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A SURVEY OF SYSTEMIC RISK INDICATORS

by Antonio Di Cesare* and Anna Rogantini Picco**

Abstract

The aim of this survey is to provide a rigorous, but not so technical, introduction to several systemic risk indicators frequently used in official publications by institutions involved in macroprudential analysis and policy. The selected indicators are classified using three taxonomies. The first one adopts the point of view of regulators and policy-makers, whose attention is usually focused on the implementability and forward-looking nature of the indicators. The second taxonomy highlights the features that are most relevant for researchers, i.e. the reliance on a sound theoretical background and the use of advanced analytical techniques. The third taxonomy classifies the indicators according to the specific aspects of systemic risks that are captured. For each indicator both general and technical descriptions are provided, as well as specific examples.

JEL Classification: G21, G28, G14, C13.
Keywords: systemic risk, financial stability, systemic risk indicators.

Contents

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1 Introduction[1]

The consequences of the global financial and economic crisis have prompted in-depth studies on how to identify, measure and mitigate systemic risk, i.e. the risk that the financial system, or part of it, may become so impaired that severe negative consequences on the overall economic activity would be inevitable. This risk is by nature multi-faceted and difficult to capture in a unique, compact framework (see e.g. Hansen, 2014). In order to grasp its main features, an extensive number of indicators has been proposed in a broad and heterogeneous body of literature.

The aim of this survey is to provide a rigorous, though not overly technical introduction to several systemic risk indicators by organizing and reviewing them in a unified and consistent way. More specifically, the contribution of this survey is threefold.

First, the survey reports the indicators that are more frequently used in official publications by several institutions involved in macroprudential analysis and policy, such as the International Monetary Fund (IMF), the European Systemic Risk Board (ESRB), the European Central Bank (ECB), the Bank of England (BoE), and the Bank of Italy (BoI). To the best of our knowledge, previous surveys do not provide such an extensive coverage – with the remarkable exception of Bisias et al. (2012) – and only focus on relatively smaller sets of indicators (see e.g. Blancher et al., 2013).

Second, the selected indicators are organized according to three different taxonomies. The first one adopts the point of view of regulators, whose attention usually focuses on the implementability and the forward-looking nature of systemic risk indicators. The second taxonomy takes the point of view of researchers, who are mainly interested in indicators with solid theoretical foundations and advanced analytical features. The last taxonomy highlights the specific features of systemic risk that the selected indicators are able to capture.

Finally, this survey contributes to the literature by presenting the selected indicators in a systematic way with the aim of making them accessible to a wide range of readers, such as researchers, regulators, and practitioners. To this end, the description of each indicator is structured as follows:

[1] Our goal was to make this survey as comprehensive as possible and to give the most extensive credit to the incredible amount of work that has been done in this ever-growing field. We are sorry for any relevant piece of analysis and research that we may have missed. The hyperlinks in the references are accurate as of 1 August 2018. The views expressed in the article are those of the authors and do not necessarily reflect those of the Bank of Italy.

1. A *general description* illustrates the main features, the advantages and disadvantages of the indicator;

2. A *technical description* focuses on the analytical aspects of the indicator;

3. An *empirical example* taken from published documents provides further clarifications on the actual use and interpretation of the indicator.

Moreover, the description of each indicator includes a table that provides information on the author(s), the institution(s) that use(s) it, its strengths and weaknesses, and the kind of data that is required to implement it. This systematic way of presenting each indicator serves the purpose of satisfying multiple needs of the readers. In fact, while the general description gives a broad overview of each indicator for those who like to have a bird's-eye view, the technical description adds further details for readers who want to go more in depth. The summary table helps to wrap up the main features of each indicator and provides immediate access to references.

This survey builds upon some already existing reviews of systemic risk indicators. IMF (2009a) discusses 'complementary approaches to assessing direct and indirect financial sector systemic linkages', but it focuses on network indicators and overlooks several indicators that capture other features of systemic risk. IMF (2009b) provides a non-systematic analysis of some tools, which are aimed at detecting systemic risk as well as helping policy makers to take the necessary measures to mitigate it. The Office of Financial Research of the US Department of the Treasury provides an excellent survey of systemic risk analytics (Bisias et al., 2012) and develops Matlab codes for their implementation. Bisias et al. (2012) are also the first to present a survey that organizes the indicators according to the 'supervisory perspective' and the 'research perspective'. In this paper we point out different features within each of the two taxonomies and add a third, new taxonomy, which highlights the specific features of systemic risk that are taken into account by each indicator. A further review of systemic risk indicators can be found in Blancher et al. (2013), who, however, only focus on the toolkit of indicators used by the IMF. Arsov et al. (2013) build a metric for systemic financial stress and then compare it with other near-coincident indicators to test the validity and robustness of their new metric. Benoit et al. (2016) try to connect the literature on systemic risk with the regulatory debate. In particular, they identify two main approaches to tackling systemic risk: the first one focuses on specific sources of systemic risk and draws on confidential data, while the second one derives global measures of systemic risk from market data. They recognise a gap between the two approaches, which they hope will be bridged in the future.

In addition to the above-mentioned surveys, it is worth mentioning two online laboratories – the NYU Stern Volatility Lab (V-Lab) and the Center for Risk Management at Lausanne (CRML) – that provide weekly updates of the SRISK (a systemic risk indicator presented by Brownlees and Engle, 2017; see Section 5) for systemic financial institutions from all over the world.

Both the surveys and the online laboratories contribute to the lively debate about systemic risk by presenting cutting-edge research on systemic risk.

The remainder of the paper is organized as follows. Section 2 discusses three taxonomies that can be used to classify systemic risk indicators. Section 3 to Section 15 describe the individual indicators. Section 16 concludes.

2 How to Classify Systemic Risk Indicators

In the aftermath of the global financial crisis, an extensive and heterogeneous literature on systemic risk and systemic risk indicators has developed. As noted in Section 1, the heterogeneity of this literature is mainly due to the multifaceted nature of systemic risk, which makes it difficult to encompass its numerous features within a compact framework. Given the wide range of existing systemic risk indicators, it is worth identifying possible criteria to classify them according to their characteristics. This survey presents three ways to do so.

The first taxonomy is organized according to the features of systemic risk indicators that are most relevant from a regulatory perspective:[2] the capacity to anticipate systemic events (ex-ante vs. near-coincident vs. ex-post), the simplicity of implementation (easy vs. difficult), and the possibility of updating them frequently (yes vs. no).

The second taxonomy adopts the point of view of the researchers, who are interested in the theoretical foundations of the indicators and in the analytical techniques that are used. According to this taxonomy, the indicators are organized into the following groups: contingent claim analysis, probability and mathematical methods, interconnection analysis, and composite measures.

The third taxonomy focuses on the main features of systemic risk that each indicator is able to capture, and organizes the indicators into the following classes: indicators of expected losses in case of default of financial institutions; indicators

[2] The term 'regulators' is used in this paper to refer to the public institutions that have oversight, regulatory or supervisory powers over the financial system or some of its components. We thus refer not only to national and supranational regulatory and supervisory authorities, but also to international institutions with oversight capacity over the financial system, such as the Financial Stability Board, the International Monetary Fund, the Bank of International Settlements, and the European Systemic Risk Board.

of the probability of default of financial institutions when the system is in distress; indicators that look at specific mechanisms of contagion; and indicators of the overall level of distress in the system.

Clearly, there is an inherent dimension of subjectivity in the criteria just introduced. While we believe that they may help the reader to have a systematic view of the several available measures, alternative approaches are certainly possible.

2.1 The regulator-oriented perspective

For regulatory purposes, it is important that systemic risk indicators have at least three characteristics. In order to allow regulators to adopt policy measures to prevent or reduce the risks associated with financial instability, systemic risk indicators should be able to reliably indicate the build-up of risks well in advance. Moreover, because regulatory activity is usually subject to transparency and accountability requirements, it would also be useful if the indicators were relatively easy to calculate. Finally, as regulatory decisions during or close to a crisis have to be taken rapidly, it is important that the indicators can be updated quickly.

More in detail, from the point of view of regulators, the 'optimal' systemic risk indicator should have the following characteristics:

1. **Temporal dimension**. Regulators need to rely on ex-ante indicators, which are able to quantify the build-up of systemic risk. However, while identifying and measuring upsurges in systemic risk is indeed extremely helpful, it may not be enough to reveal when a systemic crisis is about to break out. That is why near-coincident indicators may also provide crucial warnings of an imminent crisis and compel authorities and systemic institutions to take action to mitigate the crisis. Finally, the ex-post analysis of the indicators is important to monitor the development of a crisis and the effect of policy measures (Schwaab et al., 2011, Bisias et al., 2012).

2. **Implementability**. This is crucial for an indicator to be relevant from a regulatory perspective. In particular, data availability and ease of calculation are two necessary requirements for an indicator to be implementable as well as available to regulators in due time.

3. **Possibility of frequent updates**. The possibility of updating the indicators frequently improves their accuracy and performance, thus helping regulatory authorities to take more timely policy measures.

Table 1 reports the main characteristics of the indicators analysed in this survey according to the regulator-oriented taxonomy. All the ex-ante indicators are easy

Table 1 – Regulator-oriented taxonomy

Main characteristics of the indicators illustrated in this article from the regulator's point of view.

Indicator	Temporal dimension	Implementability	Possibility of frequent updates
ΔCoVaR	near-coincident	easy	yes
CoRisk	near-coincident	easy	yes
SES and SRISK	ex-ante	easy	yes
DIP	ex-ante	easy	yes
PCA	ex-ante	easy	yes
Granger causality	ex-ante	easy	yes
Option-iPoD	near-coincident	difficult	no
JPoD	near-coincident	difficult	no
BSI	near-coincident	difficult	no
DiDe	near-coincident	difficult	no
PCE	near-coincident	difficult	no
Systemic CCA	near-coincident	difficult	no
Network analysis	ex-ante	easy	yes
Default intensity model	ex-ante	easy	yes
SWARCH	ex-ante	easy	yes
CISS	near-coincident	difficult	yes
RAMSI	near-coincident	difficult	yes

to implement and update. On the contrary, the majority of the near-coincident indicators are difficult to implement. ΔCoVaR and CoRisk show the best combination in terms of implementability and frequency among the near-coincident indicators. CISS and RAMSI, while being difficult to implement owing to the amount of data they require, are easy to update once the data become available. Option-iPoD, JPoD, BSI, DiDe, PCE, and Systemic CCA, which are also near-coincident, are difficult both to implement and update. Therefore, on the basis of this taxonomy, the latter indicators seem to be less advantageous compared with the other near-coincident indicators. However, the other two taxonomies illustrated in Section 2.2 and Section 2.3 show that the more complex near-coincident indicators are based on different theoretical foundations and capture features of systemic risk that escape the simpler near-coincident indicators.

Some remarks need to be added. First, in this review an indicator is classified as ex-ante if it is able to signal the build-up of systemic risk at least one quarter before the outbreak of a systemic crisis. On the other hand, an indicator is said to be near-coincident if it is able to signal the outbreak of a crisis about one month in advance. An additional remark concerns the criteria on the basis of which indicators are defined as easy or difficult. These criteria relate to difficulties in accessing the necessary data as well as in implementing the computer programs for calculating the indicators. Finally, the update frequency is determined by the frequency with which the required data are available and the time needed to calculate the indicators.

2.2 The researcher-oriented perspective

While from a regulatory perspective the most important characteristics of systemic risk indicators are their implementability and their ability to provide policy prescriptions, from a research perspective other characteristics gain importance, such as the use of sound theoretical frameworks or advanced analytical techniques. This section classifies the indicators according to whether they mainly rely on:

1. **Contingent claim analysis**. Indicators based on Merton's balance sheet approach (Merton, 1973), where the value of equity can be seen as a call option on assets with a strike price equal to the on-balance-sheet liabilities.

2. **Probability and mathematical methods**. Indicators based on the probability distribution of asset returns and default rate models.

3. **Interconnection analysis**. Indicators based on techniques – such as quantile regressions, network analysis, principal components analysis, and Markov-regime switching models – that are built to capture interconnections among financial institutions.

4. **Composite measures**. Indicators that are the result of the aggregation of several sub-indices, which are combined together according to different criteria in order to provide an aggregate measure of systemic risk across several markets or risk factors.

Table 2 classifies the indicators from a researcher's perspective. Both the Option-iPoD and Systemic CCA methodologies are based on Merton's balance sheet approach. In addition, Option-iPoD makes use of the concept of minimum cross-entropy, whereas Systemic CCA uses techniques from extreme value theory. Eight indicators are computed using probability and mathematical methods. In particular, ΔCoVaR measures the systemic spillover from a single financial institution to the whole financial system. SES and SRISK are based on the concept of expected

Table 2 – Researcher-oriented taxonomy

Main characteristics of the indicators illustrated in this article according to the researcher-oriented taxonomy.

Indicator	Methodology
ΔCoVaR	Probability and mathematical methods
CoRisk	Interconnection analysis
SES and SRISK	Probability and mathematical methods
DIP	Probability and mathematical methods
PCA	Interconnection analysis
Granger causality	Interconnection analysis
Option-iPoD	Contingent claim analysis
JPoD	Probability and mathematical methods
BSI	Probability and mathematical methods
DiDe	Probability and mathematical methods
PCE	Probability and mathematical methods
Systemic CCA	Contingent claim analysis
Network analysis	Interconnection analysis
Default intensity model	Probability and mathematical methods
SWARCH	Interconnection analysis
CISS	Composite measures
RAMSI	Composite measures

shortfall, i.e. the expected loss of a financial institution, conditional on the loss being greater than a given threshold. JPoD, BSI, DiDe, and PCE are built upon the notion of 'banking system multivariate density', which is the multivariate density of the banking system seen as a portfolio of banks. JPoD, for example, is a default probability measure that estimates the probability of default of all banks in the financial system. DIP is calculated using probabilities of default and asset return correlations, while the default intensity model is based on modelling the default rate.

Five indicators measure interconnections among financial institutions. CoRisk uses quantile regressions to account for non-linear patterns in common risk factors across financial institutions. PCA, Granger-causality tests, and network analysis are interconnection-based indicators. SWARCH is a regime-switching model.

Finally, two indicators are composite measures of systemic risk. CISS is a composite indicator derived by aggregating five sub-indices and RAMSI is a quantitative model of financial stability, which integrates a balance sheet approach with a network model.

Table 3 – Risk-oriented taxonomy

Main characteristics of the indicators illustrated in this article according to the specific features of systemic risk they focus on.

Indicator	Features
ΔCoVaR	Expected losses
CoRisk	Contagion
SES and SRISK	Expected losses
DIP	Expected losses
PCA	Contagion
Granger causality	Contagion
Option-iPoD	Probability of default
JPoD	Probability of default
BSI	Probability of default
DiDe	Probability of default
PCE	Probability of default
Systemic CCA	Expected losses
Network analysis	Contagion
Default intensity model	Probability of default
SWARCH	Broad measure of financial stress
CISS	Broad measure of financial stress
RAMSI	Broad measure of financial stress

2.3 The risk-oriented perspective

As observed in Section 1, the multifaceted nature of systemic risk requires the deployment of a wide range of indicators in order to obtain reliable measures of its various features. Thus, we suggest an additional classification of the indicators presented in this survey into four main categories according to the specific feature of systemic risk that they are able to capture:

1. **Probability of default** (or distress) of individual financial institutions or groups of them;
2. **Expected losses**, individual or joint, in case of financial and economic distress;
3. **Contagion** across institutions in the system;
4. **Measure of financial stress** in a broad sense.

This third taxonomy is displayed in Table 3. The first category of indicators, estimating the probability of default or distress, include Option-iPod, JPoD, BSI, DiDe, and PCE. While Option-iPoD focuses on individual probabilities of default, JPoD captures the probability of joint distress for all banks in the system. BSI measures the number of distressed banks, PCE captures the probability that at least one bank of the system will become distressed, given that another bank has become distressed, and DiDe gauges the pairwise conditional probabilities of distress.

The indicators belonging to the second category are ΔCoVaR, SES and SRISK, DIP, and Systemic CCA. In particular, ΔCoVaR, SES and SRISK measure the individual expected losses that are due to another institution or the whole financial system being in distress. Instead, DIP and Systemic CCA capture the joint expected losses in a distress scenario.

As for the third category, CoRisk, PCA, Granger-causality models, and network analysis are all indicators that estimate the degree of contagion among financial institutions. Within these indicators, CoRisk focuses on pairwise linkages, while Granger-causality tests are specifically built to capture the directionality of linkages. PCA helps to identify the interdependence across financial institutions, and network analysis makes it possible to track the impact of a credit and/or funding shock throughout the system.

The fourth category consists of broad measures of financial distress and includes SWARCH, CISS, and RAMSI. SWARCH captures regime changes in market volatility, thus helping to predict the likelihood of a crisis. CISS is a composite indicator which aims to capture and summarize in a single metric the state of instability stemming from several markets within the financial system. Finally, RAMSI is a quantitative model of financial stability, which encompasses many types of risk, namely credit risk, income risk, liquidity risk, counterparty risk, and mark-to-market risk.

The following sections describe these indicators in greater detail.

3 (Δ)Conditional Value at Risk (ΔCoVaR)

Table 4 – ΔCoVaR: Main characteristics

Author(s)	Adrian and Brunnermeier (2016)
Institution(s)	ECB (2010b, 2011, 2014b) ESRB Risk Dashboard, issues from ESRB (2012) to ESRB (2015) IMF (2011b, 2015, 2016)
Pros	High-frequency, near-coincident Able to capture systemic spillovers Easy to calculate
Cons	Bivariate only
Data	Market data

3.1 General description

The Conditional Value at Risk (CoVaR) has been proposed by Adrian and Brunnermeier (2016) as the Value at Risk (VaR) of the financial system conditional on an institution being in distress.[3] Its aim is to measure the systemic spillover from an individual institution to the whole financial system. While two institutions may be similar in terms of VaR, their contribution to systemic risk could differ substantially. As explained by Adrian and Brunnermeier (2016), ΔCoVaR – the individual contribution of an institution to systemic risk – is calculated as 'the difference between the CoVaR conditional on the distress of an institution and the CoVaR conditional on the median state of that institution'.

Intuitively, the CoVaR estimates the stock losses that the whole financial system would face (with a certain confidence level) conditional on the stock returns of an individual institution. In turn, ΔCoVaR estimates how the potential losses for the whole financial system would increase when the individual institution shifts from being in a normal condition to being in trouble.

If the conditioning is reversed, i.e. if the ΔCoVaR of an individual institution conditional on the financial system being in distress is calculated, it is possible to quantify the exposure of a single institution to systemic financial distress. This indicator is called 'exposure ΔCoVaR'.

[3] Let us recall that the VaR of institution i at the level of confidence level $1 - q$ is defined as the maximum loss that i may suffer with probability q.

Since its introduction, CoVaR has been widely used by public institutions (ECB, 2010b, 2011, 2014b; ESRB Risk Dashboard issues from ESRB 2012 to ESRB 2015; IMF, 2011b, 2015, 2016), as well as researchers (Blancher et al., 2013 and Bisias et al., 2012). According to several stress tests, it is considered one of the most accurate systemic risk indicators (see e.g. IMF, 2011b; Arsov et al., 2013). CoVaR is not simply a tail indicator like the VaR, but is a tail indicator conditional on a bad event. It is exactly that conditioning that helps identify as systemically significant some institutions that would have not been recognized as such otherwise.

3.2 Technical description

The VaR of institution i (VaR_i) calculated for the variable X_i (usually stock returns) at the level of confidence $1 - q$ is formally defined as

$$\mathbb{P}(X_i \leq \mathrm{VaR}_i) = q. \tag{1}$$

$\mathrm{CoVaR}_{j|i}$ is then defined by Adrian and Brunnermeier (2016) as the VaR of institution j conditional on the occurrence of an event $A(X_i)$ concerning institution i, i.e.

$$\mathbb{P}(X_j \leq \mathrm{CoVaR}_{j|i} \mid A(X_i)) = q. \tag{2}$$

Often the event $A(X_i)$ is given by the case in which $X_i = \mathrm{VaR}_i$, where the variable X_i represents the stock returns of institution i.

Given $\mathrm{CoVaR}_{j|i}$, $\Delta\mathrm{CoVaR}_{j|i}$, which is defined as the contribution of institution i to the risk of j, may be expressed as

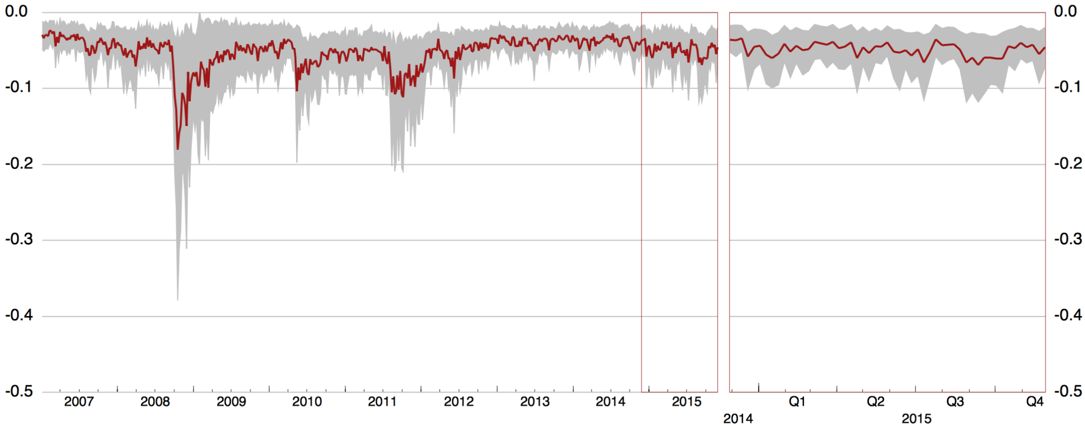
$$\Delta\mathrm{CoVaR}_{j|i} = \mathrm{CoVaR}_{j|X_i=\mathrm{VaR}_i} - \mathrm{CoVaR}_{j|X_i=\mathrm{median}(X_i)}. \tag{3}$$

Adrian and Brunnermeier (2016) use quantile regressions (see Section 4.2) to estimate ΔCoVaR, but they also show that the indicator can be estimated by using other techniques, such as GARCH models.

3.3 Example

Since CoVaR is a bivariate indicator, it can be calculated for the financial system conditional on one institution at a time being in distress. Figure 1 displays the average ΔCoVaR of the log stock returns of 52 European banks listed in the STOXX Europe 600 index. The shaded area shows the interval between the 5th and the 95th percentiles. The indicator shows several troughs during the periods of major distress (11 September 2001, the 2008 financial crisis, and the 2010–12

Figure 1 – ΔCoVaR



Source: ESRB (2015).
Data based on the log stock prices of 52 European banks listed in the STOXX Europe 600 index. The red line is the mean, while the shaded area shows the interval between the 5^{th} and the 95^{th} percentile.

sovereign debt crisis). This highlights how the market was particularly sensitive to some institutions being in distress during those periods.

4 CoRisk

Table 5 – CoRisk: Main characteristics

Author(s)	IMF (2009a)
Institution(s)	IMF (2009a)
Pros	Near-coincident Captures non-linear comovements
Cons	Bivariate only
Data	Market data

4.1 General description

Introduced by IMF (2009a), CoRisk is a measure of risk interdependence across financial institutions that accounts for common risk factors and potential nonlinear effects.

Intuitively, CoRisk is the percentage difference between a default risk measure of an institution conditional on the default risk of another institution and some common drivers of default risk and its unconditional counterpart. Using an OLS regression model to estimate the conditional default risk measure would only capture a linear relationship between the default risk measures of the two institutions. To avoid this drawback, CoRisk is estimated by running quantile regressions, that can capture potential non-linearities in the conditional distribution of default risk (Chan-Lau, 2009). CoRisk is an alternative indicator to other models that capture non-linear comovements based on extreme-value theory (e.g. Systemic CCA – see Section 10) or regime switching estimations (e.g. SWARCH – see Section 13).

CoRisk can be estimated using different datasets. IMF (2009a) uses CDS spreads. In particular, CDS spreads below the 5$^{\text{th}}$ quantile of their empirical distribution are assumed to indicate a very favourable regime, while CDS spreads above the 95$^{\text{th}}$ quantile are assumed to indicate a regime of distress.

4.2 Technical description

IMF (2009a) uses the following quantile regression to estimate CoRisk:

$$\mathrm{CDS}_i = \alpha_\tau + \sum_{m=1}^{K} \beta_{\tau,m} R_m + \beta_{\tau,j}\mathrm{CDS}_j + \epsilon_i \tag{4}$$

where CDS_i, the credit default swap spread of institution i, depends on the CDS spread of institution j (CDS_j) and K common risk factors (R_m), such as the implied volatility index (VIX) calculated by the Chicago Board Option Exchange (a measure of the general risk appetite), a LIBOR spread (which measures the default risk in the interbank market), and the slope of the US yield curve (a business cycle indicator). The parameter τ is the quantile for the estimation.

The parameters are estimated using the quantile regression method introduced by Koenker and Basset Jr. (1978) as an extension of the standard linear regression. The linear regression model focuses on modelling the average relationship between a dependent variable y and a vector of regressors x, i.e. the conditional mean of y given x ($\mathbb{E}(y|x)$). However, the conditional mean may not be sufficient to describe the whole relationship between y and x. For example, in a skewed distribution the median may be a more appropriate measure of central tendency than the mean, and the relationship between the dependent variable and the regressors can be better modelled by using a conditional median regression than a conditional mean regression.

The quantile regression model is a generalization of the median regression model, as the latter is simply the former when run at the 50^{th} quantile. In particular, the quantile regression estimates the effects of regressors on various quantiles in the conditional distribution. If ϵ_i is the model prediction error, ordinary least squares (OLS) minimize $\sum_i \epsilon_i^2$. The median regression minimises the sum of the least absolute deviations, $\sum_i |\epsilon_i|$. The quantile regression minimizes a sum that gives asymmetric penalties based on the quantile τ: it assigns weight τ when $\epsilon_i > 0$ and weight $(\tau - 1)$ when $\epsilon_i \leq 0$. Therefore, the minimization problem to find the quantile regressor estimators at quantile τ is:

$$\min_{\alpha_\tau, \beta_{\tau,m}, \beta_{\tau,j}} \sum_i \rho_\tau \left(\mathrm{CDS}_i - \alpha_\tau - \sum_{m=1}^{K} \beta_{\tau,m} R_m - \beta_{\tau,j} \mathrm{CDS}_j \right) \tag{5}$$

where ρ_τ is the weighting function defined as

$$\rho_\tau = \begin{cases} \tau & \text{for} \quad \epsilon_i > 0 \\ \tau - 1 & \text{for} \quad \epsilon_i \leq 0 \end{cases} \tag{6}$$

and $\tau \in (0, 1)$. Once the quantile regression coefficients have been estimated, it is possible to estimate CoRisk for any given quantile τ as

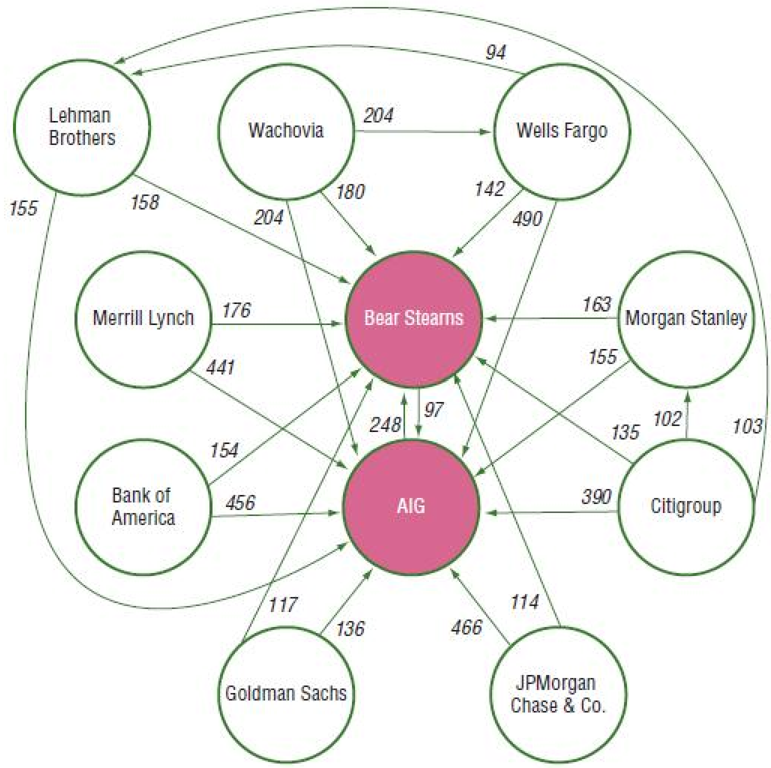
$$\mathrm{CoRisk}_{i,j}^{\tau} = 100 \left(\frac{\alpha_\tau + \sum_{m=1}^{K} \beta_{\tau,m} R_m + \beta_{\tau,j} \mathrm{CDS}_j(\tau)}{\mathrm{CDS}_i(\tau)} - 1 \right) \tag{7}$$

where $\mathrm{CDS}_i(\tau)$ and $\mathrm{CDS}_j(\tau)$ are the CDS spreads of institutions i and j corresponding to the τ^{th} percentile of their empirical sample, and the coefficients α_τ, $\beta_{\tau,m}$, and $\beta_{\tau,j}$ are the parameters of the τ^{th} quantile regression in Equation (4). As one is usually interested in measuring CoRisk in periods of distress, this metric is often calculated for high quantiles, such as the 95^{th}. A high CoRisk indicates an increased sensitivity of the default risk of institution i to the default risk of institution j.

4.3 Example

Figure 2 shows the CoRisk estimates of a subset of systemically important US financial institutions in March 2008 (IMF, 2009a). Only the values of CoRisk that exceed 90 per cent are reported. The CoRisk of A conditional on B is calculated as the percentage difference between the estimated and the observed CDS spreads of A at the 95^{th} empirical percentile. The estimated CDS spread of A is computed using the 95^{th} empirical percentile CDS spread of B as an input in the 95^{th} quantile regression of A on B. The numbers reported next to the arrows are the CoRisk measures. For example, the risk of Wells Fargo conditional on the risk of AIG

Figure 2 – CoRisk



Source: IMF (2009a).
The numbers on the arrows are the CoRisk measures between two institutions.

is almost five times (490 per cent) higher than the risk corresponding to the 95$^{\text{th}}$ percentile of the empirical distribution of Wells Fargo.

5 Systemic Expected Shortfall (SES) and SRISK

Table 6 – SES and SRISK: Main characteristics

Author(s)	SES, Acharya et al. (2017) SRISK, Brownlees and Engle (2017)
Institution(s)	ECB (2010b, 2013, 2014a) IMF (2014b, 2016)
Pros	Ex-ante measure Model-based, with theoretical justification Additivity
Cons	Measures only the average, linear, bivariate dependence
Data	Balance sheet and market data

5.1 General description

The Systemic Expected Shortfall (SES) was introduced by Acharya et al. (2017) 'to bridge the gap between economic theory and actual regulation'. For this reason, the SES combines the features of being theoretically micro-founded and practically relevant for regulators.

The SES measures how much a given financial institution is undercapitalised when the whole financial system is undercapitalised. More specifically, by being undercapitalised it is meant by how much a bank's equity drops below a given fraction of its assets when the aggregate banking capital drops below a given fraction of the aggregate banking assets.

The SES has two main components: the leverage (LVG) and the marginal expected shortfall (MES) of the institution under consideration. The latter is defined as the losses of a firm 'in the tail of the aggregate sector's loss distribution'. While the LVG can be calculated from balance sheet data, the MES needs to be estimated. Acharya et al. (2017) propose computing MES as the average stock return of a financial institution in the 5 per cent worst days of any given year.

Building upon the theoretical framework of Acharya et al. (2017), Brownlees and Engle (2017) propose another indicator, the SRISK, which is also computed from the LVG and the MES. The main difference between the SRISK and the SES lies in the econometric techniques used for the estimation of the MES. While Acharya et al. (2017) compute the MES using historical market data from the previous year, Brownlees and Engle (2017) use advanced time series models to calculate the

SRISK. Estimates of the SRISK for major financial institutions across the world are published online on the NYU Stern Volatility Lab (V-Lab) website.[4]

5.2 Technical description

In order to define the SES, some concepts need to be introduced. To begin with, Expected Shortfall (ES) is the expected loss of a bank, conditional on the loss being greater than the Value at Risk calculated at a given level of confidence $1 - \alpha$:

$$\mathrm{ES}_\alpha = -\mathbb{E}[R \mid R \leq -\mathrm{VaR}_\alpha]. \tag{8}$$

For several reasons, such as risk management or strategic capital allocation, firms may decompose their firm-wide losses R into contributions r_i from each individual group or trading desk i, which are weighted by y_i (Acharya et al., 2017). ES can then be written as the following weighted average:

$$\mathrm{ES}_\alpha = -\sum_i y_i \mathbb{E}[r_i \mid R \leq -\mathrm{VaR}_\alpha]. \tag{9}$$

The marginal expected shortfall for the trading desk i, MES_i, is defined as

$$\mathrm{MES}^i_\alpha = \frac{\partial \mathrm{ES}_\alpha}{\partial y_i} = -\mathbb{E}[r_i \mid R \leq -\mathrm{VaR}_\alpha]. \tag{10}$$

This risk management framework for a single institution can be extended to the whole financial system 'by letting R be the return of the aggregate banking sector or the overall economy' (Acharya et al., 2017). In this case, the conditioning event is a systemic event, which is thought of as the 5 per cent worst days of any given year in terms of stock returns.

The Systemic Expected Shortfall (SES_i) is defined as

$$\mathrm{SES}_i = -\mathbb{E}[za_i - w_i \mid W < zA]. \tag{11}$$

In words, 'SES_i is the amount a bank's equity w_i drops below its target level – which is a fraction z of assets a_i – in case of a systemic crisis when aggregate banking capital W is less than z times aggregate assets A'.

[4] The V-Lab (https://vlab.stern.nyu.edu) 'provides real time measurement, modelling and forecasting of financial volatility, correlations and risk for a wide spectrum of assets. V-Lab blends together both classic models as well as some of the latest advances proposed in the financial econometrics literature. The aim of the website is to provide real time evidence on market dynamics for researchers, regulators, and practitioners.'

Once MES_i and SES_i are defined, Acharya et al. (2017) compute the following statistical relationship between the two indicators:

$$\text{SES}_i = \beta_0 + \beta_1 \text{LVG}_i + \beta_2 \text{MES}^i_{5\%}, \tag{12}$$

which shows that MES_i and its level of leverage LVG_i are predictors of SES_i, i.e. the contribution to systemic risk of institution i.

For empirical purposes, MES_i is estimated by Acharya et al. (2017) by averaging the stock return of the 5 per cent worst days of the year for institution i, while LVG is computed from balance sheet and market data as follows:

$$\begin{aligned} \text{LVG}_i &= \frac{\text{quasi-market value of assets}}{\text{market value of equity}} \\ &= \frac{\text{book value of assets} - \text{book value of equity} + \text{market value of equity}}{\text{market value of equity}}. \end{aligned} \tag{13}$$

The main difference between the SES and the SRISK lies in the estimation of the MES. Taking a step back, the SRISK of firm i at time t is defined in Brownlees and Engle (2017) as

$$\text{SRISK}_{i,t} = \mathbb{E}_t[z a_{i,t+h} - w_{i,t+h} \mid R_{t+1:t+h} < C], \tag{14}$$

where $R_{t+1:t+h}$ is the stock return between $t+1$ and $t+h$ and C is a threshold for market decline over time horizon h. As before, $a_{i,t}$ is the value of firm i's assets, $w_{i,t}$ is the value of firm i's equity, and z is the prudential capital fraction. In words, the SRISK measures the expected capital shortfall conditional on the systemic event $R_{t+1:t+h} < C$. Given the balance sheet identity $a_{i,t} = d_{i,t} + w_{i,t}$, i.e. the value of assets $a_{i,t}$ equals that of debt $d_{i,t}$ plus equity $w_{i,t}$, SRISK can be written as follows:

$$\begin{aligned} \text{SRISK}_{i,t} &= \mathbb{E}_t[z a_{,it+h} - w_{,it+h} \mid R_{t+1:t+h} < C] \\ &= z\mathbb{E}[d_{i,t+h} \mid R_{t+1:t+h} < C] - (1-z)\mathbb{E}[w_{i,t+h} \mid R_{t+1:t+h} < C]. \end{aligned} \tag{15}$$

Assuming that, when a systemic event materialises, debt cannot be renegotiated, i.e. $\mathbb{E}[d_{i,t+h} \mid R_{t+1:t+h} < C] = d_{i,t}$, then

$$\text{SRISK}_{i,t} = z d_{i,t} - (1-z)\mathbb{E}[w_{i,t+h} \mid R_{t+1:t+h} < C]. \tag{16}$$

Defining leverage as $\text{LVG}_{i,t} = (d_{i,t} + w_{i,t})/w_{i,t}$ and long-run MES as $\text{LRMES} = -\mathbb{E}[R_{i,t+1:t+h} \mid R_{t+1:t+h} < C]$, where $R_{i,t+1:t+1}$ is firm i's stock return between $t+1$ and $t+h$, SRISK then becomes

$$\text{SRISK}_{i,t} = w_{i,t}[z\text{LVG}_{i,t} + (1-z)\text{LRMES} - 1]. \tag{17}$$

Figure 3 – SRISK

(a) Bank of America Corp

(b) Deutsche Bank AG

Source: V-Lab
Data in millions of US dollars.

Given that $\text{LVG}_{i,t}$ can be computed as in Acharya et al. (2017), the issue is how to estimate LRMES. While Acharya et al. (2017) estimate the MES by directly using market data, Brownlees and Engle (2017) develop more advanced econometric techniques for estimating LRMES. In particular, they propose a bivariate conditionally heteroskedastic model to determine the dynamics of the log stock returns of both firm i and the whole market on a given day t. The specification requires an estimation of time-varying volatility and correlation, as well as non-linear tail dependence. A multi-step Generalized Autoregressive Heteroskedasticity (GARCH) approach and a Dynamic Conditional Correlation approach (see Engle, 2002) are proposed for the first two, while a non-parametric kernel estimator is used to estimate tail dependence.

5.3 Example

Figure 3 shows the SRISK of Bank of America Corp and Deutsche Bank AG over the period July 2008 - July 2016. The SRISK is computed over a period of six months for the threshold $C = -40\%$, i.e. with a market decline of at least 40 per cent in half a year.[5] As shown in Figure 3a, after declining just before the collapse of Lehman Brothers, the SRISK of Bank of America Corp started to rise in September 2008 and peaked above \$154 billion in April 2009, following the acquisition of Merrill Lynch in January 2009. After a temporary decline, the indicator hovered around \$130 billion in the last months of 2011. Since then, the SRISK of Bank of America Corp. has declined and in summer 2016 it was around \$80 billion. Figure 3b shows the SRISK of Deutsche Bank AG. This indicator, while less volatile than that of Bank of America Corp, still shows significant variations. The two highest peaks were reached in autumn 2010 and in the last months of 2011, when the SRISK rose to \$130 billion. Since then, the SRISK of Deutsche Bank AG has also declined, hovering around \$87 billion in summer 2016.

6 Distress Insurance Premium (DIP)

Table 7 – DIP: Main characteristics

Author(s)	Huang et al. (2009)
Institution(s)	ECB (2010b)
Pros	Ex-ante, forward-looking Economically intuitive
Cons	Incorporates liquidity appetite in addition to systemic risk
Data	CDS spreads and equity returns

6.1 General description

The Distress Insurance Premium (DIP) was proposed by Huang et al. (2009) as an indicator of systemic risk that 'is equivalent to a theoretical premium to a risk-based deposit insurance scheme that guarantees against most severe losses for the banking system'. The indicator represents the expected value of portfolio

[5] The graph shown in Figure 3 can be found at https://vlab.stern.nyu.edu/analysis/RISK.WORLDFIN-MR.GMES#risk-graph.

credit losses that are equal or exceed a minimum share of the total liabilities in the banking sector.

Two components are necessary to calculate the DIP: the probability of default (PD) for individual banks and the correlation of asset returns. PDs are derived from single-name CDS spreads, while asset return correlations are derived from the comovements of equity returns. The DIP increases in both components: a higher DIP may be driven by either an increased probability of default of individual banks or a greater exposure to common risk factors. In addition, since CDS spreads and equity prices are available in real time, the DIP is a forward-looking indicator that can be updated very frequently.

The DIP is further extended by Huang et al. (2012) along three lines. First, asset return correlation is not assumed to be time invariant, but is estimated using a Dynamic Conditional Correlation model (Engle, 2002). Second, PDs are derived from the expected default frequency from Moody's KMV. Unlike CDS spreads, which provide risk-neutral PDs, the expected default frequency is a physical measure of PDs based on balance-sheet information and equity price data. Third, the analysis focuses not only on the aggregate level of systemic risk, but also on the marginal contribution of each bank to the systemic risk of the entire banking system.

6.2 Technical description

The DIP is calculated starting with two components: risk-neutral PDs and asset return correlations. Huang et al. (2009) suggest calculating the PD for bank i at time t from single-name CDS spreads as follows:

$$\mathrm{PD}_{i,t} = \frac{a_t s_{i,t}}{a_t \mathrm{LGD}_{i,t} + b_t s_{i,t}}, \tag{18}$$

where $s_{i,t}$ is the observed CDS spread, $\mathrm{LGD}_{i,t}$ is the loss given default, $a_t = \int_t^{t+T} e^{-r\tau} d\tau$, $b_t = \int_t^{t+T} \tau e^{-r\tau} d\tau$, and r is the risk-free rate. LGD is assumed to be independent from the PD process and to follow a symmetric triangular distribution with mean 0.55 and range [0.1,1]. The mean is derived from the Basel II internal ratings-based approach and is also consistent with the historical data.

As for the estimation of asset return correlations, Huang et al. (2009) point out that two methodologies are adopted in the literature. The first one estimates the correlations directly from historical data on defaults. However, since defaults are rare events, this may lead to estimation errors, thus potentially limiting this approach. The second methodology, which is the one used by the authors to calculate the DIP, derives correlations from equity returns: 'The logic behind this approach is that equity is a call option on the underlying firm assets. Hence, the comovements in equity prices tend to reflect the comovement among underlying asset values'.

In particular, Hull and White (2004) suggest using equity return correlation as a proxy for asset return correlation. Equity return correlation is a good proxy for asset return correlation if firm leverage is constant. However, when firm leverage is time varying, the relationship between equity and asset return correlation breaks down and the discrepancy depends on the comovements between asset returns and leverages (Huang et al., 2009). The assumption of constant leverage is more likely to hold in the short run and is tested empirically by the authors, who find that it is not rejected for eleven out of the twelve American banks in their sample over the period 2001–08.[6] Since the DIP aims at being a forward-looking indicator, Huang et al. (2009) use forecasted correlations instead of historical ones. The forecasted correlation is calculated as follows, by considering a time unit of one week:

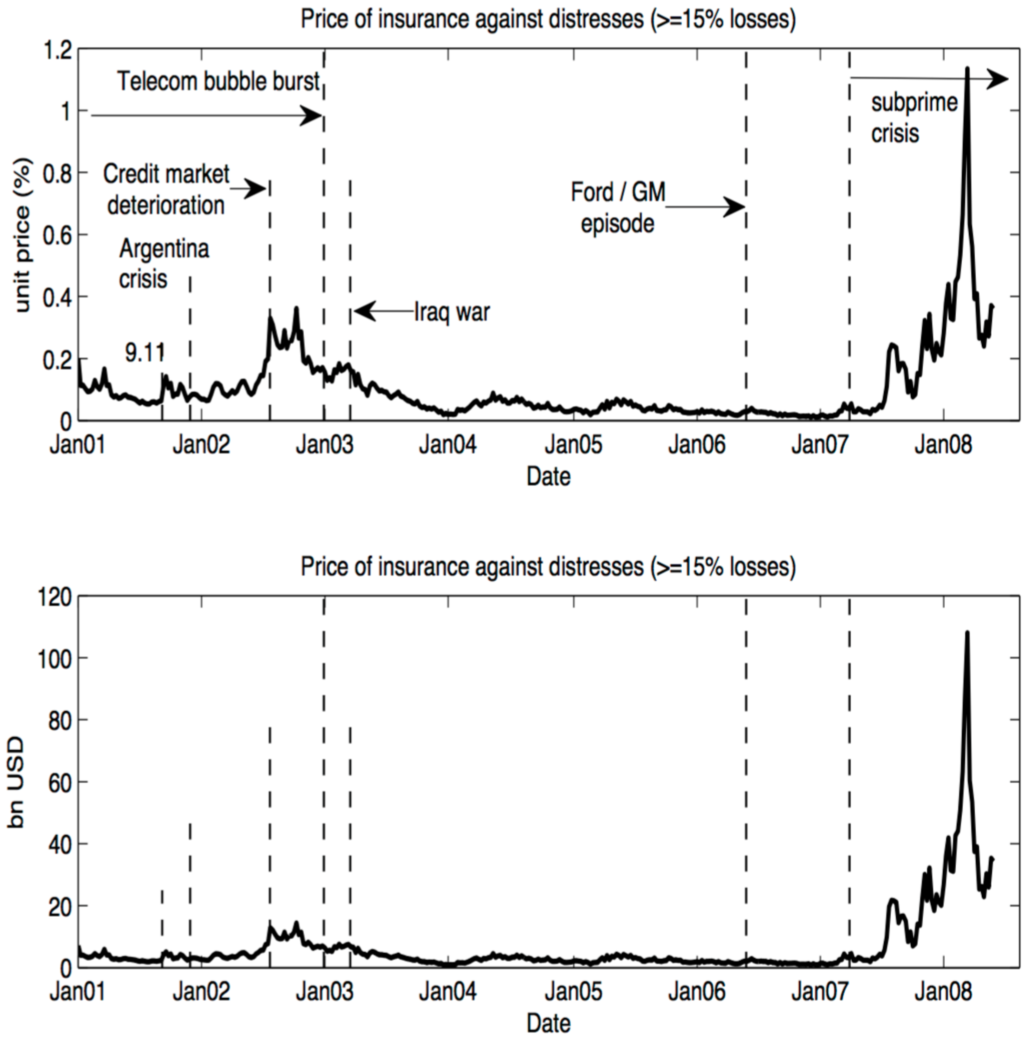
$$\rho_{t,t+12} = c + k_1 \rho_{t-12,t} + \sum_{i=1}^{l} k_{2i} \rho_{t-i,t-i+1} + \eta X_t + \nu_t, \tag{19}$$

where ρ indicates the average asset return correlation and its subscripts refer to the time horizon over which it is calculated, while X_t is a set of financial market variables, namely the one-quarter return of the S&P500 and its current implied volatility (VIX), the Fed funds rate and the term spread, which is defined as the difference between 10-year and 3-month constant maturity Treasury rates.

Once individual PDs and forecasted asset return correlations have been calculated, it is possible to build the DIP. For the estimation of the DIP, Huang et al. (2009) consider a hypothetical debt portfolio that consists of the liabilities (deposits, debts, and others) of all banks. The DIP is the theoretical insurance premium against any loss of that portfolio above a certain threshold over the following 12 weeks. It is calculated as the risk-neutral expectation of credit losses that equal or exceed a minimum share of the total liabilities of the sector. The share is set at 15 per cent of the total liabilities of the banking system. In order to calculate the expected credit losses of the portfolio, Montecarlo simulations based on the model of Tarashev and Zhu (2008) are implemented in two steps. First, the joint default scenario is simulated by using individual PDs and asset return correlations. Second, conditional on defaults occurring in the first step, the realization of LGDs and the overall credit losses of the whole portfolio are simulated.

[6] The twelve banks in the sample are: Bank of America, Bank of New York, Bear Stearns, Citibank, Goldman Sachs, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Morgan Stanley, State Street Corp, Wachovia and Wells Fargo.

Figure 4 – DIP



Source: Huang et al. (2009).

6.3 Example

Figure 4 shows the DIP, the theoretical insurance premium against a credit loss of at least 15 per cent of the banking sector's liabilities in the United States over the period 2001–08. The price of insurance is shown as the cost per unit of exposure to the banking sector's liabilities. That price increases significantly in correspondence with the telecom bubble burst in the early 2000s and the starting of the subprime crisis in mid-2007; it dramatically spikes in March 2008, one of the worst months in the subprime crisis.

7 Principal Component Analysis (PCA) and Granger-Causality Networks

Table 8 – PCA and Granger-Causality Networks: Main characteristics

Author(s)	Billio et al. (2012a)
Institution(s)	ECB (2010b)
Pros	Ex-ante, forward-looking Measures the interconnectedness and the direction of causality Statistically unconditional
Cons	Granger-causality tests are vulnerable to common factors
Data	Stock returns

7.1 General description

Billio et al. (2012a) propose two econometric measures of systemic risk based on principal component analysis (PCA) and Granger-causality tests. The first is aimed at identifying common components among different sectors of the financial markets, whereas the second is used to detect the direction of causality between pairs of financial sectors.

In a nutshell, PCA decomposes the stock return volatility of a sample of financial institutions into different components. The number of components which are necessary to explain a given fraction of stock return volatility becomes lower as the interconnection among financial institutions increases. These components can thus be used to measure the interconnectedness across institutions. To measure not only the degree, but also the directionality of interconnections across financial institutions, Billio et al. (2012a) propose to use the statistical notion of Granger causality, which is based on the relative forecast power of two time series. More precisely, a series is said to Granger-cause another one when past values of the former contain information that helps to predict the latter. The authors use Granger causality to assess whether lagged returns of a financial institution have forecast power for present returns of another institution.

Both measures are calculated using stock returns of four types of financial institutions: hedge funds, banks, brokers/dealers, and insurance companies. The PCA shows interdependence among the four sectors at the height of the financial crisis; in addition, the Granger-causality tests highlight that this interdependence is asymmetric and that the returns of banks and insurance companies are found to have a higher impact on the returns of hedge funds and brokers/dealers than viceversa.

The analysis conducted by Billio et al. (2012a) on financial institutions is further developed in Billio et al. (2012b) along two directions. First, instead of stock returns, they use fair-value CDS spreads derived from expected loss ratios (ELRs), which are calculated by following a contingent claim analysis (CCA) approach.[7] Second, they extend the analysis to sovereigns in order to study the interconnections between sovereigns and financial institutions, as well as among different sovereigns.

7.2 Technical description

7.2.1 Principal component analysis

PCA is a non-parametric method in data analysis that allows valuable information to be extracted from a dataset which is noisy and redundant.[8] The dataset of Billio et al. (2012a) consists of the stock returns matrix R of the financial institutions under consideration. PCA operates a linear transformation of the original dataset R by re-expressing the data as a linear combination of its basis vectors. This transformation yields a decomposition of the variance-covariance matrix of R into an orthogonal matrix of eigenvectors, and a diagonal matrix of eigenvalues.

Let R_i be the stock return for institution i, $i = 1, \ldots, N$, $\mathbb{E}[R_i] = \mu$, and $\mathbb{V}ar[R_i] = \sigma_i^2$. Then, the variance of the financial system σ_S^2 is given by

$$\sigma_S^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j \mathbb{E}[z_i z_j], \tag{20}$$

where $z_k = \frac{R_k - \mu_k}{\sigma_k}$ is the standardized return of institution k. It is now possible to introduce N zero-mean uncorrelated variables ζ_k, whose variance-covariance matrix is

$$\mathbb{E}[\zeta_k \zeta_l] = \begin{cases} \lambda_k & \text{if } k = l, \\ 0 & \text{if } k \neq l, \end{cases} \tag{21}$$

where λ_k is the k-th eigenvalue, and for which all the higher-order co-moments are equal to those of the z_k. The standardized returns can then be expressed as a

[7] See Section 10 for an explanation of the CCA approach.
[8] For more details on PCA, see Jolliffe (2002).

linear combination of the ζ_k

$$z_k = \sum_{k=1}^{N} L_{i,k}\zeta_k, \tag{22}$$

where $L_{i,k}$ is a factor loading of ζ_k for institution i.

Therefore, the variance-covariance matrix can be decomposed into the orthonormal matrix of loadings and the diagonal matrix of eigenvalues

$$\begin{aligned}\mathbb{E}[z_i z_j] &= \mathbb{E}\left[\left(\sum_{k=1}^{N} L_{i,k}\zeta_k\right)\left(\sum_{l=1}^{N} L_{j,l}\zeta_l\right)\right] \\ &= \sum_{k=1}^{N}\sum_{l=1}^{N} L_{i,k}L_{j,l}\mathbb{E}[\zeta_k\zeta_l] = \sum_{k=1}^{N} L_{i,k}L_{j,k}\lambda_k,\end{aligned} \tag{23}$$

and the variance of the system becomes

$$\sigma_S^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} \sigma_i\sigma_j L_{i,k}L_{j,k}\lambda_k. \tag{24}$$

Billio et al. (2012a) build an indicator of interconnectedness based on the observation that only a few principal components are sufficient to explain the largest part of the variability of the stock returns. In particular, only the first $n < N$ eigenvalues explain most of the variation of the system, especially in crisis periods when the majority of returns tend to move together. Periods when the subset n of principal components explain more than some fraction H of the total volatility are indicative of increased interconnectedness between financial institutions. Once the total risk of the system has been defined as $\Omega = \sum_{k=1}^{N} \lambda_k$ and the risk associated with the first n principal components as $\omega_n = \sum_{k=1}^{n} \lambda_k$, it is possible to compare the ratio of the two, i.e. the cumulative risk fraction, to the pre-specified critical threshold H in order to capture periods of increased interconnectedness:

$$\frac{\omega_n}{\Omega} = h_n \geq H. \tag{25}$$

When the financial institutions of the system are highly interconnected, only a few principal components are able to explain a large proportion of its variability and, hence, h_n is more likely to exceed the threshold H. The rise of new linkages can then be detected through the time variations of h_n.

In addition to the cumulative risk fraction, Billio et al. (2012a) define a univariate measure of connectedness for each company i, the principal component analysis systemic risk measure, PCAS, as follows:

$$\text{PCAS}_{i,n} = \frac{1}{2}\frac{\sigma_i^2}{\sigma_S^2}\frac{\partial \sigma_S^2}{\partial \sigma_i^2}\bigg|_{h_n \geq H}. \tag{26}$$

$\text{PCAS}_{i,n}$ captures the contribution of institution i to systemic risk, conditional on the condition $h_n \geq H$ being verified, i.e. conditional on a strong common component across the returns of financial institutions. Billio et al. (2012a) show that this measure also corresponds to the exposure of institution i to the total risk of the system. The latter is measured as the weighted average of the square of the factor loadings of the single institution i to the first n principal components, where the weights are the eigenvalues:

$$\text{PCAS}_{i,n} = \frac{1}{2}\frac{\sigma_i^2}{\sigma_S^2}\frac{\partial \sigma_S^2}{\partial \sigma_i^2}\bigg|_{h_n \geq H} = \sum_{k=1}^{n}\frac{\sigma_i^2}{\sigma_S^2}L_{ik}^2\lambda_k\bigg|_{h_n \geq H}. \tag{27}$$

Since the focus is on endogenous risk, $\text{PCAS}_{i,n}$ measures both the contribution and the exposure of the i-th institution to the overall risk of the system, given a strong common component across the returns of all institutions.

7.2.2 Granger-causality tests

Granger-causality tests can be used to identify the directionality of interlinkages between financial institutions. The time series x is said to Granger-cause the time series y if past values of x help to predict future values of y. In Billio et al. (2012a) the time series taken into account are the stock returns of two financial institutions, R_i and R_j,

$$R_{i,t+1} = a_i R_{i,t} + b_j R_{j,t} + e_{i,t+1} \tag{28}$$

$$R_{j,t+1} = a_j R_{j,t} + b_i R_{i,t} + e_{j,t+1}, \tag{29}$$

where $e_{i,t+1}$ and $e_{j,t+1}$ are uncorrelated white noise processes and a_i, a_j, b_i, and b_j are the parameters of the model. The number of lags included in the model is chosen on the basis of the Bayesian information criterion (BIC). If b_j is significantly different from zero, j is said to Granger-cause i. If b_i is significantly different from zero, i is said to Granger-cause j. If both coefficients are significantly different from zero when an F-test is performed, there is a feedback relationship between i and j. Billio et al. (2012a) then define an indicator of causality $(j \to i)$, which is 1 if j Granger-causes i and 0 otherwise. Starting from this indicator, the authors build several network-based measures as follows:

1. *Degree of Granger Causality*: it is the fraction of statistically significant Granger causality relationships among all the pairs of financial institutions.

2. *Number of Connections*: it measures the number of institutions that are Granger-caused by a given institution j, the number of institutions that Granger-cause institution j, and the sum of the previous two measures.

3. *Sector Conditional Connections*: it is similar to the previous indicator, but it is conditional on the type of financial institution.

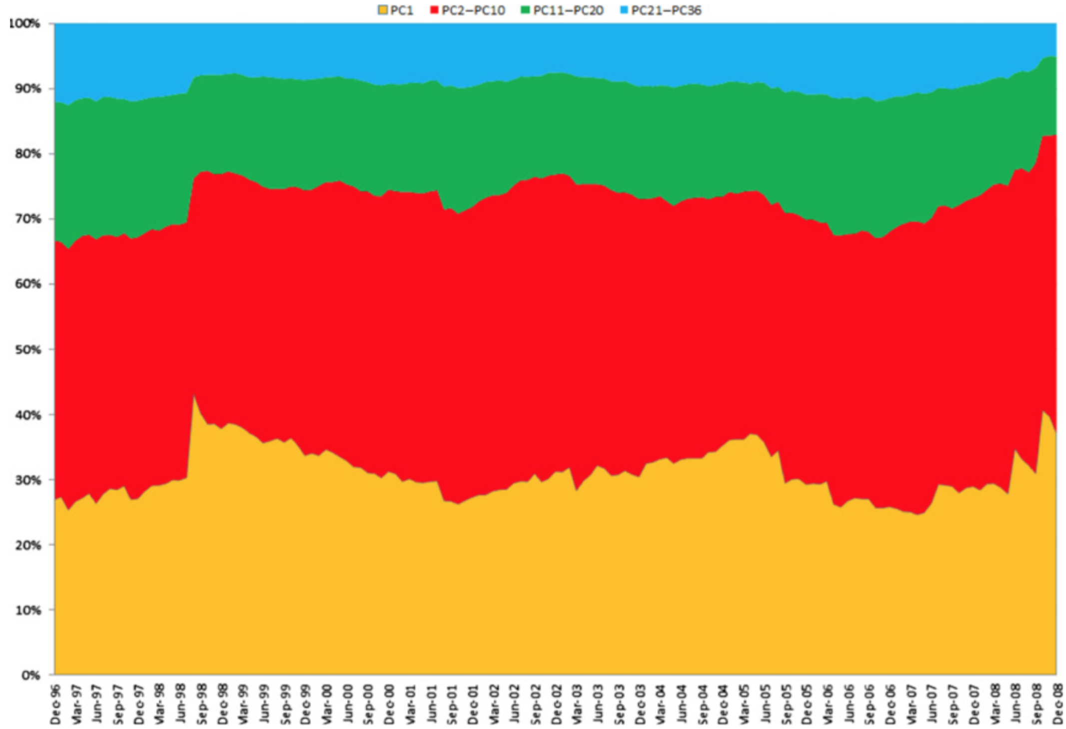
4. *Closeness*: it 'measures the shortest path between a financial institution and all other institutions reachable from it, averaged across all other financial institutions'. In other words, it measures the average of the shortest distance between each pair of institutions after having defined a concept of distance in terms of causality path.

5. *Eigenvector Centrality*: it 'measures the importance of a financial institution in a network by assigning relative scores to financial institutions based on how connected they are to the rest of the network'. It does so by defining an adjacency matrix and then by calculating its eigenvectors.

7.3 Example

Figure 5 displays 36-month rolling windows for the cumulative risk fraction ω_n/Ω over the period January 1994 – December 2008. The financial institutions used in the sample are the hedge funds included the Lipper TASS database, and the banks, brokers/dealers, and insurers in the University of Chicago's Center for Research in Security Prices Database. The graph shows that the variability explained by the first principal component (PC1) – the yellow area – rises remarkably in correspondence with the LTCM crisis in 1998 and in 2005, when the Fed raises interest rates. It then peaks in 2008, when PC1 alone is able to explain almost 40 per cent of the variability of stock returns. From the figure it is also clear that 20 out of 36 PCs are enough to explain roughly 90 per cent of the stock return variability, and even more than 90 per cent in the last part of the sample.

Figure 5 – Principal components



Source: Billio et al. (2012a).

8 Option-iPoD

Table 9 – Option-iPoD: Main characteristics

Author(s)	Capuano (2008)
Institution(s)	IMF (2009b)
Pros	Makes use of non-parametric density function Default barrier determined endogenously
Cons	Gives no information when in the default state
Data	Equity options

8.1 General description

Introduced by Capuano (2008), the option-implied probability of default (Option-iPoD) is a market-based indicator that measures the probability of default for an institution based on the prices of its equity options.

In this approach, the probability of default is defined as the probability that the value of a firm's assets drop below a threshold level (the default barrier). To estimate this probability, both the default barrier and the density function of the asset value are needed. Since equity options are used to estimate the two components, the probability of default measure proposed by Capuano (2008) is called Option-iPoD. More specifically, the model is based on Merton's balance sheet approach (Merton, 1973) according to which the value of equity can be seen as a call option on the firm's assets with strike price value equal to the on-balance sheet liabilities.

The advantages of the Option-iPoD are twofold: first, it does not make any distributional assumption on the asset value, which is estimated non-parametrically; and second, the default barrier is estimated endogenously. The Option-iPoD is calculated by applying the principle of minimum cross-entropy and by solving a constrained optimization problem. Cross-entropy 'can be interpreted as a measure of relative distance between the prior and the posterior density function of the value of an asset' (Capuano, 2008).

The main limitation of the Option-iPoD is that it is not able to describe the probability density function of the value of assets in the default state, i.e. the state when the asset value V_T is lower than the default barrier D. Therefore, Capuano (2008) proposes an extension of the model that uses zero-coupon bonds in place of options, given that bonds are senior claims with respect to equity and their prices also provide information on the default state. Nonetheless, even though this extension may be theoretically appealing, it has some empirical limitations, such as the mismatch between the maturity of the zero-coupon bonds and that of the option contracts, or the existence of several types of bonds of the same company with a different seniority structure. In practice, these limitations hamper the actual implementability of the proposed extension.

8.2 Technical description

The Probability of Default (PoD) is defined by Capuano (2008) as

$$\mathrm{PoD}(X) = \int_0^X f_V(v)\mathrm{d}v \tag{30}$$

where V is the value of an asset, f_V is the probability density function of V, and X is the default barrier, i.e. the value below which a financial institution defaults. The goal is to estimate X and f_V. In order to do this, equity call options are used.

In general, the payoff of a call option written on an asset is given by

$$C_T = \max(A_T - K; 0), \tag{31}$$

where K is the strike price and A_T is the price of the asset at the maturity date T. In particular, when a call option is written on a stock, its payoff is

$$C_T = \max(E_T - K; 0), \tag{32}$$

where E_T is the price of the stock at the maturity date T. Since Capuano (2008) adopts the balance sheet approach, a stock can be seen as an option on the firm's assets with a strike price equal to the on-balance sheet value of liabilities:

$$E_T = \max(V_T - D; 0) \tag{33}$$

where V_T is the value of the firm's assets and D is the value of the debt. Therefore, an option on a stock can be seen as an option on an option:

$$\begin{aligned} C_T &= \max(E_T - K; 0) \\ &= \max(\max(V_T - D; 0) - K; 0) \\ &= \max(V_T - D - K; 0). \end{aligned} \tag{34}$$

In order to calculate the Option-iPoD(D), it is necessary to estimate D, i.e. the default threshold that corresponds to the value of debt, and the probability that the value of the assets V_T will fall below D. In order to solve for PoD(D), Capuano (2008) uses the principle of the minimum cross-entropy, which is a generalization of the principle of maximum entropy introduced by Kullback and Leibler (1951).

The objective function of the optimization problem is given by:

$$\min_D \left(\min_{f(V_T)} \int_{V_T=0}^{\infty} f(V_T) \log \left[\frac{f(V_T)}{f_0(V_T)} \right] \mathrm{d}V_T \right), \tag{35}$$

where $f_0(V_T)$ is the prior probability density function of the value of the assets, representing the prior knowledge of $f(V_T)$, which is the posterior density. The function to be minimized is the relative-entropy between $f_0(V_T)$ and $f(V_T)$, that is the uncertainty around $f(V)$. This framework also envisages the particular case in which there is no prior knowledge and which corresponds to the assumption of $f_0(V_T)$ being uniform. The minimization is subject to three constraints:

1. Balance sheet constraint

$$\begin{aligned} E_0 &= \mathrm{e}^{-rT} \int_{V_T=0}^{\infty} \max(V_T - D; 0) f(V_T) \mathrm{d}V_T \\ &= \mathrm{e}^{-rT} \int_{V_T=D}^{\infty} (V_T - D) f(V_T) \mathrm{d}V_T, \end{aligned} \tag{36}$$

i.e. the theoretical value of the stock price has to correspond to the value of the stock price today, E_0;

2. Observable options price constraint

$$\begin{aligned} C_0^i &= \mathrm{e}^{-rT} \int_{V_T=0}^{\infty} \max(V_T - D - K; 0) f(V_T) \mathrm{d}V_T \\ &= \mathrm{e}^{-rT} \int_{V_T=D+K}^{\infty} (V_T - D - K_i) f(V_T) \mathrm{d}V_T, \end{aligned} \tag{37}$$

i.e. the posterior probability density has to be able to price the observable option prices. In other words, the present value of the expected call option payoff at the maturity date must correspond to the price of the call option observed today, C_0^i;

3. Normalization constraint

$$1 = \int_{V_T=0}^{\infty} f(V_T) \mathrm{d}V_T, \tag{38}$$

i.e. the probability density function must integrate to 1.

The problem is first solved by finding $f(V_T)$ as a function of D and then, given the optimal $f(V_T)$, is solved for D. Once D is found, it can be substituted back into the optimal equation for $f(V_T)$. The Lagrangian is given by:

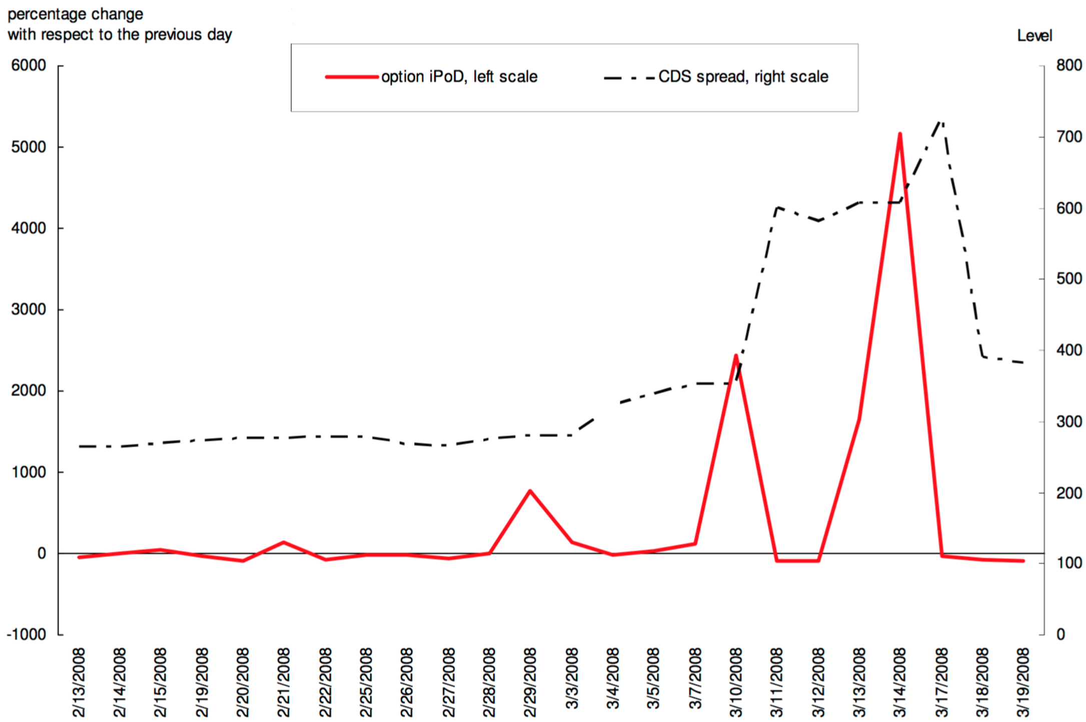
$$\begin{aligned} L = &\int_{V_T=0}^{\infty} f(V_T) \log\left[\frac{f(V_T)}{f_0(V_T)}\right] \mathrm{d}V_T + \lambda_0 \left[1 - \int_{V_T=0}^{\infty} f(V_T)\mathrm{d}V_T\right] \\ &+ \lambda_1 \left[E_0 - \mathrm{e}^{-rT} \int_{V_T=D}^{\infty} (V_T - D) f(V_T) \mathrm{d}V_T\right] \\ &+ \sum_{i=1}^{n} \lambda_2 \left[C_0^i - \mathrm{e}^{-rT} \int_{V_T=D+K}^{\infty} (V_T - D - K_i) f(V_T) \mathrm{d}V_T\right]. \end{aligned} \tag{39}$$

The FOC for $f(V_T)$ is found by equalizing the Fréchet derivative of L to zero. The solution $f^*(V_T, D)$ is a function of the default barrier D. To solve for D it is possible to substitute $f^*(V_T, D)$ back into Equation (39). The optimal D is determined by:

$$\lim_{\Delta \to 0} \frac{L(f^*(V_T, D + \Delta)) - L(f^*(V_T, D))}{D + \Delta} = 0 \tag{40}$$

In order to calculate Option-iPoD empirically, it is necessary to have at least two option contracts: one is used to solve the first optimization problem in Equation (35) to shape the probability density function $f^*(V_T, D)$, while the other is used

Figure 6 – Option-iPoD for Bear Sterns



Source: Capuano (2008).

to pin down D so that it satisfies the constraint in Equation (37) and solves the second optimisation problem in Equation (35).

It is also possible to solve the problem for options with different maturities so as to infer a term structure of Option-iPoDs.

8.3 Example

Figure 6 is taken from Capuano (2008) and shows the Option-iPoD and the CDS spread for Bear Sterns for the period from 12 February 2008 to 19 March 2008. The collapse of Bear Sterns took place on 14 March 2008. The Option-iPoD started showing some variations on 21 February. It then displayed a first peak on 29 February, which was followed by a week of calm. On 10 March the Option-iPoD of Bear Sterns peaked at a value which was four times bigger than the first spike recorded on 29 February. The maximum level was reached on 14 March, but decreased over the days following the Fed's announcement of a rescue plan. It is worth observing that in the sample analysed, Option-iPoD appears to be a leading indicator compared with the CDS spread.

9 Joint Distress Indicators

Table 10 – Joint Distress Indicators: Main characteristics

Author(s)	Segoviano and Goodhart (2009)
Institution(s)	ECB (2008, 2010b) IMF (2009b, 2011b) BoI (2011)
Pros	Embeds linear and non-linear time-varying dependence Uses non-parametric density function Measures banks' interdependence structure
Cons	CDS may overshoot, as they also incorporate liquidity risk and risk aversion into the financial system
Data	CDS spreads

9.1 General description

Segoviano and Goodhart (2009) present a set of joint distress indicators that are built upon the concept of the Banking System Multivariate Density (BSMD). In particular, they treat the banking system as a portfolio of banks and they estimate its multivariate density. From the latter they calculate four indicators of joint distress:

1. The *Joint Probability of Distress* (JPoD), which measures 'the probability of all banks in the system (portfolio) becoming distressed, i.e. the tail risk of the system';

2. The *Banking Stability Index* (BSI), which 'reflects the expected number of banks becoming distressed given that at least one bank has become distressed';

3. The *Distress Dependence Matrix* (DiDe), which is a matrix that 'contains the probability of distress for the bank specified in the row given that the bank specified in the column becomes distressed';

4. The *Probability of Cascade Effects* (PCE), which is the probability that 'at least one bank becomes distressed, given that a specific bank becomes distressed'.

These indicators analyse the stability of the system from different perspectives, namely the common distress of the banks in the system (JPoD), the distress between specific banks (DiDe), and the distress of the system generated by a given bank (BSI and PCE).

A major advantage of these indicators is that they are able to capture both linear and non-linear distress dependencies among the banks in the system. In addition, the linear and non-linear dependencies are allowed to change throughout the economic cycle. Given their ability to capture non-linear time-varying distress dependencies, these indicators have been used, among others, by the European Central Bank (ECB, 2008, 2010b) and the Bank of Italy (BoI, 2011).

9.2 Technical description

In order to calculate the four indicators, it is necessary to recover the BSMD. Section 9.2.1 describes how to calculate the BSMD, while Section 9.2.2 reports more technical details for each indicator.

9.2.1 How to calculate the Banking System Multivariate Density

Segoviano and Goodhart (2009) propose a minimum cross-entropy approach, which is the same methodology used to calculate the Option-iPoD described in Section 8. According to this approach, the cross-entropy between a prior and a posterior joint probability density function is minimized with respect to the posterior joint density function. In particular the minimization problem is:

$$\min_{p(x_1,\ldots,x_n)} \int \cdots \int p(x_1,\ldots,x_n) \log \frac{p(x_1,\ldots,x_n)}{q(x_1,\ldots,x_n)} \mathrm{d}x_1 \ldots \mathrm{d}x_n, \tag{41}$$

where $(x_1,\ldots,x_n)$ are the logarithmic asset returns of the n banks that make up the banking system, $p(x_1,\ldots,x_n)$ is the posterior joint density function of the portfolio of the banks' asset returns, and $q(x_1,\ldots,x_n)$ is the parametric prior joint density function of the same portfolio. The minimization problem is subject to the following constraints:

1. *Probability of default (PoD) constraints*:

$$\mathrm{PoD}_t^i = \int \cdots \int p(x_1,\ldots,x_n) 1_{x_i \geq x_i^d} \mathrm{d}x_1 \ldots \mathrm{d}x_n, \tag{42}$$

   $\forall i = 1,\ldots,n$, where $1_{x_i \geq x_i^d}$ is the indicator function that is equal to 0 when bank i's asset return is below its default threshold x_i^d and 1 otherwise. These constraints on the PoD are necessary to guarantee that the posterior joint density function is consistent with the empirically estimated PoDs.

2. *Normalization constraint*:

$$\int \cdots \int p(x_1, \ldots, x_n) \mathrm{d}x_1 \ldots \mathrm{d}x_n = 1, \tag{43}$$

i.e. the BSMD has to integrate to 1.

The Lagrangian associated with the minimization problem is:

$$\begin{aligned} L = & \int \cdots \int p(x_1, \ldots, x_n) \log \frac{p(x_1, \ldots, x_n)}{q(x_1, \ldots, x_n)} \mathrm{d}x_1 \ldots \mathrm{d}x_n \\ & + \sum_{i=1}^{n} \lambda_i \left[\int \cdots \int p(x_1, \ldots, x_n) 1_{x_i \geq x_i^d} \mathrm{d}x_1 \ldots \mathrm{d}x_n - \mathrm{PoD}_t^i \right] \\ & + \mu \left[\int \cdots \int p(x_1, \ldots, x_n) \mathrm{d}x_1 \ldots \mathrm{d}x_n - 1 \right]. \end{aligned} \tag{44}$$

The posterior joint density function is obtained by solving the problem through the calculus of variations:

$$p(x_1, \ldots, x_n) = q(x_1, \ldots, x_n) \exp \left\{ - \left(1 + \mu + \sum_{i=1}^{n} \lambda_i 1_{x_i \geq x_i^d} \right) \right\}. \tag{45}$$

It is important to observe that the calculation of $p(x_1, \ldots, x_n)$ requires several inputs. First of all, a prior multivariate distribution and a default barrier for each bank must be given. The authors assume a standard normal distribution as prior and follow Segoviano (2006) for the calculation of the default barrier. In addition, a PoD for each bank has to be empirically estimated. Several methods can be used for this estimation. Segoviano and Goodhart (2009) discuss the advantages and disadvantages of different methodologies, namely the structural approach, CDS spreads, and out-of-the-money option prices. They conclude that the least flawed methodology is the estimation using CDS spreads, which does not require the distribution of asset prices to be modelled or volatilities to be explicitly estimated. However, the main drawback of CDS spreads is that they may also incorporate the liquidity risk in the CDS market as well as a generalized risk aversion in the financial market.

9.2.2 The four banking stability indicators

Once BSMD is obtained, the four distress dependence indicators can be calculated:

1. The *Joint Probability of Distress* (JPoD) is the probability that all banks in the system become distressed at the same time. It represents the tail risk of the system and captures the linear as well as the non-linear changes in the distress dependence among banks:

$$\mathrm{JPoD}_t = \int_{x_1^d}^{\infty} \cdots \int_{x_n^d}^{\infty} p(x_1, \ldots, x_n) \mathrm{d}x_1 \ldots \mathrm{d}x_n. \tag{46}$$

2. The *Banking Stability Index* (BSI) is the expected number of banks in distress given that one bank has become distressed:[9]

$$\mathrm{BSI} = \frac{\sum_{i=1}^{n} \mathbb{P}(x_i \geq x_i^d)}{1 - \mathbb{P}(x_1 < x_1^d, \ldots, x_n < x_n^d)}. \tag{47}$$

3. The *Distress Dependence Matrix* (DiDe) presents the pairwise conditional probabilities of distress. The conditional probabilities do not imply causation but they highlight the linkages between pairs of banks. For each entry (i, j) of the matrix, the conditional probability of bank i being in distress given that j is in distress is:

$$\mathbb{P}\left(x_i \geq x_i^d \mid x_j \geq x_j^d\right) = \frac{\mathbb{P}\left(x_i \geq x_i^d, x_j \geq x_j^d\right)}{\mathbb{P}\left(x_j \geq x_j^d\right)}. \tag{48}$$

4. The *Probability of Cascade Effects* (PCE) gauges the cascade effects on the system of a specific bank being in distress. In the case of four banks R, X, Y, and Z, if we suppose that bank R becomes distressed, the PCE is defined as:

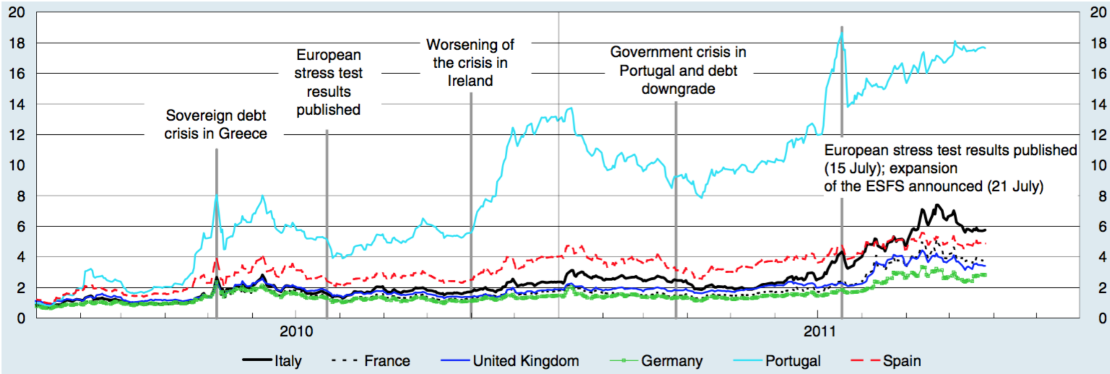
$$\begin{aligned} \mathrm{PCE} =& \mathbb{P}(X \mid R) + \mathbb{P}(Y \mid R) + \mathbb{P}(Z \mid R) + \mathbb{P}(X \cap Y \cap Z \mid R) \\ & - [\mathbb{P}(X \cap Y \mid R) + \mathbb{P}(X \cap Z \mid R) + \mathbb{P}(Y \cap Z \mid R)]. \end{aligned} \tag{49}$$

9.3 Example

Figure 7 and Figure 8 show the JPoD and the BSI of a sample of large international banks over the period 1 January 2010 - 31 October 2011. In particular, Figure 7 shows the JPoD for a set of intermediaries from six European countries: Italy, France, Germany, Portugal, Spain and the United Kingdom. A first peak in the JPoD of each bank is registered in correspondence with the sovereign debt crisis

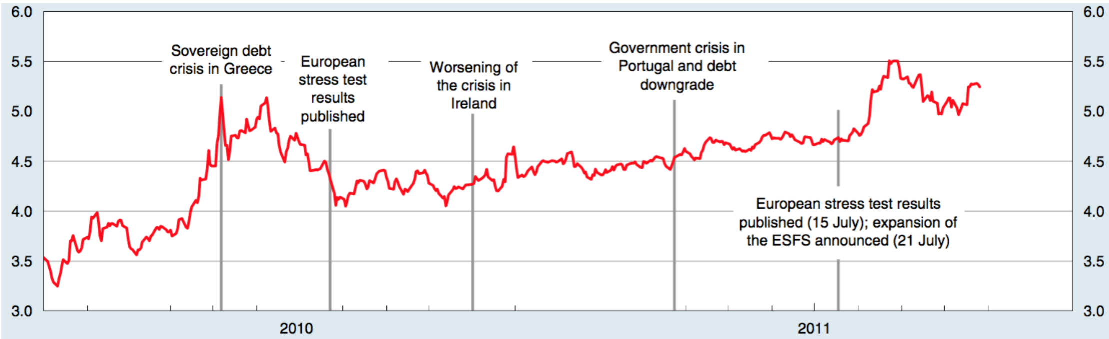
[9] A similar indicator is calculated by the Bank of Italy in its Financial Stability Report (BoI, 2010). The main difference is that the Bank of Italy uses stock returns instead of CDS spreads. In particular, the Bank of Italy calculates the expected number of banks that have stock returns lower than the 5th percentile of their distribution, given that at least one bank has returns below that threshold.

Figure 7 – JPoD of large European banks



Source: BoI (2011).
Daily data, per cent. The banks included in the sample are the following. France: BNP Paribas and Société Générale; Germany: Deutsche Bank and Commerzbank; Italy: UniCredit and Intesa Sanpaolo; Portugal: Banco Espirito Santo and Banco Comercial Português; Spain: Santander and Banco Bilbao Vizcaya Argentaria; and United Kingdom: Barclays and Royal Bank of Scotland.

Figure 8 – BSI of ten large cross-border banking groups



Source: BoI (2011).
Daily data. The ten banking groups are: UniCredit, Intesa Sanpaolo, Banca Monte dei Paschi di Siena, Santander, BNP-Paribas, Deutsche Bank, UBS, Barclays, Lloyds TSB and Citigroup.

in Greece. The Portuguese JPoD is the highest of all countries, reaching a value of 14 per cent by the end of 2010. It declines slightly at the beginning of 2011, before climbing up to roughly 19 per cent in July 2011. The JPoDs of the other countries also increase in the second half of 2011, but they only reach their peaks in October. Figure 8 reports the BSI for ten large European banking groups. The indicator spikes to 5 with the sovereign debt crisis in Greece. It is even higher in August 2011 after the publication of the European stress tests, when it peaks at 5.5, a value that it had reached only after the Lehman Brothers had gone bankrupt.

10 Systemic Contingent Claim Analysis (Systemic CCA)

Table 11 – Systemic CCAs: Main characteristics

Author(s)	Gray and Jobst (2013)
Institution(s)	IMF (2009b)
Pros	Integrates market-implied expected losses in a multivariate specification of joint default risk Default barrier determined endogenously
Cons	Not easy to calculate Its validity depends on the underpinning valuation models
Data	Balance sheet data on outstanding liabilities, and market data on equity and equity options

10.1 General description

Systemic CCA was proposed by Gray and Jobst (2013) as a new measure of systemic risk founded on two main theories: CCA and extreme value theory (EVT). It has been defined as 'a forward-looking, market data-based analytical framework for measuring systemic solvency risk by means of a multivariate extension to CCA paired with the concept of extreme value theory'. CCA is a generalization of the option pricing theory pioneered by Black and Scholes (1973) and Merton (1973), which is necessary in order to estimate each firm's expected losses. EVT is used to combine the individual firms' expected losses through a dependence measure in order to derive the joint expected losses as a measurement of systemic riskiness.

More in detail, CCA is a risk-adjusted balance-sheet framework based on three main principles: i) the values of liabilities can be derived from the value of assets; ii) assets follow a stochastic process; and iii) liabilities have different seniorities. These principles allow one to compute the value of equity as the value of an implicit call option on assets, and the value of risky debt as the difference between default-free debt and a guarantee against default. This guarantee can be calculated as the value of a put option on assets. The value of the put option measures the expected losses of a financial institution. Once the expected losses of individual institutions are estimated, Gray and Jobst (2013) propose to aggregate them using a time-varying dependence structure through EVT. In this framework, the financial sector is viewed as a portfolio of individual expected losses in which the marginal distributions of individual expected losses are combined with a time-varying dependence structure to generate the multivariate distribution of joint expected losses of all sample firms.

CCA can be used to perform stress tests and assess capital adequacy. According to Gray and Jobst (2013), the main strengths of Systemic CCA are that it can be used to derive market-implied expected losses and that it endogenizes the loss given default. On the other hand, the major drawbacks are that it is not easy to calculate and that it relies on the specification of the option pricing model. Therefore, a flaw in the specification of the latter would invalidate the estimation of the systemic indicator.

10.2 Technical description

Several steps are needed to estimate Systemic CCA:

1. Calculating the financial institutions' expected losses through CCA;
2. Estimating the marginal distribution of expected losses through EVT;
3. Estimating the dependence structure of individual expected losses;
4. Estimating the joint distribution of expected losses;
5. Estimating a tail risk measure for joint expected losses.

Once the joint potential losses have been estimated, it is possible to calculate their individual contributions.

Starting from the first step, we go through the estimation procedure for the Systemic CCA.

1. *Calculating the financial institutions' expected losses through CCA*

   The market values of assets and debts usually differ from their accounting values and can rarely be observed. However, the market value of equity is easily observable. CCA derives the market value of assets and debts from the market value of equity through the option pricing theory. In particular, given that shareholders have a residual claim on assets once all the outstanding debt has been paid, equity can be thought of as a call option on assets with a strike price equal to the accounting value of outstanding debt. Similarly, risky debt, i.e. debt that is not default-free, can be modelled as an implicit put option on assets, with a strike value equal to the balance sheet value of debts. The cost of the put option is reflected by the credit spread above the risk-free rate, which compensates bondholders for holding risky debt.

The risk-adjusted balance sheet is given by:

$$A_t = D_t + E_t, \tag{50}$$

where A_t is the implied market value of assets, D_t is the implied market value of debts and E_t is the observable market value of equity. In order to estimate the expected loss, it is important to observe that the value of the put option rises with the increase in the probability of the implied asset value falling below the present value of debt, i.e. the default barrier, over a pre-defined horizon (Gray and Jobst, 2013). Hence, the expected losses are estimated according to the Black-Scholes-Merton pricing equation for a put option over the time horizon $T - t$:

$$P_E(t) = Be^{-r(T-t)}\Phi(-d - \sigma_A\sqrt{T-t}) - A(t)\Phi(-d), \tag{51}$$

where r is the risk-free discount rate, B is the present value of debt and the strike price of the option on the asset value $A(t)$, Φ is the normal cumulative distribution, and d is the leverage:

$$d = \frac{\left(\log\frac{A(t)}{B}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{T-t}}. \tag{52}$$

The asset volatility σ_A can be found as follows. The asset value $A(t)$ at time t is assumed to evolve under the risk-neutral probability measure Q according to the following stochastic differential equation:

$$\mathrm{d}A(t) = A(t)r\mathrm{d}t + A(t)\sigma_A\mathrm{d}W_Q(t), \tag{53}$$

where the diffusion is defined by a standard geometric Brownian motion $\Delta W_Q(t) \sim \phi(0, \Delta t)$. Since equity $E(t)$ is a function of assets, it is possible to derive an expression for the diffusion process of $E(t)$ by using the Itô-Döblin theorem:

$$\begin{aligned}
\mathrm{d}E(t) &= \frac{\partial E}{\partial A}\mathrm{d}A(t) + \frac{1}{2}\frac{\partial^2 E}{\partial A^2}\mathrm{d}A^2(t)\sigma_A^2\mathrm{d}t \\
&= \frac{\partial E}{\partial A}\left(A(t)r\mathrm{d}t + A(t)\sigma_A\mathrm{d}W_Q(t)\right) + \frac{1}{2}\frac{\partial^2 E}{\partial A^2}\mathrm{d}A^2(t)\sigma_A^2\mathrm{d}t \\
&= \left(\frac{\partial E}{\partial A}(A(t)r + \frac{1}{2}\frac{\partial^2 E}{\partial A^2}A^2(t)\sigma_A^2\right)\mathrm{d}t + \frac{\partial E}{\partial A}A(t)\sigma_A\mathrm{d}W_Q(t).
\end{aligned} \tag{54}$$

Equity $E(t)$ is also assumed to follow a log-normal process:

$$\mathrm{d}E(t) = E(t)r\mathrm{d}t + E(t)\sigma_E\mathrm{d}W_Q(t), \tag{55}$$

where σ_E is the observable equity volatility. By matching both addends in the right-hand side of Equation (54) and Equation (55), we get

$$E(t)r = \frac{\partial E}{\partial A}A(t)r + \frac{1}{2}\frac{\partial^2 E}{\partial A^2}A^2(t)\sigma_A^2 \tag{56}$$

and

$$E(t)\sigma_E = \frac{\partial E}{\partial A}A(t)\sigma_A. \tag{57}$$

It is possible to solve Equation (57) for the asset volatility σ_A:

$$\sigma_A = \frac{E(t)\sigma_E}{A(t)\Phi(d)} = \left[1 - \frac{Be^{-r(T-t)}\Phi(d - \sigma_A\sqrt{T-t})}{A(t)\Phi(d)}\right]\sigma_E, \tag{58}$$

where the latter equality is given by the equation for the equity value $E(t)$ as the value of an implicit call option on the assets $A(t)$.

2. *Estimating the marginal distribution of expected losses through EVT*

   The marginal distribution of expected losses is derived by means of EVT. Let

$$X^n = \left(P^n_{E,1}(t), \ldots, P^n_{E,m}(t)\right) \tag{59}$$

   be the vector of independent and identically distributed observations of expected losses (i.e. a total of n daily put option values $P^n_{E,m}(t)$ up to time t) estimated according to Equation (51) for m financial institutions. By following a parametric approach and assuming i.i.d. observations of expected losses, the asymptotic tail behaviour of each financial institution is modelled according to the Fisher-Tippett-Gnedenko theorem (Fisher and Tippett, 1928; Gnedenko, 1943). This theorem states that the maximum of a sample of i.i.d. normalized random variables converges in distribution to one of the following three types of distributions: Gumbel, Fréchet or Weibull. Therefore, the distribution of the normalized component-wise maximum of X^n converges to one of the abovementioned distributions. These three distributions can be combined into a unitary framework, which is called the Generalized Extreme Value (GEV) distribution (Jenkinson, 1955):

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} \exp\left[-(1+\xi\frac{x-\mu}{\sigma})^{-1/\xi}\right], & \text{if} \quad \xi \neq 0 \quad \text{and} \quad 1 + \frac{\xi(x-\mu)}{\sigma} > 0, \\ \exp[-\exp(-\frac{x-\mu}{\sigma})], & \text{if} \quad \xi = 0. \end{cases} \tag{60}$$

   From the GEV it is possible to derive the three limiting distributions:

   - for $\xi = 0$ the Gumbel family is obtained;

- for $\xi \geq 0$ the Fréchet family is obtained;
- for $\xi \leq 0$ the Weibull family is obtained.

The i^{th} univariate marginal density function of each series of expected losses is:

$$y_i(x) = \left(1 + \hat{\xi}_i \frac{x - \hat{\mu}_i}{\hat{\sigma}_i}\right)^{-1/\hat{\xi}_i}, \tag{61}$$

for $i = 1, \ldots, m$, where the argument in parenthesis is non negative, $\hat{\sigma}_i \geq 0$ is the scale parameter, $\hat{\mu}_i$ is the location parameter, and $\hat{\xi}_i$ is the shape parameter. The three unknown parameters $(\hat{\mu}, \hat{\sigma}, \hat{\xi})$ can be estimated through maximum likelihood.

3. *Estimating the dependence structure of individual expected losses*

   The dependence function between the marginal distributions of the expected losses is given by a multivariate extension of Pickands' bivariate logistic method (Pickands, 1981) with margins adjusted according to Hall and Tajvidi (2000):

$$\Upsilon_t(\omega) = \min\left\{1, \max\left\{n\left(\sum_{i=1}^{n} \Lambda_{j=1}^{m} \frac{y_{i,j}/\hat{y}_{\cdot,j}}{\omega_j}\right)^{-1}, \omega, 1-\omega\right\}\right\}, \tag{62}$$

   where $\hat{y}_{\cdot,j} = \sum_{i=1}^{n} y_{i,j}/n$ is the average marginal density of all put options $i \in n$ and $\Upsilon(\omega_j)$ is such that $0 \leq \max\{\omega_1, \ldots, \omega_{m-1}\} \leq \Upsilon(\omega_j) \leq 1$, for all $0 \leq \omega_j \leq 1$. $\Upsilon(\cdot)$ is a convex function on $[0, 1]$ with $\Upsilon(0) = \Upsilon(1) = 1$.

4. *Estimating the joint distribution of expected losses*

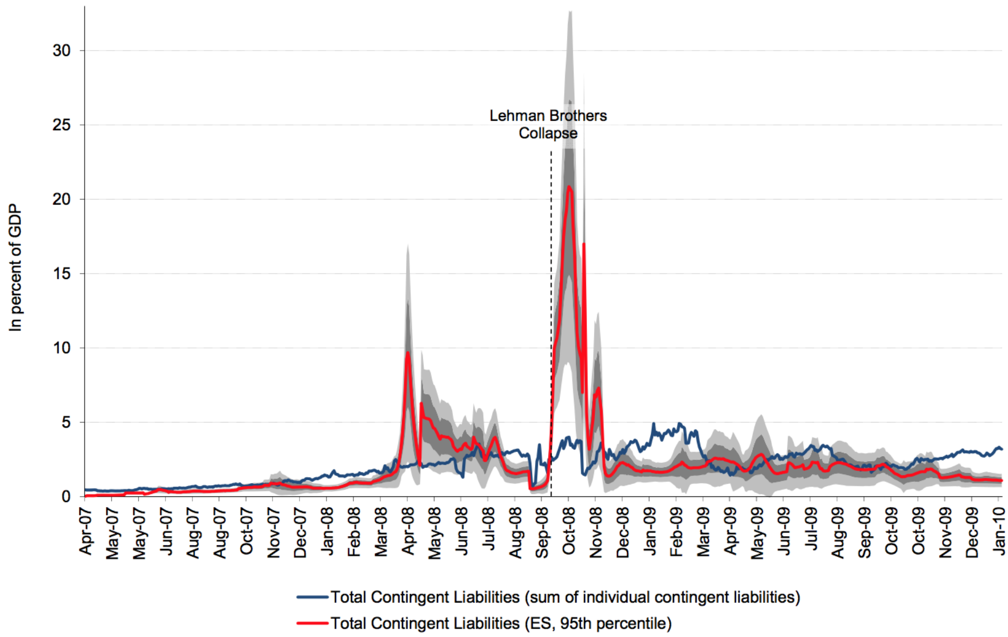
   The joint distribution of the expected losses is obtained by combining the marginal distributions with their dependence structure in accordance with Sklar's theorem (Sklar, 1959) on constructing joint distributions with arbitrary marginal distribution functions via copula functions. The resulting cumulative distribution function $G_{t,m}(x)$ and probability density function $g_{t,m}(x)$ are:

$$G_{t,m}(x) = e^{-\left(\sum_{j=1}^{m} y_{t,j}\right)\Upsilon_t(\omega)}, \tag{63}$$

$$g_{t,m}(x) = \hat{\sigma}_{t,m}^{-1}\left[\left(\sum_{j=1}^{m} y_{t,j}\right)\Upsilon_t(\omega)\right]^{\hat{\xi}_{t,m}+1} e^{-\left(\sum_{j=1}^{m} y_{t,j}\right)\Upsilon_t(\omega)}, \tag{64}$$

   where $\hat{\xi}_{t,m}$ is the shape parameter and $\hat{\sigma}_{t,m}$ is the scale parameter.

Figure 9 – Systemic CCA estimates for the United States



Source: Gray and Jobst (2013).

5. *Estimating a tail risk measure for joint expected losses*

   Finally, as a measure of conditional tail expectation, Gray and Jobst (2013) compute the joint expected shortfall as follows:

$$\mathrm{ES}_{t,m,a} = \mathbb{E}\left[\zeta | \zeta \geq G_{t,m}^{-1}(a) = \mathrm{VaR}_{t,a}\right] \tag{65}$$

   where $\zeta \in \mathbb{R}$, $G_{t,m}^{-1}(a)$ is the point estimate of the joint potential losses of m financial institutions at quantile $1-a$ and time t, and is given by:

$$G_{t,m}^{-1}(a) = \hat{\mu}_{t,m} + \frac{\hat{\sigma}_{t,m}}{\hat{\xi}_{t,m}}\left[\left(-\frac{\log(a)}{\Upsilon_t(\omega)}\right)^{-\hat{\xi}_{t,m}} - 1\right], \tag{66}$$

   and $\mathrm{VaR}_{t,a}$ is:

$$\mathrm{VaR}_{t,a} = \sup\left\{G_{t,m}^{-1}(a) | \mathbb{P}\left[\zeta > G_{t,m}^{-1}(a)\right] \geq a\right\}. \tag{67}$$

10.3 Example

Figure 9 exhibits the total contingent liabilities of thirty-six US financial institutions. The blue line shows the total level of contingent liabilities, which reflects 'the concurrent realization of individual distress at an average degree of severity without consideration of their conditional probability of default'. However, when estimating the systemic risk arising from the joint probability of default, it becomes essential to consider the intertemporal changes in the dependence structure of risk-adjusted default risk. This is taken into account by the red line, which shows the expected shortfall for the sample of the same thirty-six US financial institutions at the 95$^{\text{th}}$ percentile threshold within a confidence band of one and two standard deviations – the dark and light grey areas respectively. The multivariate density is generated by univariate marginals that conform to the GEV distribution. The red line shows that during the exceptionally distressed period of Lehman Brothers failure, market prices of sample institutions implied joint contingent liabilities of more than 20 per cent of GDP. The magnitude of this tail risk dropped to 2 per cent at the end of 2008.

11 Network Analysis

Table 12 – Network Analysis: Main characteristics

Author(s)	Espinosa-Vega and Solé (2011)
Institution(s)	IMF (2009a, 2011a) ECB (2010b)
Pros	Easy to calculate Allows most systemic and vulnerable institutions to be identified
Cons	Static Requires non-public supervisory data
Data	Inter-institution exposures

11.1 General description

Network analysis is a useful approach for identifying financial interlinkages among institutions, as well as for tracking the reverberation of a credit and/or funding event throughout the system. It is a valuable tool for identifying both the most systemic institutions – the ones that trigger the stronger domino effects in case of default – and the most vulnerable institutions – those that are most seriously affected by the default of other institutions. In addition, it is helpful in quantifying the potential capital losses of a contagious event. The main drawback of this approach is that it carries out a static analysis, without taking the dynamic changes in the interlinkages into account. Furthermore, it requires access to data on inter-institution exposures, which are usually only available to supervisors.

As an illustration of the network approach, Espinosa-Vega and Solé (2011) present a network analysis for cross-border financial sector surveillance by simulating credit and funding shocks in the banking systems of a selection of countries. In particular, they simulate the default of a financial institution and study the impact of this default on the balance-sheet of the other institutions in the network, which are connected by borrowing and lending relationships.

11.2 Technical description

Espinosa-Vega and Solé (2011) analyse the transmission of two types of shocks:

1. a *credit shock*, where the initial default of an institution triggers a domino effect in the banking system;
2. a *credit-plus-funding shock*, where the default of an institution also causes a liquidity squeeze that triggers a spiral of fire sale losses.

The analysis is based on the following stylized bank balance sheet:

$$\sum_j x_{ji} + a_i = k_i + b_i + d_i + \sum_j x_{ij}, \tag{68}$$

where x_{ji} are bank i's loans to bank j, a_i are the other assets of bank i, k_i is bank i's capital, b_i is bank i's borrowing, d_i is bank i's deposits, and x_{ij} is bank i's borrowing from bank j.

11.2.1 Transmission of credit shocks

The transmission of credit shocks is simulated by assuming the individual default of each bank i of the sample. A bank is said to default if its capital is not enough to cover the loss due to the credit shock. In particular, considering the loss-given default λ, institution i is said to default as a consequence of the default of bank h if $k_i - \lambda x_{hi} < 0$. Bank i's balance sheet after the credit event, i.e. the default of bank h, becomes:

$$\sum_{j \neq h} x_{ji} + a_i + (1-\lambda) x_{hi} = k_i - \lambda x_{hi} + b_i + d_i + \sum_j x_{ij}. \tag{69}$$

For each of the simulations, a network algorithm is implemented that checks whether a credit event in one institution triggers the default of the other institutions in the system. The algorithm is repeated several times in order to analyse how the event propagates throughout the system until there are no further failures.

11.2.2 Transmission of credit-plus-funding shocks

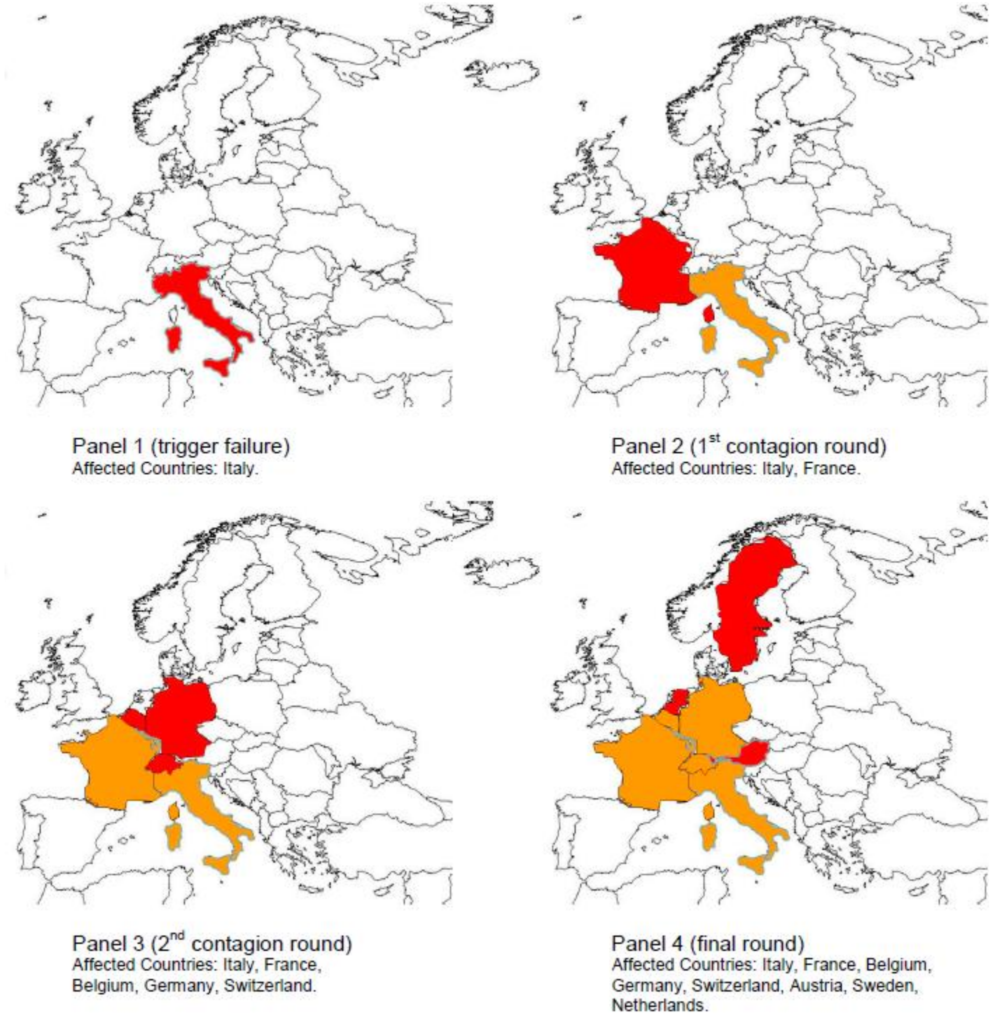
This simulation adds a tight liquidity market to the credit event, where it is difficult to obtain liquidity. In this framework, a credit event may have even more severe consequences than the previous scenario without liquidity constraints. When a bank is not able to obtain liquidity on the money market, it has to sell some assets in order to re-establish its balance sheet identity. However, if several banks try to sell their assets at the same time, they cause a decline in prices and they can only trade their assets at a discount. Espinosa-Vega and Solé (2011) study a situation in which bank i is only able to replace a fraction $1 - \rho$ of the lost funding from bank h. Its assets trade at a discount, therefore it is forced to sell assets worth $(1 + \delta)\rho x_{ih}$ in book-value terms. The loss, $\delta\rho x_{ih}$, is absorbed by bank i's capital. The new balance sheet identity is:

$$\sum_j x_{ji} + a_i - (1 + \delta)\rho x_{ih} = (k_i - \delta\rho x_{ih}) + b_i + d_i + \sum_j x_{ij} - \rho x_{ih}. \tag{70}$$

11.3 Example

Figure 10 represents how network analysis can be used to track the reverberation of a possible credit event throughout the banking system. It shows the contagion path triggered by the hypothetical default of Italy's cross-border interbank loans. At the first contagion round, the shock spills over to France, which then defaults. The default of France, in turn, triggers the default of Germany, Belgium and Switzerland. In the final round, Austria, Sweden and the Netherlands are also affected.

Figure 10 – Contagion path triggered by the default of Italy as a consequence of a credit shock



Source: Espinosa-Vega and Solé (2011).

12 Default Intensity Model

Table 13 – Default Intensity Model: Main characteristics

Author(s)	Giesecke and Kim (2011)
Institution(s)	IMF (2009a) ECB (2010b)
Pros	Captures effects of direct and indirect interlinkages among institutions Is regime-dependent
Cons	Does not distinguish between direct and indirect linkages
Data	Default data

12.1 General description

Giesecke and Kim (2011) present a reduced-form statistical model of the timing of banking default events, which is designed to capture the regime-dependent behaviour of the default rate of financial institutions. The model is formulated in terms of a default rate or 'intensity', which is why it is called 'default intensity model'.

In brief, the default rate is modelled as a continuous time process which jumps at default events, thus reflecting the increased likelihood of further default events due to spillover effects. The jump size specification guarantees that the impact of an event increases with the default rate prevailing at the time of the event (Giesecke and Kim, 2011). This is consistent with the clustering behaviour of defaults and the fact that the impact of default events tend to be 'regime-dependent'. The impact of an event fades over time.

While the advantage of this model is that it captures both direct and indirect linkages among financial institutions, it is not able to disentangle between the two.

12.2 Technical description

Let T_n be the sequence of the arrival times of default in the universe of Moody's-rated institutions. Let N_t be the number of defaults that have occurred by time t. The conditional default intensity λ_t is measured in defaults per year and is assumed to evolve through the following continuous time equation:

$$\mathrm{d}\lambda_t = K_t(c_t - \lambda_t)\mathrm{d}t + \mathrm{d}J_t, \tag{71}$$

where $K_t = k\lambda_{T_{N_t}}$ is the decay rate at which the intensity reverts to the level $c_t = c\lambda_{T_{N_t}}$ at t, and $\lambda_0 > 0$ is the value of the intensity at the beginning of the period. J_t is the jump response process:

$$J_t = \sum_{n \geq 1} \max(\gamma, \delta\lambda_{T_{N_t^-}}) 1(T_n \leq t), \tag{72}$$

where $1(T_n \leq t)$ is the indicator function, which has a value of 1 if $T_n \leq t$ and 0 otherwise. The quantities $K > 0$, $0 < c < 1$, $\delta > 0$, and $\gamma > 0$ are constant proportional factors. Giesecke and Kim (2011) show that in order for the counting process N_t to be nonexplosive, the condition $c(1+\delta) < 1$ has to be satisfied. Equation (71) states that default intensity jumps whenever there is a default, reflecting the increase in the likelihood of further events. The magnitude of the jump depends on the default intensity right before the event and therefore guarantees the regime-dependence of the impact of default events. After being bolstered by an event, the default intensity decays exponentially to the level $c\lambda_{T_{N_t}}$ at rate $K\lambda_{T_{N_t}}$. The vector of parameters to be estimated is $\theta = (k, c, \delta, \gamma, \lambda_0)$. The estimation is performed through maximum likelihood. The maximum likelihood problem for $\lambda_t = \lambda_t(\theta)$ is:

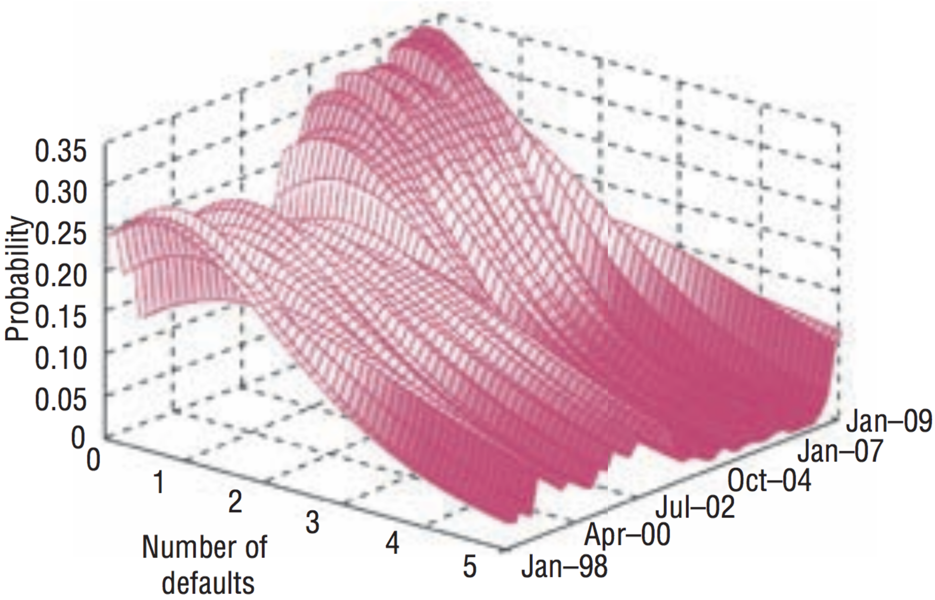
$$\max_{\theta \in \Theta} \left[\int_0^\tau \log\lambda_{s^-}(\theta)\mathrm{d}N_s - \int_0^\tau \lambda_s(\theta)\mathrm{d}s \right], \tag{73}$$

where Θ is the set of admissible vectors of parameters.

12.3 Example

Figure 11 shows a time series of quarterly forecasts for the one-year distributions of the number of defaults in the US banking sector estimated from the model for the banking-wide default rate (IMF, 2009a). It depicts a fatter tail, i.e. a higher probability of a joint default of several US banks, during the 2008 financial crisis.

Figure 11 – Default intensity model: Default rate probability and number of defaults



Source: IMF (2009a).

13 Markov-Regime Switching Model (SWARCH)

Table 14 – Markov-Regime Switching Model: Main characteristics

Author(s)	González-Hermosillo and Hesse (2011)
Institution(s)	IMF (2009b)
Pros	Easy to update State-dependent parameters
Cons	Univariate only
Data	High frequency market-based data

13.1 General description

González-Hermosillo and Hesse (2011) present a Markov regime switching autoregressive conditional heteroskedastic model (SWARCH) for assessing financial volatility and the likelihood of a crisis. In particular, they model the dynamics of proxies for global market conditions – such as the VIX index, the TED spread and the EUR-USD forex swap[10] – as ARCH models with state-dependent parameters. This allows them to differentiate between states of low, medium, and high volatility. The probability of switching from one state to another is modelled as a Markov chain, which can be estimated, together with the other parameters of the model, through maximum likelihood.

The main strength of the SWARCH model is that it allows for state-dependent parameters. However, the model is univariate only and its estimation requires high-frequency data.

13.2 Technical description

The Markov regime switching model chosen by González-Hermosillo and Hesse (2011) is the ARCH Markov-switching model by Hamilton and Susmel (1994). It is an ARCH model in which the parameters are state-dependent so as to differentiate between multiple volatility states. In particular,

$$\mathbb{P}(s_t = j \mid s_{t-1} = i, s_{t-2} = k, \ldots, y_{t-1}, y_{t-2}) = \mathbb{P}(s_t = j \mid s_{t-1} = i) = p_{ij} \tag{74}$$

is the equation that describes the Markov chain where y_t is a vector of observed variables, s_t is an unobserved random variable, and p_{ij} is the probability of transition from state i to state j. In the SWARCH model the mean equation is an AR(1) and the variance is time-varying with the ARCH parameters being state-dependent:

$$y_t = \alpha + \phi y_{t-1} + \epsilon_t, \tag{75}$$

$$\epsilon_t = \sqrt{g_{S_t}}\tilde{\epsilon}_t, \tag{76}$$

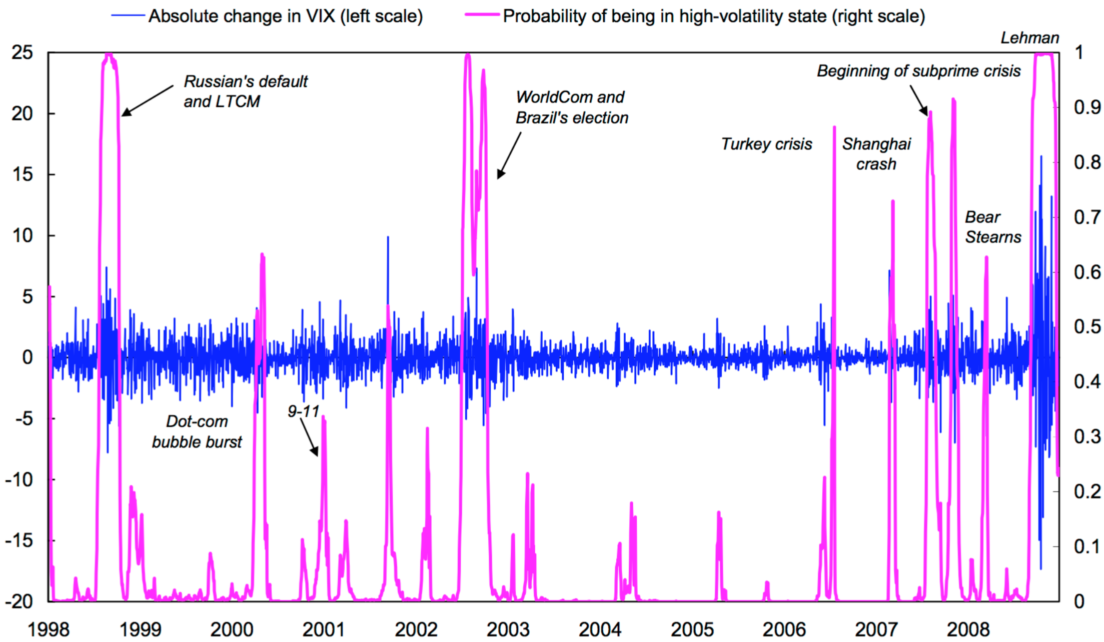
$$\tilde{\epsilon}_t = h_t v_t, \tag{77}$$

$$h_t^2 = a_0 + \sum_{i=1}^{q} a_i \tilde{\epsilon}_{t-i}^2 + \delta d_{t-1}\tilde{\epsilon}_{t-1}^2, \tag{78}$$

where h_t^2 is the time-varying variance, $v_t \sim N(0,1)$, $S_t \in \{1,2,3\}$ is the set of states, and d_{t-1} is a dummy variable which is equal to 1 if $\tilde{\epsilon}_t \leq 0$ and 0 otherwise. The ARCH parameters are state-dependent due to multiplication with the scaling

[10] The VIX index is the Chicago Board Options Exchange volatility index, which measures the implied volatility of the S&P 500 index options over the next 30 days; the TED spread is the difference between the three-month LIBOR and the three-month Treasury bill rate. The VIX index is usually interpreted as a proxy for market uncertainty, the TED spread as a measure for stress in the interbank market, and the EUR-USD forex swap as an indicator of US dollar funding pressures in international financial markets.

Figure 12 – Markov-switching ARCH model of the VIX index



Source: González-Hermosillo and Hesse (2011).

factor g_{S_t}, which is normalized to 1 for low volatility regimes. Variables are first-differenced in order to eliminate non-stationarity.

13.3 Example

Figure 12 shows the results of a daily SWARCH model for the VIX index over the period 1998–2008. A probability of being in the high state equal to 1 is reached for the Russian and LTCM defaults in 1998, for the WorldCom scandal and Brazil's election in 2003, and for Lehman's collapse in 2008. After Lehman's collapse the probability of being in the high state remained high for a significant period.

14 Composite Indicator of Systemic Stress (CISS)

Table 15 – CISS: Main characteristics

Author(s)	Holló et al. (2012)
Institution(s)	ECB (2010a, 2011) and all following issues of the ECB Financial Stability Review ESRB (2012) and all following issues of the ESRB Risk Dashboard
Pros	Calculated in real time Available for a broad set of countries and for long data samples Robust to the arrival of new information
Cons	Not based on a structural model that includes systemic risk Has problems of comparability with other indicators Not an early warning indicator
Data	Market-based, with two exceptions*

*Monetary financial institutions' emergency lending to Eurosystem central banks and book-to-price ratio for the financial sector equity market index.

14.1 General description

The Composite Indicator of Systemic Stress (CISS, pronounced 'kiss') is introduced in ECB (2010a, 2011) and then thoroughly explained by Holló et al. (2012). The euro-area CISS is included in the ECB's analytical toolkit, i.e. the set of analytical instruments used by the ECB to support its macroprudential functions.

As suggested by its name, the CISS is a composite indicator built to 'measure the current state of instability, i.e. the current level of frictions, stresses and strains (or the absence thereof) in the financial system and to summarize it in a single statistic' (Holló et al., 2012). As highlighted by ECB (2011), it is designed not only to identify systemic risk within the financial system (the 'horizontal view', according to which systemic risk pervades the whole financial system), but also to consider systemic risk stemming from the interaction between the financial system and the real economy (the 'vertical view', according to which systemic risk spills over into the real sector).

The CISS is a coincident indicator that is built up through a process of aggregation.

Five categories within the financial system are considered: the equity market, the bond market, the money market, the foreign exchange market, and the financial intermediaries' sector. The construction of the CISS builds upon a vast literature on composite indicators (see e.g. Illing and Liu, 2006; Caldarelli et al., 2011; Nelson and Perli, 2007). The major novelty of the CISS is the procedure through which the five categories are aggregated. This procedure borrows from the standard portfolio theory that uses time-varying cross-correlations to weigh the assets constituting a portfolio. It is through this aggregation process that systemic risk is incorporated into the indicator. In fact, while a simple average of the categories would assume a perfect correlation between them, an average weighted with time-varying cross-correlations allows emphasis to be put on situations in which stress prevails in several market segments at the same time (Holló et al., 2012).

One of the main strengths of the CISS is that, since it consists mainly of market-based indicators, it can be calculated in real time and is available for a broad set of countries. Moreover, it is robust to the arrival of new information, i.e. the addition of new information does not significantly change the estimates produced by the CISS (Holló et al., 2012).

With the other composite indicators, it shares the drawback of being barely comparable and 'often not informative about the origins and transmission channels for widespread instability' (ECB, 2010a). In addition, it is not founded on a solid theoretical model encompassing systemic risk.

14.2 Technical description

CISS results from the aggregation of five sub-indices, each representing one market, i.e. the equity market, the bond market, the money market, the foreign exchange market, and the financial intermediaries' sector. Each sub-index, in turn, comes from the aggregation of three raw indicators of different market segments (for the complete list of the 15 raw indicators, see Holló et al., 2012). In order to obtain a sub-index which is unit-free and measured on an ordinal scale, an empirical cumulative distribution function is calculated by using order statistics. Given the vector of n observations of the raw indicator $x_t = (x_{1,t}, \ldots, x_{n,t})$, the respective ordered sample is $(x_{[1],t}, \ldots, x_{[n],t})$, where $x_{[n],t}$ is the sample maximum. The empirical cumulative distribution is then calculated as

$$F_n(x_t) = \begin{cases} r/n, & \text{for} \quad x_{[r]} \leq x_t < x_{[r+1]}, \quad r = 1, 2, \ldots, n-1, \\ 1, & \text{for} \quad x_t \geq x_{[n]}. \end{cases} \tag{79}$$

In case of two or more equal observations, the average of the rankings of the equal observations is taken. The empirical cumulative distribution provides a unit-free indicator, which is measured in the interval (0,1]. Furthermore, it can be easily calculated for a bigger sample of $n+T$ observations by simply substituting n with $n+T$ in the above formula.

The three raw indicators obtained are then aggregated by taking their arithmetic mean. Hence from 15 raw indicators, five sub-indices are obtained.

The most innovative part of the CISS, which is where systemic risk comes into play, lies in the aggregation of the five sub-indices in order to obtain the final index. For this last step, a standard portfolio approach is applied by weighing the five components with the time-varying cross-correlations. Reporting the formula of Holló et al. (2012):

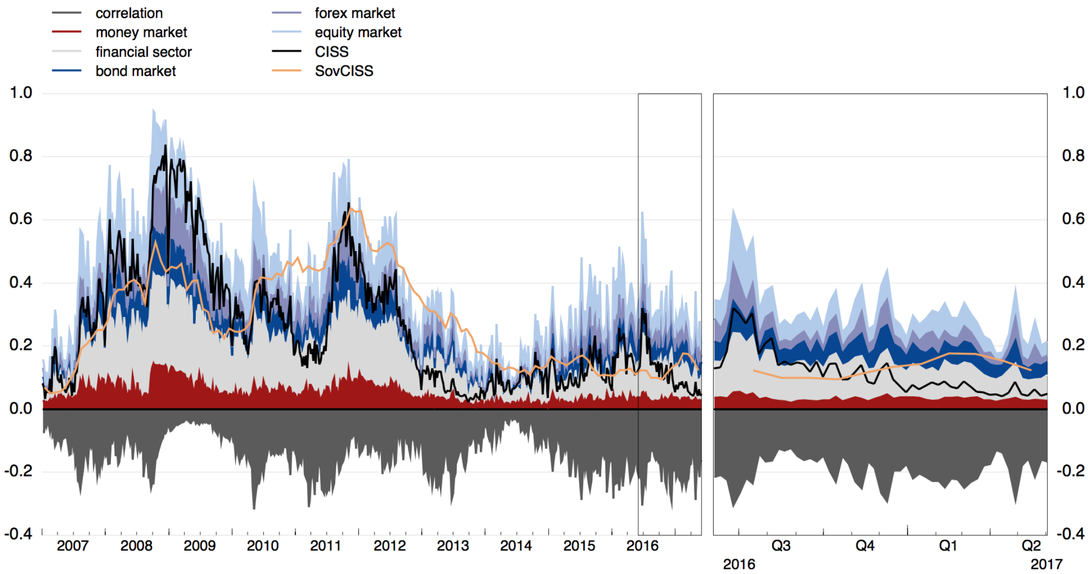
$$\mathrm{CISS}_t = (w \circ s_t) C_t (w \circ s_t), \tag{80}$$

where s_t is the vector of sub-indices, C_t is the time-varying cross-correlation matrix, and $\circ$ is the Hadamard product (i.e. an element-by-element multiplication). The vector w_t is a vector of weights for sub-indices, which is calculated 'on the basis of their average relative impact on the industrial production growth measured by the cumulated impulse responses from a variety of standard linear VAR models' (Holló et al., 2012). The time-varying cross correlations are calculated recursively by estimating the respective variances and covariances with an exponentially weighted moving average (EWMA), which is initialized at the average values of variances and covariances and uses 0.93 as a smoothing parameter (Holló et al., 2012). In the case of perfect correlation between the five sub-indices, matrix C_t is an identity matrix and the CISS coincides with the square of the simple arithmetic average of the five sub-indices, which is an upper bound for the CISS.

14.3 Example

Figure 13 exhibits the CISS (the black line) for the euro-area members. It clearly shows how systemic stress increased from mid-2007 and escalated after the Lehman Brothers' bankruptcy. The chart also displays the stacked contributions from each of the five sub-indices, which make up the CISS. The upper border of the upper area is equivalent to the weighted average of the five sub-indices assuming perfect correlation across all of them. The difference between the CISS and this weighted average is plotted in the area below the zero line and reflects the impact of cross correlation across the sub-indices. It is worth observing that when markets are either extremely calm or extremely distressed, the difference between the CISS and the weighted average with perfect correlation is small, meaning that the five sub-indices are highly correlated. In intermediate situations, the CISS is able to evaluate an increase in cross-correlation among the sub-indices, which is a signal of

Figure 13 – CISS



Source: ESRB (2017).

increased systemic risk. The figure also shows the Sovereign CISS (the yellow line), which applies the same methodology of the CISS to the sovereign bond markets. It comprises the following six sub-indices: the yield spread against the euro swap interest rate of comparable maturity, the one-week realized volatility of daily yield changes, and the bid-ask bond price spread as a percentage of the mid-price, all computed for sovereign bonds with a 2-year and 10-year maturity respectively. SovCISS reaches its peak at the height of the sovereign debt crisis in the second half of 2011.

15 Risk Assessment Model for Systemic Institutions (RAMSI)

Table 16 – RAMSI: Main characteristics

Author(s)	Aikman et al. (2011)
Institution(s)	IMF (2009a) ECB (2010b)
Pros	Very comprehensive
Cons	Univariate only
Data	Balance sheet and market-based data

15.1 General description

RAMSI is a quantitative model of financial stability that aims to assess institution-specific and system-wide vulnerabilities. The most recent version of RAMSI is presented in Aikman et al. (2011) as an extension with liabilities-side feedback on the prototype version built by Alessandri et al. (2009) at the Bank of England, which, in turn, is based on a framework developed by the Oesterreichische National Bank (2006) for the Austrian banking system.

RAMSI relies on a modular approach where 'a macroeconomic model is combined with models that describe how the risk profiles of key financial institutions respond to changes in macroeconomic conditions' (Alessandri et al., 2009). In particular, RAMSI adopts a balance sheet approach and integrates it with a network model in order to account for interaction and contagion among banks. The model allows for macro-credit risk, interest and non-interest income risk, network interaction and feedback effects emerging on both the asset and the liability side of the balance sheet (Aikman et al., 2011). Systemic risk arises because of the feedback loop that may be triggered by the failure of a bank heavily connected with the rest of the system. As a result of counterparty risk, fire sales and confidence contagion, the failure of one bank spills over to other banks, thus generating a cascade effect.

15.2 Technical description

RAMSI adopts a modular framework where a balance sheet approach is integrated with a network approach. The balance sheets of the largest banks – only UK banks are considered in Aikman et al. (2011) – are highly disaggregated both on the asset and the liability side. The model is run on a three-year horizon, which is usually a sufficient time for adverse shocks to be reflected in credit losses and is also the horizon usually used by central banks in their stress tests (Aikman et al., 2011).

The evolution of macroeconomic and financial variables is captured by a large-scale Bayesian vector autoregression (BVAR), which is the only source of shocks in RAMSI. The BVAR has two lags and is estimated using quarterly data for 12 macroeconomic and financial variables for the domestic country and 12 analogous variables for the foreign country – the UK and the US respectively in Aikman et al. (2011). The BVAR is used to determine the yield curve, the PDs and the LGDs on banks' exposures.

Several risk factors and their impact on banks are then modelled, such as credit losses, net interest income, and other operating expenses. The impact of these risk factors on each bank is referred to as the first-round impact. The deterioration of bank fundamentals may trigger a downgrade of its credit rating and an increase in its funding costs. The consequences could be so severe that the bank may be shut out of the short-term funding markets. If the bank defaults, there is a good chance that it will trigger a feedback loop for other banks, thus generating second-round consequences for the whole financial system. Counterparty credit losses, mark-to-market losses, and confidence contagion trigger a cascade effect on the whole financial system.

The following subsections analyse the first- and second-round effects on banks in more detail.

15.2.1 First-round impact on banks

The first impact on banks is modelled through: credit risk, net interest income, non-interest income, operating expenses, profits, taxes and dividends.

1. Credit losses are derived as the product of PD, LGD, and each bank's total exposure to households and firms. PD and LGD are determined through the BVAR.

2. Interest income is modelled endogenously by following the risk-neutral asset-pricing model of Drehmann et al. (2010). Banks usually price their loans on the basis of the prevailing yield curve and the perceived riskiness of their debtors. However, banks' repricing ability is constrained by the maturity structure of their balance sheet, i.e. they can only reprice at maturity. Since it is unlikely that asset and liability maturities are perfectly matched, income risk may arise due to this mismatch. Let A be a risky asset with a repricing maturity equal to T, i.e. asset A pays a fixed coupon C for the next T

periods. The value of asset A today is the discounted value of its future coupon payments plus the principal:

$$V(A_0) = \sum_{t=1}^{T} D_t C A_0 + D_T A_0. \tag{81}$$

The discount factors are:

$$D_t = \prod_{i=1}^{t} (1 + R_{i-1,i})^{-1}, \tag{82}$$

$$R_{i-1,i} = \frac{r_{i-1,i} + \mathrm{PD}_{i-1,i} \cdot \mathrm{LGD}_{i-1,i}}{1 - \mathrm{PD}_{i-1,i} \cdot \mathrm{LGD}_{i-1,i}}, \tag{83}$$

where $r_{i-1,i}$, $\mathrm{PD}_{i-1,i}$, and $\mathrm{LGD}_{i-1,i}$ are respectively the forward risk free interest rate, the expected PD and the expected LGD between time $i-1$ and i. By using Equation (81) it is possible to calculate a coupon such that $V(A_0) = A_0$:

$$C = \frac{1 - D_T}{\sum_{t=1}^{T} D_t}. \tag{84}$$

At any repricing maturity nT with $n \in \mathbb{N}$, the bank can adjust the coupon C to any change in the discount factors. Instead, at any time nT with $n \notin \mathbb{N}$, i.e. at any time that does not correspond to a repricing maturity, the bank cannot adjust the fixed coupon C to changes in interest rate, PD or LGD. Therefore, a mismatch between assets and liabilities arises.

3. Non-interest income is considered to be procyclical, as suggested by Stiroh (2004). Its growth rate is modelled as being dependent on four of its lags and four lags for the GDP growth rate. Instead, operating expenses are found to be less procyclical and are estimated according to the following equation, with only one lag:

$$\left(\frac{\text{operating expense}}{\text{operating income}}\right)_t = \beta_0 + \beta_1 \left(\frac{\text{operating expense}}{\text{operating income}}\right)_{t-1} + \beta_2 \Delta \ln(\mathrm{GDP}_t). \tag{85}$$

4. As for profits, they are assumed to be proportional to the size of each bank's portfolio. Profits are then computed as the sum of all sources of income, net of expenses and credit losses. Post-tax post-dividend profits are assumed to increase Tier 1 capital directly.

15.2.2 Funding liquidity risk and bank failure

Given how important funding liquidity risk can be in triggering a bank failure (Brunnermeier and Pedersen, 2009), funding liquidity effects were included in RAMSI by Aikman et al. (2011). In particular, they observe that while the relationship between a deterioration in bank fundamentals and an increase in funding costs is roughly linear during normal times, it becomes non-linear in times of crisis. This non-linearity is included in RAMSI through bank ratings and through a 'danger-zones' approach.

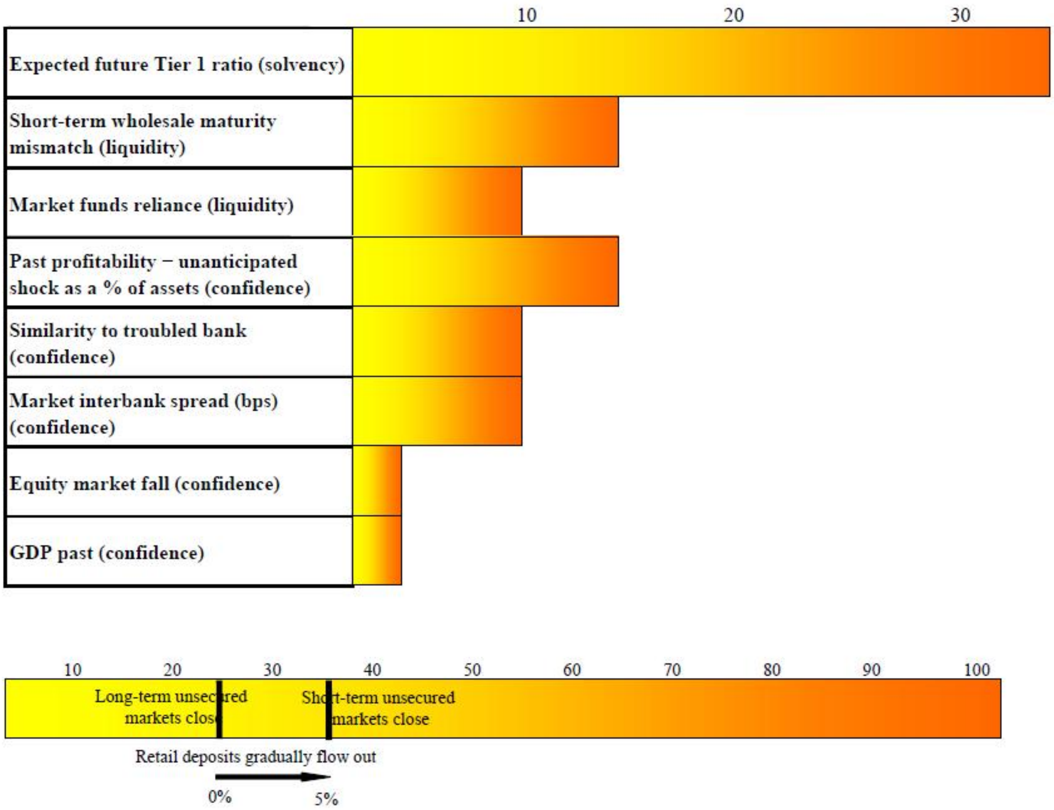
1. Aikman et al. (2011) use a probit model to examine the sensitivity of Moody's senior unsecured ratings to a number of key bank performance indicators and macroeconomic variables and to estimate ratings for each bank every quarter. The assigned ratings are then mapped to credit spreads by using Merrill Lynch's indices for bond spreads associated with different credit ratings.

2. In order to model the closure of funding markets, Aikman et al. (2011) also adopt a 'danger-zones' approach, 'under which banks accumulate points as liquidity conditions deteriorate, and face the prospect that certain funding markets may be closed to them as their score crosses particular thresholds'. In particular, the closure of certain funding markets to an institution may have a double effect on the institution itself: on the one hand, it may compel it to rely on short-term funding, thus worsening the liquidity position; on the other hand, it may affect the institution simply through a confidence channel. Figure 14 shows the eight indicators that are used to assign the scores, along with the thresholds at which funding markets are assumed to close to a given institution. In brackets there are the three underlying factors that the indicators are trying to proxy: solvency, liquidity and confidence effects. Roughly equal weight is placed on each of the three underlying factors. A funding crisis can be caused either by extreme scores for one of the three triggering factors or by a combination of moderate scores across different factors. In particular, an institution is assumed to default if it scores 35 danger zone points and at the same time is shut out of short-term unsecured funding markets. The costs incurred by a defaulting institution are assumed to be 10 per cent of its remaining assets.

15.2.3 Second-round effects and contagion

A bank in distress usually sells its assets to restore its capital-to-assets ratio and to obtain the necessary liquidity. Massive sales may result in a price fall with severe

Figure 14 – RAMSI danger zones



Source: Aikman et al. (2011).

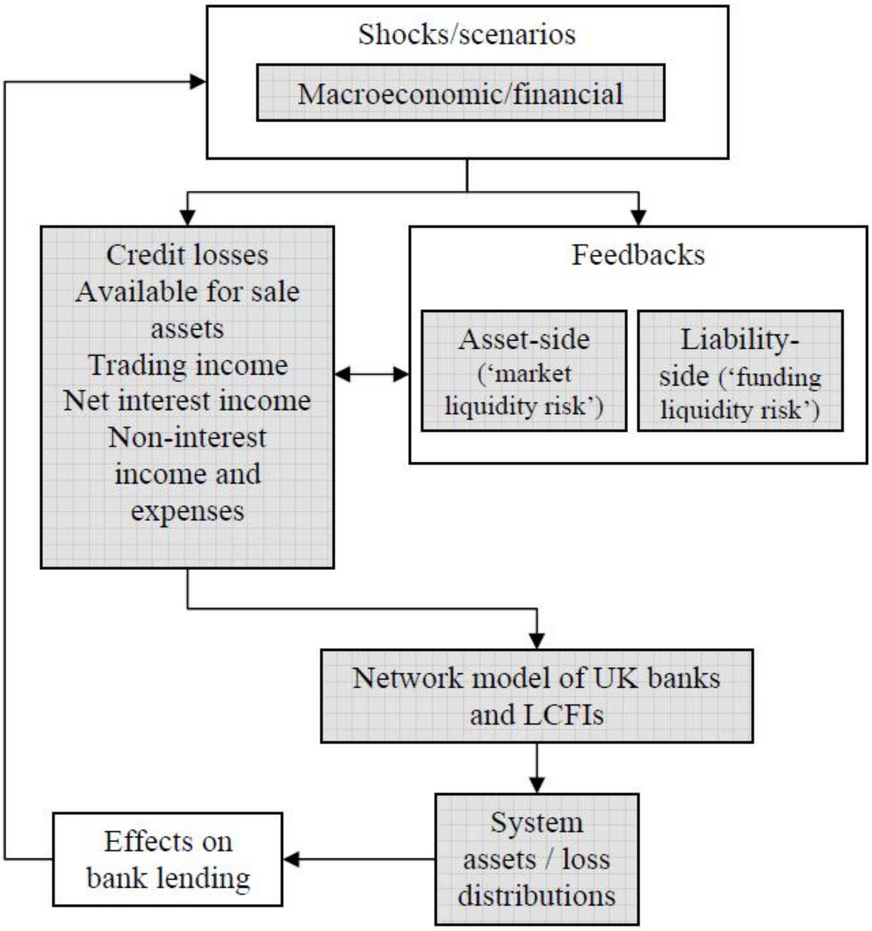
consequences for other banks that incur mark-to-market losses. The relationship between prices and the magnitude of fire sales is taken to be concave and is given by:

$$\hat{P}_j = \max\left\{0, P_j\left[2 - \exp\left(\theta\frac{S_j}{M_j + \epsilon_j}\right)\right]\right\}, \tag{86}$$

where $\hat{P}_j$ is the price of asset j after the fire sale and P_j is the price of the asset before the fire sale. P_j is multiplied by the discount factor in round brackets, which is a function of the value of assets sold in the fire sale (S_j), the depth of the market in normal times (M_j) and a parameter (θ) that reflects frictions such as search problems. Market depth M_j can also be shocked by ϵ_j, which captures changes in market depth as a consequence of changes in macroeconomic conditions.

When a bank defaults, it triggers credit losses in its counterparts. Counterparty credit losses are determined through a network model and a matrix of interbank exposures is built. Once counterparty credit losses and mark-to-market losses are

Figure 15 – RAMSI framework



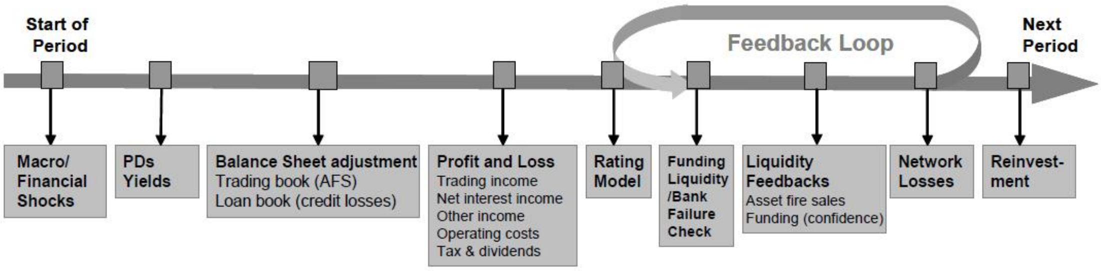
Source: Aikman et al. (2011).

accounted for, the danger zone scores are updated for banks that initially survived. If a bank's score reaches 35, the bank is assumed to default and the feedback mechanism is iterated. If no bank defaults, the balance sheets are updated to absorb the losses deriving from counterparty risk. In addition, asset prices recover to their pre-feedback level so that mark-to-market losses are not carried over.

15.3 Example

Figure 15 shows the modular structure of RAMSI. A macroeconomic or financial shock is transmitted to the system via several feedback channels, which act through balance sheet interdependencies and network effects. In particular, three sources of risk are identified by Aikman et al. (2011): counterparty credit risk, mark-to-

Figure 16 – RAMSI model dynamics



Source: Aikman et al. (2011).

market risk, and confidence contagion. The dynamics of the model are shown in Figure 16.

The macroeconomic model determines the yield curve, PD, and LGD on banks' credit exposures. For each combination of risk factors, a first-round impact on each bank is modelled through distinct modules that account for: credit losses, net interest income, non-interest income and other operating expenses. In case of shocks to a bank's fundamentals, its credit rating may deteriorate, thus worsening its funding conditions. In case of default, the bank triggers a feedback loop through counterparty credit losses, mark-to-market losses and confidence contagion. These further losses may generate a cascade effect on other connected banks. In contrast, in the absence of bank failures, the balance sheets of surviving banks are updated through some reinvestment rules.

16 Conclusion

This survey illustrates the systemic risk indicators that are most frequently used by the IMF, the ESRB, the ECB, the BoE and the BoI. Given the remarkable heterogeneity of systemic risk indicators, this survey seeks to identify different criteria according to which they can be organized. In particular, the selected indicators are classified according to three different taxonomies. The first one adopts the point of view of regulators and policy makers, whose attention is focused on the implementability and the time-horizon of indicators. The second taxonomy is organized according to the features which are important for researchers, i.e. the theoretical grounding and the techniques used to compute the indicators. Last but not least, the third taxonomy classifies the indicators according to the specific features of systemic risks which are captured by each measure. In this regard, while a group of indicators aims at gauging the individual or joint probability of default for one or more financial institutions, another estimates the expected losses in case of default, a third one measures the network structure of the financial system, and a fourth class evaluates the overall level of distress in the system.

Given the limitations that recurring to a single systemic risk measure would imply in terms of the accuracy and reliability of the estimates, it is highly recommended that a wide range of indicators is deployed when trying to detect systemic risk. This is what ECB (2015) and Nucera et al. (2016) try to do when they propose a systemic risk ranking for financial institutions derived from a pool of alternative systemic risk indicators. Only those regulatory authorities equipped with a variety of indicators may identify a rise in systemic risk well in advance and take the necessary counteractive measures in due time.

This survey may contribute to helping regulators, researchers and practitioners have a clearer picture of major systemic risk indicators and reconcile their different aspects in a unified framework.

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Glossary

ARCH	Autoregressive conditional heteroskedasticity
BoE	Bank of England
BoI	Bank of Italy
BSI	Banking stability index
BSMD	Banking system multivariate density
BVAR	Bivariate vector autoregression
CCA	Contingent claim analysis
CDS	Credit default swap
CISS	Composite indicator of systemic stress
CoVaR	Conditional value at risk
CRSP	Centre for Research and Security Prices
DiDe	Distress dependence
DIP	Distress insurance premium
ECB	European Central Bank
ESRB	European Systemic Risk Board
EVT	Extreme value theory
EWMA	Exponentially weighted moving average
FOC	First order conditions
FPC	Financial Policy Committee
FSOC	Financial Stability Oversight Council
GARCH	Generalized autoregressive conditional heteroskedasticity
GEV	Generalized extreme value
JPoD	Joint probability of distress
IMF	International Monetary Fund
LIBOR	London interbank offered rate
LGD	Loss given default
LVG	Leverage
MES	Marginal expected shortfall
NYU	New York University
OFR	Office of Financial Research
Option-iPoD	Option-implied probability of default
PCA	Principal component analysis
PCE	Probability of cascade effects
PD	Probability of default
PoD	Probability of default
RAMSI	Risk assessment model for systemic institutions
SES	Systemic expected shortfall
SWARCH	Markov-regime switching autoregressive conditional heteroskedasticity
TASS	Trading advisor selection system
VaR	Value at risk
VAR	Vector autoregression
VIX	Volatility index

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