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The Cambridge School of Monetary Theory: an Empirical Analysis for Italy

by Francesco Montaruli[*] and Roberto Rinaldi[**]

Abstract

This paper investigates the monetary theory of the Cambridge School, which emerged from the contributions of Alfred Marshall and Arthur Cecil Pigou between the late 19th and early 20th century. While still grounded in Fisher's quantity equation, the Cambridge School brought significant innovations to monetary theory. It emphasized the various functions of money beyond its role as a means of payment, which was the key insight of the quantity theory of money. The Cambridge School paved the way for new developments, eventually leading to John Mainard Keynes' 'A Treatise on Money' and 'The General Theory'. Specifically, Pigou examined the sum of currency and demand deposits as a ratio to nominal GDP, known as the *k* ratio. The Cambridge *k* is influenced by the current state of the economy and expectations regarding the purchasing power of money. Our analysis uses yearly time series data from Italy, spanning from its unification in 1861 to the introduction of the euro. We test the relationship between the *k* ratio and nominal interest rates. Our findings indicate that *k* follows a non-stationary process, challenging the notion of a stable velocity of circulation. However, when combined with two nominal non-stationary interest rates, we detect a cointegrating relationship which can be interpreted as a long-term equilibrium. The result of a Vector Error Correction Model (VECM) estimated over the entire time span supports the theoretical predictions of the Cambridge School.

JEL Classification: B10, B13, C32, E41
Keywords: the Cambridge School and "*k*"; money; integration; cointegration; VECM

Contents

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1.Introduction[1]

The Cambridge School of monetary theory has received relatively little interest in the economic literature; one reason may be that the theory was developed between two milestones in monetary theory: The Purchasing Power of Money by Fisher (1911) and the General Theory of employment, interest and money by John Mainard Keynes (1936).

In this study, we revisit insights of the School originated by Marshall and Pigou to claim that, although linked to the quantity theory of money, the School was the origin of significant innovations in the theoretical monetary framework. It introduced the liquidity preference concepts that were eventually fully developed by Keynes, changing the future perspective on liquidity demand and its nexus with different market rates. For example, we shall better see that already Marshall did not limit the analysis of money only to the transaction motive and introduced the concept of a "standard" of deferred payments, while Pigou then referred to the "precautionary motive". In our view, the Cambridge School, whose main actors are initially Marshall and Pigou, constitutes a fundamental step of the theoretical pathway from Fisher to Keynes.

In this work, after having discussed main theoretical positions in the School and the Italian historical setting in the last century, we analyze from an empirical viewpoint one of the crucial variables pointed out by the School, using annual historical data from 1862 to 1998: the ratio k (introduced by Pigou in 1917). The ratio k is defined as the sum of currency and demand deposits, divided by nominal GDP (M1 according to present statistical definitions, but we also tested the relationships with M2). For Italy, we conduct an empirical analysis on k from 1862 to 1998 on yearly data, the aim is to verify empirically the main theoretical Cambridge School predictions in the extensive interpretation from Marshall to Keynes. The investigation will be conducted on two subsamples (from 1862 to 1943 and from 1948 to 1998) and findings compared to the full period. Considering the length of economic historical time series, we expect that the most significant quantitative results are those related to the years from the end of World War II until 1998. We selected 1998 as the ending point because the adoption of the euro in 1999 was a profound structural break. The euro has, indeed, modified the monetary and financial system in Europe and influenced the underlying forces governing the variables that we examine.

To start our time series analysis, we investigate whether the variable k follows a stationary process, as suggested by the quantity theory of money, and, if not, whether there exists an equilibrium relation between k and two nominal short-term interest rates. More

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specifically, k is traditionally expected by the scholars of the Cambridge school and lately mainly by Keynes to be negatively related to a bond interest rate and positively related to the remuneration of deposits. Should the empirical analysis confirm such a relation,[2] we can conclude that the insights of the Cambridge School are overall consistent with the Italian data over the full period and its two subperiods. Moreover, considering the estimated parameters and simulations from the econometric models we can add some insights regarding mutual roles of main variables, their dynamics and the timing and persistence of those relations. To the best of our knowledge, this is a novelty that can help to close the gap and complement on the liquidity preference side the extensive analysis of money demand already conducted on Italy (see Barbiellini Amidei et al. 2016, for a chronological summary of money demand studies in Italy).

This work is organized as follows. In the next Section we analyze the Cambridge School's contributions to monetary theory. In Section 3 we describe the relevant time series for the Italian case, which we will study through the "lens" of the Cambridge approach; in this context we will briefly make a reference to the institutional and monetary implementation changes. In Section 4 we report the econometric analysis on integration and cointegration. Section 5 concludes.

2. The Cambridge School

The early Cambridge School of monetary economics brought forward two famous and impactful economists: Alfred Marshall and Arthur Cecil Pigou. These figures contributed at Cambridge University to the development of the theory of money between the end of the nineteenth century and the beginning of the last century. Marshall, the initiator of the School, with his most important students, Pigou and Keynes, contributed to the work of the School with important pieces of monetary analysis. Marshall outlined his view on money in "Money, Credit, and Commerce" (1929) and contributed to the development of the theory with several memorials and official papers.[3]

Before the publication of Marshall's book, Pigou wrote a major contribution with "The Value of Money", published in 1917 in the Quarterly Journal of Economics. We will focus below on the analyses of these two economists, before lightly touching on Keynes' thoughts.

In the following excerpt from Money Credit and Commerce (cit.), Marshall introduces for the first time qualitatively the concept of k, namely defined as the fraction of total resources in the economy that the private sector holds as money:

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[2] The analysis is based on the degree of integration and cointegration of the time series, on preliminary simple regressions for *M1* and GDP, vector autoregression and vector error correction models (VECM) adding alternative interest rates. Considering the dynamic relations that we investigate, there is not such a relevant difference using *M1* or *M2* in terms of empirical results obtained.

[3] See Eshag E. (1963). In particular, see the section "Key to Abbreviations" at p. xix where the most important contributions on monetary theory by Marshall are reported.

"In every state of society there is some fraction of (their) income which people find it worth while to keep in the form of currency; it may be a fifth, or a tenth, or a twentieth. A large command of resources in the form of currency renders their business easy and smooth, and puts them at an advantage in bargaining; but, on the other end, it locks up in a barren form resources that might yield an income of gratification if invested, say, in extra furniture; or a money income, if invested in extra machinery or cattle." (p. 45).

It is easy to translate this passage into the following simple algebraic form:

(1) $$k = M/PY$$

where *M* is the stock of money, *P* the price level and *Y* real GDP. The equation (1) is the one proposed by Pigou, once properly rearranged in his "The Value of Money" (cit.; see below).

Marshall did not exclusively consider money as a medium of spot exchange. From the same book, p. 16:

"… the chief functions of money fall under two heads. Money is, firstly, a medium of exchange for bargains that are completed almost as soon as they are begun; it is a "currency"; it is a material thing carried in purses, and "current" from hand to hand, because its value can be read at a glance. This first function of money is admirably discharged by gold and silver and paper based on them. The second function of money is to act as a standard of value, or standard for deferred payments – that is, to indicate the amount of general purchasing power, the payment of which is sufficient to discharge a contract, or other commercial obligation, that extends over a considerable period of time: and for this purpose stability of value is the one essential condition."

Marshall introduces the issue of changes in the level of prices over time that influence deferred value. Deferred payments open in turn the issue of money as a form of investing savings and point to the relevance of the stability of purchasing power (inflation and deflation) as one factor influencing the propensity to hold money.

Marshall and his School adopted as monetary framework the exchange identity or quantity equation according to which by construction receipts are equal to expenditures and, in addition, supported the quantity theory of money as proposed by Fisher (cit.):

"The total value of a country currency, multiplied into the average number of times of its changing hands for business purposes in a year, is of course equal to the total amount of business transacted in that country by direct payments of currency in that year. But this identical statement does not indicate the causes that govern the rapidity of circulation of currency: to discover them we must look to the amounts of purchasing power which the people of that country elect to keep in the form of currency." (p. 43).

Concerning the causes that govern the rapidity of circulation (i.e. the causal relation between money and prices):

"…there is a certain volume of their resources which people of different classes, taken one with another, care to keep in the form of currency; and if everything else remains the same, then there is this direct relation between the volume of currency and the level of prices, that, if one is increased by ten per cent, the other also will be increased by ten per cent." (p. 45).

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Pigou's analysis in the 1917 essay was more technical than that of Marshall. Pigou defined k as the fraction of total resources of the economy, R, expressed in bushels of wheat, held in form of money M ("the number of units of legal tender"); $P(w)$ the price of one unit of legal tender in bushels of wheat. The demand for units of legal tender M is:

(2) $$P(w) = k\, R/M$$

a rectangular parabola in the (M, P(w)) space once k and R are given.

The titles of legal tender M are:

"*actual legal tender (for practical purposes token coins may be reckoned as part of this), bank notes, and bank balances against which cheques can be drawn*". (Pigou, cit., p. 41).

The variable k is related to the ease in exchanges and to the avoidance of risks due to a lack of liquidity since payments and cash inflows are not synchronized (precautionary motives are introduced). K is not constant but can be considered as relatively stable, being influenced by factors linked to the state of the economy and to expected inflation or deflation.[4]

Pigou did not explicitly introduce interest rates in his equation, nor did he mention them when dealing with the factors influencing k. The reason why he (and the Cambridge School) did not explicitly mention bonds as an alternative to monetary assets has been open to debate. In our analysis, we do link k to interest rates, making reference to the subsequent theoretical and policy developments and supported by the following, among others:

"*although the formal version of the Cambridge demand-for-money function does not include an interest rate variable, it is nevertheless true that one of the main contributions of the Cambridge School to monetary theory was to call attention to the fact that such variables*

[4] The role of the expected change in prices is clear: high inflation reduces the purchasing power of cash and demand deposits; it lowers money demand and k in order to invest it in goods or assets that hold their value better over time. The opposite applies in case of expected deflation.

such as the rate of interest may be important determinants of the demand for money". (Laidler, 1977; p. 62).[5]

The approach by Pigou emphasizes, indeed, the role of money as a medium of exchange, as in Fisher's traditional approach and in his exchange equation (Fisher, op. cit.), but also, as we saw, as a precautionary instrument. Pigou argues that his equation, which determines the price of a unit of legal tender in bushels of wheat is coherent with the exchange equation. The two equations (Pigou's and Fisher's) imply that:

(3) $$PV = T/R$$

where V is (Fisher's) velocity of circulation and T is total transaction.[6]

The interesting question is whether Pigou's and Fisher's approach are truly identical. Pigou's analysis is within the Fisherian approach but the quality and angle of the analysis are different. When comparing the approaches Pigou writes:

"… it does not follow that there is nothing to choose between them. Mine is not, of course any "truer" than the rival. They are both equally true. The claim that I make on behalf of mine is merely that is a somewhat more effective engine of analysis." (Pigou, cit. p.54).

Pigou argues that his approach does not concentrates on the "velocity of circulation". His approach pays more attention to the "volition" of consumers and firms rather than on a seemingly arbitrary technical concept (velocity of circulation):

"*I offer this specification of it in order than those interested in monetary theory may test its powers in actual work upon concrete problems*" (Pigou, cit, p. 54).

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As pointed out by Laidler (cit.) the issue of whether Keynes was fully part of the Cambridge School is subject to debate. While 'A Tract on monetary reform' (1923) is considered within the Cambridge tradition, 'A Treatise on money' is, from several viewpoints, a departure from the School, even though it still makes reference to the exchange equation. The reference is not strictly one of adherence; in 'A Treatise' the supply of money does influence prices but not in the mechanistic and proportional way implied by the quantity

[5] See also Laidler (2004). Patinkin (1972) explains that the absence in the Cambridge monetary theory of an explicit causal link from the interest rate to k is due to the fact that the School did not consider the demand for money as part of a more general process of wealth allocation, influenced by the rates of return of alternative assets. Concerning the issue of the portfolio balance approach and Marshall' thought, see also Bridel (1987).

[6] It can be simpler to assume that k is the inverse of velocity, that the price level in terms of units of legal tender is $P=1/P(w)$ and one can find that the two equations are identical; this is a more stringent criterion than "consistent".

theory. Vicarelli (1977), offers a profound analysis for this and other interesting interpretations of Keynes positions and progresses in the debate.

Our view is that overall ‘A Treatise’ is not in the Cambridge (Marshallian-Pigouvian) narrow tradition; an exception to this may be only the book dealing with the demand for money, which is more within the School mainstream interpretation. Foreign to that tradition, for example, is the part of ‘A Treatise’ where Keynes debates Pigou’s equation $P = k\ R/M$. We think that A Treatise, with several parts devoted to savings and investments and business fluctuations, is the opening-up of the revolutionary new context of The General Theory.

For more details on the link between the Cambridge School and Keynes, see Vicarelli (cit). The interpretation of the parameter k has originated a large body of literature, which extends its strict original interpretation to include financial market alternatives among different liquidity assets and yields. According to Vicarelli, already in The General Theory Keynes formalized the relationship between the demand for money and interest rates through his liquidity preference theory. Demand for cash and liquidity instruments reflects not only the transactions, but also precautionary and speculative motives for holding money. This implies that an increased demand for money, if not matched immediately by an increase in money supply, will lead to higher interest rates, as people are willing to pay more to hold cash. More recently, economists like Roncaglia (2009) and Kregel (2024) have expanded its interpretation. They include a broader range of financial market products and alternative liquidity management choices; explicitly considering trade-offs between holding different types of liquid assets (like cash, bonds, or stocks) and their respective yields.

3. The money to GDP ratio and its interplay with interest rates up to the start of EMU

In the empirical section of our study, building on the Cambridge approach to monetary theory and particularly on the role of the parameter k, our objective is straightforward: to investigate whether a long-term relationship exists between k and two alternative short-term interest rates (namely, the interest on bank deposits and the interest on short-term government bonds) and to analyze how this relationship influences the dynamics of k, also in a modern perspective.

This may be expressed as the simple linear relation that follows:

(4) $$\ln(k) = \ln(M/GDP) = \text{a} + \text{b int\_dep} - \text{c short\_bond}$$

where k is the average fraction of year in which money is held (see below); int_dep is the rate on bank deposits and short_bond a rate on governments bonds.

It may be useful to discuss why in the empirical analysis we underline the temporal dimension of k. Since velocity V gives how many times a monetary unit circulates in a given period (usually is intended a year), k = 1/V represents the average time (or fraction of time) that a unit of money is held before being spent. If V is measured in cycles per year (the number of times a monetary unit is spent in a year), then *k* has units of years per cycle, or simply years. This interpretation provides a direct temporal dimension to *k*, framing it as a measure of the holding time for money.[7]

Interpreting *k* as a fraction of time (e.g., years or months) adds practical insights: a higher *k* (lower V) means money circulates more slowly, with individuals holding onto money longer; a lower *k* (higher V) means money circulates more rapidly, since individuals are spending money more quickly. This temporal interpretation links *k* directly to macroeconomic dynamics such as economic stagnation (high *k*) or rapid consumption and investment cycles (low *k*).

The analysis we shall carry out relates specifically to the variables *k* and the two interest rates. Nonetheless, it may be argued - formally - that equation (4), once the log of nominal GDP is moved to the right of the equation, is, indeed, equivalent to a money demand relation with unitary elasticity with respect to nominal GDP. In our context, such unitary elasticity we do not think is strictly necessary to be tested, because we focus directly on *k* and not on money demand. However, in the preliminary analysis, it was tested and it was found equal to one over the whole sample. Recall again that our goal is to relate the three variables and not estimate a standard money demand equation, a subject that has already received considerable attention in the literature (see again for an extensive survey for Italy Barbiellini et al., op. cit.). Hence, we simply construct and use the variable *k* as it is defined in the literature. It was, indeed, necessary to investigate for long-term cointegration between M1 and GDP and verify in preliminary regressions its unitary elasticity.[8]

We start our empirical analysis with a description of Figure 1 where in panel (a) we report our measure of *k* for Italy: *M1* (currency and demand deposits) as a ratio of nominal gross domestic product, or as we proposed, interpretable as the average time a monetary unit is held in portfolios. In the same panel, we also show currency - banknotes and coins - as a ratio of GDP. The most important period for our time series analysis of the Cambridge School is coincident with the complete data set over the entire historical period (from 1862 to 1998); at first the empirical strategy aims at testing for stationarity and cointegration. Once the long-term relationship between *k* and interest rates will be verified by finding at least one cointegrating relationship, the theory will be verified by employing an appropriate econometric model. This model will be analyzed for its consistency with theoretical predictions and also used to run impulse response simulations to represent correlation signs and directions of dynamics among *k*, and two short-term rates. We will discuss their dynamics

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[7] For instance, if V=5 each unit of money circulates 5 times per year, then *k*=0.2 years. This means that, on average, a monetary unit is held for 0.2 years (or about 2.4 months) before being spent.

[8] Concerning the value of the elasticity to nominal GDP in money demand equations we think that appropriate value for the long run should indeed produce a unitary elasticity for nominal income. Less than unitary values pose a problem for the long run: if the economy in the steady state is growing, the ratio of *M* to GDP approaches zero.

in graphical scenarios of impulse responses mainly for the second subsample, from 1945 to1998. In the years before Italy was, indeed, a country of high volatility after the unification in 1861, with incomplete capital markets for the first years of the century and ruled by an autarkic regime from 1922 to 1943.

From 1861, the country adopted the lira as its national currency replacing various regional currencies. It was a period of deregulation in printing money and some financial crises. The first decade of last century was marked by relatively stable conditions, but with World War I Italy financed its participation in war through borrowing and printing money, leading to significant inflation. After the war, efforts were made to stabilize the currency, but the economy faced challenges due to high debt and inflation first and then also deflation for the artificial revaluation of the lira and the absorption of liquidity. During the fascist regime from 1922 to 1943, the government implemented policies to control inflation and stabilize the economy, including the revaluation of the lira in 1927, a measure adopted to strengthen the currency and restore confidence in the Italian economy. The regime pursued autarky (economic self-sufficiency) and increased state control over the economy.

Considering the variable *k* and the alternative interest rates we can distinguish the 1862 to 1943 sample when the conditions of the financial markets functioning were limited as described before, from the rest of the sample (one can see different patterns in all the panels of figure 1 after 1948). The initial post World War II vertical drop of the ratio *k* is caused by a period of very high inflation rates in 1945-47: inflation reached 100 per cent in 1945 and 60 per cent in 1947 (we eliminate those data in subsample analysis). In the mid-sixties, the liquidity ratio started to increase and accelerated, reaching a maximum of 50 per cent in 1973, the year of the first oil shock. Since then, the *k* ratio, with some fluctuations, has declined.

Through the period, currency to nominal GDP decreased almost linearly, at a modest pace, as the lower blue line in the chart of panel (a) of Figure 1 shows. In the same panel 1973 constitutes an exception: the turbulence and uncertainty following the first oil shock determined a jump in currency and demand deposits as safe assets.

In the eighties and nineties major changes took place in the institutional set-up and in the operational framework of monetary policy. The reforms and the changes in procedures gave increasing weight to the transmission of monetary policy based on market determined interest rates with the Bank of Italy controlling the very short end of the term structure of interest rates.

We can summarize the process as one in which:

- the Bank of Italy increasingly acquired control of the creation of monetary base through the so called "Treasury channel" (in 1981, end of residual buyer at Treasury auctions; the so-called "divorce"), creating a divide between the financial needs of the public sector and the supply of liquidity (in 1993, abolition of direct financing of the Treasury that previously took place through an overdraft facility);

- the control of the traditional instruments of monetary policy was transferred unilaterally to the Bank - in February 1992 the Bank was given the power to fix in autonomy the discount rate; in 1993 reserve requirements were set independently by the Bank;
- monetary authorities (i.e. the Treasury and the Bank) took action to improve the functioning of public bonds auctions (in 1988-89 the price floor at bill auctions were abolished), diversify the supply of Treasury securities (through CCT and BTP), further the depth of the secondary markets for liquidity and securities (reforms of the MID and MTS markets).

In panel (b) of figure 1 we show the remuneration of demand deposits (demand deposits in Italy have always offered an interest linked, with lags, to short rates), the interest rate on short term Treasury securities in panel (c) and their markdown in panel (d). During the most recent subsample the Treasury nominal rate was influenced by the inflation rate. The peak was reached in the wake of the second oil crisis of 1979-80 following the Iran-Iraq war. Spikes in short rates were determined also by the exchange rate crisis in 1992 and by the fallout of the crisis of the Mexican peso in 1995.

The Treasury rate was also influenced by the institutional and operational changes described above: they enhanced the flexibility of interest rates in response to shocks; the indirect influence on the yield curve became more impactful due to the full control of policy rates and the depth and efficiency of secondary markets.

We should also recall that from the beginning of the 80's, following the participation of the lira in the Exchange Rate Mechanism (ERM), real interest rates have been constantly and significantly positive.

In panel (d) of figure 1, we end our brief description of the relevant time series by showing the "markdown", i.e. the spread between the interest rates on demand deposits and the Treasury bond. The troughs correspond to periods of turbulence in financial and forex markets that we recalled earlier when describing the Treasury bond rate evolution. Note that the volatility of the markdown increased significantly starting in the seventies: short-terms rates were the underlying factor, the result of a much more active monetary policy to rein in inflation and counter financial and forex instabilities.

4. The empirical analysis

The data set for Italian historical time series is public and available from the website of the Bank of Italy. We tried a variety of different historical yearly economic variables available related to money, interest rates, and GDP and used them in preliminary integration, autoregression and cointegration analysis. Eventually, for the sake of a parsimonious approach and for granting adherence to the theoretical setting, we consider that the most suitable yearly time series variables for our analysis (see also Table 6) are the following:

M1 = (narrow money) is the most liquid measure of money, comprising currency (banknotes and coins) in circulation and overnight deposits. Includes currency and sight deposits at banks and post office.

M2 = (intermediate money) includes *M1* plus deposits with an agreed maturity of up to two years and deposits redeemable at notice of up to three months. Note that *M2* produces nearly equivalent empirical results and we will refer to *M1* onwards.

GDP_M_PR = Nominal GDP.

LIQ_RATIO_1 = M1/GDP_M_PR. This variable represents the Cambridge *k*.

LIQ_RATIO_2 = M2/GDP_M_PR. Cambridge *k* alternative.

INT_DEPOSIT = Interest rate on bank deposits.

SHORT_BOND = Interest rate on short government bond (up to 1 year).

LONG_BOND = Interest rate on long government bond.

MARK_DOWN = INT_DEPOSIT - SHORT_BOND.

INFLA_ISTAT = Historical statistic ISTAT. Percentage changes in the national consumer price indices - Years 1862-2015 (the changes are compared to the previous year).

For the monetary aggregates *M1* and *M2* the reference for the sources of the historical data and their reconstruction is again in Barbiellini Amidei et al. (2016). For the GDP the reference source is in Alberto Baffigi (2011, 2013, 2015). The yield of short-term government securities is available annually from 1862 to 2016, thanks to the historical reconstruction by Piselli and Vercelli (2023).[9]

In this work the pivotal variable of the analysis is *k* (defined as *M1* divided by nominal GDP), and its interactions with the interest rate on sight deposits and a short government bond rate.[10] According to the Cambridge theoretical framework and even more to its later key Keynesian interpretations the ratio *k* should be equally influenced by a change in banking

[9] Throughout the entire period, BOTs (Buoni del Tesoro Ordinari) with a maturity of one year or less are considered. The series were obtained as a simple average of the yields at issuance for each maturity until 1976, and as a weighted average thereafter. They are published on the web site of the Bank of Italy, quoting the different original sources.

[10] The same analysis is conducted on other alternative variables, mainly longer-term yields, but we decided to keep the alternative between two short-term rates, one being on bank deposits, the other on government bonds. This is in line with the liquidity preference arbitrage choice, for an evaluation of statistical correlations and a reference to other studies, even though those are often more focused on money demand than *k* see Barbiellini et al., 2016.

deposits rate and a change of the opposite sign of government bond rate. An alternative to the three variable approach was also conducted by using the spread between the deposit rate and government bonds rates (so called "markdown") to measure its effect on *k*.[11] All in all, our goal is to find the best empirical model to check if the Italian historical data mimic the Cambridge predictions.

As shown in Figure 1, the variable *k* moves slowly while interest rates fluctuate significantly. These asymmetric paths can obscure deeper long-term relationships between variables, complicating the choice of the correct empirical model. Our empirical strategy operates on multiple levels, initially testing for integration and cointegration. To assess the existence of the long-term relationship between *M1* and GDP (the *k* assumption), we first test for cointegration between *M1* and GDP and find it, as expected (see Table 10, in the Appendix). We then validate the Cambridge *k* hypothesis by testing the elasticity value, regressing GDP on *M1*, and consistently we found a significant coefficient close to one (see Table 11).

A parallel step to choose an appropriate econometric model for our purpose is to investigate the (non)stationary nature of *k* and that of all other financial time series. It is worth noting that testing for *k* also clarifies whether the velocity of money circulation is a stationary process, as *k* is coincident to the inverse of money velocity.

Given large non-stationarity found in all series, we check for cointegration of *k* with interest rates, to identify equilibrium relationships economically interpretable and to choose the best empirical model. The analysis spans from 1862 to 1998 and includes also two subsamples (i.e.: 1862-1943 and 1948-1998). After testing the single series for integration, then only in case of cointegration, we will use a Vector Error Correction Model (VECM), which captures the long-term equilibrium path, while modeling short-term adjustments.

4.1 Integration

We first check for the degree of integration of *k* and nominal interest rates. In Table 1 we report the Augmented Dickey Fuller (ADF) tests for our set of variables (see Dickey and Fuller, 1979). These tests are validated and consistent over the full time series and produce equivalent results also in the two subsamples. Considering the empirical literature about non-stationarity and appropriate stationarity tests for financial variables and interest rates, we do not report random walks with a deterministic trend and we also give less importance to models with a drift.[12]

[11] In preliminary regressions and short-term (S)VAR models, we already considered the impact of markdowns on *k*. The founding was in favour of the fundamental Cambridge relationships. However, we consider the VECM analysis with the two separated interest rates and *k* more robust and appropriate.

[12] In addition to this, those alternative tests with trends were also conducted and they were overall in line with those we considered the most appropriate one, which we report here in the annexes.

For instance, the ADF statistic for the natural log of the liquidity ratio (ln k) is -1.584; being to the right of critical values, hence we do not reject the null hypothesis of an I(1) process. Similarly, the test on the logarithm of velocity (statistically the inverse of k) suggests that we also cannot consider its inverse $1/k$ a stationary process. The same result that confirms non-stationarity is obtained for the other variables at stake in our Cambridge equation formula: short-term interest on bank deposits is non-stationary and also short-bond rate is not stationary. All the tests of stationarity have been repeated also according to the Philips and Perron (1988) tests and they give same results of ADF, for all the different samples (see the Table 1).[13]

We can summarize the results of the integration tests as follows: the ADF tests with no drift and no deterministic trend indicate that we cannot reject the null hypothesis that the logarithm of k as well as the two interest rates (on sight deposits and on short-term bonds) are following non-stationary I(1) processes. The results are consistent across the different subsamples (1862-1943; 1948-1998 and the full sample from 1862 to 1998). We report only results over the full sample for the sake of simplicity. Note also that from the viewpoint of monetary analysis the result concerning k (and hence its inverse V) is of notable relevance: velocity of circulation of money, taken in isolation, is not a stationary process (not fluctuating around a constant value): not surprisingly, one of the fundamental assumptions of the Fisher approach is not supported by our empirical analysis. The empirical literature on the degree of integration of velocity in Italy is not extensive. However, the literature that concerns more countries is available. All in all, such literature (see Benati, 2020; Jung, 2024) indicates that the velocity of circulation is an integrated non-stationary process of order one.

At this stage, we could follow two routes to investigate the dynamic interplay of the three variables for testing the School theoretical prediction. (i) The first is to build VAR or structural VAR models of the stationary changes of the original series; (ii) the second is to test for cointegration among the I(1) processes and eventually, in case we found cointegration relationships, build up a VECM. After some preliminary estimations of structural VAR models, we have considered that a VECM approach could have been more appropriate to verify our extensive interpretation of Cambridge school theory.[14] The VECM technique seems better since all the variables have shown to be non-stationary and an equilibrium relationship between k and the interest rates is very likely at play.

[13] The ADF test for the short-term rate gives a statistic of -0.683, as against a critical value of -1.950 at the 95 per cent confidence level. The null hypothesis of I(1) is not rejected as well. The same result applies for the deposit rate.

[14] The OIRF of a preliminary SVAR are reported in the Appendix (see Figure 5). Notwithstanding the limitation of the short-term horizon of investigation, (S)VARs furnished simulations in line with the Cambridge predictions VECM models. However, given non-stationarity, the cointegration relationship, and a need to capture the dynamic interactions among the variables, both in the short and long term, the VECM technique has been considered in this work encompassing the (S)VAR approach.

4.2 Cointegration

Given non-stationarity of the time series, we think it is useful to analyze cointegration among them and we do it following the traditional analysis of Engle and Granger (1987). In a previous study Rinaldi and Tedeschi (1996) have shown the potential use of the error correction technique to model the complementary topic of money demand. Many economic and financial variables, even if they show volatility in the short term, are linked over the long term. Hence a VECM, that can handle multiple variable dynamics simultaneously, will enable us to represent the relationships between bank deposit interest rates, short bond yields, and the liquidity ratio more accurately. A VECM model representation can better measure the effect of interest on deposit and interest on short bond on *k*, revealing signs of the effects, the role of the equilibrium relationship and the timing of the interplay.

First, to proceed with a VECM approach, we need to prove cointegration among the variables. Cointegration indicates a long-term equilibrium relationship, allowing us to model the short-term dynamics while maintaining the steady state path. We will initially test for cointegration, then estimate the model and eventually interpret and simulate it. For cointegration testing of *k* and interest rates we conducted Johansen (1995) trace tests and information criteria tests.

We perform a set of cointegration tests over the whole sample from 1862 to 1998 among variable couples and then all the three variables together (*k* and the two rates jointly).[15] On the one hand, based on the trace statistics and information criteria, we conclude that there is not a strong cointegrating relationship only between the two interest rates: interest on banking deposit and interest on short-term bond. In this case, as reported in Table 4, only one out of three tests indicate cointegration, meaning that the interest rates, as represented in our data, do not share a long-term equilibrium relationship.

On the other hand, the test applied to *k* and the interest on bank deposits (see Table 2) or to *k* and the short-term bond rate (see Table 3) both confirm one cointegrating relationship. An identical result of only one cointegrating relationship is obtained testing for cointegration jointly on the three variables, as reported in Table 5. We consider this final test as the most relevant to define the rank of the VECM model, since it means this is the only one relationship strong enough to be considered significant in a three variables VECM model. Hence, we estimate the three-variable VECM, of rank one and with two lags, as suggested by the *ad hoc* criteria to choose lag length (see Table 7). The rank one corresponds to the unique cointegrating relation. The information criteria (SBIC, HQIC, AIC) also confirm rank one, indicating that this is the most appropriate VECM specification.

[15] Note that consistency with those results is found also testing over the second subsample in the dataset. In the first subsample also, even though tests are less significant in this subsample. Those results are available from the authors on request. In the Tables, the trace test indicates that there is one cointegrating relationship at the five per cent significance level, as the trace statistic for rank zero exceeds the critical value.

By imposing rank one to the VECM, the model will capture a single long-term equilibrium relationship among the variables. The short-term dynamics indicate how the variables adjust to restore this equilibrium. Here is the formula of the VECM estimated over the full sample:

(5) $$\Delta y_t = \prod y_{t-1} + \sum_{i-1}^{p-1} \Gamma_i \; \Delta y_{t-1} + C d_t + \varepsilon_t$$

Where: Δy_t is the first difference of the variables in vector y; Π is a coefficient matrix of cointegrating relationships; Γ_i are coefficient matrices of the lags of differenced variables of y; C is a coefficient matrix for deterministic terms, d_t is a vector of deterministic terms and ε_t is an error term with zero mean and variance-covariance matrix Σ.

The matrix Π can be decomposed into $\alpha\beta'$, where α is the loading matrix and β is the cointegration matrix. The economic interpretation of the model implies that the error correction terms indicate how quickly variables adjust to restore equilibrium after a shock. The lagged differences show the short-term dynamics and how past changes in one variable affect current changes in another. Eventually, the cointegrating equation reflects the long-term equilibrium relationship among the variables. In our case, after the Johansen normalization of k to one (denominated lg_LIQ_RATIO_1) the cointegrated equation is[16]

(6) lg_LIQ_RATIO_1 = 1.34 - 0.12 SHORT_BOND + 0.13 INT_DEPOSIT

Overall, in equation (6) VECM results support the Cambridge equation focus on the liquidity preferences, showing how changes in bond yields and deposit rates influence the liquidity ratio (see in the Appendix Table 8, Table 9 and Figure 4 for details on the estimated model and its parameters; on residuals' autocorrelation and stability conditions). In terms of k (or lg_LIQ_RATIO_1 that represents the proportion of income held as liquid assets) the negative coefficient (-0.12) in SHORT_BOND suggests that an increase in short-term bond yields is associated with a decrease in the liquidity ratio. This supports the idea that higher bond yields might reduce the need to hold liquid assets. In INT_DEPOSIT the positive coefficient (0.13) indicates that an increase in interest deposits is associated with a rise in the liquidity ratio. This suggests that higher deposit rates encourage holding more liquid assets, as higher returns on deposits make holding cash more attractive. For the error correction term in D_lg_LIQ_RATIO_1 equation the coefficient (-0.1048) is significant, indicating that deviations from the long-term equilibrium are corrected by about 10.5 per cent, each period. This shows a moderate speed of adjustment towards equilibrium. In D_SHORT_BOND the coefficient (0.80) is significant, indicating a stronger adjustment towards the long-term

[16] In Stata's VECM framework, the Johansen normalization to one is a method used to identify the cointegrating vectors. This standardization ensures that one of the variables in the cointegrating equation is set to have a coefficient of one, which simplifies the interpretation of the cointegrating relationships.

equilibrium. In D_INT_DEPOSIT the coefficient (0.17) is not significant, suggesting a slower or less direct adjustment.

Furthermore, in our view, the VECM results support the Keynesian perspective that liquidity preference plays a crucial role in determining interest rates and bond yields. An increase in liquidity preference (namely a higher lg_LIQ_RATIO_1) raises, indeed, both interest rates on deposits (INT_DEPOSIT) and short-term bond yields (SHORT_BOND). This correlation can be interpreted as higher liquidity preference requires higher interest rates to persuade agents to hold less liquid assets.

Finally, after testing successfully for stability conditions and autocorrelation (see Appendix 1), orthogonalized impulse response functions (OIRF) and forecast error variance decomposition (FEVD) are calculated from the VECM, to check by graphic inspection the dynamic interplay of the three variables. OIRFs have been computed and analyzed by us in the three subsamples. Overall, reflecting the coefficients of the VECM, we reckon their prevalent dynamics are broadly confirming theoretical Cambridge's school predictions. Among these simulations those more strongly in favor of Cambridge School's modern predictions are in the last sub-sample period (i.e. after the World War II).

Considering OIRFs in different subperiods and over the full period regressions, we have found that the orthogonalized responses of shock to interest deposit on liquidity k are broadly positive. Responses of liquidity k to a shock on short bonds are generally negative, as we expected. In the VECM simulations for the first subsample estimates (from 1862 to 1944 and not included in the graphs), however, these responses are less neat for a shock on short-term interest deposit. This can be due to the stronger substitution effect of cash holding given the less developed banking system, the higher volatility and the autarkic regime in Italy, lasting for many years of the last century. These factors may have offset the short-term interest deposit drive and its interplay, somewhat reducing the interpretability of the OIRF at that time. In other words, from 1862 to 1944 cash was a more significant part of the money supply (*M1* aggregate). As a result, the responses of liquidity k to shocks in interest deposits were less predictable, because the interplay between cash and banking money was more complex.

Hence, for the sake of simplicity we report here in Figure 2 only the result of the most convenient OIRFs, those calculated from the model estimations of the second subperiod (from 1948 to 1998). In these years, the conditions of financial markets, of the banking system and the reconstruction of data are more stable and reliable. Accordingly, simulations from the VECM are mirroring closely the Cambridge School theoretical predictions. OIRFs computed over horizons of ten steps and their signs are reflecting quite clearly our Cambridge liquidity preference interpretation. In this subperiod, the response of k to a shock to the deposit rate is steady positive as indicated by the model coefficients and graphs, while the response of k is always negative when the shock concerns the short-term bond, see the responses in upper panels (a) and (b) in Figure 2. Note also, as reported in lower panels (c) and (d) of Figure 2, that in the dynamic simulations represented by the OIRF of the VECM, positive shocks to *k* have positive impacts on interest rates and bond yields.[17] This result is read by us as in line with Keynes liquidity preference theory and its more recent interpretations. A positive shock on liquidity demand leads to higher interest rates and bond yields, reflecting the need to compensate for liquidity shortage.[18]

Regarding the forecast error variance decompositions (see Figure 3), they quantify how much of the forecast error variance of each variable can be explained by shocks to each variable in the system over time. The results of the simulations confirm the relevance of *k* in the long term for explaining the variance of the two interest rates and point to a stronger effect of the short-term bond rate and a moderate influence of both interest rates on *k*.

5. Conclusions

In this work, we have theoretically discussed and empirically analyzed, on Italian data from 1862 to 1998, the Cambridge School's ratio *k* and its yearly interplay with a short-term bond and a bank deposit short-term interest rate. The Cambridge *k* is a concept introduced by Pigou in the beginning of the 20th century and further developed by his prominent scholars. We believe its importance has been somewhat undervalued, especially when compared to the broader role of the Fisher equation in monetary debates. Initially, Pigou analyzed the sum of currency and demand deposits as a ratio of nominal GDP. Consequently, *k* is influenced by economic variables such as the expected future purchasing power of money and can be interpreted in various new directions. Specifically, *k* originates from a broader view of money than merely interpreted as a neutral transaction instrument. Although initially the founders of the Cambridge School did not mention explicitly bonds as substitutes for monetary assets, from a more modern and Keynesian perspective, we consider it plausible that they already intended to view short-term bonds as alternatives to monetary assets.

We believe that a suitable sample for testing our view is the historical yearly economic data set of Italy, recently made available from the country's unification in 1861 to the introduction of the euro in 1998. This unique data set, reconstructed by the Bank of Italy's Economic History Division, provides an excellent test for the Cambridge theory. It spans nearly 140 years, encompassing various political and monetary policy regimes, and includes periods marked by numerous exogenous shocks. We applied several quantitative techniques over different subsamples to identify the best model to prove the theory.

[17] We interpret the positive shock on *k* as a demand shock, since banking liquidity adjusts more slowly to an unpredicted shock on liquidity, given a certain amount of liquidity provision; also, in the VECM *k* comes before in the variable order.

[18] The OIRS of the subsamples can be made available on request to the authors; those on the first subsample indicate a puzzle in case of interest on banking deposits, where those of the full sample are consistent with the Cambridge theory for the first three steps, showing the correct signs. Moreover, while studying the dynamic of the liquidity preference, in a VECM, the authors found its OIRFs generally slightly less sensitive to the ordering of variables compared to a VAR model.

After testing for stationarity and running a set of preliminary multivariate and autoregressive regressions we considered that the most suitable empirical strategy to verify the Cambridge theoretical predictions could be testing for cointegration and then using VECM models. This technique includes short-term adjustments and long-term equilibrium relationships, which seems suitable to mimic the Cambridge liquidity preference viewpoint. In this econometric context, the Cambridge *k*, the fraction of time a monetary unit is held in cash or in banking money, appears in VECM models to be negatively influenced by a nominal short-term interest bond rate and positively associated with a banking short-term deposit rate.

Does the data set overall conform to the Cambridge School? We think that the empirical evidence of this work supports a positive answer. This is particularly evident when adopting an empirical strategy that extends beyond the short-term approach, aiming at identifying also fundamental long-term relationships.

Overall, the subsample analysis reveals that the Cambridge theoretical predictions were not fully met solely in the initial subsample from 1862 to 1944; a period marked by substantial volatility and significant shocks. The empirical findings and model estimations are broadly consistent, indeed, over the full sample and more robustly supported also by graphical simulations after the Second World War. In addition to this, the analysis from 1948 to 1998 suggests interpretations in favor of the innovative views of John Maynard Keynes. Keynes interpreted *k* from the demand side perspective, viewing liquidity preference as the driving force for short-term interest rates, particularly in the event of an unexpected liquidity shortage.

As a potential avenue for future research, it would be highly interesting to test the liquidity preference theory with Cambridge *k* across different countries and compare the consistency of the results from a broader cross-country historical perspective.

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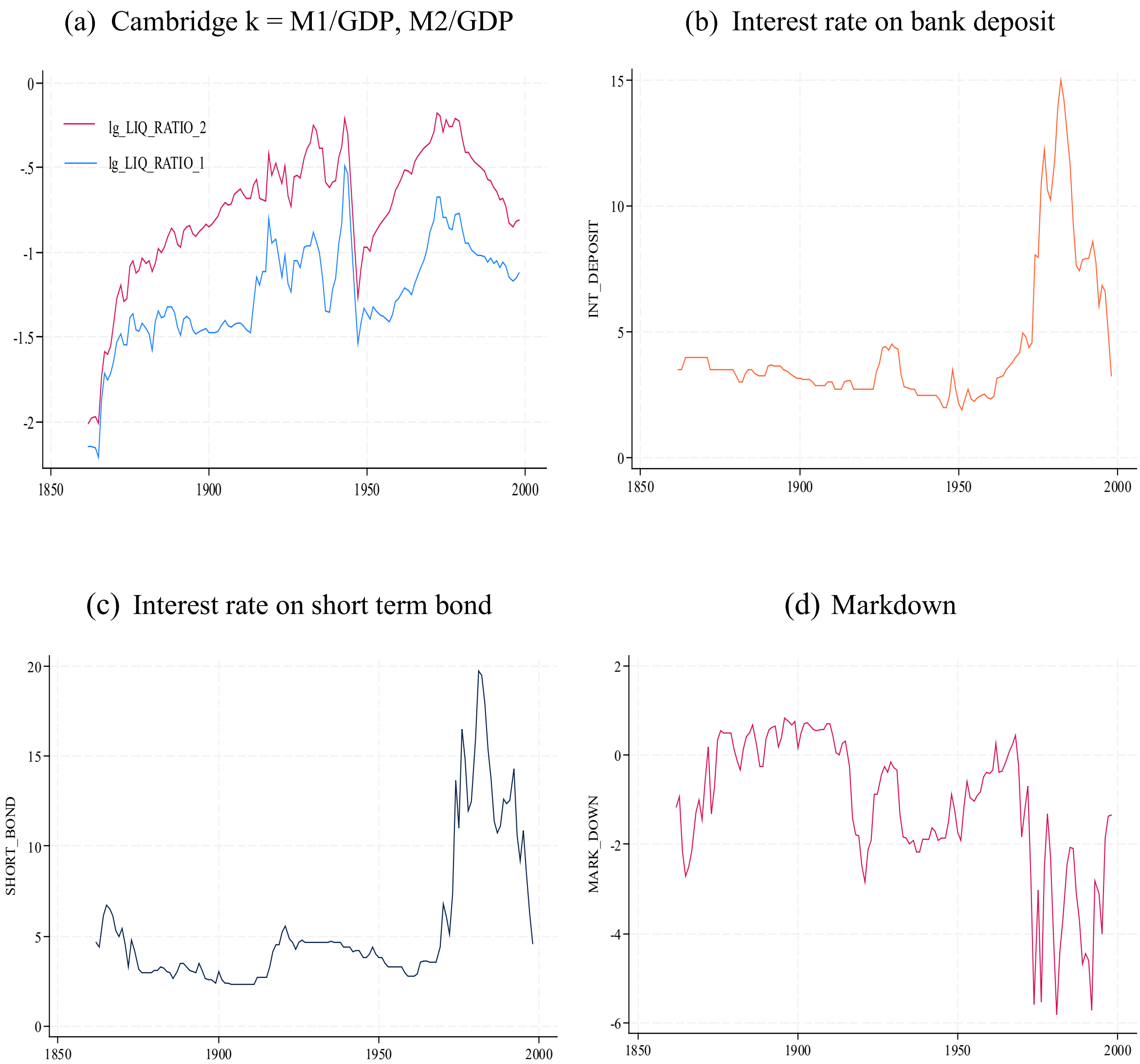
Tables and figures

Figure 1. Main economic variables

(sample: 1944 - 1998) (1)

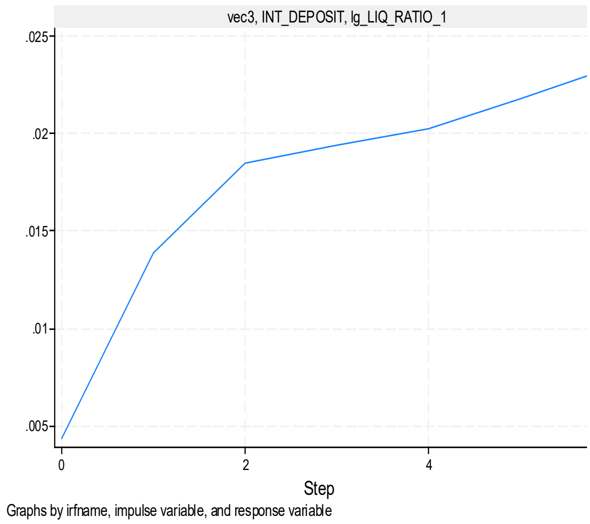


(1) Sources are reported in the empirical analysis in section 4.

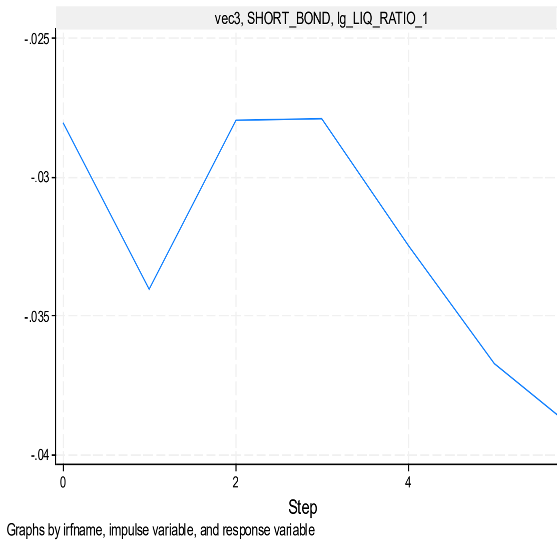
Figure 2. OIRF from the VECM model

(sample: 1944 - 1998) (1)

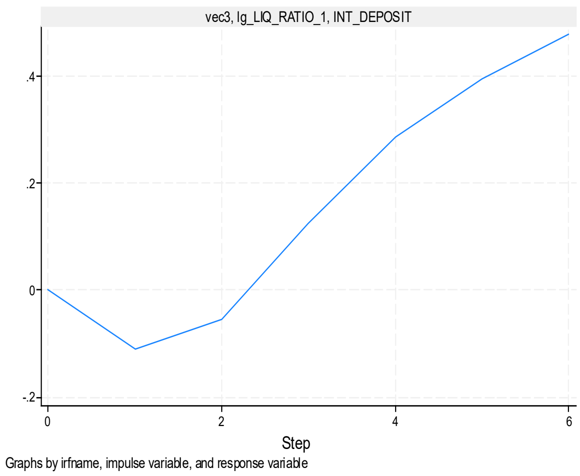
(a) Shock to Int deposit response on *k*



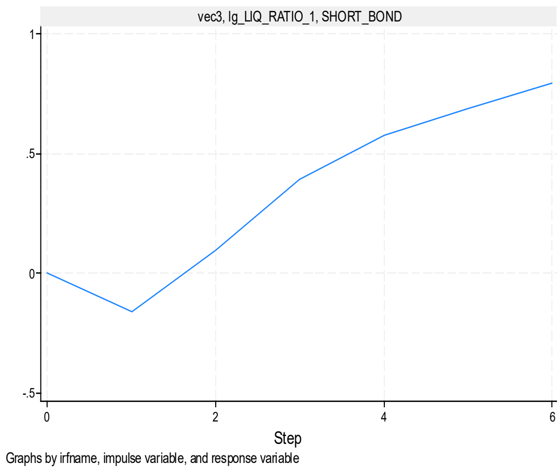
(b) Shock to Short bond response on *k*



(c) Shock to *k* response on Int dep



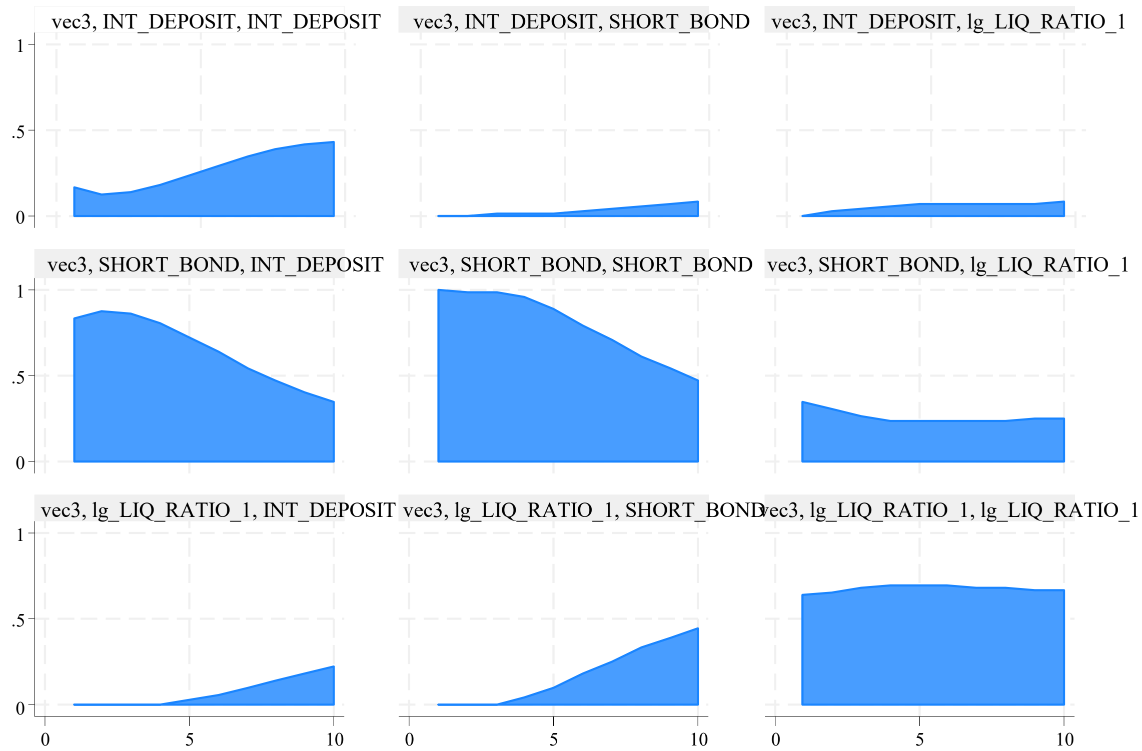
(d) Shock to *k* response on short bond



(1) Model identification with normalization with respect to variable short bond in the cointegrating equation.

Figure 3. FEVD from the VECM

(sample: 1944 - 1998)



Graphs by irf name, impulse variable, and response variable

Table 1. Stationarity tests

(1862 -1998 - 136 observations)

me series variables	Test st.	Significance levels* 1%	5%	10%
D.Fuller test for unit root				
Lg_LIQ_RATIO_1 (Cambridge k = M1/GDP nominal)	(-1.474)	-2.595	-1.950	-1.612
INT_DEPOSIT (Interest on bank deposit)	(-1.176)	-2.595	-1.950	-1.612
SHORT_BOND (interest on short term bond)	(-1.076)	-2.595	-1.950	-1.612
P.Perron test for unit root				
Lg_LIQ_RATIO_1 (Cambridge k = M1/GDP nominal)	(-1.514)	-3.498	-2.888	-2.578
INT_DEPOSIT (Interest on bank deposit)	(-1.689)	-3.498	-2.888	-2.578
SHORT_BOND (interest on short term bond)	(-1.857)	-3.498	-2.888	-2.578

* The null hypothesis of a random walk without drift cannot be rejected at all conventional significance levels.

Table 2. Cointegration tests: *k* and Deposit interest rate

(1862 -1998 - 136 observations)

Test: lg_LIQ_RATIO_1 and INT_DEPOSIT		Trace statistic	Critical value 5%
Johansen test for cointegration			
0		21.2508	15.41
1 **Selected rank** (1)		5.0193	3.76
2			
	SBIC	HQIC	AIC
Information criteria for cointegration			
0	.3104439	.2337926	.1813205
1 **Selected rank** (1)	.2992169*	.18424*	.1055319
2	.2983723	.1706202	.0831667

Table 3. Cointegration tests: *k* and Short bond rate

(1862 -1998 - 136 observations)

Test: lg_LIQ_RATIO_1 and SHORT_BOND		Trace statistic	Critical value 5%
Johansen test for cointegration			
0		21.5767	15.41
1 **Selected rank** (1)		4.0463	3.76
2			
	SBIC	HQIC	AIC
Information criteria for cointegration			
0	1.537975	1.461323	1.408851
1 **Selected rank** (1)	1.517126*	1.402149*	1.323441
2	1.523489	1.395736	1.308283

(1) The trace test indicates that there is one cointegrating relationship at the 5% significance level, as the trace statistic for rank 0 exceeds the critical value. The information criteria (SBIC, HQIC, AIC) also suggest that rank 1 is the most appropriate model.

Table 4. Cointegration tests: Short bond and Deposit interest rate

(1862 -1998 - 136 observations)

Test: SHORT_BOND and INT_DEPOSIT		Trace statistic	Critical value 5%
Johansen test for cointegration			
0 **Selected rank** (2)		12.6714*	15.41
1		2.8307	3.76
2			
	SBIC	HQIC	AIC
Information criteria for cointegration			
0 **Selected rank** (2)	3.933642*	3.856991	3.804519
1	3.969754	3.854777*	3.776069
2	3.985121	3.857369	3.769915

Table 5. Cointegration tests: *k*, Deposit interest rate and Short bond rate

(1862 -1998 - 136 observations)

Test: Lg_LIQ_RATIO, SHORT_BOND and INT_DEPOSIT		Trace statistic	Critical value 5%
Johansen test for cointegration			
0		30.9804	29.68
1 **Selected rank** (1)		12.4942*	15.41
2		3.5854	3.76
3			
	SBIC	HQIC	AIC
Information criteria for cointegration			
0	2.207324*	2.054021	1.949077
1 **Selected rank** (1)	2.252066	2.034887*	1.886216
2	2.295080	2.039576	1.864669
3	2.304857	2.036578	1.852926

(1) The trace test indicates that there is one cointegrating relationship at the 5% significance level, as the trace statistic for rank 0 exceeds the critical value. The information criteria (SBIC, HQIC, AIC) also suggest that rank 1 is the most appropriate model.

(2) The trace test indicates that there is not one cointegrating relationship at the 5% significance level, as the trace statistic for rank 0 does not exceed the critical value. Only the HQIC would suggest that rank 1 is the most appropriate model, which is not enough evidence for us.

APPENDIX

The VECM and preliminary empirical analysis

1. Vector Error Correction Model (1862 – 1998)

Table 6. Descriptive statistics

Variable	Obs	Mean	Std. dev.	Min	Max
GDP_M_PR	137	100384	252398	4.429678	1135500
LIQ_RATIO_1	137	.3003692	.0886454	.1099295	.610177
LIQ_RATIO_2	137	.5029046	.1601542	.1344192	.8354164
INT_DEPOSIT	137	4.327885	2.72695	1.93	15.03
SHORT_BOND	137	5.452468	3.893307	2.333333	19.7025
LONG_BOND	137	6.851616	3.319189	3.62	20.21
INFLA_ISTAT	137	8.678832	31.85531	-14.4	344.4
MARK_DOWN	137	-1.124583	1.552919	-5.8105	.8213242

Table 7. Lag-order selection criteria test

Sample: 1866 thru 1998 Number of obs = 133

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-692.658				7.01035	10.461	10.4875	10.5262
1	22.6483	1430.6	9	0.000	.000171	-.160124	-.054152	.100659
2	102.277	159.26*	9	0.000	.000059*	-1.22221*	-1.03676*	-.76584*
3	105.026	5.4983	9	0.789	.000065	-1.12821	-.863282	-.476254
4	107.456	4.8596	9	0.846	.000072	-1.02941	-.685002	-.181867

* optimal lag equals 2

Table 8. VECM model

VECM equation $\Delta y_t = \prod y_{t-1} + \sum_{i-1}^{p-1} \Gamma_i \ \Delta y_{t-1} + Cd_t + \varepsilon_t$

Where y_t = *k* = *M1*/GDP, *Short bond interest rate, deposit interest rate (1864 -1998)*

```
Sample: 1864 thru 1998                         Number of obs     =        135
                                               AIC               =   1.886216
Log likelihood = -110.3196                     HQIC              =   2.034887
Det(Sigma_ml)  =  .0010289                     SBIC              =   2.252066

Equation            Parms       RMSE      R-sq       chi2      P>chi2
-----------------------------------------------------------------
```

```
D_lg_LIQ_RATIO_1      5     .092436   0.1672   26.10188   0.0001
D_SHORT_BOND          5     1.18208   0.0780   10.99234   0.0515
D_INT_DEPOSIT         5     .631647   0.1949   31.46282   0.0000
----------------------------------------------------------------
------------------------------------------------------------------------------------
                 | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+------------------------------------------------------------------
D_lg_LIQ_RATIO_1 |
            _ce1 |
             L1. |  -.1047597   .0296974    -3.53   0.000    -.1629655   -.0465539
                 |
  lg_LIQ_RATIO_1 |
             LD. |   .3173863   .0824282     3.85   0.000       .15583    .4789426
                 |
      SHORT_BOND |
             LD. |  -.0085133   .0104911    -0.81   0.417    -.0290755     .012049
                 |
     INT_DEPOSIT |
             LD. |   .0201432   .0184601     1.09   0.275    -.0160378    .0563243
                 |
           _cons |   .0052333     .00798     0.66   0.512    -.0104071    .0208738
-----------------+------------------------------------------------------------------
D_SHORT_BOND     |
            _ce1 |
             L1. |   .8045519   .3797722     2.12   0.034     .0602122    1.548892
                 |
  lg_LIQ_RATIO_1 |
             LD. |   .2709251   1.054097     0.26   0.797    -1.795067    2.336917
                 |
      SHORT_BOND |
             LD. |   .3117193   .1341614     2.32   0.020     .0487678    .5746708
                 |
```

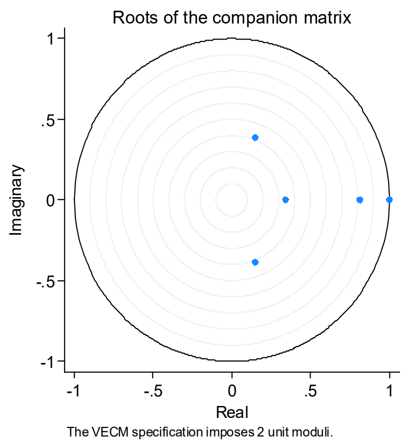
```
     INT_DEPOSIT |
             LD. |  -.5710526   .2360686    -2.42   0.016    -1.033738   -.1083667
                 |
           _cons |   .0010938   .1020483     0.01   0.991    -.1989173    .2011048
-----------------+----------------------------------------------------------------
D_INT_DEPOSIT    |
            _ce1 |
             L1. |   .1698571   .2029327     0.84   0.403    -.2278836    .5675978
                 |
  lg_LIQ_RATIO_1 |
             LD. |  -.4161897   .5632605    -0.74   0.460     -1.52016    .6877806
                 |
      SHORT_BOND |
             LD. |   .2493296   .0716896     3.48   0.001     .1088205    .3898387
                 |
     INT_DEPOSIT |
             LD. |   .0031079   .1261441     0.02   0.980      -.24413    .2503458
                 |
           _cons |  -.0019532   .0545299    -0.04   0.971    -.1088299    .1049234
----------------------------------------------------------------------------------
Cointegrating equations
Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                   2    7.37395   0.0250
-------------------------------------------
Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
--------------------------------------------------------------------------------
          beta | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------+----------------------------------------------------------------
_ce1           |
```

lg_LIQ_RATIO_1	1	.	.	.	.	.
SHORT_BOND	-.1236328	.0551929	-2.24	0.025	-.2318089	-.0154568
INT_DEPOSIT	.1325646	.0781496	1.70	0.090	-.0206057	.285735
_cons	1.341414	.	.	.	.	.

Where:

- _ce1 represents the cointegrating equation and captures the long-term equilibrium relationships between the variables.
- L1 denotes the first lag of the variable and LD stands for the first difference of the variable.
- In this output, _ce1 L1 refers to the first lag of the first cointegrating equation, while LD refers to the first difference of the respective variables.

Figure 4. Eigenvalue Stability conditions



In a VECM with 3 variables and a cointegration rank of 1, having two eigenvalues with a modulus of 1 can be explained by the structure of the model:

- Cointegration Rank: A rank of 1 indicates there is one cointegrating relationship among the three variables.
- Unit Roots: The presence of two unit roots (eigenvalues with modulus 1) is typical in this scenario. One unit root corresponds to the cointegrating relationship, and the other reflects the fact that the system has three variables but only one cointegrating vector. The remaining two variables are integrated of order I(1), which means they have a stochastic trend.
- Therefore, the two eigenvalues with modulus 1 are expected and consistent with having one cointegrating relationship in a system with three variables. This setup ensures that the system can adjust to maintain the long-term equilibrium defined by the cointegrating vector.

Table 9. Lagrange-multiplier test

lag	chi2	df	Prob > chi2
1	5.3332	9	0.80435
2	9.8810	9	0.36021

H0: no autocorrelation at lag order

- Lag 1: The p-value is 0.80435, which is much higher than the common significance levels (e.g., 0.05). This means we fail to reject the null hypothesis, indicating no significant autocorrelation at lag 1.
- Lag 2: The p-value is 0.36021, also higher than common significance levels. Again, we fail to reject the null hypothesis, indicating no significant autocorrelation at lag 2.

Overall, these results suggest that there is no significant autocorrelation in the residuals of the model at the tested lag orders.

2. Preliminary tests, regressions and vector autoregressive analysis

Table 10. Cointegration tests: *M1* and GDP

(*1864 -1998 - 135 yearly observations*) (1)

Test: M1 and GDP_M_PR		Trace statistic	Critical value 5%
Johansen test for cointegration			
0		45.7327	15.41
1 **Selected rank** [(1)]		0.0913*	3.76
2			
	SBIC	HQIC	AIC
Information criteria for cointegration			
0	38.4685	38.39185	38.33938
1 **Selected rank** [(1)]	38.23942*	38.12445*	38.04574
2	38.27508	38.14733	38.05988

[(1)] The trace test indicates that there is one cointegrating relationship at the 5% significance level, as the trace statistic for rank 0 exceeds the critical value. The information criteria (SBIC, HQIC, AIC) also suggest that rank 1 is the most appropriate model.

Table 11. Preliminary regression of log *M1* and and log GDP

(*sample: 1862 - 1998*) (1)

Source	SS	df	MS
Model	2766.6213	1	2766.6213
Residual	8.44771883	135	.062575695
Total	2775.06901	136	20.4049192

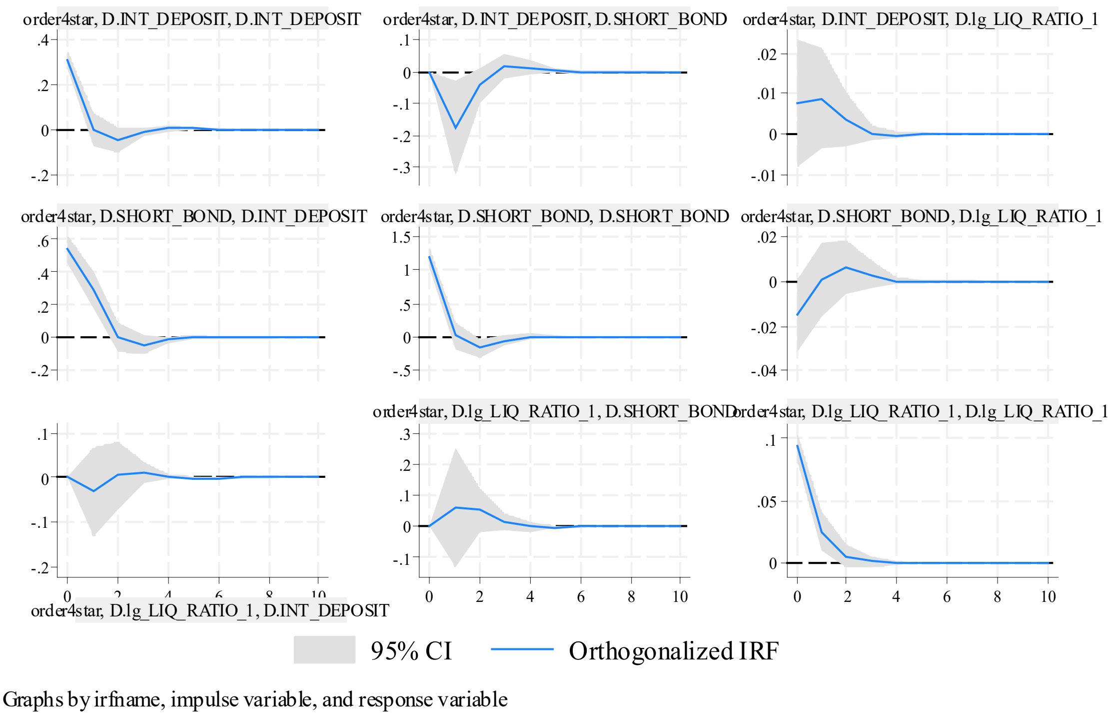
Number of obs	=	137
F(1, 135)	=	44212.39
Prob > F	=	0.0000
R-squared	=	0.9970
Adj R-squared	=	0.9969
Root MSE	=	.25015

log_M1	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
log_GDP_M_PR	1.039477	.0049436	210.27	0.000	1.0297	1.049254
_cons	-1.482783	.0365002	-40.62	0.000	-1.554969	-1.410597

(1) The positive and statistically significant relationship between log GDP_M_PR and log *M1* suggests that as GDP increases, the demand for money also increases with elasticity nearly equal to one, which is in line with the liquidity preference theory. The requirement for the elasticity of money demand with respect to GDP to be equal to one ensures that the Cambridge equation for liquidity preference accurately reflects the proportional relationship between income and money demand.

Figure 5. OIRF from a preliminary short term SVAR model

(*sample: 1862 - 1998*)(1)



(1) Note that the signs of the OIRF are consistent in the short term with the theoretical predictions. A shock to the interest rate on bank deposit has a positive impact on *k* (D.lg_LIQ_RATIO_1) and a shock to the short bond interest rate has a negative impact on *k*.

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