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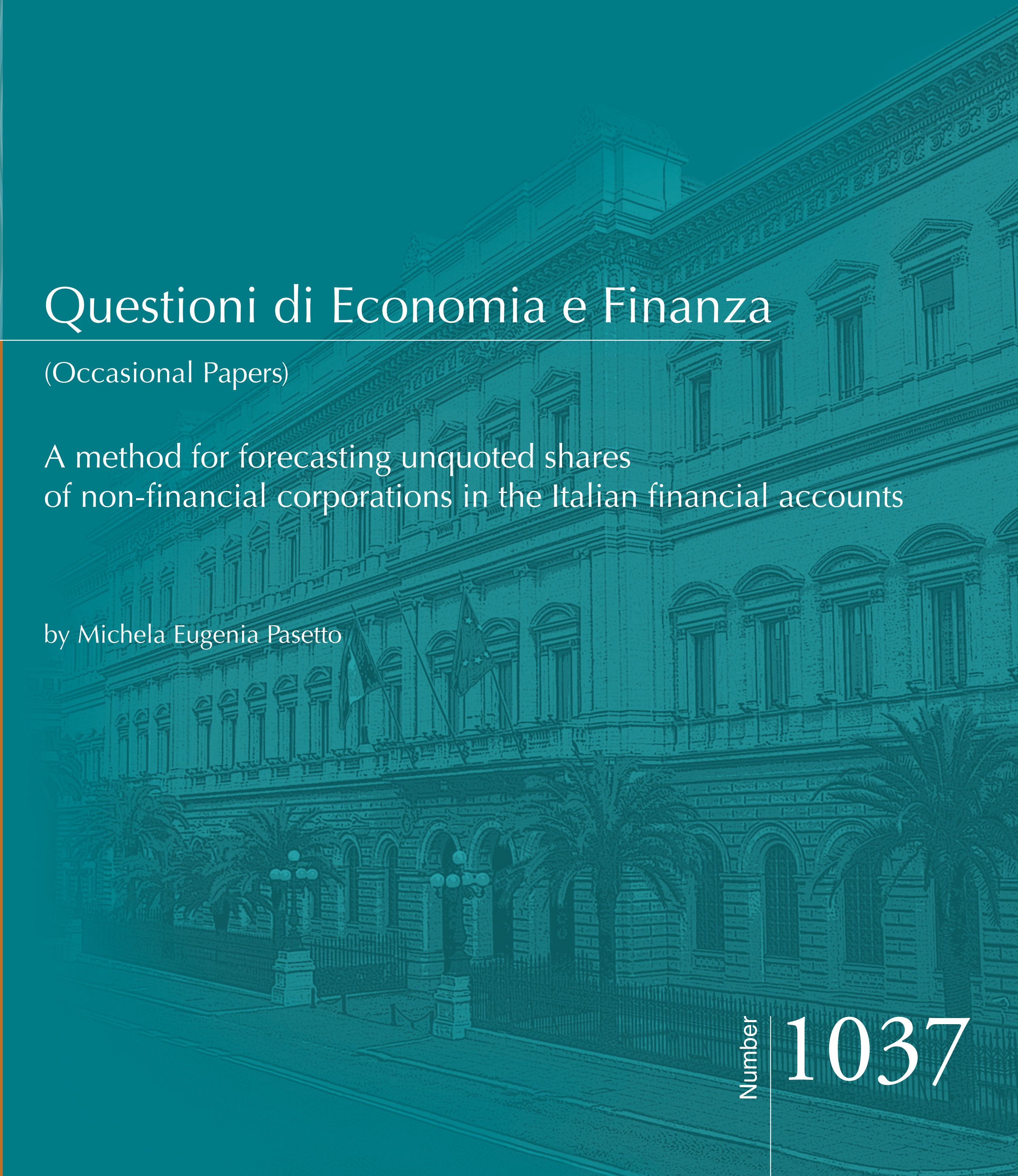

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(Occasional Papers)

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A METHOD FOR FORECASTING UNQUOTED SHARES OF NON-FINANCIAL CORPORATIONS IN THE ITALIAN FINANCIAL ACCOUNTS

by Michela Eugenia Pasetto*

Abstract

The accurate valuation of unquoted shares on the liability side of non-financial corporations' balance sheets is of primary importance, as equity issuance often constitutes a strategic choice for raising capital to finance investments. At macroeconomic level, quarterly data on equity issuance are published in financial accounts statistics and rely on estimation techniques. This paper proposes a new estimation method based on a Bayesian vector autoregression model to forecast unquoted equities issued by Italian non-financial corporations on a quarterly basis. The model is applied to both outstanding amounts and transactions. We compare the performance of the proposed model in its point forecasts and variability with five alternative econometric approaches, showing it performs very well in out-of-sample forecasting.

JEL Classification: C32, C53, E27, E66.

Keywords: Bayesian VAR, forecasts, financial accounts, equity.

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1 Introduction[1]

An accurate valuation in unquoted shares is of primary importance, especially for non-financial corporations[2]. Issuing equity is often a strategic choice alongside internal savings or debt instruments to finance investments, and may proxy firms' expectations about future economic conditions. The economic rationale behind equity issuance has been extensively discussed in the literature. For example, the seminal work of Myers and Majluf (1984) argues that firms tend to prefer debt over equity due to information asymmetries between managers and investors. In contrast, Dittmar and Thakor (2007) model the empirical observation that equity issuance often occurs when stock prices are high, suggesting that issuing equity can be more advantageous than issuing debt.

Regardless of a firm-specific underlying motivations, equity issuance remains a fundamental aspect of corporate financial strategy and liability management. At the macroeconomic level, data on liabilities in quoted and unquoted shares, valued at market prices, are published quarterly in financial accounts statistics. Quoted shares are listed on an recognised stock exchange, with prices continuously recorded and market value readily observable (i.e. the number of shares multiplied by the end-of-day market price). Conversely, unlisted shares are equities not traded on a public exchange. Therefore information on those financial instruments is scarce.

On an annual basis, data on the outstanding amounts of unquoted shares by non-financial corporations can be derived from firms' balance sheets through the own funds item. However, balance sheets are compiled at nominal value and the valuation at market prices requires the use of an estimation technique. National accounts handbooks (e.g. the System of national accounts, SNA25) suggest to employ a capitalization ratio method. This approach involves assigning to an unquoted firm the capitalization ratio of a comparable quoted company, thereby approximating the market value of own funds based on firms' similar characteristics such as the sector of economic activity. Nevertheless, on a quarterly basis, information on unquoted shares is unavailable at nominal value as well. Moreover, not only data on own funds are accessible just at the annual level, but they are also typically released with a delay of 15 to 18 months. As a result, the quarterly compilation of unquoted equity in financial accounts heavily relies on statistical estimation methods and faces several challenges. First, annual data must be temporally disaggregated into quarterly frequency. Second, due to the lag in the availability of firms' balance sheets, own funds for four to eight quarters following the end of the fiscal year need to be forecasted. Third, the equities' market value should be estimated.

Despite the relevance and difficulty of the estimation of unquoted equity, there is no definite practical instruction on its compilation in the system of accounts. In the past, a dedicated

[1] I am grateful to Massimiliano Affinito, Laura Bartiloro, Todd Clark, Luigi Infante and Francesco Vercelli for their valuable suggestions. All remaining errors are mine alone. The views expressed in this paper are my own and do not necessarily represent those of the Bank of Italy or the Eurosystem.

[2] For brevity, in this paper the terms "firms" and "corporations" are used interchangeably to refer to non-financial corporations.

working group dealt exclusively with the estimation of market values.[3] Nevertheless, among the steps necessary to estimate unquoted equity, the forecasting component is especially delicate and demands a reliable methodology. In this context, we propose an estimate based on Bayesian Vector Autoregression (BVAR) models. BVARs have been employed extensively in forecasting macroeconomic variables and numerous alternative specifications have been proposed. In this paper, we follow the method suggested by Giannone, Lenza, and Primiceri (2015) (hereafter GLP) as their model maximizes the one-step-ahead out-of-sample prediction under certain conditions. The approach proved to have a forecasting performance highly competitive with other models and has been extensively used in applied research (e.g. in Baumeister and Kilian 2016; Altavilla, Boucinha, and Peydró 2018; Altavilla, Pariès, and Nicoletti 2019).

In this paper, we first perform a temporal disaggregation of annual balance sheets' to get quarterly data. Second, we propose a forecasting model to recover timely estimates to include in quarterly financial accounts. Our forecast encompasses both outstanding amounts and transactions in unquoted equity and is based on the hierarchical BVAR approach proposed by GLP. The model includes three conjugate prior distributions often used in the literature (the Minnesota, sum-of-coefficients and dummy-initial-observation priors) and it is estimated on a data set constructed with firms' main real and financial aggregates. We assess the BVAR's out-of-sample predictions in terms of point and density forecasts and evaluate the robustness of the results with five different econometric models. The BVAR model provides an estimate at nominal values and, as a last step, we include a market valuation of the outstanding amounts.

The novelty of this work lies in the proposal of a forecasting method for unquoted equity. Until now, efforts in this area have focused on market value estimation. Prior to this work, the Italian financial accounts had already incorporated some of the European System of Accounts (ESA) recommendations for the annual estimation of equity on the liability side of firms. In this paper, we describe the methodology followed on an annual basis with regards to data sources and market prices estimation. Nonetheless, this paper extends the existing annual estimation framework, and its main contribution is the development of new quarterly data sources and a corresponding forecasting technique. Finally, this work documents how the hierarchical BVAR method can be applied in financial account statistics.

The rest of the paper is organized as follows. In Section 2 we describe data on own funds and paid-in capital of non-financial corporations at annual frequency and the sources for the quarterly forecast, while Section 3 shows how we assign a market value to balance sheets' items. In Section 4 we explain the BVAR method for the quarterly estimation of unquoted shares and the competitive econometric models. Results are presented in Section 5. Section 6 concludes.

[3] In 2002, Eurostat launched a working group on unquoted shares limiting the analysis to the valuation of the market price (Durant and Massaro 2004). In 2019, ECB established a virtual group on unlisted equity to develop an inventory of European Union Member States' equity data sources and evaluate different approaches to assigning market value estimations. As of today, European countries follow different methods for the estimation of unquoted shares.

2 Data description

Balance sheets' data of Italian non-financial corporations are available from the Central balance sheet office (Cerved), which covers almost the whole universe of Italian companies. From these balance sheets, we extract information on total own funds and the subcomponent of paid-in capital. The own funds item includes paid-in capital and reserves (the latter incorporating retained earnings), and is the basis to estimate the outstanding amounts of unquoted shares. The paid-in capital in first differences, instead, is used for estimating transactions, as in the system of accounts only the variation in paid-in capital represents a transaction in equity. Non-distributed profits, instead, increase the outstanding amounts of issued equity. However, since those profits are not generated by mutual agreement between counterparts, they do not represent a transaction and are recorded within the revaluation accounts.

Our data spans from 1995 to 2023[4] and includes both quoted and unquoted companies. Listed non-financial corporations are identified by the Italian stock exchange. Importantly, for those companies we observe their market value, calculated as the number of shares multiplied by the end-of-day market price. While Cerved provides the statistical classification of economic activities (NACE), it does not include that of institutional sectors according to the ESA regulation.[5] The ESA counterpart is therefore obtained by merging fiscal IDs of firms with the Italian Entities register.

We classify unlisted corporations into two distinct groups. We define *priced* firms those whose own funds will be adjusted to recover the market valuation. These represent the largest unlisted firms. Since we want to apply a market value based on prices observed for similar listed corporations, *priced* companies are those for which their own funds exceed the first quartile of the listed companies' own funds distribution within the same NACE sector. In contrast, *non-priced* corporations correspond to the smallest unlisted companies. The way the market value is assigned to *priced* companies is explained in Section 3. Figure 1 illustrates the breakdown of own funds into paid-in capital and reserves (including retained earnings) for both priced and non-priced unquoted corporations. In 2023, the overall number of Italian non-financial unquoted corporations available in Cerved stood at 908,254, of which about 38 per cent were quite small (own funds less than 100,000 euros). Quoted firms amounted to only 267. However, the priced corporations' own funds account for approximately 70 per cent of overall own funds of Italian unquoted firms. The sharp increase in the own funds of priced firms since 2018 is primarily driven by the service sector.[6] As mentioned in Section 1, however, those data have low frequency, late timeliness and outstanding amounts are reported at nominal value.

During the quarterly release of financial accounts, annual data on own funds and paid-in capital of non-financial corporations, separated in quoted, priced unquoted and non-priced unquoted

[4]For the years 1995-2007, data on quoted firms have been reconstructed with Reuters database. From 2008 onwards, the data source is Cerved.

[5]Regulation EU No. 549/2013, 21 May 2013, L 174.

[6]Legislative decree 126 of 2020 allowed for a revaluation of the assets recorded on the balance sheet of firms, thus implying a revaluation of the reserves at nominal value.

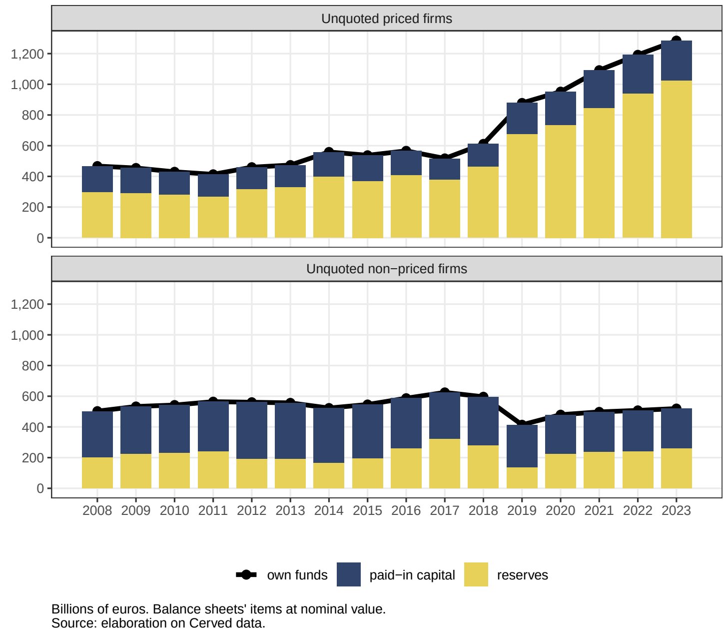


Figure 1: Own funds, paid-in capital and reserves at nominal value of Italian unquoted non-financial corporations classified into priced and non-priced groups.

companies, are disaggregated at quarterly frequency. Temporal disaggregation is carried out using the Chow-Lin method (Chow and Lin 1971), with the high-frequency indicator being the total amount of loan liabilities available at quarterly frequency in the financial accounts.[7] For the forecast, we construct a data set with time series on gross fixed capital formation, gross operating surplus, gross saving, net lending/net borrowing, distributed income of corporations,[8] issuance of debt securities and of quoted shares as well as total loans liability side.[9]

Variables are selected from the non-financial corporations' sequence of accounts based on their economic relevance to equity issuance. National accounts, indeed, are timely available at quarterly frequency and provide detailed information on the financial resources firms require within each quarter. These data can offer insights into the amount of shares companies might need to issue based on their gross savings and investment needs. Moreover, since the issuance of equity contributes to raise capital when needed (Dittmar and Thakor 2007), we expect its dynamics to be partly linked with that of debt securities and borrowing though loans, whose

[7]The Covid-19 pandemic did not introduce outliers or structural breaks in the time series of own funds or paid-in capital. Conversely, Decree 126/2020 led to a structural break, specifically a mean shift, in the series related to unquoted priced corporations. Despite this, the temporal disaggregation based on outstanding liable loans remains statistically significant.

[8]All sectoral national accounts data are seasonally-adjusted and retrieved from the Italian National Institute of Statistics (Istat).

[9]From financial accounts data, we employ time series of stocks to forecast own funds, and time series of transactions to predict paid-in capital and the net issuance of unquoted shares.

data are timely available. However, for flexibility, we prefer to not make assumptions on the sign of the link between those instruments. Under the assumption that unquoted companies, especially those *priced*, may emulate the practices of their quoted counterparts, we include the account of quoted shares. The next Section describes how we assign a market value to balance sheets' items.

3 Estimation of the market value at annual frequency

We define the listed companies' capitalization ratio by NACE branch at time t, r_t, as the market value of a firm (MV_t) over its own funds (OF_t):

$$r_t = \frac{MV_t}{OF_t}. \tag{1}$$

The basic assumption is that capitalization ratios are similar between listed and unquoted non-financial corporations within the same branch of business. Therefore it should hold that:

$$\begin{aligned} \frac{MV_l}{OF_l} &\sim \frac{MV_u}{OF_u}, \\ MV_u &\sim \frac{MV_l}{OF_l} \times OF_u, \end{aligned}$$

where subscript l represents a listed company and u an unlisted corporation.

Nonetheless, the assumption that listed and unlisted corporations exhibit similar capitalization ratios is non-trivial. In fact, such comparability necessitates that firms share certain characteristics, typically related to their economic activity. This assumption holds in countries where the number of listed and unlisted companies is relatively balanced.[10] Instead, the Italian industrial sector is composed by few large firms and many small companies, so that the difference between the number of listed and unlisted companies is very high. In practice, the method of capitalization ratios can be mostly applied to large unlisted corporations. For a small firm, the market value will be approximated by its own funds at nominal value, as suggested by the European system of accounts handbook. Finally, even if some quoted and unquoted companies might be similar based on certain criteria, the market price of unlisted shares should reflect a lack of liquidity. Shareholders may indeed attach a lower value to unlisted shares than to that of listed because selling the first is more difficult than selling the latter. The capitalization ratio should then be discounted by a liquidity premium. Durant (2005) estimates a 25 per cent discount premium in a similar context.

[10] The issue of the representativeness of a country's stock exchange for estimating unlisted equity is also discussed in the ECB virtual group on unlisted equity.

Considering the advantages and limitations of the capitalization ratio method, we can turn to the groups of unquoted corporations introduced in Section 2. We define *priced* firms those for which their own funds are higher than the first quartile of the listed companies' own funds distribution by NACE sector. Those firms are *priced* in the sense that we assign them the median capitalization ratio of listed companies (again, by NACE sector). Then the median capitalization ratio applied to unquoted firms is discounted by a liquidity premium of 25 per cent (Durant 2005). Conversely, *non-priced* corporations have own funds smaller than the first quartile and receive a capitalization ratio of one.[11] The outstanding amounts of unlisted shares on the liability side is then estimated as own funds times the discounted capitalization ratio.

Instead, the annual value of transactions in unlisted equity is estimated by taking first differences of non-financial corporations' paid-in capital.

Figure 2 compares the nominal value and the estimated market value of own funds for unquoted financial companies. The own funds at market value of all unquoted firms correspond to the outstanding amount of issued shares in Italy on an annual basis.

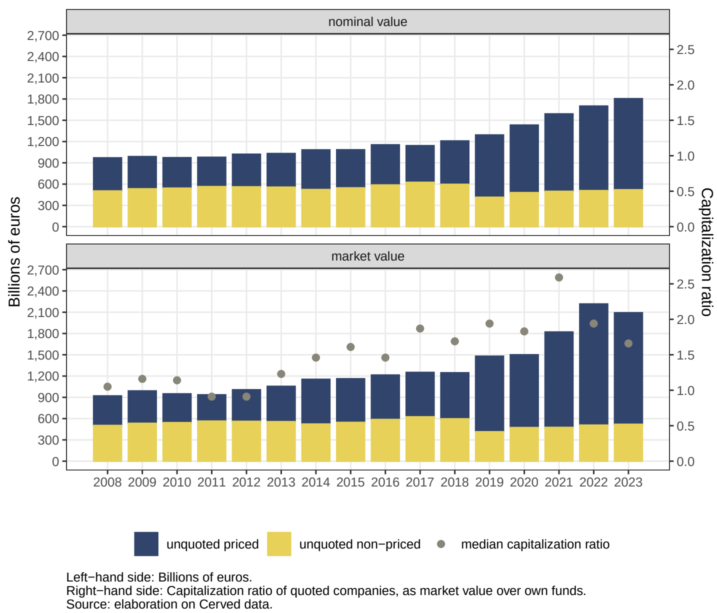


Figure 2: Own funds at nominal and market values of Italian non-financial corporations divided into unquoted priced and unquoted non-priced companies (lhs) and median of the capitalization ratio of listed corporations (rhs).

[11] Few other sectors receive a capitalization ratio equal to one independently of the value of their own funds. This applies to operational holdings (NACE no. 7415 "società di gestione - holding operative") because we do not know exactly in what sector those holdings operate, and "SpA with unique shareholder" (legal form no. 53). Since in Italy there is only one listed company in the agricultural sector (NACE sectors no. 1-5), all other unlisted firms in the same sector receives a capitalization ratio of one as well.

4 Forecasting unlisted shares at quarterly frequency

In this Section, we first introduce the proposed BVAR model and subsequently discuss the competing econometric models. Finally, we outline the simulation design.

4.1 Bayesian VAR model

Bayesian VAR models have been extensively applied to the forecasting of macroeconomic variables, with a variety of model specifications proposed in the literature. The seminal work of Doan, Litterman, and Sims (1984) and Litterman (1986) documented the effectiveness of the approach and stimulated a subsequent vast body of literature, both empirical and theoretical. Recent development of the methodology extended the study on the number of dependent variables to include (e.g. Carriero, Kapetanios, and Marcellino 2010; Koop 2013), while De Mol, Giannone, and Reichlin (2008) discussed how such models can outperform other common econometric techniques.

BVAR models entail a risk of over-parametrization and shrinkage of various sorts has been found to greatly improve their performance (Koop 2013). In the spirit of hierarchical modelling, it became common to impose additional structure on a given family of prior distributions, so as to reduce the number of parameters to be specified to a lower-dimension vector of so-called hyperparameters (Kilian and Lütkepohl 2016). Various heuristics on the best method to set hyperparameters have been proposed. For instance, Litterman (1980) set priors and hyperparameters to maximize the out-of-sample forecasting performance over a pre-sample. Del Negro and Schorfheid (2004), Del Negro and Schorfheid (2011) and Carriero, Clark, and Marcellino (2015) chose parameters that maximize the marginal likelihood of the model within a grid of possible values. Bańbura, Giannone, and Reichlin (2010) defined a targeted in-sample fit to find the most suitable hyperparameters. More recently, Giannone, Lenza, and Primiceri (2015) generalized the approach to the optimal selection by conditioning on a set of hyperprior distributions. GLP stress that under certain conditions their model maximizes the one-step-ahead out-of-sample prediction.

We consider the following Bayesian $VAR(p)$ model for n variables at time $t = 1, \dots, T$:

$$\begin{aligned} y_t &= C + B_1 y_{t-1} + \dots + B_p y_{t-p} + \varepsilon_t, \\ \varepsilon_t &\sim N(0, \Sigma), \end{aligned} \tag{2}$$

where ε_t is an $n \times 1$ vector of exogenous shocks, C is an $n \times 1$ intercept vector, and $B_1, \dots, B_p$ and Σ are $n \times n$ matrices containing the unknown parameters of the model. The $n \times 1$ vector of endogenous variables, y_t, comprises quarterly disaggregated data of balance sheets' item along with the aforementioned time series from national and financial accounts. The balance sheets' item is either own funds or paid-in capital. In practice, we fit Equation 2 two times. If we are forecasting the stock of unquoted shares, y_t includes disaggregated data of own funds and the national and financial accounts time series. In particular, financial accounts data employ time series of outstanding amounts. Conversely, when forecasting transactions, the model in Equation 2 presents paid-in capital, data from national accounts and time series on transactions from financial accounts.

Equation 2 can be rewritten as

$$y_t = X_t\beta + \varepsilon_t,$$

where $X_t \equiv I_n \times x_t'$ with $x_t \equiv [1, y_{t-1}', \ldots, y_{t-p}']'$ and I_n is an $n \times n$ identity matrix. The unknown parameter vector is $\beta \equiv vec([C, B_1, \ldots, B_p]')$, and the number of coefficients to estimate is $n + n^2p$.

In this work we follow the GLP strategy and focus on prior distributions belonging to the normal-inverse-Wishart family:

$$\begin{aligned}\Sigma \sim & IW(\Psi, d),\\ \beta|\Sigma \sim & N(b, \Sigma \times \Omega),\end{aligned}$$

where $\Psi = diag(\psi_i)$, b and Ω are functions of a hyperparameter vector γ. As in Kadiyala and Karlsson (1997), to guarantee the existance of the prior mean of Σ, we set the degrees of freedom of the inverse-Wishart distribution to $d = n + 2$. The elements on the main diagonal of Ψ, ψ_i, should be loosely related to the scale of the model and they govern the decay rate. As regards the conditional Gaussian prior for β, we combine the Minnesota prior introduced by Litterman (1980), the sum-of-coefficients dummy prior (SOC) and the dummy-initial-observation prior (SUR). The Minnesota's key hyperparameters defining the tightness of the prior mean b is λ, while the hyperparameters governing the SOC and SUR distributions are μ and δ, respectively.

For the forecast on own funds, we build two BVAR models in which variables enter either in log-levels (we label this model "BVAR log") or in growth rates (called "BVAR growth"). Conceptually, Bayesian econometrics is able to deal with non-stationary variables by imposing prior distributions with unit roots or sum of coefficients. The theoretical grounds for non-stationary BVARs are discussed in Sims (1988). Several practitioners prefer to leave variables in log levels and capture possible cointegrating relationship across the variables, as e.g. Bańbura, Giannone, and Reichlin (2010) or Giannone et al. (2010). However, other works use models in differences or growth rates, for instance Del Negro and Schorfheid (2004) or Koop (2013), and demonstrate that in some cases stationarity can improve the forecasting results in presence of instability (Clements and Hendry 1996; Diebold and Kilian 2000). Carriero, Clark, and Marcellino (2015) show that while models in levels might have an average gain in forecast accuracy slightly above the model in differences, the latter provides sizable forecasting gains at longer horizons. Overall, the choice of the model in levels or stationary variables might depend on the research question. Forecasting few steps-ahead could be better achieved by capturing the existence of cointegration, while growth rates may best represent longer lasting dynamics. In this work, we evaluate which of the two approaches most suits the issuance of unquoted shares. Finally, by imposing SOC and SUR priors in the model in log-levels, we are assuming the existence of unit roots and cointegration among the variables. In order to check for this assumption and highlight the role played by dummy observation priors, we also include a BVAR with a dogmatic Minnesota prior which models a random walk with drift (labelled RW).

The forecast can be computed either unconditional or conditional to some additional information. To some extent, conditional forecast has also been considered a scenario analysis, as it implies that the distribution of the shock changes according to the dynamics of some variables. We condition our prediction based on the issuance of debt securities, and we evaluate how such additional information is relevant for forecasting unquoted shares.

The BVAR models are constructed as specified in the Appendix. The point forecast in our model is the median of the predictive density. The code is based on the `BVAR` package (Kuschnig and Vashold 2021), which is designed to implement the GLP procedure. However, the BVAR proposed by Giannone, Lenza, and Primiceri (2015) is constructed to maximize the one-step-ahead forecast, but in our setting we must always forecast between four and eight quarters-ahead. Thus at each model's iteration t we plug in the median of the posterior density estimated at $t-1$, so as to update the estimation sample and re-estimate the posterior of the hyperparameters.

The quarterly forecast from the BVAR model provides the estimates of own funds at nominal values. As a last step, to estimate the outstanding amount of unquoted shares (uq) at quarter t for unquoted priced (uqp) and unquoted non-priced corporations ($uqnp$), we must assign a market value. We do so according to the formula:

$$\text{stock}_{uq,t} = (r_t * \text{lp}) * \text{own funds}_{uqp,t} + \text{own funds}_{uqnp,t}, \tag{3}$$

where r_t is the capitalization ratio defined in Equation 1 and lp denotes the liquidity premium introduced in Section 3, set equal to 25 per cent as in Durant (2005). Empirically, the quarterly capitalization ratio is the market value of listed non-financial corporations over the own funds' quarterly forecast for the same quoted companies.

Based on the forecasts for paid-in capital, instead, the estimate of transactions in unquoted shares is:

$$\text{transactions}_{uq,t} = (\text{capital}_{uqp,t} - \text{capital}_{uqp,t-1}) + (\text{capital}_{uqnp,t} - \text{capital}_{uqnp,t-1}). \tag{4}$$

4.2 Competing models

The forecasting performance of the BVAR is compared against several alternative approaches: a standard VAR estimated by Ordinary Least Squares (OLS), and three single-equation models. These include: (1) an Autoregressive Distributed Lag (ARDL) model, (2) an ARDL model augmented with principal components (AugPCA), and (3) an automatic Autoregressive integrated moving average (auto-ARIMA) model. In all models, variables are transformed into first differences to ensure stationarity.

The ARDL model might somewhat be considered the "benchmark" model, as it was used in the compilation of Italian financial accounts until around 2022. It incorporates quarterly time series of first differences of own funds' of quoted, priced unquoted, and non-priced unquoted firms. Each own funds series is modeled as an autoregressive process of order two without an intercept and as exogenous variable the net lending/net borrowing of the overall corporate sector.

The PCA-augmented ARDL model enhances the original ARDL specification by replacing the net lending/net borrowing variable with principal components extracted from a broader set of macroeconomic indicators, some of which include variables already discussed as gross fixed capital formation, gross saving, net lending/net borrowing, distributed income (uses side), and the own funds of quoted, priced unquoted, and non-priced unquoted corporations. The first three principal components derived from this set are used as exogenous variables in place of net lending/net borrowing.

Lastly, the auto-ARIMA model follows the procedure developed by Hyndman and Khandakar (2008). This method systematically searches across a wide space of possible ARIMA specifications, with a maximum lag order of five, and it selects the optimal model based on the corrected Akaike Information Criterion (AICc).

4.3 Design of the simulation exercise

To evaluate forecasting performance in a simulation setting, we define the initial training sample as the period from the first quarter of 1995 to the last quarter of 2017 (hereafter, we use the notation 1995Q1-2017Q4). The evaluation is conducted recursively over eight-quarter horizons, as in the financial accounts' case the prediction is always necessary for at least four quarters but no further than eight quarters-ahead. The first testing period then covers from 2018Q1 to 2019Q4. After each iteration, both the training and testing windows are shifted forward by one year. This results in the following sequence of test sets: 2018Q1-2019Q4, 2019Q1-2020Q4, 2020Q1-2021Q4, 2021Q1-2022Q4, 2022Q1-2023Q4. Each forecast is performed in a pseudo-real-time, i.e. only the information available up to the forecast origin is used and future data is excluded to simulate real-time forecasting conditions. While for each model variables enter with different transformations, the forecast results are reported in units (millions of euros).

In the context of yearly updates of data, when firms' balance sheet data are released, the eight quarters previously estimated become an other simulation set and we re-evaluate which model best estimated the latest dynamics of own funds. Indeed, every new data point may indicate a different data generating process, and therefore we should prefer the model best capturing the most recent process.

We evaluate the point forecast ability of the models in terms of root mean squared forecast error (RMSFE), mean absolute deviation (MAD) and mean absolute percentage error (MAPE), defined as follows. Let $\hat{y}^{(i)}_{t+h}(M)$ be the forecast for the ith variable $y^{(i)}_{t+h}$ made by model M at horizon h. The evaluation measures for all H forecast produced are:

$$\begin{aligned}
\mathrm{RMSFE}^{(M)}_{i,h} &= \sqrt{\frac{1}{H}\sum(\hat{y}^{(i)}_{t+h}(M)-y^{(i)}_{t+h})^2}, \\
\mathrm{MAD}^{(M)}_{i,h} &= \frac{1}{H}\sum|\hat{y}^{(i)}_{t+h}(M)-y^{(i)}_{t+h}|, \\
\mathrm{MAPE}^{(M)}_{i,h} &= 100\times\frac{1}{H}\sum\frac{|\hat{y}^{(i)}_{t+h}(M)-y^{(i)}_{t+h}|}{y^{(i)}_{t+h}}.
\end{aligned} \tag{5}$$

The accuracy of the density forecast is measured by using the log-predictive score, which is the logarithm of the predictive density evaluated at the realized values. Practically, if one model, say M_1, has a higher average log predictive score than another model M_2, the values predicted by M_1 were a priori more likely according to M_1 than M_2. As in Carriero, Clark, and Marcellino (2015), the log-predictive score is measured using a gaussian approximation:

$$s_t(y^{(i)}_{t+h}) = -0.5[\ln(2\pi)+\ln(V^{(i)}_{t+h|t})+(y^{(i)}_{t+h}-\bar{y}_{t+h|t})^2/V^{(i)}_{t+h|t}],$$

where $\bar{y}_{t+h|t}$ and $V^{(i)}_{t+h|t}$ denote the posterior mean and variance, respectively, of the simulated forecast distribution h steps-ahead. The average score for model M is therefore the average of predictive likelihoods:

$$\mathrm{SCORE}^{(M)}_{i,h} = \frac{1}{H}\sum s_t(y^{(i)}_{t+h})$$

To compare models against each other (e.g. model M_1 versus M_2) we consider the percentage gains in terms of RMSFE and score:

$$\begin{aligned}
\text{percentage gains in RMSFE} &= 100\times(1-\mathrm{RMSFE}^{(M_1)}_{i,h}/\mathrm{RMSFE}^{(M_2)}_{i,h}), \\
\text{percentage gains in score} &= 100\times(\mathrm{SCORE}^{(M_1)}_{i,h}-\mathrm{SCORE}^{(M_2)}_{i,h}).
\end{aligned}$$

For an indication of the statistical significance of different forecasting estimates, we provide results of the Diebold and Mariano (1995) test in the modified version proposed by Harvey, Leybourne, and Newbold (1997) which includes a variance adjustment that enters the statistics.

5 Results

In the next two Sections, we evaluate the forecasting ability of the different models for own funds and paid-in capital.

5.1 Own funds

This Section shows the results of the different models with regards to the forecast of own funds. The estimation of outstanding amounts of unquoted shares in financial account data is obtained by applying Equation 3 to the time series forecasted.

We start evaluating the models' performance by looking at the last simulation round (that is 2022Q1:2023Q4). Under the hypothesis that today's own funds prosecute their most recent trend, this simulation set provides an indication on how the models' performance would be "right now" in the quarters currently missing. Table 1 and Table 2 show the models' accuracy of point forecasts according to the different metrics presented above for unquoted corporations.

Table 1: Deviation measures of non-financial unquoted priced corporations' own funds in the simulation set 2022Q1:2023Q4. Billions of euros for RMSFE and MAD, percentages for MAPE.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
RMSFE	12.4	62.8	41.9	75.9	76.5	65.9	41.2
MAD	11.1	56.4	38.0	64.0	75.9	53.9	36.9
MAPE	1.3	6.6	4.5	7.5	9.1	6.6	4.5

Table 2: Deviation measures of non-financial unquoted non-priced corporations' own funds in the simulation set 2022Q1:2023Q4. Billions of euros for RMSFE and MAD, percentages for MAPE.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
RMSFE	22.4	36.1	52.9	35.1	358.5	327.8	17.4
MAD	21.4	30.6	48.7	32.5	356.0	327.4	14.7
MAPE	2.4	3.4	5.5	3.6	40.2	37.1	1.7

Table 3: Percentage gains in terms of RMSFE for priced unquoted non-financial corporations in the simulation set 2022Q1:2023Q4.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
BVAR log	-	-	-	-	-	-	-
BVAR growth	80.2	-	-	-	-	-	-
OLS	70.4	-49.7	-	-	-	-	-
RW	83.6	17.3	44.7	-	-	-	-
AugPCA	81.2	4.7	36.4	-15.1	-	-	-
ARDL	83.8	17.9	45.2	0.8	-16	-	-
auto-ARIMA	69.9	-52.2	-1.7	-84.1	-85.4	-59.9	-

Table 4: Percentage gains in terms of RMSFE for non-priced unquoted non-financial corporations in the simulation set 2022Q1:2023Q4.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
BVAR log	-	-	-	-	-	-	-
BVAR growth	37.8	-	-	-	-	-	-
OLS	57.6	31.8	-	-	-	-	-
RW	36.1	-2.9	-50.8	-	-	-	-
AugPCA	93.2	88.9	83.9	89.3	-	-	-
ARDL	93.7	89.9	85.2	90.2	-9.4	-	-
auto-ARIMA	-28.9	-107.5	-204.1	-101.7	-1960.5	-1783.6	-

The BVAR in log-levels outperforms by far the other models. The BVAR in growth rates, instead, shows a fairly good accuracy for the estimation of the non-priced unquoted companies' own funds, similar to that of the RW model. For the same period 2022Q1:2023Q4, Table 3 and Table 4 summarize the percentage gains in terms of RMSFE in which the columns present the reference models and the rows the competing approaches. The BVAR in log-levels gains above all other models, being approximately 80 per cent for the group of unquoted priced corporations. For unquoted non-priced corporations, the auto-ARIMA model apparently gains over the BVAR in log-levels. However, as it will be further discussed later on in this paragraph, the auto-ARIMA estimates a flat line with quite high variance; although it seems that the overall RMSFE is lower than that of the other methods, in practice the model does not capture any dynamics and returns as forecast the variables' historical sample means.

Table 5 and Table 6 show the RMSFE for all simulation sets and models. For the estimation of unquoted priced firms' own funds, the accuracy of the BVAR in log-levels strongly improves over time. In the period 2018-2020, this model does not thoroughly capture the swift surge of corporations' own funds, but, once the trend stabilizes into a stable growth, the forecast error greatly shrinks. We argue that the own funds' steep surge might be a cause (Figure 1).

Table 5: RMSFE of unquoted priced corporations' own funds by simulation set.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
2018Q1-2019Q4	131.3	131.8	139.8	142.8	248.2	221.9	127.7
2019Q1-2020Q4	175.3	139.5	166.8	193.7	327.5	295.5	168.9
2020Q1-2021Q4	77.8	66.5	52.4	64.6	116.4	114.1	38.3
2021Q1-2022Q4	75.4	12.8	70.7	51.1	209.0	216.8	114.8
2022Q1-2023Q4	12.4	62.8	41.9	75.9	76.5	65.9	41.2

Table 6: RMSFE of unquoted non-priced corporations' own funds by simulation set.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
2018Q1-2019Q4	93.2	127.7	66.3	84.9	262.0	282.6	90.0
2019Q1-2020Q4	103.0	98.7	87.6	85.6	255.4	237.5	100.0
2020Q1-2021Q4	28.0	103.7	120.4	66.7	337.1	303.4	41.8
2021Q1-2022Q4	22.7	8.1	28.9	41.0	344.8	320.8	30.4
2022Q1-2023Q4	22.4	36.1	52.9	35.1	358.5	327.8	17.4

As discussed in Villani (2009), historical shifts in the mean of the data can adversely affect the performance of the BVAR estimation.[12] Interestingly, the BVAR in growth rates better captures the own funds' sharp increase. As discussed in Section 4.1, this model may best represent the overall trend of the variables, although the quarterly estimates are less precise. With regards to the estimation of unquoted non-priced companies, the RMSFE of the BVAR in log-levels is consistently the smallest (or the second smallest). This confirms that when upwards (or downwards) growth rates are stable, the BVAR in log-levels provides the most reliable estimation. Finally, the auto-ARIMA model seems to show smaller errors across the simulation sets. However, when looking at the actual point forecasts and confidence bands, those do not capture any trend in the data. We compute the Diebold and Mariano (1995) test modified by Harvey, Leybourne, and Newbold (1997) to evaluate whether the differences in RMSFE among models are statistically significant. Results of the test are shown in Table A1 and Table A2 in the Appendix, highlighting that the auto-ARIMA results are not statistically significantly better than those of the other models.

Table 7 reports the percentage gains in forecast scores for the simulation period 2022Q1-2023Q4. This metric is designed to assess the quality of predictive distributions and is therefore applied exclusively to Bayesian models. Here we compare different Bayesian specifications and conditional versus unconditional forecasts. On the one hand, the scores indicate that the BVAR-log achieves higher gains compared to other Bayesian alternatives. On the other hand,

[12] Allowing time variation in means often improves forecast accuracy and can be achieved by introducing steady-state information in the prior distributions. However, we do not implement this scenario and leave it for further development.

it remains unclear whether conditioning on the issuance of debt securities leads to a meaningful improvement in forecast variability. To explore this further, Figure A1 in the Appendix displays the posterior distributions under both conditional and unconditional scenarios for the BVAR log model. The unconditional forecast relies solely on historical data and the model's internal dynamics, whereas the conditional forecast incorporates additional information about the future path of the conditioning variable (the issuance of debt securities). The two distributions should exhibit similar shapes so that conditioning does not distort the underlying forecast structure. Both distributions are centered around the same values but the conditional distribution appears more dispersed around the point forecast. This means that the median of the predictive density, which is our point forecast, is consistent in both specifications. Therefore running a scenario analysis based on the issuance of debt securities does not introduce bias in the point estimation. However, conditioning seems to introduce more variability around the posterior median. Since so far we use the point estimation for retrieving outstanding amounts in unquoted shares, the increased variability does not affect the estimation directly. Nevertheless, we must take this into consideration when evaluating the credible intervals around the median.

Table 7: Percentage gains in terms of scores of BVAR models in the simulation set 2022Q1:2023Q4.

	BVAR log vs growth	BVAR log vs RW	BVAR growth vs RW	BVAR log cond vs uncond	BVAR growth cond vs uncond	RW cond vs uncond
Unquoted-priced	1.9	7.5	5.7	-2.1	1.5	6.3
Unquoted-non-priced	3.7	-0.4	-4.1	1.8	1.5	5.8

5.2 Paid-in capital

This Section evaluates the forecasting results for transactions in unquoted shares on the non-financial corporations' liability side.

For forecasting the firms' quarterly issuance in unquoted shares we consider the BVAR model in Equation 2 in which all variables are specified in first differences. Hence, the BVAR (labelled "BVAR diff") uses only the Minnesota prior, as stationary variables do not need hyperpriors capturing cointegration or trend components. The competing models remain the same: OLS, ARDL, ARDL with augmented PCA, and auto-ARIMA. The simulation settings is unchanged as well. The final estimation of total issuance of unquoted shares is obtained by calculating first differences in firms' forecasted paid-in capital as indicated in Equation 4.

Table 8 and Table 9 reports the RMSFE values for the simulation sets. As in previous cases, the BVAR in first differences performs better than the other models.

Finally, Table 10 presents the gain in terms of score of the conditional forecast compared to the unconditional prediction. In contrast with the results obtained for the estimation of stocks, conditioning to the issuance of debt securities shows clear benefits in terms of variance of forecast.

Table 8: RMSFE of unquoted priced corporations' paid-in capital by simulation set.

	BVAR diff	OLS	AugPCA	ARDL	auto-ARIMA
2018Q1-2019Q4	10.0	119.5	75.9	85.0	86.8
2019Q1-2020Q4	8.9	142.4	79.3	91.2	82.9
2020Q1-2021Q4	4.1	216.1	126.4	148.4	151.9
2021Q1-2022Q4	4.3	206.3	129.8	137.5	147.9
2022Q1-2023Q4	6.1	281.9	166.3	202.5	180.3

Table 9: RMSFE of unquoted non-priced corporations' paid-in capital by simulation set.

	BVAR diff	OLS	AugPCA	ARDL	auto-ARIMA
2018Q1-2019Q4	8.4	306.0	207.3	234.1	304.4
2019Q1-2020Q4	11.7	334.5	223.1	252.8	329.2
2020Q1-2021Q4	5.2	268.0	191.5	210.4	287.0
2021Q1-2022Q4	3.2	268.9	175.2	190.9	258.9
2022Q1-2023Q4	3.7	268.1	176.2	203.4	268.9

Table 10: Percentage gains in terms of scores of BVAR in first differences by simulation set.

	Unquoted priced	Unquoted non-priced
2018Q1-2019Q4	8.4	8.4
2019Q1-2020Q4	8.4	7.9
2020Q1-2021Q4	8.0	8.5
2021Q1-2022Q4	8.4	8.5
2022Q1-2023Q4	8.2	8.5

6 Conclusions

In this paper, we discuss several models for forecasting stock and transactions in unquoted equity on the liability side of Italian firms at quarterly frequency. First, we present how we assign a market value to own funds and paid-in capital in corporates' balance sheets. Second, using the Bayesian perspective discussed in Giannone, Lenza, and Primiceri (2015), we propose a BVAR model against competing modelling specifications. We evaluate whether conducting a forecast conditional to issuance of debt securities affects the point estimate and its variability.

For forecasting outstanding amounts, the BVAR in log-level specification currently yields the most reliable estimates. Nevertheless, we will continue to reassess its performance annually, as changes in the data-generating process may affect the modelling settings.

With regards to transactions, the BVAR model in first differences outperforms alternative approaches. In this case, conditioning significantly reduces the variability of the estimates.

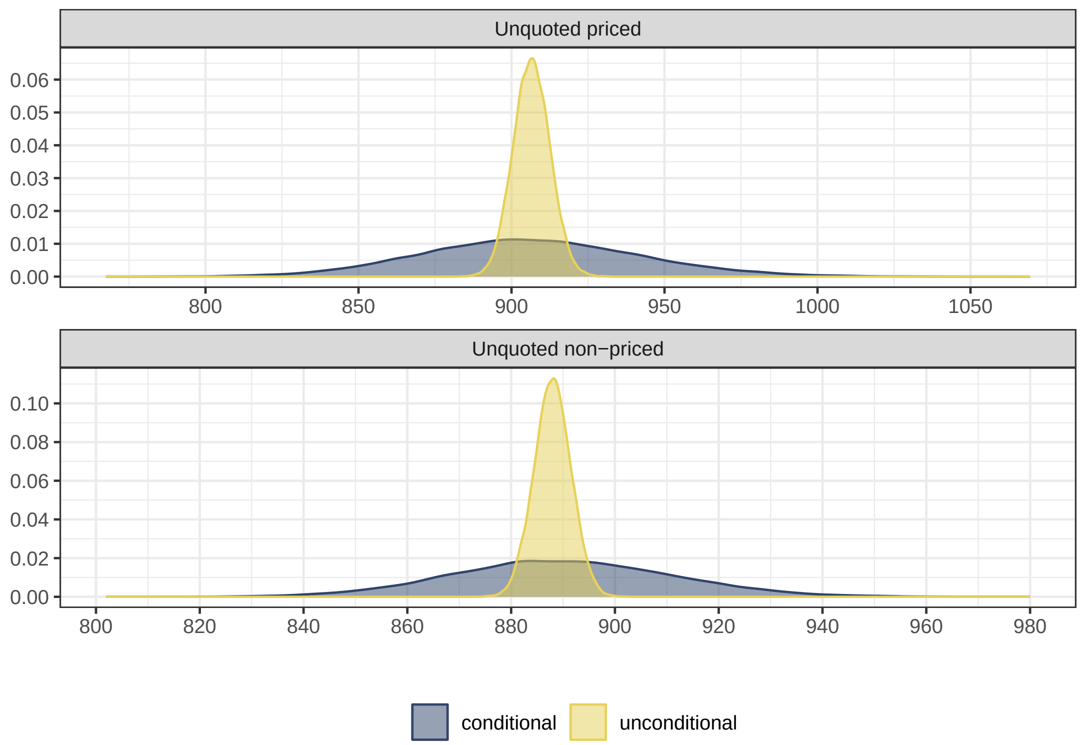
Finally, we document how BVAR models can be effectively applied in the context of official statistics. Such techniques can in the future enhance the estimation and forecast of some critical aspects in national and financial accounting.

A Appendix

A.1 Construction of the BVAR models

The BVAR models are constructed as follows. The number of lags is set to two. As in the Minnesota's distribution provided in Litterman (1986), the hyperpriors for λ follow Gamma densities with mode equal to 1 for the model in log-levels and 0 for the BVAR in growth rates; the standard deviation is set to 0.001 in both models. The parameter μ for the SOC hyperprior follows Gamma density with mode 1 and standard deviation 0.1, while δ of SUR hyperprior is a Gamma with mode 0.3 and standard deviation of 1. Those hyperpriors are specified only for the model in log-levels. The choice of the hyperparameters' distributions follows standard practice in the literature. The elements of Ψ are related to the scale of the variables and are set to the square-root of the innovations variance of an $AR(p)$ process fitted to the data.

A.2 Additional figures



Billions of euros.

Figure A1: Conditional and unconditional density distributions of the BVAR in log-levels for the simulation set 2022Q1:2023Q4.

A.3 Additional tables

Table A1: Percentage of cases in which the forecasts from the competing models are statistically different according to the Diebold and Mariano (1995) test with Harvey et al. (1997) adjustment for unquoted priced companies.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
2018Q1-2019Q4	100	67	0	83	0	33	0
2019Q1-2020Q4	100	67	0	83	0	0	0
2020Q1-2021Q4	100	67	17	83	0	0	0
2021Q1-2022Q4	100	67	17	83	0	0	0
2022Q1-2023Q4	100	67	17	83	0	0	0

Table A2: Percentage of cases in which the forecasts from the competing models are statistically different according to the Diebold and Mariano (1995) test with Harvey et al. (1997) adjustment for unquoted non-priced companies.

	BVAR log	BVAR growth	OLS	RW	AugPCA	ARDL	auto-ARIMA
2018Q1-2019Q4	100	67	0	83	0	0	0
2019Q1-2020Q4	100	67	0	83	0	0	0
2020Q1-2021Q4	100	67	17	83	0	33	0
2021Q1-2022Q4	83	67	17	83	0	50	0
2022Q1-2023Q4	83	67	17	83	0	50	0

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