Inflation and Growth Risk: Balancing the Scales with Surveys

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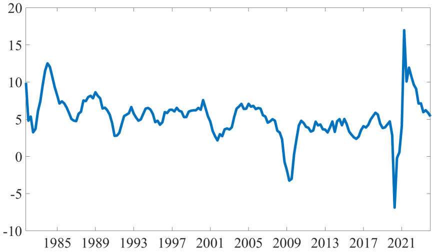
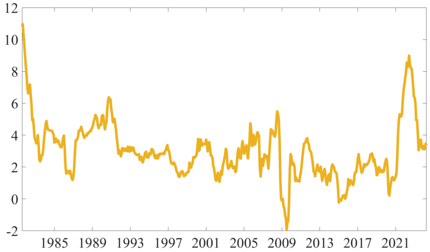
Large and volatile macroeconomic fluctuations

- Q2-2020: largest decline in economic activity in a century.
- Q2-2021: fastest recovery in decades.
- Q4-2021: inflation has surged around the globe.



Annual GDP Growth



Year-on-year Inflation Rate

New sources of macro risk that influence monetary policy

*"The pandemic and war have underscored the need for the **risk management** framework to take full account of both **upside and downside risks to inflation**, as well as to the possibility that **serious tensions** may arise between the objectives of **price stability** and employment or **growth**."* [Gita Gopinath, Jackson Hole Symposium, August 26, 2022]

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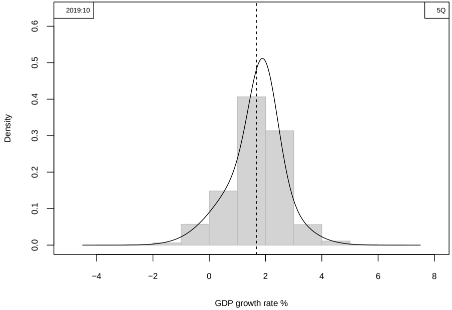
- Assess the risks weighing on prices and production with a focus on tail risk
- SPFs are a rich source of information
  - Subjective expectations of informed market participants
  - Often used to nourish monetary policy decisions (minutes/statements)
  - Perception of (extreme) risks without these having to materialize
  - Tails are the first to move when there is uncertainty [Reis, 2021]

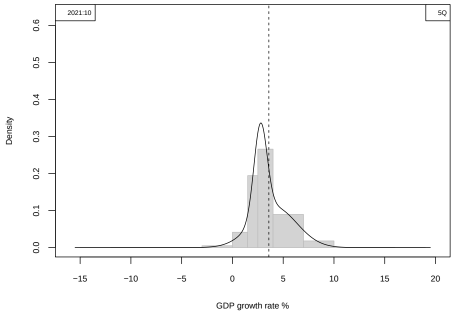
High levels of uncertainty and tail risk



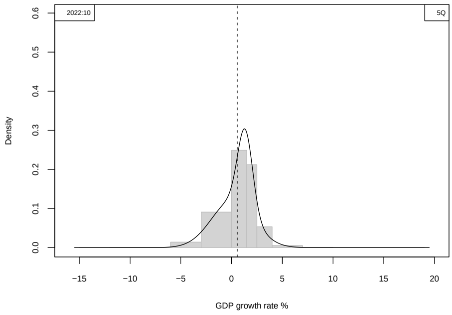
2019

Survey-based distribution of US Growth (1Y ahead)

High levels of uncertainty and tail risk



2021



2022

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Overcoming data limitations

Surveys are an imperfect source of information:

- Inflation and GDP surveys are disjoint
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- Inflation and GDP surveys are disjoint
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We overcome these issues and exploit SPFs by proposing a model that disciplines the data

- Inflation and growth can be affected by the same factors $\Rightarrow$ study their joint dynamics
- Categorize factors to decompose inflation and growth into demand & supply components
- Perform this decomposition not only on actual variables but also on their expectations

Are macro fluctuations attributable to demand or supply?

- COVID-19: multiple large and concurrent demand and supply shocks
- Essential for setting the appropriate policy mix
- Standard identification:

	Inflation	GDP growth
Demand	$+$	$+$
Supply	$-$	$+$

Contribution

Extend identification beyond central tendencies

- Use probabilistic responses of surveys
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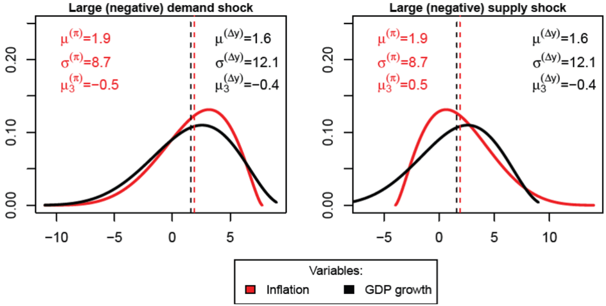

Our Approach

Inputs

(Realized and survey data)

- Inflation
- GDP Growth
- SPF*

→ Estimation of the dynamic factor model →

Outputs

- Conditional distributions of future inflation and growth rates
- Joint expectations of inflation and growth rates

*: First-, second-, third- and fourth-order moments

Our Approach

- Dynamic factor model featuring time-varying uncertainty, asymmetry and fat tails
- Identify demand and supply factors using expectations of inflation and GDP growth
- Allow for a trend/cycle decomposition

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- GDP and surveys available quarterly (while inflation is monthly)

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- Trend-cycle decomposition for $log(P)$ and $log(GDP)$:

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- Quarterly inflation and growth rate are respectively:

$$\pi_{t-1,t} = \Delta T_t^{(\pi)} + C_t^{(\pi)} - C_{t-1}^{(\pi)}$$
$$\Delta y_{t-1,t} = \Delta T_t^{(\Delta y)} + C_t^{(\Delta y)} - C_{t-1}^{(\Delta y)}.$$

Model

- The dynamics of the trend and cycle are linear combinations of $\mathcal{Y}_t$ (stationary):

$$\Delta T_t^{(\pi)} = \rho^{(\pi)} + \delta_T^{(\pi)\prime} \mathcal{Y}_t$$
$$\Delta T_t^{(\Delta y)} = \rho^{(\Delta y)} + \delta_T^{(\Delta y)\prime} \mathcal{Y}_t$$

and

$$C_t^{(\pi)} = \delta_C^{(\pi)\prime} \mathcal{Y}_t$$
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where $\mathcal{Y}_t$ are unobserved latent factors

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$\Rightarrow$ Factors are **common** to GDP and inflation allowing us to analyze the **joint dynamics**

Model

- The joint model for inflation and GDP growth can be specified as:

$$\begin{bmatrix} \pi_{t-1,t} \\ \Delta y_{t-1,t} \end{bmatrix} = \begin{bmatrix} \rho^{(\pi)} \\ \rho^{(\Delta y)} \end{bmatrix} + \begin{bmatrix} \delta^{(\pi_{\mathbf{1}})\prime} \\ \delta^{(\Delta y_{\mathbf{1}})\prime} \end{bmatrix} Y_t,$$

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- Y_t follows:

$$Y_t = \Phi_Y Y_{t-1} + \Theta(z_t - \bar{z}) + \Sigma(z_t)\varepsilon_{Y,t}, \quad \varepsilon_{Y,t} \sim \mathcal{N}(0, I),$$

where z_t is an exogenous vector of factors driving Y_t's conditional expectation, variance, asymmetry and fat tails

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- Y_t features stochastic volatility, asymmetry and fat tails $\Rightarrow$ **uncertainty** and **tail risk**

Model

- z_t follows a multivariate auto-regressive gamma process
  $\approx$ time-discretized CIR process
  [Gouriéroux and Jasiak, 2006], [Monfort et al., 2017], [Grishchenko et al., 2017]

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- VAR representation:

$$z_t = \mu_z + \Phi_z z_{t-1} + \Omega(z_{t-1})\varepsilon_{z,t},$$

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  where $\varepsilon_{z,t}$ has a conditional zero mean and an *Id* conditional covariance matrix.
- z_t is an **affine process** (conditional log-Laplace transform is affine)
  $\Rightarrow$ Tractability of the model

Summary of model factors

	Vol.	Skew.	Kurt.	$z^s_{p,t}$	$z^s_{n,t}$	$z^d_{p,t}$	$z^d_{n,t}$	$z_{v,t}$	$C^{(\pi)}$	$C^{(\Delta y)}$	$T^{(\pi)}$	$T^{(\Delta y)}$
$\mathcal{Y}_{1,t}$	✓	✓	✓	✓	✓				$-$	$+$	0	0
$\mathcal{Y}_{2,t}$	✓	✓	✓			✓	✓		$+$	$+$	0	0
$\mathcal{Y}_{3,t}$	✓		✓					✓	?	$+$	?	$+$
$\mathcal{Y}_{4,t}$	✓		✓					✓	?	$+$	?	$+$

State-space representation

- The state vector X_t follows a VAR process with stochastic volatility and asymmetry:

$$X_t = \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \mu_X + \Phi_X \begin{bmatrix} Y_{t-1} \\ z_{t-1} \end{bmatrix} + \Sigma_X(z_{t-1})\varepsilon_{X,t},$$

where $\varepsilon_{X,t}$ is a unit-variance martingale difference sequence
[$\Sigma_X(z_{t-1})$ varies through time, and the distribution of $\varepsilon_{X,t}$ is not Gaussian]

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- Denote by S_t the vector of observations, which is affine in X_t:

$$S_t = A + B'X_t + diag(\sigma^S)\eta_t^S$$

where $S_t = [\pi_t, \Delta y_t, ESPF_t, VSPF_t, SSPF_t, KSPF_t]'$ and $\mathbb{V}ar(\eta_t^S) = Id$

Estimation

Key property: X_t is an **affine process**

- Cond. moments of $\forall$ linear combination of future X_t are affine & available in closed-form
- The model admits a **linear state-space** representation
  (The conditional cumulants of any future linear combination of X_t is affine in X_t.)
- The model is estimated by quasi-maximum likelihood, using the Kalman filter:
  - simultaneously estimate the model parameters and the latent factors X_t
  - handle missing observations

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Empirical application: United States, 1981Q3-2024Q1

Survey Data

Horizon	Description	Index	Rate Definition	Frequency	Sample
Panel A: US SPF for inflation					
$\bar{\pi}_{t,t+4}$	Density	GDP deflator	Annual average/annual average	Q	1981Q3-2024Q1
$\bar{\pi}_{t,t+5}$	Density	GDP deflator	Annual average/annual average	Q	1981Q3-2024Q1
$\bar{\pi}_{t,t+6}$	Density	GDP deflator	Annual average/annual average	Q	1981Q3-2024Q1
$\bar{\pi}_{t,t+7}$	Density	GDP deflator	Annual average/annual average	Q	1981Q3-2024Q1
$\bar{\pi}_{t,t+8}$	Density	GDP deflator	Annual average/annual average	Q	1981Q3-2024Q1
Panel B: US SPF for GDP growth					
$\tilde{p}_{t,t+4}$	Density	GDP growth	Annual average/annual average	Q	1981Q3-2024Q1
$\tilde{p}_{t,t+5}$	Density	GDP growth	Annual average/annual average	Q	1981Q3-2024Q1
$\tilde{p}_{t,t+6}$	Density	GDP growth	Annual average/annual average	Q	1981Q3-2024Q1
$\tilde{p}_{t,t+7}$	Density	GDP growth	Annual average/annual average	Q	1981Q3-2024Q1
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Model fit and stylised facts

Model fit:

- Preliminary step: use Gaussian mixture to obtain moments from raw data
- Our model matches the term structure of all moments (up to 4th order) for inflation and output growth (some trade-offs across forecasting horizons)
- Higher-order moments are <u>crucial</u> to fit distributions of expectations (π & Δy)

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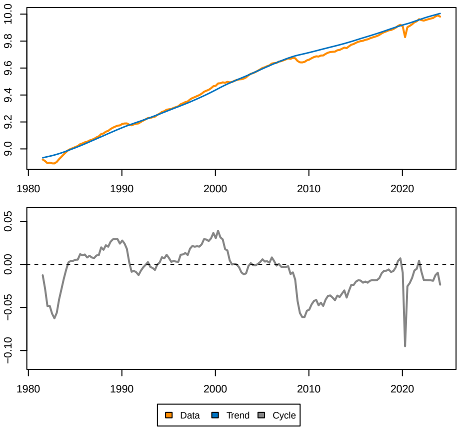
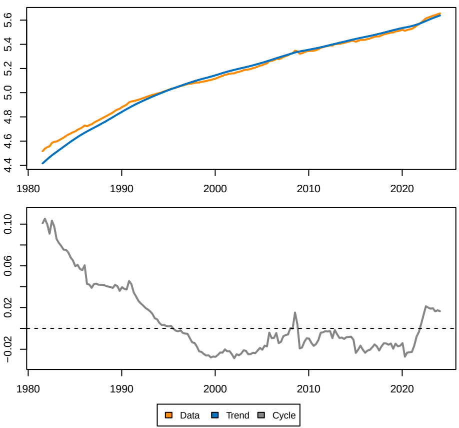
Stylised facts for US SPF expectations:

- Asymmetry on either side and excess kurtosis (leptokurtotic distributions)
  - Inflation: 80s and in 2008
  - GDP: 80s and since COVID

Cyclical and trend component of GDP and Prices

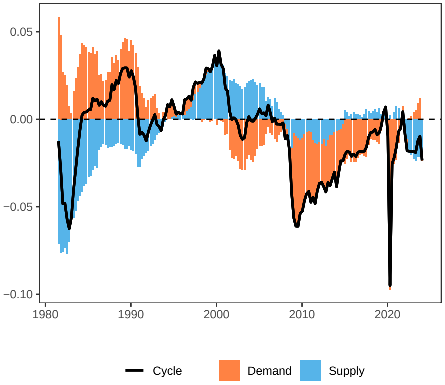


log GDP

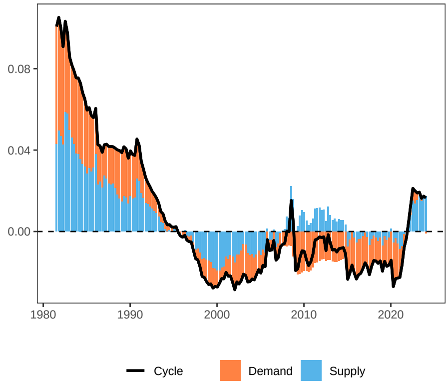


log P

Decomposition of cyclical components

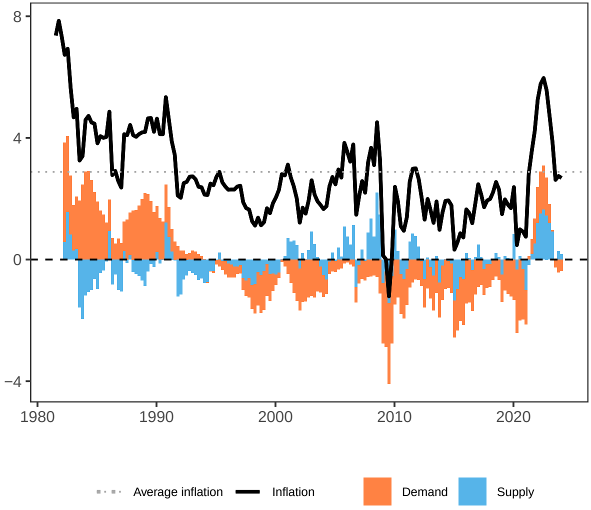


log GDP

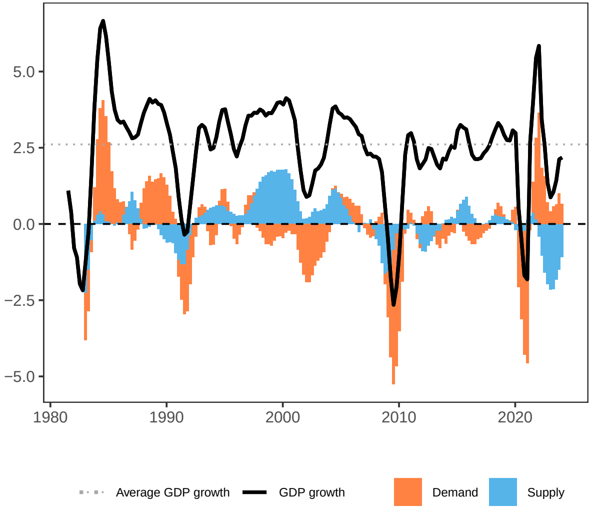


log P

Decomposition of US inflation



Decomposition of US growth

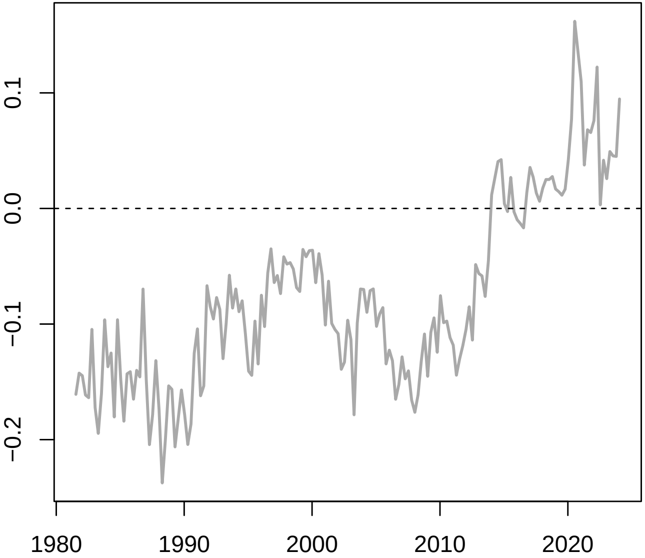


Supply-driven outputs are in line with supply indicators

Regressions of supply-driven outputs on:

- Commodity Factors/Indices ✓
- Oil Supply Shocks ✓
- Oil Price Expectation Surprises ✓
- Crude Oil (WTI) and Gas Prices ✓
- Global Supply Chain Pressure Index ✓

Correlation between π and Δy



Conclusion

Propose a framework to assess the risks on inflation and growth with a focus on tail risk

- Inflation and growth can be affected by the same factors $\Rightarrow$ study their joint dynamics
- Categorize factors to decompose inflation and growth into demand & supply components
- Perform this decomposition not only on realized variables but also on their expectations

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Findings in the US suggest that:

- Great Recession and COVID are demand driven
- Supply factors are very important until the Great Recession and since COVID
- The post-COVID period is characterised by a mix of demand and supply drivers

References I

Gouriéroux, C. and Jasiak, J. (2006).
Autoregressive gamma processes.
Journal of Forecasting, 25:129–152.

Grishchenko, O. V., Mouabbi, S., and Renne, J.-P. (2017).
Measuring Inflation Anchoring and Uncertainty : A US and Euro Area Comparison.
Finance and Economics Discussion Series 2017-102, Board of Governors of the Federal Reserve System (U.S.).

Monfort, A., Pegoraro, F., Renne, J.-P., and Roussellet, G. (2017).
Staying at Zero with Affine Processes: An Application to Term-Structure Modelling.
Journal of Econometrics, 201(2):348–366.

Reis, R. (2021).
Losing the Inflation Anchors.
Brookings Papers on Economic Activity, 52(2 (Fall)):307–379.

— Appendix —

Uncertainty

- Uncertainty: the variance of the aggregate probability distribution function
- $\sigma^2_{agg,th}$: conditional variance of the aggregate distribution; σ^2_{ith}: individual variances
- Then, the proxy for uncertainty is given by:

$$\sigma^2_{agg,th} = \frac{1}{N}\sum_{i=1}^{N}(f_{ith} - f_{.th})^2 + \frac{1}{N}\sum_{i=1}^{N}\sigma^2_{ith},$$

  where N is the number of forecasters, f_{ith} is the forecast at time t, for horizon h of individual i and $f_{.th}$ is the consensus forecast.

- The conditional variance of the aggregate distribution is equal to the sum of disagreement and of the average of individual variances (law of total variance).

Autoregressive Gamma Processes

- The vector z_t follows a multivariate $\text{ARG}_\nu(\varphi, \mu)$ process.
- Conditionally on $\underline{z_{t-1}} = \{z_{t-1}, z_{t-2}, \dots\}$, the different components of z_t, denoted by $z_{i,t}$, are independent and drawn from non-centered Gamma distributions:

$$z_{i,t}|\underline{z_{t-1}} \sim \gamma_{\nu_i}(\varphi_i' z_{t-1}, \mu_i),$$

  where ν, μ, φ_1, ..., φ_{q-1} and φ_q are q-dimensional vectors.
- Recall that W is drawn from a non-centered Gamma distribution $\gamma_\nu(\varphi, \mu)$, iif there exists an exogenous $\mathcal{P}(\varphi)$-distributed variable Z such that $W|Z \sim \gamma(\nu + Z, \mu)$ where $\nu + Z$ and μ are, respectively, the shape and scale parameters of the gamma distribution.

Model

- The joint model (inflation and GDP growth) can be specified as:

$$\begin{bmatrix} \pi_{t-1,t} \\ \Delta y_{t-1,t} \end{bmatrix} = \begin{bmatrix} \rho^{(\pi)} \\ \rho^{(\Delta y)} \end{bmatrix} + \begin{bmatrix} \delta^{(\pi_1)\prime} \\ \delta^{(\Delta y_1)\prime} \end{bmatrix} Y_t,$$

with,

$$\delta^{(\pi)} = \begin{bmatrix} \delta_T^{(\pi)} + \delta_C^{(\pi)} \\ -\delta_C^{(\pi)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad \delta^{(\Delta y)} = \begin{bmatrix} \delta_T^{(\Delta y)} + \delta_C^{(\Delta y)} \\ -\delta_C^{(\Delta y)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad Y_t = \begin{bmatrix} \mathcal{Y}_t \\ \mathcal{Y}_{t-1} \\ \hat{\pi}_{t-2,t-1} \\ \hat{\pi}_{t-3,t-2} \\ \hat{\pi}_{t-4,t-3} \\ \Delta \hat{y}_{t-2,t-1} \\ \vdots \\ \Delta \hat{y}_{t-7,t-6} \end{bmatrix}.$$

Model

- We consider 4 factors $\mathcal{Y}_t$ and 5 factors z_t. The equations for $\mathcal{Y}_t$ are the following:

$$
\begin{aligned}
\mathcal{Y}_{1,t} &= \phi_{1,1}\mathcal{Y}_{1,t-1} + \theta^s(z^s_{p,t} - z^s_{n,t}) + \sqrt{\Gamma_{1,\mathcal{Y},0}}\epsilon_{1,\mathcal{Y},t},\\
\mathcal{Y}_{2,t} &= \phi_{2,2}\mathcal{Y}_{2,t-1} + \theta^d(z^d_{p,t} - z^d_{n,t}) + \sqrt{\Gamma_{2,\mathcal{Y},0}}\epsilon_{2,\mathcal{Y},t},\\
\mathcal{Y}_{3,t} &= \phi_{3,3}\mathcal{Y}_{3,t-1} + \sqrt{\Gamma_{3,\mathcal{Y},0} + \Gamma_{[3,5],\mathcal{Y},1} z_{v,t}}\epsilon_{3,\mathcal{Y},t},\\
\mathcal{Y}_{4,t} &= \phi_{4,4}\mathcal{Y}_{4,t-1} + \sqrt{\Gamma_{4,\mathcal{Y},0} + \Gamma_{[4,5],\mathcal{Y},1} z_{v,t}}\epsilon_{4,\mathcal{Y},t},
\end{aligned}
$$

  where θ^s and θ^d have positive signs, and $\phi_{i,i} \in (0, 0.99)$.

- For the z_t, we consider five different factors:

$$
z_t = (z^s_{p,t}, z^s_{n,t}, z^d_{p,t}, z^d_{n,t}, z_{v,t})',
$$

  where $z^s_{p,t}$ and $z^s_{n,t}$ are equivalent processes as well as $z^d_{p,t}$ and $z^d_{n,t}$. A fifth factor $z_{v,t}$ is introduced to model time-varying variances and fat tails.

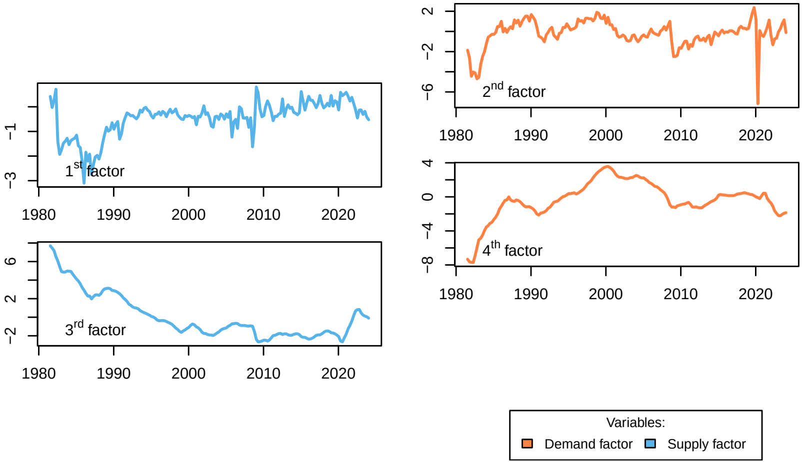
Model

- For the sake of identification, different elements of δ are set to 1.

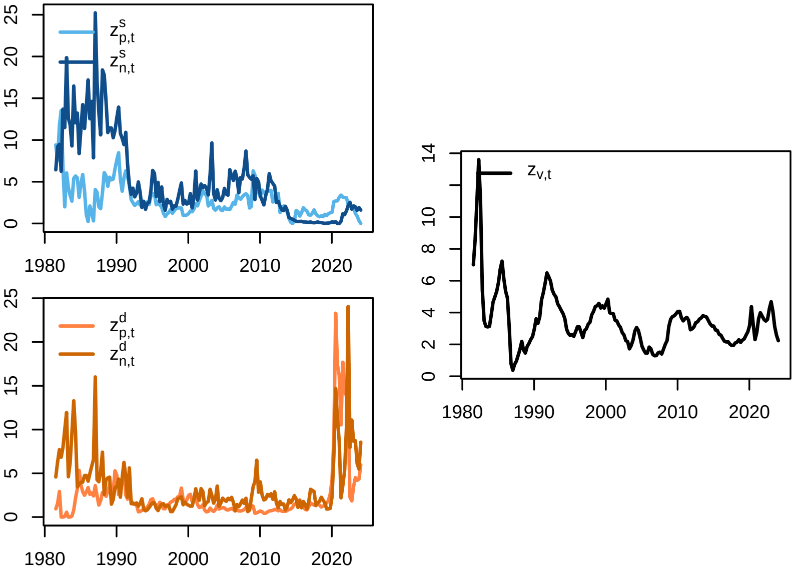
$$\delta_T^{\pi} = \begin{bmatrix} 0 \\ 0 \\ \delta_{3,T}^{(\pi)} \\ \delta_{4,T}^{(\pi)} \end{bmatrix}, \quad \delta_T^{\Delta y} = \begin{bmatrix} 0 \\ 0 \\ \delta_{3,T}^{(\Delta y)} \\ \delta_{4,T}^{(\Delta y)} \end{bmatrix}, \quad \delta_C^{\pi} = \begin{bmatrix} -\delta_{1,C}^{(\pi)} \\ +\delta_{2,C}^{(\pi)} \\ \delta_{3,C}^{(\pi)} \\ \delta_{4,C}^{(\pi)} \end{bmatrix}, \quad \delta_C^{\Delta y} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

- The elements of δ^{π} with positive signs identify demand factors, while those with negative signs identify supply factors.

Latent Factors: Y_t



Latent Factors: z_t



Fit of price and output gap