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COMPARING THE SYSTEMIC RISK OF ITALIAN INSURERS AND BANKS

by Michele Leonardo Bianchi* and Federica Pallante**

Abstract

In this paper we assess the systemic risk of listed Italian insurers and banks by estimating four different measures based on conditional value-at-risk and marginal expected shortfall. Daily estimates in the period from 2007 to 2023 are obtained by assuming a parametric model able to capture volatility clustering phenomena. We keep the framework as simple as possible to get closed formulas or straightforward simulations for the estimation of the risk measures empirically studied in this paper. This allows us to compare the systemic risk of the entities in our sample without resorting to complex model calibration and risk measure evaluation, and to explore the dynamics of systemic risk on more than 4,000 daily observations for the 14 banks and 4 insurers in our sample. Our findings, partly justified by the composition of our sample (i.e. the few insurance companies considered have significantly higher market capitalizations compared with some of the banks in the sample), reveal that, across three out of four risk metrics, insurance entities exhibit slightly higher systemic risk levels than banks, on average. The least systemic banks consistently maintain lower risk profiles compared with the least systemic insurance companies. Conversely, the most systemic bank is slightly riskier than the most systemic insurance company. However, there is more variability among banks compared with insurance companies.

JEL Classification: C63, G21, G22, G17.

Keywords: systemic risk, value-at-risk, conditional value-at-risk, marginal expected shortfall, banks, insurers.

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Introduction[1]

In an increasingly interconnected financial landscape, the assessment of systemic risk within the banking and insurance sectors has become of increasing importance. As past financial crises have shown, the failure or distress of a single financial institution can trigger a domino effect, posing a significant threat to the stability of an entire system (financial system). In light of this, our study delves into an empirical examination of the systemic risk of the Italian financial sector over the past 16 years, focusing on both listed insurances and banks. The primary objective of this work is to estimate systemic risk using four distinct measures, based on the conditional value-at-risk (CoVaR) introduced by Adrian and Brunnermeier [2016] and the marginal expected shortfall (MES) as defined by Acharya et al. [2017] (see also Bernard et al. [2012]). The CoVaR measures how much the system value-at-risk (VaR) increases when a particular institution is in distress. The MES is the expected shortfall (ES) of the system (a particular institution) conditional on the occurrence of a distress of a particular institution (the system). While CoVaR is based on VaR, MES is based on the ES. The International Association of Insurance Supervisors [2021] includes both measures among the reduced-form approaches that could be used by supervisors to identify systemically risky insurers.

Assuming that the CoVaR and the MES are possibly good candidates to measure systemic risk, the aim of this paper is twofold. First, we analyze the behavior of these measures during a large time span covering the last 16 years. Second, we compare the systemic risks of Italian insurances and banks and analyze their dynamics and differences, if any, over time.

To achieve daily risk estimates, we implement a bivariate parametric model, carefully designed to capture the volatility clustering behavior of stock log returns. We keep our modeling framework as simple as possible and this allows us to deal only with closed-form solutions and straightforward simulations for the estimation of the risk measures. The estimates can be obtained without complex algorithms to calibrate the model and to evaluate the risk measures. This reduces the computation burden and ensures that the empirical assessment of systemic risk is accessible without a deep knowledge of multivariate analysis.

The rest of the paper is organized as follows. In Section 2, we recall the definition of the systemic risk measures we focus on in this study. After having provided some details on the input data, we describe the parametric model and the estimation approach implemented in our risk assessment framework (see Section 3). Thus, in Section 4 we describe the main findings of the empirical analysis. Section 5 concludes.

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1 Systemic risk measures

In this section we recall the definition of CoVaR, both the original one introduced by Adrian and Brunnermeier [2016] and the more appropriate alternative (see Bernard et al. [2012] and Bianchi et al. [2023]), and MES as proposed by Acharya et al. [2017].

For each institution j, the random variable y_t^j represents the log returns of the market value of equity S_t^j, that is $y_t^j = \log(S_t^j / S_{t-1}^j)$. Superscript *sys* denotes the entire system (financial system), i.e. the capitalization-weighted portfolio of all financial institutions in the selected sample or an index that is representative of the stock market (e.g. the S&P 500 index or the Euro Stoxx 50 index).

At time t, the $\text{VaR}_{\alpha,t}^j$ of financial institution j, with tail level α, is defined as

$$P\left(y_t^j \leq \text{VaR}_{\alpha,t}^j\right) = \alpha.$$

As defined by Adrian and Brunnermeier [2016], when α is 1% (2.5% or 5%) and market returns of institution j are equal to its VaR_α, a distress for institution j occurs. Thus, for a given tail level β, the $\text{CoVaR}^=$ of the system conditional on institution j being in distress can be obtain by the following equality

$$P\left(y_t^{sys} \leq \text{CoVaR}^{=j}_{\beta,\alpha,t} \mid y_t^j = \text{VaR}_{\alpha,t}^j\right) = \beta. \tag{1.1}$$

The $\Delta\text{CoVaR}^=$ is defined as the difference between the CoVaR in equation (2.1) and the CoVaR with $\alpha = 0.5$ representing the median state, that is

$$\Delta\text{CoVaR}^{=j}_{\beta,\alpha,t} = \text{CoVaR}^{=j}_{\beta,\alpha,t} - \text{CoVaR}^{=j}_{\beta,0.5,t},$$

and measures the institution j contribution to systemic risk.

A more appropriate definition of CoVaR is also possible, that is, for a given tail level β, the $\text{CoVaR}^\leq$ of the system conditional on institution j being in distress is equal to

$$P\left(y_t^{sys} \leq \text{CoVaR}^{\leq sys|j}_{\beta,\alpha,t} \mid y_t^j \leq \text{VaR}_{\alpha,t}^j\right) = \beta. \tag{1.2}$$

In equation (2.2) a distress occurs when market returns of institution j are less or equal to its VaR_α. Under the definition in equation (2.2), it makes no sense to set $\alpha = 0.5$ to define the median state, as done under the definition in equation (2.1). For this reason it is necessary to introduce a different benchmark state to define the $\Delta\text{CoVaR}^\leq$. By following Girardi and Ergün [2013], we define the benchmark state as the event $\{\mu_t^j - \sigma_t^j \leq y_t^j \leq \mu_t^j + \sigma_t^j\}$ with probability of occurrence p_t^j, where μ_t^j and σ_t^j are, respectively, the conditional mean and the conditional standard deviation of institution j. We refer to this event as the *one-sigma* event, in which the log returns of the institution j are far no more than one standard deviation (e.g. one-sigma) from the mean. Thus, as in equation (2.2), we define

$$P\left(y_t^{sys} \leq \text{CoVaR}^{\leq sys|j}_{\beta,\sigma,t} \mid \mu_t^j - \sigma_t^j \leq y_t^j \leq \mu_t^j + \sigma_t^j\right) = \beta. \tag{1.3}$$

Under the definition in equation (2.2), the institution j contribution to systemic risk is defined by

$$\Delta\text{CoVaR}^{\leq j}_{\beta,\alpha,t} = \text{CoVaR}^{\leq j}_{\beta,\alpha,t} - \text{CoVaR}^{\leq j}_{\beta,\sigma,t}. \tag{1.4}$$

It captures the negative externalities that institution j imposes on the system. It should be noted that while the first term of the right-hand side of the equation (2.4) represents the tail event (there is α), the second term represents the *one-sigma* event (there is σ).

At time t, given the $\mathrm{VaR}^{sys}_{\alpha,t}$ of the system (financial system), with tail level α, the marginal expected shortfall $\mathrm{MES}^{j}_{\alpha,t}$ of financial institution j, with tail level α, was defined by Acharya et al. [2017] as

$$\mathrm{MES}^{j|sys}_{\alpha,t} = \mathrm{E}\left[y^j_t \,\middle|\, y^{sys}_t \leq \mathrm{VaR}^{sys}_{\alpha,t}\right]. \tag{1.5}$$

Equation (2.5) represents the expected loss of financial institution j when the system is in distress. This distress is represented through the inequality $\{y^{sys} \leq \mathrm{VaR}^{sys}_{\alpha}\}$. Similarly to equation (2.2), for a given tail level β, the $\mathrm{CoVaR}^{\leq}$ of the institution j conditional to the system being in distress is equal to

$$P\left(y^j_t \leq \mathrm{CoVaR}^{\leq j|sys}_{\beta,\alpha,t} \mid y^{sys}_t \leq \mathrm{VaR}^{sys}_{\alpha,t}\right) = \beta, \tag{1.6}$$

and, as shown by Banulescu et al. [2021], the following equality holds

$$\mathrm{MES}^{j|sys}_{\alpha,t} = \int_0^1 \mathrm{CoVaR}^{\leq j|sys}_{\beta,\alpha,t} d\beta. \tag{1.7}$$

It is possible to define a MES with the same conditioning event of the $\mathrm{CoVaR}^{\leq}$ in equation (2.2), that is

$$\mathrm{MES}^{sys|j}_{\alpha,t} = \mathrm{E}\left[y^{sys}_t \mid y^j_t \leq \mathrm{VaR}^{j}_{\alpha,t}\right]. \tag{1.8}$$

Equation (2.8) represents the expected loss of the system when the financial institution j is in distress, that is the event $\{y^j \leq \mathrm{VaR}^j_{\alpha}\}$.

In Adrian and Brunnermeier [2016] the distress of a financial institution j is defined as the event $\{y^j = \mathrm{VaR}^j_{\alpha}\}$. Here we also consider the conditional value-at-risk ($\mathrm{CoVaR}^{\leq}$) measure, that is the systemic risk measure where the conditioning event is the distress represented through the inequality $\{y^j \leq \mathrm{VaR}^j_{\alpha}\}$. Mainik and Schaanning [2014] demonstrates that the $\mathrm{CoVaR}^{\leq}$ in equation (2.2) gives a much more consistent response to dependence than the $\mathrm{CoVaR}^{=}$ in equation (2.1). Additionally, as shown in Banulescu et al. [2021], the $\mathrm{CoVaR}^{\leq}$ can be backtested (see also Girardi and Ergün [2013] and Bianchi et al. [2023]), by using the standard tests developed for the VaR, and the MES can be written in terms of this systemic risk measure. Conversely, the original $\mathrm{CoVaR}^{=}$ of Adrian and Brunnermeier [2016] is simple to estimate, but not so simple to backtest, because the tests to backtest the VaR cannot be applied. As shown in Bianchi and Sorrentino [2020], if the model for stock log returns is based on the multivariate normal random variable, a close formula for $\mathrm{CoVaR}^{=}$ is available.

2 Implementation

The measurement of the systemic risk measures described in Section 2 is divided into three steps: (1) the estimate of the univariate models on the time series of log returns; (2) the estimate of the bivariate models on the data extracted from the first step; (3) the computation of the risk measure (VaR) and of the systemic risk measures (i.e. CoVaR$^{=}$, CoVaR$^{\leq}$, and two different definitions of MES). It should be noted that the definitions of the four systemic risk measures provided in Section 2 are valid for any bivariate model for the dynamics of log returns.

In this section we describe the data input of the empirical analysis (Section 3.1), the parametric model (Section 3.2) and the approach followed to estimate the systemic risk measures (Section 3.3).

2.1 Data

We estimate the systemic risk measure with daily data from January 2002 to September 2023, by considering in the estimation of model parameters rolling windows with a 5-year length. The dividend-adjusted closing stock prices and the market capitalization for listed Italian insurance companies and banks are obtained from Datastream (LSEG). We consider all insurance companies and banks listed at the end of September 2023 and with at least five years of daily observations (i.e. 4 insurances and 14 banks).

The time period in this analysis includes the high volatility period after the Lehman Brothers filed for Chapter 11 bankruptcy protection (September 15 2008), the eurozone sovereign debt crisis, during which, in November 2011, the spread between the 10-year Italian BTP and the German Bund with the same maturity exceeded 500 basis points, the turmoil after the Italian political elections in 2018, the outbreak of Covid-19 when the Italian stock market fell by 16.92 percent in a single day (March 12, 2020), and the recent financial market turmoil caused by the conflict between Russia and Ukraine.

In the empirical study we consider two different definitions of system. First, we assume that the it is represented by the capitalization-weighted portfolio of all financial institutions in the selected sample, that is both insurance companies and banks, and we refer to it as *weighted portfolio*. Then, as an alternative, we also assume the system represented by FTSE MIB index, in which some banks and insurances are included,[2] and we refer to it as *index*. Given the inclusion of non-financial companies in the index, the risk measures are evaluated within a broader system, going beyond just the financial sector. Thus, we study how the assumption in the definition of system affects the estimates of the systemic risk measures.

2.2 Parametric model

In this section we provide a concise overview of the parametric model on which our systemic risk metrics are based. We assume for univariate log return processes an AR(1)-GARCH(1,1) model with Glosten-Jagannathan-Runkle (GJR) dynamics for the volatility, that is

$$\begin{aligned} y_t &= a y_{t-1} + \sigma_t \varepsilon_t + c \\ \sigma_t^2 &= \xi_0 + \xi_1 \left(|\sigma_{t-1}\varepsilon_{t-1}| - \gamma \left(\sigma_{t-1}\varepsilon_{t-1} \right) \right)^2 + \eta_1 \sigma_{t-1}^2 \end{aligned} \tag{2.1}$$

[2] For the complete list of components readers are reffered to `https://www.borsaitaliana.it/borsa/azioni/ftse-mib/lista.html?lang=en`.

where ε_t is a collection of independent and normally distributed random variables with zero mean and unit variance (see Glosten et al. [1993]).

The univariate autoregressive GARCH model featuring GJR dynamics for the log returns of both the system and individual institutions is estimated by means of the *garchFit* function of the *fGarch* package of R. In each iteration of the estimation phase, we assess the statistical significance of the autoregressive component. If it fails to meet the significance threshold, we proceed to estimate the model without the autoregressive component.

Once we have estimated the univariate discrete-time dynamic volatility model, as specified in equation (3.1), for each individual institution and the system as a whole, we proceed to extract the innovations. Subsequently, at each time t, we estimate the parameters of the multivariate normal random variable having as margins the system and the financial institutions. This is equivalent to the estimation of the constant conditional correlation (CCC) model. As an alternative, it is possible to consider the dynamic conditional correlation (DCC) model with multivariate normal (see Bauwens et al. [2006]) or non-normal (see Bianchi et al. [2023]) innovations.

Following the completion of these two estimation steps, we proceed to predict the one-day ahead volatility based on the AR-GARCH parameters we have found out. These forecasted volatilities serve as the input data for the evaluation of the risk metrics in Section (3.3).

2.3 Risk measures estimation

The first step in the estimation of the systemic risk measures discussed in Section 2 is the computation of the VaR. Under the modeling framework defined in Section 3.2, as observed in Kim et al. [2011], the following equality holds

$$\mathrm{VaR}^{y}_{\alpha,t+1} = a y_t + \sigma_{t+1}(\mathrm{VaR}^{\varepsilon}_{\alpha,t+1}) + c. \tag{2.2}$$

In practical terms, this implies that the VaR of y is the quantile of a standardized random variable ε. Within the modeling framework described in Section 3.2, ε is a standardized normal random variable.

From equations (17) and (18) in Adrian and Brunnermeier [2011], if one assumes a multivariate GARCH model with normal innovations, the $\Delta\mathrm{CoVaR}^{=}$ becomes a function of the volatility of the system and the correlation between the institution j and the system (see Bianchi and Sorrentino [2020]), that is,

$$\Delta\mathrm{CoVaR}^{=j}_{\beta,\alpha,t} = \phi^{-1}(\alpha)\rho^{j}_{t}\sigma^{sys}_{t}, \tag{2.3}$$

where ϕ^{-1} is the inverse of the cumulative distribution function of a standardized normal random variable, ρ^j_t is the correlation between the residuals of institution j and those of the system, and σ^{sys}_t is one-day ahead forecast at day t of the volatility in the system estimated in Section 3.2. As observed in Adrian and Brunnermeier [2016], for jointly and normally distributed random variables, the ΔCoVaR estimate strictly depends on the correlation between institution j and the system, that is, systemic banks are those with the highest correlation with the system.

The estimates of $\text{CoVaR}^{\leq}$ and MES are more challenging to obtain. Even under the bivariate normal distribution assumption, numerical procedures are needed to estimate these systemic risk measures (see Bernard et al. [2012]). In both cases there are not closed formulas and the estimates are obtained by evaluating multiple integrals, by finding the zeros of a function (see Bianchi et al. [2023]) and (or) by numerical inversion.

However, as observed by Brownlees and Engle [2017], it is straightforward to implement a simulation based procedure to obtain exact MES (and $\text{CoVaR}^{\leq}$) estimates. Even if this simulation approach is not elegant from a purely theoretical perspective, it remains valid for any bivariate model for the dynamics of log returns. Additionally, the same random samples can be used to obtain predictions of both systemic risk measures, reducing the simulation bias in the analysis of the differences between risk measures. The simulation approach seems particularly convenient when dealing with normal random variables, as in our case, since they are simple to simulate.

Assuming that, for each t and each j, we are able to generate N random variates $y^{j}_{t,i}$ and $y^{sys}_{t,i}$, where i ranges from 1 to N, we define the following events

$$\begin{aligned} M &= \left\{ y^{sys}_{t,i} \leq \text{CoVaR}^{\leq sys|j}_{\beta,\alpha,t} \right\}, \\ D^{j} &= \left\{ y^{j}_{t,i} \leq \text{VaR}^{j}_{\alpha,t} \right\}, \\ D^{sys} &= \left\{ y^{sys}_{t,i} \leq \text{VaR}^{sys}_{\alpha,t} \right\}, \\ D^{\sigma_j} &= \left\{ \mu^{j}_{t} - \sigma^{j}_{t} \leq y^{j}_{t,i} \leq \mu^{j}_{t} + \sigma^{j}_{t} \right\}. \end{aligned}$$

An estimate of the $\text{CoVaR}^{\leq}$ in equation (2.2) can be obtained by finding the value of $\text{CoVaR}^{\leq sys|j}_{\beta,\alpha,t}$ which satisfies the following equality

$$\frac{\sum_{i=1}^{N} I_M I_{D^j}}{\sum_{i=1}^{N} I_{D^j}} = \beta, \tag{2.4}$$

where $I_{[\cdot]}$ is an indicator function. To estimate the $\text{CoVaR}^{\leq}$ in equation (2.3), one considers the event D^{σ_j} instead of D^{j}. Similarly, an estimate of the MES in equation (2.5) can be obtained through the following equality

$$\text{MES}^{j|sys}_{\alpha,t} = \frac{\sum_{i=1}^{N} y^{j}_{t,i} I_{D^{sys}}}{\sum_{i=1}^{N} I_{D^{sys}}}, \tag{2.5}$$

and the same approach can be used to estimate the $\text{MES}^{sys|j}$ in equation (2.8).

3 Empirical findings

As observed in Section 3, the systemic risk measure estimation is divided into three steps. In the first step, we estimate a univariate AR-GARCH model on the time series of log returns. In the second step, we estimate the dependence structure. In the third step, we compute the VaR at the given tail level α and then, by setting

α (or σ) and β, we compute the four systemic risk measures. While the $\Delta\text{CoVaR}^{=}$ is computed by applying the formula in equation (3.3), the other three systemic risk measures are obtained through Monte Carlo simulation. As already observed, given the bivariate discrete-time model discussed in Section 3.2, it is simple to generate random draws for y_t^j and y_t^{sys} and, then, to evaluate (3.4) and (3.5). For each institution j and each time step t, we generate one million simulations for the bivariate model. To speed up the evaluation phase, we rely on an efficient R code making use of the packages *foreach* and *doParallel* and run it on a multi-core platform (a Linux based system with Intel processors).

The systemic risk measures are estimated on the basis of the time series from the previous five years. For example, the $\text{CoVaR}^{=}$ for December 6, 2018 is estimated from the data for the period from December 6, 2013 to December 5, 2018. For each intermediary and each model we consider 4,253 estimations from January 3, 2007 to September 30, 2023. It should be noted that all the data presented in this section consists of moving averages calculated over a two-month rolling period.

In this empirical analysis the comparison is conducted under different standpoints. First, we investigate if and how the definition of system affects the estimates. Second, we assess how the systemic risk depends on the definition of risk metrics (i.e. $\Delta\text{CoVaR}^{=}$, $\Delta\text{CoVaR}^{\leq}$, $\text{MES}^{j|sys}$, and $\text{MES}^{sys|j}$). Third, the systemic risk may vary across different types of institutions (i.e. insurances vs banks).

As observed in Section 3.1, we consider two different definitions of system. While the first one is represented by the capitalization-weighted portfolio of all financial institutions in the selected sample (Figures 1), the second one is represented by FTSE MIB index (Figures 2).

In Figures 1 and 2 we show the behavior of the $\Delta\text{CoVaR}^{=}$, $\Delta\text{CoVaR}^{\leq}$ and of two MES, the first measuring the expected loss of the institution j given the distress of the system (i.e. $\text{MES}^{j|sys}$) as proposed by Acharya et al. [2017], the second measuring the expected loss of the system given the distress of the institution j (i.e. $\text{MES}^{sys|j}$).

In both figures, on the left (right) side, the black lines represent the highest and lowest values achieved by the insurances (banks) during a specific trading day. In contrast, the magenta (red) lines denote the peak values attained by the banks (insurances) within the sample on a given trading day, while the green (blue) lines depict the corresponding minimum values.

Then in Tables 1 and 2 for the four systemic risk measures and by distinguish between insurances (I) and banks (B), we report some summary statistics computed across institutions and over time. We first compute, for each trading day, minimum, maximum and mean values across institutions, then we evaluate various statistics of these time series over all trading days considered in this study. This allow us to evaluate the differences between Italian insurances and banks.

Even if, by construction the $\Delta\text{CoVaR}^{=}$ is lower than the $\Delta\text{CoVaR}^{\leq}$, we do not observe remarkable differences between the two measures. This is also caused by the choice of the parametric model which is not able to capture possible tail events (see Bianchi et al. [2023]). Since the $\text{CoVaR}^{\leq}$ and the $\text{MES}^{sys|j}$ have a similar definition and the same conditioning event, there are not remarkable differences between these two measures and their dynamics are almost indistinguishable. Conversely, the $\text{MES}^{j|sys}$, which measures the expected shortfall (ES) of the insti-

				low	high	mn	md	σ	s	k
ΔCoVaR$^{=}$	weighed portfolio	B	min	0.17	2.44	0.97	0.95	0.39	0.69	1.79
			max	1.28	5.94	2.70	2.52	0.89	1.16	1.35
			mean	0.75	4.29	1.86	1.78	0.63	1.13	1.88
		I	min	0.50	4.42	1.73	1.63	0.63	1.05	2.33
			max	1.30	5.31	2.44	2.26	0.79	1.23	1.38
			mean	0.81	4.71	2.00	1.88	0.67	1.19	1.98
	index	B	min	0.15	2.21	0.78	0.76	0.31	0.98	2.85
			max	0.93	5.00	2.02	1.81	0.70	1.57	2.68
			mean	0.57	3.65	1.44	1.32	0.50	1.41	2.51
		I	min	0.41	3.96	1.38	1.26	0.50	1.58	4.77
			max	0.92	4.71	1.96	1.74	0.69	1.55	2.37
			mean	0.61	4.23	1.60	1.43	0.56	1.59	3.43
ΔCoVaR$^{\leq}$	weighed portfolio	B	min	0.21	2.99	1.19	1.17	0.47	0.70	1.81
			max	1.54	6.97	3.18	2.96	1.04	1.17	1.33
			mean	0.91	5.11	2.23	2.12	0.76	1.13	1.85
		I	min	0.62	5.29	2.09	1.97	0.75	1.06	2.30
			max	1.55	6.24	2.89	2.67	0.94	1.23	1.34
			mean	0.99	5.62	2.39	2.25	0.80	1.19	1.93
	index	B	min	0.18	2.70	0.96	0.93	0.38	0.97	2.78
			max	1.11	5.89	2.38	2.12	0.82	1.59	2.73
			mean	0.69	4.38	1.73	1.58	0.60	1.43	2.54
		I	min	0.50	4.74	1.66	1.52	0.60	1.58	4.65
			max	1.10	5.57	2.32	2.05	0.82	1.57	2.47
			mean	0.74	5.04	1.91	1.72	0.67	1.60	3.41

Table 1: Summary statistics of the estimated systemic risk measures between January, 2007 and September, 2023. All values are changed in sign. We first compute minimum, maximum and mean values (min, max and mean) across banks (B) and insurances (I), then we evaluate various statistics of these time series: minimum (low), maximum (high), mean (mn), median (md), standard deviation (σ), skewness (s) and kurtosis (k).

				low	high	mn	md	σ	s	k
$\text{MES}^{j\|sys}$	weighed portfolio	B	min	0.03	2.01	1.06	1.13	0.46	-0.50	-0.35
			max	2.02	11.89	5.57	5.00	2.08	0.98	0.25
			mean	1.07	6.24	3.01	2.94	0.90	0.57	1.01
		I	min	0.56	4.09	2.18	2.13	0.69	0.43	0.19
			max	1.55	18.28	3.41	2.90	2.10	4.41	24.63
			mean	1.07	9.77	2.75	2.49	1.21	2.68	10.72
	index	B	min	0.03	2.01	1.07	1.13	0.44	-0.57	-0.06
			max	2.20	11.17	5.42	4.79	1.98	1.07	0.33
			mean	1.14	6.05	3.00	2.90	0.87	0.64	0.81
		I	min	0.64	4.30	2.25	2.18	0.72	0.56	0.15
			max	1.57	19.42	3.53	2.98	2.26	4.38	24.11
			mean	1.13	10.39	2.85	2.58	1.29	2.75	10.92
$\text{MES}^{sys\|j}$	weighed portfolio	B	min	0.16	3.07	1.21	1.21	0.50	0.60	1.55
			max	1.56	7.42	3.36	3.16	1.11	1.13	1.32
			mean	0.88	5.40	2.32	2.23	0.81	1.07	1.77
		I	min	0.58	5.52	2.16	2.05	0.79	1.00	2.21
			max	1.57	6.68	3.05	2.83	1.00	1.18	1.29
			mean	0.96	5.88	2.49	2.36	0.84	1.13	1.85
	index	B	min	0.18	2.75	1.00	0.97	0.40	0.89	2.33
			max	1.16	6.21	2.53	2.28	0.88	1.53	2.48
			mean	0.71	4.54	1.82	1.67	0.63	1.35	2.24
		I	min	0.51	4.91	1.74	1.60	0.63	1.53	4.39
			max	1.15	5.85	2.46	2.19	0.87	1.50	2.17
			mean	0.77	5.25	2.01	1.81	0.70	1.53	3.13

Table 2: Summary statistics of the estimated systemic risk measures between January, 2007 and September, 2023. All values are changed in sign. We first compute minimum, maximum and mean values (min, max and mean) across banks (B) and insurances (I), then we evaluate various statistics of these time series: minimum (low), maximum (high), mean (mn), median (md), standard deviation (σ), skewness (s) and kurtosis (k).

tution j conditional on the occurrence of a distress of the system, has a different pattern in comparison with the other three measures. As shown in both figures, the dynamics of this latter measure is strongly affected by an episode occurred to a specific insurance in 2012.

For three over four risk metrics (i.e. with the exception of the $\text{MES}^{j|sys}$), the systemic risk of insurances is, on average, slightly higher than the risk of banks (see the rows labeled as *mean* in Tables 1 and 2). On all trading days in our observation period, the less systemic banks has a risk much lower than the less systemic insurance (see the rows labeled as *min* in Tables 1 and 2). This finding remains true across all perspectives examined in this study, even if it could be due to the fact that the few considered insurance companies are larger than the smallest bank in the sample. Conversely, the most systemic bank is slightly riskier than the most systemic insurance (see the rows labeled as *max* in Tables 1 and 2). However, this empirical results are not true if one looks at the $\text{MES}^{j|sys}$. From the perspective provided by this latter risk measure, the impact of the system on the most systemic banks is much larger that the corresponding impact on most systemic insurances. This seems to suggest that, during distress periods, the system and the most systemic banks influence each other, even if with a different impact (i.e. on average $\text{MES}^{j|sys}$ is larger than $\text{MES}^{sys|j}$ as reported in the rows labeled as *mean* in Table 2). For most systemic insurances this only partially true: if a distress for the system occurs, the expected loss of most systemic insurances is lower than the expected loss of most systemic banks (see the rows labeled as *max* related to the estimates of $\text{MES}^{j|sys}$ in Table 2).

As far as the heterogeneity across institutions is concerned, while we observe considerable dispersion across banks, the differences in the systemic risk of insurances are lower, with the exception of a specific episode occurred at single institution. These empirical findings are also motivated by the fact that there are 14 banks and only 4 insurances in our sample. The insurance companies included in our empirical analysis have significantly higher market capitalizations compared to some of the banks in the sample. During the analyzed period, the average total market capitalization was 43.1 billion for the four insurance companies, 77.1 billion for the four largest banks, and 1.5 billion for the four smallest banks.

From a visual comparison of the dynamics reported in Figures 1 with those shown in Figures 2, it appears clear that the assumption on the variable representing the system (weighted portfolio vs index) partially affects the dynamics of the estimates. The major differences can be observed when a distress more affecting the financial sector occurs, like in 2016. This is also confirmed by the value of σ reported in Tables 1 and 2, which is higher when the weighted portfolio is considered to represent the system. The levels of the risk measures computed on the basis of the weighted portfolio are higher. This may be due to the fact that, as observed above, the constituents of the FTSE MIB index are not only financial, even if financial companies have a significant weight in the index. There are risk factors affecting the behavior of the index and not related to financial companies. We recall that by construction the companies included in the weighted portfolio are only the institutions in the sample considered in this study, that is all listed Italian insurances and banks.

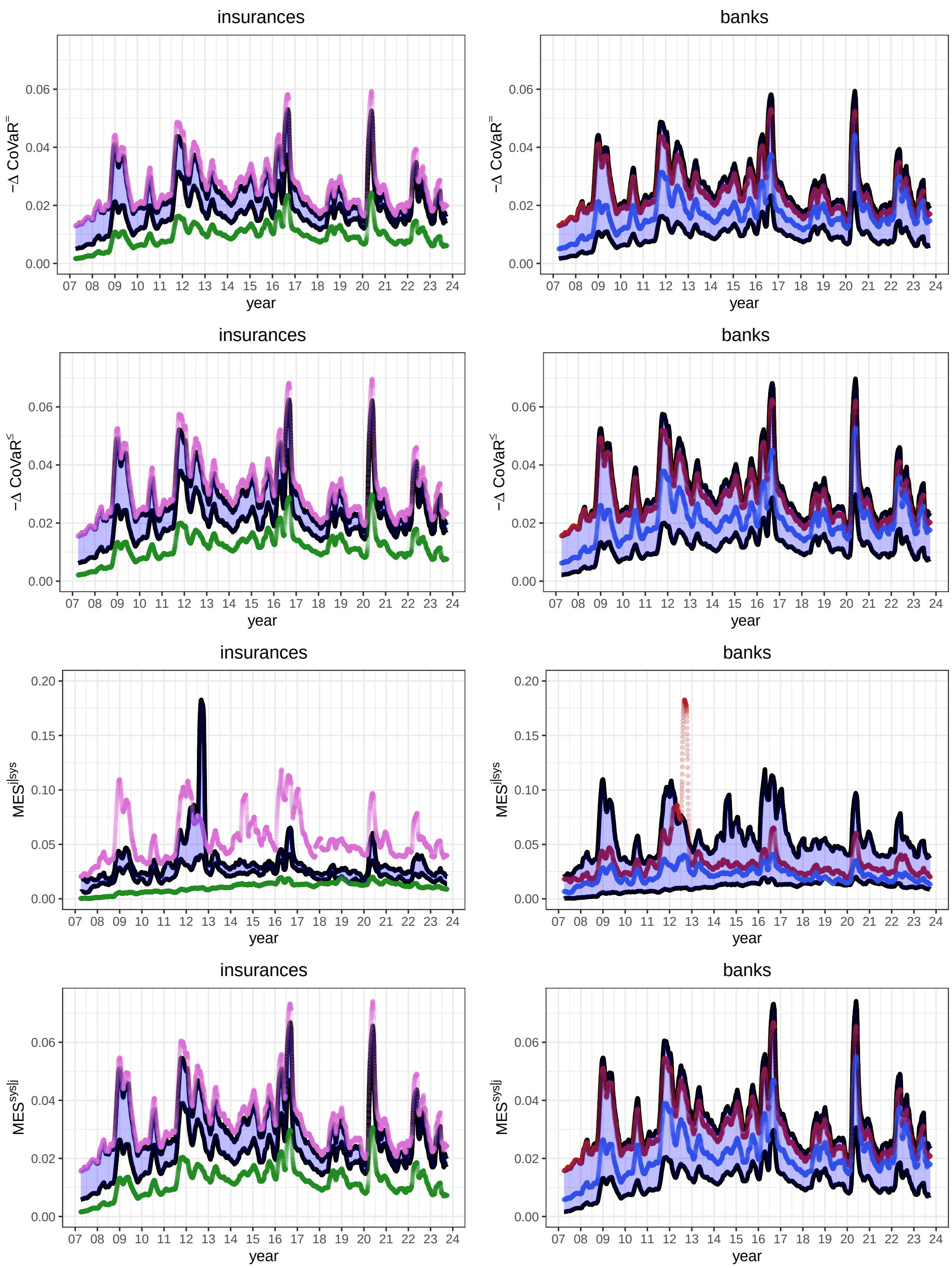

Figure 1: We report the time series from January 2, 2007 to September 30, 2023 of the 3-month rolling window estimates of $\text{CoVaR}^{=}$, $\text{CoVaR}^{\leq}$ and MES, with $\alpha = \beta = 0.05$. We assume that the system is represented by the capitalization-weighted portfolio of all financial institutions in the selected sample (weighed portfolio). All values are changed in sign. On the left (right) side, the black lines identify the maximum and minimum value reached by the insurances (banks) in a given trading day. While the magenta (red) depicts the maximum value reached in a given trading day by the banks (insurances) in the sample, the green (blue) depicts the minimum one.

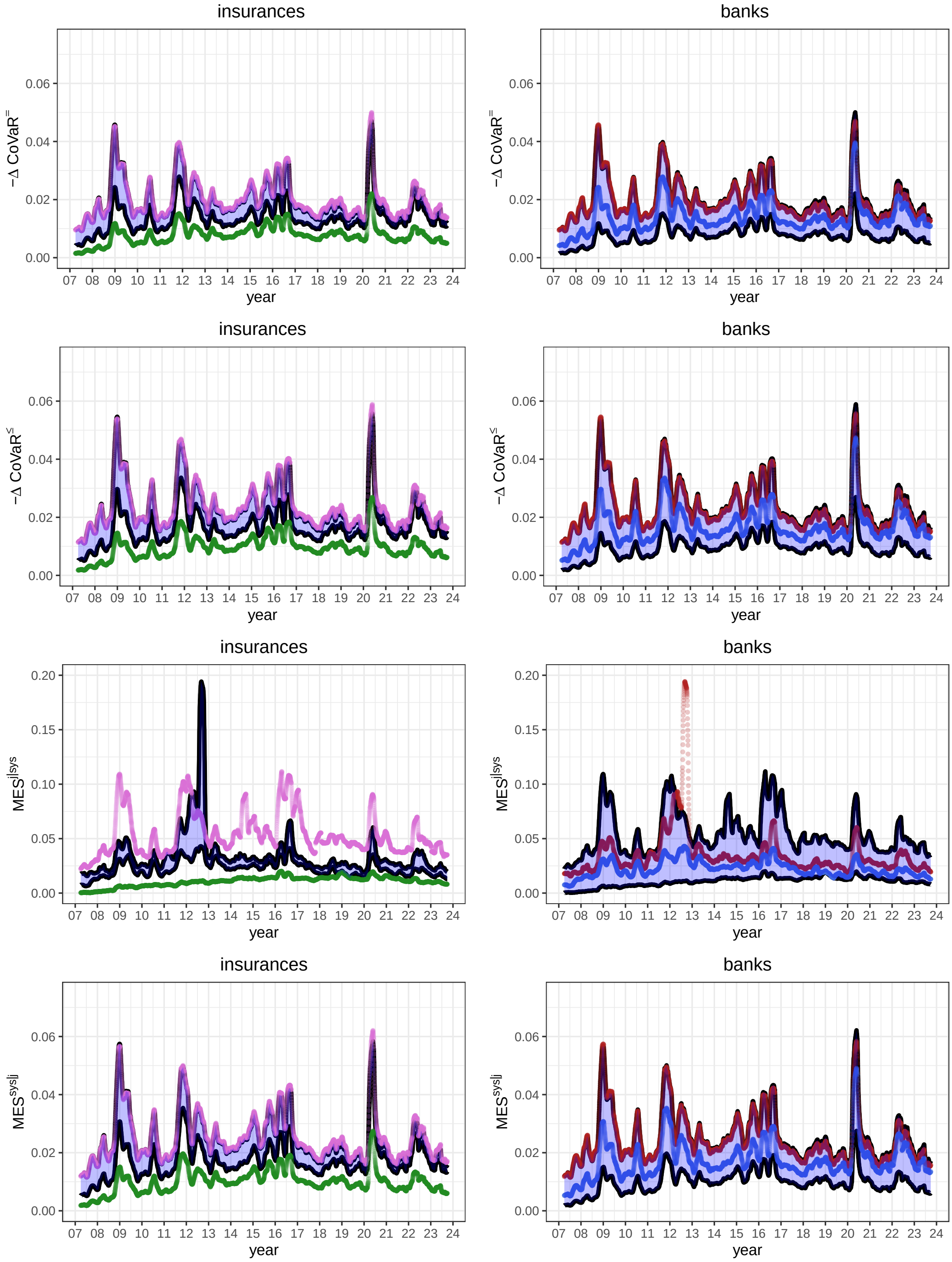

Figure 2: We report the time series from January 2, 2007 to September 30, 2023 of the 3-month rolling window estimates of $\text{CoVaR}^{=}$, $\text{CoVaR}^{\leq}$ and MES, with $\alpha = \beta = 0.05$. . We assume that the system is represented by the FTSE MIB index (index). All values are changed in sign. On the left (right) side, the black lines identify the maximum and minimum value reached by the insurances (banks) in a given trading day. While the magenta (red) depicts the maximum value reached in a given trading day by the banks (insurances) in the sample, the green (blue) depicts the minimum one.

4 Conclusions

In this work we assess the systemic risk in the financial sector to monitor the potential of a single institution distress to trigger a wider financial turmoil of the system. We also measure the extent to which a distress for the system results in a severe event for a specific institution within the system. The study focuses on Italian insurances and banks over the last 16 years with the aim to estimate systemic risk measures belonging to the CoVaR and the MES family. The study employs a parametric model for daily risk estimates, which simplifies both the model calibration phase and risk measure evaluations. The paper has two main objectives: to analyze these measures behavior over time and to compare systemic risks between insurances and banks. In this empirical analysis, we approach the comparison from distinct perspectives. Initially, we examine the potential impact of how we define the system on our estimations. We show that the assumption on the variable representing the system (weighted portfolio vs index) plays a role in shaping the dynamics of the estimates. Subsequently, we assess the extent to which the systemic risk depends on the choice of risk metrics. When both CoVaR and MES are defined based on the same conditioning event, notable discrepancies between these two measures are not observed, and their dynamics appear almost identical. Differently, the MES displays significant sensitivity to the specific definition of the conditioning event (i.e. a distress for the entire system or an individual institution). Lastly, we explore possible heterogeneity in the systemic risk of different categories of financial institutions, specifically, insurance companies and banks. Across three out of four risk metrics, insurance entities exhibit slightly higher systemic risk levels compared to banks, on average. The least systemic banks consistently exhibit lower risk profiles than the least systemic insurance companies. In contrast, the most systemic bank is slightly riskier than the most systemic insurance company. Moreover, there exists a significant degree of variability among banks, whereas the differences in systemic risk among insurance companies are comparatively smaller. These findings are influenced by our sample, which includes 14 banks and 4 insurance firms. Throughout the analyzed period, the four largest banks have higher market capitalizations compared to the considered insurance companies, which in turn have much higher average market capitalizations compared to the smallest banks.

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