0000	3.8408	0.0232	65.4613
		(33.7288)	(0.0000)	(0.0798)	(0.1283)	(0.0989)	(0.0000)	(1.4745)	0.0385	
15	Elect. Engineering	-147.1293	0.0000	0.4435	0.6658	-0.5006	0.0000	2.3990	0.0183	71.9402
		(32.1978)	(0.0000)	(0.0625)	(0.0576)	(0.0616)	(0.0000)	(2.1855)	0.0438	
16	Ship Building	-35.2346	0.0000	0.2755	0.8839	0.0000	0.0000	2.3188	0.0273	60.4602
		(10.6987)	(0.0000)	(0.0596)	(0.0340)	(0.0000)	(0.0000)	(0.3136)	0.0717	
17	Motor Vehicles	-80.4445	-0.5104	0.3540	0.2526	-0.2730	0.5787	0.8899	0.0212	69.1531
		(45.5710)	(0.1434)	(0.0414)	(0.0976)	(0.0684)	(0.1277)	(0.0000)	0.0292	
18	Aerospace Equipment	300.4468	-0.4353	0.1133	0.3192	0.0000	-0.3018	3.2089	0.0269	60.1555
		(84.7415)	(0.1426)	(0.0663)	(0.1488)	(0.0000)	(0.1227)	(0.1579)	0.0134	
19	Other Vehicles	-130.8584	-0.2149	0.3656	0.7052	-0.1870	0.2579	1.3417	0.0315	58.4228
		(87.2358)	(0.2020)	(0.0788)	(0.1090)	(0.0636)	(0.2139)	(0.0000)	0.0885	
20	Metal Goods nes	-38.9423	0.0000	0.3711	0.6497	-0.2014	0.0000	5.4613	0.0169	74.0920
		(22.1737)	(0.0000)	(0.0396)	(0.0712)	(0.0227)	(0.0000)	(0.3004)	0.0862	
21	Textiles	-190.0781	0.0000	0.3607	0.5449	-0.4710	0.3909	1.5493	0.0188	71.7816
		(41.9485)	(0.0000)	(0.0735)	(0.0839)	(0.0621)	(0.1172)	(0.1020)	0.0206	
22	Leather, Clothing et	-16.7573	-0.3231	0.3659	0.3654	-0.3769	0.1333	4.1750	0.0129	82.6064
		(24.7463)	(0.1598)	(0.0666)	(0.1003)	(0.0716)	(0.0828)	(0.0000)	0.0000	
23	Bricks	-204.2333	-0.2881	0.3679	0.6358	-0.4899	0.3084	2.5290	0.0169	75.2218
		(58.5311)	(0.1464)	(0.1136)	(0.0818)	(0.0788)	(0.1986)	(0.0000)	0.0418	

Table 2
(contd.)

Part 2

		INPT	TIME	LY	LEH1	LW	SLY	BOXP/RST	SE/LM1	LLF
24	Timber & Furniture	113.0376	-0.3327	0.2940	0.2198	-0.1579	0.1333	1.5096	0.0135	81.2999
		(32.2402)(	0.1102)(	0.0572)(	0.0952)(	0.0512)(	0.1164)(	0.0000)	0.0044	
25	Paper & Board	-17.1064	0.0000	0.5755	0.2931	-0.3485	0.0000	5.8812	0.0184	71.7713
		(17.0776)(	0.0000)(	0.0531)(	0.0817)(	0.0264)(	0.0000)(	1.4927)	0.3680	
26	Printing & Publishin	84.9130	0.0000	0.3074	0.6204	-0.1660	-0.2623	3.7091	0.0207	69.1133
		(37.1731)(	0.0000)(	0.1219)(	0.1160)(	0.0979)(	0.1234)(	0.0110)	0.3051	
27	Other Manufacturing	-34.8621	-0.8107	0.5440	0.6313	0.0000	0.0000	4.8782	0.0196	70.0762
		(24.8857)(	0.0802)(	0.0638)(	0.0952)(	0.0000)(	0.0000)(	0.8867)	0.2560	
28	Construction	-90.9475	0.0000	0.5854	0.3001	-0.6651	0.0000	10.0811	0.0226	66.1794
		(33.7546)(	0.0000)(	0.0796)(	0.0985)(	0.0824)(	0.0000)(	2.0332)	0.4860	
29	Gas	-101.6012	-0.4864	0.0514	0.6667	-0.1705	0.4874	3.6843	0.0247	65.0320
		(63.7289)(	0.1606)(	0.0386)(	0.1250)(	0.0398)(	0.1083)(	0.0000)	0.0125	
30	Electricity	102.4103	0.0000	0.2770	0.6381	-0.1743	-0.3528	3.0545	0.0237	65.4847
		(42.8713)(	0.0000)(	0.1058)(	0.1066)(	0.0655)(	0.1553)(	0.6913)	0.0498	
31	Water	-5.7136	0.5776	1.3500	0.2529	-0.4686	-0.7631	3.8952	0.0440	49.4163
		(78.8126)(	0.2350)(	0.4401)(	0.1625)(	0.1197)(	0.2748)(	0.0000)	0.2023	
32	Rail	-72.6607	0.0000	0.4099	0.8840	0.0000	0.0000	7.0756	0.0263	61.5458
		(18.7943)(	0.0000)(	0.0974)(	0.0285)(	0.0000)(	0.0000)(	3.5557)	0.0083	
33	Road	-24.7397	-0.2445	0.1006	0.8193	-0.0657	0.1465	3.4204	0.0166	75.7122
		(75.7105)(	0.1067)(	0.0709)(	0.1377)(	0.0630)(	0.1103)(	0.0000)	0.3575	
34	Other Transport	-9.6232	0.0000	0.2546	0.6499	-0.2249	0.0000	5.9906	0.0167	74.2998
		(63.7328)(	0.0000)(	0.0778)(	0.1183)(	0.0507)(	0.0000)(	0.0380)	0.1205	
35	Communication	-32.5031	-0.6150	0.4743	0.6237	-0.1050	0.0000	9.0106	0.0199	70.1603
		(63.3698)(	0.2304)(	0.1723)(	0.1233)(	0.0996)(	0.0000)(	0.0107)	0.6006	
36	Distribution	67.5730	0.2837	0.0000	0.6909	-0.2725	0.0000	0.4881	0.0154	76.4538
		(34.4184)(	0.1272)(	0.0000)(	0.0820)(	0.0887)(	0.0000)(	2.3529)	0.0098	
37	Business Services	201.3484	0.7484	0.0000	0.4314	-0.1374	0.0000	8.2349	0.0153	76.7630
		(70.8778)(	0.1822)(	0.0000)(	0.1689)(	0.0526)(	0.0000)(	0.7364)	0.0465	
38	Professional Service	179.2274	0.6116	0.0000	0.4273	-0.1871	0.0000	3.6567	0.0191	70.7778
		(55.4294)(	0.1440)(	0.0000)(	0.1526)(	0.0575)(	0.0000)(	0.3334)	0.0003	
39	Misc. Services	-72.1981	0.0000	0.4162	0.8153	-0.1325	-0.1151	2.9192	0.0178	73.1912
		(62.4282)(	0.0000)(	0.1276)(	0.0984)(	0.0752)(	0.0812)(	1.7960)	0.0097	

Table 3

Disaggregate Employment functions
Manufacturing industries (Unrestricted)

		INPT	TIME	LY	LEH1	LW	SLY	BOXP/RST	SE/LM1	LLF
5	Food Manufacturing	-69.9065	-0.2113	0.4083	0.6541	-0.1612	0.2075	2.3353	0.0143	79.8058
		(46.4236)	(0.1097)	(0.1686)	(0.1779)	(0.0628)	(0.0637)	(0.0000)	0.0012	
6	Drink	37.8777	-0.0240	-0.0633	0.6878	-0.0225	0.1709	2.3854	0.0264	63.1817
		(42.5710)	(0.2150)	(0.3053)	(0.1672)	(0.0555)	(0.2249)	(0.0000)	0.0622	
7	Tobacco	59.9267	-0.4048	-0.0587	0.2969	0.0441	0.4002	1.3067	0.0370	54.0657
		(63.3658)	(0.1414)	(0.2556)	(0.3029)	(0.0439)	(0.1300)	(0.0000)	0.2964	
8	Coal Products	-102.6437	-0.4507	0.5505	0.0545	-0.4116	0.5755	3.7458	0.0396	52.2817
		(28.5429)	(0.1250)	(0.1706)	(0.1805)	(0.0471)	(0.1930)	(0.0000)	0.0090	
9	Petroleum Products	-46.3237	-0.2844	0.1177	0.5663	-0.2684	0.3013	3.7762	0.0582	41.8678
		(63.0851)	(0.1663)	(0.3037)	(0.1629)	(0.1313)	(0.5396)	(0.0000)	0.0081	
10	Chemicals	9.0338	0.3114	-0.1667	0.4881	-0.3249	0.5069	1.7416	0.0137	80.8669
		(33.4605)	(0.1176)	(0.1369)	(0.1224)	(0.0578)	(0.1530)	(0.0000)	0.0156	
11	Iron & Steel	-101.2095	-0.2384	0.1845	0.5752	-0.3533	0.5849	1.3203	0.0264	63.2336
		(25.7308)	(0.1127)	(0.0850)	(0.1017)	(0.0909)	(0.1734)	(0.0000)	0.0124	
12	Non-ferrous Metals	38.5559	-0.1520	0.5621	0.4168	-0.0922	-0.2026	9.2425	0.0220	68.0841
		(17.4252)	(0.0577)	(0.1126)	(0.0866)	(0.0354)	(0.1342)	(0.0000)	0.2915	
13	Mech. Engineering	-118.2132	0.2620	0.3924	0.5747	-0.7397	0.1558	6.0826	0.0200	70.7006
		(45.9703)	(0.1422)	(0.1102)	(0.0970)	(0.1900)	(0.1329)	(0.0000)	0.1918	
14	Instr. Engineering	-29.0963	0.0169	0.1796	0.5157	-0.3635	0.2304	4.5045	0.0225	67.4877
		(23.8623)	(0.1167)	(0.1326)	(0.1251)	(0.1195)	(0.1450)	(0.0000)	0.0267	
15	Elect. Engineering	-94.5352	0.0254	0.4694	0.6783	-0.5408	-0.0363	1.6317	0.0190	72.0684
		(27.3875)	(0.1267)	(0.1189)	(0.0665)	(0.1304)	(0.1043)	(0.0000)	0.1036	
16	Ship Building	-11.7981	-0.0166	0.2805	0.8954	0.0350	-0.0374	3.1626	0.0290	60.6709
		(36.1130)	(0.0883)	(0.0884)	(0.1280)	(0.0771)	(0.1178)	(0.0000)	0.0692	
17	Motor Vehicles	-31.9425	-0.0981	0.3037	0.2407	-0.2896	0.5550	1.3943	0.0221	67.9997
		(26.6771)	(0.0605)	(0.0511)	(0.1031)	(0.0715)	(0.1340)	(0.0000)	0.0366	
18	Aerospace Equipment	128.7433	-0.3205	0.1264	0.3920	-0.0406	-0.1668	3.1780	0.0302	59.5682
		(49.4535)	(0.0882)	(0.0702)	(0.1413)	(0.0967)	(0.0985)	(0.0000)	0.0112	
19	Other Vehicles	-87.9142	-0.0832	0.3672	0.6766	-0.2462	0.2984	1.3204	0.0305	59.3497
		(42.4140)	(0.0889)	(0.0744)	(0.0923)	(0.0755)	(0.1709)	(0.0000)	0.0854	
20	Metal Goods nes	-48.7532	-0.0153	0.2632	0.6279	-0.2999	0.2008	4.1928	0.0168	75.4260
		(30.2581)	(0.0552)	(0.0856)	(0.0821)	(0.1054)	(0.1392)	(0.0000)	0.1348	
21	Textiles	-75.1687	-0.0705	0.2705	0.4100	-0.4512	0.4750	1.9783	0.0158	77.0691
		(21.1369)	(0.1604)	(0.0861)	(0.1108)	(0.0563)	(0.1358)	(0.0000)	0.0000	
22	Leather, Clothing et	5.0914	-0.1854	0.3423	0.3194	-0.3707	0.1376	4.8940	0.0126	83.1847
		(17.1097)	(0.0840)	(0.0696)	(0.1066)	(0.0703)	(0.0722)	(0.0000)	0.0009	
23	Bricks	-113.3054	-0.1076	0.2175	0.5689	-0.4906	0.5007	2.4113	0.0129	82.6079
		(19.2596)	(0.0474)	(0.0808)	(0.0594)	(0.0597)	(0.1135)	(0.0000)	0.0031	
24	Timber & Furniture	73.6216	-0.1446	0.2186	0.1170	-0.2186	0.2419	0.7143	0.0125	83.4382
		(17.0026)	(0.0330)	(0.0639)	(0.1033)	(0.0557)	(0.1067)	(0.0000)	0.0007	
25	Paper & Board	-18.2006	0.0609	0.5793	0.3085	-0.4008	-0.0199	4.8250	0.0186	72.6048
		(14.9974)	(0.0569)	(0.1052)	(0.0862)	(0.0530)	(0.1205)	(0.0000)	0.3691	
26	Printing & Publishin	32.8236	-0.0993	0.4207	0.5899	-0.2250	-0.2959	3.0112	0.0205	70.0483
		(17.6484)	(0.0638)	(0.1476)	(0.1151)	(0.1318)	(0.1292)	(0.0000)	0.1787	
27	Other Manufacturing	6.3166	-0.3731	0.1335	0.3035	0.0202	0.6794	5.5287	0.0156	77.3350
		(38.0470)	(0.0960)	(0.1174)	(0.1274)	(0.0961)	(0.1829)	(0.0000)	0.2166	

Table 4

Disaggregate Employment functions
Manufacturing industries (Restricted)

		INPT	TIME	LY	LEH1	LW	SLY	BOXP/RST	SE/LM1	LLF
5	Food Manufacturing	-69.9065	-0.2113	0.4083	0.6541	-0.1612	0.2075	2.3353	0.0143	79.8058
		(46.4236)(	0.1097)(	0.1686)(	0.1779)(	0.0628)(	0.0637)(	0.0000)	0.0012	
6	Drink	20.9370	0.0000	0.0000	0.7258	-0.0625	0.1272	2.3951	0.0256	62.7993
		(34.5354)(	0.0000)(	0.0000)(	0.1597)(	0.0207)(	0.0698)(	0.6034)	0.0883	
7	Tobacco	44.3500	-0.3258	0.0000	0.3534	0.0000	0.3490	1.0884	0.0362	53.4260
		(40.8661)(	0.1033)(	0.0000)(	0.2969)(	0.0000)(	0.1168)(	1.0191)	0.3201	
8	Coal Products	-97.0666	-0.4539	0.5822	0.0000	-0.4179	0.5596	3.6448	0.0387	52.2232
		(21.7750)(	0.1246)(	0.1345)(	0.0000)(	0.0421)(	0.1857)(	0.0913)	0.0076	
9	Petroleum Products	-21.1400	-0.3517	0.2717	0.5235	-0.3106	0.0000	3.3607	0.0573	41.6689
		(44.1095)(	0.1147)(	0.1271)(	0.1438)(	0.1074)(	0.0000)(	0.3118)	0.0008	
10	Chemicals	-9.7935	0.1831	0.0000	0.5694	-0.3073	0.3346	2.3732	0.0139	79.9467
		(29.6699)(	0.0520)(	0.0000)(	0.1025)(	0.0560)(	0.0581)(	1.4812)	0.0001	
11	Iron & Steel	-101.2095	-0.2384	0.1845	0.5752	-0.3533	0.5849	1.3203	0.0264	63.2336
		(25.7308)(	0.1127)(	0.0850)(	0.1017)(	0.0909)(	0.1734)(	0.0000)	0.0124	
12	Non-ferrous Metals	38.5559	-0.1520	0.5621	0.4168	-0.0922	-0.2026	9.2425	0.0220	68.0841
		(17.4252)(	0.0577)(	0.1126)(	0.0866)(	0.0354)(	0.1342)(	0.0000)	0.2915	
13	Mech. Engineering	-103.0669	0.2526	0.4920	0.5574	-0.7188	0.0000	5.0700	0.0202	69.8450
		(44.1170)(	0.1419)(	0.0702)(	0.0959)(	0.1892)(	0.0000)(	1.3740)	0.2192	
14	Instr. Engineering	-27.0728	0.0000	0.1899	0.5133	-0.3570	0.2184	4.1306	0.0220	67.4742
		(19.3240)(	0.0000)(	0.1118)(	0.1240)(	0.1108)(	0.1188)(	0.0209)	0.0274	
15	Elect. Engineering	-86.7686	0.0000	0.4435	0.6658	-0.5006	0.0000	2.3990	0.0183	71.9402
		(19.8576)(	0.0000)(	0.0653)(	0.0602)(	0.0644)(	0.0000)(	0.2003)	0.1065	
16	Ship Building	-20.7794	0.0000	0.2755	0.8839	0.0000	0.0000	2.3188	0.0273	60.4602
		(6.3070)(	0.0000)(	0.0596)(	0.0340)(	0.0000)(	0.0000)(	0.3304)	0.0778	
17	Motor Vehicles	-31.9425	-0.0981	0.3037	0.2407	-0.2896	0.5550	1.3943	0.0221	67.9997
		(26.6771)(	0.0605)(	0.0511)(	0.1031)(	0.0715)(	0.1340)(	0.0000)	0.0366	
18	Aerospace Equipment	140.0893	-0.3346	0.1188	0.3900	0.0000	-0.1867	3.1505	0.0296	59.4556
		(41.3993)(	0.0816)(	0.0678)(	0.1412)(	0.0000)(	0.0863)(	0.1759)	0.0169	
19	Other Vehicles	-87.9142	-0.0832	0.3672	0.6766	-0.2462	0.2984	1.3204	0.0305	59.3497
		(42.4140)(	0.0889)(	0.0744)(	0.0923)(	0.0755)(	0.1709)(	0.0000)	0.0854	
20	Metal Goods nes	-53.6337	0.0000	0.2734	0.6405	-0.3178	0.1856	4.4205	0.0164	75.3769
		(24.5768)(	0.0000)(	0.0773)(	0.0685)(	0.0831)(	0.1279)(	0.0765)	0.1183	
21	Textiles	-82.9097	0.0000	0.2964	0.4493	-0.4656	0.4279	2.0671	0.0155	76.9454
		(11.6979)(	0.0000)(	0.0627)(	0.0655)(	0.0459)(	0.0835)(	0.1933)	0.0000	
22	Leather, Clothing et	5.0914	-0.1854	0.3423	0.3194	-0.3707	0.1376	4.8940	0.0126	83.1847
		(17.1097)(	0.0840)(	0.0696)(	0.1066)(	0.0703)(	0.0722)(	0.0000)	0.0009	
23	Bricks	-113.3054	-0.1076	0.2175	0.5689	-0.4906	0.5007	2.4113	0.0129	82.6079
		(19.2596)(	0.0474)(	0.0808)(	0.0594)(	0.0597)(	0.1135)(	0.0000)	0.0031	
24	Timber & Furniture	73.6216	-0.1446	0.2186	0.1170	-0.2186	0.2419	0.7143	0.0125	83.4382
		(17.0026)(	0.0330)(	0.0639)(	0.1033)(	0.0557)(	0.1067)(	0.0000)	0.0007	
25	Paper & Board	-19.4379	0.0562	0.5644	0.3119	-0.3984	0.0000	4.7433	0.0182	72.5873
		(12.9946)(	0.0491)(	0.0542)(	0.0837)(	0.0510)(	0.0000)(	0.0273)	0.3754	
26	Printing & Publishin	32.8236	-0.0993	0.4207	0.5899	-0.2250	-0.2959	3.0112	0.0205	70.0483
		(17.6484)(	0.0638)(	0.1476)(	0.1151)(	0.1318)(	0.1292)(	0.0000)	0.1787	
27	Other Manufacturing	-1.2506	-0.3557	0.1334	0.3169	0.0000	0.6880	5.5883	0.0153	77.3065
		(12.4952)(	0.0487)(	0.1174)(	0.1102)(	0.0000)(	0.1782)(	0.0443)	0.2161	

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Appendix A

The generalised goodness-of-fit criterion with instrumental variables estimation

In this appendix the validity of the generalised choice criterion proposed in Section 4 of the paper is shown when the aggregate and the disaggregate models are estimated by the IV method under parameter constraints. The disaggregate model is given as before by

$$\text{(A1)}\quad \tilde{H}_d: \begin{cases} \underset{\sim}{y}_i = X_i \underset{\sim}{\beta}_i + \underset{\sim}{u}_i, \\ R_i \underset{\sim}{\beta}_i = \underset{\sim}{r}_i \end{cases} \qquad i = 1,2, \ldots, m$$

and the aggregate model by

$$\text{(A2)}\quad \tilde{H}_a: \begin{cases} \bar{\underset{\sim}{y}} = Z\underset{\sim}{\gamma} + \bar{\underset{\sim}{u}}_a \\ R\underset{\sim}{\gamma} = \underset{\sim}{r} \end{cases}$$

but now parameter estimation is by constrained 2SLS using the sets of instruments W_i for the disaggregate specifications and W for the aggregate equation.

The constrained 2SLS estimates of the parameters of the disaggregate model are given by

$$\text{(A3)}\quad \tilde{\underset{\sim}{\beta}}_i = D_i \hat{X}_i{}' \underset{\sim}{y}_i + \underset{\sim}{g}_i$$

where

$$\text{(A4)}\quad G_i = I_n - X_i D_i \hat{X}_i{}' = I_n - Q_i$$

$$\text{(A5)}\quad D_i = (\hat{X}_i{}'X_i)^{-1} - (\hat{X}_i{}'X_i)^{-1} R_i{}' C_i^{-1} R_i (\hat{X}_i{}'X_i)^{-1}$$

(A6) $\quad C_i = R_i(\hat{X}_i'X_i)^{-1}R_i'$

(A7) $\quad \underset{\sim}{g}_i = (\hat{X}_i'X_i)^{-1}R_i'C_i^{-1}\underset{\sim}{r}_i$

(A8) $\quad \hat{X}_i = W_i(W_i'W_i)^{-1}W_i'X_i.$

Similarly for the aggregate model we have

(A9) $\quad \tilde{\underset{\sim}{\gamma}} = D\hat{Z}'\bar{\underset{\sim}{y}} + \underset{\sim}{g}$

(A10) $\quad G = I_n - ZD\hat{Z}' = I_n - Q$

(A11) $\quad D = (\hat{Z}'Z)^{-1} - (\hat{Z}'Z)^{-1}R'C^{-1}R(\hat{Z}'Z)^{-1}$

(A12) $\quad C = R(\hat{Z}'Z)^{-1}R'$

(A13) $\quad \underset{\sim}{g} = (\hat{Z}'Z)^{-1}R'C^{-1}\underset{\sim}{r}$

(A14) $\quad \hat{Z} = W(W'W)^{-1}W'Z.$

Consider now the generalised goodness-of-fit criteria

(A15) $\quad s_a^2 = \bar{\underset{\sim}{e}}_a'\bar{\underset{\sim}{e}}_a/Tr(G'G)$

$\quad\quad = \bar{\underset{\sim}{e}}_a'\bar{\underset{\sim}{e}}_a/(n - 2\,\tilde{k} + Tr(Q'Q))$

and

(A16) $\quad s_d^2 = \sum_{i,j=1}^{m} \hat{\sigma}_{ij}$

where

(A17) $\quad \hat{\sigma}_{ij} = \{n - \tilde{k}_i - \tilde{k}_j + Tr(Q_i'Q_j)\}^{-1}\underset{\sim}{e}_i'\underset{\sim}{e}_j,$

with $\bar{\underset{\sim}{e}}_a = \bar{\underset{\sim}{y}} - Z\tilde{\underset{\sim}{\gamma}}$, $\underset{\sim}{e}_i = \underset{\sim}{y}_i - X_i\tilde{\underset{\sim}{\beta}}_i$, $\tilde{k}_i = k_i - q_i$, and $\tilde{k} = k - q$.

Under $\tilde{H}_d$

(A18) $\quad \underset{\sim}{e}_i = G_i \underset{\sim}{u}_i$

and

(A19) $\quad \bar{\underset{\sim}{e}}_a = G(\bar{\underset{\sim}{u}} + \underset{\sim}{\xi}) + Z(\hat{Z}'Z)^{-1}R'C^{-1}(R\underset{\sim}{\gamma} - \underset{\sim}{r})$,

so that, taking expectations conditional on X, Z, W_i and W,

(A20) $\quad E_d(s_d^2 | X, W_i) = \bar{\sigma}^2$

(A21) $\quad E_d(s_a^2 | X, W_i, Z, W) = \bar{\sigma}^2 + \frac{x'x}{Tr(G'G)}$,

where $x = G\underset{\sim}{\xi} + Z(\hat{Z}'Z)^{-1}R'C^{-1}(R\underset{\sim}{\gamma} - \underset{\sim}{r})$ which justifies, as before, the use of the following choice critèrion for discrimination between the aggregate and disaggregate models

Choose $\tilde{H}_d$ if $s_d^2 < s_a^2$, otherwise choose $\tilde{H}_a$.

Appendix B

A proof of the asymptotic validity of the proposed test of perfect aggregation

In this appendix we provide a proof of the theorem stated in the paper. Let

$$\text{(B1)} \qquad \underset{\sim}{q}_m = \left(\sum_{i=1}^{m} \hat{\sigma}_{ii} H_i^2\right)^{-\frac{1}{2}} (\bar{\underset{\sim}{e}}_a - \bar{\underset{\sim}{e}}_d)$$

where $\bar{\underset{\sim}{e}}_a$, $\bar{\underset{\sim}{e}}_d$, H_i and $\hat{\sigma}_{ii}$ are already defined in the text by the relations (3.1), (3.2), (5.5) and (5.10) respectively. Then the test statistic in the theorem can be written as

$$\text{(B2)} \qquad \underset{\sim}{a}_m = \underset{\sim}{q}_m' \underset{\sim}{q}_m .$$

Consider now the probability limit of $\hat{\psi}_m = \sum_{i=1}^{m} \hat{\sigma}_{ii} H_i^2$ as $m \to \infty$. Under (2.1) we obtain

$$\text{(B3)} \qquad \hat{\psi}_m = (n - k)^{-1} \sum_{i=1}^{m} (\underset{\sim}{u}_i' M_i \underset{\sim}{u}_i) H_i^2 .$$

But since M_i is an idempotent matrix of rank $n - k$, we can also write

$$\text{(B4)} \qquad \sigma_{ii}^{-1} \underset{\sim}{u}_i' M_i \underset{\sim}{u}_i = \sum_{t=1}^{n-k} \varepsilon_{it}^2, \qquad i = 1,2, \ldots, m$$

where ε_{it} represent scalar random variables distributed independently across i and t with zero means and unit variances. Substituting (B4) in (B3) yields

$$\text{(B5)} \qquad \hat{\psi}_m = (n - k)^{-1} \sum_{t=1}^{n-k} \left(\sum_{i=1}^{m} \sigma_{ii} \varepsilon_{it}^2 H_i^2 \right).$$

But, noting that $H_i = A_i - \bar{A}$, we have

(B6) $$\sum_{i=1}^{m} \sigma_{ii}\varepsilon_{it}^2 H_i^2 = f_m\bar{A} + F_m - F_m\bar{A} - \bar{A}F_m$$

where

$$f_m = \sum_{i=1}^{m} \sigma_{ii}\varepsilon_{it}^2$$

$$F_m = \sum_{i=1}^{m} \sigma_{ii}\varepsilon_{it}^2 A_i$$

Now under assumption 3 it readily follows that

$$\operatorname*{plim}_{m\to\infty}(f_m) = \operatorname*{plim}_{m\to\infty}(m^{-1}\sum_{i=1}^{m} \tau_{ii}\varepsilon_{it}^2) \leq \tau^2 \operatorname*{plim}_{m\to\infty}(m^{-1}\sum_{i=1}^{m} \varepsilon_{it}^2),$$

and since ε_{it} are identically and independently distributed random variables, then by the law of large numbers
$m^{-1}\sum_{i=1}^{m} \varepsilon_{it}^2 \overset{p}{\to} 1$, and

(B7) $$\operatorname*{plim}_{m\to\infty}(f_m) \leq \tau^2 < \infty.$$

Similarly, under assumptions 2 and 3 we have

$$\operatorname*{plim}_{m\to\infty}(\sum_{i=1}^{m} \sigma_{ii}\varepsilon_{it}^2 A_i) = \operatorname*{plim}_{m\to\infty}\{m^{-1}\sum_{i=1}^{m} \tau_{ii}\varepsilon_{it}^2 P_i\}.$$

Hence

(B8) $$\operatorname*{plim}_{m\to\infty}(F_m) \leq \tau^2 P < \infty$$

where P is already defined by assumption 2. The results (B7) and (B8) establish the existence of the probability limits of f_m and F_m, as $m \to \infty$, and this in turn establishes [using (B6)] that

$$\underset{m\to\infty}{\text{plim}}\left(\sum_{i=1}^{m} \sigma_{ii}\varepsilon_{it}^{2}H_{i}^{2}\right) = \lim_{m\to\infty}\left(\sum_{i=1}^{m} \sigma_{ii}H_{i}^{2}\right).$$

Using this result in (B5) we finally obtain

$$\text{(B9)} \qquad \hat{\psi}_m = \sum_{i=1}^{m} \hat{\sigma}_{ii}H_i^2 \overset{P}{\to} \lim_{m\to\infty}\left(\sum_{i=1}^{m} \sigma_{ii}H_i^2\right) = \psi.$$

Therefore, asymptotically we have[1]

$$\underset{\sim}{q}_m \overset{a}{\sim} \psi^{-\frac{1}{2}}(\bar{\underset{\sim}{e}}_a - \bar{\underset{\sim}{e}}_d).$$

But under (2.1) on the assumption that H_ξ holds

$$(\bar{\underset{\sim}{e}}_a - \bar{\underset{\sim}{e}}_d) = \sum_{i=1}^{m} H_i\underset{\sim}{u}_i.$$

Hence,

$$\text{(B10)} \qquad \underset{\sim}{q}_m \overset{a}{\sim} m^{-\frac{1}{2}} \sum_{i=1}^{m} \underset{\sim}{z}_i,$$

in which

$$\underset{\sim}{z}_i = (m\,\sigma_{ii})^{\frac{1}{2}}\, \psi^{-\frac{1}{2}}H_i\, \underset{\sim}{v}_i,$$

---

[1] Note that by assumption 4, matrix ψ is non-singular.

and $\underset{\sim}{v}_i = \underset{\sim}{u}_i/\sqrt{\sigma_{ii}}$. We now show that under the assumptions of the theorem, as $m \to \infty$, the sum $S_m = m^{-\frac{1}{2}} \sum_{i=1}^{m} \underset{\sim}{z}_i$ tends to a multivariate normal distribution with mean zero and the covariance matrix I_n; an identity matrix of order n. For this purpose it is sufficient to demonstrate that for any fixed vector $\underset{\sim}{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)'$, the limiting distribution of $\underset{\sim}{\lambda}'S_m$ is $N(0, \underset{\sim}{\lambda}'\underset{\sim}{\lambda})$.

Let

$$(B11) \qquad d_m = \underset{\sim}{\lambda}'S_m = \sum_{i=1}^{m} w_i,$$

in which

$$(B12) \qquad w_i = (m\,\sigma_{ii})^{\frac{1}{2}}\, \underset{\sim}{\lambda}'\psi^{-\frac{1}{2}} H_i \underset{\sim}{v}_i, \qquad\qquad i = 1, 2, \ldots, m$$

is now a scalar random variable. We have, for all i,

$$E(w_i) = 0$$

$$V(w_i) = m\,\sigma_{ii}\, \underset{\sim}{\lambda}'\psi^{-\frac{1}{2}} H_i^2 \psi^{-\frac{1}{2}} \underset{\sim}{\lambda}.$$

Setting $\underset{\sim}{\mu} = \psi^{-\frac{1}{2}}\underset{\sim}{\lambda}$, then

$$(B13) \qquad C_m^2 = \sum_{i=1}^{m} V(w_i) = m\,\underset{\sim}{\mu}'\Big(\sum_{i=1}^{m} \sigma_{ii} H_i^2\Big)\underset{\sim}{\mu}.$$

Denoting the (t, t') element of matrix H_i by $h_{i,tt'}$, we also have [using (B12)]

$$w_i = (m\,\sigma_{ii})^{\frac{1}{2}} \sum_{t'=1}^{n} \Big(\sum_{t=1}^{n} \mu_t\, h_{i,tt'} \Big) v_{it'}.$$

Therefore, since by assumption ψ is non-singular and $h_{i,tt'}$ are bounded in absolute value for all i, we can also write

$$|w_i| \leq n\kappa(m\ \sigma_{ii})^{\frac{1}{2}}\ |\sum_{t'=1}^{n} v_{it'}|,$$

where $|\mu_t h_{i,tt'}| < \kappa < \infty$. Consequently

$$E|w_i|^3 \leq n^3\kappa^3(m\ \sigma_{ii})^{3/2}\ E|\sum_{t=1}^{n} v_{it}|^3.$$

However, since the random variables v_{it} are i.i.d. with finite third order moments, then $E|\sum_{t=1}^{n} v_{it}|^3 \leq n\theta^3$, where $\theta^3 = E|v_{it}|^3$, and

(B14) $$E|w_i|^3 \leq n^4\kappa^3\theta^3(m\ \sigma_{ii})^{3/2}.$$

We are now in a position to apply the Liapunov Central Limit Theorem to the sum d_m defined by (B11).(1) Setting

$$B_m^3 = \sum_{i=1}^{m} E|w_i|^3,$$

then using (B14) it follows that

$$B_m^3 \leq (n^4\kappa^3\theta^3)\sum_{i=1}^{m}(m\ \sigma_{ii})^{3/2}$$

which together with (B13) yields(2)

---

(1) See, for example, Rao (1973, p. 127)

(2) Notice that $\lim_{m\to\infty}\{\underset{\sim}{\mu}'(\sum_{i=1}^{m}\sigma_{ii}H_i^2)\underset{\sim}{\mu}\} = \underset{\sim}{\mu}'\psi\underset{\sim}{\mu} = \underset{\sim}{\lambda}'\underset{\sim}{\lambda}$.

$$\lim_{m\to\infty}\left(\frac{B_m}{C_m}\right) \leq \left\{\frac{n^{4/3}\kappa\theta}{(\lambda'\lambda)^{\frac{1}{2}}}\right\} \lim_{m\to\infty}\left(\sum_{i=1}^{m} \sigma_{ii}^{3/2}\right)^{1/3}$$

But under assumption 3

$$\lim_{m\to\infty}\left(\sum_{i=1}^{m} \sigma_{ii}^{3/2}\right) = 0$$

and for fixed n $\lim(B_m/C_m) = 0$ as $m \to \infty$, and the condition of the Liapunov theorem will be met. Hence

$$\underset{\sim}{q}_m \overset{a}{\sim} S_m \overset{a}{\sim} N(0, I_n).$$

Now using (B2) we have

$$\underline{a}_m = \underset{\sim}{q}_m' \underset{\sim}{q}_m \overset{a}{\sim} \chi_n^2. \qquad \text{Q.E.D.}$$

Appendix C

Data Sources and Definitions

The data used in the empirical estimation in section 7 are annual UK observations obtained from the Cambridge Growth Project Databank. The data on industry man-hours, employment, wages and salaries and employers' contributions were originally provided by the Manpower Research Group at Warwick; data on industry output were provided by the CSO whilst data on the producer price indexes of industry output were obtained from a number of published sources including the Department of Industry and Trade's publication 'British Business', the 'Annual Abstract of Statistics' and the 'Energy Review'.

Some of the 39 industry groups used in the empirical analysis are identical to the 'Industrial Orders' distinguished in the 1968 Standard Industrial Classification. However in view of the significant differences between them in a large number of cases, the groups are listed in Table C1, using as a reference the Minimum List Headings of the 1968 Standard Industrial Classification. In the analysis of the manufacturing sector industry groups 5 to 27 inclusive are included.

Table C1

Classification of industry groups (based on the 1968 Standard Industrial Classification)

Industries		Minimum List Headings
1.	Agriculture, forestry and fishing	001, 002, 003
2.	Coal mining	101
3.	Mining nes	102, 103, 109
4.	Petroleum and natural gas	104
5.	Food manufacturing	211-219, 221, 229
6.	Drink	231, 232, 239
7.	Tobacco	240
8.	Coal products	261
9.	Petroleum products	262, 263
10.	Chemicals etc.	271-279
11.	Iron and steel	311-313
12.	Non-ferrous metal	321-323
13.	Mech. engineering	331-339, 341, 342, 349
14.	Inst. engineering	351-354
15.	Elect. engineering	361-369
16.	Shipbuilding	370
17.	Motor vehicles	381
18.	Aerospace equipment	383
19.	Other vehicles	380, 382, 384, 385
20.	Metal goods nes	390-396, 399
21.	Textiles	411-419, 421-423, 429
22.	Leather, clothing etc.	431-433, 441-446, 449, 450
23.	Bricks, pottery, glass	461-464, 469
24.	Timber and furniture	471-475, 479
25.	Paper and board	481-484
26.	Printing and publishing	485, 486, 489
27.	Other manufacturing	491-496, 499
28.	Construction	500
29.	Gas	601
30.	Electricity	602
31.	Water	603
32.	Rail transport	701
33.	Road transport	702-704
34.	Other transport	705-707, 709
35.	Communications	708
36.	Distribution	810-812, 820, 821, 831, 832
37.	Financial services	861-866
38.	Professional services	881-899
39.	Misc. services	901, 906

For empirical estimation, the man hours employed (LEH_i) are defined as a product of the actual hours worked per week in 39 industries and the numbers employed, including self employed ('000s) in these industries. Industry output (LY_i) is gross value added by industry in 1975 prices (£m). Average real wage rate (LW_i) is a measure of the real product wage by industry. It is obtained by first deflating an industry's total labour costs including both employees' wages and salaries and employers' national insurance contributions (£m) by the price index of industry output (1975 = 1.00). This is then divided by the man hours employed in that industry to obtain the average real wage rate.

All the data are annual covering the period 1954-81 with both the aggregate and disaggregate equations estimated over the period 1955-81. These data, and the computer programmes used both in estimation and in the computation of the choice criterion and the test of aggregation, are available on request from the authors.

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