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Estimation of complete demand systems: the trinomial expenditure system in comparison with alternative demand systems

ESTIMATION OF COMPLETE DEMAND SYSTEMS: THE TRINOMIAL EXPENDITURE SYSTEM IN COMPARISON WITH ALTERNATIVE DEMAND SYSTEMS (*)

by

Carlo Andrea Bollino

In this paper, I consider the issue of how total expenditure enters in the consumer demand equations and the impact of demographic characteristics. I introduce a new class of demand systems (labeled "trinomial expenditure system", TES) which nests, among others, the linear expenditure system, the quadratic expenditure system and the PIGL system. Empirical results are based on Italian household budget data (1973-78). While previous studies in Italy have focused upon time series data, this is the first attempt to view the Italian household behavior from a different perspective. On the basis of the estimates I first confirm the validity of the TES with respect to alternative forms and I next consider the effect of family size on income and price elasticities. The comparison is carried on in terms of likelihood values, regularity conditions and accuracy of estimates both inside and outside the sample.

ESTIMATION OF COMPLETE DEMAND SYSTEMS: THE TRINOMIAL EXPENDITURE SYSTEM IN COMPARISON WITH ALTERNATIVE DEMAND SYSTEMS(*)

1 - Introduction

The purpose of this paper is to estimate a complete demand system grouped from household budget data, addressing the issue of choice among functional forms and the impact of demographic characteristics. A new functional form, the Trinomial Expenditure System, is introduced and used in the estimation stage. All the empirical results are based on Italian household budget data for the period 1973-1978. While

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previous studies [1] of Italian consumption patterns have focused upon time series data from National Income Accounts, the present study is a first attempt to view the Italian household behavior from a different perspective.

In this paper I refer to "complete demand systems" as the result of a well behaved model of consumer behavior, i.e., those systems which are derived from well defined optimization hypotheses. The purpose of the exercise, then, is to characterize consumer responses to both total expenditure and relative price variations.

In particular, I explore demand systems which are non-linear in total expenditure and I introduce a new class, whose relevant properties are summarized in a theorem.

In addition, I consider the problem of demographic effects on household consumption, using theoretically plausible procedures such as "demographic translating", i.e., parametric representations which are consistent with the well behaved nature of the underlying consumer behavior model.

The structure of the paper is as follows: Section 2 briefly discusses some existing demand systems in relation with the Trinomial Expenditure System (TES), which is a generalization of both Quadratic Expenditure System (QES), and the Price Independent Generalized Linearity (PIGL) system. Section 3 describes the data and the estimation procedure. Section 4 discusses the relative empirical performance of alternative demand systems in terms of likelihood values (where nested testing is appropriate) and in terms of forecasting accuracy outside the sample. Section 5 is a brief summary of the results.

2

2 - Theory

Many demand systems have been discussed and estimated in the literature since the pioneering Linear Expenditure System (LES), characterized by Klein and Rubin (1947), Samuelson (1947), Geary (1949) and estimated by Stone (1954), Pollak and Wales (1969) and others. The Addilog System was proposed by Houthakker (1960). The Generalized Constant Elasticity Substitution (CES) was proposed by Christensen (1967) and Pollak (1967) and estimated by Wales (1971). The Quadratric Expenditure System (QES) has been estimated by Pollak and Wales (1978) and discussed by Howe, Pollak and Wales (1979). The basic Translog System (BTL) was estimated by Christensen Jorgensen and Lau (1975) and the Generalized Translog (GTL) was introduced and estimated by Pollak and Wales (1980).

Most of these studies share a common line of approach, namely the use of theoretical restrictions as a maintained hypothesis for estimation purposes. In other words, a parametric representation of demand functions is fitted to a particular data set in order to analyze the consumer reaction to given price-expenditure situations. However, whether the estimation is based on time series of per capita data derived from national income statistics or on time series of cross section of random samples of households, a problem of aggregation arises [2].

In this respect, a maintained functional form derived from utility theory is primarily viewed in this paper as a useful construct for organizing a priori assumptions and analyzing and interpreting the regularities in the data. It would not seem appropriate, however, to use it tout court to confirm or refute the underlying theory, which applies to individual rather than market phenomena.[3] This leads to the problem of choosing a particular function to analyze the available data. In other words, a set of parameters derived from theoretical demand restrictions is used to characterize and test the consumer's allocation mechanism.

Three main issues have been addressed in the literature with regard to the specification of the number of independent parameters needed to characterize a demand system. First, there is the issue of functional flexibility (e.g.., Diewert (1974), Christensen Jorgenson and Lau (1975), Berndt and Khaled (1979), Appelbaum (1979), Deaton and Muellbauer (1980). Second, there is the issue of functional separability (e.g., Houthakker (1960), Pollak (1972), Blackorby, Primont and Russel (1977). Third, there is the issue of how expenditure enters the demand equations (e.g., Pollak (1971), Muellbauer (1975), Howe, Pollak and Wales (1978).

In this paper I shall focus primarily on the third issue. It is worth noting that the parametric effect of total expenditure on the demand functions impinges directly upon the direction and magnitudes of the income effect in the Slutzky equation and of the derived income elasticities of each commodity.

From an empirical viewpoint, this issue seems crucially relevant in the present analysis of household budget data, for the large variability of total expenditure has to be accounted for with sufficient precision in the estimation stage. It seems natural to begin with the case of expenditure linearity. Gorman (1961) has characterized the class of demand systems which are linear in total expenditure:

$$(1) \quad h^i(p,\mu) = A^i(p) + B^i(p)\,\mu \qquad i=1,\dots n$$

where: P is a vector of prices
μ is total expenditure.

Equation (1) depicts a linear relationship in the consumption-expenditure space. The coefficients A and B are, in general, functions of all prices and embody suitable restrictions derived from a well behaved optimization problem. Howe, Pollak and Wales (1979) have investigated quadratic forms and they have characterized the class of such demand systems:

$$(2) \quad h^i(p,\mu) = A^i(p) + B^i(p)\,\mu + C^i(p)\,\mu^2 \quad i=1,\dots n$$

The terminology associated with equation (2) is self explanatory, for the C coefficient subsumes an expenditure effect of the second order on the optimal consumption choice.

Muellbauer (1975), in the discussion of the aggegation problem, has characterized a demand system of the form:

$$(3) \quad h^i(p,\mu) = B^i(p)\,\mu + C^i(p)\,\mu^\varepsilon \qquad \varepsilon\neq 0 \quad i=1,\dots n$$

which includes (2) when $A^i(p) = 0$ and $\varepsilon = 2$.

Equation (3) represents the "Price Independent-Generalized Linearity" (PIGL) demand system, which allows exact aggregation across consumers.

Finally, Gorman (1981) has discussed demand functions which are polynomial in expenditure form:

$$(4) \quad h^i(p,\mu) = \sum_{k=1}^{K} f^k(p)\, g^k(\mu) \qquad i = 1,\dots n;\ k=1,\dots K$$

showing that little can be gained when $K > 3$.

Quadratic systems and in general polynomial functions in expenditure have been critized by Houthakker (1952), first on theoretical and second on empirical grounds. In fact, quadratic systems do not satisfy the non-negativity condition for all price-expenditure situations. In addition, in the context of empirical estimation a variable and its powers are highly intercorrelated, possibly weakening the precision of the estimated results.

While the above remarks are undoubtedly valid, Howe, Pollak and Wales (1979) argue that there is no objection in principle to systems "locally quadratic in expenditure," if we confine ourselves to a subregion of all possible price-expenditure situations. This regions would be spanned by the "committed quantities" vector, which is generally defined as the minimum subsistence bundle of the household.([4]) Moreover, the collinearity of several total expenditure terms in demand functions is no more serious a problem than the usual price collinearity, insofar as suitable parametric restrictions are imposed on the demand functions, as is done by the translog system, for one From an empirical viewpoint, investigation of non-linear forms in total expenditure could be justified on the basis of the data under study, if there is enough evidence of departure from linearity in the consumption-expenditure space.

The crucial problem in the specification of a demand system is, therefore, to find a parameterization that is theoretically plausible, capable of interpreting the data with sufficient precision yet not excessively demanding in terms of dimensionality of the parameter space.

With this consideration in mind, I propose an expression which involves three terms in total expenditure:

$$(5) \quad h^i (p,\mu) = A^i (p) + B^i (p) \mu + C^i (p) \mu^{1+\lambda}$$

The characterization of the class of (5), named Trinomial Expenditure System (TES), is discussed in Appendix A. This allows us to derive a parameterization of the TES demand functions such as:

$$(6) \quad h^i(p,\mu) = \gamma_i + \frac{(\mu-\Sigma p\gamma)}{p_i} [a_i + \delta(c_i - a_i) \, \Pi p_k^{-\lambda c_k} (\mu - \Sigma p\gamma)^\lambda] \quad i=1,\ldots n$$

corresponding to the indirect utility function:

$$(7) \quad \Psi(p,\mu) = - \frac{k(p)}{[\mu-g(p)]^\lambda} - \frac{t(p)}{k(p)}$$

where: $g(p) = \Sigma_k p_k \gamma_k$

$k(p) = \pi_k p_k^{\lambda a_k} \qquad \Sigma_k a_k = 1$

$t(p) = \delta k(p)^2 / \pi p_k^{\lambda c_k} \qquad \Sigma_k c_k = 1$

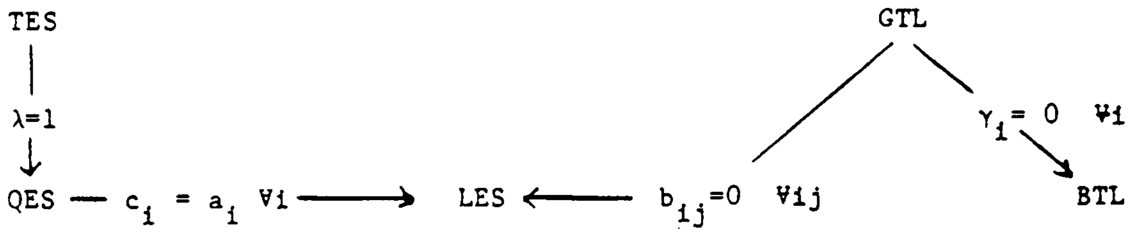
It is evident that the restriction $\lambda = 1$ in (6) and (7) yields the QES of Pollak and Wales (1980) which belongs to the class of (2), while the restriction $c_i = a_i$ $\forall i$ yields the LES. Alternatively, the restriction $\gamma_i = 0$ $\forall i$ yields the PIGL (Muellbauer, 1975), which belongs to the class of (3).

From an empirical viewpoint, the TES seems to be a manageable functional form since it adds only one parameter to the QES and thus shares with this latter the characteristic of being "parsimonious" in the parameters required as the number of commodities increases. Inspection of (6) reveals that the TES contains 3n independent parameters. On the other hand, among other nonlinear systems, we find that the BTL requires the estimation of $(n + 3n - 2)/2$ independent parameters and the GTL - a system of the translog family obtained by introducing "committed quantities" in the BTL - contains $(n^2 + 5n - 2)/ 2$ parameters.(5)

It is interesting to note that the LES can be obtained in two alternative ways, as a special case of either the GTL or the TES. This is not surprising, for the LES is nested in each of the above systems. This point highlights the difference between the path of assumptions leading from the LES to the GTL versus the alternative path to the TES. Figure 2.1 illustrates the nesting structure that interrelates the alternative systems estimated in this paper. Arrows in the figure represent the direction of nesting suitable for testing.

FIGURE 2.1

Alternative Demand System



where:

LES: $h^i(p,\mu)=\gamma_i+(a_i/p_i)\,(\mu-\sum_k p_k\gamma_k)$ $\qquad \Sigma a_i=1$

QES: $h^i(p,\mu)=\gamma_i+(a_i/p_i)\,(\mu-\sum_k p_k\gamma_k)+\dfrac{\delta(c_i-a_i)\Pi p_k^{-c_k}\,(\mu-\sum_k p\gamma)^2}{p_i}$ $\qquad \Sigma a_i=1$, $\Sigma c_i=1$

TES: $h^i(p,\mu)=\gamma_i+(a_i/p_i)\,(\mu-\sum_k p\gamma_k)+\dfrac{\delta(c_i-a_i)\prod_k p_k^{-\lambda c_k}\,(\mu-\sum_k p_k\gamma_k)^{1+\lambda}}{p_i}$ $\qquad \Sigma a_i=1$, $\Sigma c_i=1$

BTL: $h^i(p,\mu)=\dfrac{\mu}{p_i}\;\dfrac{a_i+\sum_j b_{ij}\lg p_j/\mu}{\Sigma a_k+\sum_k\sum_j b_{jk}\lg p_k/\mu}$ $\qquad \sum_k a_k+\sum_k\sum_j b_{kj}=1$, $b_{kj}=b_{jk}$

GTL: $h^i(p,\mu)=\gamma_i+\dfrac{(\mu-\Sigma p_j\gamma_j)}{p_i}\;\dfrac{a_i+\sum_j b_{ij}\lg\left(\dfrac{p_j}{\mu-\sum_k p_k\gamma_k}\right)}{\sum_k a_k+\sum_k\sum_j b_{kj}\lg\left(\dfrac{p_j}{\mu-\sum_k p_k\gamma_k}\right)}$ $\qquad \sum_k a_k+\sum_k\sum_j b_{kj}=1$, $b_{kj}=b_{jk}$

3 - Data and estimation procedure

The data set used in this paper is reported in the series : "Consumi delle Famiglie" - Supplemento al Bollettino di Statistica - ISTAT. This publication reports, among other data, total expenditure, expenditures on about 79 consumption items (such as meat, bread, vegetables, etc.) and some demographic characteristics of Italian households, aggregated from a survey of about 30,000 households. The set thus consists of grouped rather than micro-level data.

The sample is chosen every year by the Italian Central Statistical Institute (ISTAT) as representative of the population. Data collection is carried on in two steps. First, food expenditure are recorded by the household in a booklet for a period of ten days. At the end of this period, non food expenditures for the entire month are recorded by an interviewer during an interview session with the household members. Although collaboration with ISTAT is compulsory by law, it is conceivable that data on non-food expenditures are less accurate than on food expenditures [6].

I use data for the five years 1973-1978 - the longest interval over which the available published data are homogeneously defined [7]. Specifically, data are reported in two alternative ways. In the first data set, there are 18 expenditure classes in 1973-1975 and 19 classes in 1976-1978 for a total of 111 price expenditure observations. In the second data set, for all six years there is a cross-classification by four classes of economic condition and five classes of family size, for a total of 120 price-expenditure observations [8].

The price vector has been derived from the Consumer Price Index series ("Annuario di Statistica" - ISTAT), which follows a definition of consumption categories consistent with the household survey. For computational simplicity, I have analyzed three consumption categories: "food", "clothing", and "miscellaneous", excluding durables [9]. The treatment of such data requires a set of assumptions for the estimation.

First, saving is not considered explicitly, for only total expenditure is available. Moreover, the household allocation procedure is viewed as a static one, which seems appropriate, given the relatively short period considered.

Second, non-durable goods are assumed to be separable from services of durables. This assumption is really dictated by the lack of reliable data on the flows of durable services ([10]). In addition, the data report "allowances" to children. Exclusion of this category has to be justified by assuming that it is separable from the rest of the consumption categories.

Third, as far as health care expenditures are concerned, some hospital and medical services are provided under the direct control of the Italian state. Inclusion of these categories could be justified by assuming that there is no rationing.

Fourth, all households are assumed to face the same prices. This is equivalent to assuming that there are no regional price differences and that all classes face the same prices.

Finally, prices are assumed to be exogenous. This last assumption, which may be controversial in the realm of aggregate analyses based on National Income Account data, seems plausible in the present context, where grouped household budget data are considered ([11]).

The stochastic specification of the demand system is assumed to be additive in the share form of each equation. Specifically, $E(u_i) = 0$ and $E(u_i u_i') = \Omega^*$ where u_i are normally and independently distributed. Ω^* is obviously singular, reflecting the budget identity. The estimated form is given for each system by the equations of Fig. 2.1 with an error term appended. The estimation is thus performed on the (n - 1) subsystem of equations without loss of efficiency or estimate invariance (Barten (1969) and Pollak-Wales (1969)). All the estimates discussed in the next section are obtained with a non linear FIML procedure (Berndt, Hall, Hall and Hausman (1974)) of TSP.

4 - Empirical results

Table 4.1 presents the logarithmic likelihood value (a constant additive factor is omitted in each term) for the various systems that have been estimated. Detailed parameter estimates are presented in Appendix B. I attempted to answer three basic questions concerning the behavior of Italian households in reference to Table 4.1.

First, which demand system is most appropriate for discerning the regularities in the data? Second, is the consumption pattern influenced in a significant way by the size of the household? Third, given observations of different price-expenditure situations outside the sample used for estimation, what could be inferred about the predictive accuracy of these systems?

The first column of Table 4.1 presents the likelihood values for alternative systems. The demand equations (in expenditure form) have been estimated for the classification by expenditure classes (111 semple points) (Bollino (1982)). On the basis of the likelihood ratio test (LRT), it can be concluded that the QES, the TES and the GTL are all statistically significant generalizations of the LES at the 99% level. Also, the GTL is a significant generalization of the BTL (99% level).

In addition, Column 1 provides a new result in empirical demand analysis. A comparison between the QES and the TES likelihood values shows that the latter system is a significant generalization of the former at the 99% level.

Column 2 of Table 4.1 presents the likelihood values of the estimates based on the data cross-classified by economic condition and family size (120 sample points). The demand equations have been estimated in share forms without explicit treatment of demographic effects. In general, the findings of Column 1 are confirmed except that the LRT between the GTL and the BTL is significant only at the 90% level and between the TES and the QES is significant only at the 95% level. Notice that the TES is also a statistically significant generalization of the PIGL system at the 99% level ([12]).

In order to consider the influence of family size, it is possible to introduce demographic effects into a demand system. The procedure involves two assumptions. First, a new set of parameters is introduced in the specification of the demand equations. Second, it is assumed that there is an explicit functional relationship between the demographic variables and these parameters. The problem of demographic effects has been extensively discussed in the literature, since the classic study by Prais and Houthakker (1955). A general discussion is found in Pollak and

Wales (1981), where alternative procedures fot incorporating demographic variables are tested. In this paper, I have used linear demographic translating - a procedure which introduces a parameters in a given demand system. Specifically, in each system in Figure 2.1 the s are defined by:

(8) $\gamma_i = \bar{\gamma}_i + \alpha_i f$

where α_i = demographic translating parameters
f = size of the household

(8) implies that the demographic variable affects consumption only through the parameters α_i and it is a maintained hypothesis in this paper.

Column 3 of Table 4.1 reports the likelihood values for linear demographic translating of the LES, QES TES and GTL (13). A comparison with Column 2 shows that the introduction of family size has a significant effect on consumption patterns for all four functional forms (at the 99% level) (14). It is also interesting to compare the LES with the other systems. Both the GTL and the TES are significant generalizations of the LES at the 99% level, while the QES is not.

Although no formal comparison can be made (on the basis of the likelihood ratio test) between the TES and the GTL, given their non-nested relationship ([15]), it could be concluded, from an empirical viewpoint, that the TES performance is satisfactory with a limited number of parameters. The relevance of this latter consideration is potentially strengthened as the number of commodities under study increases because the number of parameters increases linearly with the number of goods.

As far as price and income elasticities are concerned, Tables 4.2 and 4.3 report selected estimated values for 1977 for different family sizes. Food appears to be moderately price elastic for the LES and for one-person households for the QES. It is price inelastic and less so as family size increases for the TES. In the LES a price elasticity greater than one in absolute value results from a negative "committed quantity". This is perhaps surprising, for food is generally price inelastic and previous estimates of the LES for Italy (e.g. Leoni (1967) and Bollino (1981)) show a positive "committed quantity". For the other categories, all systems show price elasticities greater than one in absolute value for all family sizes. The absolute value is inversely related to family size for both clothing and miscellaneous in the case of the QES, while the relation is mixed for the other systems.

A characteristic common to all systems is the notion of income inelasticity for food and the reverse for the other categories. Moreover, the highest point estimate for income elasticity for food occurs in the case of two-person households (except for the GTL). This is perhaps not surprising since ISTAT classifies "purchases of meals outside the house" within this category. As far as clothing and miscellaneous income elasticities are concerned, all systems show values substantially above unity with a tendency to increase for larger households.

It is noteworthy that, with few exceptions, the above findings agree qualitatively with previous studies of Italian consumption patterns (e.g., Bollino (1981), Rossi (1981), Vinci (1970)). However since the level of disaggregation of household composition has never been investigated before in the framework of complete demand system estimation, no further comparison seems fruitful.

Before leaving the discussion on the estimates, it is important to mention that for the QES and the TES the estimated Slutzky matrix is negative semi-definite in all sample points either with or without demographic effects (see Table 4.1). In the GTL and BTL the regularity conditions fail in 91 and 69 of the 120 situations without demographic effects, respectively. In the GTL with demographic translating the rate of failure is 40 out of 120. Table 4.4 reports some common measures of goodness of fit for estimated expenditures with demographic translating both inside and outside the sample. A glance at the values of root mean square error and mean absolute error shows a satisfactory performance of all models in the sample and tends to confirm the ranking established on the basis of the likelihood values ([16]).

The forecasting for the 1979 and 1980 price-expenditure situations is on average substantially less accurate on the basis of the above mentioned indicators. This result was expected, on the basis of the discussion in the previous section (see footnote 7).

Nevertheless, it is interesting to note that a comparison between the GTL and the TES shows that in 1979 the latter performs better in the case of food and clothing and about equally for miscellaneous. In 1980 the TES outperforms the GTL in forecasting expenditures for clothing. Although further investigation is necessary, this confirms the satisfactory performance of the TES in empirical analysis of Italian consumption patterns.

5 - Conclusions

In this paper I have estimated five alternative demand systems for Italian household budget data for the period 1973-1978, and analyzed the impact of family size on consumption patterns.

Four of the five systems - the LES, the QES, the BTL and the GTL - have been discussed before in the literature, while the TES is a new system which includes the QES and the LES as special cases. For purposes of empirical analysis, the distinctive feature of the TES is the relatively low number of parameters required to characterize non-linearity in total expenditure, in comparison for instance with the translog systems.

In general, the likelihood ratio test indicates that the LES is inferior to both the GTL and the TES. Moreover, the TES appears to be a statistically significant generalization of the QES.

As far as the impact of demographic characteristics is concerned, the estimates based on demographic translating show that family size does have a significant influence on consumption patterns for all systems. Although demographic translating has not been tested against more general procedures ([17]) for incorporating demographic effects, the results are generally satisfactory. In fact, regularity conditions are met at all sample points (except in the GTL) and both price and income elasticity estimates are within plausible ranges.

The forecasting accuracy of all the systems decreases considerably outside the sample, possibly due to discontinuities in the data series. Nevertheless, the relatively good performance of the TES in comparison with the GTL confirms that the TES is potentially a promising demand system for further analysis of more disaggregated consumption items for household budget data.

FOOTNOTES

(1) Leoni (1967), Vinci (1970), Schianchi (1979), Rossi (1983), Bollino (1981).

(2) The problem of aggregation across consumers would not exist, if panel data were available.

(3) It follows, in this context, that tests among functional forms may be considered, at best, as a way to assess their relative performance and plausibility in interpreting the available data, rather than as a way to test alternative demand theories. On the contrary, stronger conclusions could be derived from the analysis of panel data - where the same household is sampled through time.

(4) The same problem arises, for instance, when "committed quantities" are allowed in a Cobb-Douglas function to yield a Linear Expenditure System.

(5) The linear homogeneous translog (Lau and Mitchell (1971)) has not been considered in this paper.

(6) For a discussion of reliability versus representativeness in household surveys see: Houthakker and Taylor (1970) p. 238.

(7) The 1969 survey reports a different expenditure breakdown. The major revision of the statistics collection procedures undertaken by ISTAT has caused some delay in data publication and, infact, the 1979-1980 data have not yet been published in full. Therefore I decided to use the 1979-1980 data only for a preliminary analysis of forecasting performance.

(8) The classes of economic condition are assumed to represent separate expenditure classes. They are: employed in agriculture, self-employed in agriculture, employed in non-agriculture, self-employed in non-agriculture. The five classes of family size are: 1, 2, 3, 4 and 5, and 6 or more member households.

(9) "Food" includes food and beverages. "Clothing" includes clothing, footwear and leather goods related to clothing. "Miscellaneous" includes tobacco, health care, recreation (e.g., hotels, toys, radio ant TV licenses and rentals), education (e.g., training expenses), cultural expenses (e.g. books, magazines, stationery, theaters, sports events) and other goods and personal services. The three categories accounted for approximately 60% of total consumption expenditures in 1980. Housing is excluded on the ground of the rationing existing in this market (Rossi (1983)).

(10) From an empirical viewpoint, one must be careful in defining durables. For instance, clothing can be assumed non-durable by reason of the frequent turnover of Italian fashion, etc.

(11) In other words, I have not considered geographical location or the fact that households are surveyed in different months of the year, and I ignore error correlation across households.

(12) Values for PIGL are not reported in Table 4.1 (although available upon request), because this system is not suitable for estimation with demographic variables. See footnote 13.

(13) I have not estimated the BTL with demographic translating because it does not contain "committed quantities". Introduction of demographic translating in such a system could be misleading in case there is misspecification in the original system (see Pollak and Wales (1981)).

(14) Use of "size of household" is a cruder way to introduce demographic characteristics than age structure or number of children, but it is the only choice available.

(15) For a more general discussion of non-nested hypothesis testing between the TES and the GTL see: Bollino (1983).

(16) However, the measures of fit are very close for all systems while the estimated price elasticities differ in some cases in a non-negligible way.

(17) See, for instance, Pollak and Wales (1981).

TABLE 4.1

ALTERNATIVE DEMAND SYSTEMS ESTIMATION RESULTS
LOG LIKELIHOOD VALUES

	1 Expenditure Classes	2 Economic conditions by family size - no demographic effect	3 Economic conditions by family size - demographic translating
LES	388.112 (5) [0]	694.081 (5) [0]	733.484 (8) [0]
QES	528.152 (8) [0]	703.018 (8) [0]	777.670 (11) [0]
TES	562.251 (9) [38]	705.064 (9) [0]	780.213 (12) [1]
BTL	600.604 (8) [0]	702.902 (8) [69]	- -
GTL	612.884 (11) [27]	706.072 (11) [91]	782.358 (14) [40]

Notes:
A common additive constant is omitted in each column.
In parenthesis: number of parameters
In brackets: number of sample points where regularity conditions fail
Column 1: sample = 111
Columns 2,3: sample = 120

TABLE 4.2

INCOME ELASTICITIES - 1978

LES FAM SIZE	ELY1	ELY2	ELY3
1 MEMBER	.701261	1.44484	1.40902
2 MEMBER	.705307	1.42936	1.39992
3 MEMBER	.704019	1.43105	1.40465
4-5 MEMBER	.685853	1.48626	1.45871
6+ MEMBER	.646200	1.63864	1.60438

QES FAM SIZE	ELY1	ELY2	ELY3
1 MEMBER	.700533	1.41192	1.43485
2 MEMBER	.704140	1.39823	1.42650
3 MEMBER	.702555	1.40000	1.43263
4-5 MEMBER	.684078	1.44986	1.49297
6+ MEMBER	.644009	1.58559	1.65673

TES FAM SIZE	ELY1	ELY2	ELY3
1 MEMBER	.695101	1.38565	1.46546
2 MEMBER	.699336	1.37403	1.45094
3 MEMBER	.698363	1.37858	1.45191
4-5 MEMBER	.680441	1.42983	1.50854
6+ MEMBER	.641056	1.56688	1.66619

GTL FAM SIZE	ELY1	ELY2	ELY3
1 MEMBER	.693071	1.51556	1.38221
2 MEMBER	.685159	1.54873	1.39864
3 MEMBER	.680654	1.56559	1.40964
4-5 MEMBER	.663087	1.63790	1.46006
6+ MEMBER	.631115	1.79571	1.57366

ELY1: FOOD
ELY2: CLOTHING
ELY3: MISCELLANEOUS

TABLE 4.3

OWN PRICE ELASTICITIES - 1978

LES FAM SIZE	ELP11	ELP22	ELP33
1 MEMBER	-1.13743	-2.19763	-1.93621
2 MEMBER	-1.10551	-2.07995	-1.84902
3 MEMBER	-1.08623	-2.03384	-1.81540
4-5 MEMBER	-1.07314	-2.11611	-1.88108
6+ MEMBER	-1.05906	-2.39125	-2.09697

QES FAM SIZE	ELP11	ELP22	ELP33
1 MEMBER	-1.02295	-1.92448	-1.74099
2 MEMBER	-.975788	-1.78022	-1.62751
3 MEMBER	-.944943	-1.70910	-1.57164
4-5 MEMBER	-.912952	-1.71525	-1.53242
6+ MEMBER	-.865753	-1.80042	-1.66811

TES FAM SIZE	ELP11	ELP22	ELP33
1 MEMBER	-.626375	-1.15144	-1.30629
2 MEMBER	-.692437	-1.24737	-1.35554
3 MEMBER	-.731099	-1.31154	-1.38966
4-5 MEMBER	-.751836	-1.40911	-1.46661
6+ MEMBER	-.767693	-1.61675	-1.64918

GTL FAM SIZE	ELP11	ELP22	ELP33
1 MEMBER	-.853923	-2.80027	-1.06374
2 MEMBER	-.811773	-3.00818	-.970099
3 MEMBER	-.768109	-3.12990	-.869716
4-5 MEMBER	-.726153	-3.54363	-.776978
6+ MEMBER	-.644627	-4.45199	-.562452

ELP11: FOOD
ELP22: CLOTHING
ELP33: MISCELLANEOUS

TABLE 4.4

COMPARISON OF ACTUAL AND PREDICTED VALUES

		1973-78		1979	1980
		RMSE	MAE	MAE	MAE
LES					
	1	0.02172	0.01574	0.1585	0.2007
	2	0.01318	0.01006	0.0965	0.0836
	3	0.01881	0.01446	0.2192	0.2578
QES					
	1	0.02139	0.01550	0.1599	0.1902
	2	0.01270	0.00950	0.1000	0.0822
	3	0.01888	0.01454	0.7196	0.2430
TES					
	1	0.02187	0.01595	0.1534	0.1706
	2	0.01270	0.00949	0.0874	0.0477
	3	0.01833	0.01422	0.2154	0.2312
GTL					
	1	0.02087	0.01559	0.1618	0.1551
	2	0.01236	0.00969	0.1209	0.0762
	3	0.01826	0.01424	0.2121	0.1934

1: Food
2: Clothing
3: Miscellaneous

RMSE: Root Mean Square Error
MAE: Mean Absolute Error

Appendix A

Characterization of the class of TES

Consider a "textbook" theoretical demand system which is trinomial in expenditure:

(1) $$h^i(p,\mu) = A^i(p)\,\mu^{\lambda+1} + B^i(p)\,\mu + C^i(p)$$

Lemma: If (1) is theroretically plausible, then:

(2a) $$\Sigma p_k A^k = 0$$

(2b) $$\Sigma p_k B^k = 1$$

(2c) $$C^k = 0$$

(3a) $$A^i_j + \lambda B^j A^i = A^j_i + \lambda B^i A^j$$

(3b) $B^i_j + B^j B^i = B^j_i + B^i B^j$

Proof:

Use budget identity:

$$\Sigma p_k h^k = \Sigma p_k A^k \mu^{\lambda+1} + \Sigma p_k B^k \mu + \Sigma p_k C^k = \mu$$

to get (2a) and (2b), use non-negativity:

$$C^k \geq 0 \text{ and } \Sigma p_k C^k = 0 \text{ to get (2c).}$$

Use Slutzky symmetry condition $k_{ij} = k_{ji}$ where:

$$k_{ij} = (\lambda+1)A^i A^j \mu^{2\lambda+1} + (A^i_j + A^j B^i + (\lambda+1)B^j A^i)\mu^{\lambda+1} + (B^i_j + B^j B^i)\mu$$

to get (3a) and (3b).

<u>Theorem</u>:

Any TES of the form (1) can be written as:

$$(4) \qquad h^i\,(p,\mu) = \frac{i}{\lambda g^2}\left(f_i - \frac{g_i}{g}\,f\right)\mu^{\lambda+1} + \frac{g_i}{\lambda g}\,\mu$$

where f(p) and g(p) are homogeneous of degree λ . Equation (4) is derived from the indirect utility function:

$$(5) \qquad \psi\,(p,\mu) = -\,\frac{g(p)}{\mu^{\lambda}} - \frac{f(p)}{g(p)}$$

Proof:

(i) There exist n functions k^i such that (4) can be written as:

$$h^i\,(p,\mu) = A^i\mu^{\lambda+1} + k^i\mu$$

provided that $\lambda \neq 0$, thus $k^i = B^i$

(ii) There exists a function $g(p)$ homogeneous of degree λ such that:

$$k^i = \frac{g_i}{\lambda g}$$

Define: $\phi^i(p, z) = k^i(p)z$ (Hurwicz - Uzawa (1971))

Then: $\phi^i_j + \phi^j \phi^i_z = k^i_j \, z + k^j z k^i = z(k^i_j + k^j k^i) = z(B^i_j + B^j B^i)$

which is symmetrical from (3b).

Use (2b) to substitute:

$$\Sigma p_k \frac{g_k}{\lambda g} = \Sigma p_k B^k = 1$$

$\Sigma p_k g_k = \lambda g$ which establishes the omogeneity of $g(p)$.

(iii) There exists a function $f(p)$ homogeneous of degree λ such that:

$$A^i = \frac{1}{\lambda g^2} \left(f_i - \frac{g_i}{g} f\right)$$

Manipulation yields:

$$f_i = \lambda g^2 A^i + \frac{g_i}{g} f$$

Define: $$\phi^i(p,z) = \lambda g^2 A^i + \frac{g_i}{g} z$$

Then:

$$\phi^i_j + \phi^j \phi^i_z = \lambda(g^2 A^i_j + 2gA^i g_j) + \frac{z}{g^2}(g_{ij} g - g_i g_j) + \lambda(g^2 A^j + \frac{g_j}{\lambda g} z) \frac{g_i}{g} =$$

$$= \lambda(g^2 A^i_j + 2g^2 \lambda A^i \frac{g_j}{\lambda g}) + \lambda g^2 A^j \frac{g_i}{\lambda g}) \quad) + \frac{z g_{ij}}{g} =$$

$$= \lambda g^2((A^i_j + \lambda A^i B^j) + \lambda(A^i B^j + A^j B^i)) + \frac{g_{ij}}{g}$$

The first term in brackets is symmetrical from (3a) and the other terms are clearly symmetrical.

Use (2a) to substitute:

$$\Sigma p_k f_k = \lambda g^2 \Sigma p_k A^k + \lambda f \Sigma p_k B^k$$

$\Sigma p_k f_k = \lambda f$ which establishes the homogeneity of f(p).

(iv) Application of Roy's identity to (5) immediately yelds (4).

Finally, let us consider the problem of introducing "committed quantities" in a theoretically plausible demand system with the following corollary.

Corollary:

Let : $\bar{h}^i(p,\mu) = z^i + h^i(p,\bar{\mu}) \qquad i = 1,\ldots,n$

$\bar{\mu} = \mu - \Sigma p_k z^k$

z^k: committed quantities.

If: $h^i = h^i(p,\mu)$ is a theoretically plausible demand system, then: $\bar{h}^i = \bar{h}^i(p,\bar{\mu})$ is also theoretically plausible.

Proof:

Consider the indirect utility function: $\psi(p,\mu)$. Roy's identity yields:

$$h^i(p,\mu) = -\frac{\psi_i(p,\mu)}{\psi_\mu(p,\mu)}$$

The transformation $\psi(p,\mu) = \phi(p,\bar{\mu})$ yields:

$$\bar{h}^i(p,\mu) = -\left[\frac{\phi_i(p,\bar{\mu}) + \phi_{\bar{\mu}}(p,\bar{\mu}),\ \frac{\partial\bar{\mu}}{\partial p_i}}{\phi_{\bar{\mu}}(p,\bar{\mu}),\ \frac{\partial\bar{\mu}}{\partial\mu}}\right]$$

$$= -\frac{\partial\bar{\mu}}{\partial p_i} - \frac{\phi_i}{\phi_{\bar{\mu}}}$$

$$= z^i + h^i(p,\bar{\mu}).$$

<u>Corollary</u>:

Introduction of "committed quantities" (see the discussion of Houthakker's reservation in Section 2) in (4) yields:

(4a)

$$h^i(p,\mu) = \alpha_i + \frac{g_i}{\lambda g}(\mu - \alpha(p)) + \frac{1}{\lambda g^2}(f_i - \frac{g_i}{g}f)(\mu - \alpha(p))^{\lambda+1}$$

where $\alpha(p)$ is a function homogeneous of degree 1. Equation (4a) is derived from the indirect utility function:

$$(5) \qquad \psi(p,\mu) = - \frac{g(p)}{(\mu-\alpha(p))^{\lambda}} - \frac{f(p)}{g(p)}.$$

As is intuitively clear from the specification of a trinomial expression in total expenditure for consumption expenditure, the degree of non-linearity of the Engel curve in expenditure-consumption space depends upon the magnitude of the highest power, while the concavity depends upon the sign of its coefficient. It can be noted immediately by plotting the function for different parameters values that the TES is able to approximate any curvature in the relevant range. This characteristic can be contrasted with the translog family of systems, where essentially the non-linear income response is achieved through a logarithmic function.

Appendix B

Estimated demand equations

The following tables report the estimated values of the parameters. The notation for the parameters corresponds to that of Figure 2.1 in the text. All the numbers in parenthesis are ratios of parameters to their asymptotic standard errors. The subscripts F, C, M, refer to Food, Clothing and Miscellaneous, respectively. For the QES and the TES, a_M is given by one minus the sum of the other a's and c_M is given by one minus the sum of the other c's. For the GTL a_M is given by one minus the sum of the other a's and all the b's.

Table B.1 gives estimates from data corresponding to Column 1 of Table 4.1 in the text.

Table B.2 gives estimates from data corresponding to Column 2 of Table 4.1 in the text.

Table B.3 gives estimates from data corresponding to Column 3 of Table 4.1 in the text. The notation for the translating parameters corresponds to that of equation (8) in the text.

Notice that the parameter λ in the TES is significant in all three tables. This is equivalent to saying that the likelihood ratio test is significant in the comparison between the QES and the TES, for λ is the only additional parameter in the latter system with respect to the former.

TABLE B.1

PARAMETER ESTIMATES - EXPENDITURE CLASSES

	QES		TES		GTL	
γ_F	-3.49	(-3.58)	.087	(.29)	-1.355	(-3.78)
γ_C	-4.50	(-12.9)	-.190	(-.54)	.216	(2.21)
γ_M	-.399	(-5.00)	.048	(.76)	.669	(3.00)
a_F	.979	(15.6)	1.31	(10.2)	2.750	(3.15)
a_C	.045	(1.33)	-.041	(-.64)	-.288	(-1.69)
c_F	.976	(15.6)	1.26	(10.7)	—	
c_C	.046	(1.36)	-.079	(-.479)	—	
δ	12.5	(2.03)	11.98	(.20)	—	
λ	—		.246	(5.54)	—	
b_{FF}	—		—		.914	(2.59)
b_{CC}	—		—		-.538	(-2.59)
b_{MM}	—		—		-.673	(-2.68)
b_{CF}	—		—		-.051	(-.87)
b_{CM}	—		—		.235	(2.26)
b_{FM}	—		—		-.302	(-2.07)

TABLE B.2

PARAMETER ESTIMATES - ECONOMIC CONDITIONS BY FAMILY SIZE

(no demographic effect)

	TES		QES		GTL	
γ_F	.434	(1.57)	-.616	(-2.39)	.238	(.19)
γ_C	.037	(.46)	-.249	(-3.31)	-.251	(-1.31)
γ_M	.089	(.56)	-.460	(-4.26)	-.343	(-1.04)
a_F	.558	(35.8)	.562	(38.8)	.323	(2.00)
a_C	.158	(17.6)	.162	(23.0)	.274	(.56)
c_F	-8.01	(-1.25)	10.33	(1.41)	—	
c_C	-7.62	(-1.08)	8.41	(1.16)	—	
δ	-.0031	(-.61)	.00045	(.50)	—	
λ	-.566	(-1.83)	—		—	
b_{FF}	—		—		.028	(0.07)
b_{CC}	—		—		.112	(.47)
b_{MM}	—		—		-.126	(-.24)
b_{CF}	—		—		-.187	(-.31)
b_{CM}	—		—		+.056	(.57)
b_{FM}	—		—		.158	(.39)

TABLE B.3

PARAMETER ESTIMATES - ECONOMIC CONDITIONS BY FAMILY SIZE

(demographic translating)

	QES		TES(*)		GTL	
λ_F	-.151	(-.99)	.731	(8.59	-12.75	(-.81)
λ_C	-.285	(-3.35)	.137	(3.04)	-7.21	(-.82)
λ_M	-.543	(-4.36)	.303	(3.66)	-11.7	(-.82)
a_F	.411	(36.7)	.407	(32.64)	.406	(30.1)
a_C	.214	(19.36)	.213	(22.99)	.221	29.4
c_F	-1.21	(.37)	-2.42	(-1.47	—	
c_C	19.5	(1.03)	-5.66	(-1.40)	—	
δ	.00031	.373	-.0063	(-.77)	—	
λ	—		-.915	(-10.86)	—	
b_{FF}	—		—		.217	(14.4)
b_{CC}	—		—		.156	(12.3)
b_{MM}	--		—		.207	(13.7)
b_{CF}	—		—		-.080	(-10.5)
b_{CM}	--		—		-.130	(-10.7)
b_{FM}	—		—		-.073	(-9.04)
α_F	.106	(1.47)	.041	(.48)	-12.39	(-1.60)
α_C	-.023	(-.59)	-.055	(-1.26)	-6.99	(-1.62)
α_M	-.049	(-.81)	-.109	(-1.52)	-11.34	(-1.63)

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