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Danger rate, the issue of the rating as a risk differentiation driver

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Overview

Some banks, which have been granted permission to use internal models for credit risk, are introducing in the loss given default (LGD) calculation a risk driver representing the obligors' creditworthiness: the rating (i.e. PD). This is a significant novelty in the LGD estimation process, as it introduces a dynamic component as a driver and a strong correlation effect between the PD and LGD parameters. Through the use of real data collected from the private customers of all Italian banks over the years 2020-23, the impact of including the rating variable in the estimation of the LGD model was assessed, showing a decrease in total RWAs of approximately 9 per cent. Although the interdependency between PD and LGD is not explicitly forbidden in the Basel framework and the current regulations in force, the RWA calculation formula does not take it into account. The analysis concludes by discussing how the incorporation of the rating as an LGD driver may be questionable in the absence of adequate supporting analysis for monitoring and testing performance.

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1 Introduction

Some banks (supervised entities, SEs) that have been granted permission to use internal models for credit risk are enhancing (or considering to enhance) the methodology for quantifying the loss given default (LGD) parameter in order to introduce variables that represent the obligors' creditworthiness, i.e. the rating. This is a significant novelty in the LGD parameter estimation process, since it introduces:

i. A dynamic component as a driver. Before this, the most relevant LGD risk drivers were selected from a basket of variables relating to the specific characteristics of the type of exposure, such as: transaction-related risk characteristics (e.g. type of product, type of collateral, geographical location of the collateral), obligor-related risk characteristics (e.g. size, industrial sector), and external factors (e.g. legal framework and other factors influencing the expected length of the recovery process). These variables are typically stable if observed at the moment of default and also within a year before default.[1] By contrast, the rating typically worsens as the moment of default approaches.

ii. A strong correlation effect between the PD and the LGD parameters, hence a potential pro-cyclical effect in the LGD estimation; as discussed below, this effect could be mitigated by a proper downturn adjustment.[2]

More concretely, by considering an LGD performing model as the product of two main components, the Loss Given Sofferenza (Loss Given doubtful status; as for Italian intermediaries a doubtful loan classification is a relevant step in initiating the hard collection process) and the Danger Rate, this note introduces the rating as a driver of the probability of transition to Doubtful from first entering into default ($P_{doubtful}$), which is the main element of the Danger Rate.[3]

First, this is a unique aspect that needs to be thoroughly investigated. The risk differentiation is performed on a sample of defaulted positions, while it is then applied to a portfolio mainly composed of performing positions. Indeed, this could lead to implausible results for the $P_{doubtful}$ component, e.g. with respect to the best-rated obligors who are unrepresented or scarcely represented in the estimation sample. Furthermore, the rating tends to be highly volatile in the months preceding the default, potentially leading the SE to estimate a statistical impact that is not fully representative of the characteristics of the performing position stock.

1 See ECB guide to internal models (February 2024), Credit risk – 6.2 LGD structure, and EBA Guidelines on PD and LGD – EBA/GL/2017/16 (November 2017).

2 See section 3 for details.

3 The concept of 'LGD – doubtful loan' was introduced for modelling the Italian peculiarities of the recovery process, it is conceptually similar to the Loss Given Loss. The Danger Rate can be seen as the probability to migrate to a default (doubtful) status, where a loss is observed (i.e. the complement to 1 of the Cure Rate, which is the probability to come back to a performing status). Thus, without loss of generality and for the sake of simplicity, P_*doubtful* is considered equal to the Danger Rate in this note.

We review real data from the private customers of all Italian banks[4] and analyze the impact of the rating variable inclusion into the LGD model estimation. More specifically, a what-if simulation of LGD and Risk Weighted Assets (hereinafter RWAs) with and without the adoption of the rating driver in the Danger Rate module is provided. The main outcome is that the inclusion of the rating variable determines a decrease in total RWAs of around 9 per cent in the simulated scenario.

Finally, given the cyclical nature of the rating, this note concludes with an analysis of the Basel Committee on Banking Supervision (BCBS) regulatory principles and of Regulation (EU) No 575/2013 – Capital Requirements Regulation (henceforth, CRR). More specifically, we focus on the sections that call for the need to estimate the LGD with ratings derived from the conditional stressed PD_S, as opposed to unconditional ones such as those used by SEs.

As observed above, the analysis of risk drivers in the LGD model estimation process has a practical interest for Supervisory Authorities as well as for SEs. It should be noted that SEs invest time and resources in collecting granular data useful for conducting a robust econometric analysis in their model development framework, as well as in validation activities, to end up with a selection of those variables proposed as drivers for the estimation of credit risk parameters.

Empirical studies on banking practices are still at an early stage. Starting from the issue under discussion, in the next paragraphs we will highlight which are the potential criticalities of such an approach, also introducing some useful tools and checks that can be implemented for model assessment or monitoring.

2. Effects and issues

2.1 The data

In order to study the case under analysis, real data from the private customers of all Italian banks are considered. The PDs adopted are the ones calculated on a yearly basis by the Bank of Italy for its in-house credit assessment system (see Giovannelli et al. (2020)) for the whole of Italian obligors.

In order to define the estimation population, all the exposures of the whole Italian banking system to obligors classified as 'producer households'[5] were considered for the period December 2020 - December 2023. The default status was observed each month according to the following three classifications: P Performing; D Default; S Doubtful ('Sofferenza'). The statistical unit is given by the Accounting Date, the Identification of the Obligor and the Issuing Bank; all the exposures of a specific bank to an Obligor are summarized.

4 In details, 'famiglie produttrici' (producer households) over the time span December 2020 – December 2023.

5 Sole proprietorships, simple partnerships and de facto corporations, producers of non-financial goods and services intended for sale, with up to 5 employees.

The December 2020 and December 2021 cohorts were considered, all the performing statistical units at the starting date of the cohort were taken into account and the following information was retrieved:

1. Total amount granted at the starting date;
2. Total amount on the balance sheet at the starting date;
3. TimeToDef, the number of months, with respect to the given cohort, after which the exposure was classified as defaulted (0 if no transition to default is observed);
4. TimeToSoff, the number of months, with respect to the given cohort, after which the exposure was classified as doubtful (0 if this does not occur);
5. The Bank of Italy-estimated PD at the cohort starting date.

Finally, the estimation sample was built considering statistical units with TimeToDef greater than 0 and less or equal to 12.

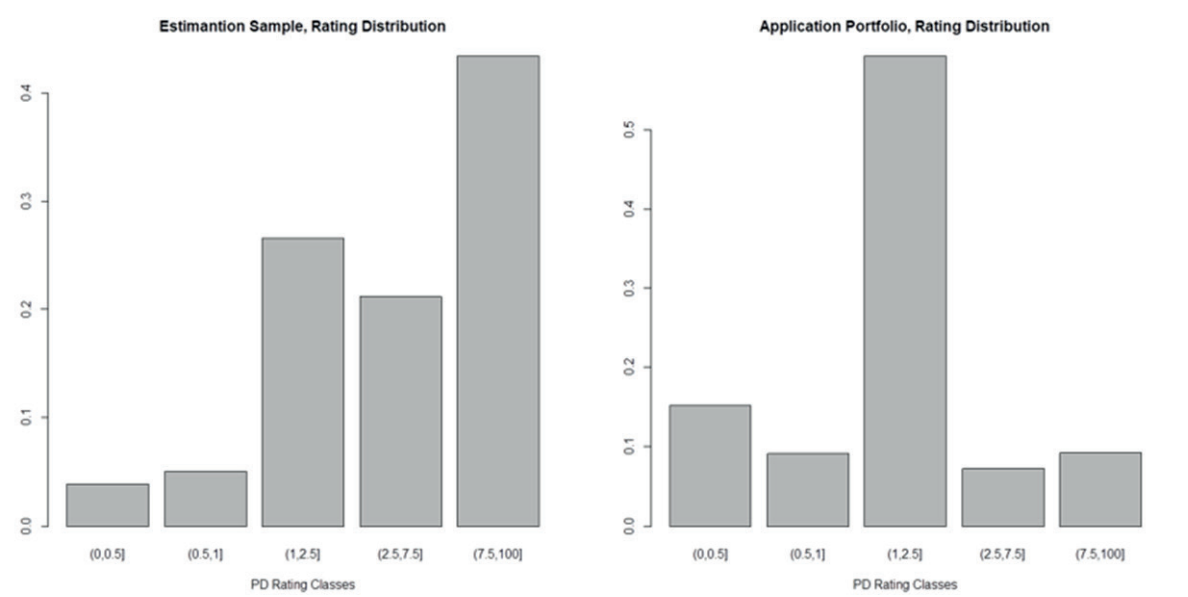
The application portfolio was identified considering only the performing positions at the accounting date of December 2023. The following variables were also used:

1. Total amount granted at December 2023;
2. Total amount on the balance sheet at December 2023;
3. The Bank of Italy-estimated PD at December 2023.

2.2 *Representativeness*

In the next picture the relative distributions by rating of the estimation sample and the application portfolio are presented. The PDs are grouped according to the 5 classes defined by the AQR methodology (Table 41 of the Asset Quality Review - Phase 2 Manual).

Figure 1: Estimation population and application portfolio distribution by rating class



It is clear that the two frequency distributions have not a similar shape, the estimation population is concentrated in the higher risk classes (roughly 65 per cent of the exposures); on the contrary, the highest percentage of observation in the application portfolio is in the medium-low risk segment (85 per cent). This analysis highlights the first issue of representativeness in terms of differences between the estimation population and the application portfolio.

The data used for creating the previous figures are reported in the following table:

Table 1: Estimation population and application portfolio data by rating class

PD Buckets	Estimation Population		Application Portfolio	
	% EAD	% No. Obligors	% EAD	% No. Obligors
(0-0.5]	4.103369	3.86	15.214666	15.18
(0.5-1]	4.525464	5.06	7.892345	9.13
(1-2.5]	26.166579	26.55	60.987815	59.23
(2.5-7.5]	20.99355	21.15	7.025768	7.22
(7.5-100]	44.211039	43.35	8.879407	9.22

What needs to be emphasized here is not a lack of representativeness tout court, but huge differences in the two populations (estimation vs application). This could imply a potentially inappropriate dataset selection for the SE model development, as it anticipates a low model performance on the application portfolio.

2.3 *Simulation of the impact*

The purpose of this section is to illustrate the potential huge impact of the rating variable inclusion into the LGD model; hence, a what-if simulation analysis has been carried out.

The next table shows the observed Danger Rates as a function of the riskiness retrieved from the estimation sample. The transition to Doubtful was considered if it occurred within 24 months from the default date only in order to make the two cohorts homogeneous. The Danger Rate was hence estimated as the percentage of Default exposures that migrate to Doubtful within 24 months.

The PDs are grouped according to the 5 classes defined by the AQR methodology (Table 41 of the Asset Quality Review - Phase 2 Manual).

Table 2: Danger Rate results from the estimation sample

PD Buckets	Danger Rate
(0-0.5]	0.0653266
(0.5-1]	0.1149425
(1-2.5]	0.1101633
(2.5-7.5]	0.123011
(7.5-100]	0.1425586
Total	0.1254369

The LGD is calculated with a simplified method according to the following expression:[6]

$$LGD = Danger\ Rate \cdot LGS \qquad (1)$$

Where *LGS* is the loss in case the exposure has not cured. The *Danger Rate* is the probability of a position in default to migrate to a 'Doubtful' status, meaning that it cannot cure anymore and the bank must carry out a hard collection process in order to recover the defaulted exposure.

The asset correlation in the application portfolio is computed, as for retail exposures, as a function of the average PD by bucket in accordance with the regulation (see art. 154 of the CRR). The LGS is set as constant at 0.55.[7]

The table below reports the what-if simulation results. More specifically, the application portfolio is split into 5 groups according to the PD buckets. The column 'N (K)' reports the number of statistical units, columns 'T RWA rating' and 'T RWA no rating' report the total risk-weighted assets computed according to the RWA formula set out in article 154 of the CRR and adopting the LGD resulting from Table 2 or the overall LGD respectively.

Table 3: What-if analysis with and without the rating variable as a risk driver

PD Buckets	PD	EAD (M)	N (K)	T RWA rating (M)	T RWA no rating (M)
(0-0.5]	0.00324	10,841	112	43	83
(0.5-1]	0.00715	5,624	67	62	68
(1-2.5]	0.01524	43,458	436	628	715
(2.5-7.5]	0.04184	5,006	53	98	100
(7.5-100]	0.30120	6,327	68	254	223
Total		71,257	736	1085	1189

It is worth noticing that the rating variable used as a risk driver determines a decrease in total RWAs of around 8.7 per cent compared with the case where it is not considered.

3. Logic of the ARSF Model and the interdependence between PD and LGD

In this section we will explore the methodological background that suggests that including the rating variable as a driver of the LGD model requires robust ex-ante methodological analysis and ex-post performance evaluation. This is crucial as it would determine two unintended consequences:

i. Embedding an evident interdependence between the LGD and PD risk parameters;

[6] Please refer to section 1 for details.

[7] The value is fully judgmental and chosen for explanation purposes only. Different values would not affect the result. The level of simplification is intentionally very high because of the purpose of this analysis, that is to disentangle the effect of the rating variable only on the Danger Rate.

ii. Calibrating the LGD on a baseline scenario and violating the Downturn definition.

Regarding the first point, it is possible to argue that the interdependence between the conditional PD and LGD is simply ignored in the regulation, and not explicitly assumed, hence there is no reason to exclude a PD-related risk parameter in the LGD calibration. However, an argument in favour of questioning the inclusion of the rating in the LGD calibration is that the regulatory framework implicitly assumes the mutual independence of the PD and LGD, since it is ignored in the methodological framework itself (see Basel Committee on Banking Supervision (2006)). The reason is that the methodological framework is taken from the original paper by Gordy, M. B. (2003) where the LGD is considered stochastic but independent of the probability of default, which is a generalization from the models that treat LGD as a constant parameter (Boston (1997)) and is consistent with the models where it is considered as a stochastic variable independent of the probability of default (see for example Gupton et al. (1997), Crosbie and Bohn (2003)). Therefore, the introduction of a link between the PD and LGD parameters (via the inclusion of the rating as a risk driver of the danger rate in the LGD parameter) could lead to a violation of the underlying theoretical model and consequently to an underestimation of the expected prudential requirement.[8]

In any case, even if the correlation between PD and LGD has been empirically proven (see for example Van Vuuren, et al. (2017)), this does not mean it can be considered into the current prudential regulation. In this case, the guide published by the ECB,[9] aimed at ensuring a common and consistent approach to internal model matters, already specifies that the selection of reference dates for risk drivers should align with the expected distribution of defaults over a one-year horizon and the corresponding changes in the value of the risk driver for the relevant exposures. If risk drivers fluctuate over time, using a fixed time horizon before default should be avoided, especially if it's less than 12 months, unless the institution can prove that it does not result in an underestimation of final LGD estimates due to lack of representativeness. The adherence to these principles effectively serves as a minimum safeguard to ensure sufficient representativeness with respect to the default moment.

When accepting the rating as a risk driver, the second point remains valid; it is clear that using it as derived from the unconditional PD means to calibrate the LGD as in the case of a baseline economic scenario and not of an adverse one. This is because in the CRR the Risk Weighted formula is as follows (see ***Appendix*** for details):

$$RW = [(PD_S - PD) \cdot LGD] \cdot MFC \tag{2}$$

Where:

8 This argument would seem to bear resemblance to the concept of 'granularity adjustment', which was introduced to compensate for the assumption that idiosyncratic risk is fully diversified in a bank's portfolio, and could be eventually managed by a Pillar 2 add-on.

9 See ECB guide to internal models (February 2024), Credit risk – 6.2 LGD structure.

- PD_S is the conditional PD derived from average PD plugging in an economic adverse stress scenario derived from an adaptation of Merton's (1974) single asset model to credit portfolios (see also Gordy (2003)).
- MFC is a maturity correction factor.

Let us consider the calculation of the RW for a performing position in the application portfolio, if the LGD is calibrated using the actual rating of such position. It means that the LGD is calibrated consistently with the average unconditional PD in (2) and not with PD_S. Consequently, the LGD is calibrated by a parameter (PD) based on a baseline scenario and not on 'an adverse economic scenario'.

The Asymptotic Single Risk Factor (ASRF) model is the standard theoretical framework adopted by the BCBS to develop the Supervisory Formula to calculate the risk weight for credit assets under the Internal Rating Based Approach (IRB-A).

The European regulatory framework ignores the interdependence between PDs and LGDs (BCBS, 2006) and requires calculating downturn LGD in order to compensate for not explicitly modelling the relation between the two.

Indeed, in BCBS (2005) it is reported that:

"Under the implementation of the ASRF model used for Basel II, the sum of UL and EL for an exposure (i.e. its conditional expected loss) is equal to the product of a conditional PD and a "downturn" LGD. As discussed earlier, the conditional PD is derived by means of a supervisory mapping function that depends on the exposure's average PD. The LGD parameter used to calculate an exposure's conditional expected loss must also reflect adverse economic scenarios. During economic downturn losses on defaulted loans are likely to be higher than those under normal business conditions because, for example, collateral values may decline."

Within the European regulatory framework, clear guidelines are provided regarding the requirements for calculating the downturn effect. For models incorporating risk drivers sensitive to economic cycles, it is imperative for institutions to ensure that the resulting downturn LGD estimates do not disproportionately react to economic fluctuations. To address this concern, institutions should analyze the variation between the distribution of exposures across facility grades or pools, or appropriate intervals for continuous facility scales, within the current portfolio, and the expected distribution influenced by the relevant downturn period selected. In addition, if a significant disparity is identified through the analysis, institutions should implement adjustments to their downturn LGD estimates.[10]

[10] See EBA Guidelines for the estimation of LGD appropriate for an economic downturn - EBA/GL/2019/03 (March 2019).

4. Conclusion and proposals

In this analysis, we demonstrated the significant impact of the rating variable used as risk driver in the LGD model estimation. In a nutshell, the induction of a strong correlation between PD and LGD results in a significant decrease in RWAs for the performing application portfolio.

Even though the correlation between PD and LGD is not explicitly forbidden in the Basel framework (and in the CRR), the RWA calculation formula does not take it into account; hence, the incorporation of the rating as an LGD driver may be questionable. If such a risk driver is accepted, additional analyses should be considered.

Firstly, a representativeness analysis of the calibration population and the application portfolio should be performed. Then, the sensitivity of the LGD final estimates to the date at which the rating was retrieved needs to be tested, considering that this variable is volatile in the months immediately preceding the default event. Furthermore, according to the BCBS definition, the resulting LGD should incorporate an economic downturn; hence, the rating grade that drives it has to be the one retrieved from the conditional stressed PD_S and not from the unconditional one.

Appendix

The formula in articles 153 and 154 of the CRR is as follows:

$$RW = \left(LGD \cdot N\left(\frac{1}{\sqrt{1-R}} G(PD) + \sqrt{\frac{R}{1-R}} G(0.999) \right) - LGD \cdot PD \right) \cdot MF \cdot 12{,}5 \cdot 1{,}06 \qquad (3)$$

Where $MF = \frac{1+(M-2{,}5)\cdot b}{1-1{,}5\cdot b}$ (article 153) and $MF = 1$ (article 154).

(3) can be rewritten as:

$$RW = [(PD_S - PD) \cdot LGD] \cdot MFC$$

Where:

- $PD_S = N\left(\frac{1}{\sqrt{1-R}} G(PD) + \sqrt{\frac{R}{1-R}} G(0.999) \right)$
- $MFC = MF \cdot 12{,}5 \cdot 1{,}06$
- N(x) denotes the cumulative distribution function for a standard normal random variable (i.e. the probability that a normal random variable with zero mean and variance of one is less than or equal to x);
- G(Z) denotes the inverse cumulative distribution function for a standard normal random variable (i.e. the value x such that N(x) = z);
- R denotes the coefficient of correlation (the asset correlation in the ARSF formulation).

The economic stress is captured in the PD stressing the systemic factor argument weighted by the asset correlation R, in analytical terms:

$$Economic.Stress = \sqrt{\frac{R}{1-R}} G(0.999).$$

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