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AGGREGATE DYNAMICS AND MICROECONOMIC HETEROGENEITY: THE ROLE OF VINTAGE TECHNOLOGY

by Giuseppe Fiori* and Filippo Scoccianti**

Abstract

We study how the timing of technology adoption through capital accumulation shapes firm-level productivity dynamics and quantify its aggregate implications in a model of heterogeneous firms. Using data on the census of incorporated Italian firms and exploiting the lumpiness of capital accumulation, we document that large investment episodes lead to productivity gains at the firm and sectoral level due to vintage effects. In a general equilibrium model of firm heterogeneity, we find that the presence of vintage technology constitutes a powerful microeconomic-based amplification mechanism of aggregate shocks relative to a benchmark real business cycle model.

JEL Classification: D24, E22, E32.
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1 Introduction

In the aftermath of the Global Financial Crisis[1], advanced economies have experienced stagnant productivity accompanied by slow capital accumulation (OECD, 2015). Since the work of Johansen (1959) and Solow (1960), a large body of the theoretical literature has emphasized the role of investment for productivity dynamics through technology adoption, as the newer vintages are of better quality and may enhance the efficiency of existing capital.

In this paper, we study the aggregate consequences of firm heterogeneity in technology adoption and advance the existing literature along three dimensions.

First, we bring firm-level evidence to bear on the role of capital accumulation for productivity dynamics. Using data on the census of incorporated Italian firms, we document that significant capital expenditures, or investment *spikes*, lead to productivity gains: Firms that have experienced recent spikes have higher productivity than firms that experiences spikes less recently. Controlling for a plethora of confounding factors, including reverse causality, we estimate this gap at about three-quarters of a percentage point per year for total factor productivity (TFP). Our evidence indicates that technology adoption through capital accumulation is a source of productivity heterogeneity across firms.

Second, we document that firms investment decisions contribute to sectoral TFP dynamics. Driven by an increase in the share of firms experiencing spikes, which is the extensive margin of investment, a strong pace of capital accumulation at the sectoral level leads to higher sectoral TFP through the improved efficiency of newer capital.

Third, we highlight the importance of microeconomic heterogeneity for aggregate outcomes, as investment slumps contribute to slow TFP growth with negative effects on GDP. To quantify the role of vintage technology for aggregate fluctuations, we formulate a state-of-the-art model of firm heterogeneity that explicitly accounts for the link between capital accumulation and TFP dynamics at the firm level. In the model, a non-convex adoption

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cost prevents firms from adopting the most recent technology in every period and reproduces the pattern of capital accumulation at the firm level. New and old vintages coexist in equilibrium, yielding a non-degenerate distribution of capital stocks and technologies across firms. Following aggregate shocks, shifts in this distribution result in endogenous procyclical movements in economy-wide productivity that *amplify* macroeconomic fluctuations beyond the effect of the initial shock.

In the empirical analysis, we employ firm-level data that cover over two-thirds of the value added in the Italian economy and span 30 years.[2] The dimension of our panel, both in the cross section and in the time series, allows us to quantify how the timing of large capital expenditures affects various measures of productivity controlling for many potential confounding factors, such as firms' age and size; aggregate-, industry-, and firm-specific effects; and the potential endogeneity of investment spikes.

We start by documenting the nature of capital accumulation at the firm level and its importance for aggregate investment dynamics. In line with many studies across advanced economies, investment at the firm level is a large and infrequent, or *lumpy*, episode. On average, only 18 percent of firms exhibit investment spikes or an investment rate above 20 percent, but they account for about two-thirds of total investment in our data.

To study whether the timing of investment contributes to productivity gains at the firm level, we use investment age, the time elapsed between the firm's investment spikes, to capture the vintage technology available to firms.

We consider an instrumental variable (IV) approach to tackle the potential endogeneity of investment spikes that may be driven by firm-specific idiosyncratic shocks. Exploiting the richness of our data, we instrument current investment age with its own lags taken at different frequencies and find that the negative relationship between investment age and TFP is not driven by present or *past* realizations of productivity innovations.

Our results highlight that technology adoption through capital accumulation is a pivotal contributor to firms' productivity dynamics: firms with a lower investment age are,

[2]The coverage in terms of value added has been increasing over time, from around 60 percent at the beginning of the '90s, to around 80 percent at the end of the sample period.

other things being equal, more productive than firms with a higher investment age. This finding is robust to alternative definitions of spikes, the sample composition, and the age of the firms in our sample. At the industry level, we find evidence of vintage effects in sectors for which equipment goods are an essential component of their production process.

One significant implication of the empirical analysis is that the timing of firms' investment decisions leads to productivity heterogeneity across firms and, through this channel, contributes to determining aggregate productivity. Measuring the macroeconomic relevance of this microeconomic behavior requires a theoretical framework that (i) agrees with the micro evidence on capital accumulation and the timing of investment spikes, (ii) reproduces the link between investment and productivity, and (iii) takes into account general equilibrium effects. As detailed in Section 1.1, despite the advancements in the investment literature, little to no theoretical work aims to quantify the implications of vintage technology for aggregate productivity. We aim to fill this gap in the existing literature.

Our approach follows in the footsteps of Khan and Thomas (2008) and builds on their work on firm heterogeneity. They formulate a general equilibrium framework with rich firm heterogeneity that reproduces the pattern of capital accumulation at the firm level but relies on exogenous TFP at the firm level. To account for the causal link between investment and TFP, we introduce a quality ladder model in the form of a vintage technology structure to the framework in Khan and Thomas (2008) that makes firm-level TFP dependent on capital accumulation.

In the model, the firm's productivity includes *(i)* the *permanent* vintage component, which is endogenous to the timing of technology adoption, and *(ii)* a temporary but persistent idiosyncratic component, which is fully exogenous. The vintage component evolves as in a quality ladder model. Firms optimally decide if and when to adopt the latest vintage, i.e., the latest technology. As this choice is subject to a non-convex adoption cost, the firm's policy functions are of the (S,s) type: some firms adopt the latest technology, while others postpone it. Conditional on the adoption decision, firms optimally choose the capital stock. In equilibrium, technologies of different quality coexist.

This microeconomic heterogeneity determines aggregate productivity. Instead, when the adoption cost is set to zero, all the firms find it optimal to adopt the latest technology in every period. Thus, our framework boils down to a standard real business cycle (*RBC*) model where firms' and aggregate productivity coincide and are exogenous.

To study the role of microeconomic heterogeneity in shaping aggregate fluctuations, we parameterize the model to reproduce the cross-sectional distribution of investment rates and investment age in the data. Our framework has a novel set of theoretical implications relative to the benchmark Khan and Thomas framework. In response to aggregate shocks, microeconomic heterogeneity *amplifies* macroeconomic dynamics. The presence of vintage effects at the *firm level* is critical for this result: shifts in the distribution of capital stocks and technologies across firms induced by the aggregate shock lead to fluctuations in the economy-wide TFP. In turn, this endogenous response of productivity constitutes an additional force that contributes to amplifying macroeconomic dynamics beyond the effect of the initial shock. We also note that the link between investment decisions and productivity results in a countercyclical TFP dispersion as in Bloom (2009) and Bloom et al. (2018).

We turn to the quantitative implications of the model to consider the aggregate dynamics following a shock that leads to a temporary deterioration in financial conditions, making it more expensive for firms to undertake capital expenditures. The ensuing drop in investment also results in a fall in productivity, leading to output losses 40 percent larger than those predicted by a standard *RBC* that abstracts from vintage effects. Also, the vintage technology model accounts for about one-third of the drop in Italian aggregate productivity observed in the data (productivity is exogenous and remains constant in the *RBC* model). Our model-results support the view that an investment slump, such as the one observed in the aftermath of the Great Recession, contributed to a decline in productivity growth and raises concerns about the speed of the recovery from the Covid-19 pandemic.

We also consider a pure technology shock. In the context of our model, this amounts to a shock to the efficiency of newer vintages relative to the previous one.[3] When the growth

[3]When the cost of adoption is zero; an exogenous innovation to the efficiency to the efficiency of the

rate of productivity of newer vintages slows down, firms postpone adopting the latest technology and reduce capital expenditures, as the productivity gap with the technology frontier increases less than expected. In the presence of vintage effects, pure technology shocks increase the volatility of the growth rates of aggregate series (such as output and investment) by about one-half relative to the standard model.

Our paper is organized as follows. In Section 1.1, we describe our contribution relative to the existing literature. We document the nature of capital accumulation at the firm level in Section 2 and the relationship between capital accumulation and productivity in Section 3. In Sections 4 and 5, we outline the model and its quantitative performance relative to the data. In Section 6, we quantify the role of vintage capital for aggregate dynamics. Section 7 concludes.

1.1 Literature Review

Our work connects to different strands of the existing literature. Our empirical analysis investigates the link between capital accumulation and productivity using firm-level data. After Gordon (1990) and Cummins and Violante (2002), who use product-level and sectoral data, most of the existing literature on vintage capital has focused on aggregate data; see for instance Hulten (1992), Wolff (1996), Greenwood, Hercowitz and Krusell (1997), and Greenwood, Hercowitz and Krusell (2000). The central insight of these papers is that, under some conditions, the growth rate of the price of investment goods (relative to the one of consumption) can be interpreted as a measure of investment-specific technological progress. There is, instead, little systematic evidence on the role of capital accumulation for productivity dynamics at the *firm level* partly because a rigorous analysis requires a set of data not commonly available to researchers. The exceptions provide mixed evidence on vintage effects; see Licandro, Maroto Illera and Puch (2005), Power (1998), and Sakellaris and Wilson (2004). Using U.S. manufacturing data, Power (1998) finds no evidence that investment spikes contribute to increasing a firm's productivity. Using similar data Sakellaris (2004), Sakellaris and Wilson (2004), and, more recently (and using Spanish manu-

latest technology in the model with vintage technology is equivalent to an aggregate productivity shock in the *RBC* model.

facturing data) Licandro, Maroto Illera and Puch (2005) find the opposite result. Relative to these studies, we explicitly tackle the thorny issue of reverse causality and broaden the analysis to all the sectors in the economy as our data consists of a cross-section 10 times as large and a sample 2 times as long.[4] Also, we consider the empirical relevance of this microeconomic behavior for industry-level TFP dynamics highlighting the importance of the extensive margin of investment, i.e., fluctuations in the share of firms that experience large capital expenditures and therefore adopt the latest technology.

In studying how capital accumulation contributes to productivity dynamics, our analysis sheds light on the determinants behind the significant TFP heterogeneity observed in the data. As discussed in (Syverson, 2011), many factors affect TFP from managerial skills (Bloom et al., 2013) to firms' innovation (Doraszelski and Jaumandreu, 2008) to trade and foreign competition (Bloom, Draca and Reenen, 2016). Our study focuses on technology adoption and shows that about 15 percent of the productivity heterogeneity in the sample depends on the timing of firms' investment behavior.

From a theoretical standpoint, our paper relates to the literature that studies sectoral and aggregate dynamics in models with rich firm heterogeneity; see, for instance, Cooper and Haltiwanger (1993), Caballero and Engel (1999), Khan and Thomas (2008), and Bachmann, Caballero and Engel (2013), to name a few. We retain several elements that have determined the quantitative success of this class of models in accounting for the pattern of capital accumulation at the firm level. Also, in our model, TFP is endogenous as firms decide if and when to adopt the latest vintage and, conditional on this choice, the next-period stock of capital.

The literature has also debated on the relevance of accounting for the rich cross-sectional dynamics in investment for aggregate dynamics, see Thomas (2002), House (2014), Fiori (2012), Khan and Thomas (2008), Bachmann, Caballero and Engel (2013), and recently Winberry (2021). Our results indicate that technology adoption motives constitute a power amplification mechanism of aggregate disturbances providing a channel through which microeconomic heterogeneity determines aggregate dynamics. Interestingly, it also generates a countercyclical dispersion in TFP without relying on stochastic volatility of

[4] The sample excludes the financial and the banking sector.

the productivity as in Bachmann and Bayer (2014) and Bloom et al. (2018).

The main difference with models that study vintage capital based on Solow (1960), is that, to be consistent with the empirical analysis, we focus on the total effect of embodied technical change on firms' TFP. The quantitative focus of our analysis distinguishes our work from the abundant literature on vintage capital. As in Solow (1960), in general, the dynamics of capital vintage models cannot be captured through a representative firm unless knife-edge conditions are met—for instance, constant returns to scale in production. As a result, the number of studies that have confronted vintage models with microeconomic data has been limited to non-existant.[5] For a complete list of references and a historical perspective on the evolution of the literature on vintage capital, see the extensive surveys of Boucekkine, de la Croix and Licandro (2011) and Boucekkine and de Oliveira Cruz (2015).

2 Microeconomic Evidence on Capital Adjustment

In this section, we describe the data set employed to document the role of investment for productivity dynamics. We first provide details about the source of our data. Next, we report descriptive statistics on the age composition of the census of incorporated Italian firms. Finally, we document the pattern of capital accumulation at the firm level.

2.1 Data Set

We obtained our data set combining different sources. To construct the variables of interest, firm-level investment rates and measures of productivity, we require information on payroll, gross value-added, and employment (see Appendix A and B for detailed information on data sources and variables construction). The sample spans a period of 30 years, from 1986 to 2015. The data set includes 5,004,894 firm-year observations from 395,169 different firms. On average, the number of firms in the cross section of any given

[5] Cooley, Greenwood and Yorukoglu (1997) study the balanced growth path and the transitional dynamics of a deterministic model with two sectors and vintage capital and compare it with the neoclassical growth model. Samaniego (2006) formulates a model that emphasizes the role of organizational capital as friction that prevents firms from adopting newer technology.

year is 169,223. The time series and the cross-sectional dimensions of our panel make these data ideal to study the role of investment for productivity dynamics. Our data match the size, and the distribution of Italian firms accounting for up to 80 percent of the value added produced in the Italian economy. In Table $A.1$, we report the composition of the data set by sector. Sectors are identified following the statistical classification of economic activities in the European Community, abbreviated as NACE. Consistent with their share of the economy, the manufacturing and the trade sectors constitute more than one-half of the observations in the data.

2.2 Age Distribution of Firms

We now turn to study the age composition of the firms in our sample. The ability to distinguish between firm age and investment age will be essential in Section 3, where we measure how the investment pattern at the firm level accounts for differences in productivity across firms.

Table 1: Descriptive Statistics - Age

Firm Age	Share in Data Set (A)	Share of Output (B)	Share of Investment (C)	Share of Employment (D)
$0-5$ years old	29.90%	13.90%	16.69%	15.74%
$5-10$ years old	23.05%	17.20%	17.81%	16.60%
$10-20$ years old	24.97%	25.80%	24.87%	25.04%
$20+$ years old	22.08%	43.10%	40.63%	42.62%
Total	100.00%	100.00%	100.00%	100.00%

Note: The sample period is 1998 to 2015. Statistics are computed as averages over the sample period considered.

Table 1 reports aggregate statistics conditioning on firms' age. Table entries are averages over the sample period from 1998 to 2015 but are representative even if the sample ends in 2009.[6] We follow Fort et al. (2013) and denote firms that have an age below five

[6]The sample period is consistent with the analysis in Section 3. There, we use data over the period

years as "young firms". Also, we consider age groups for mature and old firms: 5-10, 10-20 and over 20 years old. The average (median) firm is 11 years old (10 years old). As shown in column A, our data set includes young, mature, and old firms alike. Young firms account for 30 percent of the firms in the sample. As expected, mature and old firms represent more than 80 percent of output, investment and labor, with firms over 20 years old accounting for about one-half of output, investment, and employment. These shares are stable over the sample period we consider. We now turn to study the nature of capital accumulation at the firm level.

2.3 The Lumpy Nature of Capital Accumulation

In this subsection, we document that the nature of capital accumulation at the firm level is lumpy: capital adjustment is large and infrequent. We compute the distribution of investment rates for the 1998-2015 sample. As is customary in the literature, we calculate real capital stocks applying a perpetual inventory method from balance sheets data (see Appendix B for details). Following Bloom (2009), we define the investment rate for a given firm f at time t as $ik_{f,t} = \frac{I_{f,t}}{0.5(K_{f,t-1}+K_{f,t})}$, where $I_{f,t}$ is real investment net of disinvestment.[7] Investment $I_{f,t}$ includes expenditures on equipment and structures.[8] Table 2 reports the empirical distribution of $ik_{f,t}$ in our sample. As in Bachmann and Bayer (2014), we define lumpy adjusters as those firms that exhibit a spike, i.e., an investment rate above 20 percent. These investors account for 61 percent of total investment. Instead, firms that experience small capital adjustments (defined as in Øivind and Schiantarelli (2003) as experiencing $ik_{f,t}$ between negative 5 and 5 percent) account for only 6 percent of total investment.[9] While the share of investment accounted for by these two groups of

1986 to 1997 to initialize the distribution of investment age that constitutes the variable of interest in our empirical analysis.

[7]Doms and Dunne (1998) define investment rates net of capital depreciation while Cooper and Haltiwanger (2006) and Gourio and Kashyap (2007) follow the convention that $ik_{f,t} = I_{f,t}/K_{f,t}$. The lumpy nature of the capital accumulation process in our data does not depend on the specific formula used to compute investment rates.

[8]We cannot separately identify the fraction of equipment investment from the one on structures.

[9]The lumpy nature of the capital accumulation process is a feature of the data also in other countries. Doms and Dunne (1998) report evidence for the United States; Bachmann and Bayer (2014) for Germany; Licandro, Maroto Illera and Puch (2005) for Spain; Øivind and Schiantarelli (2003) for Norway; and Gourio and Kashyap (2007) for Chile.

firms differs substantially, the share of output and employment are instead equivalent.

Table 2: Cross-Sectional Distribution of Investment Rates

Investment Rate	Share in Data Set (A)	Share of Output (B)	Share of Investment (C)	Share of Employment (D)
$ik \geq 20\%$	18.81%	26.77%	61.04%	27.52%
$-5\% \leq ik \leq 5\%$	34.19%	25.67%	5.76%	27.01%
$ik \leq -20\%$	3.11%	1.98%	-6.65%	2.14%

Note: ik denotes the investment rate. See the main text for the definition. The distribution of investment rates is computed over the sample period 1998 to 2015.

Higher moments of the cross-sectional distribution of investment rates exhibit positive skewness (0.88). The existing literature interprets this evidence as suggesting the presence of investment lumpiness at the firm level (see Caballero, Engel and Haltiwanger (1995)).[10]

To shed light on the source of the type of capital adjustment cost at the firm level, we estimate the probability of the firm experiencing an investment spike given the time elapsed since the last investment spike. As is customary in the existing literature, we refer to this probability as the *hazard rate*. As discussed in Haltiwanger, Cooper and Power (1999) and Øivind and Schiantarelli (2003), a positively sloped hazard function is consistent with non-convex capital adjustment costs. Instead, convex capital adjustment costs will not, in general, imply upward-sloping hazard rates.

To estimate hazard rates from the distribution of spells between spikes, we use the semi-parametric heterogeneity model proposed by Heckman and Singer (1984) and employed in Haltiwanger, Cooper and Power (1999).

This approach consists of fitting a particular functional form for the hazard rate: the proportional hazard model. This class of models is flexible because it can (i) estimate the hazard using the distribution of investment spells defined over a fixed number of

[10]The cross-sectional dispersion of firm-level investment rates is pro-cyclical with a correlation of 0.34 with the growth rate of GDP. This feature of the data echoes the finding of Bachmann and Bayer (2014) for the German economy.

discrete intervals and (ii) control for unobserved heterogeneity. As shown in Figure $A.1$, the hazard rates are upward sloping. This result is consistent with the presence of non-convexities at the firm level (see Appendix C for details about the estimation).

3 Investment Age and Productivity

In this section, we document the link between investment and TFP using detailed firm-level data. Consistent with the idea that technological progress is embodied in new equipment, our main result is that firms that have experienced a recent large investment episode (spike) are, other things being equal, more productive than firms that have such an episode less recently.

The richness of the data set in terms of firm size, industry and coverage makes it suited to study the relationship between investment and productivity. A key feature of our study is accounting for the potential endogeneity of investment spikes and confounding factors at the firm, industry, and aggregate levels.

We start by describing our empirical strategy. We then show that investment spikes, proxying the vintage technology, lead to higher TFP at the firm level. Our sectoral analysis reveals that vintage effects are more pronounced in industries for which capital is an essential component of their production process. Finally, we show that firm-level investment dynamics determine sectoral TFP. At the industry level, an increase in the fraction of firms experiencing spikes, i.e., the extensive margin of investment, leads to higher industry-wide productivity.

3.1 Empirical Specification

To quantify the link between productivity and investment, we estimate the following equation that constitutes our baseline specification:

$$log(TFP)_{f,t} = \alpha + \beta Inv.Age_{f,t} + Controls_{f,t} + \epsilon_{f,t}. \tag{1}$$

The dependent variable is the log of TFP for firm f in year t. TFP is measured through the Solow residual. As the stock of capital is not quality-adjusted, our measure does not distinguish between neutral or investment-specific technological progress.[11] Appendix B reports additional details on the construction of the variables.

The coefficient of interest β captures the vintage of capital for firm f in year t. The vintage technology available is constructed using the time elapsed since the last investment spike. As discussed in Section 2.3, we follow the convention in Bachmann and Bayer (2014) (and the existing literature) and define an investment spike using a threshold of 20 percent ($ik_{f,t} \geq 20 percent$). In Section 3.4, we show that our results are not sensitive to this choice.

When a firm experiences an investment spike, the variable $Inv.Age$ equals zero and it progressively increases by one every year until the same firm experiences an investment spike.

To characterize the differences between specific vintage technologies, we follow Power (1998) and consider a more flexible specification in which $Inv.Age$ is discretized into a set of dummies ($Inv.Age_{j,f,t}$) and the coefficient β is vintage specific:

$$log(TFP)_{f,t} = \alpha + \sum_{j=1}^{J} \beta_j Inv.Age_{j,f,t} + Controls_{f,t} + \epsilon_{f,t}. \tag{2}$$

To avoid collinearity of the regressors, we exclude the dummy $Inv.Age_{0,f,t}$, i.e., investment age for firms that have just experienced a spike are not part of the equation. As a result, the β_j coefficients measure the productivity gap of older capital vintages relative to the newest vintage. We denote with J the maximum investment age, which, as in Power (1998), is the category 6+ years.

Given the long time-series dimension in our data, we split the sample and use the first part (1986-1997) to initialize the distribution of $Inv.Age$. We then estimate the two specifications using the second part of the sample (1998-2015).

The set of controls includes firm- and industry-specific effects. To capture aggregate shocks related to fiscal or monetary policy as well as time-varying industry factors, we

[11] The Solow residual is computed assuming a Cobb-Douglas production function. Following Bachmann and Bayer (2014), we estimate the output elasticities of the production function as median factor expenditures share over gross value-added within each industry.

also include time- and time-industry effects. Finally, to avoid confounding vintage capital effects with the firm's age-specific effects, we include five dummies for the age and six for the size of the firm.[12] We estimate equations 1 and 2 using ordinary least squares (OLS) and IV estimators.

3.2 Investment Leads to TFP Gains

Table 3 reports estimates of the baseline specification in equation 1. Our results show that the coefficients are precisely estimated and document a negative relationship between firms' productivity and investment age: investment leads to productivity gains at the firm level. OLS estimates, displayed in columns A through C, quantify the productivity gains at about 0.8 percent per year. In columns D through F, we tackle the potential endogeneity of investment spikes (and therefore investment age) using an IV approach where we instrument current $Inv.Age_{f,t}$ with its first lag. IV estimates corroborate the causal link between investment and TFP, pointing to productivity gains of about 1 percent.

The IV approach allows us to confirm that our results are not driven by current idiosyncratic shocks. While idiosyncratic shocks may result in investment spikes, instrumenting the current $Inv.Age$ with lags of the same variable allows to distinguish between spikes that are part of the regular investment cycle of the firm (and can be predicted by the time elapsed since the last spike) and spikes driven by current idiosyncratic shocks. Importantly, in Section 3.4 we also show that our results are not driven by *past* idiosyncratic shocks, instrumenting current investment age with lags of the same variables. When we consider as a dependent variable the growth rate of productivity, we do not find any discernible effect of investment age (not shown). This result points to a "level" effect of embodied technological change rather than a growth rate effect.

We find that vintage effects account for about 15 percent of the heterogeneity in labor and TFP measured in the data.[13]

[12] Age dummies define categories for the age of the firm at any given point in time given intervals of 0-5, 5-10, 10-20, 20-30, and above 30 years of age. The range for firm size dummies is 1-5, 5-20, 20,-50, 50-100, 100-300, and above 300 employees.

[13] To obtain this number, we take the ratio between the average contribution investment age of productivity and the standard deviation of the productivity residual of the estimated regression. Using the interquartile range of the estimated residuals yields an equivalent number.

Table 3: Investment Age and Total Factor Productivity

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)	$TFP_{f,t}$ (F)
$Inv.Age_{f,t}$	-0.853^{***}	-0.777^{***}	-0.816^{***}	-0.987^{***}	-0.959^{***}	-0.975^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. of obs.	4,058,036	3,404,387	3,675,873	3,540,352	3,242,758	3,373,532
R^2	0.739	0.749	0.734			
Estimator	OLS	OLS	OLS	IV	IV	IV
Firm FE	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Time×Ind. FE	✓	✓	✓	✓	✓	✓
Sample	All	$Age_{f,t} \geq 3$	$N_f \geq 4$	All	$Age_{f,t} \geq 3$	$N_f \geq 4$

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an investment rate above 20 percent. Each equation also includes firm size and age as additional controls. Columns D through F report estimates obtained instrumenting $Inv.Age_{f,t}$ using its first lag. Entries expressed in percent. $Age_{f,t}$ indicates the age of the firm. N_f denotes the number of observations for firm f. The sample period is 1997 to 2016.

3.3 Discretized Investment Age and Sectoral Analysis

To study whether TFP differences vary with investment age, we estimate equation 2. We report in Figure 1 the coefficients on the set of investment age dummies β_j, and we confine to Table $A.7$ in Appendix F the full set of coefficients and description of the estimation details. In the figure, the horizontal axis measures $Inv.Age_j$, the years elapsed since the last investment spike. The vertical axis reports the point estimates of the TFP gap between the latest and previous vintages of capital expressed in percent. Circles indicate that the coefficient is significant at the 5 percent level. OLS estimates (blue continuous line) indicate that postponing investment spikes increases the distance from the technological frontier by about 0.5 percentage points. IV results (red dashed line) are quantitatively similar.

To further corroborate the presence of vintage effects related to investment in physical capital, we consider a sectoral perspective by fitting our empirical specification to every

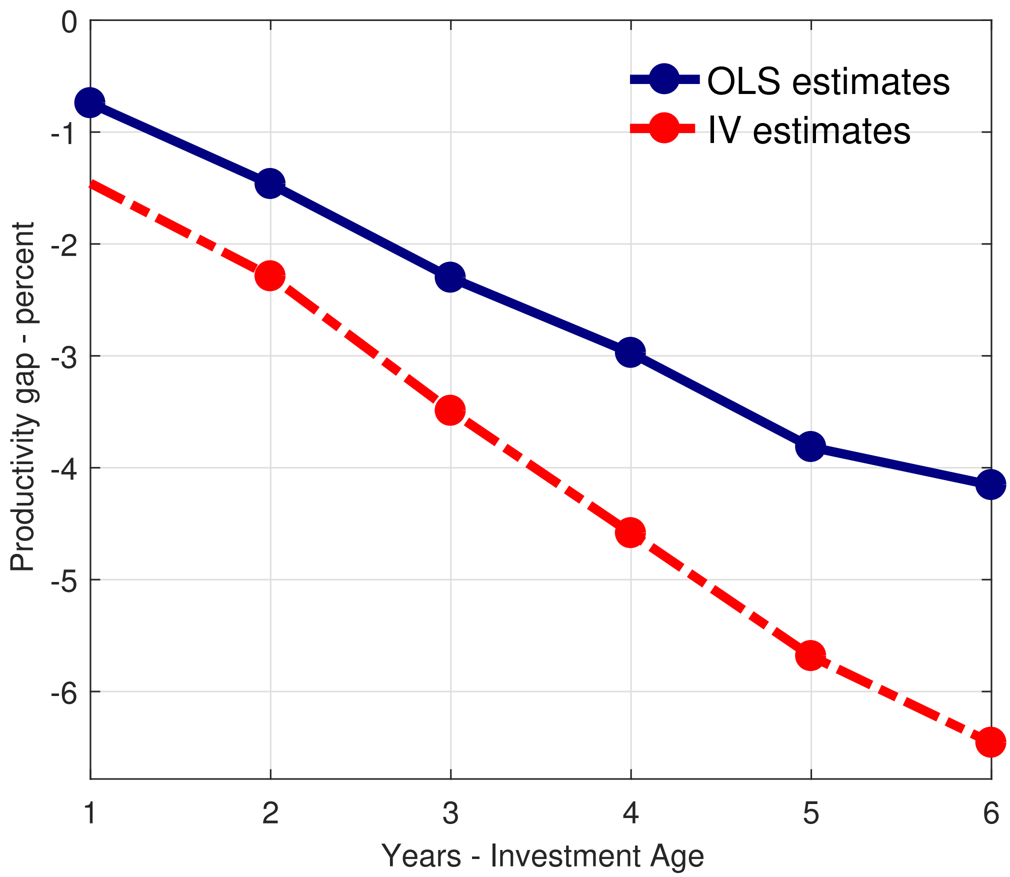

Figure 1: Investment Age and Total Factor Productivity

Notes: The figure reports estimates of the β_j coefficients in equation 2. Circles denote coefficients significant at 95 percent level. The panel reports estimates obtained with ordinary least squares (OLS) and instrumental variables (IV) including fixed-, industry-, year-effects, year-industry effects, and a set of dummies for firm's age and size. Circles indicate that coefficients are statistically significant at a 5 percent confidence level.

industry in the sample (see Appendix A). After confirming that the lumpiness of capital adjustment at the firm level is a characteristic of all the sectors, we find evidence of vintage effects in sectors for which physical capital is a critical input of production, such as manufacturing. Instead, We find smaller vintage effects in the hotel and food services. Figure $A.2$ in Appendix G reports the findings.

3.4 Results Robust to Alternative Spikes Definition, Sample Composition, and Timing of IV Instruments

We now show that our results do not depend on the definition of spikes, the sample composition, or the timing of the instrument in our IV regressions.

As there is no firm consensus in the existing literature as to what constitutes a large investment episode, and as the spike is an empirical convention, we consider alternative measures to identify the vintage technology available to the firm. We proceed in two ways. First, we increase the threshold that identifies an investment rate from 20 to 30 percent as in Doms and Dunne (1998) (see Table $A.2$ in Appendix D). Second, in the spirit of Power (1998), we employ a measure that does not rely on capital. A firm exhibits an *absolute* investment spike if its real investment exceeds the 80th percentile of the firm-specific distribution of real investment (Table $A.3$ in Appendix D). In both cases, we estimate a negative relationship between TFP and investment age.

To verify that firm entry and exit dynamics do not drive our results, we follow Bachmann and Bayer (2014) and consider only firms three years old. We also assess the robustness of our results concerning the sample composition by including only firms that are observed for, at least, five consecutive periods. (Results are reported in columns B, C, E, and F in Table 3 and in Appendix D).

Finally, we show that instrumenting $Inv.Age$ with lags of the same variable other than the first does not affect the negative relationship between investment age and TFP. Considering an alternative timing of $Inv.Age$ instruments allows to exclude that our results may be driven not only by current realizations of idiosyncratic disturbances, but also by past shocks (up to four years). Considering further lags reduces the number of observations and progressively weakens the significance of the instrument. Because of these two forces, the magnitude of vintage effects is sensitive to the lag of the instrument considered and aligning with estimates of vintage effects in Gordon (1990) and, generally, on the lower end of the existing literature. Estimates are reported in Appendix E.

3.5 Industry-Level Analysis: The Extensive and the Intensive Margin of Investment

We now turn to evaluate the *aggregate* relevance of the causal relationship between investment and productivity uncovered at the *firm level*. Using industry variables, we test whether an increase in investment at time $t-1$ due to the extensive margin (number of firms experiencing spikes) or the intensive margin (the average size of a spike) leads to higher TFP between time $t-1$ and t. To do so, we perform an industry-based analysis in which industry-wide TFP is regressed on the fraction of spikes adjusters and the average adjustment undertaken by those investors. The purpose of this approach is two-fold. First, estimating such an empirical specification allows us to measure if the firm-level evidence contributes to determining productivity at the industry level. Second, we can disentangle whether this occurs through the extensive or the intensive margin. Notice that, despite the evidence of vintage effects at the firm level, a priori there is no reason why this firm-level characteristic should be a *quantitatively* relevant determinant of productivity at the industry level.

The sample includes 55 industries and runs from 1987 to 2015 (see Appendix A for details). The number of years is larger than the sample employed in Sections 3.2 and 3.3 because there is no need to initialize the distribution of investment age. As in Bachmann and Bayer (2014), the extensive margin is measured as the fraction of firms that experience spikes (or equivalently lumpy adjusters) in a given year. In column A, we use a count of firms, and, in column B, we weight firms by their capital. The intensive margin consists of the average adjustment of those same firms. Table 4 reports the estimated coefficients. The extensive margin of capital accumulation is a quantitatively relevant determinant of productivity at the industry level. The coefficients are both statistically and economically significant. To get a sense of the magnitude implied by our estimates, a one-time decrease in the fraction of lumpy adjusters equal to a standard deviation leads to lower TFP by about 0.7 percent for TFP. The same experiment yields 1.1 percent when we measure the extensive margin as the weight of capital accounted for by lumpy adjusters. The magnitude of these effects is economically significant and indicates that an investment slump

Table 4: The Extensive Margin of Investment and Productivity

	$\underline{TFP_{s,t}}$ (A)	$\underline{TFP_{s,t}}$ (B)
$ExtensiveMargin_{s,t-1}$	0.36^{**}	
Lumpy-Adjusters Count	(2.13)	
$ExtensiveMargin_{s,t-1}$		0.15^{***}
Lumpy-Adjusters K-weighted		(3.67)
$IntensiveMargin_{s,t-1}$	0.03	0.05
	(0.41)	(0.62)
N. of obs.	2073	2073
R^2	0.76	0.79

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level; t-statistics in parentheses. Each equation is estimated using ordinary least squares, and it includes industry-specific fixed effects and year dummies. The dependent variable is the log of total factor productivity measured at the two-digit NACE classification. The sample period is 1987 to 2015.

leads to stagnant productivity. However, these estimates must be interpreted with care since, neglecting general equilibrium effects, they likely constitute an upper bound of the true effects. Indeed, a sizable shift in the number of lumpy adjusters may affect factor prices and, hence, firms' investment decisions. We take into account these considerations in the next section, where we develop a theory of technology adoption through capital accumulation to assess the role of microeconomic heterogeneity for the dynamics of aggregate productivity.

4 Technology Adoption through Capital Accumulation

In this section, we describe the theoretical framework that we develop to study how microeconomic heterogeneity contributes to the dynamics of aggregate productivity in response to macroeconomic shocks. Our starting point is the neoclassical growth model of Khan and Thomas (2008), the benchmark for quantitative analysis involving firm dynamics. This framework allows us to capture the features of capital accumulation at the firm level discussed in Section 2 in a general equilibrium framework. We endogenize firms' TFP by introducing a vintage technology structure based on capital accumulation. The firm's productivity not only depends on idiosyncratic factors, but also on the technological vintage available to the firm. In other words, vintage technology is a defining characteristic of the firm. The firm's problem consists of deciding the optimal timing to obtain the latest vintage and, if an investment is made, how much to invest. This option is subject to a non-convex adjustment cost that, in equilibrium, leads to the coexistence of vintages of different quality and contributes to reproducing the firm-level evidence on capital accumulation. In this respect, we follow the existing literature since Cooper and Haltiwanger (1993), Haltiwanger, Cooper and Power (1999), and Caballero and Engel (1999), as well as the empirical evidence in Section 2.3 that supports for the presence of non-convex capital adjustment cost. In our vintage model, this cost captures any friction, whether real or financial, that prevents large capital adjustment and delays the adoption of new technology.

Our framework contributes to the existing literature that studies the role of investment indivisibility in general equilibrium; recent contributions include Thomas (2002), Khan and Thomas (2008), Bachmann, Caballero and Engel (2013), and Fiori (2012). In Sections 4.1 and 4.2, we outline the tradeoffs that determine the production and investment decision of each firm. Sections 4.3 and 4.4 describe the households' problem, and Section 4.5 details the recursive equilibrium of the economy. In Sections 4.6 and 4.7, we discuss the implications of the model for aggregate productivity and the mapping between the model and the evidence presented in Section 3.

4.1 Production

The economy consists of a continuum of firms that is normalized to one.[14] The economy features one commodity that can be consumed or invested. Each firm has access to an

[14] As discussed in the sensitivity analysis in Section 3, our results are not driven by entry and exit. In light of this evidence, we abstract from entry and exit dynamics.

increasing and concave production function that combines predetermined capital stock k with its available technology to produce output y:

$$y = \varepsilon z k^{\theta}, \tag{3}$$

where $0 < \theta < 1$.[15] The efficiency of production depends upon two variables, ε and z. ε denotes the idiosyncratic productivity that is exogenous to the firm. z identifies the current vintage of technology available to each production unit and is optimally chosen by the firm. Every period, firms decide whether to pay the cost ξ and adopt the latest vintage or to postpone it. The technological frontier grows deterministically at the gross rate of $\gamma_A > 1$. Along the balanced growth path, z_0 indicates the latest vintage or the technological frontier that, along the balanced growth path deflated by its trend, is equal to 1. A firm that chooses not to obtain the latest vintage keeps its current technology that becomes more obsolete relative to the frontier at a per-period rate γ_A, so that $z' = z/\gamma_A$.[16] We describe in the next section the economic tradeoff associated with the investment choice. As in Khan and Thomas (2008), $\varepsilon \in \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_{N_\varepsilon}\}$ where $Pr(\varepsilon = \varepsilon_m | \varepsilon = \varepsilon_l) \equiv \pi^{\varepsilon}_{lm} \geq 0$, and $\sum_{m=1}^{N_\varepsilon} \pi^{\varepsilon}_{lm} = 1$ for each $l = 1, ..., N_\varepsilon$. In each period, a firm is defined by its vintage productivity z, its idiosyncratic productivity level $\varepsilon \in \mathcal{E} \equiv \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_{N_\varepsilon}\}$, its predetermined stock of capital $k \in \mathbf{R}_+$, and its cost associated with vintage adoption $\xi \in [0, \bar{\xi}]$, which is denominated in units of output.

4.2 Firm's Adoption and Investment Decision

In every period, each firm faces the choice between keeping its current vintage or adopting the latest available technology. This choice consists of choosing whether to pays its current adoption cost ξ. By paying ξ, the firm obtains the latest vintage z_0 and optimally chooses the stock of capital k', where primes denote next-period variables. The firm's cap-

[15]Variables reported are deflated by their respective trends. Along the balanced growth path, γ_A denote the gross trend growth rate of the technology frontier. Consumption and capital grow at a gross rate $\gamma = \gamma_A^{1/(1-\theta)}$.

[16]Given that the frontier of z_0 evolves deterministically at a rate of γ_A, z indicates the time elapsed since the firm has adopted the latest vintage.

ital stock evolves according to $k^{'} = (1 - \delta - \delta_S)/\gamma k + i$, where i is its current investment and $\delta \in (0, 1)$ is the rate of physical capital depreciation. The parameter δ_S captures potential incompatibility of existing capital with the old and new vintage in a parsimonious way.[17] When δ_S is equal to zero, this choice amounts to assume a full retrofitting, which is that the productivity associated with the new vintage applies to the capital stock of the firm already installed. When δ_S is positive, adopting the new technology comes at the cost of scrapping δ_S units of its current k. Also, a positive δ_S amounts to a rescaling of the fixed cost of adoption that makes it firm specific, as it depends on the current capital stock available to the firm. Specifically, by forfeiting ξ units of current output, the firm can invest in any future capital $k \in \mathbf{R}_+$ and upgrade technology z to the latest vintage of technology z_0.

Firms that postpone paying the adjustment cost keep their current vintage and can undertake investment i^{NA}, where NA stands for non-adoption. In this case, the firm's distance from the technological frontier or the degree of technological obsolescence increases so that $z^{'} = z/\gamma_A$, and $k^{'} \in \Omega \subseteq \mathbf{R}_+$, where

$$\Omega(k) \equiv \left[\frac{1 - \delta + a}{\gamma}k, \frac{1 - \delta + b}{\gamma}k\right]. \tag{4}$$

Introducing the possibility of frictionless capital adjustment detaches the technology adoption decision from the capital accumulation choice. Precisely, the parameters a and b determine the length of the capital region over which firms can invest or disinvest at no cost, without obtaining the latest vintage.

Table 5 summarizes the decision set available to the firm regarding its capital stock and vintage technology between two consecutive periods, from k to $k^{'}$ and z to $z^{'}$.

As in Khan and Ravikumar (2002), the adoption adjustment cost ξ is non-convex, and

[17] As in Solow (1960), the dynamics of vintage models cannot be represented through a representative firm unless knife-edge conditions are met. This case is not valid in models that feature a vintage structure and non-convex adjustment costs. Motivated by our empirical evidence in Section 3, our identifying assumption is that the firm vintage, linked to the capital accumulation decision of the firm, is a defining characteristic of the firm. Khan and Thomas (2003) employ a different identifying assumption. They assume that the latest vintage technology applies only to the most recent investment.

Table 5: Firm's Vintage Adoption and Investment Decision

Technology Adoption	Adoption Cost Paid	Future Technology $z^{'}$	Future Capital $k^{'}$	Total Investment
Yes	ξ	z_0	$k^{'} > 0 \in \mathbf{R}_{+}$	$\gamma k^{'} - (1 - \delta - \delta_S)k$
No	0	z/γ_A	$k^{'} > 0 \in \Omega(k)$	$\gamma k^{'} - (1 - \delta)k$

its modeling strategy follows Caballero and Engel (1999) and the subsequent literature on lumpy investment. Thus, the decision to adopt the latest vintage involves a non convexity; conditional on adjusting capital and upgrading technology, the cost ξ incurred is independent of the scale of adjustment. As in Thomas (2002), we assume that ξ is independently and identically distributed across firms and across time. Every period, each firm draws its current cost of vintage adoption $\xi \geq 0$ (denominated in units of output) from the time-invariant distribution G common to all production units. As the firm's current adjustment cost has no implication for its future adjustment, the distribution of firms is summarized by (ε, z, k) : the idiosyncratic productivity ε, vintage technology z, and capital stock k. To characterize the distribution of firms over (ε, z, k), we use the probability measure μ defined on the Borel algebra S for the product space $S = \mathcal{E} \times \mathbf{R}_{+} \times \mathbf{R}_{+}$. The distribution of firms evolves over time according to a mapping (defined below) Γ : $\mu^{'} = \Gamma(\mu)$.

4.3 Firm's Dynamic Programming Problem

To describe the adoption and the investment decision of the firm, as is customary in the literature, we adopt the approach in Khan and Thomas (2008) and state the problem in terms of utils of the representative households (rather than physical units) and denote the marginal utility of consumption by $p = p(\mu)$. This variable denotes the pricing kernel used by firms to price output streams. Given the i.i.d. nature of the adjustment cost ξ, continuation values can be integrated out of future continuation values.

Let $v^1(\varepsilon_l, z, k, \xi; \mu)$ denote the expected discounted value of a firm entering the period with (ε_l, z, k) and drawing an adjustment cost ξ when the aggregate state of the economy is μ. The dynamic optimization problem for the typical firm is described using a functional equation defined by equations 5, 6, and 7. First, we define the beginning-of-period expected value of a firm before the realization of its fixed cost draw, but after the determination of (ε_l, z, k):

$$V^0(\varepsilon_l, z, k; \mu) = \int_0^{\bar{\xi}} V^1(\varepsilon_l, z, k, \xi; \mu) dG(\xi). \tag{5}$$

The firm's profit-maximization problem, which takes as given the evolution of the firm distribution, $\mu = \Gamma(\mu)$, is then described by

$$V^1(\varepsilon, z, k, \xi; \mu) = \max_{k^*,\, k^{NA}} \left\{ \begin{array}{c} [F(\varepsilon, z, k) + (1-\delta)k]\, p(\mu) + \\ \max \left[\begin{array}{c} -\xi p(\mu) - \delta_S k + R(\varepsilon, z_0, k^*; \mu'), \\ R(\varepsilon, z/\gamma_A, k^{NA}; \mu') \end{array} \right] \end{array} \right\} \tag{6}$$

$s.t.\ k^* \in \mathbf{R}_+$ and $k^{NA} \in \Omega(k)$,

where $R(\varepsilon, z', k'; \mu')$ represents the continuation value associated with a given combination of the idiosyncratic shock, the vintage, and the stock of capital:

$$R(\varepsilon, z, k'; \mu') \equiv -\gamma k' p(\mu) + \beta \sum_{m=1}^{N_\varepsilon} \pi^{\varepsilon}_{lm} V^0(\varepsilon_m, z', k'; \mu') \tag{7}$$

Every period, the firm decides whether to pay the fixed cost (ξ) and upgrade its vintage and, accordingly, adjust its capital stock. Otherwise, the firm keeps its current vintage. For notational convenience, as in Khan and Thomas (2008), rather than subtracting investment from current profits, the value of undepreciated capital augments current profits, and the firm is seen to repurchase its capital stock each period.[18] Because of the perfect mapping between technology adoption and the capital stock decision, we find it more transparent to focus on the capital decision. Thus, we let $K(\varepsilon, z, k, \xi; \mu)$ represent the

[18]This approach is equivalent but notationally more convenient.

choice of capital for the next period by firms of type (ε, z, k) with adjustment cost ξ, as this choice subsumes the adoption decision.

4.4 Households

The economy features a continuum of identical households that have access to a complete set of state-contingent claims. As there is no heterogeneity across households, these assets are in zero net supply in equilibrium. Moreover, they own shares in the production units, denoted by the measure $\lambda(\varepsilon, z, k; \mu)$ and value $\rho_0(\varepsilon, z, k; \mu)$. Given the value for their current shares, $\rho_0(\varepsilon, z, k; \mu)$, households maximize their lifetime expected utility by choosing current consumption c as well as the numbers of $\lambda(\varepsilon, z, k)$ to purchase at prices $\rho_1(\varepsilon, z, k; \mu)$:

$$W(\lambda; \mu) = \max_{C, \lambda'} \left[U(c) + \beta W(\lambda'; \mu') \right] \tag{8}$$

subject to

$$\begin{aligned} & c + \int_S \rho_1(\varepsilon, z, k; \mu) \lambda' \left(d \left[\varepsilon' \times z' \times k' \right] \right) \\ \leq & \int \rho_0(\varepsilon, z, k; \mu) \lambda \left(d \left[\varepsilon \times z \times k \right] \right). \end{aligned} \tag{9}$$

The household optimality condition yields:

$$p(\mu) = U_C(c) \tag{10}$$

Let us denote the optimal choices for the household as $C(\lambda; \mu)$ and $\Lambda^h(\varepsilon', z', \lambda, k'; \mu)$.

4.5 Recursive Equilibrium

A recursive competitive equilibrium is a set of functions (p, v^1, K, W, C, Λ^h, Γ) that satisfy firms' and households' problem and clear the markets for assets, labor, and output:

(i) Firm's optimality: Taking p as given, $V^1(\varepsilon, z, k, \xi; p)$ solves equations 5, 6, and 7 and the corresponding policy functions $K = K(\varepsilon, z, k, \xi; p)$.

(ii) Household's optimality: Taking p as given, the household's consumption satisfies equation 10 and (C, Λ^h).

(iii) $\Lambda^h(\varepsilon_m, z, k; \mu) = \mu(\varepsilon_m, z, k)$ for each $(\varepsilon_m, z, k,) \in S$.

(iv) Commodity market clearing: $C = \int y d\mu - \int \int_0^{\bar{\xi}} [\gamma K(\varepsilon, z, k, \xi; p) - (1-\delta) K] \, dG d\mu$ $- \int \xi dG d\mu$.

(v) Model-consistent dynamics: The evolution of the cross-sectional distribution that characterizes the economy, $\mu' = \Gamma(\mu)$, is induced by the adjustment decision and the exogenous processes for ε. Conditions (i), (ii), (iii), and (iv) define an equilibrium given Γ, while condition (v) determines the equilibrium condition for Γ. We confine to Appendix H the discussion about the (S,s) decision rule for the firm upgrading and investing decision and the details on the evolution of the cross-sectional distribution of firms' productivity and capital stocks.

4.6 Firm-Level Vintage Technology and Aggregate Productivity

In this section, we discuss the role of the vintage structure for the aggregate economy. The presence of non-convex adoption cost implies that the firm's technology adoption decision follows an (S,s) rule: Some firms adopt the latest vintage, while others postpone it.[19] Conditioning on the adoption decision, firms decide next-period capital stock. The degree of obsolescence of the vintage currently available to the firm (the productivity gap from the technological frontier) as well as the realization of the idiosyncratic and potentially aggregate shocks affect the timing of technology adoption at the firm level. Thus, in equilibrium, vintages of different quality coexist, implying that the distribution of productivity across firms determines the economy-wide production efficiency. Shifts in the cross-sectional distribution, determined by variations in the firms' adoption decision, result in fluctuation in aggregate productivity. We highlight that at the firm level, there is no one-to-one mapping between the size of the investment adjustment and the adoption of the latest vintage. As a result, the response of aggregate investment (or measuring vintage effects at the firm level) is not a sufficient statistic to characterize the evolution of

[19]See Appendix H for additional details.

aggregate productivity and its role for aggregate dynamics in response to shocks.

4.7 Mapping Between the Model and the Evidence on Vintage Effects

Before discussing the calibration strategy, we emphasize one crucial aspect related to the mapping between the model and the data. The empirical evidence in Section 3 highlights the relationship between recent investment spikes and productivity: Firms with lower investment age (measured as the time elapsed between investment spikes) are, other things being equal, more productive than firms with higher investment age. This link is not hard-wired into the model. Adopting the latest vintage entails a non-convex adoption cost independent of the size of the investment necessary to reach the target capital k^*.

5 Taking the Model to the Data

In this section, we take the model to the data. We start by describing the parameterization of the model in Section 5.1. Given our focus on the role of microeconomic heterogeneity for aggregate dynamics, one critical aspect in evaluating the empirical performance of the model is its ability to fit the cross-sectional distribution of investment rates and the timing of investment spikes across firms, i.e., the empirical proxy for vintage technology. We discuss these issues in Sections 5.2 and 5.3.[20]

5.1 Parameterization

Following the business cycle literature, we calibrate the model to fit key first-order moments of the Italian economy. Table 6 summarizes parameter values, targeted moments, and data sources. We are to assign values to 11 parameters related to the growth rate of aggregate variables (γ and γ_A), the production process (δ, δ_S and θ), individual preferences (β), the adjustment cost function and boundaries to the investment process ($\bar{\xi}$, a and b), and the idiosyncratic productivity process (ρ_ε, and σ_ε). We first describe the set

[20] Appendix I reports details about the computation of the stationary equilibrium of the model.

of parameters that are externally calibrated, i.e., using independent evidence. Then, we focus on those estimated within the model.

Externally Calibrated. One period in the model represents one year, which corresponds to the frequency of the data employed in Section 2. The depreciation rate is taken from the Italian National Statistical Institute and is equal to 9 percent. We set the average growth rate of output to 0.7 percent, consistent with its sample average computed using data until 2012. To calibrate the productivity increase of the latest vintage relative to the previous one (γ_A), we use our estimates of the vintage effects in Section 2, so that $\gamma_A = 1.004$. We set δ_S to zero because firms that experience a negative gross investment rate are, on average, less than 3 percent. Also, more than 95 percent of all the firms in the sample report a disinvestment rate of less than 1.5 percent.[21]

Internally Calibrated. The discount factor β is set to 0.975 to reproduce the real annual interest rate in the data. The elasticity of output to capital is set to 0.4 to satisfy the balanced growth path restriction that links the growth rate of output and the one on technology.

Table 6: Benchmark Calibration

Parameter		**Value**	**Target**
Depreciation rate	δ	0.091	Data
Scrapping rate	δ_S	0	Data
Growth rate of productivity	γ_A	1.004	Estimated productivity gap
Growth rate of output	γ	1.007	Data
Discount factor	β	0.975	Annual real interest rate = 2.3%
Elasticity of output w.r.t. capital	θ	0.4	Balanced growth path restriction
Persistence idiosyncratic productivity	ρ_ε	0.872	Data
St. dev. idiosyncratic productivity	σ_ε	0.066	*ik* distribution
Upper support adj. cost distribution	$\bar{\xi}$	0.007	*ik* distribution
Upper bound of Ω	b	0.050	*ik* distribution
Lower bound of Ω	a	-(1-δ)	Free disinvestment

We set the persistence of the idiosyncratic productivity process (ρ_ε) equal to its data counterpart of 0.872. This value is obtained fitting an autoregressive process of order one to firm-level TFP. We set a, the lower bound of the frictionless investment range Ω, to

[21]See Lanteri (2018) for a model of endogenous irreversibility.

-(1-δ), so that firms can disinvest at no cost. To select the remaining parameters, we follow Khan and Thomas (2008). We set the upper support of the adjustment cost function ($\bar{\xi}$), the upper bound of Ω (b), and the standard deviation of the idiosyncratic productivity process (σ_ε) to reproduce selected moments of the cross-sectional distribution of investment rates (ik) in the data. Results are reported in Section 5.2. Before reviewing the empirical performance of the model, it is worth examining the role of each parameter separately. The upper support of the adjustment cost distribution ($\bar{\xi}$) determines the magnitude of the adjustment costs. The higher $\bar{\xi}$, the higher the potential cost of adopting the latest vintage. Increasing this parameter leads to a higher average investment age. The persistence and the standard deviation of the idiosyncratic process interact with the vintage effect in shaping the economic incentives that make the firm adopt the latest vintage and choose capital. Finally, the parameters a and b determine the extent of the investment frictions without adopting the latest vintage of productivity.

5.2 Model Fit: Cross-Sectional Distribution of Investment Rates

We now turn to examine the model performance in accounting for the cross-sectional distribution of investment rates. Given our focus, the model must reproduce the firm-level pattern of capital accumulation observed in the data. As in Cooper and Haltiwanger (2006) and Khan and Thomas (2008), the cross-sectional distribution is summarized using five groups: inaction, positive and negative investment, and positive and negative spikes. A specific threshold for the investment rate identifies each group.[22] Before examining the results in Table 7, we note that our definition of the inaction region is broader than the definition employed in the existing theoretical literature and more in line with the empirical literature (see Øivind and Schiantarelli 2003). This choice allows us to capture the small investment rates occurring in about one-third of the firms in the sample.

The model provides an excellent account of the cross-sectional distribution of investment rates. Of course, this result is the byproduct of the calibration strategy discussed in

[22] The Inaction region is identified as the fraction of investors with an investment rate less than or equal to 5 percent in absolute terms. Positive (negative) investors are defined as firms experiencing an investment rate above (or below negative) 5 percent. An investment rate above (below) 20 percent identifies positive (negative) spikes.

Table 7: Distribution of Firm Investment Rates

	Inaction	Positive Spikes	Negative Spikes	Positive Investment	Negative Investment
	(A)	(B)	(C)	(D)	(E)
Data	34.19%	18.81%	3.11%	59.81%	6.00%
Model	35.07%	18.57%	0.15%	61.08%	3.85%

Note: Each entry reports the fraction of firms that, on average, exhibit investment rates that fall in each category. See the text for the definition of inaction, positive and negative spikes, and positive and negative investment.

Section 5.1. While the model accurately reproduces the fraction of firms in the inaction region and those exhibiting positive spikes, it does slightly less well in accounting for the behavior of downsizing investors. This feature obtains because the realizations of the idiosyncratic process and the gap from the vintage frontier have conflicting effects on the adoption decision of the firm. Unfavorable realizations of the idiosyncratic process tend to make firms postpone the adoption decision. However, postponing the adoption of the latest vintage increases the distance from the productivity frontier and reduces the capital available to the firm. This second effect is quantitatively stronger and implies that firms adopting the latest vintage do so with positive investment.

5.3 External Validation: Investment Age Distribution and Time-Series Properties of Investment Rates

We now turn to validate the model across dimensions that are left untargeted in the calibration. We proceed in two ways.

First, we compute the model-based distribution of investment age, calculated as the time elapsed since the last time the firm experienced an investment spike. Reproducing this dimension of the data is essential because investment age is the variable at the core of the empirical strategy in Section 3 to identify vintage effects in the data.

Figure 2 reports the result. Firms that experience an investment spike have an invest-

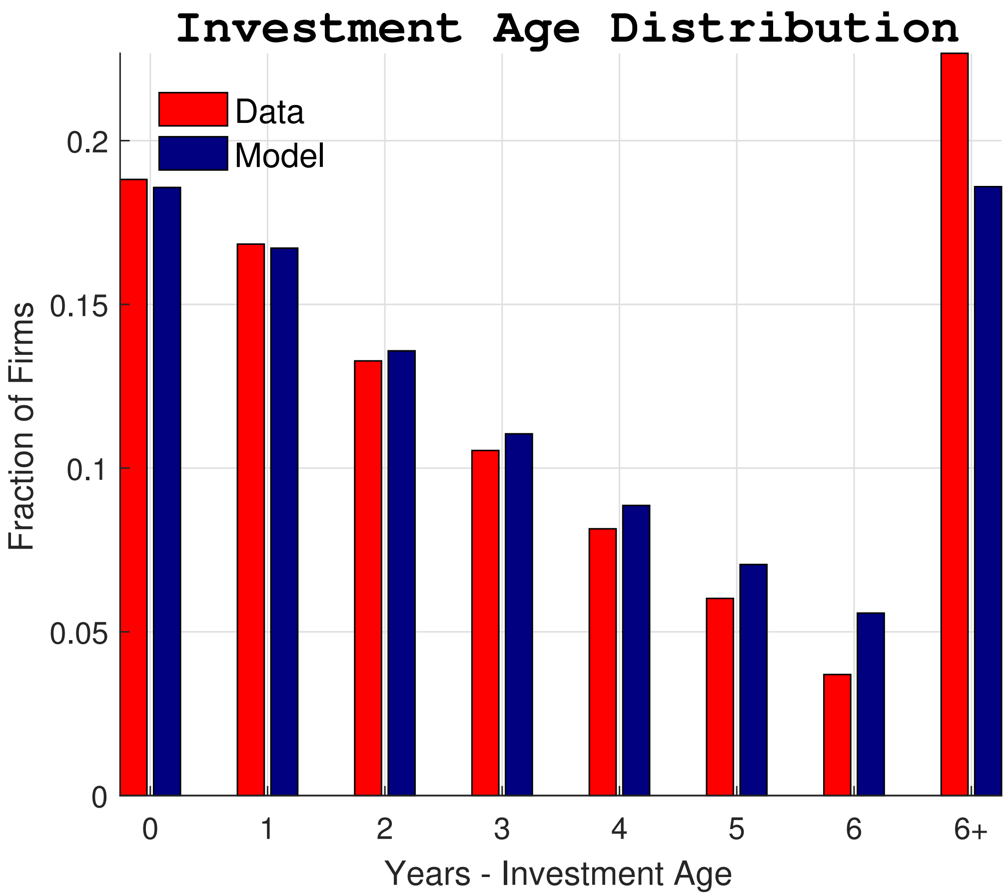

Figure 2: Comparison between the Empirical and the Model-Based Investment Age Distribution

ment age of zero. They move along the distribution until they exhibit another spike. As shown by the figure, the model-based distribution displays less mass on the right tail relative to the data. It is not surprising then that the average investment age in the model is three years while it is about four-and-a-half years in the data. Overall, we consider the performance of the model quite satisfactory. We notice that the ability of the framework to reproduce the timing of investment spikes across firms is distinct from the model's success to account for the cross section of investment rates. While the fraction of firms with investment age zero coincides by construction to the fraction of firms exhibiting spikes, the fraction of firms with age one and above depends on the investment behavior of firms, determined by the realization of individual states.

Second, we show that the model reproduces the time-series properties of investment rates. Specifically, we show that the model reproduces the absence of autocorrelation of investment rates at the firm level. This statistic takes the value of negative 0.10 in the model, a close match for its data counterpart of negative 0.04.

After establishing the empirical success of the model in accounting for the pattern of capital accumulation at the firm level, we turn to assess the relevance of vintage effects for aggregate dynamics.

6 Vintage Technology Amplifies Aggregate Fluctuations

In this section, we use our framework to quantify the relevance of microeconomic heterogeneity induced by vintage technology in the propagation of macroeconomic shocks. Toward this goal, we compare the properties of the aggregate series obtained in our baseline model with a positive adoption cost and a benchmark with zero adoption cost, i.e., the standard *RBC* with idiosyncratic shocks. In Section 6.1, we consider a deterioration in financial conditions to study the role of vintage effects in accounting for the stagnant productivity observed in developed economies following the Great Recession. Then, in Section 6.2, we characterize business cycle dynamics in the presence of technology shocks.

Unlike the *RBC* model, when the technology available to each firm also depends on investment age, shocks that alter the timing of capital expenditures at the individual firms also affect the production possibility frontier of each firm. As a result, the joint distribution of capital stocks and technology across firms determines the *aggregate* efficiency of the economy. As firms postpone capital expenditures, they also postpone introducing the latest technology vintage. In a parameterized version of the model that closely reproduces the cross-sectional distribution of investment rates and investment age, this mechanism amplifies macroeconomic dynamics in response to standard aggregate shocks. We now discuss in detail our main exercises.

6.1 Financial Shock - Perfect Foresight

Our first experiment characterizes the model dynamics in response to an aggregate deterioration in financial conditions. Following the approach in Gavazza, Mongey and Violante

(2018), we study the *perfect foresight* transitional dynamics of the model in response to a one-time, unexpected temporary aggregate shock. In so doing, we follow several recent papers that assume firms did not foresee the aggregate shocks of the 2012 crisis (see for instance Gavazza and Lanteri, 2018). This scenario is obtained assuming a temporary increase in the real cost of investment goods for firms that experience an investment rate above the threshold level of 5 percent. The economy starts in steady state, and the path of the shock always reverts to its initial value, so the economy also returns to its initial steady state.[23]

Rather than providing a microfounded characterization of financial frictions in the model, we assess how shocks that increase the cost of undertaking large capital expenditure to individual firms affect the endogenous dispersion of capital and productivity across firms and contribute to aggregate outcomes.[24] In the spirit of Gomes (2001), firms are subject to an additional real cost equal to $\lambda_t i_{f,t}$ when they choose an investment rate $ik_{f,t}$ larger than 5 percent.

To discipline our exercise, we rely on firm-level Italian data. We parameterize the process for λ_t so that the model reproduces the fluctuations in the distribution of investment age consistent with the data, i.e., a reduction in the fraction of firms experiencing spikes observed in the data.[25] Thus, the share of firms with investment age equal to zero matches exactly its data counterpart. To quantify the role of vintage effects, we then compare the dynamics implied of our baseline model with the nested *RBC*, the typical benchmark of quantitative macroeconomic analysis.

Table 8 reports the impulse response function of the two models. In the *RBC* model, a persistent increase in the cost of investment makes capital expenditures more expensive, leading to a drop in aggregate investment that affects the economy for a few periods. As capital is predetermined and is the only input of production, output is unaffected on impact but declines over time due to the lack of net investment. As financial conditions

[23]Details about the computation of the transitional dynamics are reported in Appendix I.

[24]The literature on financial frictions is large. For an explicit characterization of financial frictions in macroeconomic models see the seminal contribution of Bernanke, Gertler and Gilchrist (1999) and, in a model featuring production heterogeneity, Khan and Thomas (2013).

[25]The fraction of firms experiencing spikes drops by 4 percentage points in 2012, relative to its value in 2011. The initial shock increases the price of investment by roughly 1 percent. The persistence of λ is equal to 0.75.

Table 8: Financial Shock - Aggregate Responses

	GDP		Investment	
	RBC	Vintage	*RBC*	Vintage
	(A)	(B)	(C)	(D)
Impact	0.00%	0.00%	-4.88%	-4.36%
Period 1	-0.18%	-0.60%	-3.60%	-3.05%
Period 2	-0.31%	-0.84%	-2.65%	-2.81%
Period 3	-0.41%	-0.76%	-1.95%	-2.23%
Period 4	-0.38%	-0.67%	-1.42%	-1.50%

Note: Each entry is in percent relative from trend values. *RBC* refers to the model with zero adoption cost, while Vintage refers to the baseline model.

revert to the steady state, the economy recovers and returns to the initial stationary equilibrium. While the qualitative pattern of the aggregate series in the *RBC* and the baseline model are similar, the *quantitative* predictions of the two models are starkly different. In the baseline model, vintage effects amplify aggregate dynamics. The distribution of productivity and capital stocks is responsible for this difference. As capital expenditures become more expensive, the average capital expenditure (intensive margin) and the number of investors (extensive margin) shrink. Despite the increase in the cost of investment, a significant fraction of firms finds optimal paying the non-convex adjustment cost and adopting the latest vintage. (By virtue of the calibration strategy, the model matches the fraction of spikes adjusters.) Nonetheless, such investors choose a lower target capital relative to the stationary equilibrium. Current financial conditions reduce the number of firms that adopt the latest technology. On impact, the drop in investment is virtually the same in both economies. Over time, in the baseline model, the response is amplified. As a byproduct of the drop in capital accumulation, the productivity of non-adjusting firms stagnates. As the distance of non-adjusting firms from the technological frontier increases, the economy-wide productivity dispersion increases. Lower average productivity and capital deepen the recession. Over time, the two economies recover at a similar speed.

In Table 9, we report the response of aggregate productivity across models and in the data, following the 2012 recession.[26] Model-based measures of TFP are computed imposing a Cobb-Douglas production function.

Table 9: Financial Shock - TFP Response

	Total Factor Productivity		
	Data (A)	Vintage (B)	*RBC* (C)
2012	-1.27%	-0.43%	0.00%
2013	-1.08%	-0.57%	0.00%
2014	-1.15%	-0.40%	0.00%
2015	-0.89%	-0.26%	0.00%

Note: Each entry is in percent relative from trend values. TFP is computed using an aggregate production function.

Through the lens of our model, parameterized to reproduce the micro and the macro pattern of capital accumulation in the data, vintage effects account for a third of the decline in aggregate productivity with respect to trend.[27] Increasing the size of the shock increases the decline in aggregate investment and aggregate productivity. In the nested *RBC*, the response of aggregate productivity is exogenous and constant over time.

6.2 Technology Shock - Aggregate Uncertainty

In this section, we extend the baseline model in two dimensions. First, we introduce aggregate uncertainty by assuming that the rate at which technology evolves is stochastic

[26]TFP data in Table 9 is computed as the cumulative growth rates of TFP from 2012-on, netting out the effect of TFP growth trend measured as the average over the sample 1992-2011.

[27]Results are equivalent if TFP is computed as a weighted average of individual TFP across firms. Moreover, the measure of TFP in the data is adjusted for the utilization rate in the economy; this ensures comparability between the model and the data.

rather than deterministic. Thus, newer vintages are more productive than the previous one at a gross rate of $\gamma_{A,t}$ that is now time varying according to an autoregressive process of order one. Second, we include a labor supply decision on the household side. Both modifications allow us to study how microeconomic heterogeneity contributes to the propagation of technology shocks. While productivity and technology are often used interchangeably to label stochastic disturbances to production efficiency, the same is not true in the context of our baseline model, as shocks affect only the *current* vintage. Adding aggregate uncertainty complicates the solution of the model as the distribution (and its evolution) enters the state space of the model. To solve the model, we adapt the solution method in Khan and Thomas (2008), described in Appendix J.2. The calibration strategy follows the same targets outlined in Section 5.1 and discussed in Appendix J.1.

We perform the conventional business cycle exercise by simulating the model in response to technology shocks.[28]

Table 10: Business Cycle Statistics
Technology Shock

	ΔGDP	ΔC	ΔI	L
	(A)	(B)	(C)	(D)
RBC				
σ_X	0.27%	0.27%	3.47%	0.39%
Vintage				
σ_X	0.44%	0.20%	4.26%	0.41%

Note: Each entry represents the volatility of the respective variable. Δ indicates the growth rate. C, I, and L refer to consumption, investment, and labor, respectively.

As shown in Table 10, vintage effects amplify aggregate dynamics relative to the nested *RBC* model. The standard deviation of GDP increases by about 50 percent. This result obtains because the sensitivity of aggregate investment to technology shocks is higher in the vintage model. On average, large capital expenditures also entail produc-

[28]We discretize the aggregate technological process so that the realizations of the shock are such that there is no technological regress, i.e., the growth rate of technological efficiency, $\gamma_{A,t}$, is always non-negative.

tivity gains. The gap from the technological frontier is now stochastic and depends upon the realization of the aggregate technology shock. When the efficiency of the latest vintage is higher than expected, more firms pay the adoption cost and, on average, incur more significant capital expenditures. The ensuing shift in the distribution magnifies the response of GDP and investment.

The vintage model is also consistent with the cyclicality of the dispersion of TFP and investment rates across firms, discussed in Bloom et al. (2018) and Bachmann and Bayer (2014). This feature does not come from a countercyclical dispersion in idiosyncratic shocks faced by firms, but it endogenously arises in equilibrium in response to first-moment shocks. In good times, as more firms adopt the latest technology, the dispersion of TFP decreases. Moreover, as adopting the latest technology, on average, involves larger capital expenditure, the dispersion of investment rates increases as well.

7 Concluding Remarks

Using firm-level data, we provide evidence of technology adoption through capital accumulation. Controlling for concurrent determinants of investment spikes, including their dependence on current and past productivity innovations, we find that the timing of investment adjustment is a source of TFP heterogeneity across firms. Firms that have undertaken a recent large capital adjustment are, other things equal, more productive than firms that have done so less recently.

The negative relationship between TFP and investment spikes carries important aggregate implications. Shocks that induce firms to postpone their investment also contribute to a productivity slowdown, as firms delay adopting new efficiency-enhancing technology. Shifts in the distribution of capital stocks and vintage technologies across firms induced by aggregate shocks also lead to fluctuations in the economy-wide TFP. In turn, the endogenous response of productivity amplifies macroeconomic dynamics beyond the effect of the initial shock. We find this amplification effect quite significant in response to persistent negative shocks, like a financial crisis or a slowdown in the exogenous growth rate of technology efficiency.

In an application to the Italian financial recession of 2012, we show that the shift in the distribution of investors recorded during the crisis can account for about one-third of the missing productivity growth of the Italian economy in subsequent years and increase GDP losses relative to a benchmark RBC model. Thus, our results support the view that a prolonged investment slump, like the one experienced by many advanced economies in the decade after the Great Recession, significantly contributes to dampening aggregate productivity growth, raising questions about the recovery after the Covid-19 pandemic.

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Online Appendix to "Aggregate Dynamics and Microeconomic Heterogeneity: The Role of Vintage Technology"

A Data Sources

Detailed information on yearly balance sheets comes from Cerved Group S.P.A. (Cerved Database), while data on employment and wages are obtained from the Italian National Social Security Institute (INPS). Industry-specific price deflators and depreciation rates are obtained from the Italian National Statistical Institute (ISTAT). Sectors are constructed aggregating available data from two-digit industries, according to the 2007 NACE classification. The agriculture sector includes industries 1, 2, 3, and 8. The manufacturing sector comprises industries 10, 11, and 13-33.

Table $A.1$: Sectoral Data

Sector	No. of Obs.
Agriculture, forestry, and fishing	96,087
Manufacturing	1,487,826
Electricity and gas supply	12,324
Water supply	40,249
Construction	614,258
Wholesale and retail trade	1,324,078
Transportation and storage activities	189,789
Accommodation and food service	267,581
Information and communication	223,826
Financial and insurance activities	25,160
Real estate activities	60,759
Professional, scientific, and technical activities	224,766
Administrative and support service activities	172,656
Public administration and defense	31,138
Education	121,044
Human health and social work	66,950
Other activities	46,403

The electricity and gas supply includes industry 35. The water supply sector includes industries 36-39. The construction sector includes industries 41-43. The wholesale and retail trade sector includes industries 45-47. The transportation and storage activities sector includes industries 49-53. The accommodation and food service sector includes industries 55 and 56. The information and communication sector includes industries 58-63. The financial and insurance activities sector includes industry 66. The real estate activities sector includes industry 68. The professional, scientific, and technical activities sector includes industries 69-75. The administrative and support service activities sector includes industries 77-82. The public administration and defense sector includes industry 85. The education sector includes industries 86-88. The human health and social work sector includes industries 90-93. The other activities sector includes industries 95 and 96. The composition of the data set by sector is reported in Table $A.1$.

B Investment Rates and Total Factor Productivity

Our measure of interest is TFP together with investment age. Next, we discuss the construction of intermediate variables. Our computations follow the prevalent practice in the existing literature.

B.1 Total Factor Productivity

As in Bloom et al. (2018), we measure value-added $v_{f,t}$ for each firm f at year t as

$$v_{f,t} = Q_{f,t} - M_{f,t}, \tag{A.1}$$

where $Q_{f,t}$ is nominal output and $M_{f,t}$ is cost of materials. Nominal quantities are deflated by the corresponding sectoral deflators to obtain a measure of real value-added. Concerning labor input, we directly observe the wage bill and the number of employees for the firm at a given time t. We follow Bloom et al. (2018) and define value-added-based TFP as

$$log(\hat{z}_{f,t}) = log(v_{f,t}) - \theta_f log(k_{f,t}) - \nu_f log(N_{f,t}), \tag{A.2}$$

where $v_{f,t}$ denotes real value added, $k_{f,t}$ the real capital stock, and $N_{f,t}$ labor input, and θ and ν are the cost shares for capital and labor, respectively. We follow Bachmann and Bayer (2014) and estimate θ and ν by the median of the firm average share of factor expenditure in total value-added, as defined by

$$\begin{aligned}\hat{\theta}_f &= T^{-1}\sum_t \frac{wn_{f,t}}{v_{f,t}} \text{ and} \\ \hat{\nu}_f &= T^{-1}\sum_t \frac{(r_{f,t}+\delta_{f,t})k_{f,t}}{v_{f,t}},\end{aligned} \tag{A.3}$$

where $wn_{f,t}$ is the real wage bill and $r_{f,t}$ the real cost of funds for the corporate sector and is estimated using the average real interest rate on banking loans for the corporate sector. As in Becker et al. (2006) and most of the existing literature, we construct the real capital stock series using the perpetual inventory method so that

$$k_{f,t} = (1-\delta_{f,t})k_{f,t-1} + i_{f,t}, \tag{A.4}$$

where $i_{f,t}$ is real net investment (deflated using sectoral deflators for capital expenditures) on tangible and intangible assets. To initialize the recursion, we estimate the real stock of capital using the book value of fixed assets net of funds amortization. The depreciation rate δ is common within sectors.

C Hazard Rates - Estimation Details

In this section, we describe the procedure employed to estimate investment hazard rates. Our discussion follows Haltiwanger, Cooper and Power (1999) and Cameron and Trivedi (2005).

Let us denote by $h_f(t)$ the hazard for firm f at time t. The hazard is parameterized using the popular proportional hazard form:

$$h_f(t) = h_0(t)exp\{z_f(t)\beta\} \tag{A.5}$$

where $h_0(t)$ is the baseline hazard rate at time t (which is unknown), z is a vector of covariates, and β is the vector of unknown parameters. The probability that a spell lasts until $t+1$ given it has lasted for t periods is defined by:

$$P[T_f \geq t+1 | T_f \geq t] = exp[-exp(z_f(t)'\beta + \gamma(t) + \mu)] \tag{A.6}$$

where

$$\gamma(t) = ln\left[\int_t^{t+1} h_0(u)du\right] \tag{A.7}$$

The likelihood function for a sample of N firms can then be written in terms of $A.6$ as:

$$L = \prod_{f=1}^{N} \sum_{j=1}^{J} \alpha_j L_f \tag{A.8}$$

where

$$L_f = \left[1 - exp\{-exp[\gamma k_f + z_f(k_f)'\beta]^{d_f}\}\right] \times \prod_{t=1}^{k_f-1} \left[exp\{-exp[\gamma t + z_f(t)'\beta + \mu_j]\}\right] \tag{A.9}$$

where C_f is the censoring time, d_f if $T_f \leq C_f$ and 0 otherwise, and k_f is the minimum between the observed length of the spell and the C_f. The first term in $A.9$ represents the probability that the firm exhibits an investment spike in the interval $[k_f, k_f+1]$ given that the spell has lasted until k_f. The second term in $A.9$ represents the probability that a spell lasts until k_f. The unobserved heterogeneity across firms is approximated by a discrete distribution with a finite number of points. The μ_j are the J points of support of the distribution where each of them has an associated probability α_j. μ_j and α_j are estimated jointly with the other parameters by maximizing the log of the likelihood in $A.8$.[29] As discussed in Cameron and Trivedi (2005), one can interpret the j as a discrete number of unobserved types from which the data are assumed to be drawn. We consider investment spells in the 1986 and 2015 period, and we then track the firm's investment age until the next spike. All firms that have an investment age over 12 years are censored. For computational reasons, we consider only firms for which we have at least 15 consecutive

[29] In the estimation μ_0 is normalized to zero, and the sum of the α_j is constrained to one.

observations. Figure $A.1$ reports the results from estimating $A.8$ with our data. We report results considering three groups, i.e., J is equal to 3. The estimation yields three groups.[30]

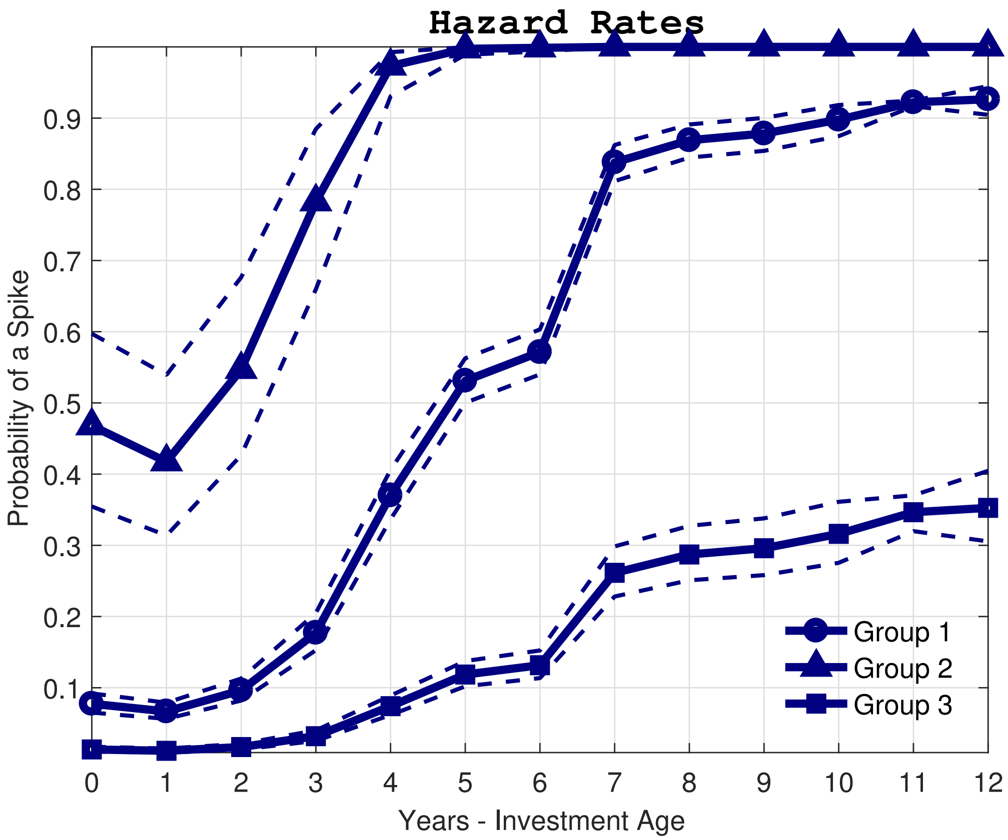

Figure $A.1$: Semi-Parametric Estimates

Notes: Appendix C reports details about the estimation procedure. The weight to the first group is 0.42, the weight to the second group is 0.06, and the weight to the third 0.52.

In all cases, the hazard rates are upward-sloping, although the slopes are different across groups. Each group is to be interpreted as a type. The weight associated with each group is the probability that the sample is drawn from one of the groups. Group 1 has a weight of 0.42, group 2 of 0.06, and group 3 of 0.52. For low levels of investment age up to five years, the hazard rates in groups 1 and 3 are considerably lower than the ones for group 2. Over time, while the probability of a spike increases dramatically for group 1, the one

[30]The estimation requires specifying the number of groups. As there is no guiding theory on the choice of J, we follow standard practice in the existing literature. We start with $J = 2$ and increase the number of groups until the log-likelihood does not change significantly. When we consider four groups, the log-likelihood is similar to the case with three groups, and the estimates associated with the fourth group virtually reproduce the ones for group 1.

for group 3 increases at a slower pace. Overall, the probability that a firm exhibits an investment spike is rising with the time since the last spike.

D Robustness I: Alternative Measures of Spikes and Sample Composition

In this section, we show that the negative relationship between TFP and $Inv.Age$ does not depend on the definition of investment spikes. Tables $A.2$ and $A.3$ report estimates for a 30 percent threshold and an absolute spike.

Table $A.2$: Investment Age and Total Factor Productivity: Spikes $i/k \geq 30\%$

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)	$TFP_{f,t}$ (F)
$Inv.Age_{f,t}$	-0.522^{***}	-0.462^{***}	-0.497^{***}	-0.621^{***}	-0.597^{***}	-0.615^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. of obs.	4,058,036	3,404,387	3,675,873	3,540,352	3,242,758	3,373,532
R^2	0.734	0.742	0.737			
Estimator	OLS	OLS	OLS	IV	IV	IV
Firm FE	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Time×Ind. FE	✓	✓	✓	✓	✓	✓
Sample	All	$Age_{f,t} \geq 3$	$N_f \geq 4$	All	$Age_{f,t} \geq 3$	$N_f \geq 4$

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an investment rate above 30 percent. Each equation also includes firm size and age as additional controls. Columns D through F report estimates obtained instrumenting $Inv.Age_{f,t}$ using its first lag. Entries expressed in percent. $Age_{f,t}$ indicates the age of the firm. N_f denotes the number of observations for firm f. The sample period is 1997 to 2016.

Table $A.3$: Investment Age and Total Factor Productivity: Absolute Spikes

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)	$TFP_{f,t}$ (F)
$Inv.Age_{f,t}$	-0.541^{***}	-0.502^{***}	-0.490^{***}	-0.504^{***}	-0.499^{***}	-0.486^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. of obs.	4,058,036	3,404,387	3,675,873	3,540,352	3,242,758	3,373,532
R^2	0.747	0.756	0.740			
Estimator	IV	IV	IV	IV	IV	IV
Firm FE	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Time×Ind. FE	✓	✓	✓	✓	✓	✓
Sample	All	$Age_f \geq 3$	$N_f \geq 4$	All	Age ≥ 3	$N_f \geq 4$

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an absolute spike (see the text for details). Each equation also includes firm size and age as additional controls. Columns D through F report estimated obtained instrumenting $Inv.Age_{f,t}$ using its first lag. Entries expressed in percent. N_f denotes the number of observations for firm f. The sample period is 1997 to 2016.

E Robustness II: Alternative Timing of IV Instruments

Below we show that the negative relationship between TFP and $Inv.Age$ is not sensitive to the timing of the instrument considered in the IV estimates. Tables $A.4$, $A.5$, and $A.6$ report the estimated coefficients of equation 1 obtained using an IV approach.

Table $A.4$: Inv. Age and TFP: Timing IV Instrument Spikes $i/k \geq 20\%$

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)
$Inv.Age_{f,t}$	-0.987^{***}	-1.217^{***}	-1.688^{***}	-2.058^{***}	-3.058^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. of obs.	3,540,352	2,963,790	2,532,833	2,153,884	1,814,877
IV Instrument	First lag of Inv. Age	Second lag of Inv. Age	Third lag of Inv. Age	Fourth lag of Inv. Age	Fifth lag of Inv. Age
Firm FE	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Time×Ind. FE	✓	✓	✓	✓	✓
Sample	All	All	All	All	All

Notes: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an investment rate above 20 percent. Each equation also includes firm size and age as additional controls. Columns A through D report estimated obtained instrumenting $Inv.Age_{f,t}$ using different lags of the dependent variable. Entries expressed in percent. The sample period is 1997 to 2016.

Table $A.5$: Inv. Age and TFP: Timing IV Instrument Spikes $ik \geq 30\%$

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)
$Inv.Age_{f,t}$	-0.621^{***}	-0.739^{***}	-0.984^{***}	-1.364^{***}	-3.496^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. of obs.	3,540,352	2,963,790	2,532,833	2,153,884	1,814,877
IV Instrument	First lag of Inv. Age	Second lag of Inv. Age	Third lag of Inv. Age	Fourth lag of Inv. Age	Fifth lag of Inv. Age
Firm FE	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Time$\times$Ind. FE	✓	✓	✓	✓	✓

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an investment rate above 30 percent. Each equation also includes firm size and age as additional controls. Entries expressed in percent. The sample period is 1997 to 2016.

Table $A.6$: Inv. Age and TFP: Timing IV Instrument Absolute Spikes

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)
$Inv.Age_{f,t}$	-0.504^{***}	-0.482^{***}	-0.504^{***}	-0.753^{*}	-0.709^{***}
	(0.00)	(0.00)	(0.00)	(0.06)	(0.00)
N. of obs.	3,540,352	2,963,790	2,532,833	2,153,884	1,814,877
IV Instrument	First lag of Inv. Age	Second lag of Inv. Age	Third lag of Inv. Age	Fourth lag of Inv. Age	Fifth lag of Inv. Age
Firm FE	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Time×Ind. FE	✓	✓	✓	✓	✓

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an absolute spike (see text details). Each equation also includes firm size and age as additional controls. Entries expressed in percent. The sample period is 1997 to 2016.

F Discretized Investment Age Estimates

Table $A.7$ reports estimates of equation 2 using OLS and IV estimators.

Table $A.7$: Discretized Investment Age and Total Factor Productivity

	$TFP_{f,t}$ (A)	$TFP_{f,t}$ (B)	$TFP_{f,t}$ (C)	$TFP_{f,t}$ (D)	$TFP_{f,t}$ (E)	$TFP_{f,t}$ (F)
$Inv.Age\ 1_{f,t}$	-1.127^{***}	-0.762^{***}	-0.743^{***}	-1.755	-1.064	-1.458
	(0.00)	(0.00)	(0.00)	(0.23)	(0.55)	(0.37)
$Inv.Age\ 2_{f,t}$	-1.825^{***}	-1.365^{***}	-1.465^{***}	-2.818^{**}	-2.034	-2.290
	(0.00)	(0.00)	(0.00)	(0.05)	(0.24)	(0.14)
$Inv.Age\ 3_{f,t}$	-2.486^{***}	-2.032^{***}	-2.303^{***}	-3.896^{***}	-3.174^{*}	-3.492^{**}
	(0.00)	(0.00)	(0.00)	(0.01)	(0.08)	(0.03)
$Inv.Age\ 4_{f,t}$	-3.121^{***}	-2.662^{***}	-2.974^{***}	-5.001^{***}	-4.272^{**}	-4.586^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)
$Inv.Age\ 5_{f,t}$	-3.931^{***}	-3.488^{***}	-3.816^{***}	-6.098^{***}	-5.357^{***}	-5.684^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Inv.Age\ 6_{f,t}$	-4.231^{***}	-3.816^{***}	-4.156^{***}	-6.864^{***}	-6.092^{***}	-6.459^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Inv.Age\ 6+_{f,t}$	-3.139^{***}	-3.060^{***}	-3.294^{***}	-7.517^{***}	-6.719^{***}	-7.106^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. of obs.	4,058,036	3,404,387	3,675,873	3,540,352	3,242,758	3,373,532
R^2	0.737	0.750	0.730			
Estimator	OLS	OLS	OLS	IV	IV	IV
Firm FE	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Time×ind. FE	✓	✓	✓	✓	✓	✓
Sample	All	$Age_{f,t} \geq 3$	$N_f \geq 4$	All	$Age_{f,t} \geq 3$	$N_f \geq 4$

Note: * p<0.10, ** p<0.05, and *** p<0.01, where p is the marginal probability level and is reported in parentheses. The dependent variable is the log of total factor productivity (TFP) for firm f at time t. $Inv.Age_{f,t}$ measures the time elapsed between investment spikes, defined as the firm experiencing an investment rate above 20%. Each equation also includes firm size and age as additional controls. Columns D through F report estimated obtained instrumenting the set of $Inv.Age_j f,t$ using the first lag of the set of dummies. Entries expressed in percent. $Age f,t$ indicates the age of the firm. N_f denotes the number of observations for firm f. The sample period is 1997 to 2016.

G Sectoral Analysis - Estimates

In this section we provide empirical evidence of the link between investment age and productivity using industry-level data. We fit equation 2 to firms in each sector separately. Figure $A.2$ plots the estimated coefficients of $Inv.Age_j$ obtained by fitting equation 1 to each sector. This exercise sheds light on which sectors are driving our results and whether vintage effects are a defining characteristic of firm-level productivity in all sectors.

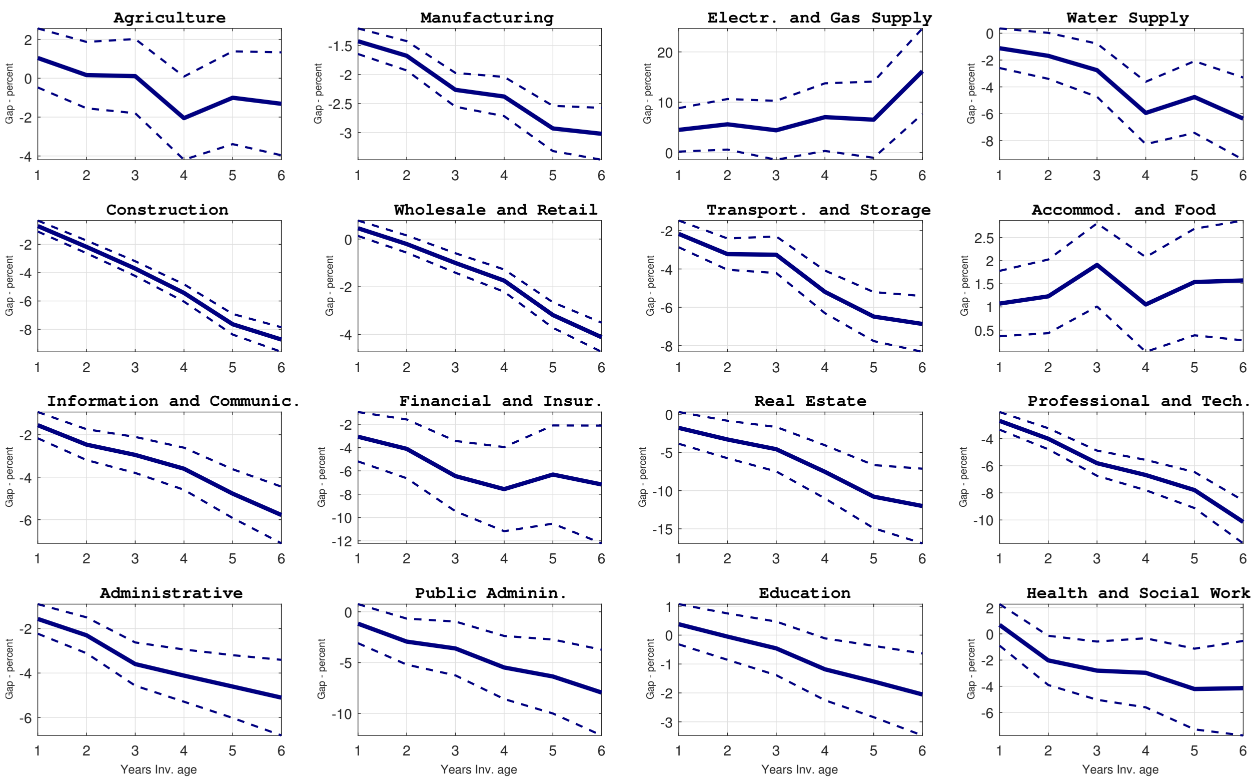

Figure $A.2$: Investment Age and Total Factor Productivity - Sectoral Analysis

Notes: The figure reports the estimated β_j coefficients in equation 1 for each sector. Dashed lines are 95 percent confidence bands. Each equation is estimated with ordinary least squares, and firm- and sector-specific effects, year effects, and a series of dummies for a firm's age and size.

H Equilibrium and (S,s) Decision Rules

Given the presence of fixed cost, the adoption and investment decision is akin to exercising an option. Consider a firm of a type (ε, z, k) drawing adjustment cost ξ. Define the value associated with the value of action $V^A(\varepsilon, z, k; \mu)$ and the one with the inaction choice $V^I(\varepsilon, z, k; \mu)$ as

$$V^A(\varepsilon, z_0, k; \mu) \equiv \max_{k' \in \mathbf{R}_+} R(\varepsilon, z_0, k'; \mu), \tag{A.10}$$

$$V^I(\varepsilon, k; \mu) \equiv \max_{k' \in \Omega(k)} R(\varepsilon, z_0, k'; \mu), \tag{A.11}$$

Next, define the firm's target capital k^* as the optimal choice of k—when the firm obtains the latest vintage—that solves the right-hand side of (A.10). The solution to the problem in (A.10) is independent of the current stock of capital k and ξ, but not ε (and of course z_0), given persistence in firm-specific productivity. As a result, all firms with current productivity ε and pay their fixed costs to upgrade to their latest vintage choose a common target capital for the next period, $k^* = k(\varepsilon, \mu)$, and achieve a common gross value $V^A(\varepsilon, z_0, k; \mu)$. By contrast, firms that do not pay adjustment costs have value $V^I(\varepsilon, z, k; \mu)$. In this case, the firm keeps its current vintage z that becomes more obsolete (i.e., more distant from the technological frontier) at a rate γ_A. The firm gets to adjust its stock of capital, that is constrained to be included in $\Omega(k)$.

A firm will pay the fixed cost if $V^A(\varepsilon, k; \mu) - p(\xi + \delta_S k)$—the value of adjusting—is at least as great as $V^I(\varepsilon, z, k; \mu)$ — the value of inaction. Given continuity in the adjustment cost ξ, it is possible to identify threshold value such that a type (ε, z, k) firm indifferent between action and inaction:

$$-p(\mu)(\hat{\xi}(\varepsilon, z, k; \mu) + \delta_S k) + V^A(\varepsilon, k; \mu) = V^I(\varepsilon, z, k; \mu). \tag{A.12}$$

To summarize the adoption and investment decision define $\xi^T(\varepsilon, z, k; \mu) \equiv \min[\,\bar{\xi}, \hat{\xi}(\varepsilon,z,k;\mu)\,]$ so that $0 \leq \xi^T(\varepsilon, z, k; \mu) \leq \bar{\xi}$. Any firm (ε, z, k) that draws an adjustment cost at or below its type-specific threshold $\xi^T(\varepsilon, z, k; \mu)$ will pay the fixed cost and adjust k and

z.

Thus, for a given group of firms of type $(\varepsilon, z, k; \mu)$, a fraction $G\left[\xi^T(\varepsilon, z, k; \mu)\right]$ pay their fixed cost to adopt the latest vintage and optimally choose capital. Thus, the market-clearing levels of consumption required to determine p using equation 10 is given by

$$\begin{aligned} C \;=\; & \int_S F(\varepsilon, z, k) && (A.13) \\ & -G\left[\xi^T(\varepsilon, z, k; \mu)\right] J(\xi \leq \xi^T(\varepsilon, z, k; \mu))(i-\xi) \\ & -\left[1 - G\left[\xi^T(\varepsilon, z, k; \mu)\right]\right] J(\xi > \xi^T(\varepsilon, z, k; \mu)) \left[i^{NA}\ \mu\left(d\left[\varepsilon \times z \times k\right]\right)\right], && (A.14) \end{aligned}$$

where it is understood that i and i^{NA} depend upon the firm's current state. Finally, we turn to the evolution of the firm distribution, $\mu^{'} = \Gamma(\mu)$. It is useful to define the indicator function $J(x) = 1$ if x is true, and 0 otherwise. For each $(\varepsilon_m, z, k) \in S$

$$\begin{aligned} & \mu^{'}(\varepsilon_m, z, k) \\ = \; & \sum_{l=1}^{N_\varepsilon} \pi_{lm}^{\varepsilon} \left[\begin{array}{c} \int J(\xi \leq \xi^T(\varepsilon, z, k; \mu)) G\left[\xi^T(\varepsilon_l, z, k; \mu)\right] \mu(\varepsilon_l, z, dk) \\ + \int \left[1 - \int G\left[\xi^T(\varepsilon_l, z, k; \mu)\right] \mu(\varepsilon_l, z, dk)\right] J(\xi > \xi^T(\varepsilon, z, k; \mu)) \end{array} \right] \end{aligned}$$

I Computational Details - Stationary Equilibrium

I.1 Value Function and Steady State

The value function to solve the firm's problem defined in equation 5, 6, and 7 are the basis of our numerical solution of the economy. The solution algorithm involves repeated application of the contraction mapping implied by equation 5, 6, and 7 to solve for firms' value function, given the price functions $p(\mu)$. More specifically, the firm's problem amounts to find the next-period value of capital $k^{'}$. To do so, we use a golden section search to allow for continuous control. We discretize the state space using a fine grid between 0.1 and 8.5 for capital k and between 0.2 and 1 for z. The process for the idiosyncratic process ε is approximated using the procedure in Tauchen (1986) over 13 possible values. We compute the value function exactly at the grid points above and interpolate for in-between values. This procedure is implemented using a multidimensional cubic splines procedure, with a so-called "not-a-knot"-condition to address the large number of degrees of freedom problem, when using splines (see Judd, 1998). With the firm's policy function at hand, we compute the stationary distribution and verify that the guessed price is consistent with market clearing. We update the guessed price function $p(\mu)$ until convergence, i.e., until the guessed and the market-clearing price converges.

I.2 Transitional Dynamics

We solve for the transitional dynamics as follows. We specify a path for $\{X_t\}_{t=0}^{T}$ with $X_0 = X_T = \bar{X}$, where $\bar{X}$ indicate the steady-state value of X. Following the approach in Ríos-Rúll (1999), we conjecture a path for the marginal utility of consumption $\{p_t\}_{t=1}^{T}$. Assuming that at time T the economy is in steady-state, we can solve backward the expected value function at all dates T-1, T-2,... till 1. Using the expected value function, the value of the shocks, and the conjecture path, we explicitly solve for the market clearing in the goods market in every time period and obtain new values for $\{p_t^{MktClearing}\}_{t=1}^{T}$. We iterate until the proposed path $\{p_t\}_{t=1}^{T}$ and equilibrium path $\{p_t^{MktClearing}\}_{t=1}^{T}$ for the marginal utility of consumption converge.

I.3 Aggregate Process for the Financial Shock

We study the aggregate and cross-sectional dynamics in response to a macroeconomic shock under two alternative scenarios. First, we consider a deterioration in financial conditions. In the spirit of Gomes (2001), firms (that adopt the latest vintage) are subject to an additional financial cost equal to $\lambda_t i_{f,t}$. The processes for lambdas are parameterized so that the model reproduces the micro and macro behavior of investment during the 2012 recession. More specifically, the shock processes result in a drop in the fraction of firms experiencing spikes and in aggregate investment observed in the data.[31]

Let X indicate either shock, depending on the experiment. The path for $\{X_t\}_{t=0}^{T}$ is

[31] The fraction of firms experiencing spikes drops of 4 percentage points in 2012, relative to its value in 2011.

Table $A.8$: Calibration of the Financial Shock

Variable	$\bar{X}$	X_1	ΔX	ρ_X
	(A)	(B)	(C)	(D)
λ	0	0.008	0.008	0.75

such that the economy is initialized at the steady state, and once the shock is absorbed, the economy reverts to the steady state. We assume that X is an autoregressive process of order one. More formally, this results in $X_0 = X_T = \bar{X}$ and $(X_t - \bar{X}) = \rho_X(X_{t-1} - \bar{X})$ for $t \in \{1, ...T-1\}$, where $\bar{X}$ is the value taken in the steady state. Table $A.8$ reports the details of the calibration exercise.

J Computational Details - Aggregate Uncertainty

J.1 Calibration

We retain the parameters in Table 6, with few exceptions. We introduce labor supply considerations by assuming a perfectly elastic labor supply (see Hansen, 1985 and Thomas, 2002). A denotes the parameter that governs the disutility of labor in the utility function and is set to 0.79 to ensure that aggregate labor is, on average, equal to one. We estimate the elasticity of output with respect to capital (θ) and the one to labor (ν) from the data using the procedure detailed in Appendix B. In the sample, θ is equal to 0.18 and ν to 0.64. To calibrate the process of idiosyncratic shocks and the upper support of the adjustment cost distribution, we follow the same strategy in Section 5.1 and choose these parameters to reproduce the cross-sectional distribution of investment rates. This yields ρ_ε and σ_ε equal to 0.86 and 0.0163, respectively. The upper support of the distribution (ξ) is set to 0.026. Finally, the log of $\gamma_{A,t}$ follows an autoregressive process centered around its mean value $\bar{\gamma_A}$: $log(\gamma_{A,t}) = (1-\rho_{\gamma_A})\ log(\bar{\gamma_A}) + \rho_{\gamma_A} log(\gamma_{A,t-1}) + \sigma_{\gamma A}\varepsilon_{\gamma_A}$, where ε_{γ_A} is a normally distributed i.i.d. process with standard deviation $\sigma_{\gamma A}$. In the absence of empirical guidance for the evolution of the technological frontier, we choose ρ_{γ_A} and $\sigma_{\gamma A}$ so that average TFP in the model matches the persistence and the volatility of its data counterpart for the sample 1992-2007. We then set ρ_{γ_A} equal to 0.15 and $\sigma_{\gamma A}$ to 0.0026.

J.2 Solution Algorithm

When the growth rate of technology is stochastic, the endogenous distribution of capital stocks and productivity enters the state space of the model. To solve the model, we follow the approach in Khan and Thomas (2008). This strategy replaces the aggregate law of motion for the distribution with a forecast rule. Typically, to predict prices and the future proxy aggregate state, agents use the mean capital stock. In our framework, two endogenous distributions for the capital stocks and the vintage technologies are available to the firms. In theory, this could complicate the solution algorithm by requiring agents to forecast the behavior of two distributions rather than one. In practice, when the persistence of the shock is relatively low, the standard rule that uses the mean of the capital stocks as a regressor works very well, yielding an accurate forecast of prices and future proxy aggregate state. We forecast the mean capital K' and the marginal utility of consumption p using $log(K')$=β_0 + $\beta_1 log(K)$ + ε_K and $log(p)$=β_0 + $\beta_1 log(K)$ + ε_p. As we approximate $\gamma_{A,t}$ using the discretization procedure in Tauchen (1986) using a grid with seven points, we estimate a regression conditional on each realization of the aggregate process, $\gamma_{A,t}$.

In Table A.9 and A.10 we assess the accuracy of the forecasting rule for both models. We find that the algorithm yields a very accurate solution as testified by the high R^2 and small standard errors. As discussed by Den Haan (2010), R-squares are averages and scaled by the variance of the dependent variable. To provide a robust statistic, we report the maximum forecast error for each regression. For the vintage model, the maximum percentage errors are 0.031 percent for p and 0.015 for K'. For the RBC model, the maximum percentage errors are is 0.007 percent for p and 0.013 percent for for K'. We conclude that the forecasting rules are extremely precise.

Table $A.9$: Forecasting Rules - Vintage Model

	Technology	β_0	β_1	$S.E.$	Adj. R^2
	(A)	(B)	(C)	(D)	(E)
Forecasting K'					
	$\gamma_{A,1}$	0.01570	0.74045	2.84e-05	0.99995
	$\gamma_{A,2}$	0.01241	0.73526	7.15e-05	0.99997
	$\gamma_{A,3}$	0.00904	0.72849	0.00011	0.99966
	$\gamma_{A,4}$	0.00550	0.72144	0.00031	0.99874
	$\gamma_{A,5}$	0.00182	0.71175	0.00038	0.99712
	$\gamma_{A,6}$	-0.00192	0.70161	0.00049	0.99242
	$\gamma_{A,7}$	-0.00566	0.69424	0.00050	0.99176
Forecasting p					
	$\gamma_{A,1}$	0.52492	-0.28663	1.01e-05	0.99862
	$\gamma_{A,2}$	0.52455	-0.27706	5.98e-05	0.99931
	$\gamma_{A,3}$	0.52420	-0.27296	6.76e-05	0.99821
	$\gamma_{A,4}$	0.52385	-0.27114	6.24e-05	0.99775
	$\gamma_{A,5}$	0.52335	-0.26741	8.88e-05	0.99750
	$\gamma_{A,6}$	0.52275	-0.26385	6.49e-05	0.99636
	$\gamma_{A,7}$	0.52212	-0.26312	6.41e-05	0.99777

Table A.10: Forecasting Rules - RBC Model

	Technology	β_0	β_1	$S.E.$	Adj. R^2
	(A)	(B)	(C)	(D)	(E)
Forecasting K'					
	$\gamma_{A,1}$	0.01858	0.71162	1.9982e-05	0.99999
	$\gamma_{A,2}$	0.01507	0.71045	2.0342e-05	0.99999
	$\gamma_{A,3}$	0.01152	0.71082	2.0835e-05	0.99999
	$\gamma_{A,4}$	0.00798	0.71072	2.4525e-05	0.99999
	$\gamma_{A,5}$	0.00444	0.71065	2.3658e-05	0.99999
	$\gamma_{A,6}$	0.00090	0.71041	1.9454e-05	0.99999
	$\gamma_{A,7}$	-0.00264	0.71078	1.7023e-05	0.99999
Forecasting p					
	$\gamma_{A,1}$	0.52663	-0.29804	9.3465e-06	0.99998
	$\gamma_{A,2}$	0.52585	-0.29815	1.0727e-05	0.99999
	$\gamma_{A,3}$	0.52507	-0.29855	7.8238e-06	0.99999
	$\gamma_{A,4}$	0.52429	-0.29889	8.0911e-06	0.99998
	$\gamma_{A,5}$	0.52351	-0.29927	9.0340e-06	0.99998
	$\gamma_{A,6}$	0.52273	-0.29959	8.8113e-06	0.99998
	$\gamma_{A,7}$	0.52195	-0.29991	7.0511e-06	0.99999