BANCA D'ITALIA
EUROSISTEMA

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The series is available online at www.bancaditalia.it .

ISSN 1972-6627 (print)
ISSN 1972-6643 (online)

Printed by the Printing and Publishing Division of the Bank of Italy

THE EURO-AREA OUTPUT GAP THROUGH THE LENS OF A DSGE MODEL

by Lorenzo Burlon* and Paolo D'Imperio**

Abstract

The paper provides estimates of the euro-area output gap, based on a relatively standard medium scale DSGE model estimated recursively with Bayesian techniques over the period 1985-2016. The main findings can be summarized as follows. First, our measure of output gap identifies episodes of expansion and recession generally in line with the official business cycle dating of the CEPR. Second, unlike measures of output gap obtained by means of statistical filtering techniques, real-time DSGE-based estimates are remarkably stable and hence are less prone to ex-post revisions. According to our results, the euro-area output gap was -3.4% in 2016, more negative than assessed by most economic analysts and institutions (spanning a range between from 0 and to -2%), but arguably more consistent with the still weak dynamics of both labour costs and core inflation.

JEL Classification: C11, E32, E37, E66.
Keywords: output gap, potential output, DSGE modelling, Bayesian estimation, euro area.

Contents

* Bank of Italy, DG Economics, Statistics and Research.
** Sapienza University of Rome.

1 Introduction[1]

Economic analysts often rely on the output gap (i.e., the deviation of output from its potential) as an indicator of the overall state of the economy. This is true also for fiscal and monetary authorities, which explicitly include the output gap among the indicators used to ground their policies. However, since the output gap is a latent variable, its estimation and especially its validation are challenging. In this work, we show that the estimate of the euro-area output gap obtained with the (nowadays standard) DSGE model developed by Smets and Wouters (2003) can overcome some of the limitations encountered using the most common methodologies based on statistical filtering and the aggregate production function.

Several methodologies have been developed to obtain a reliable and economically meaningful measure of the output gap.[2] At the same time, new approaches based on the estimation or direct measurement of the supply capacity of an economy are continuously being proposed.[3] Despite the numerous available methodologies, several studies have questioned the reliability of the output gap estimates in real-time. In particular, Orphanides and Van Norden (2002) apply alternative detrending methods to the US data and conclude that the ex-post revisions can be as big as the estimated output gaps. In the same vein, Marcellino and Musso (2011) find that the sign and the magnitude of the euro-area real-time estimates are characterized by a high degree of uncertainty. Recent studies on this topic confirm that there is still no consensus on the best methodology to use for the estimation of the output gap.[4]

This paper documents the advantages of considering a measure of the output gap based on an estimated structural model. The output gap is defined as the deviation of actual output from its potential, where the latter is defined as the outcome of an economy with flexible prices, flexible wages, and constant markups.[5] The more stringent structure imposed to the data by the DSGE

[1]This paper was realized during Paolo D'Imperio's internship at the Bank of Italy. We thank Fabio Busetti, Alessandro Notarpietro, Massimiliano Pisani, Luigi Federico Signorini, and Stefano Siviero for their comments and suggestions. The views expressed in this paper are those of the authors alone and should not be attributed to the Bank of Italy or the Eurosystem. All remaining errors are ours.

[2]See, e.g., Hall (1980), Beveridge and Nelson (1981), Hodrick and Prescott (1997), Kuttner (1994), Christiano and Fitzgerald (2003), and Laubach and Williams (2003).

[3]See, among others, De Masi (1997) and Denis et al. (2006).

[4]See, among others, Borio et al. (2016) and Berger et al. (2015). Turner et al. (2016) offer an account of the ongoing debate in international institutions.

[5]See Woodford (2003).

model helps to provide stable and reliable measures of the output gap compared to reduced-form methods, especially in periods of greater uncertainty. At the same time, the relatively parsimonious parametrization of the model we adopt, that is, the Smets & Wouters (henceforth, SW) model, does not overcharge the data with cross-equation restrictions, achieving a balance between structure and flexibility.

We use standard Bayesian methods to recursively estimate the model on euro-area data at a quarterly frequency over the period 1985-2016, and compare the DSGE-based measure obtained in (pseudo) real time with that of international institutions.[6] Our main findings can be summarized as follows. First, our measure of output gap identifies episodes of expansion and recession generally in line with the official business cycle dating of the CEPR. Second, unlike measures of output gap obtained by means of statistical filtering techniques, real-time DSGE-based estimates are remarkably stable and hence are less prone to ex-post revisions. According to our results, the euro-area output gap was -3.4% in 2016, more negative than assessed by most economic analysts and institutions (spanning a range between 0 and -2%), but arguably more consistent with the still weak dynamics of both labor costs and core inflation.

The rest of the paper is organized as follows. Section 2 reviews relevant issues in the literature. Section 3 summarizes the main features of the model and defines a model-consistent concept of potential output. Section 4 reports the estimation strategy. Section 5 studies the properties of our measure of output gap. Section 6 compares our estimates with other standard measures in terms of their stability and reliability. Section 7 concludes.

2 Literature review

A growing literature employs New Keynesian structural models to estimate the potential output and the relative output gap. These are often compared with more traditional measures used as a benchmark. Among others, Edge et al. (2008) show that output gap estimates derived with a structural model of the US economy signal the same recession periods of those obtained with the production function approach but tend to diverge over the rest of the sample. In a recent

[6]For a review of the techniques used by international institutions, see Giorno et al. (1995) for the IMF, De Masi (1997) for the OECD, and Havik et al. (2014) for the European Commission.

study, Justiniano et al. (2013) find that output gaps for the US are highly procyclical and that discrepancies with standard indicators are mainly due to differences in the definition of potential output. Along this line, Kiley (2013) and Fueki et al. (2016) show that DSGE-based output gaps resting on different potential output definitions for, respectively, the US and Japan can be mapped with those obtained with standard techniques. We document that similar properties apply to DSGE-based output gaps for the euro area.

Estimated general equilibrium models have been used to address a number of relevant issues related to the potential output, especially in its relation with the Phillips curve. Among others, Neiss and Nelson (2005) use the theory-consistent output gaps derived from a DSGE model to estimate the Phillips curve parameters for the US, the UK, and Australia. Riggi and Venditti (2015) show how the weak dynamic of euro-area inflation in the recent years can only be explained by either a flattening of the Phillips curve or by a much larger output gap than what is suggested by international institutions. Jarociński and Lenza (2018) find that measures of the euro-area output gap that increase the predictability of inflation highlight a large EA output gap in the most recent period. Consistent with these studies and with the observed inflation developments, we obtain an updated measure of the euro-area output gap that is larger than what may be obtained with other methods.

The inclusion of additional features in an otherwise simple DSGE model can produce different estimates of the output gap, as potential sources of misspecification become accounted for. The absence of these features is likely to imply a different combination of structural shocks with which the model interprets the observed data. For example, the open-economy dimension is explored by Coenen et al. (2009) and Vetlov et al. (2011), who analyze the output gap produced by the New Area-Wide Model (NAWM). They find that the model-based estimates of the output gap are broadly in line, but more volatile, than those produced with traditional techniques, and that point estimates are characterised by a rather low level of uncertainty. Both studies point out that estimated output gaps have a good predicting power for inflationary pressure although not necessarily the best when compared with alternative models. Sala et al. (2008), Sala et al. (2010), and Galí et al. (2011) shed light on the relation between the output gap and labor market frictions, and Furlanetto et al. (2017) stress the importance of financial frictions in the estimation of model-based output gaps, challenging the concept of potential output that should be relevant for optimal monetary or fiscal policy. Lindé et al. (2016) take the zero lower bound explicitly into account when estimating a S&W model for the euro area, and find that the inclusion of this feature changes the composition of the filtered shocks, with little repercussions on parameter estimates. Interestingly though, they find that the alternative composition of shocks changes the balance of risks for the output gap to the downside. Thus, an alternative measure of the output gap that takes into account the zero lower bound may highlight an even wider negative output gap in the recent years.

3 The model

In this section we present a dynamic stochastic general equilibrium (DSGE) model of the euro-area, based on the model developed by Smets and Wouters (2003).

We model the euro-area as a closed economy. The model features sticky prices and wages, external habit persistence, and adjustment costs on investment. Business cycle fluctuations are captured by the means of nine supply, demand, monetary, and markup shocks. The model includes some of the standard extensions to the SW model that can be found also in Christoffel et al. (2008) and Gomes et al. (2012).[7] All salient details are provided in what follows.

3.1 Households

There is a continuum of households in the economy indexed by $i \in [0, 1]$, which derive utility from consumption $C_t(i)$ and disutility from labor $L_t(i)$. The preference of households are given by the following utility function:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[\frac{1-k}{1-\sigma} \zeta_{t+k}^c \left(\frac{C_{t+k}(i) - kC_{t+k-1}}{1-k} \right)^{1-\sigma} - \frac{L_{t+k}(i)^{1+\tau}}{1+\tau} \right] \right\}, \quad (1)$$

where σ is the risk aversion parameter ($\sigma > 0$), τ is the inverse of the Frisch elasticity ($\tau > 0$), and ζ_t^c is a preference shock on consumption. The function features external habit formation

[7] In particular, our model features a common stochastic trend for real variables, a standard parametrization of the rule followed by the monetary authority, and the absence of capital utilization, disutility of labor, and risk premium shocks.

in consumption. External habits imply that utility increases with the gap between individual consumption and the lagged aggregate consumption. In each period, households maximize their expected utility subject to the following budget constraint:

$$P_tC_t(i) + P_tI_t(i) + B_t(i) \leq W_tL_t(i) + (\Pi_t^p + R_t^kK_{t-1}(i)) + R_{t-1}B_{t-1}(i) + T_t, \qquad (2)$$

where P_t is the price of the only good in the economy, I_t are investments, and B_t is a short-term bond which pays an interest equal to R_t after one period. The variable K_t is the stock of physical capital which is rented to intermediate firms at the nominal rental rate R_t^k. Finally, Π_t^p are profits that firms transfer to households and T_t are lump-sum taxes or transfers from the government. In each period, households optimally choose their amount of consumption, investment, the level of capital, and the amount of resources to save. They also set their wages subject to a specific labor demand schedule, committing to supply the amount of labor demanded by firms. The capital stock is owned by households and evolves according to the following rule:

$$K_{t+1}(i) = (1-\delta)K_t(i) + \Upsilon_t(1-\Gamma^I(i))I_t(i), \qquad (3)$$

where δ is the depreciation rate, Υ_t is a shock affecting the marginal efficiency of investment (see Justiniano et al. (2011)), and $\Gamma^I(i)$ are adjustment costs based on the gross rate of change in investment, that is,

$$\Gamma^I(i) = \frac{\psi}{2}\left(\frac{I_t(i)}{I_{t-1}(i)} - g_{z,t}\right)^2, \qquad (4)$$

where $\psi > 1$ and $g_{z,t}$ is the gross rate of labor augmenting productivity.

3.1.1 Wage setting

Households supply differentiated labor inputs to intermediate firms, which bundles them according to the following technology:

$$L_t(j) = \left[\int_0^1 L_t^i(j)^{\frac{\theta_w-1}{\theta_w}}\,di\right]^{\frac{\theta_w}{\theta_w-1}}, \qquad (5)$$

where θ_w is the elasticity of substitution between labor varieties. The total demand for labor variety i depends on the relative wage and on the total demand for labor (see Section 3.2.1),

$$L_t^i = \int_0^1 L_t^i(j)dj = \left(\frac{W_t(i)}{W_t}\right)^{-\theta_w} L_t. \tag{6}$$

Following Erceg et al. (2000), in each period only a fraction $(1-\xi_w)$ of households can optimally reset their nominal wages. Those who cannot adjust are allowed to update their wages according to the following indexation scheme:

$$W_t(i) = g_{z,t}\Pi_{t-1}^{\chi_w}\bar{\Pi}_t^{1-\chi_w}W_{t-1}(i). \tag{7}$$

The updated wages will depend on the geometric average of past gross inflation ($\Pi_{t-1} \equiv \dfrac{P_{t-1}}{P_{t-2}}$) and the central bank inflation objective $\bar{\Pi}_t$, whose relative importance depends on the indexation parameter χ_w with $0 \leq \chi_w \leq 1$. Wages are also indexed to gross labor productivity growth $g_{z,t}$. The nominal wage of each i household is

$$W_t(i) = \begin{cases} \widetilde{W}_t(i) & \text{if optimizes} \\ g_{z,t}\Pi_{t-1}^{\chi_w}\bar{\Pi}_t^{1-\chi_w}W_{t-1}(i) & \text{otherwise.} \end{cases} \tag{8}$$

Each household allowed to re-optimize chooses the optimal wage that maximizes its utility subject to the budget constraint, the total demand for its labor variety L_t^i, and the indexation scheme. The result is the following first order condition:

$$E_t\left[\sum_{k=0}^{\infty}(\beta\xi_w)^k\left(\Lambda_{t+k}(i)\prod_{s=1}^{k}\left(\frac{g_{z,t+s}\Pi_{t+s-1}^{\chi_w}\bar{\Pi}_{t+s}^{1-\chi_w}}{\Pi_{t+s-1}}\right)\frac{\widetilde{W}_t(i)}{P_t} - \Theta_{w,t}L_{t+k}(i)^{\tau}\right)L_{t+k}(i)\right] = 0, \tag{9}$$

where $\Theta_{w,t} = \frac{\theta_w}{\theta_w - 1}\mu_{w,t}$ is the time varying wage markup and $\mu_{w,t}$ evolves according to an exogenous stationary process. The optimal wage set by households will be increasing in expected future prices and costs, the latter being the subjective disutility of labor. The aggregate wage index evolves according to

$$W_t = \left[\xi_w \left(g_{z,t}\Pi_{t-1}{}^{\chi_w}\bar{\Pi}_t^{(1-\chi_w)}W_{t-1}\right)^{1-\theta_w} + (1-\xi_w)\widetilde{W}_t^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}}. \tag{10}$$

3.2 Firms

A continuum of monopolistically competitive intermediate goods producers, indexed by $j \in [0,1]$, sell their differentiated goods $Y_t(j)$ to a competitive final good firm, which bundles them according to the following technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{(\theta_p-1)}{\theta_p}} dj\right)^{\frac{\theta_p}{\theta_p-1}}, \tag{11}$$

where θ_p is the elasticity of substitution between goods. The final good firm profit maximization yields the demand for intermediate good j,

$$Y_t^j = \left(\frac{P_t(j)}{P_t}\right)^{-\theta_p} Y_t, \tag{12}$$

where the demand for a specific intermediate good Y_t^j depends on its relative price and on the total demand Y_t. The (aggregate) price for the final good is

$$P_t^{1-\theta_p} = \int_0^1 P_t(j)^{1-\theta_p} dj. \tag{13}$$

3.2.1 Intermediate goods firms

We assume a constant-elasticity-of-substitution production function for intermediate goods production, that is,

$$Y_t(j) = a_t \left[\gamma^{\frac{1}{\alpha}} K_t(j)^{\frac{\alpha-1}{\alpha}} + (1-\gamma)^{\frac{1}{\alpha}} (z_t L_t(j))^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}, \tag{14}$$

where γ is the bias towards capital and α the elasticity of substitution between factors of production. Following Christoffel et al. (2008), a_t is a transitory technology shock, while z_t is a permanent technology shock affecting labor productivity. We define the stationary growth rate of labor augmenting productivity as $g_{z,t} = \frac{z_t}{z_{t-1}}$. Capital is rented from households in competitive markets, while differentiated labor is hired in monopolistically competitive markets. Intermediate producers take nominal wage for each single labor input as given and choose the (optimal) amount of each labor type that minimizes total labor costs. Since firms are symmetric, they will all choose the same amount for each specific labor variety. Aggregating over the continuum of firms, the total demand for labor input i takes the following form:

$$L_t^i = \int_0^1 L_t^i(j)dj = \left(\frac{W_t(i)}{W_t}\right)^{-\theta_w} L_t. \tag{15}$$

3.2.2 Price setting

Following Calvo (1983), in each period only a fraction $(1 - \xi_p)$ of firms are allowed to optimally reset their prices, while all the others will follow an indexation scheme based on past inflation and the central bank inflation target $\bar{\Pi}_t$. Thus,

$$P_t(j) = \begin{cases} \widetilde{P}_t(j) & \text{if optimize} \\ \Pi_{t-1}^{\chi_p} \bar{\Pi}_t^{1-\chi_p} P_{t-1}(j) & \text{otherwise,} \end{cases} \tag{16}$$

where χ_p is the indexation parameter $(0 < \chi_p < 1)$.

Each firm j maximizes its expected nominal profits,

$$E_t \left[\sum_{k=0}^{\infty} (\xi_p)^k \Lambda_{t,t+k} \left(P_t(j) Y_t(j) - MC_{t+k} Y_{t+k}(j) \right) \right], \tag{17}$$

subject to the indexation scheme and the demand for type-j product Y_t^j. Nominal marginal costs, MC_t, are equal across firms, $\Lambda_{t,t+k}$ is the stochastic discount factor while ξ_p^k is the probability that the chosen price will still be valid after k periods. The maximization problem yields the following first order condition:

$$E_t \left\{ \sum_{k=0}^{\infty} (\xi_p)^k \Lambda_{t,t+k} \left[\prod_{s=1}^{k} \left(\Pi_{t+s-1}^{\chi_p} \bar{\Pi}_{t+s}^{1-\chi_p} \right) \widetilde{P}_t(j) - \Theta_{p,t} MC_{t+k} \right] Y_{t+k} \right\} = 0, \tag{18}$$

where $\Theta_{p,t} = \dfrac{\theta_p}{(\theta_p - 1)} \mu_{p,t}$ is the time varying price markup and $\mu_{p,t}$ is the price markup shock related to the substitutability between goods. The fraction $(1-\xi_p)$ of re-optimizing firms set prices in order to equate expected discounted revenues to expected discounted costs.

The aggregate price index is given by

$$P_t = \left[\xi_p \left(\Pi_{t-1}{}^{\chi_p} \bar{\Pi}_t^{1-\chi_p} P_{t-1} \right)^{1-\theta_p} + (1-\xi_p) \widetilde{P}_t{}^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}}. \tag{19}$$

3.3 Monetary and fiscal authorities

The government raises funds through the issue of new debt $(B_t - B_{t-1})$, on which it pays a nominal interest R_t, and consumes an exogenous portion of output G_t. The monetary authority sets interest rates according to a standard Taylor rule, where the annualized (gross) nominal interest depends on its lagged values, the annualized consumer price inflation $\Pi_{4,t}$, and the quarterly output growth rate:

$$\left(\frac{R_t}{\bar{R}_t} \right)^4 = \left(\frac{R_{t-1}}{\bar{R}_t} \right)^{4\rho_R} \left(\frac{\Pi_{4,t}}{\bar{\Pi}_t^4} \right)^{(1-\rho_R)\rho_\pi} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{(1-\rho_R)\rho_{GDP}} \epsilon_t^R, \tag{20}$$

where R_t is the gross monetary policy rate, $\bar{R}_t$ the steady-state gross nominal policy rate, and $\bar{\Pi}_t$ the time-varying long-run inflation target. The parameter ρ_R captures inertia in interest rate setting $(1 < \rho_R < 0)$, while ρ_π and ρ_{GDP} are the weights for inflation and output growth. The variable ϵ_t^R is an i.i.d shock on the gross nominal interest rate, while $\bar{\Pi}_t$ evolves according to an exogenous autoregressive process.

3.4 Exogenous processes

All the exogenous processes outlined in the text evolve according to the stationary first order autoregressive process

$$log\, X_t = \rho_X \, log\, X_{t-1} + (1-\rho_X)\, log\, \bar{X} + \epsilon_{X,t}, \quad \epsilon_{X,t} \sim N(0, \sigma_X^2), \tag{21}$$

where X_t is the generic exogenous process, $\bar{X}$ its steady state level, ρ_X its persistence parameter, and σ_X the standard deviation of its innovations. $\bar{X}$ is equal to the steady state level of government spending G for the process related to public expenditures, to the steady state level of inflation $\bar{\Pi}$ for the exogenous process of the monetary authority's target, and to the steady-state growth in labor-augmenting technology g_z for the process related to the permanent technology shock . For the remaining exogenous processes, $\bar{X} = 1$.

3.5 Market clearing

The market for physical capital is in equilibrium when the demand by intermediate firms matches households' supply. The labor market is in equilibrium when the aggregate labor supply L_t^s equals the sum of labor demand by variety:

$$L_t^s = \int_0^1 L_t^i \, di \,. \tag{22}$$

Plugging in the demand for each labor variety, we obtain

$$L_t^s = S_t^w L_t, \tag{23}$$

where $S_t^w = \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\theta_w} di$ is a measure of wage dispersion.

The final goods market is in equilibrium when the usual aggregate accounting identity applies, where aggregate demand equals (public and private) consumption and investment:

$$Y_t = C_t + I_t + G_t Y_t. \tag{24}$$

Total demand for intermediate goods has to be equal to total production, that is,

$$\int_0^1 a_t \left[\gamma^{\frac{1}{\alpha}} K_t(j)^{\frac{\alpha-1}{\alpha}} + (1-\gamma)^{\frac{1}{\alpha}} (z_t L_t(j))^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} dj = Y_t \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta_p} dj. \tag{25}$$

Since firms are symmetric and markets clear, we have

$$a_t \left[\gamma^{\frac{1}{\alpha}} K_t^{\frac{\alpha-1}{\alpha}} + (1-\gamma)^{\frac{1}{\alpha}} (z_t L_t)^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} = Y_t S_t^p, \tag{26}$$

where $S_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta_p} dj$ is a measure of price dispersion. Finally, equilibrium in the bond market is achieved when the total supply of bonds is matched by the total demand at the interest rate set by the monetary authority.

3.6 Potential output

The main objective of this paper is to study the properties of the output gap generated by the structural model and to compare it against more standard measures. Following Justiniano et al. (2013), we define the potential output as the equilibrium level of output produced by an economy with flexible prices and wages and constant elasticity of substitution between goods and labor varieties (that is, without markup shocks). Accordingly, we define the output gap as the difference between actual and potential output. We do not consider the efficient output, that is, the equilibrium level of output in a perfectly competitive economy with flexible prices, flexible wages, and perfectly substitutable goods and labor types. The reason behind this choice is the assumption that in the short-run the central bank can in principle undo the fluctuations stemming from nominal rigidities but not the ones arising from the structural imperfections of the economy; it follows that the attainable and relevant target for the central bank is the potential output as previously defined.[8]

With imperfect substitutability of intermediate goods and labor varieties, fully flexible wages ($\xi_w = 0$), and no markup shocks, the real wage is equal to a constant markup $\mu_w = \frac{\theta_w}{\theta_w - 1}$ over the competitive wage (the marginal rate of substitution between consumption and leisure). Equation (9) becomes

[8] See Smets and Wouters (2003), Galí et al. (2011), Justiniano et al. (2013), and Furlanetto et al. (2017) on this point.

$$\frac{\widetilde{W}_t}{P_t} = \mu_w \frac{L_t(i)^\tau}{\Lambda_t(i)^{-1}}. \tag{27}$$

Similarly, with flexible prices ($\xi_p = 0$) and no markup shocks, firms set their prices equal to a constant markup $\mu_p = \frac{\theta_p}{\theta_p - 1}$ over marginal costs. Equation (18) becomes

$$\widetilde{P}_t = \mu_p MC_t. \tag{28}$$

Under this setup, actual and potential output steady-states coincide and are both affected by the distortion induced by the imperfect substitutability of goods and labor varieties.

4 Estimation

Following the seminal paper of An and Schorfheide (2007), we estimate the model parameters using Bayesian methods. The methodology is an extension of the maximum likelihood approach, where prior distributions are combined with the likelihood function. The posterior mode is obtained by numerical methods while the parameter (posterior) distributions are estimated with the Metropolis-Hastings algorithm, a sampling-like method. In what follows we describe the data, the calibrated parameters, the choice of the prior distributions, and the results of the Bayesian estimation.

4.1 Data

We estimate the model on seven quarterly variables for the euro-area. All variables are taken from the Area Wide Model (AWM) database (see Fagan et al. (2001)). We cover a period of 32 years, from 1985q1 to 2016q4. We use quarterly growth rates of real GDP, real consumption, real investment, the consumption deflator, and the nominal wage per head. Total employment is measured in log-deviation from its linear trend, where the latter is estimated from 1985q1 to 2007q4, that is, the beginning of the recent crisis period.[9] Finally, we use the annualized 3-month

[9] See Subsection 4.2 on the calibration for the reasons behind this choice. An additional robustness exercise that we report in the appendix assumes the trend in labor to be best described by the observations from 1985q1 to 2016q4, with little impact on parameter estimates.

Euribor for the nominal interest rate. As in Smets and Wouters (2003), we use the following auxiliary equation to link the variable on hours worked L_t to the one on employment E_t:

$$E_t = E_{t+1}^{\frac{\beta}{1+beta}} E_{t-1}^{\frac{1}{1+\beta}} L_t^{\frac{(1-\beta\xi_E)(1-\xi_E)}{(1+\beta)\xi_E}} E_t^{-\frac{(1-\beta\xi_E)(1-\xi_E)}{(1+\beta)\xi_E}},$$

where $(1-\xi_E)$ is the fraction of firms allowed to adjust their level of employment to match their optimal total labor input. Finally, the following measurement equations link the seven time series to their relative variables in the model:

$$\begin{cases} Y_t^{obs} = \left(\frac{Y_t}{Y_{t-1}} g_{z,t} - 1\right) + \epsilon_Y^{me} \\ C_t^{obs} = \left(\frac{C_t}{C_{t-1}} g_{z,t} - 1\right) + \epsilon_C^{me} \\ I_t^{obs} = \left(\frac{I_t}{I_{t-1}} g_{z,t} - 1\right) + \epsilon_I^{me} \\ \Pi_t^{obs} = (\Pi_t - 1) + \epsilon_\Pi^{me} \\ R_t^{obs} = R_t^4 - 1 \\ E_t^{obs} = log\left(\frac{E_t}{\bar{E}}\right) \\ W_t^{obs} = \frac{W_t}{W_{t-1}} - 1, \end{cases} \tag{29}$$

where Y_t^{obs}, C_t^{obs}, I_t^{obs}, Π_t^{obs}, R_t^{obs}, E_t^{obs}, W_t^{obs} are the time series previously described, and the variable $\bar{E}$ is the steady-state level of employment. We also include normally distributed measurement errors on output, consumption, investment, and the consumption deflator, with mean zero and constant variance.

4.2 Calibration

We calibrate a subset of parameters to match the model's steady-state values to the long-run means before the financial crisis. This choice allows us to remain agnostic about the nature of the developments after the global financial crisis. If we use the means up to 2016q4 we implicitly assume that the nature of the shock that hit the economy from 2009 onward was mostly permanent. We prefer to take a position on what the balanced growth path of the economy is and to let the data speak for the last period.[10] Consistently with the observed short-term interest rate, the discount factor β is set to 0.996. We proxy the balanced-growth path of the economy with the long-run growth rate of real GDP, so we set the steady-state growth in labor-augmenting technology $\bar{g}_z$ to 0.063. The quarterly monetary authority's inflation target is set to 0.05. In line with the investment-to-GDP ratio, we set the bias towards capital γ to 0.314. Following the existing literature, we calibrate the capital depreciation δ to 2.5%, the elasticity of inter-temporal substitution $\frac{1}{\sigma}$ to 1, and the inverse Frisch elasticity τ to 0.5.[11] We calibrate the elasticity of substitution between labor varieties θ_w to 4.3 and the one between goods varieties θ_p to 6, corresponding to a steady-state wage markup of 1.3 and a steady-state price markup of 1.2, respectively. The last calibrated parameter is the elasticity of substitution between factors of production α, which we set to 0.98. The calibrated parameters are reported in Table 1 and the implied steady-states in Table 2. We calibrate the standard deviations of the measurement errors related to the consumption deflator to 0.3 while the remaining standard deviations are calibrated to 0.1.

4.3 Prior distributions

We follow the literature and estimate the subset of parameters that govern the model's dynamics (see, e.g., Christoffel et al. (2008) and Forni et al. (2012)). We choose prior distributions as in SW and Coenen et al. (2009). The standard deviations of the shocks to the exogenous processes and the investment adjustment parameter ψ have an inverse gamma (G) distribution. Parameters bounded between 0 and 1 have a beta (B) distribution, namely, the habit formation k, the Calvo parameters ξ_p and ξ_w, the indexation parameters χ_p and χ_w, the so-called Calvo-style parameter ξ_E, the auto-regressive coefficients, and the interest rate smoothing ρ_R. The two unbounded parameters, that is, the response of the monetary authority to inflation ρ_π and GDP growth rate ρ_{GDP}, have a normal distribution. Prior means are set to 0.001 for the standard deviations of the shocks and to 0.75 for all the auto-regressive coefficients, with the exception of the processes related to price and wage markup shocks which are equal to 0.65. We set the mean of the interest rate smoothing to 0.9, and those of the interest reaction to inflation and output growth rate to

[10] In the Appendix we report an alternative estimation of the output gap that takes into account the sample 1985q1:2016q4 rather than the benchmark pre-crisis sample. The estimates do not differ substantially.

[11] See, e.g., Christoffel et al. (2008), Gomes et al. (2012), and Forni et al. (2012).

1.7 and 0.0625, respectively. Habit formation is set to 0.7, investment adjustment to 5, and the Calvo-style parameter to 0.5. The prior means of the Calvo and the indexation parameters are set to 0.75 and 0.5, respectively. Standard deviations are set to 0.05 for the nominal rigidities and to 0.15 for the parameters governing the indexation schemes. The standard deviation of habit formation is equal to 0.1 and the one of the Calvo-Style parameter to 0.15, while the prior for the investment adjustment parameter has a standard deviation of 1.5. Standard deviations are set homogeneously across exogenous shocks and auto-regressive coefficients. Finally, the parameters of the Taylor rule have standard deviations of 0.1 for the response to inflation and 0.05 for the parameters governing the interest rate smoothing and the reaction to output growth rates. The details on prior distributions are reported in Table 3.

4.4 Estimation results

Table 3 reports the results of the estimation exercise. The mode is obtained through the numerical maximization of the (log) posterior distribution.[12] The posterior distributions of the estimated parameters are obtained through 500000 draws of the Metropolis-Hastings algorithm after discarding the first 100000 iterations. In what follows we comment on the mean values. First, we find a value of 0.897 for the wage stickiness parameter and a slightly higher value for the price stickiness 0.910. The two results imply that wages and prices are adjusted on average every 10 and 11 quarters. The indexation parameter is lower for wages 0.328 than for prices 0.412. The mean for the habit formation parameter distribution is 0.753, while the one for the investment adjustment parameter is equal to 6.278. The mean of the parameters governing the Taylor rule is equal to 1.891 for the reaction to inflation, 0.089 for the reaction to the output growth rate, and 0.624 for the interest rate smoothing. The exogenous processes have a persistence in a range that goes from 0.660 of the permanent technology shock to the 0.903 of the government spending shock. Overall, the estimation seems to deliver informative results, with the exception of the wage markup process and the standard deviations related to the monetary policy shocks. SW obtain similar results: the parameter related to the wage markup is not identified and the posterior distributions for the standard deviations related to the Taylor rule are largely determined by the assumed priors. The weak

[12]We use the csminwel routine developed by Chris Sims.

identification issue related to the parameters governing the Phillips curve and the monetary policy function is a well known result in the literature.[13] More generally, Canova and Sala (2009) and Iskrev (2010) analyze the results of several estimated DSGE models (including the one of SW) and conclude that identification issues often undermine the reliability of these estimations. As outlined by Adolfson and Lindé (2011), however, the weak identification of a subset of parameters does not necessarily affect the overall validity of the Bayesian estimation. The smoothed structural shocks, the smoothed measurement errors, the impulse response functions, and shock decompositions of the observables are reported in the appendix.

5 Output gap estimation

Since potential output and output gap are unobserved variables, we use the Kalman filter to extract them from the observables included in the estimation. We adopt the posterior mean of the parameters to extract point estimates. Moreover, we explore the uncertainty surrounding these point estimates exploiting the full posterior distribution of the parameters and the filter uncertainty.

We first focus on the point estimate of potential output to analyze the timing of expansions and recessions implied by our measure. Figure 1 shows on a log scale the actual output and the model-implied potential output. For comparison, we report in the same figure also the Hodrick-Prescott (HP) trend and the peak-to-trough euro-area recessions.[14] Potential output expands at a rather constant pace throughout the sample, with only a slight acceleration in correspondence of the start of the convergence process in the euro-area. There are three major recessions in our sample: the currency crisis in the first half of the 1990s, the global financial crisis in 2008-2009, and the sovereign debt crisis in 2011-2013. Recessions bring actual GDP at or below the level of potential output, and lead to an extended period of negative gaps between actual GDP and potential output. The HP trend provides a similar dating of recessions and expansions until mid-2010.[15] However, our estimation highlights a persistently negative gap between actual GDP and

[13]See, on this point, Schorfheide (2011).

[14]The chronology of recessions and expansions is based on the CEPR euro-area Business Cycle Dating Committee's findings. See, e.g., CEPR (2009).

[15]Vetlov et al. (2011) reach similar conclusions for the period 1999q1:2010q4. We also find that potential output

its potential level after 2010, while the HP trend identifies a positive gap right before the sovereign debt crisis in 2011. Moreover, the two measures diverge substantially after 2011. While the HP trend slows down compared to its pre-crisis performance and even crosses actual GDP in 2016, our model-based potential output keeps expanding in the last years, driven by positive technology shocks that stimulate the productive potential of the economy while exerting downward pressure on prices.[16]

In Figure 2 we study the output gap and the uncertainty surrounding its measurement. The point estimate highlights a positive gap at the beginning of the sample which is driven into negative territory by the currency crisis. The potential output accelerates in this period due to positive permanent technology shocks. These originate from the interpretation that the model provides of the simultaneous occurrence of lower nominal wage growth, progressively lower inflation, and future lower interest rates. Throughout the 1990s actual GDP and potential output grow at the same pace, so the gap remains at the level reached with the currency crisis. The acceleration in actual GDP in the late 1990s narrows the gap to negligible values, where it remains up until 2005. The pre-crisis expansion drives the gap well above 5% at the peak. The global financial crisis causes a drop of almost 9 percentage points of output gap, below -3% at the end of 2009. The short-lived recovery in 2010 is not sufficient to bring back the GDP to its potential, and the onset of the sovereign debt crisis in late 2010 adds an additional downward shift of 6 percentage points, bringing the output gap to a trough close to -7% by 2013. In the last part of the sample the GDP grows at a faster pace than its potential. The output gap gradually decreases, reaching around -3% in 2016q4.

In order to explore the uncertainty surrounding the point estimates of the output gap, we repeatedly draw from the multivariate posterior distribution of parameters and we run a Kalman filter over the estimation sample for each draw, storing the mean estimate from the filter for each draw.[17] This process yields a distribution of the output gap that takes into account also the uncertainty coming from the model's parameters ('parameter uncertainty'). A credibility set of 99 percentage points

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and HP trend converge during the financial crisis while they are quantitatively different in the rest of the sample.

[16]Other permanent shocks that we do not include in our analysis, such as those related to demographic trends in the euro-area, are arguably more likely to produce their effects in the years to come.

[17]We make 1200 draws from the multivariate distribution described by the chains of the Metropolis-Hastings algorithm used to explore the posterior distribution.

suggests that uncertainty characterizes especially the output gap's peaks and troughs. Nevertheless, the timing of recessions and recoveries would not be affected even if we considered such extremes of the distribution. Only in the case of the short-lived recovery of 2010-2011 the 90th percentile of the output gap's distribution assigns a slightly positive gap to those quarters.

6 Output gap revisions

Estimates of the output gap provided by standard methodologies, such as unobserved component models (e.g., the HP filter) and production function models, face two key issues. First, when new data come in, previous estimates tend to be significantly revised, particularly at the end of the sample (Orphanides and Van Norden (2002)). Second, they tend to underestimate the difference between actual and potential output during phases of severe expansion or contraction of the economy (Berger et al. (2015)).[18] In this section we compare our model-based output gap estimates with the ones provided by the IMF and the OECD and those obtained with a standard HP filter.[19] In the first subsection we focus on a case study, comparing the output gap estimates produced with data up to the onset of the financial crisis with those obtained using data until the end of 2016. In the second we move to a comparison of the real-time estimates with those based on the full-sample. In the final subsection we look at how the reliability of the output gap estimates evolves over time.

6.1 Case study: pre-crisis and full-sample estimates

Exploiting the case study of the financial crisis, in Figure 3 we quantify how our DSGE-based measure of the output gap compares with others. Before the start of the crisis, official estimates did not point to large positive gaps, while later estimates did revise the assessment, bringing the output gap to a pre-crisis peak of at least 3%. Thus, we look at two estimation vintages, in 2007q4 ('pre-crisis') and in 2016q4 ('full-sample'), and check whether the assessment of the output gap in the pre-crisis peak of 2007 significantly changes between vintages. Together with our measure,

[18] See also Turner et al. (2016) on this topic for approaches based on the production function framework.

[19] Output gap estimates for the two institutions are from different historical vintages of the OECD Economic Outlook and the IMF World Economic Outlook. See IMF (2017), ?, and previous releases.

we consider the output gap obtained with a standard HP filter and the official estimates provided by the OECD and the IMF.[20] Official estimates are available at a yearly frequency from 1991, accordingly, we take annual averages of the DSGE-based and the HP filter-based output gaps starting from the same year.

Figure 3.a reports the HP filter-based output gap. The pre-crisis output gap estimated for 2007 is equal to 0.7%, while the full-sample estimate for the same quarter is 2.4%, more than 3 times larger than the first.[21] Figure 3.b and Figure 3.c show that the pre-crisis estimations of OECD and IMF did not capture the overheating of the economy at the end of 2007. Indeed, there is a striking difference between the pre-crisis and full-sample estimations. The pre-crisis one suggests that output was below potential before the financial crisis with a gap close to zero at the end of 2007. Conversely, the full-sample estimation reports a largely positive output gap for the same year. Against this backdrop, we report our DSGE-based measure in Figure 3.d. We use data from 1985q1 to 2007q4 for the pre-crisis estimation and data from 1985q1 to 2016q4 for the full-sample estimation.[22] The pre-crisis estimation of the output gap in 2007 is 3.3%, while the full-sample estimate is 4.3%.

While our estimation is not immune to revisions, the difference between the pre-crisis and the full-sample results is smaller compared to other estimates. Most importantly, our model estimated at the end of 2007 delivers a positive and high output gap whereas the other estimates suggest a gap close to zero. The dynamics of our full-sample output gap are qualitatively similar to the ones of the two institutions. However, there are two substantial differences. First, our full-sample estimation for 2007 is higher compared to that of the two official estimates. Second, our estimation diverges from the others starting in 2011, indicating a gap of -6.6% in 2013 against -3.6% for the IMF and -2.8% for the OECD. Accordingly, our latest estimation for 2016 is quite large (-3.4%) compared to the results (-1.5% on average) of the two institutions. It is worth noting that the output gap we would have obtained in 2007 is in line with the ones published after 2016 by the

[20] We run the HP filter on quarterly real GDP in logs with a smoothing parameter set to 1600 over two subsamples: 1985q1:2007q4 and 1985q1:2016q4. Official estimates come from the IMF World Economic Outlook 2008/1 and the OECD Economic Outlook 2008/1 for the pre-crisis vintages and from the IMF World Economic Outlook 2017/1 and the OECD Economic Outlook 2017/1 for the full-sample vintages. The historical vintages for the European Commission are available since 2011 and are not reported.

[21] Orphanides and Van Norden (2002) find qualitatively similar results for univariate and more complex multivariate UC models over the period 1965-1997.

[22] We reestimate the model's parameters over the pre-crisis subsample.

IMF and the OECD. Our results for the years following the financial crisis are close to the ones presented by Jarociński and Lenza (2018). The two authors develop and compare several measures of the euro-area output gap finding that the one most consistent with the persistently low level of inflation recently observed estimates an average gap close to -6% in 2013-2014.[23]

In Figure 4 we generalize the insights from the case study of the financial crisis and present the output gap estimates coming from the recursive estimation of the model each year (in the fourth quarter of each year, from 1985q1:1998q4 to 1985q1:2016q4 for a total of 19 vintages). We report the quarterly profiles of each series. We assess our measure's performance against two forms of instability. First, the larger the gap, the higher the instability of its estimate over time, although the revisions are minor as a percentage of the original estimate. Second, there is almost no case in which the gap changes sign across vintages (the only exception is in 2005, from negative to positive).

6.2 Real-time and full-sample estimates

In order to give an overall reliability measure of the output gap we compare real-time estimates with the ones based on the full sample. The real-time series are the most timely estimates available, namely those obtained with information up to year t in real-time.[24] Figure 5 plots the two series for the OECD, the IMF, the DSGE, and the HP filter over the period 2007-2016.[25] The results that

[23]In their exercise, Jarociński and Lenza (2018) find that the best performing model includes the largest set of real economy variables (GDP, private investment, imports, exports, unemployment, consumer confidence, and capacity utilization) coupled with measures of long-term inflation expectations and a random walk representation for the trends in the real variables. It is thus a specification that features low variation in potential output growth over time.

[24]While official estimates are provided in actual real-time, our estimate and the HP filter-based estimate are in pseudo-real-time. For instance, we use the vintage of historical data available today for the subsample up to 2007 rather than the vintage of data available at the end of 2007 to compute the output gap that would have been computed at that time. Thus, we do not take into account revisions of historical data. Orphanides and Van Norden (2002) find, however, that such revisions play a minor role in ex-post revisions of the output gap as opposed to the role played by additional data points. Marcellino and Musso (2011) find that the role of data revisions is even lower for the euro-area.

[25]The OECD and IMF real-time series are constructed using the latest output gap estimates for each data vintage published from 2008 to 2017. The 2007 real-time output gap comes from the vintage published in the first semester of 2008, the 2008 real-time output gap is the one published in the first semester of 2009, and so on. The full-sample series, as before, is the one published in the first semester of 2017. To obtain the HP output gap in real-time we (i) detrend the log real output series over a four-quarter-expanding-window from 1985q1:2007q4 to 1985q1:2016q4 (smoothing parameter: 1600); (ii) take the average of the last four quarters for each iteration to obtain the annual real-time estimate. As an illustration, the annual real-time HP output gap for 2007 is the average of the last four quarterly estimates obtained applying the filter on the sample 1985q1:2007q4. The full-sample series is obtained applying the filter on the full-sample 1985q1:2016q4 and taking annual averages from 2007q1 to 2016q4, our period of interest. The DSGE real-time series is obtained as follows: (i) we estimate the model recursively over a four-

emerge are similar to the ones reported in the previous subsection: there are sizable revisions across the different estimates, but the difference between the real-time and the full-sample estimates of the DSGE model is smaller than the others'. To formalize the latter finding, we follow Orphanides and Van Norden (2002) and present two tables. Table 4 reports the descriptive statistics for the time series of full-sample and real-time output gaps, together with the revisions of the output gaps, that is, the difference between the two estimates. The statistics are computed for the annual output gap series of the OECD, the IMF, the DSGE, and the HP filter over the period 2007-2016. All the estimations but the DSGE-based have revisions of the same magnitude as the real-time output gap. In Table 5 we report several reliability measures based again on the comparison of real-time and full-sample output gap estimates. The first (a form of Noise-to-Signal ratio, NS) is the ratio between the standard deviation of the revisions and the standard deviation of the full-sample estimates. The second (NSR) is the Root Mean Square (RMS) of the revisions over the standard deviation of the full-sample output gap. The last (COR) is the correlation between the real-time and the full-sample estimates. The DSGE has the highest correlations and the lowest noise-to-signal ratios. In the last columns we report the percentage of times with which the real-time and full-sample estimates have different signs. As for the IMF and the OECD, there are no periods where the DSGE has estimates of different sign.[26]

6.3 Output gap reliability over time

In this subsection, we shed some light on how the output gap reliability evolves over time looking at a longer sample of 19 years from 1998 to 2016. When available, we also report the results based on the estimates provided by the European Commission (EC).[27] The IMF, the OECD, and the EC publish their estimates biannually.

For each institution, there are two output gap publications each year. Thus, we consider two output

quarter-expanding-window from 1985q1:2007q4 to 1985q1:2016q4; (ii) for each iteration we apply the Kalman filter to obtain a quarterly measure of the output gap and take averages of the last four quarters to obtain the latest real-time estimate. For instance, the annual real-time DSGE output gap for 2007 is obtained estimating the model and applying the Kalman filter on the sample 1985q1:2007q4, after taking averages of the last four quarters; the DSGE output gap for 2008 comes from the estimation of the model over the period 1985q1:2008q4, after taking the average of the last four quarters. For the full-sample series we estimate the model over the period 1985q1:2016q4, apply the Kalman filter on the same sample and take annual averages from 2007q1 to 2016q4.

[26] As a robustness check, we report in the appendix the same results over different time periods, adding the results based on the estimates provided by the European Commission in the AMECO database when available.

[27] See ECFIN (2017) and previous releases.

gap estimates per year also for the HP filter and the DSGE model. Let us call $G^I_{T,T+1/2+H/2}$ the output gap of year T published in semester $T+1/2+H/2$, where $T = 1998,\dots,2012$, $H = 1,\dots,8$, and $I \in \{OECD, IMF, EC, HP, DSGE\}$. For example, $G^{OECD}_{1998,1999}$ indicates the OECD estimate of the output gap of year 1998 published in the first semester of 1999, $G^{OECD}_{1998,1999.5}$ indicates the OECD estimate of the output gap of year 1998 published in the second semester of 1999, and so on. Any estimate where $H = 1$ is what we call 'real-time estimate,' as it is the first published estimate of the output gap in a given year. All estimates where $H > 1$ for the same T are revisions of the original, real-time estimate.

In order to align as much as possible the HP- and DSGE-based estimates with those coming from the various institutions, we consider the estimate for 1998 with data up to 1998Q4 as the 'real-time' estimate of the 1998 output gap ($T + 1/2 + H/2 = 1998 + 1/2 + 1/2 = 1999$). The first revision of the 1998 estimate is realized with data up to 1999Q2 ($T+1/2+H/2 = 1998+1/2+2/2 = 1999.5$), the second revision with data up to 1999Q4 ($T + 1/2 + H/2 = 1998 + 1/2 + 3/2 = 2000$), and so on. In the case of the DSGE model, we reestimate the model's parameters in each year (so with data up to T), and use those estimates to apply the Kalman filter and provide the output gap $G^I_{T,T+1/2+H/2}$ for $H = 1$ and $H = 2$. For $H = 3$, we use the parameter estimates provided by the estimation with data up to year $T + 1$, and so on.

Similarly to the measures reported in Table 5 based on Orphanides and Van Norden (2002), we consider as a measure of reliability the (realized) standard deviation of the first eight estimates of the output gap for a given year divided by the (realized) standard deviation of the full-sample estimates for all years.[28] Thus, the reliability R^I_T of the output gap estimate of year T provided by the institution/model I is

$$\text{Reliability}^I_T = \frac{\sqrt{\frac{1}{7}\sum_{H=1}^{8}\left(G^I_{T,T+1/2+H/2} - \frac{1}{8}\sum_{H=1}^{8} G^I_{T,T+1/2+H/2}\right)^2}}{\sqrt{\frac{1}{18}\sum_{A=1998}^{2016}\left(G^I_{A,\text{full-sample}} - \frac{1}{19}\sum_{A=1998}^{2016} G^I_{A,\text{full-sample}}\right)^2}},$$

where the full-sample vintage $G^I_{A,\text{full-sample}}$ for $A = 1998,\dots,2016$ is defined as the output gap es-

[28] Using the ex-post revisions rather than the actual estimates does not change the results, as noted also in Orphanides and Van Norden (2002).

timates for all years either published in the first semester of 2017 (for the international institutions, that is, $I \in \{OECD, IMF, EC\}$) or realized with data up to 2016Q4 (for the HP filter and the DSGE model, that is, $I \in \{HP, DSGE\}$). We consider values of H up to 8 to guarantee sufficient observations for a given standard deviation and, at the same time, cover a sufficient number of year. With H up to 8, we can consider only the years up to 2012, as $2012 + 0.5 + 8/2 = 2016.5$ is the last available vintage of estimates in our database (the second and last estimate published in 2016, with data up to 2016Q4). According to this definition, the higher is the value of the measure, the higher is the variance between estimates provided in real-time and those provided in the following four years (thus, the lower is its "reliability" in real-time).

Figure 6 shows the results of this exercise (higher values correspond to lower reliability).[29] Three results emerge from the graph: (1) overall, the output gap reliability decreases during periods of strong economic expansion and recession, confirming the point highlighted by Berger et al. (2015); (2) the output gap estimates produced with the DSGE model have an higher reliability over the whole sample compared to the ones published by the OECD, the IMF, and the EC; the comparison with the HP filter is less clear cut, considering that the DSGE provides less reliable results in three occasions; (3) the DSGE-based output gaps outperform the other estimates before and during the financial crisis (except for the 2009 estimates of OECD and IMF).[30]

7 Conclusions

In this paper we provide a measure of the output gap derived from a standard medium-scale DSGE model of the euro-area. We estimate the model with Bayesian methods using observables over the period 1985-2017. We adopt a standard DSGE-based measure of the output gap that exploits the presence of nominal rigidities in the model, and analyze its features both full-sample and in (pseudo) real-time. We compare our estimates with those obtained using other standard methodologies or provided by international institutions. We find that the DSGE-based output gap identifies the same episodes of expansion and recession of the official dating of the CEPR.

[29] Results starts in 2005 for the IMF and in 2010 for the EC due to data availability.

[30] Similar considerations hold if we compare our estimates with those produced by the European Central Bank within its regular Macroeconomic Projection Exercise (MPE), which we do not report because of confidentiality reasons.

Moreover, the DSGE-based estimates tend to be less affected by ex-post data revisions and hence relatively more stable across vintages.

We adopt a standard DSGE model because it strikes a good balance between rigor and flexibility. Nonetheless, further modeling extensions are paramount for a DSGE-based estimation of the output gap. For example, several contributions point at the importance of financial factors to quantify correctly the economic slack of an economy, like Borio et al. (2016).[31] Another relevant aspect is the open-economy dimension, as pointed out by, e.g., Darvas and Simon (2015). Moreover, additional permanent shocks other than the labor-augmenting one would enrich the stochastic structure of the balanced-growth path, allowing additional long-run developments in output to impact the potential output.[32] Lastly, mechanisms of endogenous growth might translate cyclical developments into longer-run dynamics without the need of additional structural shocks, allowing to study the impact of phenomena such as the secular stagnation on the measurement of the output gap.[33] We plan to explore the implications of these refinements for the estimate of the output gap in future research.

[31] Our current set-up most probably captures the effects of financial shocks via investment-specific shocks. A relatively parsimonious modelling of financial frictions like Bernanke et al. (1999) might help to identify these financial shocks.

[32] Neri and Gerali (2018) find little improvement with the inclusion of shocks, e.g., to the supply of labor (which may capture demographic characteristics).

[33] See, e.g., Cova et al. (2017).

Tables and figures

Table 1: Calibrated parameters

Parameter	***Description***	**Value**
$\frac{1}{\sigma}$	Intertemporal elasticity of substitution	1
τ	Inverse of Frisch elasticity of labor supply	2
α	Substitution between factors of production	0.98
δ	Capital depreciation	0.025
β	Discount factor	0.996
γ	Bias towards capital	0.314
θ_w	Elasticity of substitution labor varieties	4.3
θ_p	Elasticity of substitution goods varieties	6

Table 2: Steady-state relationships

Parameter	***Description***	**Value**
$\overline{\Pi}$	Inflation rate (quarterly)	0.5%
$\overline{R}$	Nominal interest rate (quarterly)	1.5%
g_z	Growth rate (quarterly)	0.63%
C/Y	Consumption-to-output ratio	0.58
G/Y	Public expenditure-to-output ratio	0.20
I/Y	Investment-to-output ratio	0.22

Table 3: Prior and posterior moments of the parameters

		Prior			Posterior			
		Type	**mean**	**s.d.**	**Mode**	**Mean**	**5%**	**95%**
	Preferences							
k	Habit formation	B	0.7	0.1	0.749	0.753	0.703	0.804
	Employment							
ξ_E	Calvo-style parameter	B	0.5	0.15	0.876	0.877	0.860	0.895
	Adjustment costs							
ψ	Investment	G	5	1.5	5.015	6.278	3.988	8.493
	Monetary policy							
ρ_R	Interest rate smoothing	B	0.9	0.05	0.595	0.624	0.552	0.695
ρ_π	Resp. to inflation	N	1.7	0.1	1.889	1.891	1.745	2.033
ρ_{GDP}	Resp. to output growth	N	0.0625	0.05	0.085	0.089	0.014	0.156
	Wage and price setting							
ξ_p	Calvo:prices	B	0.75	0.05	0.917	0.910	0.875	0.945
ξ_w	Calvo:wages	B	0.75	0.05	0.900	0.897	0.883	0.911
χ_p	indexation:prices	B	0.5	0.15	0.306	0.412	0.235	0.589
χ_w	indexation:wages	B	0.5	0.15	0.416	0.328	0.138	0.508
	Shock: autoregr. coeff.							
ρ_a	Transitory tech. shock	B	0.75	0.05	0.832	0.828	0.789	0.868
ρ_{g_z}	Permanent tech. shock	B	0.75	0.05	0.670	0.660	0.593	0.729
ρ_ζ	Preferences	B	0.75	0.05	0.830	0.834	0.775	0.893
ρ_Υ	Investment	B	0.75	0.05	0.750	0.730	0.654	0.808
ρ_{μ_w}	Wage markup	B	0.65	0.05	0.652	0.651	0.570	0.734
ρ_{μ_p}	Price markup	B	0.65	0.05	0.746	0.742	0.657	0.828
ρ_G	Public expenditure	B	0.75	0.05	0.910	0.903	0.865	0.942
ρ_π	Inflation target	B	0.75	0.05	0.780	0.778	0.723	0.835
	Shock:standard deviations							
$100 * \sigma_a$	Transitory tech. shock	G	0.1	Inf	2.30	2.38	1.87	2.88
$100 * \sigma_a$	Permanent tech. shock	G	0.1	Inf	0.40	0.40	0.36	0.44
$100 * \sigma_\zeta$	Preferences	G	0.1	Inf	1.74	1.85	1.46	2.24
$100 * \sigma_\Upsilon$	Investment	G	0.1	Inf	2.19	3.02	1.56	4.47
$100 * \sigma_{\theta w}$	Wage markup	G	0.1	Inf	0.05	0.08	0.02	0.16
$100 * \sigma_{\theta_p}$	Price markup	G	0.1	Inf	1.04	1.12	0.59	1.61
$100 * \sigma_G$	Public expenditure	G	0.1	Inf	1.25	1.27	1.11	1.42
$100 * \sigma_\pi$	Inflation target	G	0.1	Inf	0.11	0.11	0.08	0.14
$100 * \sigma_R$	Monetary policy	G	0.1	Inf	0.40	0.10	0.03	0.17

Notes: this table reports the results of the Bayesian estimation. Prior distributions are the following: B=beta, G=inverse gamma, N=normal. Values referring to the standard deviations of the structural shocks are multiplied by 100 in the table.

Table 4: Summary statistics 2007-2016

	Mean	**SD**	**MIN**	**MAX**	**RMS**
OECD					
Real-time	-2.45	1.71	-5.14	0.42	
Full-sample	-1.51	2.36	-3.59	3.32	
Revisions	1.04	1.24	-0.37	3.28	1.56
IMF					
Real-time	-1.77	1.33	-3.42	0.72	
Full-sample	-0.93	2.09	-2.82	3.14	
Revisions	0.84	1.02	-0.21	3.05	1.34
DSGE					
Real-time	-2.81	2.93	-6.25	3.26	
Full-sample	-2.25	3.44	-6.52	4.32	
Revisions	0.62	0.78	-0.27	1.71	0.96
HP					
Real-time	-0.12	1.11	-2.94	0.83	
Full-sample	0.13	1.49	-2.57	2.42	
Revisions	0.29	1.19	-0.98	2.64	1.16

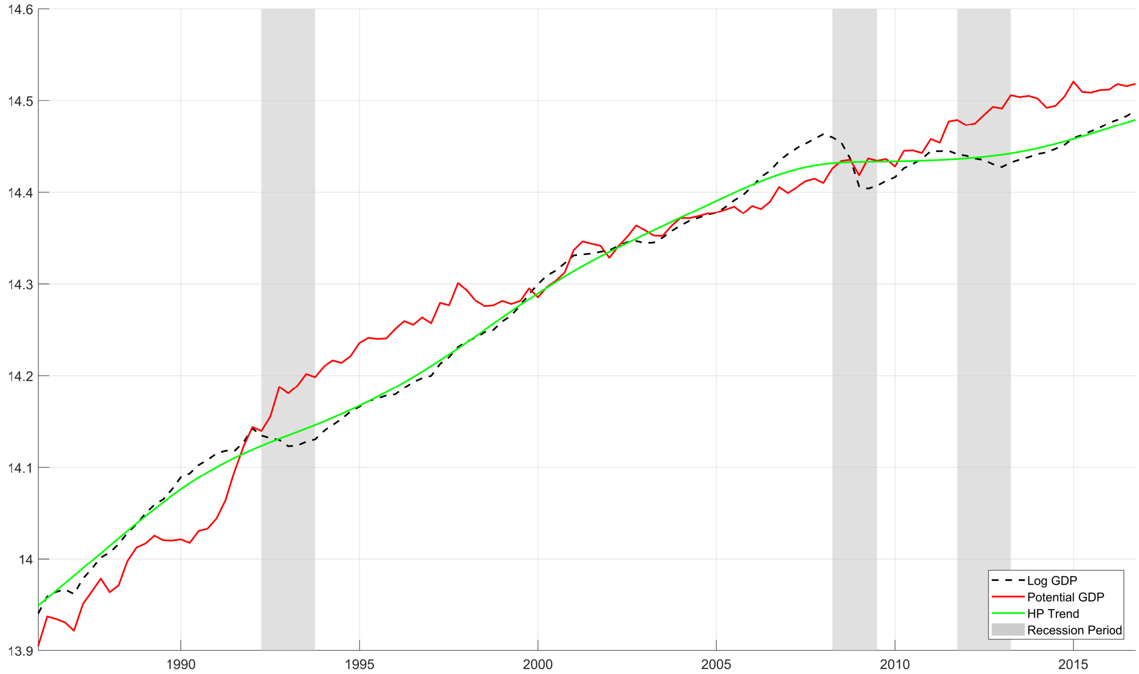
Notes: this table reports descriptive statistics of the real-time and the full-sample output gap estimates over the period 2007-2016. See Section 6 for the details. MIN is the minimum value, MAX is the maximum value, SD is the standard deviation, and RMS is the root mean square based on the difference between the real-time and the full-sample series.

Table 5: Reliability measures 2007-2016

	NS	**NSR**	**COR**	**OPSIGN**
OECD	0.53	0.66	0.87	0%
IMF	0.49	0.64	0.92	0%
DSGE	0.23	0.28	0.98	0%
HP	0.80	0.78	0.66	22%

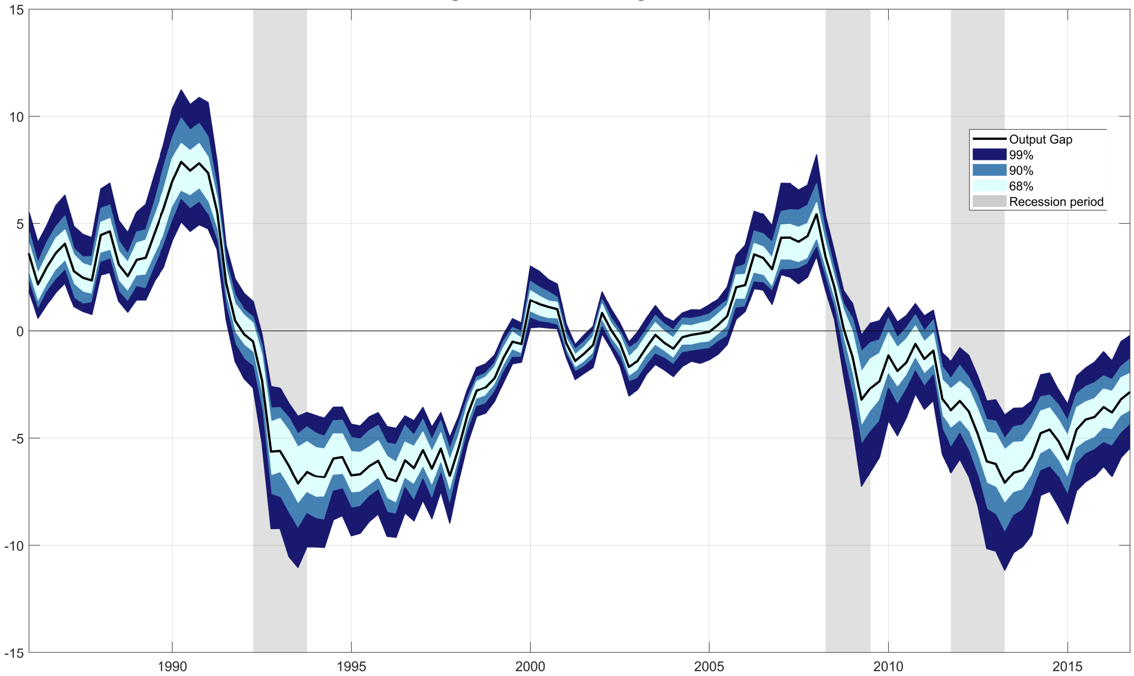
Notes: this table reports alternative measures for the reliability of the output gap estimates. COR is the correlation between the real-time and the full-sample estimate, NS is the ratio between the revision and the full-sample estimate, NSR is the ratio between the RMS of the revision and the standard deviation of the full-sample estimate. OPSIGN is the fraction of times the real-time and full-sample estimates have different sign. Standard deviations and RMSs are the ones reported in Table 4. For the details on real-time and full-sample estimates see Section 6.

Figure 1: Potential, HP, and actual output



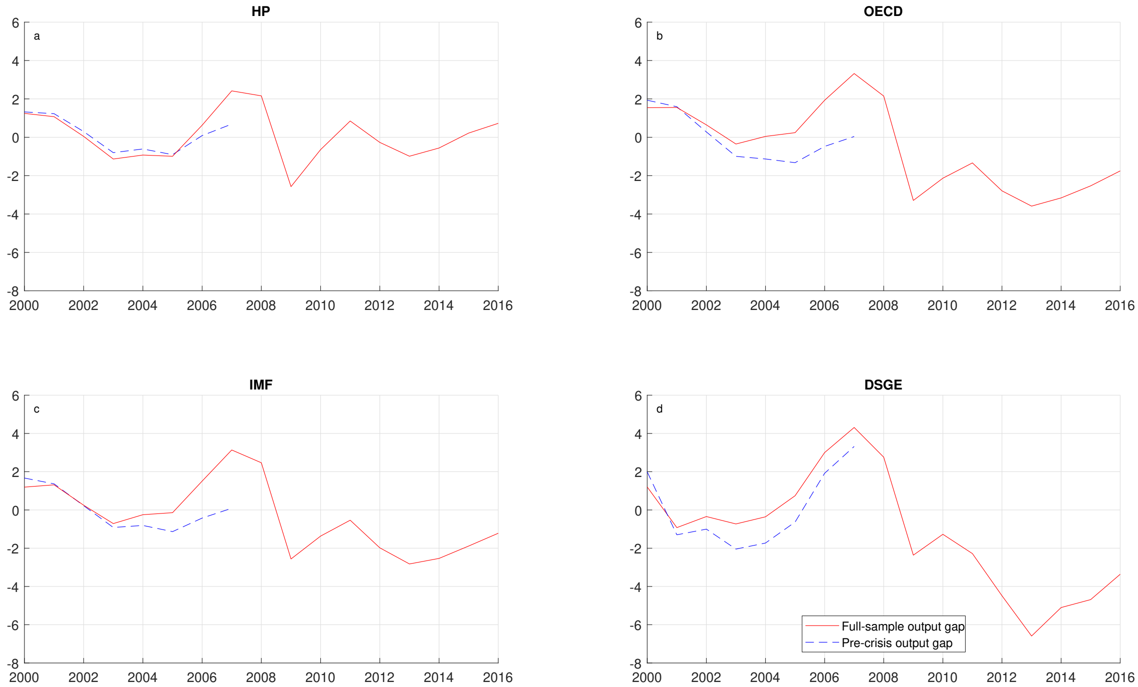
Notes: this figure shows the log GDP, the model-consistent potential output and the HP trend. The latter is obtained applying the HP filter on the log real GDP with a smoothing parameter equal to 1600. Shaded horizontal bars are euro-area recessions as defined by the CEPR euro-area business cycle dating committee. Source: authors' calculations.

Figure 2: Output gap



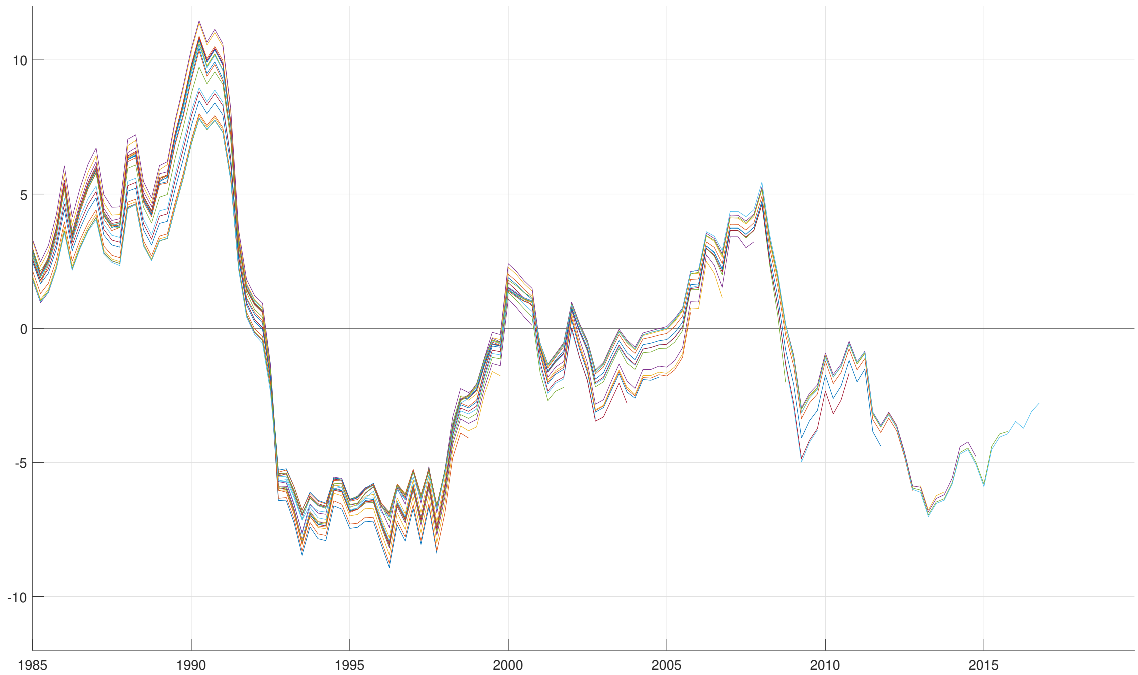
Notes: this figure shows the mean of the posterior distribution of the smoothed estimates of the output gap, together with its 99%,90%, and 68% uncertainty interval. Shaded horizontal bars are euro-area recessions as defined by the CEPR euro-area business cycle dating committee.

Figure 3: Pre-crisis and full-sample output gap estimations



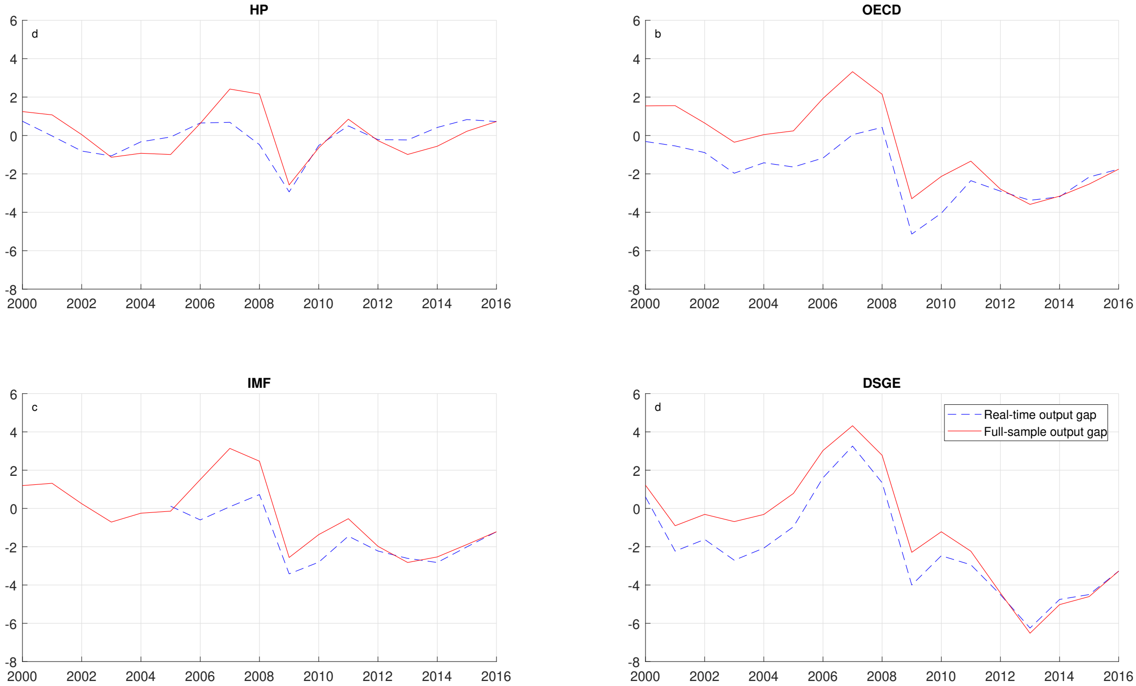
Notes: this figure shows alternative annual estimates for the output gap, defined as the percentage deviation of actual output from its potential. Pre-crisis estimates are based on data up to 2007, full-sample estimates are based on data up to 2016. In panel (a) the output gap is based on the potential output obtained applying the HP filter on the log real GDP (smoothing parameter=1600). Reported results are annual averages. Panel (b) and (c) show the output gap published by the OECD and the IMF. Panel (d) reports the model-consistent output gap as defined in Section 3.6. Reported results are annual averages. See section 6 for the details. Sources: OECD Economic Outlook; IMF World Economic Outlook; authors' calculations.

Figure 4: Output gap: recursive estimation



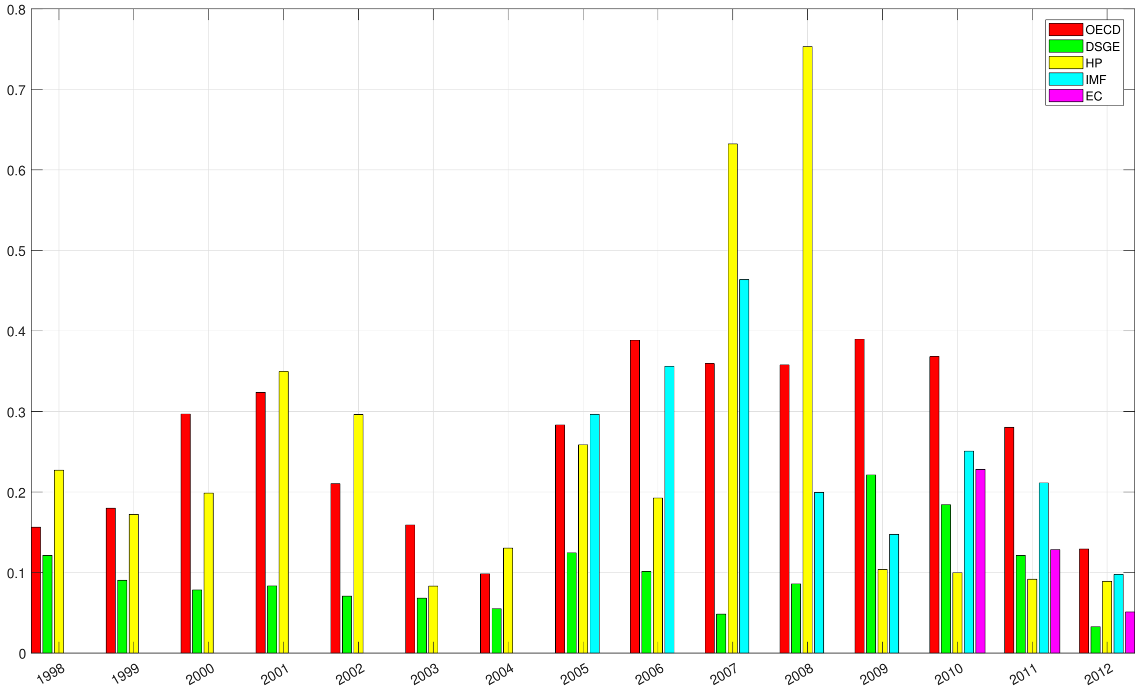
Notes: this figure shows the mean of the posterior distribution of the smoothed estimates of the output gap. These are obtained estimating the model recursively, starting from 1985q1:1998q4 and increasing the window by four quarters for each estimation up to 1985q1:2016q4.

Figure 5: Real-time and full-sample estimates



Notes: this figure shows alternative annual estimates for the output gap, defined as the percentage deviation of actual output from its potential. Real-time estimates are based on data up to year t (in real-time), full-sample estimates are based on data up to 2016. In panel (a) the output gap is based on the potential output obtained applying the HP filter on the log real GDP (smoothing parameter=1600). Reported results are annual averages. Panel (b) and (c) show the output gap published by the OECD and the IMF. Panel (d) reports the model-consistent output gap as defined in Section 3.6. Reported results are annual averages. Reported results are annual averages. See section 6 for the details. Sources: OECD Economic Outlook; IMF World Economic Outlook; authors' calculations.

Figure 6: Output gap reliability over time



Notes: this figure shows a measure of the reliability of the output gap over time for the estimates published by the OECD, the IMF, and the European Commission against the ones based on a HP filter and the DSGE model. Higher values correspond to lower reliability. Results starts in 2005 for the IMF and in 2010 for the EC. Higher values correspond to lower reliability. See Section 6 for the details.

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Appendix

A The Model

In this section we present the model's equations that were not discussed in the text, namely, the first order conditions for households and intermediate firms.

A.1 Households

The first order conditions associated with the households maximization problem are the following:

$$\zeta_t^C \left(\frac{1}{(1-k)g_z}\right)^{-\sigma} (C_t(i) - kC_{t-1}(i))^{-\sigma} = \Lambda_t, \tag{30}$$

$$\Lambda_t = \beta R_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}, \tag{31}$$

$$\begin{aligned} \Lambda_t =& \Upsilon_t \Lambda_t q_t - \Upsilon_t \Lambda_t q_t \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - g_{z,t}\right)^2 - \Upsilon_t \Lambda_t q_t \psi \left(\frac{I_t}{I_{t-1}} - g_{z,t}\right)\frac{I_t}{I_{t-1}} \\ &+ \Upsilon_{t+1}\beta\Lambda_{t+1}q_{t+1}\psi\left(\frac{I_{t+1}}{I_t} - g_{z,t+1}\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \end{aligned} \tag{32}$$

$$\Lambda_t q_t = \beta\Lambda_{t+1}\left[r_{k,t+1}\right] + \beta\Lambda_{t+1}q_{t+1}(1-\delta). \tag{33}$$

Where $\Pi_t = \dfrac{P_t}{P_{t-1}}$ is the inflation rate and $r_{k,t} = \dfrac{R_t^k}{P_t}$ the real interest rate on capital.

A.2 Intermediate Firms

In each period, intermediate producers choose the combination of capital and labor that minimizes their total costs subject to the production function and the demand for their specific good (12). The first order conditions for capital and labor associated with the minimization problem are

$$z_t L_t(j) = (1-\gamma)\left(\frac{W_t}{MC_t(j)a_t}\right)^{-\alpha}\frac{Y_t(j)}{a_t} \tag{34}$$

and

$$K(j)_t = \frac{MC_t(j)^{\alpha}{a_t}^{(\alpha-1)}Y_t(j)\gamma}{\left(R_t^k\right)^{\alpha}}, \tag{35}$$

where $MC(j)$ are intermediate firm's nominal marginal costs. Dividing the last two equations we note that the capital labor ratio is equal across intermediate producers:

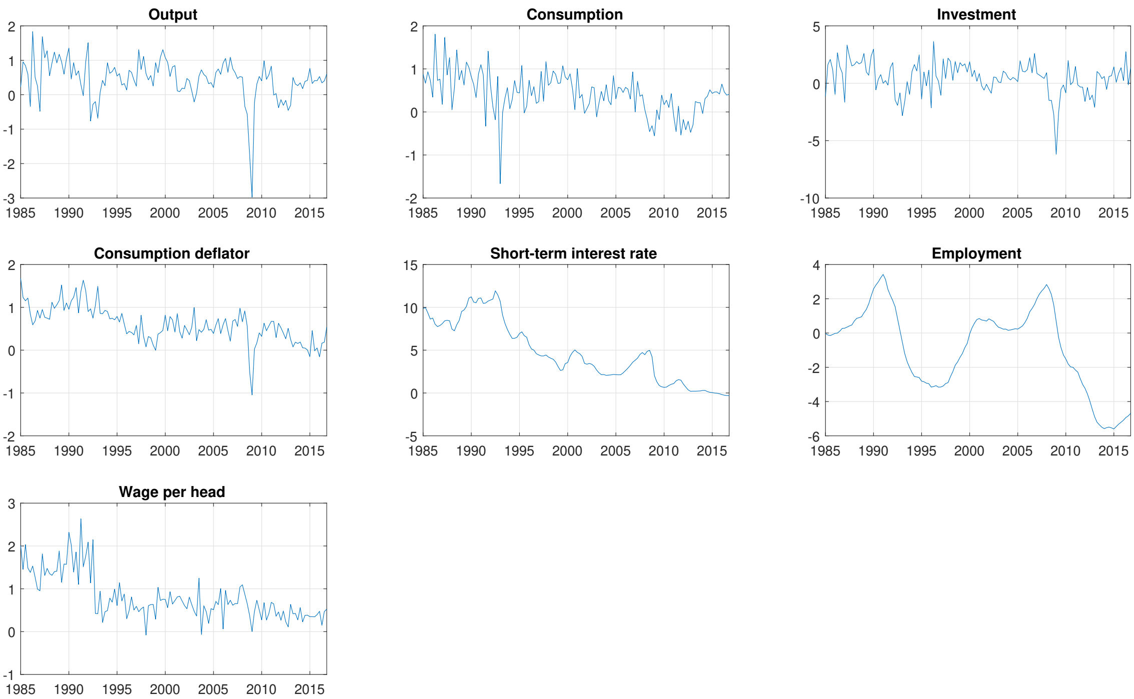
$$\frac{L_t(j)}{K_t(j)} = \frac{(1-\gamma)}{\gamma z_t}\left(\frac{W_t}{R_t^k}\right)^{-\alpha} = \frac{L_t}{K_t}. \tag{36}$$

All firms use the same technology, have same costs and hire capital and labor in the same ratio, implying equal marginal costs $MC_t(j)$ among them:

$$MC_t(j) = \frac{W_t}{{a_t}^{\frac{\alpha-1}{\alpha}}Y_t(j)^{\frac{1}{\alpha}}(1-\gamma)^{\frac{1}{\alpha}}\left(z_t L_t(j)\right)^{-\frac{1}{\alpha}}} = MC_t. \tag{37}$$

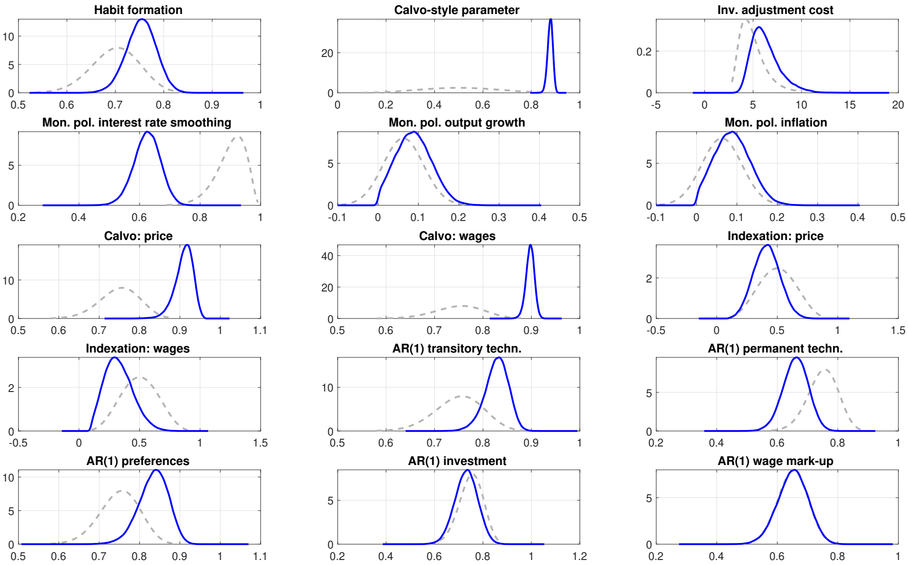
B Bayesian Estimation

Figure B.1: The data



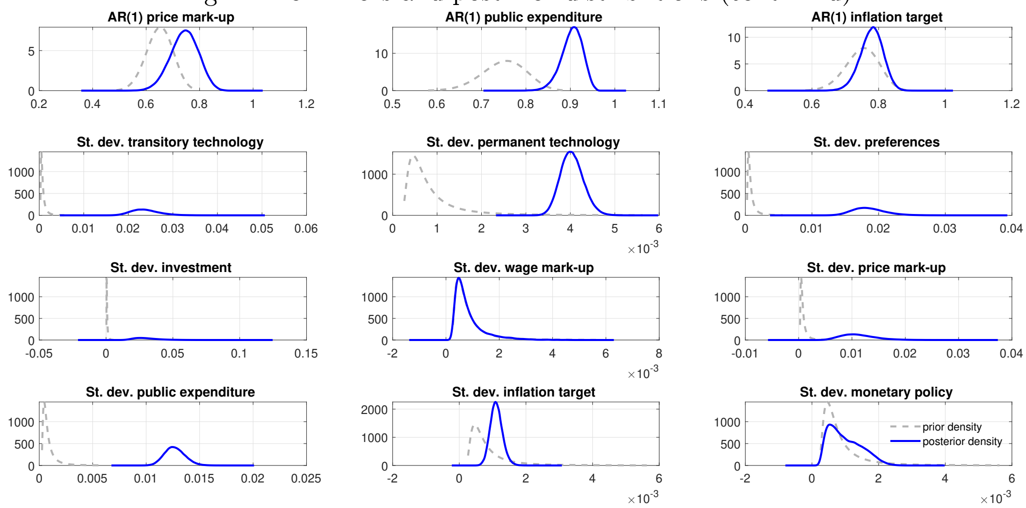
Notes: this figure shows the time series of the seven observable variables used for the estimation. Details on the series can be found in Section 4.1. Source: Area Wide Model.

Figure B.2: Priors and posterior distributions



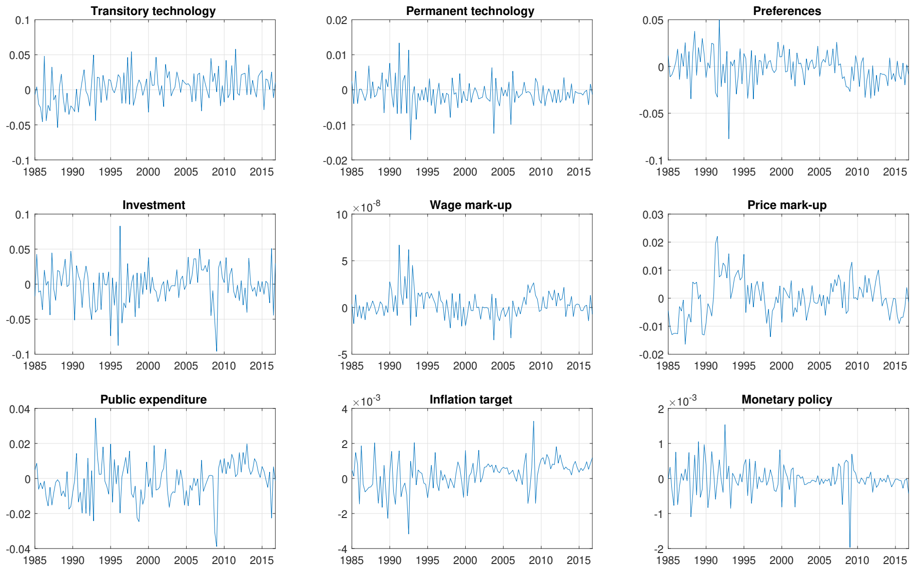
Notes: this figure shows the parameters' prior distributions and the estimated parameters' posterior distributions, obtained through the Metropolis-Hastings algorithm.

Figure B.3: Priors and posterior distributions (continued)



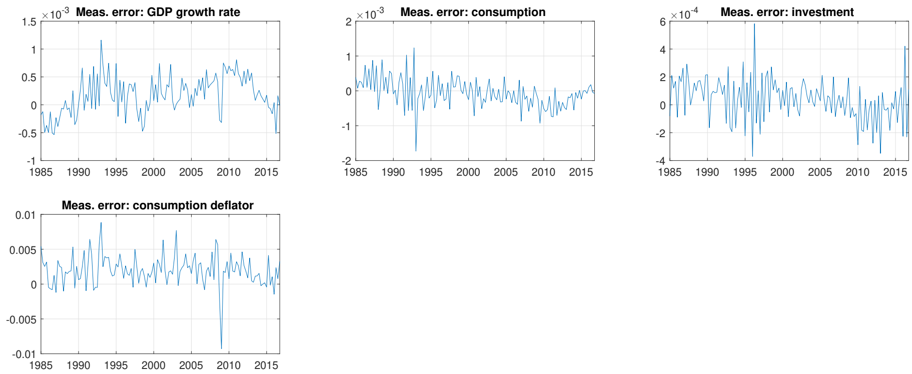
Notes: this figure shows the parameters' prior distributions and the estimated parameters' posterior distributions, obtained through the Metropolis-Hastings algorithm.

Figure B.4: Smoothed structural shocks



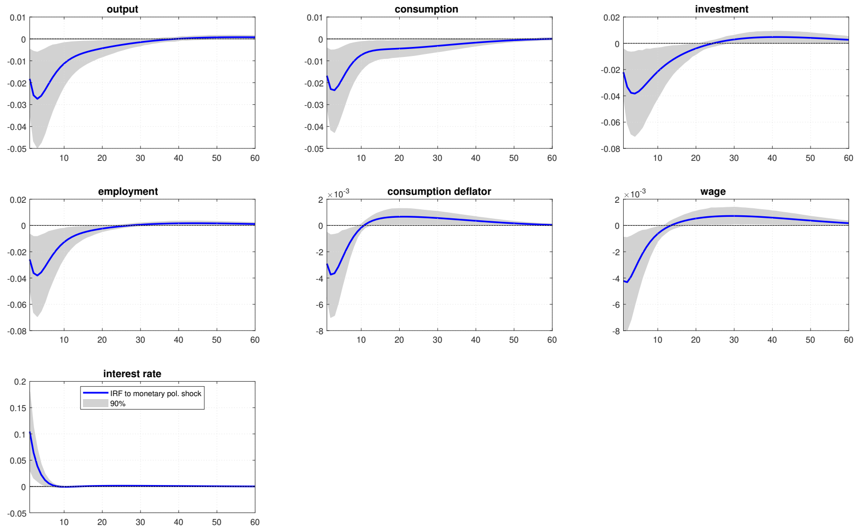
Notes: this figure shows the mean estimate of the smoothed structural shocks.

Figure B.5: Smoothed measurement error shocks



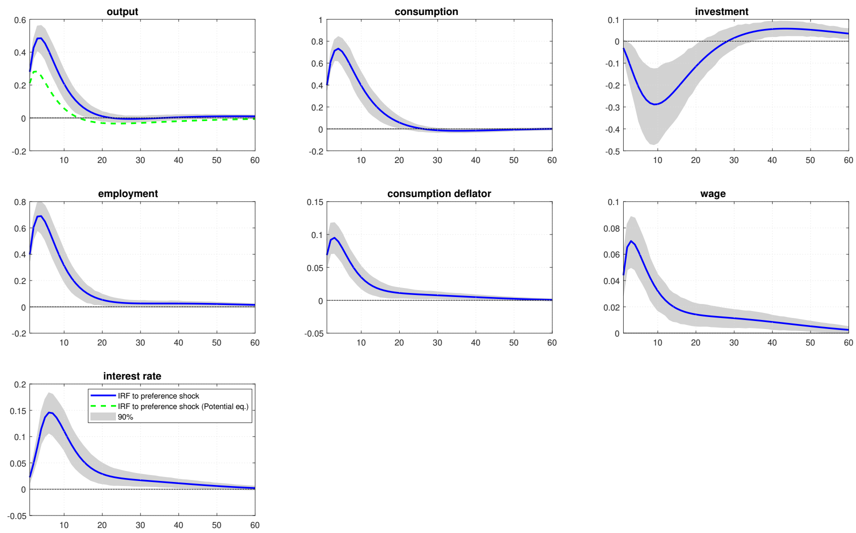
Notes: this figure shows the mean estimate of the smoothed measurement error shocks.

Figure B.6: Impulse Response Functions to a monetary policy shock



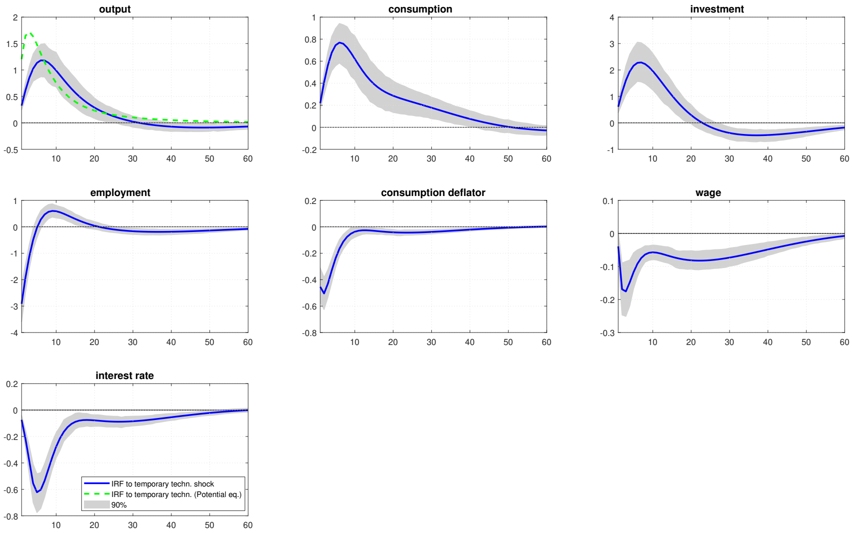
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.7: Impulse Response Functions to a preference shock



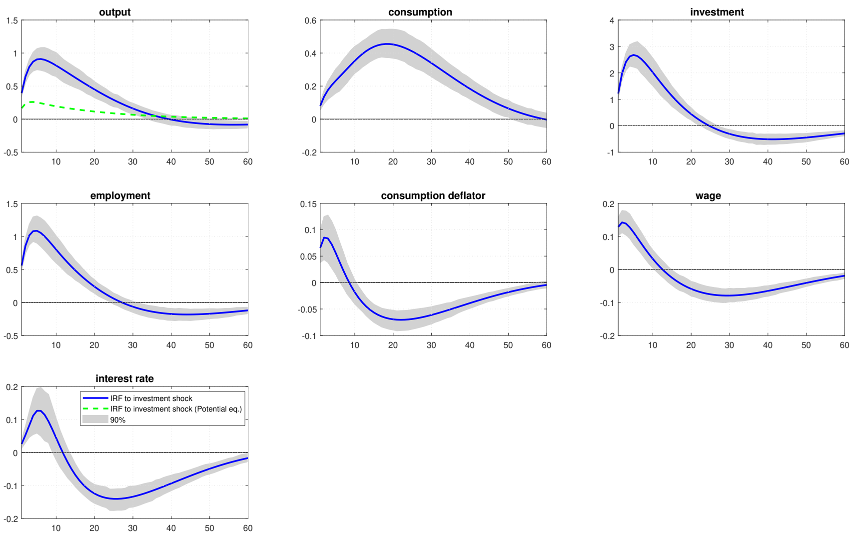
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The dashed green line refers to the potential output. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.8: Impulse Response Functions to a temporary technological shock



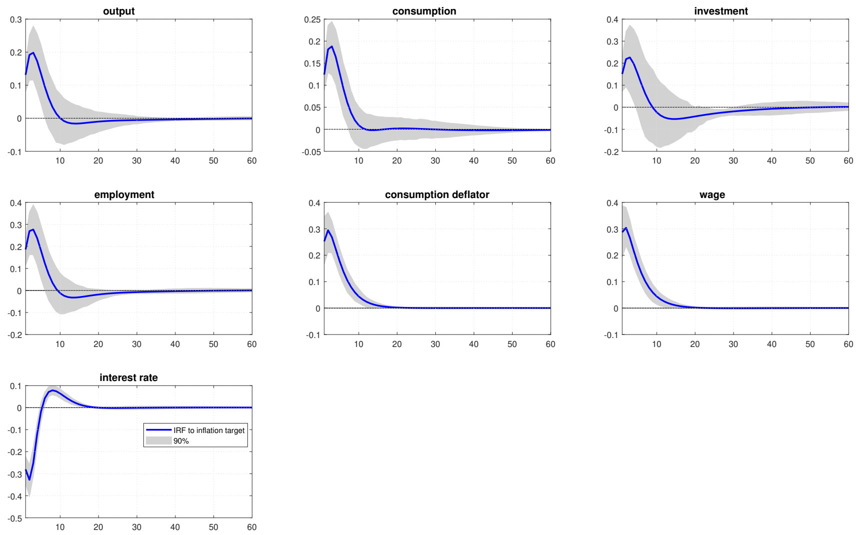
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The dashed green line refers to the potential output. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.9: Impulse Response Functions to an investment shock



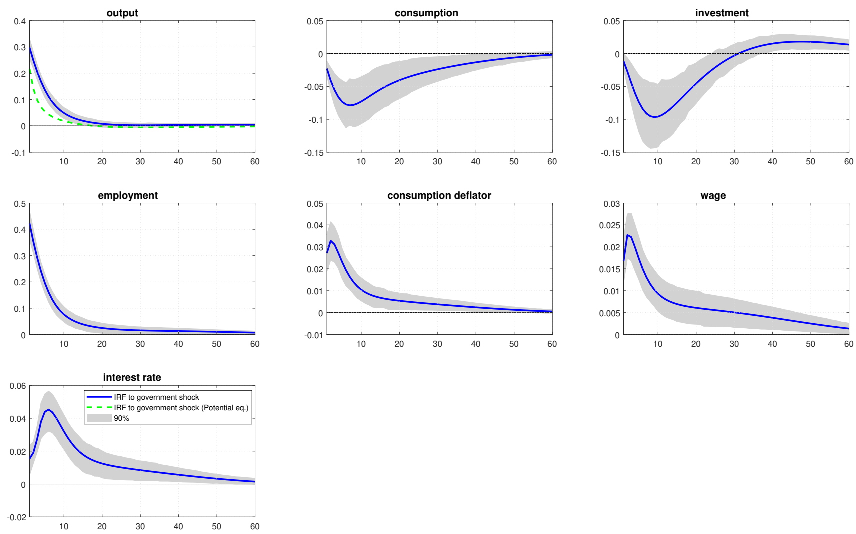
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The dashed green line refers to the potential output. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.10: Impulse Response Functions to an inflation target shock



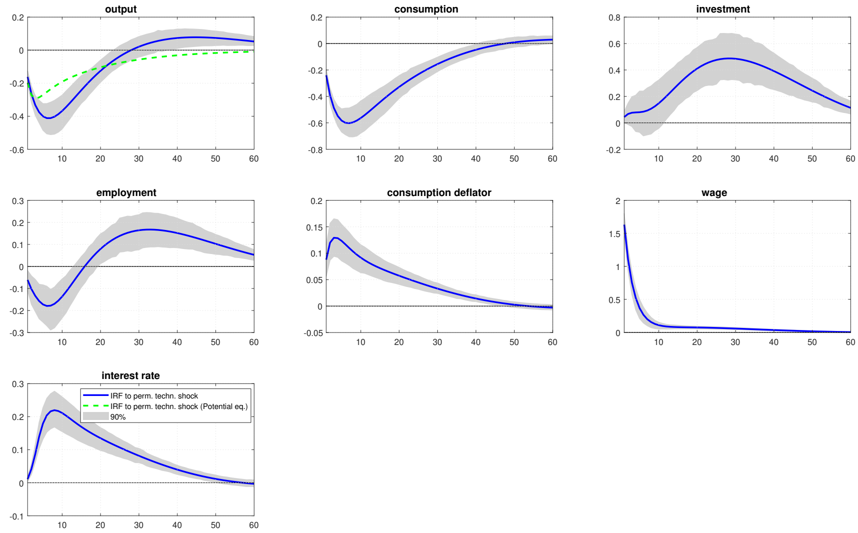
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The dashed green line refers to the potential output. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.11: Impulse Response Functions to a government spending shock



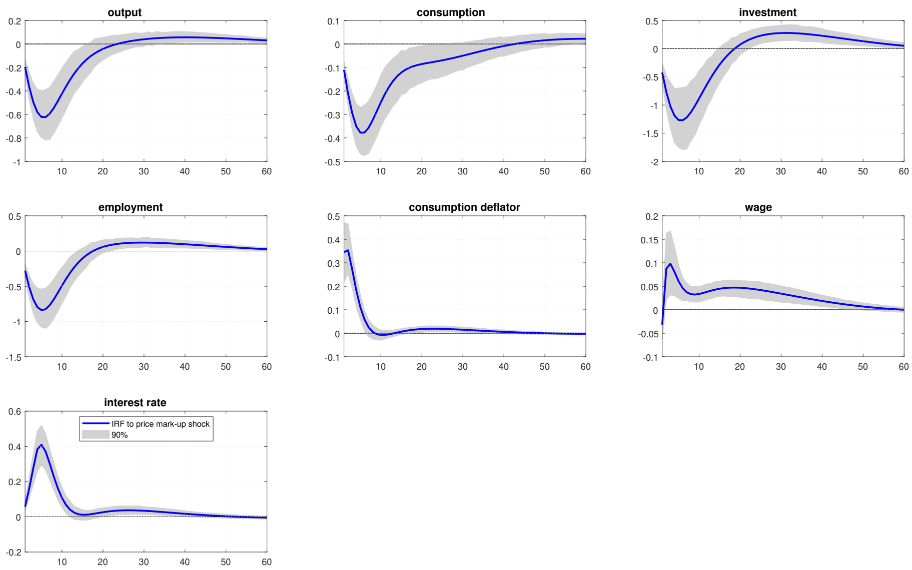
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The dashed green line refers to the potential output. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.12: Impulse Response Functions to a permanent technological shock



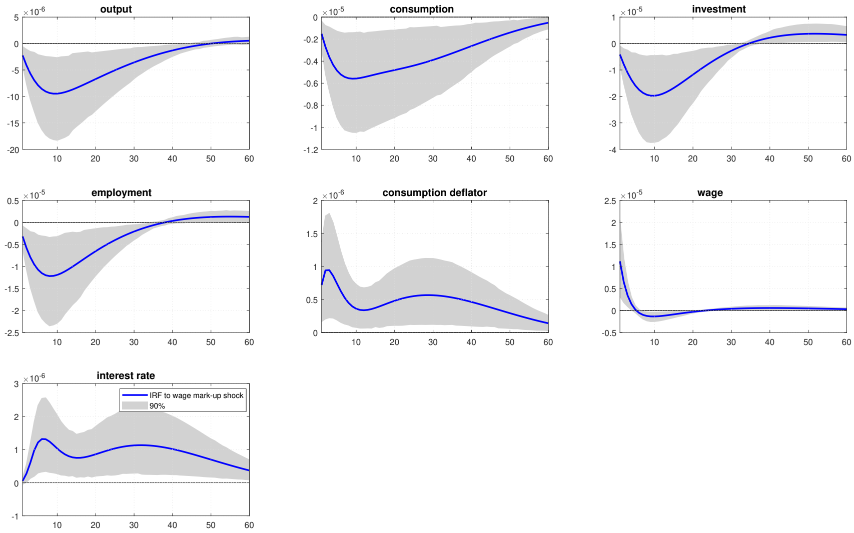
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The dashed green line refers to the potential output. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.13: Impulse Response Functions to a price markup shock



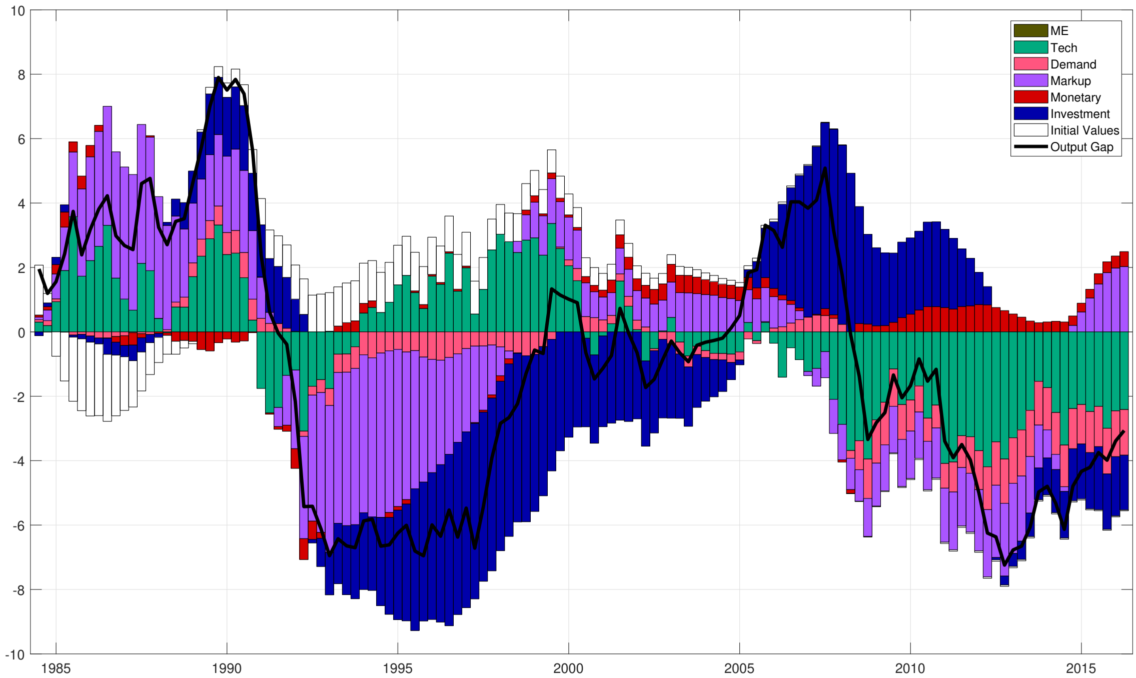
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.14: Impulse Response Functions to a wage markup shock



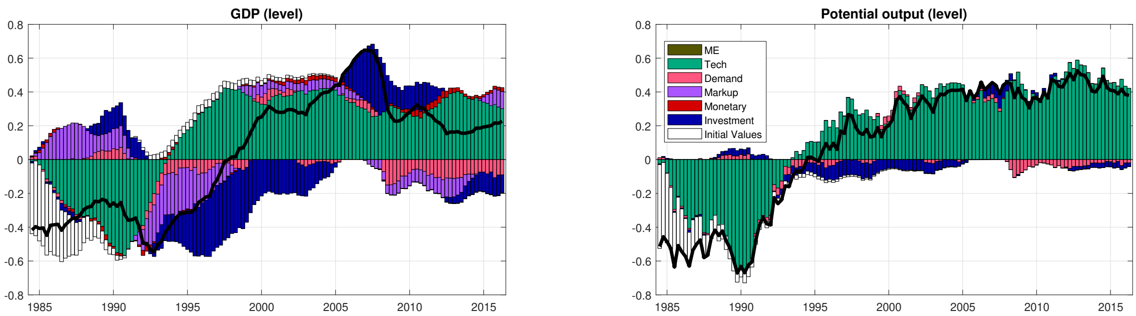
Notes: this figure shows the posterior mean impulse responses of the observable variables. Impulse responses are in percentage deviations from their steady states except for the consumption deflator, wage and interest rates that are annualized percentage-point deviations; shocks are normalized to one standard deviation. The shaded area is the 90% uncertainty interval. Horizontal axis: quarters.

Figure B.15: Shock decomposition of the output gap



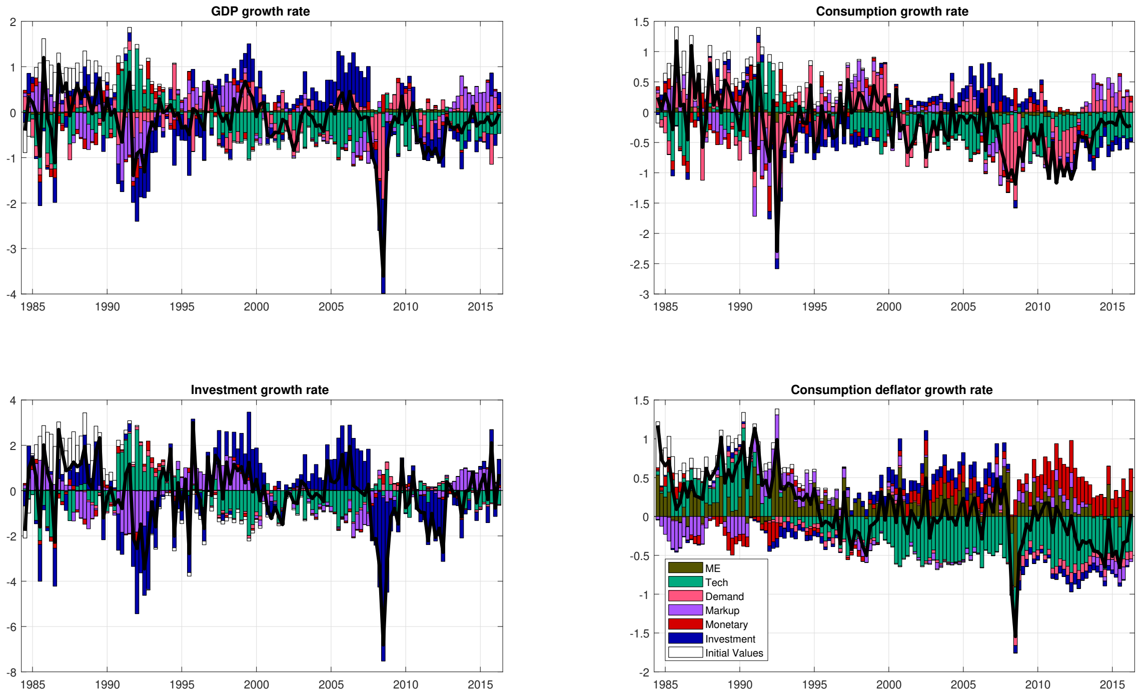
Notes: this figure shows the smoothed estimates of the output gap and the shock contribution for each quarter. Shocks are grouped as follows: ME= measurement errors; Tech= transitory and permanent technology shocks; Demand=public expenditure and consumption preference shock; Markup= price and wage mark-up shocks; Investment= investment shock.

Figure B.16: Shock decomposition: actual and potential output



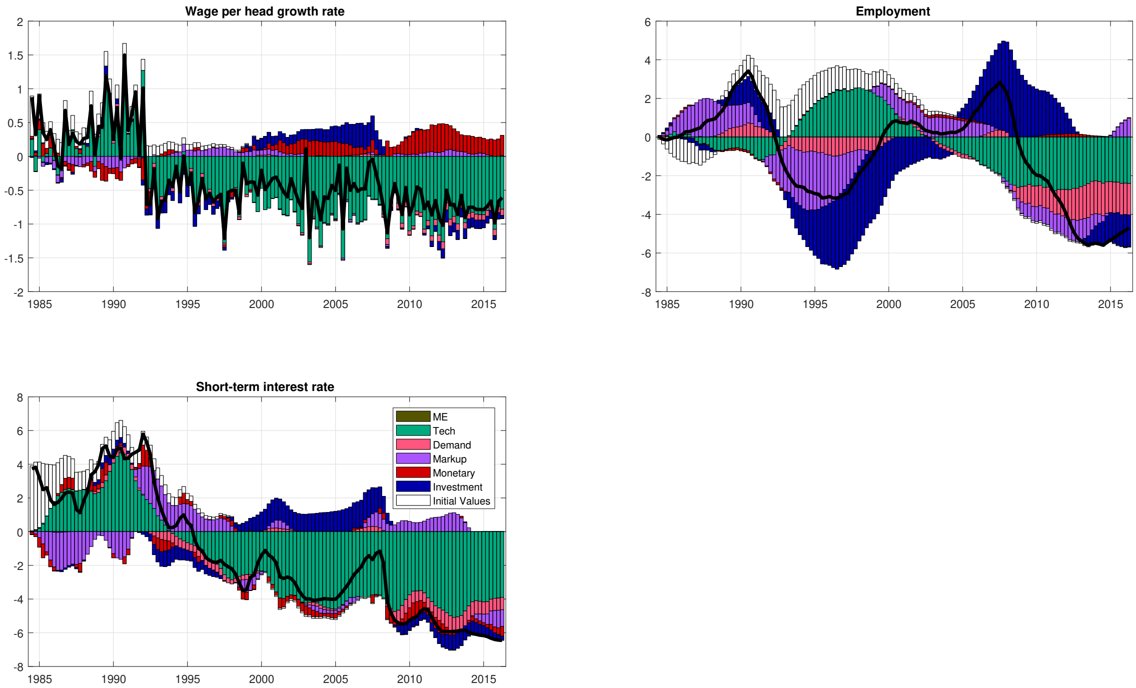
Notes: this figure shows the GDP (black line) level, the estimated potential output (black line) in deviation from their steady states, and the shocks contribution for each quarter. Shocks are grouped as follows: ME= measurement errors; Tech= transitory and permanent technology shocks; Demand=public expenditure and consumption preference shock; Markup= price and wage mark-up shocks; Investment= investment shock.

Figure B.17: Shock decomposition: observed variables



Notes: this figure shows the observed variables (black line) in deviation from their steady states and the shocks contribution for each quarter. Shocks are grouped as follows: ME= measurement errors; Tech= transitory and permanent technology shocks; Demand=public expenditure and consumption preference shock; Markup= price and wage mark-up shocks; Investment= investment shock.

Figure B.18: Shock decomposition: observed variables



Notes: this figure shows the observed variables (black line) in deviation from their steady states and the shocks contribution for each quarter. Shocks are grouped as follows: ME= measurement errors; Tech= transitory and permanent technology shocks; Demand=public expenditure and consumption preference shock; Markup= price and wage mark-up shocks; Investment= investment shock.

C Robustness

C.1 Output gap estimates

Here we report the results obtained when taking in consideration the full-sample (1985q1:2016q4) for the calibration of the model and the estimation of the linear trend we remove from the employment series. Differently from our baseline setup, under this configuration the shocks occurring after 2007q4 produce a structural break in the long-run growth path of the economy. Table C.1 and table C.2 report the calibrated parameters and the implied steady states. The results of the Bayesian estimation reported in Table C.3 are in line with those presented in the text. However, looking at the dynamic of the output gap obtained applying the Kalman Filter on this last configuration (Figure C.1) with the one of the baseline setup (Figure 2), two important differences emerge. First, the peak before the crisis occurs at an higher level while the subsequent trough is less deep. Second, and most importantly, from 2008 onwards the assumption of a structural break in the growth trend reduces the estimated potential output and the relative economic slack. Accordingly, the output gap registered for the last years in the sample is lower on average compared to the baseline model: the gap turns positive in 2011 and is close to zero (-0.7%) in the last quarter.

These latter findings underline the importance of our assumption on the growth path of the economy: the continuing (and thus higher) trend after the crisis delivers wider output gaps over the last years, far from their closure at the end of the sample. The hypothesis of a break in the trend after 2007, instead, lowers the potential output and produce output gap estimates close to zero at at the end of 2016. Discriminating between these two results is crucial to correctly gauge the most recent output gaps. In a recent paper, Jarociński and Lenza (2018) show that estimates of the output gap based on the assumption of a continuing growth trend after the financial crisis are the ones consistent with the observed low levels of inflation registered during the recovery occurred after the sovereign debt crisis. In our preferred model specification (reported in the text) we follow this line and obtain results quantitatively similar to the ones proposed by the two authors. Riggi and Venditti (2015) propose two different explanation for the low level of inflation registered after the sovereign debt crisis. The first is an increase in the elasticity of inflation to the output gap, the second an output gap lower than the one (close to -3%) estimated by the IMF and OECD for the years 2013-2014 (Figure 3). The results produced with our baseline specification are consistent with their second hypothesis.

Table C.1: Calibrated parameters (trend 1985-2016)

Parameter	***Description***	**Value**
$\frac{1}{\sigma}$	Intertemporal elasticity of substitution	1
τ	Inverse of Frisch elasticity of labor supply	2
α	Substitution between factors of production	0.98
δ	Capital depreciation	0.025
β	Discount factor	0.998
γ	Bias towards capital	0.301
θ_w	Elasticity of substitution labor varieties	4.3
θ_p	Elasticity of substitution goods varieties	6

Table C.2: Steady-state relationships (trend 1985-2016)

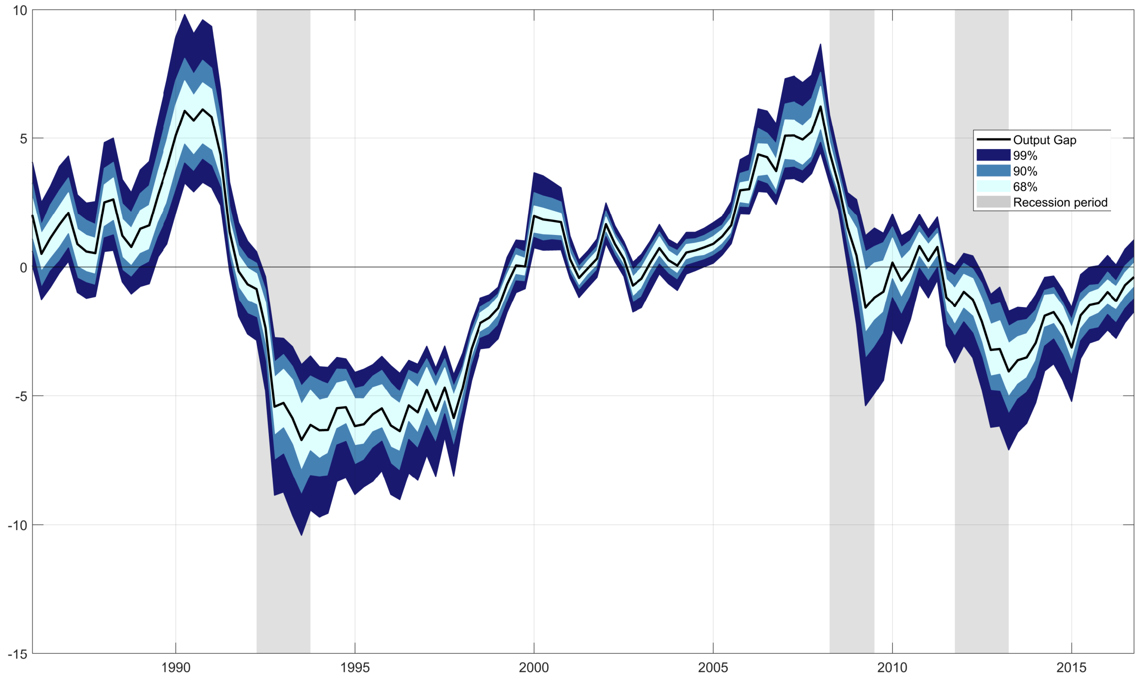
Parameter	***Description***	**Value**
$\overline{\Pi}$	Inflation rate (quarterly)	0.5%
$\overline{R}$	Nominal interest rate (quarterly)	1.15%
g_z	Growth rate (quarterly)	0.45%
C/Y	Consumption-to-output ratio	0.58
G/Y	Public expenditure-to-output ratio	0.20
I/Y	Investment-to-output ratio	0.22

Table C.3: Prior and posterior moments of the parameters (trend 1985-2016)

		Prior			Posterior			
		Type	**mean**	**s.d.**	**Mode**	**Mean**	**5%**	**95%**
	Preferences							
k	Habit formation	B	0.7	0.1	0.741	0.746	0.695	0.797
	Employment							
ξ_E	Calvo-style parameter	B	0.5	0.15	0.865	0.867	0.846	0.889
	Adjustment costs							
ψ	Investment	G	5	1.5	4.983	6.010	3.923	8.046
	Monetary policy							
ρ_R	Interest rate smoothing	B	0.9	0.05	0.610	0.623	0.552	0.694
ρ_π	Resp. to inflation	N	1.7	0.1	1.857	1.867	1.724	2.013
ρ_{GDP}	Resp. to output growth	N	0.0625	0.05	0.080	0.084	0.009	0.147
	Wage and price setting							
ξ_p	Calvo:prices	B	0.75	0.05	0.895	0.893	0.853	0.934
ξ_w	Calvo:wages	B	0.75	0.05	0.888	0.886	0.871	0.900
χ_p	indexation:prices	B	0.5	0.15	0.426	0.409	0.194	0.611
χ_w	indexation:wages	B	0.5	0.15	0.610	0.378	0.165	0.579
	Shock: autoregr. coeff.							
ρ_a	Transitory tech. shock	B	0.75	0.05	0.833	0.828	0.790	0.867
ρ_{g_z}	Permanent tech. shock	B	0.75	0.05	0.631	0.627	0.547	0.703
ρ_ζ	Preferences	B	0.75	0.05	0.821	0.820	0.764	0.878
ρ_Υ	Investment	B	0.75	0.05	0.764	0.736	0.654	0.816
ρ_{μ_w}	Wage markup	B	0.65	0.05	0.653	0.650	0.568	0.732
ρ_{μ_p}	Price markup	B	0.65	0.05	0.753	0.747	0.666	0.828
ρ_G	Public expenditure	B	0.75	0.05	0.912	0.905	0.868	0.943
ρ_π	Inflation target	B	0.75	0.05	0.753	0.752	0.694	0.809
	Shock: standard deviations							
$100 * \sigma_a$	Transitory tech. shock	G	0.10	Inf	2.22	2.31	1.80	2.81
$100 * \sigma_a$	Permanent tech. shock	G	0.10	Inf	0.39	0.39	0.35	0.44
$100 * \sigma_\zeta$	Preferences	G	0.10	Inf	1.69	1.78	1.41	2.15
$100 * \sigma_\Upsilon$	Investment	G	0.10	Inf	2.13	2.85	1.51	4.16
$100 * \sigma_{\theta w}$	Wage markup	G	0.10	Inf	0.05	0.08	0.02	0.15
$100 * \sigma_{\theta_p}$	Price markup	G	0.10	Inf	0.74	0.83	0.42	1.21
$100 * \sigma_G$	Public expenditure	G	0.10	Inf	1.25	1.27	1.11	1.42
$100 * \sigma_\pi$	Inflation target	G	0.10	Inf	0.12	0.12	0.09	0.15
$100 * \sigma_R$	Monetary policy	G	0.10	Inf	0.05	0.09	0.03	0.15

Notes: this table reports the results of the Bayesian estimation. Prior distributions are the following: B=beta, G=inverse gamma, N=normal. Values referring to the standard deviations of the structural shocks are multiplied by 100.

Figure C.1: Output gap (trend 1985-2016)



Notes: this figure shows the mean of the posterior distribution of the smoothed estimates of the output gap, together with its 99%, 90%, and 68% uncertainty interval. Shaded horizontal bars are euro-area recessions as defined by the CEPR euro-area business cycle dating committee.

C.2 Reliability measures

The table below reports the reliability measures outlined in section 6 over longer (1998-2016) and shorter (2010-2016) time periods then the one reported in the text. When available, we also report the estimates provided by the European Commission. The results confirm our previous findings: the DSGE-based output gap has higher reliability measures across the different time samples.

Table C.4: Reliability measures

	1998-2016				2010-2016			
	NS	**NSR**	**COR**	**OPSIGN**	**NS**	**NSR**	**COR**	**OPSIGN**
OECD	0.51	0.82	0.88	33%	1.11	1.14	0.47	0%
IMF	-	-	-	-	0.76	0.91	0.69	0%
EC	-	-	-	-	0.66	0.81	0.85	0%
DSGE	0.25	0.42	0.97	6%	0.35	0.34	0.98	0%
HP	0.81	0.79	0.63	33%	0.71	0.82	0.74	17%

Notes: this table reports alternative measures for the reliability of the output gap estimates. COR is the correlation between the real-time and the full-sample estimate, NS is the ratio between the revision and the full-sample estimate, NSR is the ratio between the RMS of the revision and the standard deviation of the full-sample estimate. OPSIGN is the fraction of times the real-time and full-sample estimates have different sign. Standard deviations and RMSs are the ones reported in Table 4. For the details on real-time and full-sample estimates see Section 6.