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Modelling the dynamics of nonperforming loans in Italy

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1. Summary and main conclusions

Understanding how credit quality evolves in response to macroeconomic conditions is of utmost importance to assess the resilience of the banking system, particularly in countries such as Italy where traditional credit provision is by far the most significant activity performed by banks. Furthermore, stress tests, a useful tool in assessing the stability of financial intermediaries, require conditional forecasts for credit quality measures under one or more predetermined scenarios for a set of macroeconomic variables.

The purpose of this note is to describe how a Bayesian approach can be employed to construct a parsimonious model linking a measure of credit quality to macroeconomic and financial variables. We show results for Italy. While Bayesian methods are widely employed in empirical macroeconomics,[2] they are less common in the context of credit risk forecasting. Bayesian approaches are used to analyse borrower-level default risk,[3] to estimate aggregate probabilities of loan default in the context of bank stress testing exercises,[4] and to forecast an aggregate indicator of bank vulnerability to risks arising from the real estate sector.[5]

1 Directorate General for Economics, Statistics and Research.

2 For an overview, see Steel, M.F.J. (2019), *Model Averaging and its Use in Economics*, Journal of Economic Literature; Min, C., Zellner, A. (1993), *Bayesian and non-Bayesian methods for combining models and forecasts with application to forecasting international growth rates*, Journal of Econometrics, 56.

3 Examples include Traczynski J. (2017), *Firm Default Prediction: A Bayesian Model-Averaging Approach*, Journal of Financial and Quantitative Analysis, 52, and Gonzales-Aguado C., Moral-Benito E. (2012), *Determinants of corporate default: a BMA approach*, Banco de Espana Working Paper, n. 1221.

4 Gross M., Poblaciòn J. (2017), *Implications of model uncertainty for bank stress testing*, Journal of Financial Services Research 5, also published as Gross, M., Población J. (2015), *A false sense of security in applying handpicked equations for stress test purposes*, ECB Working Papers, n.1845. See also Henry J., Kok C. (2013), *A macro stress testing framework for assessing systemic risks in the banking sector*, ECB Occasional Paper, n. 152.

5 Ciocchetta F., Cornacchia W., Felici R., Loberto M. (2016), *Assessing financial stability risks arising from the real estate market in Italy*, Bank of Italy, Occasional Papers, n. 323.

Our key variable of interest is the aggregate flow of new nonperforming loans in each quarter, normalized by the outstanding volume of performing loans at the beginning of the period (new nonperforming loan rate, or NNPL rate). Since the factors driving the default of households are likely to differ from those of nonfinancial firms, we estimate two separate regression models, one for each sector; we choose regressors among the variables suggested by the literature on credit default forecasting.

We find that the evolution of the NNPL rate over the past several years is tracked by two equations that include a small set of explanatory variables. The variables that are significant in predicting the NNPL rate of nonfinancial firms are: the interest rate on loans, the growth rates of credit and GDP, firms' leverage, and the change in unemployment. The rise in the NNPL rate of Italian firms between 2008 and 2009 is consistent with the severe recession, the credit crunch, and the high level of leverage of Italian firms at the onset of the downturn. The further increase in the NNPL rate after 2011 is consistent with the observed increase in the cost of credit and the contraction in credit. The evolution of the NNPL rate of households is predicted quite accurately by the dynamics of GDP and credit for most of the examined period; after 2013, low interest rates and renegotiations of mortgage contracts helped to keep the NNPL rate below what would have been consistent with the prevailing macroeconomic conditions.

For each of the two sectors, the model with the best fit is compared with a weighted average of models (Bayesian Model Average, BMA). Although the predicted values of the BMA and of the best models are close, the BMA is more robust to uncertainty in model selection, a desirable feature in the context of both forecasting and scenario analysis.

2. Data and variables

Our dependent variable, the NNPL rate, is calculated as the flow of loans that turn nonperforming in each quarter divided by the stock of performing loans outstanding at the beginning of that quarter, based on data reported to the Italian Credit Register by banks, credit intermediaries and securitization vehicles.[6] Quarterly data are available since 2006.

A loan is nonperforming if contractual payments are more than 90 days past-due or if the borrower is considered "unlikely to pay" by the lender.[7] The Register constructs a measure of nonperforming loans that takes into account the entire exposure towards a borrower; if the exposure that is deemed nonperforming by one or more banks reaches a predefined materiality threshold then the entire exposure to that borrower is considered nonperforming.

We focus on two broad categories of borrowers: nonfinancial firms and households. The NNPL rate for households refers mostly to mortgage debt because the Credit Register has a reporting threshold that excludes small exposures.[8] Most of the increase in the overall NNPL rate that occurred after the onset of the global financial crisis was due to

6 The NNPL rate is closer to the concept of default probability than ratios based on the stock of nonperforming loans or on loss provisions such as those employed by Virolainen K. (2004), *Macro Stress Testing with a Macroeconomic Credit Risk Model for Finland*, Bank of Finland Discussion Paper 18/2004; Wong J., Choi K., Fong T. (2006), *A framework for macro stress testing the credit risk of banks in Hong Kong*, HKMA Quarterly Bulletin; Quagliariello M. (2007), *Banks' riskiness over the business cycle: a panel analysis on Italian intermediaries*, Applied Financial Economics.

7 The definition is consistent with the one in Article 178 of the Capital Requirements Regulation (EU 575/2013).

8 Borrowers are reported if their credit exposure with a single institution is more than 30,000 euros.

the rise in defaults by nonfinancial firms. Italian households instead were quite resilient to adverse economic conditions: their highest NNPL rate was 3.4 per cent at the trough of the recession. After the peak in 2013, as the economy improved, the NNPL rate declined for both firms and households and is now below the level observed in 2006-2007.

We draw on the vast literature that investigates the determinants of borrowers' default to identify variables that could predict the NNPL rate. Theoretical models of economic distress emphasize leverage while models of financial distress focus on variables related to cash flows (e.g. profitability, income).[9] Empirical studies based on microeconomic data usually include a variety of ratios that capture both types of variables, based on balance sheet data.[10] Analyses of aggregate insolvency rates or loan defaults include aggregate measures of the financial conditions of borrowers, such as the indebtedness of the private sector and the profitability of nonfinancial firms.[11] The empirical models are augmented with standard macroeconomic indicators, typically GDP growth, inflation, unemployment, and interest rates. We select a subset of aggregate financial variables that captures the determinants suggested by economic theory, and a subset of macroeconomic indicators that takes into account the results of a previous study on Italian data.[12]

The total number of candidate variables considered is 20. We group them into three categories (see Table A1 in the Appendix for descriptive statistics). First, we consider aggregate measures of ability to repay debt, i.e. the burden of debt service with respect to income. For households we take the ratio of capital and interest expenditure over disposable income over the previous four quarters (*DSTI_HH*) while for firms, because we lack data on principal repayment, we use as a proxy the ratio of interest expenses to gross operating margin (*DSTI_NF*). Since debt service on loans with adjustable rates varies in response to changes in reference rates, we also include the 3-month Euribor rate (*Euribor*) and, for households, the share of outstanding mortgages with a fixed interest rate (*FixedRate*). In recent years, renegotiations of mortgage terms has increased substantially, particularly when interest rates on new loans declined to historical lows after 2014. Some renegotiations might have been the result of increased competition in the banking industry – resulting in clients negotiating lower rates with their banks – but others might have been aimed at extending maturities and lowering rates to avoid the financial distress of borrowers. We consider the share of outstanding mortgages for which contractual terms (e.g. interest rate, repayment schedule and maturity) were renegotiated at least once since origination. Since this share is based on cumulated renegotiations and declines only as a result of repayments, we take the natural logarithm of the variable to avoid giving excessive weight to persistently high values (*Renegotiations*).

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9 See for example Kahl, M. (2002), *Economic Distress, Financial Distress, and Dynamic Liquidation*, Journal of Finance; Leland H. (1994), *Corporate debt value, bond covenants, and optimal capital structure*, J. Finance, 49.

10 The seminal work in this field is the study by Altman, E.I. (1968), *Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy*, Journal of Finance, 23; examples of more recent work are Shumway T. (2001), *Forecasting Bankruptcy More Accurately: A Simple Hazard Model*, Journal of Business, 74; Jacobson T., Lindé J., Roszbach K. (2011), *Firm Default and Aggregate Fluctuations*, Sveriges Riksbank Research Paper Series, n. 57.

11 Castrèn O., Dees S., Zaher F. (2008), *Global macro-financial shocks and expected default frequencies in the euro area*, ECB Working Papers, n. 875; Haldane A., Hall S., Pezzini S. (2007), *A new approach to assessing risks to financial stability*, Financial Stability Paper, n. 2; Burrows O., Learmonth D., McKeown J. (2012), *RAMSI: a top-down stress-testing model*, Bank of England Financial Stability Paper, n. 17.

12 Bofondi M., Ropele T. (2011), *Macroeconomic determinants of bad loans: evidence from Italian banks*, Bank of Italy, Questioni di Economia e Finanza (Occasional Papers), n. 89.

Another group of variables captures financial structure. The aggregate leverage of nonfinancial firms is computed as the ratio of financial debt to the sum of financial debt and equity (*Leverage*), sourced from the Financial Accounts and measured at market prices. We scale liquid assets that could be easily divested to compensate for a negative income shock (cash, deposits, and securities) by total financial debt (*Liquidity_NF*). The level of indebtedness of households is measured by the sector's total financial debt scaled by total financial assets (*DebtToAssets*). Financial buffers available to households to meet income shocks are measured by the sum of financial assets reported in the Financial Accounts excluding unlisted stock and claims on insurance companies and pension funds, scaled by total financial debt (*Liquidity_HH*).

Macroeconomic conditions – the third group – are measured by variables for income, unemployment, quantities and cost of credit and asset prices. The first variable is the rate of growth of real GDP (*GDP*), which is expected to be negatively related to default rates as it affects borrowers' income. We also consider the growth rates of income for each sector: for households, real disposable income (*DispIncome*), and for non-financial firms, gross operating margin (*OperatingMargin*). Another potentially relevant variable is the growth rate of consumption of durable goods (*DurablesConsumption*) because it is more volatile than GDP and has different cyclical properties, which could enrich the model dynamics. This component of aggregate consumption is also more sensitive to credit supply conditions than others and could signal some income stress for households. Unemployment is calculated as the change in the unemployment rate with respect to the same quarter of the previous year (*Unemployment*), and should anticipate the NNPL rate with a positive coefficient. We include the growth rate of the residential property price index with respect to the same quarter of the previous year (*HousePrices*) and expect a negative correlation with loan defaults. We do not consider the change in a stock price index, which was found insignificant in another study on Italian data.[13]

We measure credit growth and cost of borrowing with reference to bank loans, since they represent the largest share of total credit to the Italian private sector. The growth rate of loans (*LoansGrowth_NF* and *LoansGrowth_HH*) directly influences the NNPL rate because new loans generally do not default soon after issuance; furthermore, a large inflow of new loans automatically reduces the ratio by increasing the denominator. In a reduced form model the coefficient of credit growth could also capture the link between changes in credit risk and credit supply conditions; an increase in risk would generate risk aversion on the part of banks and a reduction in credit growth. In the short term, the combination of the effects just described would produce a negative effect on credit growth. Over a longer time horizon, instead, fast credit growth may be associated with an increase in credit risk, as shown by several studies focusing on credit cycles;[14] we do not consider this case since we only focus on a four quarter horizon.

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[13] Bofondi and Ropele, cit.

[14] See Chavan P., Gambacorta L. (2016), *Bank Lending and Loan Quality: The Case of India,* Reserve Bank of India Working Paper Series; Jiménez G., Saurina J. (2006), *Credit Cycles, Credit Risk, and Prudential Regulation*, International Journal of Central Banking, vol. 2(2); May, Lis F.S., Pagés M., Saurina J. (2000), *Credit Growth, Problem Loans and Credit Risk Provisioning in Spain*, Banco de Espana, Working Paper 18.

The average cost of credit on new loans to firms (*Rate_NF*) and the average cost of loans to households for house purchase (*Rate_HH*) could belong in our model for different reasons. First, when loan rates rise debt service increases, with a lag that depends on the maturity structure of the outstanding debt. Second, banks adjust rates in response to changes in credit risk and an increase in interest rates on new loans could anticipate future changes in loan defaults. Third, loan interest rates incorporate bank funding conditions; to the extent that rising rates capture tightening credit supply, they could be anticipating a negative credit supply shock, which in turn increases the insolvency of borrowers that need to refinance their loans.

An inspection of the time series of the NNPL rate suggests that a dummy for the last quarter of 2014 (*DU14Q4*) should be included. The sharp increase in the NNPL rate of firms in that quarter was the result of a one-off reassessment of the exposures of banks that were subject to an asset quality review by supervisors.[15]

3. Empirical approach

For the purpose of conditional forecasting, it is sufficient to identify statistical dependencies that are stable over time and are likely to hold under a given scenario. Our empirical approach does not seek to identify causal relationships but aims at identifying a model that strikes the balance between bias and the risk of overfitting the data. Increasing the flexibility of the functional form and adding more regressors could reduce bias but increase forecast variance.[16] Results from several studies show that single equation regression models using a relatively small number of macroeconomic and financial variables, despite their simplicity, are often able to replicate quite well the time series of aggregate credit risk measures.[17]

We restrict the search for a model within the class of autoregressive distributed lag models (ADL), limiting uncertainty to which covariates and lags should be included as regressors. We model the NNPL rate with a single time-series regression equation:

$$y_t = \beta_0 + \sum_{\ell=1}^{L} \boldsymbol{\beta}_\ell \boldsymbol{X}_{t-\ell} + u_t, \qquad t = 1,2,\dots,T \tag{1}$$

where the dependent variable y_t is the NNPL rate of either households or firms, t is the quarter of reference, and $\boldsymbol{X}_{t-\ell}$ is a vector of variables with ℓ lags which may include a lag of the dependent variable.[18]

In order to select an optimal model within the space of all possible specifications within this class, we follow a Bayesian approach. Any model that potentially generates

[15] The asset quality review was part of a comprehensive assessment by the ECB and national supervisors in preparation of the operationalization of the Single Supervisory Mechanism. See the Bank of Italy's Financial Stability Report, n. 1/2015.

[16] Hastie T., Tibshirani R., Friedman J. (2017), *The Elements of Statistical Learning*, Springer.

[17] Ong Li L. (2014), *A Guide to IMF Stress Testing Methods and Model,* International Monetary Fund.

[18] Although the dependent variable is a fraction between 0 and 1, we prefer a linear formulation due to advantages in terms of computational speed and availability of standard diagnostic tests. We estimate a fractional logistic regression for robustness purposes and results are very similar to our main specification.

the data is assigned an unconditional prior probability based on beliefs and economic knowledge. The probability is updated by assessing how well the model fits the data to obtain a posterior probability (i.e. the probability that the model is the data generating process, conditional on the observed data). For the purpose of forecasting, it is then possible to employ either the model with the highest posterior probability or to use a weighted average of the models. Model averaging has the advantage that the errors of individual models tend to compensate each other, reducing the forecast error due to uncertainty in model selection.

In particular, we follow the approach proposed by Sala-i-Martin et al. (2004).[19] Their framework is appealing because any of the K possible variables is given the same prior probability p to be used as a regressor, so that the prior probabilities of candidate models (i.e. distinct combinations of regressors) only depend on how far the number of regressors included is from the expected model size $\bar{k} = p \cdot K$ chosen by the researcher. We apply a random sampling algorithm to estimate a large number of models among the possible combinations of regressors, and estimate the posterior probability of each model based on its ability to fit the data. Between two models with the same ability to fit the data, the most parsimonious one is preferred since it is less likely to suffer from overfitting. The algorithm samples from a pool of potential regressors that includes contemporaneous values and lags up to the fourth quarter of the variables described in Section 2 and the lagged dependent variable. We set $\bar{k} = 6$ because we are constrained by the small sample size financial environment and the small sample size (53 quarters).[20] More details are provided in Appendix B.

We also compute a weighted average over the pool of models estimated by the search procedure; model weights are given by their posterior probabilities (Bayesian Model Average, BMA). Since all models are linear in their parameters, the BMA is a linear equation as well. Each parameter in the BMA is a weighted average of the estimates of each single model in the pool. We assess the statistical significance of the BMA coefficients by computing their posterior inclusion probability as the sum of the posterior model probabilities for all the models that include that variable. If the posterior inclusion probability is greater than the prior probability, the estimated effect of a variable is considered statistically significant.

4. Results

The selection procedure yields a ranking of the models estimated based on the posterior model probability (PMP). We first comment on the best individual model selected by the procedure, and then consider model uncertainty results and BMA estimates.

4.1 Top ranking models

Table 1 reports the top ranking model for nonfinancial firms (col. 1). As mentioned in the previous section, the Bayesian model selection criterion based on the posterior

[19] Sala-i-Martin X., Doppelhofer D., Miller R. (2004), *Determinants of long-term growth: a Bayesian Averaging of Classical Estimates approach*, The American Economic Review, 94.

[20] An analysis of the robustness of the results to the choice of the expected model size was performed and the results are available upon request.

model probability imposes a penalty for the number of parameters, favouring more parsimonious specifications. For robustness purposes we also rank the estimated models by the Root Mean Squared Forecast Error (RMSFE), which does not take into account the number of parameters estimated.[21]

Table 1: Best linear models – nonfinancial firms

Variable	Lag	Best model by PMP	Best model by RMSFE
Intercept		-0.086***	-0.036
		[0.010]	[0.026]
DU14Q4	0	0.020***	0.018***
		[0.004]	[0.004]
GDP	0	-0.115***	-0.191***
		[0.033]	[0.057]
Rate_NF	2	0.992***	0.789***
		[0.090]	[0.132]
Leverage	3	0.245***	0.185***
		[0.022]	[0.037]
Unemployment	4	0.380***	0.358***
		[0.098]	[0.099]
LoansGrowth_NF	1	-0.194***	-0.189***
		[0.025]	[0.028]
	4		-0.021
			[0.030]
Liquidity_NF	3		-0.089**
			[0.043]
N		53	53
R2		0.976	0.978
BIC		-417.29	-414.37
PMP		0.351	0.0005
RMSFE		0.0049	0.0046
DW		0.5926	0.9515

The table shows the coefficients of the best linear specifications according to the Posterior Model Probability (PMP) and the RMFSE obtained through cross-validation. Standard errors in brackets. Statistical significance: *p<0.1, **p<0.05, ***p<0.01. The column Lag indicates the number of quarterly lags of the variable.

The best model for nonfinancial firms includes five variables, the intercept and the dummy for the last quarter of 2014. All regressors have statistically significant coefficients and display a strong co-movement with the NNPL rate. A 1 percentage point decline in GDP growth is associated with a contemporaneous increase by almost 11 basis points in the NNPL rate. A 1 percentage point variation in the average cost of credit for nonfinancial firms (*Rate_NF*) is estimated to anticipate by two quarters a movement in the NNPL rate by a similar amount. A one standard deviation increase in leverage (4 percentage points) anticipates a rise in the future NNPL rate of about 1 percentage point. A one standard deviation increase in the unemployment rate (about 1 per cent) is associated with a variation of the NNPL

[21] We compute the RMSFE by cross-validation, repeatedly fitting each model to a subset of observations and calculating the RMSFE on the observations left out.

rate by 0.38 percentage points. An increase in credit growth (*LoansGrowth_NF)* of one standard deviation (5.4 percentage points) is associated with a decline in the next quarter's NNPL rate by 1 percentage point. The model that minimizes the RMSFE is shown in column 2 of Table 1; it includes an additional lag for credit growth and an additional regressor, *Liquidity_NF,* with respect to the model in column 1, but the improvement in the RMSFE is negligible.

Table 2 shows the results for households. A one standard deviation increase in GDP (2.3 percentage points) implies a decline of 26 basis points in the NNPL rate of households, to be compared with a sample mean of the dependent variable of 2.2 percentage points. The Euribor 3-month rate has a much smaller effect than the interest rate in the regression for nonfinancial firms, while a variation in credit growth (*LoansGrowth_HH*), normalized by the standard deviation of the dependent variables, has a similar effect in the two cases. The log of the share of renegotiated loans (*Renegotiations)* has a negative coefficient, consistent with the hypothesis that renegotiations avoided some of the defaults that would have occurred under the prevailing macroeconomic conditions. The economic effect of a one standard deviation increase in renegotiated loans (4.9 percentage points) is a decline of 44 basis points in the NNPL rate of households, calculated at the mean of the values observed since 2012, when the variable is reported. The model that ranks first by RMSFE

Table 2: Best linear models – households

Variable	Lag	Best model by PMP	Best model by RMSFE
(Intercept)		-0.002 [0.001]	-0.002 [0.001]
GDP	0	-0.065*** [0.012]	-0.072*** [0.012]
Euribor	1	0.080** [0.030]	0.096*** [0.032]
Renegotiations	4	-0.009*** [0.001]	-0.009*** [0.001]
LoansGrowth_HH	3	-0.062*** [0.008]	
	4		-0.054*** [0.011]
HousePrices	2		-0.015 [0.015]
N		53	53
R2		0.931	0.93
BIC		-510.42	-506.24
PMP		0.3026	0.0007
RMSFE		0.0018	0.0017
DW		0.2315	0.1536

The table shows the coefficients of the best linear specifications according to the Posterior Model Probability (PMP) and the RMFSE obtained through cross-validation. Standard errors in brackets. Statistical significance: *p<0.1, **p<0.05, ***p<0.01. The column Lag indicates the number of quarterly lags of the variable.

(column 2 of Table 2) also contains the *HousePrices* growth as a regressor, but it does not add significant predictive power to the specification.

4.2 Model uncertainty and BMA

Our results show that the distribution of models by PMP is concentrated, with the first 10 models accounting for over 60% of the probability for firms and 70% for households. Furthermore, the models ranking the highest tend to share most of the regressors, which suggests that model uncertainty is not substantial.Table 3 reports the average

Table 3: Bayesian Model Average

A. Non-financial firms

Regressor	Lag	Prior Inclusion Probability	Posterior Inclusion Probability	Average Coefficient	St. dev. Coefficient
Intercept		1.000	1.000	-0.085	0.022
Rate_NF	2	0.136	0.998	0.999	0.181
LoansGrowth_NF	1	0.136	0.997	-0.206	0.040
Leverage	3	0.136	0.985	0.241	0.048
DU14Q4	0	0.136	0.966	0.019	0.005
Unemployment	4	0.136	0.817	0.299	0.170
GDP	0	0.136	0.763	-0.092	0.064
GDP	1	0.136	0.051	-0.005	0.024
OperatingMargin	4	0.136	0.048	-0.001	0.005
DebtService_NF	4	0.136	0.044	0.002	0.011
Liquidity_NF	4	0.136	0.043	-0.003	0.017
Leverage	2	0.136	0.040	0.004	0.029
Liquidity_NF	3	0.136	0.040	-0.003	0.016
Rate_NF	3	0.136	0.035	0.008	0.072
Liquidity_NF	2	0.136	0.032	-0.002	0.014

B. Households

Regressor	Lag	Prior Inclusion Probability	Posterior Inclusion Probability	Average Coefficient	St. dev. Coefficient
Intercept		1.000	1.000	-0.002	0.002
GDP	0	0.095	0.995	-0.064	0.015
Renegotiations	4	0.095	0.985	-0.009	0.001
Euribor	1	0.095	0.658	0.053	0.047
LoansGrowth_HH	3	0.095	0.496	-0.028	0.031
LoansGrowth_HH	4	0.095	0.330	-0.017	0.028
LoansGrowth_HH	2	0.095	0.214	-0.012	0.024
Rate_HH	2	0.095	0.128	0.012	0.037
Euribor	3	0.095	0.039	0.002	0.013
Rate_HH	1	0.095	0.031	0.003	0.022
Euribor	2	0.095	0.030	0.002	0.014
LoansGrowth_HH	1	0.095	0.021	-0.001	0.007
Unemployment	2	0.095	0.019	0.001	0.012
Euribor	0	0.095	0.018	0.001	0.008
Renegotiations	3	0.095	0.015	-0.000	0.001

The table reports the first 15 regressors ordered by posterior probability of inclusion; for those above the dotted line, the posterior probability is greater than the prior. The first two columns of the table show the prior probability of being included in a randomly selected model for each regressor and the estimated posterior probability of belonging to the data generating process. The last two columns show the weighted average and standard deviation of the coefficient value for the regressor across the estimated models.

coefficients of the first 15 regressors ordered by posterior probability of inclusion; for those above the dotted line, the posterior probability is greater than the prior. The table also shows the average and the standard deviation of each coefficient.

In the case of nonfinancial firms (Table 3.A), five variables (plus the dummy *DU1404*) are significant predictors of the NNPL rate. These variables coincide with those contained in the model that maximizes the posterior model probability; however, the absolute value of BMA coefficients is in some cases smaller than in the best model (e.g., for *GDP* and *Unemployment*), also because the BMA takes into account the effect of a larger number of variables and lags. The difference in magnitude suggests that if we only relied on the best equations we could slightly overestimate the predictive power of some regressors and underestimate others.

For households, seven variables have a posterior probability of inclusion that exceeds the prior (Table 3.B), and are represented by variables included in the top ranking model (*GDP*, *Renegotiations*, *LoansGrowth_HH*, *Euribor*) plus the cost of credit *Rate_HH*. The BMA shows that interest rates and other lags of *LoansGrowth_HH* not included in the best model have some explanatory power. The results also show that the relationship between the NNPL rate and loan growth is more persistent than in the best model.

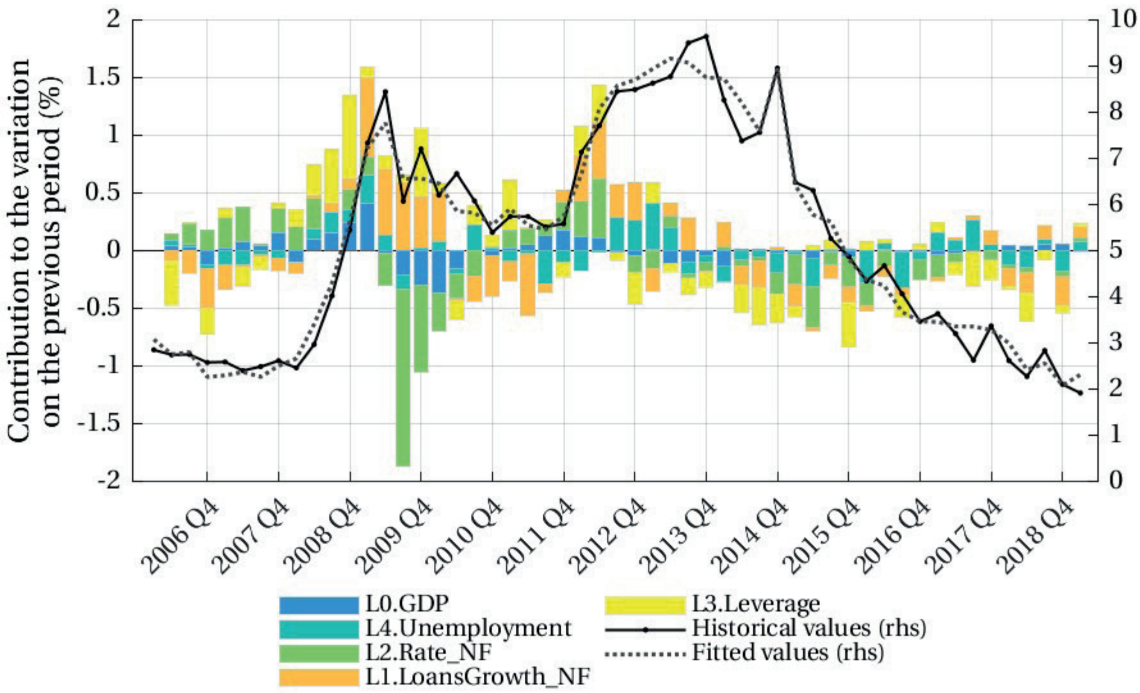
4.3 Contribution of the predictors over time

Figures 1 and 2 show the contributions of each regressor to the quarterly change in the NNPL rate (for the BMA in Figure 2, we highlight the contribution of the regressors whose posterior probability is higher than the prior). For firms, the fast increases in the NNPL rate observed in the sample (2008Q1-2009Q2 and 2011Q1-2013Q4) were associated with a sharp decline in GDP and credit growth rates, and an increase in unemployment, interest rates and leverage. In the second half of 2009 GDP recovered and interest rates started to drop while credit and leverage continued to increase until 2011. After 2014 the NNPL rate gradually fell to pre-crisis values in a context of moderate GDP growth and unemployment reduction. The decline in the NNPL rate was anticipated by a reduction in leverage and interest rates, and by a moderate recovery in credit growth.

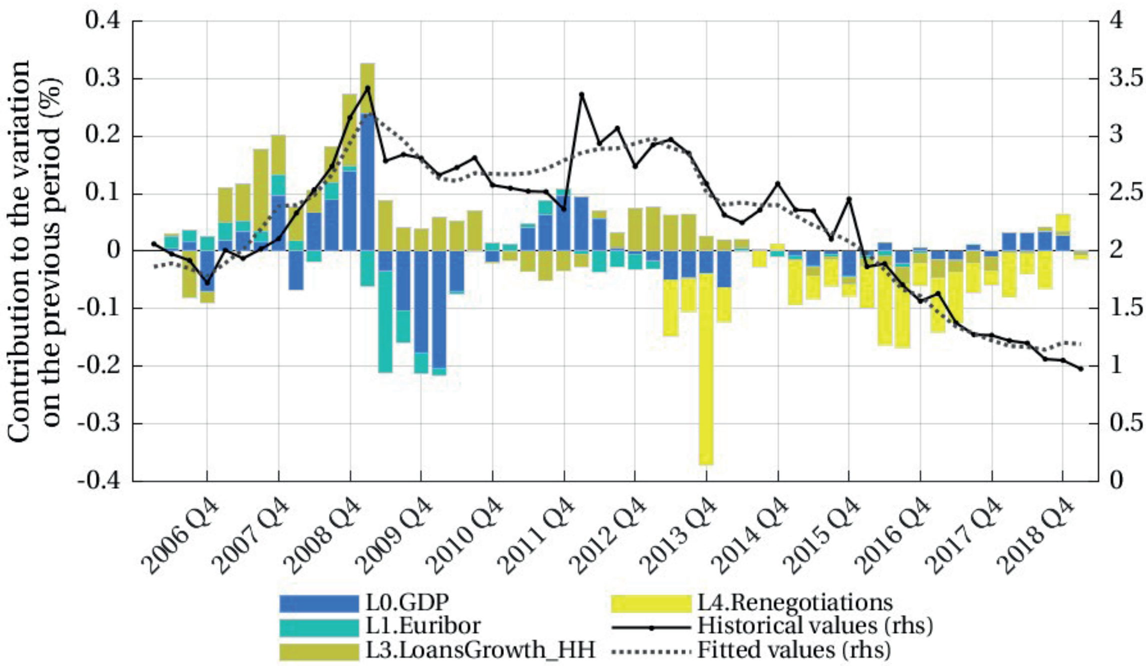
The evolution of the NNPL rate of households is related to GDP growth and is anticipated by changes in credit growth in the opposite direction until 2014. The decline in the NNPL rate after 2014, however, cannot be explained only by the macroeconomic variables, since GDP and credit remained subdued. The rise in renegotiations appears to be able to explain this decline, while the quantitative relevance of interest rates, once the other variables are taken into account, is comparatively quite small. A possible explanation is that renegotiations are capturing the impact of the low rate environment on debt service for some borrowers that would have otherwise faced financial difficulties.

Figure 1: First model by PMP – Contributions of regressors to the quarterly change in the NNPL rate

(a) Non-financial firms



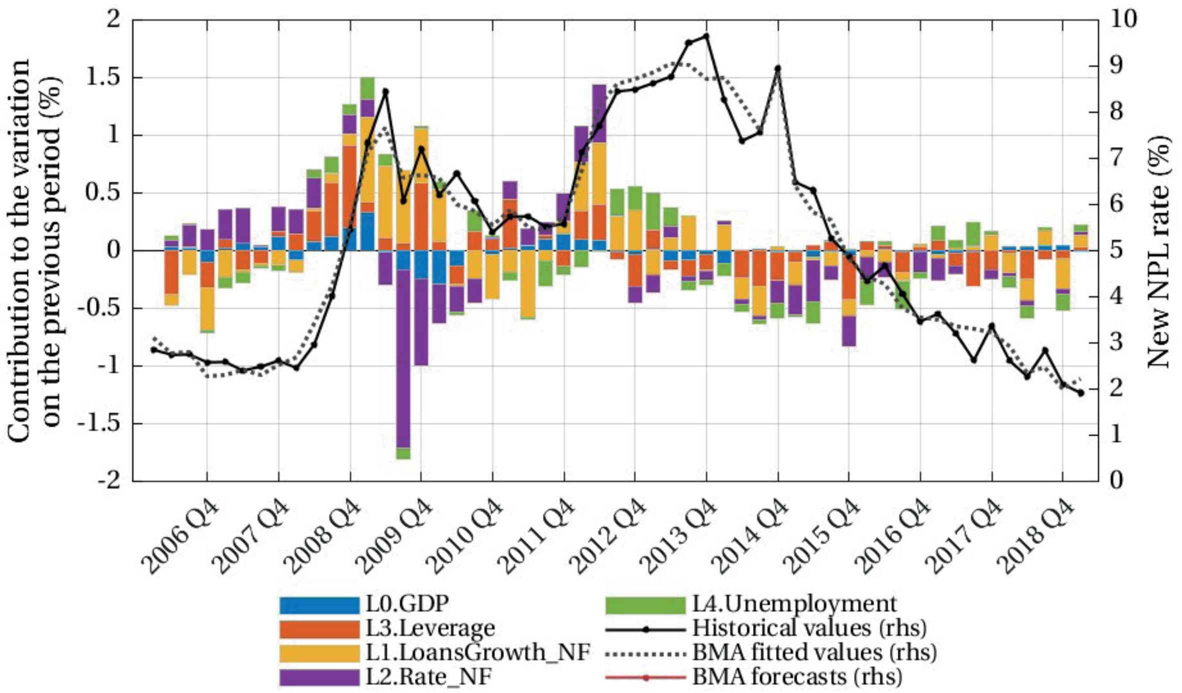
(b) Households



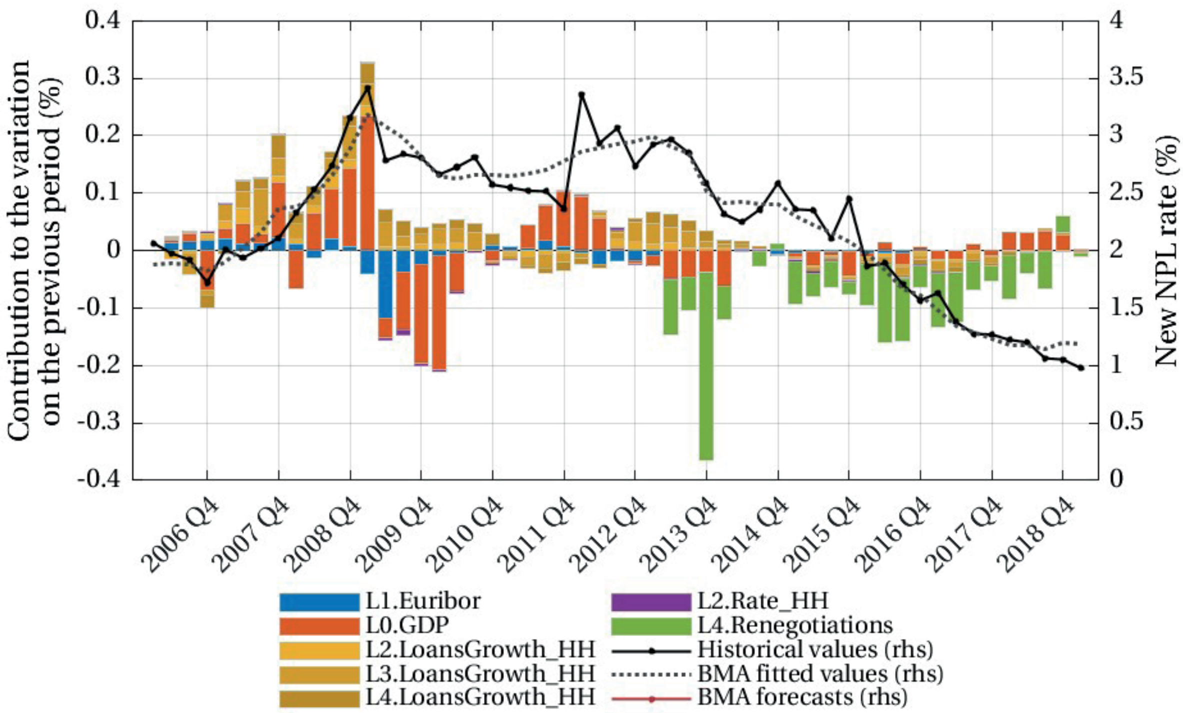
The figure shows the contribution of the regressors for which the posterior probability is greater than the prior. The effect of dummy variables and of the intercept are not shown. The regressors are ordered alphabetically rather than according to their ranking in terms of explanatory power, which is shown in Table 3. The prefix L#- indicates that the variable is lagged by # quarters.

Figure 2: Bayesian Model Average – Contributions of regressors to the quarterly change in the NNPL rate

(a) Non-financial firms



(b) Households



The figure shows the contribution of the regressors for which the posterior probability is greater than the prior. The effect of dummy variables and of the intercept are not shown. The regressors are ordered alphabetically rather than according to their ranking in terms of explanatory power, which is shown in Table 3. The prefix L#- indicates that the variable is lagged by # quarters.

Appendix A: Data

	Variable	Description (source)	mean	st. dev.
Table A1: Variables and Descriptive Statistics [1]				
Dependent variables	*NNPL_NF*	Flow of new NPLs over the stock of performing loans at the start of the quarter for nonfinancial corporations and producer households; deseasonalized and annualized (CR)	5.1	2.4
	NNPL_HH	Flow of new NPLs over the stock of performing loans at the start of the quarter for consumer households; deseasonalized and annualized (CR)	2.2	0.6
Debt service	*DSTI_NF*	Ratio of firms' interest expenses over gross operating profit; average of the previous 4 quarters (BI)	14.7	4.1
	DSTI_HH	Debt service over households' disposable income; average of the previous 4 quarters (BI)	10.7	1.2
	Euribor	Short-term interest rate (3-month Euribor rate); (EMMI)	1.1	1.7
	FixedRate	Share of existing mortgages with fixed-rate (SR)	31.1	5.3
	Renegotiations [2]	Share of existing mortgages whose terms have been renegotiated in the past (SR), natural logarithm	11.0	5.2
Financial structure	*Leverage*	Financial debt over the sum of financial debt and equity, at market value (FA)	43.1	4.2
	DebtToAssets	Ratio of total financial debt to households' total finan. assets (FA)	17.0	1.8
	Liquidity_NF	Liquid assets (currency and deposits, short term securities, bonds) over financial debt of nonfinancial firms (FA)	18.9	4.2
	Liquidity_HH	Liquid assets (financial assets excluding unlisted stock and insurance and pension funds) over total financial debt of households (FA)	228.2	18.3
Macroeconomic conditions	*GDP*	Real gross domestic product, deseasonalized; 12-month growth rate (ISTAT)	-0.1	2.2
	OperatingMargin	Gross operating margin of nonfinancial firms; 12-month growth rate (ISTAT*)	0.2	6.6
	DispIncome	Real household disposable income; 12-month growth rate (ISTAT*)	-0.3	2.0
	DurablesConsumption	Real consumption of durable goods, deseasonalized; 12-month growth rate (ISTAT*)	0.5	5.9
	Unemployment	Unemployment rate; 12-month variation (ISTAT)	0.2	1.0
	HousePrices	Average price per square meter of residential real estate; 12-month growth rate (ISTAT*)	-0.4	3.6
	LoansGrowth_NF	Bank loans to firms, adjusted for loan sales and securitizations; 12-month growth rate (BI)	2.3	5.5
	LoansGrowth_HH	Bank loans to households for house purchase, adjusted for loan sales and securitizations; 12-month growth rate (BI)	4.0	5.2
	Rate_NF	Average interest rate on new loans to nonfinancial firms (BI)	3.4	1.2
	Rate_HH	Average interest rate on new loans to households for house purchase (BI)	3.5	1.3
Other	*DU14Q4*	Dummy for the last quarter of 2014		

Legend: BI: Bank of Italy, other; CR: Credit Register; FA: Financial Accounts; SR: Supervisory Reports; ISTAT: Italian National Institute of Statistics; EMMI: European Money Markets Institute; '*': Bank of Italy's calculations on original data.
Notes: (1) Descriptive statistics refer to 2006Q1-2019Q1. (2) For *Renegotiations*, descriptive statistics refer to the level of the variable rather than its logarithm used in the regressions, and to the period 2012Q1-2019Q1.

Appendix B: Methodological notes

Model selection can be addressed as a case of model uncertainty and solved with a Bayesian approach. The vector β of parameters is considered a random variable, and a model M is an area of the parameters' support. The posterior probability that model M_i describes the joint probability distribution of the data y (which includes both the dependent variable and regressors) is:

$$P(M_i|y) = w_i / \sum_{k=1}^{n} w_k, \quad w_i = P(M_i)\, P(y|M_i) \tag{2}$$

where $P(M_i)$ is the prior probability given to a model by the researcher before examining the data and $P(y|M_i)$ is the model's marginal likelihood, which in turn depends on the prior distribution of model parameters.

Eliciting two types of priors, those on the models and those on the parameters of each possible model, requires simplifying assumptions. A possibility is to use diffuse priors for parameters, i.e. priors that tend to assign the same probability to any possible value. Sala-i-Martin et al. derive a method to use diffuse priors in the context of model averaging of multiple linear regressions, when different models may contain different sets of variables; they name it Bayesian Averaging of Classical Estimates (BACE).[22] Using a limiting argument, they show that eq. 2 converges to:

$$P(M_i|y) = w_i / \sum_{k=1}^{n} w_k, \quad w_i = P(M_i)\, T_i^{-k_i/2} SSE_i^{-T_i/2} \tag{3}$$

According to eq.3, a model posterior probability depends on its prior probability, on the number of parameters k_i included in the model (negatively), and on the goodness of fit captured by the sum of squared residuals SSE_i (positively): between two models with the same prior and the same ability to fit the data, the most parsimonious one is preferred.

For each model, we let the prior probability depend only on the number of regressors included, as suggested by Sala-i-Martin et al. (2004). The distribution of model prior probability is a binomial $\mathcal{B}(K,p)$ where K is the number of potential regressors and p the probability that any variable has of being included in the model, independent of the inclusion of any other variable, and $P(M_i) = p^{k_i}(1-p)^{K-k_i}$. A single hyperparameter, the expected the number of regressors in the model, $\bar{k} = pK$, then fully specifies the model priors.[23] This approach therefore gives the same initial probability to all potential regressors, but penalizes models with too many, or too few, parameters with respect to $\bar{k}$.[24]

[22] Another common approach in the literature is to assume a conditionally normal prior on the coefficient vector with zero mean and the prior variance proportional to the posterior sample covariance (see Zellner A. (1986), *On assessing prior distributions and Bayesian regression analysis with g-prior distributions*; in P. K. Goel and A. Zellner (Eds.), *Bayesian Inference and Decision Techniques: Essays in Honor of Bruno de Finetti*, North-Holland, pp. 233–43).

[23] An alternative widely used in the literature is represented by the approach of Fernandez C., Ley E., Steel M.F.J. (2001), *Benchmark Priors for Bayesian Model Averaging*, Journal of Econometrics, 100(2). Ley E., Steel M. (2009), *On the effect of prior assumptions in Bayesian model averaging with applications to growth regression*, Journal of Applied Econometrics 24, provide a detailed discussion that compares the prior assumptions used by Fernandez et al. (2001) and Sala-i-Martin et al. (2004). Their paper highlights the fact that the two approaches imply different penalties on model size, depending on the number of possible regressors, the prior on mean model size and number of observations. They also show that in the applied literature the typical values of these parameters are such that the two approaches give very similar results.

[24] Since $\bar{k}$ is usually chosen to be a small number, the penalty is mostly on overparametrized models.

We follow this suggestion but also assign $P(M_i) = 0$ to models for which either (a) residuals are serially correlated, or (b) the sign of one or more of the estimated marginal effects is not consistent with our prior based on economic intuition (Table A2).[25] The first condition rules out models whose predicted values would tend to deviate systematically from historical values for consecutive periods. The sign restrictions incorporate basic economic knowledge on the causal processes driving credit quality dynamics (e.g. that the same possibly unobservable factors that make GDP shrink also negatively affect credit quality): models that are coherent with these constraints are more likely to express correlations that remain stable over time. This allows us therefore to depart from purely data-driven approaches to model search.

Since the number of models that should be estimated to calculate $P(M_i|y)$ is computationally infeasible even for a moderate number of variables, we implement a stochastic algorithm to perform sampling, based on the one employed by Sala-i-Martin et al. (2004).[26] The pool of potential regressors from which the algorithm samples includes the contemporaneous values and lags up to the fourth of all the explanatory values, and one lag of the dependent variable. Since we have to balance the small number of observations (53 quarters) with a sufficiently rich description of the macroeconomic and financial environment, we set $\bar{k} = 6$.

The posterior probabilities $P(M_i|y)$ obtained after running the algorithm can be used to select a single best model, i.e. the one with the highest posterior probability. The Bayesian framework also provides a solution to address model uncertainty because the estimated posterior probability of each specification can be used as a weight to calculate a Bayesian average of models (BMA). An average of forecasts generally has a smaller forecast error than any of the individual models combined, since model averaging allows the errors of individual models to compensate each other.[27] The posterior probability distribution of the parameter vector β conditional on the observed data, $P(\beta|y)$, can be written as a weighted average of the conditional distributions of β in all the possible individual models, with the posterior model probabilities $P(M_i|y)$ as weights:

$$P(\beta|y) = \sum_{i=1}^{n} P(M_i|y)\, P(\beta|y, M_i) \tag{4}$$

Therefore, having a sufficiently large sample of models, it is possible to consistently estimate the BMA parameters as a weighted average of individual models' OLS estimates $\hat{\beta}_i^{OLS}$, with weights given by the estimated posterior model probabilities:

25 These conditions can only be tested after estimating a model, which implies that all models have positive sampling probability in the sampling algorithm but those that get discarted are given zero weight.

26 The algorithm is detailed in the online appendix to their paper.

27 Palm F.C., Zellner A. (1992), *To Combine or not to Combine? Issues of Combining Forecasts*, Journal of Forecasting, Vol. 11.

$$\hat{\beta}^{BMA} = \sum_{i=1}^{n} \hat{P}(M_i|y)\, \hat{\beta}_i^{OLS} \tag{5}$$

$$Var(\hat{\beta}^{BMA}\,|y) = \underbrace{\sum_{i=1}^{n} \hat{P}(M_i|y)\, Var(\hat{\beta}_i^{OLS}|y, M_i)}_{\substack{variance\\ within\ models}} + \underbrace{\sum_{i=1}^{n} \hat{P}(M_i|y)\, (\hat{\beta}_i^{OLS} - \hat{\beta}^{BMA})^2}_{\substack{variance\\ between\ models}} \tag{6}$$

Equation 6 for the parameters' posterior variance incorporates both the estimated variances in individual models as well as the variance in estimates across different models, highlighting the ability of BMA to account for model uncertainty.

Table A2: Sign restrictions and candidate variables

Variable	Sign	Non-financial firms	Households
NNPL_NF			x
NNPL_HH		x	
DSTI_NF	+	x	
DSTI_HH	+		x
Euribor	+	x	x
FixedRate	-		x
Renegotiations	-		x
Leverage	+	x	
DebtToAssets	+		x
Liquidity_NF	-	x	
Liquidity_HH	-		x
GDP	-	x	x
OperatingMargin	-	x	
DispIncome	-		x
DurablesConsumption	-		x
Unemployment	+	x	x
HousePrices	-		x
LoansGrowth_NF		x	
LoansGrowth_HH			x
Rate_NF	+	x	
Rate_HH	+		x
DU14Q4	+	x	

Finally, for each regressor we can calculate the posterior inclusion probability, i.e. the probability that its coefficient is not zero in the population data generating process, just as the sum of the posterior model probabilities for all the models that include that variable:

$$P(\beta \neq 0|y) = \sum_{i=1}^{n} P(M_i|y)\, \mathbb{I}(|\beta| > 0|y, M_i) \tag{7}$$

The result that the posterior inclusion probability for a regressor is greater than its prior probability can be interpreted in terms of the statistical significance of the effect (analogously to the common test of statistical significance of the coefficients performed on individual models).