Identifying Price Informativeness*

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Abstract

This paper shows how to identify and estimate price informativeness. Starting from i) an asset pricing equation and ii) a stochastic process for asset payoffs, we show how to exactly recover price informativeness from regressions of changes in asset prices on changes in asset payoffs. Applying our identification results, we estimate a panel of stock-specific measures of price informativeness for U.S. stocks. In the cross-section, large, high turnover, and high institutional ownership stocks have higher price informativeness. In the time series, the median, mean, and standard deviation of the distribution of price informativeness have steadily increased since the mid-1980s.

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1 Introduction

Financial markets play an important role by aggregating information about the fundamentals of the economy. By pooling different sources of information, asset prices act as a public signal to any external observer in the economy, potentially influencing individual decisions. This view that treats asset prices as a signal about future fundamentals is often traced back to Hayek (1945).[1] Despite the substantial theoretical literature that studies how prices aggregate information, the connection between the theoretical and empirical research on price informativeness remains understudied. The ability to measure price informativeness opens the door to understanding whether different markets aggregate dispersed information to different degrees at a particular moment or over time, which in turn makes it possible to test and discipline a variety of theories of information aggregation.

In this paper, we show how to identify and estimate exact stock-specific measures of price informativeness. To our knowledge, we provide the first identification results of price informativeness within the literature on learning in financial markets. To derive our results, we only need to postulate i) an asset pricing equation and ii) a stochastic process for asset payoffs. Our main result shows that a specific combination of R-squareds from linear regressions of changes in asset prices on changes in asset payoffs exactly identifies price informativeness. Our results are valid within a large class of models that may feature rich heterogeneity across investors' preferences, endowments, private signals, and private trading needs; competitive or strategic market structures; symmetric or asymmetric information; and that require minimal distributional assumptions.

We begin by formally defining within our framework two price informativeness measures that have been widely studied in the theoretical literature on learning in financial markets: absolute and relative price informativeness. Absolute price informativeness, which formally corresponds to the precision of the unbiased signal about the innovation to the asset payoff contained in the asset price, measures the precision of the public signal revealed by the asset price. Relative price informativeness, which corrects absolute price informativeness to account for the variability of the asset payoff, measures how much can be learned from the price relative to the total amount that can be learned. Relative price informativeness takes values between 0 and 1, making it easily interpretable and comparable across stocks. Moreover, in a Gaussian environment, relative price informativeness exactly corresponds to the Kalman gain in the updating process of a Bayesian external observer who only learns from the price. For instance, finding that relative

[1] Hayek (1945) highlights the relevance of price informativeness as follows: "The economic problem of society is (...) rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the utilization of knowledge which is not given to anyone in its totality."

price informativeness is 0.2 implies i) that the uncertainty faced by an external observer about the asset payoff is reduced by 20% after observing the price, and ii) that an external observer puts a weight of 20% on the price signal (and a weight of 80% on the prior) when forming a posterior belief over the future payoff.

We succinctly describe here our approach to identifying and estimating (relative) price informativeness. Consider the following two regressions that relate log-price changes, Δp_t, to the contemporary and future differences in log-asset payoffs, denoted by Δx_t and Δx_{t+1}, respectively:

$$\Delta p_t = \overline{\beta} + \beta_0 \Delta x_t + \beta_1 \Delta x_{t+1} + e_t \tag{R1}$$

$$\Delta p_t = \overline{\zeta} + \zeta_0 \Delta x_t + e_t^{\zeta}, \tag{R2}$$

where we denote the R-squareds of Regressions R1 and R2 by $R^2_{\Delta x, \Delta x'}$ and $R^2_{\Delta x}$, respectively. Our main result shows that the normalized difference in R-squareds

$$\frac{R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}}$$

exactly corresponds to relative price informativeness. In addition to this identification result, we show that estimating these two regressions using ordinary least squares (OLS) yields a consistent estimate of relative price informativeness. An important implication of our results is that it is possible to recover price informativeness by relying exclusively on price and payoff information, without having to observe the sources of noise in asset prices — subsumed in the error terms e_t and e_t^{ζ}. Our identification results are therefore agnostic about the nature of the noise in asset prices.

Even though we show that price informativeness can be recovered without fully specifying the model primitives, a microfounded model is necessary to understand the link between price informativeness and the primitives in the economy. For this reason, we develop several microfounded dynamic models of trading that are consistent with the asset pricing equation and the stochastic process for asset payoffs that we use to derive our identification results. First, we study a model in which investors have private signals about future payoffs and orthogonal trading motives in the form of random priors (sentiment). Subsequently, we study a representative agent model similar to those used in the macro-finance literature. Finally, we study a model with informed and uninformed investors, as in the classic literature on information and learning. Analyzing these applications has a dual purpose. First, these applications show that our identification results apply to economies i) with or without dispersed information among investors, ii) with time-varying risk aversion/risk-premia, iii) with investors who may or may not learn from prices, and iv) in which noise may arise from different sources. Second, these applications allow us to provide a structural economic interpretation to the empirical results presented in Section 5, a task we carry out in Section 6.

Even though price informativeness and price/return predictability are connected, these are conceptually different notions. We relate our results to the well-established literature on return predictability, which is based on running predictive regressions of future returns on current variables, the opposite of our approach. We explain that predictive regressions are the appropriate tool if one is interested in predicting future returns, but that our approach is the correct one if one wants to recover price informativeness. We also discuss in detail how our results relate to alternative measures of informativeness, like the posterior variance of a Bayesian external observer or the forecasting price efficiency notion of Bond, Edmans and Goldstein (2012).

Before estimating price informativeness, we extend our identification results in several dimensions. First, we describe how to account for a non-zero correlation between payoff innovations and noise. Second, we allow for the possibility of public signals about future payoffs. Third, we augment the payoff process to have an unlearnable component.[2] These extensions highlight that our results apply to more general and empirically relevant scenarios, allowing us also to refine the interpretation of our empirical results.

Finally, we use our identification results to estimate relative price informativeness. We recover a panel of stock-specific measures of price informativeness between 1980 and 2017 by running rolling time-series regressions of the form implied by Proposition 1 at the stock level using quarterly data. We find that the distribution of informativeness across stocks is right-skewed, with time-series averages of the median and mean levels of price informativeness across all stocks and years given by 1.84% and 4.25%, respectively. Our estimation exercise allows us to uncover both cross-sectional and time-series patterns about the behavior of price informativeness. In the cross section, we find that stocks that i) are larger, ii) turn over more quickly, and iii) have a higher institutional ownership share have higher price informativeness. In the time series, we find that the median and mean price informativeness have steadily increased since the mid-1980s. The standard deviation of price informativeness has also increased over this period. In the Online Appendix, we include additional results that show the robustness of our cross-sectional and time-series findings.

We conclude by providing an economic interpretation of the empirical estimates of price informativeness. If one were merely interested in knowing the precision of the signal contained in asset prices about future payoffs, our empirical results directly conclude that such signal is more precise for large, high turnover, and high institutional ownership stocks, and has become

[2]In previous versions of this paper, we also considered environments with multiple risky assets and strategic investors.

more precise on average over the last few decades. In terms of deeper primitives, through the lens of the structural models developed in Section 3, our cross-sectional empirical findings imply that investors have relatively more precise private information about large, high turnover, and high institutional ownership stocks. Our time-series findings imply that, over the last decades, on average private information has increased relative to the noise in prices.

Related Literature Our theoretical framework builds on the literature that studies the role played by financial markets in aggregating dispersed information, following Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and De Long et al. (1990), among others. Vives (2008) and Veldkamp (2011) provide systematic reviews of this well-developed and growing body of work. To our knowledge, we provide the first results on how to identify and consistently estimate price informativeness — understood as the precision of the signal about future payoffs contained in asset prices — within the literature of learning in financial markets.

While a substantial theoretical literature has studied how financial markets aggregate information, the development of empirical measures of price informativeness is more recent. There is a body of work that has proposed ad hoc variables to measure the informational content of prices. These ad hoc measures have been inspired by economic models to different degrees. Influenced by the predictions of the CAPM/APT frameworks and following the prominent Roll (1988) presidential address, Morck, Yeung and Yu (2000) study regressions of asset returns on factors and informally argue that the R^2 of such regressions can be used to capture whether asset prices are informative/predictive about firm-specific fundamentals. This ad hoc measure, sometimes referred to as price nonsynchronicity, has been used in several empirical studies that link price informativeness to capital allocation. In particular, Wurgler (2000) finds that countries with higher price nonsynchronicity display a better allocation of capital. Durnev, Morck and Yeung (2004) document a positive correlation between price nonsynchronicity and corporate investment. Chen, Goldstein and Jiang (2006) establish that there exists a positive relationship between the sensitivity of corporate investment to stock prices and two measures of the information contained in prices, price nonsynchronicity and the probability of informed trading (PIN), concluding that managers learn from the price when making corporate investment decisions. The PIN, developed in Easley, O'Hara and Paperman (1998), estimates the probability of an informed trade using high-frequency data through the lens of a model with informed and uninformed traders. See also the work of Hou and Moskowitz (2005) and Weller (2018), who propose alternative empirical measures.

While some of the existing work using ad hoc measures may uncover interesting empirical relations, Hou, Peng and Xiong (2013) forcefully highlight that a measure like Roll's R^2 (price nonsynchronicity) lacks a structural interpretation. They question the link between return price nonsynchronicity and price informativeness theoretically, in rational and behavioral settings, and empirically. In general, even if ad hoc measures of price informativeness were correlated with actual price informativeness, it would be impossible to interpret the magnitude of these ad hoc variables without a structural approach. In this paper, we address the Hou, Peng and Xiong (2013) critique of the existing literature by showing how to identify and consistently estimate exact stock-specific measures of price informativeness within a structural framework.

More recently, Bai, Philippon and Savov (2016), have considered the question of whether financial markets have become more informative over time. Even though their empirical approach is loosely motivated by a theoretical model, they do not provide identification results or show how to formally identify and estimate price informativeness in the context of a structural model. We discuss in detail how our results relate to this paper in Section A of the Online Appendix. Our results and the recent work of Farboodi et al. (2020) and Kacperczyk, Sundaresan and Wang (2020) complement each other. While our focus is to provide identification results for price informativeness (i.e., the signal-to-noise ratio in prices) in a general framework, Farboodi et al. (2020) seek to understand how changes in data processing over time have altered the amount of information (signal) incorporated in asset prices. Using our measure of price informativeness as an input in their analysis, they conclude that the divergence in price informativeness across stocks is due to an increase in the amount of information incorporated in prices of large, high growth stocks driven by an increase in data processing capacity. Kacperczyk, Sundaresan and Wang (2020) find a positive relationship between price informativeness and the ownership share of foreign institutional investors, using both ad hoc measures of informativeness and the identification results that we develop in this paper.

As in any structural model, the measure of informativeness that we recover is linked to our assumptions on the behavior of investors and the market structure. While our framework is general along several dimensions, there is scope to think about how to identify price informativeness in alternative models of trading that depart from our linearity assumptions, like the model in Albagli, Hellwig and Tsyvinski (2015). In particular, our analysis purposefully abstracts from feedback between prices and fundamentals, summarized in Bond, Edmans and Goldstein (2012) and tested in Chen, Goldstein and Jiang (2006). Incorporating two-way feedback between asset prices and payoffs unavoidably introduces non-linearities that must be addressed using full-information methods.

Outline Section 2 describes the general framework used to define price informativeness and presents our main results. Section 3 studies several microfounded models that are special cases of the general framework. Section 4 extends our results to more general environments, and Section

5 empirically implements the identification results introduced in the paper. Section 6 provides an economic interpretation of our empirical results through the lens of the general framework introduced in Section 2 and the microfounded models developed in Section 3, and Section 7 concludes. All proofs, derivations, and additional results are in the Appendix.

2 General Framework

In this section, we show how to formally identify and estimate price informativeness from an asset pricing equation and a stochastic process for asset payoffs. To stay as close as possible to the empirical implementation in Section 5, we derive the main results in the body of the paper in a log-difference-stationary environment, which is considered a better representation of reality.[3]

2.1 Environment

We consider a discrete time environment with dates $t = 0, 1, 2, \ldots, \infty$, in which investors trade a risky asset in fixed supply at a (log) price p_t at each date t. We assume that the (log) payoff of the risky asset at date $t+1$, x_{t+1}, follows a difference-stationary AR(1) process

$$\Delta x_{t+1} = \mu_{\Delta x} + \rho \Delta x_t + u_t, \tag{1}$$

where $\Delta x_t \equiv x_t - x_{t-1}$, $\mu_{\Delta x}$ is a scalar, $|\rho| < 1$, and where the innovations to the payoff difference, u_t, have mean zero, a finite variance denoted by $\mathbb{V}\mathrm{ar}\left[u_t\right] = \sigma_u^2 = \tau_u^{-1}$, and are identically and independently distributed over time. Note that the innovation to the $t+1$ payoff difference, u_t, is indexed by t — instead of $t+1$ — to indicate that investors can potentially learn about the realization of u_t at date t.

We assume that the equilibrium (log) price difference is given by

$$\Delta p_t = \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta n_t, \tag{2}$$

where $\overline{\phi}$, ϕ_0, ϕ_1, and ϕ_n are parameters and where $\Delta n_t \equiv n_t - n_{t-1}$ represents the change in the aggregate component of investors' trading motives that are orthogonal to the asset payoff, given by $\Delta n_t = \mu_{\Delta n} + \varepsilon_t^{\Delta n}$, where $\mathbb{V}\mathrm{ar}\left[\Delta n_t\right] = \sigma_{\Delta n}^2 = \tau_{\Delta n}^{-1}$. As shown in Section 3, the random variable n_t can be interpreted as a measure of investors' sentiment, risk-bearing capacity, or noise trading activity. Our timing assumes that date t variables, in particular Δx_t and u_t, are realized before the price p_t is determined. We further assume that u_t and Δn_t are independent

[3] In the Supplemental Appendix, we re-derive the main results of the paper in a level-stationary environment, which is the benchmark environment in the literature on information and learning in financial markets (Vives, 2008; Veldkamp, 2011), and also in log-level-stationary and difference-stationary environments.

— this is without loss of generality, as we show in Section 4.

In Section 3, we show that Equation (2) emerges endogenously as the solution to several fully specified dynamic models of trading. In that case, the parameters $\overline{\phi}$, ϕ_0, ϕ_1, and ϕ_n can be mapped to specific combinations of primitives. In Section 4, we extend our results to even more general environments.

2.2 Price Informativeness: Definition

Within the environment introduced in Section 2, we now formally define two related measures of informativeness: absolute and relative price informativeness. Neither of these notions is new. In fact, we focus our analysis on them because they have been widely used in the theoretical literature on information and learning. Absolute price informativeness is discussed in Section 4 of Vives (2008). Relative price informativeness corresponds to the exact notion of informativeness used in Grossman and Stiglitz (1980).[4] The contribution of this paper is to formally identify and estimate these notions. We discuss closely related measures of price informativeness in Remark 1 below.

Formally, in our context, the unbiased signal of the innovation to future payoffs u_t contained in the price is the key variable of interest from the perspective of understanding how informative are asset prices about future payoffs. This endogenous unbiased signal, which we denote by π_t, is given by

$$\pi_t \equiv \frac{\Delta p_t - \left(\overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_n \mu_{\Delta n} + (\phi_0 + \rho \phi_1) \Delta x_t\right)}{\phi_1}. \tag{3}$$

Given Equation (3), $\pi_t = u_t + \frac{\phi_n}{\phi_1}(\Delta n_t - \mu_{\Delta n})$ defines an endogenous unbiased signal about u_t, where $\frac{\phi_n}{\phi_1}(\Delta n_t - \mu_{\Delta n})$ acts as the noise contained in the price signal. This signal π_t is unbiased because $\mathbb{E}[\pi_t | u_t, \Delta x_t] = u_t$. Both absolute and relative price informativeness are the relevant measures for an external observer who uses the asset price as a signal to learn about future asset payoffs.

Definition. (Price informativeness)

a) *Absolute price informativeness,* denoted by $\tau_\pi \in [0, \infty)$, is the precision of the unbiased signal about the innovation to the asset payoff contained in the asset price. Given Equation (2), it is formally given by

$$\tau_\pi \equiv \left(\mathbb{V}\mathrm{ar}[\pi_t | x_{t+1}, \Delta x_t]\right)^{-1} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n}, \tag{4}$$

where $\tau_{\Delta n} = \mathbb{V}\mathrm{ar}[\Delta n_t]^{-1}$.

b) *Relative price informativeness*, denoted by $\tau_\pi^R \in [0, 1]$, is the ratio between absolute price

[4] Relative price informativeness, as defined in Equation (4) below, exactly corresponds to Equation (17) in Grossman and Stiglitz (1980).

informativeness and the sum of absolute price informativeness and the precision of the innovation to the asset payoff. Given Equation (2), it is formally given by

$$\tau_\pi^R \equiv \frac{\tau_\pi}{\tau_\pi + \tau_u}, \tag{5}$$

where $\tau_u = \mathbb{V}\mathrm{ar}\,[u_t]^{-1}$.

The definition of *absolute price informativeness* connects with the large body of work that follows Blackwell (1953). According to Blackwell's informativeness criterion to rank experiments/signals, a signal is more informative than another when it is more valuable to a given decision-maker. According to that criterion, in the environment considered here, absolute price informativeness induces a complete order of price signals for a decision-maker with a quadratic objective around the value of the future asset payoff. We would like to emphasize that, in general, price informativeness and social welfare can move in opposite directions. That is, it is well known that more informative signals can be associated with higher or lower social welfare depending on the environment considered — see, for instance, Angeletos and Pavan (2007, 2009). Consequently, the welfare implications of price informativeness can only be determined within a particular structural model.

Intuitively, absolute price informativeness measures the signal-to-noise ratio contained in the asset price. If the price is very responsive to x_{t+1}, perhaps because investors trade with very precise information about the future payoff, ϕ_1 and price informativeness will be higher. Alternatively, if the price is mostly driven by trading motives that are orthogonal to future payoffs, perhaps reflecting investors' sentiment, $\phi_n^2 \tau_{\Delta n}^{-1}$ will be higher and price informativeness will be lower. When price informativeness is high, an external observer receives a very precise signal about future payoffs by observing the change in the asset price Δp_t. On the contrary, when price informativeness is low, an external observer learns little about future payoffs by observing the change in the asset price Δp_t.

The definition of *relative price informativeness* corrects absolute price informativeness to account for the variability of the payoff, via τ_u. This measure captures the precision of the price signal, given by τ_π, relative to the sum of the prior and the signal precisions of an external observer who only learns from the price, given by $\tau_\pi + \tau_u$. When uncertainty is Gaussian, relative price informativeness as defined in Equation (5) corresponds exactly to the Kalman gain of a Bayesian external observer who only learns from the price, as shown in Equation (7) below. If an external observer had additional information about the future payoff in addition to the price, the Kalman gain that we identify would be an upper bound to the one used by such external observer.

Relative price informativeness is an appealing object because it provides a bounded (between

0 and 1), unit-free measure of informativeness that facilitates precise quantitative comparisons. The unit-free nature of this measure is particularly relevant when comparing informativeness across assets with different underlying payoff distributions (i.e., different τ_u), for which comparing absolute price informativeness is meaningless. In Remark 1 below, we further explain how absolute and relative price informativeness relate to other notions like posterior variances or forecasting price efficiency. In the body of the paper, we focus on the identification of relative price informativeness because it is easily interpretable and comparable across stocks. We include identification results for absolute price informativeness in the Appendix. Going forward, to simplify the exposition, we often refer to relative price informativeness simply as price informativeness.

2.3 Price Informativeness: Identification

Proposition 1 introduces the main result of the paper. It shows how to combine the R-squareds of regressions of changes in asset prices on realized and future changes in asset payoffs to recover price informativeness.

Proposition 1. (Identifying price informativeness) *Let $\overline{\beta}$, β_0, and β_1 denote the coefficients of the following regression of log-price differences on realized and future log-payoff differences:*

$$\Delta p_t = \overline{\beta} + \beta_0 \Delta x_t + \beta_1 \Delta x_{t+1} + e_t, \tag{R1}$$

where $\Delta p_t = p_t - p_{t-1}$ denotes the date t change in log-price, $\Delta x_t = x_t - x_{t-1}$ and $\Delta x_{t+1} = x_{t+1} - x_t$ respectively denote the date t and $t+1$ log-payoff differences, and where $R^2_{\Delta x, \Delta x'}$ denotes the R-squared of Regression R1. Let $\overline{\zeta}$ and ζ_0 denote the coefficients of the following regression of log-price differences on realized log-payoff differences:

$$\Delta p_t = \overline{\zeta} + \zeta_0 \Delta x_t + e^{\zeta}_t, \tag{R2}$$

where $R^2_{\Delta x}$ denotes the R-squared of Regression R2. Then, relative price informativeness, τ^R_π, defined in Equation (5), can be recovered as

$$\tau^R_\pi = \frac{R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}}. \tag{6}$$

Estimating Regressions R1 and R2 via OLS yields consistent estimates of $R^2_{\Delta x, \Delta x'}$ and $R^2_{\Delta x}$.

The proof of Proposition 1 relies on identifying the right combination of parameters in the econometric specification defined by Regressions R1 and R2 that maps into the definition of relative price informativeness, τ_π^R. We show in the Appendix that a similar logic can be used to recover absolute price informativeness. It should be evident that if one could observe the non-payoff-related determinants of prices (n_t or Δn_t), that information could be used to directly recover all the relevant primitives in Equations (1) and (2). The non-trivial economic content of Proposition 1 is that if one is interested in recovering price informativeness, it is possible to do so by relying exclusively on price and payoff information, without having to observe n_t or Δn_t.

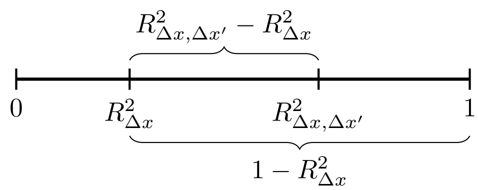

Figure 1: Interpreting relative price informativeness

Note: Relative price informativeness can be computed as the reduction in uncertainty, given by $R^2_{\Delta x,\Delta x'} - R^2_{\Delta x}$, relative to the remaining residual uncertainty about future payoffs after conditioning on the realized date t payoff, given by $1 - R^2_{\Delta x}$.

Figure 1 illustrates how to interpret Equation (6). The denominator $1 - R^2_{\Delta x}$ can be interpreted as the residual uncertainty about future payoffs after conditioning on the realized date t asset payoff. The numerator $R^2_{\Delta x,\Delta x'} - R^2_{\Delta x}$ can be interpreted as the percentage reduction in uncertainty about future payoffs after observing the asset price at date t in addition to the realized payoff Δx_t. Because $R^2_{\Delta x,\Delta x'} \geq R^2_{\Delta x}$ and $R^2_{\Delta x,\Delta x'} \in [0,1]$, it must be that $\tau_\pi^R \in [0,1]$.

As we show in the Appendix, if all random variables in the model are Gaussian, a Bayesian external observer who only learns from the price has the following posterior distribution over u_t:

$$u_t | \, \Delta p_t, \Delta x_t \sim N\left(\tau_\pi^R \pi_t, (\tau_\pi + \tau_u)^{-1}\right), \tag{7}$$

where π_t, τ_π, and τ_π^R are respectively defined in Equations (3), (4), and (5). Quantitatively, a relative price informativeness of, for instance, 0.15, implies that the initial uncertainty of an external observer who only learns from the price about the innovation to the future payoff is reduced by 15% after learning from the price — this interpretation follows from the fact that $(\tau_\pi + \tau_u)^{-1} = \left(1 - \tau_\pi^R\right)\tau_u^{-1}$.

Even though we emphasize the economic identification of price informativeness, we also address how to recover consistent estimates. It is worth highlighting that estimating Regressions R1 and R2 through OLS (ordinary least squares) yields consistent estimates of all the relevant parameters and R-squareds, since the error terms in both regressions are orthogonal to the regressors. Therefore, the estimates of price informativeness implied by Equations (3) and (4) will also be consistent.

We conclude this section with three remarks.

Remark 1. *Alternative measures of informativeness.* The notion of price informativeness defined above can be related to other variables, in particular, i) the posterior variance of the future payoff conditional on the price and the current payoff, given by $\mathcal{V}_P \equiv \mathbb{V}\text{ar}\left[u_t | \Delta p_t, \Delta x_t\right]$ and ii) forecasting price efficiency (FPE), given by $\mathcal{V}_{\text{FPE}} \equiv \mathbb{V}\text{ar}\left[\mathbb{E}\left[u_t | \Delta p_t, \Delta x_t\right]\right]$, as defined in Bond, Edmans and Goldstein (2012). Both variables are linked through the Law of Total Variance, as follows

$$\underbrace{\mathbb{V}\text{ar}\left[u_t | \Delta x_t\right]}_{\tau_u^{-1}} = \mathbb{E}\left[\underbrace{\mathbb{V}\text{ar}\left[u_t | \Delta p_t, \Delta x_t\right]}_{\mathcal{V}_P}\right] + \underbrace{\mathbb{V}\text{ar}\left[\mathbb{E}\left[u_t | \Delta p_t, \Delta x_t\right]\right]}_{\mathcal{V}_{\text{FPE}}}.$$

While $\mathcal{V}_P$ corresponds to the residual uncertainty about future payoffs after observing the price, $\mathcal{V}_{\text{FPE}}$ measures the variation of the expectation of future payoffs after observing the price. When uncertainty is Gaussian, for a Bayesian external observer who only learns from the price, both variables correspond to

$$\mathcal{V}_P = \frac{1}{\tau_\pi + \tau_u} = \frac{1 - \tau_\pi^R}{\tau_u} \qquad \text{and} \qquad \mathcal{V}_{\text{FPE}} = \frac{\tau_\pi}{\tau_\pi + \tau_u} \frac{1}{\tau_u} = \frac{\tau_\pi^R}{\tau_u}. \tag{8}$$

Equation (8) illustrates the challenge faced by both variables to identify the precision of the price as a signal: they confound the effect of uncertainty about future payoffs (τ_u^{-1}) with price informativeness (either τ_π^R or τ_π). For instance, $\mathcal{V}_P$ can be low because the payoff is not very volatile (high τ_u) or because asset prices are very informative about future payoffs (high τ_π^R or τ_π). The same ambiguous inference applies to $\mathcal{V}_{FPE}$. Theoretically, this ambiguity may not be a concern, since there is a known one-to-one mapping between these different notions. However, measuring informativeness using $\mathcal{V}_P$ of $\mathcal{V}_{FPE}$ would only recover a combination of τ_u and τ_π^R or τ_π, which makes these notions inadequate to capture how precise is the signal contained in asset prices about future payoffs.

Equation (8) also highlights that linking $\mathcal{V}_P$ or $\mathcal{V}_{\text{FPE}}$ to the precision of the information contained in prices requires making assumptions on distributions of priors, signals, and updating procedures. However, finding $\mathbb{V}\text{ar}\left[\pi_t | u_t, \Delta x_t\right]$ does not require making distributional assumptions beyond the existence of second moments. Therefore, $\left(\mathbb{V}\text{ar}\left[\pi_t | u_t, \Delta x_t\right]\right)^{-1}$ is a more desirable primitive notion of informativeness, since it can be derived without making assumptions on how an external observer updates or on the shape of the underlying distributions.

Remark 2. *Informativeness vs. predictability.* Even though price informativeness and price/return predictability may seem closely connected, they are conceptually different notions. Given the assumptions made so far, Proposition 1 shows that running regressions of prices, which are endogenous, on future payoffs, which are exogenous, allows us to recover price informativeness consistently. This entails running a regression of a date t variable, Δp_t, on a future explanatory variable, Δx_{t+1}, which contrasts with the well-established literature on return predictability (Cochrane, 2005; Campbell, 2017). One may wonder why we do not recover price informativeness using regressions of future payoffs on prices, since this type of regression can also be used for predictive purposes. This would imply reinterpreting Regression R1 as follows

$$\Delta x_{t+1} = \overline{\varphi} + \varphi_0 \Delta x_t + \varphi_1 \Delta p_t + \nu_t, \tag{R3}$$

where $\overline{\varphi} = -\frac{\overline{\beta}}{\beta_1}$, $\varphi_0 = -\frac{\beta_0}{\beta_1}$, $\varphi_1 = \frac{1}{\beta_1}$, and $\nu_t = -\frac{e_t}{\beta_1}$. The main pitfall of this regression is that the OLS estimates of the coefficients and the residual variance will produce estimates of their structural counterparts that are not consistent as long as $\mathbb{V}\text{ar}\left[\Delta n_t\right] \neq 0$, because $\mathbb{C}\text{ov}\left[\Delta p_t, \nu_t\right] = -\frac{\mathbb{V}\text{ar}[\phi_n \Delta n_t]}{\beta_1} \neq 0$. In other words, Regression R3 cannot be used to consistently recover $\mathbb{V}\text{ar}\left[\Delta x_{t+1} | \Delta x_t, \Delta p_t\right]$. In Section A.1 of the Online Appendix, we illustrate how the OLS estimate of φ_1 in Regression R3 is downward biased and inconsistent. There we also describe the relation between our results and the literature on return predictability in detail.

To clarify, predictive regressions are the right tool if one is interested in forecasting future prices/returns using current fundamentals. As forcefully expressed by Cochrane (2005), the errors in predictive regressions are by construction orthogonal to the forecasts, so there is no scope for bias or inconsistency in those cases. However, if one is interested in recovering price informativeness, which is a specific combination of structural variables, the approach developed in Proposition 1 is the adequate one.

Remark 3. *Payoff interpretation.* At the level of generality considered here, the payoff variable x_t could in principle represent any variable that satisfies Equations (1) and (2). That is, even though it may seem that, for instance, dividends are the most natural payoff measure, the results derived so far are agnostic about the exact nature of the payoff variable. We use this logic to justify the choice of earnings, instead of dividends, as the payoff measure in the empirical implementation of the results in Section 5. This observation may open the door to a higher frequency implementation of our results as data become increasingly available.

3 Structural Models

We have shown in Section 2 that it is sufficient to specify an asset pricing equation and a stochastic process for asset payoffs to identify price informativeness. In this section, we explore several fully specified environments that are consistent with Equations (1) and (2). First, we study a model in which investors have private signals about future payoffs and orthogonal trading motives in the form of random priors (sentiment). Subsequently, we study a representative agent model similar to those used in the macro-finance literature. Finally, we study a model with informed and uninformed investors, as in the classic literature on information and learning.[5]

The results in this section have a dual purpose. First, these applications show that our identification results apply to economies i) with or without dispersed information among investors, ii) with time-varying risk aversion/risk-premia, iii) in which investors may or may not learn from prices, and iv) in which noise may arise from different sources. These applications are particularly useful to highlight that our approach does not take a stance on the source of the aggregate noise. Second, these applications allow us to provide a structural economic interpretation to the empirical results presented in Section 5. In Section 6, we interpret our empirical findings through the lens of the three models studied in this section.

3.1 Sentiment as Noise

We start by considering a model in which investors' sentiment is the source of noise in the price. Starting from primitives allows us to understand which assumptions on investors' behavior endogenously determine an equilibrium pricing equation of the form assumed in Section 2.

Environment We consider a tractable overlapping generations model. Time is discrete, with dates denoted by $t = 0, 1, 2, \ldots, \infty$. The economy is populated by a continuum of investors, indexed by $i \in I$, who live for two dates. Each investor i is born with wealth w_0^i and has well-behaved expected utility preferences over his terminal wealth w_1^i, with flow utility given by $U_i\left(w_1^i\right)$, where $U_i'(\cdot) > 0$ and $U_i''(\cdot) < 0$. We assume that the distribution of initial wealth is bounded and i.i.d. across time and investor types.

There are two long-term assets in the economy: a risk-free asset in perfectly elastic supply, with gross return $R^f > 1$, and a risky asset in fixed supply Q, whose date t (log) payoff is $x_t = \ln(X_t)$ and which trades at a (log) price $p_t = \ln(P_t)$. The process followed by x_t is given by

$$\Delta x_{t+1} = \mu_{\Delta x} + u_t, \tag{9}$$

where $\Delta x_{t+1} = x_{t+1} - x_t$, $\mu_{\Delta x}$ is a scalar, and $x_0 = 0$. The realized payoff x_t is common knowledge to all investors before the price p_t is determined. The realized payoff at date $t+1$, x_{t+1}, is only revealed to investors at date $t+1$. Note that Equation (9) is a special case of Equation (1) when $\rho = 0$. We focus on the $\rho = 0$ case to simplify the exposition.

We assume that investors receive private signals about the innovation to the risky asset payoff. Formally, each investor receives a signal about the payoff innovation u_t given by

$$s_t^i = u_t + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right),$$

[5] In the Supplemental Appendix, we present conditions on investors' asset demands that are sufficient to generate an asset pricing equation of the form assumed in Equation (2).

where $\varepsilon^i_{st} \perp \varepsilon^j_{st}$ for all $i \neq j$, and $u_t \perp \varepsilon^i_{st}$ for all t and all i.

We also assume that investors have additional private trading motives coming from heterogeneous priors that are random in the aggregate. This is a particularly tractable formulation that sidesteps many of the issues associated with classic noise trading while still preventing full revelation of information — see Dávila and Parlatore (2020) for a thorough analysis of this formulation, which extends the classic DSSW model (De Long et al., 1990) to incorporate learning from prices. Formally, each investor i born at date t has a prior over the innovations to the payoff difference u_t given by

$$u_t \sim_{i,t} N\left(\overline{n}^i_t, \tau_u^{-1}\right),$$

where

$$\overline{n}^i_t = n_t + \varepsilon^i_{\overline{n}t} \quad \text{with} \quad \varepsilon^i_{\overline{n}t} \overset{\text{iid}}{\sim} N\left(0, \tau_{\overline{n}}^{-1}\right),$$

and

$$\Delta n_t = \mu_{\Delta n} + \varepsilon^{\Delta n}_t \quad \text{with} \quad \varepsilon^n_t \sim N\left(0, \tau_{\Delta n}^{-1}\right),$$

where $n_0 = 0$, $\mu_{\Delta n}$ is a scalar, and where $\varepsilon^{\Delta n}_t \perp \varepsilon^i_{\overline{n}t}$ for all t and all i. The variable n_t, which can be interpreted as the aggregate sentiment in the economy, is not observed and acts as a source of aggregate noise, preventing the asset price from being fully revealing. Without loss of generality, we assume that $u_{t+s} \sim_{i,t} N\left(0, \tau_u^{-1}\right)$ for all $s > 0$.[6]

Each investor i born at date t optimally chooses a portfolio share in the risky asset, denoted by θ^i_t, to solve

$$\max_{\theta^i_t} \mathbb{E}^i_t\left[U_i\left(w^i_1\right)\right] \tag{10}$$

subject to a wealth accumulation constraint

$$w^i_1 = \left(R^f + \theta^i_t\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\right) w^i_0, \tag{11}$$

where the information set of an investor i in period t is given by $\mathcal{I}^i_t = \left\{s^i_t, \overline{n}^i_t, \{X_s\}_{s \leq t}, \{P_s\}_{s \leq t}\right\}$.

Definition. (Equilibrium) A stationary rational expectations equilibrium in linear strategies is a set of portfolio shares θ^i_t for each investor i at date t and a price function P_t such that: i) θ^i_t maximizes the investor i's expected utility given his information set and ii) the price function P_t is such that the market for the risky asset clears at each date t, that is, $\int \theta^i_t w^i_0 di = Q$.[7]

In this class of models, it is well known that it is not possible to characterize in closed-form

[6] To simplify the analysis, we assume that investors do not learn from their priors and that the signals and priors are identically distributed across investors. Our results can be easily extended to allow for heterogeneity in τ_s, τ_u, and $\tau_{\overline{n}}$.

[7] It is well known that dynamic rational expectation models may feature multiple equilibria. Our approach is valid for any given equilibrium that may arise.

the portfolio problem solved by investors and the equilibrium price — see e.g., Vives (2008). However, we show that it is possible to find a closed-form solution to the model in approximate form.

Equilibrium Characterization In the Appendix, we show that the risky asset demand of an investor i at date t can be approximated as

$$\theta_t^i \approx \frac{1}{\gamma^i} \frac{k_0 + k_1 \mathbb{E}_t^i \left[p_{t+1} - x_{t+1}\right] + \mathbb{E}_t^i \left[\Delta x_{t+1}\right] - (p_t - x_t) - r^f}{\mathbb{V}\text{ar}_t^i \left[k_1 \left(p_{t+1} - x_{t+1}\right) + \Delta x_{t+1}\right]},$$

where $\gamma^i \equiv -\frac{w_0^i U_i''\left(w_0^i\right)}{U_i'\left(w_0^i\right)}$, $r^f = \ln\left(R^f\right)$, and k_0 and k_1 are scalars defined in the Appendix.

As we show in the Appendix, taking a first-order log-linear approximation of the first-order condition, the portfolio choice of investor i in period t can be approximated by

$$\theta_t^i \approx \alpha_x^i x_t + \alpha_s^i s_t^i + \alpha_n^i \overline{n}_t^i - \alpha_p^i p_t + \psi^i,$$

where the coefficients α_x^i, α_s^i, α_n^i, and α_p^i are positive scalars that represent the individual demand sensitivities to the contemporary payoff, the private signal, the private trading needs, and the asset price respectively, and ψ^i can be a positive or negative scalar that incorporates the risk premium. These coefficients are time invariant since we have assumed that the distribution of investor types is time invariant and the wealth distribution across time and investor type is i.i.d. Using the market clearing condition with this approximation and the information structure described above yields a log-linear approximated price given by

$$p_t \approx \frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_t + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where $\overline{\alpha_h} \equiv \int \alpha_h^i w_0^i di$ denotes the wealth-weighted cross-sectional average of a given coefficient α_h^i and $\overline{\psi} = \int \psi^i w_0^i di - Q$. Using this expression, we can map the equilibrium price process in the model to the one assumed in the general framework.

First, we take a first-order Taylor expansion of an investor's future marginal utility $U'\left(w_1^i\right)$ around the initial wealth level w_0^i. Second, we impose that terms of order $(dt)^2$, that is, terms that involve the product of two or more net interest rates, are negligible. Third, as in Campbell and Shiller (1988), we take a log-linear approximation of returns around a predetermined dividend-price ratio. Finally, we assume that the joint distribution of demand sensitivities and risk aversion is time invariant.

Lemma 1. *The price process assumed in Equation* (2) *in the general framework in Section 2 can be obtained endogenously as an approximation of the equilibrium price process in the model* *described in this section, i.e., the equilibrium price process is given by*

$$\Delta p_t \approx \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta n_t,$$

where the coefficients $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\phi_1 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, *and* $\phi_n = \frac{\overline{\alpha_n}}{\overline{\alpha_p}}$ *are determined in equilibrium.*

Lemma 1 and the payoff process assumed in this section imply that all the identification results derived in the general framework in Section 2 can be applied in the context of the fully specified model derived in this section. This connection allows us to give a structural interpretation to the coefficients recovered from Regressions R1 and R2.

For example, the sensitivity of the price to the future payoff, ϕ_1, is given by ratio of the (wealth-weighted) averages of individual demand sensitivities to information and price $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$. Therefore, the more weight individual investors put on their private signals, the more sensitive the price will be to the future payoff and, everything else equal, the higher price informativeness will be (higher $R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}$). Analogously, when investors put more weight on their orthogonal trading motives, i.e., high $\overline{\alpha_n}$, the price will be more sensitive to the aggregate sentiment and, all else equal, price informativeness will be lower (lower $R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}$).

3.2 Representative Agent

In this section, we show how to map the canonical representative agent model widely used in macro asset pricing to the setting in Section 2. This application shows that our identification results do not rely on assuming dispersed information across investors and can accommodate time-varying risk aversion.

Environment Suppose there is one representative agent in the model with sentiment introduced in the previous section, 3.1. This is the same as having all investors $i \in I$ receive the exact same signal,

$$s_t^i = u_t + \varepsilon_{st} \quad \text{with} \quad \varepsilon_{st} \sim N\left(0, \tau_s^{-1}\right),$$

have the same prior, $u_t \sim_i N\left(\overline{n}_t, \tau_u^{-1}\right)$, where

$$\overline{n}_t = n_t + \varepsilon_{\overline{n}t} \quad \text{with} \quad \varepsilon_{\overline{n}t} \overset{\text{iid}}{\sim} N\left(0, \tau_{\overline{n}}^{-1}\right),$$

and have the same initial endowment wealth, $w_0^i = w_0$, and utility, $\gamma^i = \gamma$.

Equilibrium Characterization In this case, the log-linearly approximated price is equal to

$$p_t \approx \frac{\alpha_x}{\alpha_p} x_t + \frac{\alpha_s}{\alpha_p} s_t + \frac{\alpha_n}{\alpha_p} \overline{n}_t + \frac{\psi}{\alpha_p},$$

where the coefficients α_x, α_s, α_n, and α_p are demand sensitivities and ψ is a constant.

Since all investors receive the same signal s_t and have the same prior $\overline{n}_t$, there is no asymmetric information among investors in the model and, therefore, investors do not learn from the price. However, the price contains information about the innovation u_t for an external observer who only learns from the price. The equilibrium price can be rewritten as

$$p_t \approx \frac{\alpha_x}{\alpha_p} x_t + \frac{\alpha_s}{\alpha_p} u_t + \frac{\alpha_s}{\alpha_p} \varepsilon_{st} + \frac{\alpha_n}{\alpha_p} \overline{n}_t + \frac{\psi}{\alpha_p}.$$

From the perspective of an external observer, there are two sources of noise that prevent the change in the price from being fully revealing: the noise in the signal ε_{st} and the investors' prior $\overline{n}_t$. It is easy to map the representative agent model into the framework developed in Section 2, as the lemma below shows.

Lemma 2. *The price process assumed in Equation* (2) *in the general framework in Section 2 can be obtained endogenously as an approximation of the equilibrium price process in the model described in this section, i.e., the equilibrium price process is given by*

$$\Delta p_t \approx \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta \hat{n}_t,$$

where the coefficients $\overline{\phi} = 0$, $\phi_0 = \frac{\alpha_x}{\alpha_p} - \frac{\alpha_s}{\alpha_p}$, $\phi_1 = \frac{\alpha_s}{\alpha_p}$, *and* $\phi_n = \frac{\alpha_n}{\alpha_p}$ *are equilibrium outcomes, and where* $\Delta \hat{n}_t \equiv \Delta \overline{n}_t + \frac{\alpha_s}{\alpha_n} \Delta \varepsilon_{st}$.

As in the previous section, Lemma 2 and the payoff process assumed allow us to apply all the identification results derived in Section 2 within the representative agent model. This shows that the price process in Equation (2) also encompasses models in which all investors share the same information and there is no learning from the price. In fact, our general framework does not require information to be dispersed in the economy and it can accommodate environments with and without learning.

Finally, it is worth highlighting that it is easy to introduce time-varying risk aversion in this framework — this would imply assuming that γ and consequently ψ vary over time, as γ_t and ψ_t. In that case, time-varying risk aversion manifests itself as another source of noise, in particular when Regressions R1 and R2 include none of the controls discussed in Section 4.2. Alternatively, following our interpretation of public information/signals in Section 4.2, one could add observable controls that capture changes in risk premia.

3.3 Informed, Uninformed, and Noise Traders

Noise traders are a widely used modeling device in environments with dispersed information to avoid dealing with fully revealing equilibria. The general framework in Section 2 applies to settings with noise traders. This application highlights that our identification results accommodate different forms of noise, which allows us to remain agnostic about the source of noise in the economy.

Environment Suppose that we are in the same model developed in Section 3.1 with the only difference being that there are three types of investors: informed, uninformed, and noise traders. Informed and uninformed investors share the same prior and only differ in the information they receive. Informed investors receive a perfectly informative signal of the innovation to the payoff. Uninformed investors and noise traders do not receive any signals. Mapping this to the model in Section 3.1 implies that the prior distribution of the innovation u_t for informed and uninformed investors is

$$u_t \sim_i N\left(\overline{n}_t, \tau_u^{-1}\right),$$

where $\overline{n}_t \overset{\text{iid}}{\sim} N\left(0, \tau_{\overline{n}}^{-1}\right)$ and the precision of the signals for informed investors is $\tau_{si} = \infty$ and for uninformed investors is $\tau_{si} = 0$.

Finally, noise traders have private trading motives that are orthogonal to the innovation to the payoff — these are the sole drivers of their demand. Formally, the demand of all noise traders in period t is random and given by $\delta_t \sim N\left(0, \tau_N^{-1}\right)$. The noise trader demand is only observed by noise traders.

Equilibrium Characterization In this case, the first-order log-approximated price is

$$p_t \approx \frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} \overline{n}_t + \frac{\overline{\psi}}{\overline{\alpha_p}} + \frac{\delta_t}{\overline{\alpha_p}}, \tag{12}$$

where $\overline{\alpha_h} \equiv \int_{I \cup U} \alpha_h^i w_0^i di$ denotes the wealth-weighted cross-sectional average of α_h^i over the set of informed and uninformed investors with $\alpha_s^i = 0$ for all uninformed investors, $\alpha_n^i = 0$ for all informed investors, and $\overline{\psi} \equiv \int_{I \cup U} \psi^i w_0^i di - Q$.

Lemma 3. *The price process assumed in Equation* (2) *in the general framework in Section 2, can be obtained endogenously as an approximation of the equilibrium price process in the model described in this section, i.e., the equilibrium price process is given by*

$$\Delta p_t \approx \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta \tilde{n}_t,$$

where the coefficients $\overline{\phi} = 0$*,* $\phi_0 = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$*,* $\phi_1 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$*, and* $\phi_n = \frac{\overline{\alpha_n}}{\overline{\alpha_p}}$ *are equilibrium outcomes and* $\Delta \tilde{n}_t \equiv \Delta \overline{n}_t + \frac{1}{\overline{\alpha_n}} \Delta \delta_t$*.*

Lemma 3 shows that all our identification results in Section 2 remain valid within the classic information model in Grossman and Stiglitz (1980) with inelastic noise traders. Within the model, only uninformed investors learn from the price and the only source of noise for them is the noise trader demand. However, for an external observer who only learns from the price there are two sources of noise embedded in the change in the price. The change in the noise trader demand $\Delta\delta_t$ and the change in the prior of the investors $\Delta\overline{n}_t$. Defining $\Delta\tilde{n}_t \equiv \Delta\overline{n}_t + \frac{1}{\alpha_n}\Delta\delta_t$ allows us to clearly map this model into the general framework developed in Section 2. This lemma together with the results in the previous two sections show that the price process assumed in our general framework can accommodate different sources of noise that prevent the price from being fully revealing for an external observer.

4 Extensions

Before empirically implementing our results, we extend the results derived in Section 2 in several dimensions. We describe i) how to account for a non-zero correlation between u_t and Δn_t, ii) how to allow for the possibility of public signals about future payoffs, and iii) how to implement our results when the payoff process has an unlearnable component. This section has two goals. First, it shows that our results apply to more general and empirically relevant scenarios. Second, it allows us to refine the interpretation of our empirical results.

4.1 Correlated Payoff and Noise

In Section 2, we assume that the innovation to the asset payoff u_t is uncorrelated with the aggregate source of noise Δn_t. In this section, we allow for the aggregate source of noise to be correlated with the payoff. Formally, we consider the following process for the aggregate noise:

$$\Delta n_t = \mu_{\Delta n} + \omega u_t + \varepsilon_t^{\Delta n},$$

with $\omega \neq 0$, where $\varepsilon_t^{\Delta n}$ has mean zero and finite variance and is i.i.d. across time and independent of the innovations u_t. In this case, the equilibrium price process

$$\Delta p_t = \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta n_t \tag{13}$$

can be written as

$$\Delta p_t = \overline{\phi} - \phi_n \omega \mu_{\Delta x} + \phi_n \mu_{\Delta n} + \phi_0 \Delta x_t + (\phi_1 + \phi_n \omega) \Delta x_{t+1} + \phi_n \varepsilon_t^{\Delta n}, \tag{14}$$

and absolute price informativeness is given by

$$\tau_\pi \equiv \mathbb{V}\mathrm{ar}\left[\pi_t | \, u_t, x_t \right]^{-1} = \left(\frac{\phi_1 + \phi_n \omega}{\phi_n} \right)^2 \tau_{\Delta n}. \tag{15}$$

The main difference between this extension and the baseline framework is that the sensitivity of the price to the future payoff, which determines how much information is contained in the price, has an additional component that comes from the comovement between the payoff and noise, $\phi_n \omega$, as can be seen from Equations (14) and (15). However, this sensitivity can still be recovered from the coefficient on Δx_{t+1} in Regression R1 and, as the proposition below shows, the identification result derived in the general framework remains valid.

Proposition 2. (Identifying price informativeness when payoff and noise are correlated) *Relative price informativeness can be recovered and consistently estimated from the R-squareds of Regressions R1 and R2 as follows:*

$$\tau_\pi^R = \frac{R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}}.$$

As the proposition above shows, even though the structural characterization of price informativeness is different when the payoff is correlated with the aggregate noise, price informativeness can be recovered in the same way as in the baseline general framework. Therefore, we can assume that the private trading motives are orthogonal to the asset payoff without loss of generality.

4.2 Public Signals

In our results until now, we have considered private signals as the only source of information in the economy. In this subsection, we consider the case in which investors also observe a public signal about the asset payoff. We extend the environment in the general framework in Section 2 by considering an environment in which investors observe a vector χ_t of N public signals

$$\chi_t = \overline{\omega} u_t + \overline{\varepsilon}_t^\chi,$$

where $\overline{\omega}$ is an $N \times 1$ vector and $\overline{\varepsilon}_t^\chi$ is an $N \times 1$ random vector that has mean zero, finite variance, and is i.i.d. across time and independent of the innovations u_t. In this case, we augment the price process in Equation (2) to include the private signals χ_t as follows

$$\Delta p_t = \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_\chi \cdot \Delta \chi_t + \phi_n \Delta n_t, \tag{16}$$

where ϕ_χ is an $N \times 1$ vector of coefficients.

There are two relevant notions of price informativeness that depend on the information available to the external observer. If the public signals are not available to the external observer, price informativeness is given, as in Section 2, by $\tau_\pi \equiv \mathbb{V}\mathrm{ar}\left[\pi_t | u_t, x_t\right]^{-1}$, where π_t is defined in Equation (3). If public signals are observed by the external observer, the unbiased signal contained in the price is

$$\pi_t' \equiv \frac{1}{\phi_1}\left(\Delta p_t - \left(\overline{\phi} + \phi_1\mu_{\Delta x} + \phi_n\mu_{\Delta n} + (\phi_0 + \phi_1\rho)\,\Delta x_t + \phi_\chi \cdot \Delta\chi_t\right)\right)$$

and price informativeness is given by $\tau_{\pi'} \equiv \mathbb{V}\mathrm{ar}\left[\pi_t' \middle| u_t, x_t, \chi_t\right]^{-1}$. The proposition below provides identification results for each of these two cases.

Proposition 3. (Identifying price informativeness with public signals)

a) When the public signals available to investors are not available to the external observer, relative price informativeness can be recovered and consistently estimated from the R-squareds of Regressions R1 and R2 as follows

$$\tau_\pi^R = \frac{R^2_{\Delta x,\Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}}.$$

b) When the public signals available to investors are part of the information set of the external observer, relative price informativeness can be recovered from the following regressions of prices on payoffs and the public signals

$$\Delta p_t = \overline{\beta}' + \beta_0'\Delta x_t + \beta_1'\Delta x_{t+1} + \beta_2' \cdot \Delta\chi_t + e_t' \tag{R1-PS}$$

$$\Delta p_t = \overline{\zeta}' + \zeta_0'\Delta x_t + \zeta_2' \cdot \Delta\chi_t + e_t^{\zeta}{}', \tag{R2-PS}$$

as

$$\tau_{\pi'}^R = \frac{R^2_{\Delta x,\Delta x',\Delta\chi} - R^2_{\Delta x,\Delta\chi}}{1 - R^2_{\Delta x,\Delta\chi}},$$

where $R^2_{\Delta x,\Delta x',\Delta\chi}$ and $R^2_{\Delta x,\Delta\chi}$ are the R-squareds of Regression R1-PS and Regression R2-PS, respectively.

Part a) of the proposition above follows directly from reinterpreting the unobserved terms in the price equation (16), i.e., $\phi_\chi \cdot \Delta\chi_t + \phi_n\Delta n_t$, as noise that is correlated with the payoff and using Proposition 2. This shows that our results are robust to investors having public information when this information is not available to the external observer.

Part b) shows how to extend our identification results when the public signals are part of the external observer's information set. The proof is analogous to the proof of Proposition 1 in the general framework, with the coefficients in Regressions R1-PS and R2-PS being matched to the expressions of the price process in Equations (16). A detailed proof of Proposition 3 can be found in the Appendix.

Intuitively, to capture the additional information contained in the price, the Regressions R1-PS and R2-PS need to condition for the information set of the external observer. In the general framework in Section 2 we assume that the external observer only observes the price and the contemporary payoff. If the information set of the external observer contains other information, then Proposition 3 shows that our identification results remain valid provided the regressions of prices on payoffs are augmented to include the information available to the external observer.

One can draw an analogy between the standard notions of market efficiency (weak, semi-strong, and strong) and the set of controls used in our regressions. For instance, regressions that include exclusively past payoffs as controls resemble weak-form efficiency notions. By expanding the set of controls to include public or private information, the recovered informativeness measures resemble semi-strong or strong notions of efficiency.

4.3 Learnable and Unlearnable Payoff

So far, we have considered that all components of the payoff are learnable, that is, that there is no systematic component of the payoff that deviates from the signals received by the investors. However, it is plausible to think that investors can only learn about a part of the innovation and that the remainder is unlearnable. Formally, we assume that the innovation to the payoff is given by

$$\Delta x_{t+1} = \mu_{\Delta x} + u_t, \tag{17}$$

where $\Delta x_{t+1} = x_{t+1} - x_t$, μ_x is a scalar, and $x_0 = 0$. Moreover, the innovation to the payoff is given by

$$u_t = u_t^L + u_t^U,$$

where u_t^L and u_t^U are the learnable and unlearnable components of the innovation where

$$u_t^L \sim N\left(0, \left(\tau_u^L\right)^{-1}\right) \quad \text{and} \quad u_t^U \sim N\left(0, \left(\tau_u^U\right)^{-1}\right)$$

with $u_t^L \perp u_t^U$. The main difference between these two components is that investors only receive private signals about the learnable component, u_t^L. Formally, each investor receives a signal

$$s_t^i = u_t^L + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right),$$

where $\varepsilon_{st}^i \perp \varepsilon_{st}^j$ for all $i \neq j$, $u_t^L \perp \varepsilon_{st}^i$, and $u_t^U \perp \varepsilon_{st}^i$ for all t and all i. Moreover, as in Section 2, $\Delta n_t \equiv n_t - n_{t-1}$ represents the change in the aggregate component of investors' trading motives that are orthogonal to the asset payoff, given by $\Delta n_t = \mu_{\Delta n} + \varepsilon_t^{\Delta n}$, where $\mathbb{V}\text{ar}\left[\Delta n_t\right] = \sigma_{\Delta n}^2 = \tau_{\Delta n}^{-1}$.

In this case, the price process is given by

$$\Delta p_t = \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta n_t - \phi_1 \Delta u_t^U,$$

where $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\phi_1 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and $\phi_n = \frac{\overline{\alpha_n}}{\overline{\alpha_p}}$. Using the process for the payoff, this price process can also be written as

$$\Delta p_t = \overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_0 \Delta x_t + \phi_1 u_t^L + \phi_1 u_{t-1}^U + \phi_n \Delta n_t.$$

Therefore, the unbiased signal about the change in the learnable component of the innovation contained in the price is

$$\pi_t^L \equiv \frac{1}{\phi_1}\left(\Delta p_t - \left(\overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_n \mu_{\Delta n} + \phi_0 \Delta x_t\right)\right) = u_t^L + u_{t-1}^U + \frac{\phi_n}{\phi_1}\varepsilon_t^{\Delta n},$$

and absolute and relative price informativeness are respectively given by

$$\tau_{\pi^L} \equiv \mathbb{V}\text{ar}\left[u_{t-1}^U + \frac{\phi_n}{\phi_1}\Delta\varepsilon_t^{\Delta n}\middle| u_t^L, \Delta x_t\right]^{-1} = \left(\left(\tau_u^L + \tau_u^U\right)^{-1} + \left(\frac{\phi_n}{\phi_1}\right)^2 \tau_{\Delta n}^{-1}\right)^{-1}$$

and

$$\tau_{\pi^L}^R = \left(1 + \frac{\tau_u^L}{\tau_u^L + \tau_u^U} + \left(\frac{\phi_n}{\phi_1}\right)^2 \left(\frac{\tau_{\Delta n}}{\tau_u^L}\right)^{-1}\right)^{-1}.$$

Note that the noise in the signal π_t^L is not independent of Δx_t since $\Delta x_t = \mu_{\Delta x} + u_{t-1}^L + u_{t-1}^U$. Then, conditioning on Δx_t becomes relevant in our definition of price informativeness because Δx_t contains information about the noise in the signal. Note that by conditioning on u_t^L, both definitions of price information recover the precision of the asset price about the learnable component of payoffs.

When the innovation to the asset payoff has an unlearnable component, the error term in regression R1 is given by $e_t = \phi_n \varepsilon_t^{\Delta n} - \phi_1 \Delta u_t^U$ and, therefore, is correlated with the regressors Δx_{t+1} and Δx_t. Moreover, the error term in Regression R2 is given by $e_t^{\zeta} = \phi_n \varepsilon_t^{\Delta n} + \phi_1 u_t^L + \phi_1 u_{t-1}^U$, which is correlated with Δx_t through u_{t-1}^U. Hence, to obtain consistent estimates of the parameters in both regressions, one must run Regressions R1 and R2 instrumenting for Δx_{t+1} and Δx_t with a variable that is correlated with the learnable component of the innovation but not with the unlearnable one. In addition to instrumenting for Δx_{t+1} and Δx_t to recover consistent estimates of the regression parameters, the identification procedure needs to be adjusted to take into account the extra term in the expression for absolute price informativeness and in the error term e_t. Earnings forecasts should be good candidate for an instrument, since earnings forecasts are determined by the information available to analysts about future payoffs and, therefore, are only correlated with the learnable component of payoffs. More generally, any variable correlated with the future payoff that is known before the payoff is realized is a valid candidate as an instrument in practice.

The following proposition shows how to identify and estimate price informativeness when the innovation to the asset payoff has an unlearnable component.

Proposition 4. (Identifying price informativeness about learnable component of payoffs) *When the innovation to the asset payoff has an unlearnable component, relative price informativeness about the learnable component of the innovation can be recovered as follows:*

$$\tau^R_{\pi^L} = \frac{\mathbb{V}ar\left(e^\zeta_t\right) - \mathbb{V}ar\left(e_t\right)}{\mathbb{V}ar\left(e^\zeta_t \middle| \Delta x_t\right)} \frac{1}{1 - \frac{\mathbb{V}ar\left(u^U_t\right)}{\mathbb{V}ar\left(u^L_t\right)}}, \tag{18}$$

where

$$\frac{\mathbb{V}ar\left(u^U_t\right)}{\mathbb{V}ar\left(u^L_t\right)} = \frac{1 - \frac{\mathbb{V}ar\left(e^\zeta_t\right) - \mathbb{V}ar(e_t)}{\phi_1^2 \mathbb{V}ar(u_t)}}{1 + \frac{\mathbb{V}ar\left(e^\zeta_t\right) - \mathbb{V}ar(e_t)}{\phi_1^2 \mathbb{V}ar(u_t)}} \tag{19}$$

and $\mathbb{V}ar\left(e^\zeta_t\right)$ and $\mathbb{V}ar\left(e_t\right)$ are the variances of the residuals in Regressions R2 and R1, respectively, $\mathbb{V}ar\left(e^\zeta_t \middle| \Delta x_t\right)$ is the variance of the residual in a regression of the residuals in Regression R2 on Δx_t, and $\mathbb{V}ar\left(u_t\right)$ is the variance of the residuals recovered from Equation 17. Estimating Regressions R2 and R1 consistently requires instrumenting for Δx_{t+1} and Δx_t with a variable that is correlated with the learnable component of the innovation but not with the unlearnable one.

Proposition 4 shows how to identify relative price informativeness when the innovation to the asset payoff has an unlearnable component. There are two terms that help identify price informativeness in this case. The first term in Equation (18) is a ratio of residual variances, which is analogous to the ratio of normalized difference in R-squareds in our previous identification results, once adjusted for the correlation between Δx_t and the error term e^ζ_t. In the limit, when the variance of the unlearnable component is 0, the first term in Equation (19) converges to the expression in Proposition 1 since

$$\lim_{\mathbb{V}\text{ar}\left(u^U_t\right) \to 0} \frac{\mathbb{V}\text{ar}\left(e^\zeta_t\right) - \mathbb{V}\text{ar}\left(e_t\right)}{\mathbb{V}\text{ar}\left(e^\zeta_t \middle| \Delta x_t\right)} = \frac{R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}}.$$

The second term in Equation (18) is a correction term that takes into account the presence of an unlearnable component in the innovation to the payoff. When there is no unlearnable component in the innovation, i.e., when $\mathbb{V}\text{ar}\left(u^U_t\right) \to 0$, the correction term is one, which brings us back to our identification results in our benchmark model. On the other hand, when the unlearnable component is the one driving all the change in the payoff, i.e., when $\mathbb{V}\text{ar}\left(u^U_t\right) \to \infty$, the correction term goes to zero and there is no information about the payoff in prices.

Summing up, if payoffs have an unlearnable component, the identification and estimation of price informativeness should be modified. Regarding the identification, Proposition 4 shows how it is necessary to correct the expression from our baseline model. On the estimation side, both regressions require an instrument correlated with the learnable component of the innovation but not with the unlearnable one.

Instead of looking for stock-specific instruments, we present our empirical results using Proposition 1, and use the results of this section to explore the potential bias via simulations. In Section D of the Online Appendix, we simulate a calibrated version of this model and show that the estimates of price informativeness obtained using the OLS procedure from Proposition 1 can be at times better in small samples than the estimates obtained using Proposition 4.

5 Empirical Implementation: Stock-Specific Price Informativeness

In this section, we make use of our identification results to construct and analyze measures of stock-specific relative price informativeness. We exclusively report estimates of relative price informativeness since these allow for meaningful and easily interpretable comparisons across stocks and over time. We recover a panel of stock-specific measures of price informativeness by running rolling time-series regressions of the form introduced in Proposition 1 at the stock level using quarterly data. In this section, we simply describe our approach and our findings, leaving the economic interpretation of our results to Section 6.

We find that the distribution of informativeness across stocks is right-skewed, with time-series averages of the median and mean levels of price informativeness across all stocks and years respectively given by 1.84% and 4.25%. Our approach allows us to uncover both cross-sectional and time-series patterns regarding the behavior of price informativeness. In the cross section, we find that stocks that i) are larger, ii) turn over more quickly, and iii) have a higher institutional ownership share have higher price informativeness. In the time series, we find that the median and mean price informativeness have steadily increased over time since the mid-1980s. The standard deviation of price informativeness has also increased over this period. In the Online Appendix, we include additional results that show the robustness of our cross-sectional and time-series findings.

Data Description and Empirical Specification We initially provide a brief description of the data and the sample selection procedure. The Online Appendix and the companion R notebooks include a more detailed description.[8] We obtain information on stock prices and

[8]The complete replication materials as well as the recovered measures of informativeness can be found at https://github.com/edavila/identifying_price_informativeness.

accounting measures from the CRSP/Compustat dataset, as distributed by WRDS. Our sample selection procedure follows the conventional approach described in Bali, Engle and Murray (2016). From the Center for Research in Security Prices (CRSP), we obtain data on stock prices, market capitalization, turnover, S&P500 status, and industry (SIC) classification for all common US-based stocks listed on the NYSE, NASDAQ, and AMEX. From Compustat, we obtain accounting data that includes earnings and book values, at both quarterly and annual frequencies.

In this section, we report the results of the analysis using quarterly data, available from 1961 until 2017. In the Supplementary Appendix, we report the results of the analysis using annual data, available from 1950 until 2017. To match the timing of our model and to ensure that the accounting data were public on the trading date, we merge the Compustat data with CRSP data three months ahead, although our findings are robust to using alternative windows. We use the personal consumption expenditure index (PCEPI), obtained from FRED, to deflate all nominal variables.

We implement Proposition 1 by running time-series regressions for each individual stock — indexed by j here — over rolling windows of 40 quarters. By working with the model in log-differences, we sidestep concerns associated with failures of stationary — see e.g., Campbell (2017). The use of rolling regressions makes the underlying assumption of parameter stability over a given estimation window more plausible.

We denote by p_t^j the log price of stock j, adjusted for splits. We use earnings — as measured by EBIT — as the relevant measure of payoffs, since stock-level measures of dividends are problematic for different reasons. As discussed in Section 2, our model can be flexibly interpreted to use earnings as the payoff measure. Since earnings can be negative, we compute Δx_t^j directly as a growth rate, as explained in the Online Appendix — we obtain comparable results when we compute Δx_t^j as the log-difference of the logistic transformation of standardized earnings. Formally, in a given rolling window, we run time-series regressions of the form

$$\Delta p_t^j = \overline{\beta}^j + \beta_0^j \Delta x_t^j + \beta_1^j \Delta x_{t+1}^j + d_t^{j,q} + \varepsilon_t^j \quad \Rightarrow R^{2,j}_{\Delta x, \Delta x'} \tag{20}$$

$$\Delta p_t^j = \overline{\zeta}^j + \zeta_0^j \Delta x_t^j + d_t^{j,q} + \hat{\varepsilon}_t^j \quad \Rightarrow R^{2,j}_{\Delta x}\ , \tag{21}$$

where Δp_t^j is a measure of capital gains, Δx_t^j and its one period ahead counterpart Δx_{t+1}^j are measures of earnings growth, and $d_t^{j,q}$ denote stock-specific quarterly dummies. The introduction of $d_t^{j,q}$ accounts for seasonality patterns, and can be interpreted along the lines of Section 4.2. We estimate the regression coefficients and errors using OLS. We respectively denote the R-squareds of the regressions (20) and (21) by $R^{2,j}_{\Delta x, \Delta x'}$ and the $R^{2,j}_{\Delta x}$. Hence, Regression R1 maps to Equation (20), while Regression R2 maps to Equation (21), but for the addition of the quarterly dummies,

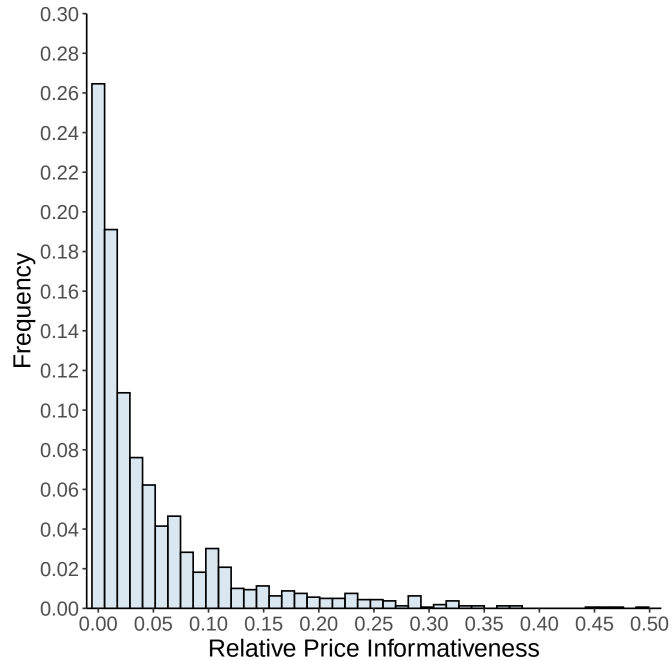

Figure 2: Price informativeness: relative-frequency histogram

Note: Figure 2 shows a relative-frequency histogram of price informativeness for a representative time period, the last quarter of 2015. Note that informativeness is computed over a rolling window of 40 quarters prior. The histogram features 1,591 stocks.

interpreted as public signals.

Consistent with Proposition 1, we recover relative price informativeness for stock j in a given period/window from Equations (20) and (21) as follows:

$$\tau_{\pi}^{R,j} = \frac{R^{2,j}_{\Delta x, \Delta x'} - R^{2,j}_{\Delta x}}{1 - R^{2,j}_{\Delta x}}.$$

After restricting our results to stocks with contiguous observations and whose maximum leverage across observations is lower than 0.95, we end up with a panel of quarterly price informativeness measures for 2,440 unique stocks. We have explored alternative criteria to deal with outliers or abnormal observations — for instance, restricting the set of observations to those with $\beta_1^j \in [0, 1]$ — but this does not change our conclusions.

Table 1 reports year-by-year summary statistics of the distribution of stock-specific price informativeness, starting in 1980. Throughout the paper, informativeness in year t is computed over a rolling window of 40 quarters prior. We illustrate our results graphically in Figure 2, which presents a relative-frequency histogram of price informativeness for a specific time period (last quarter of 2015). The shape of this histogram is representative of other periods.

The distribution of informativeness that we recover is right-skewed every year, with a mean

Table 1: Price informativeness: year-by-year summary statistics

t	$Median$	$Mean$	SD	$Skew$	$Kurt$	$P5$	$P25$	$P75$	$P95$	n
1980	0.0203	0.0405	0.0491	2.0237	4.9861	0.0011	0.0066	0.0553	0.1331	136
1981	0.0241	0.0397	0.0497	2.2526	5.6176	0.0008	0.0065	0.0531	0.1523	171
1982	0.0219	0.0422	0.0508	1.6819	2.7538	0.0005	0.0046	0.0644	0.1512	284
1983	0.0215	0.0429	0.0569	2.7807	12.0644	0.0009	0.0056	0.0587	0.1582	307
1984	0.0184	0.0387	0.0529	2.4882	7.9165	0.0005	0.0051	0.0487	0.1537	456
1985	0.0167	0.0347	0.0488	3.0589	12.8434	0.0006	0.0048	0.0459	0.1232	800
1986	0.0163	0.0364	0.0523	3.3520	17.0220	0.0005	0.0049	0.0466	0.1327	808
1987	0.0169	0.0352	0.0479	2.8658	12.0565	0.0007	0.0045	0.0484	0.1224	888
1988	0.0164	0.0353	0.0501	3.0991	13.6756	0.0003	0.0044	0.0461	0.1277	825
1989	0.0174	0.0348	0.0472	2.8804	12.6139	0.0005	0.0047	0.0442	0.1298	771
1990	0.0153	0.0327	0.0451	3.0686	15.3927	0.0007	0.0049	0.0442	0.1200	760
1991	0.0147	0.0347	0.0503	2.8258	10.7292	0.0004	0.0038	0.0458	0.1347	1094
1992	0.0153	0.0351	0.0499	2.9706	13.0878	0.0004	0.0045	0.0459	0.1285	1175
1993	0.0159	0.0361	0.0504	2.4868	7.7488	0.0005	0.0045	0.0460	0.1438	1248
1994	0.0169	0.0365	0.0511	2.6197	8.8519	0.0003	0.0045	0.0466	0.1437	1258
1995	0.0165	0.0360	0.0499	2.5047	8.0230	0.0004	0.0043	0.0456	0.1430	1301
1996	0.0166	0.0361	0.0496	2.4757	7.5962	0.0004	0.0043	0.0464	0.1370	1394
1997	0.0178	0.0362	0.0473	2.5007	8.6653	0.0005	0.0052	0.0490	0.1305	1457
1998	0.0178	0.0369	0.0515	3.1369	14.8746	0.0005	0.0054	0.0486	0.1321	1423
1999	0.0192	0.0400	0.0557	3.0497	13.8314	0.0005	0.0053	0.0540	0.1501	1352
2000	0.0220	0.0433	0.0577	2.5430	8.8059	0.0005	0.0060	0.0558	0.1634	1273
2001	0.0211	0.0428	0.0571	2.5582	9.5999	0.0005	0.0052	0.0590	0.1621	1237
2002	0.0193	0.0415	0.0565	2.5283	8.2349	0.0006	0.0052	0.0555	0.1536	1267
2003	0.0190	0.0398	0.0525	2.3117	6.5211	0.0005	0.0047	0.0548	0.1453	1405
2004	0.0181	0.0397	0.0526	2.3279	6.6881	0.0004	0.0043	0.0571	0.1448	1477
2005	0.0189	0.0391	0.0530	2.6971	10.6341	0.0004	0.0045	0.0533	0.1490	1519
2006	0.0192	0.0391	0.0523	2.5171	8.6964	0.0004	0.0047	0.0514	0.1428	1587
2007	0.0205	0.0420	0.0562	2.4062	7.1299	0.0006	0.0053	0.0542	0.1643	1611
2008	0.0228	0.0473	0.0645	2.7105	10.3734	0.0007	0.0068	0.0611	0.1823	1622
2009	0.0239	0.0538	0.0759	2.6774	9.1915	0.0006	0.0065	0.0688	0.2036	1695
2010	0.0221	0.0500	0.0710	2.7967	11.2454	0.0005	0.0052	0.0647	0.2017	1700
2011	0.0219	0.0494	0.0711	2.9496	12.6900	0.0005	0.0060	0.0621	0.1992	1703
2012	0.0221	0.0509	0.0725	2.7526	10.5131	0.0006	0.0057	0.0668	0.2067	1666
2013	0.0220	0.0499	0.0724	2.8060	10.5111	0.0005	0.0052	0.0624	0.2027	1662
2014	0.0223	0.0499	0.0727	2.8700	11.0478	0.0004	0.0057	0.0617	0.1959	1689
2015	0.0215	0.0495	0.0702	2.6747	9.2983	0.0006	0.0063	0.0630	0.1999	1720
2016	0.0207	0.0481	0.0675	2.5495	8.7341	0.0005	0.0057	0.0604	0.1960	1696
2017	0.0201	0.0481	0.0677	2.5475	8.7405	0.0004	0.0056	0.0619	0.1912	1667

Note: Table 1 reports year-by-year summary statistics on the panel of price informativeness measures recovered. It provides information on the median; mean; standard deviation; skewness; excess kurtosis; and 5th, 25th, 75th, and 95th percentiles of each yearly distribution, as well as the number of stocks in each year. Since our panel of price informativeness is quarterly, we average the measures of quarterly price informativeness at the yearly level before computing the summary statistics. We start reporting summary statistics in 1980, since we have informativeness measures for more than 100 stocks starting on that year. Informativeness in year t is computed over a rolling window of 40 quarters prior.

that is roughly 0.02 larger than the median of the distribution of informativeness. Because the distribution of informativeness is skewed, the median is often perceived as a better measure of central tendency. The 95% percentile of the distribution stays between 0.12 and 0.2, which means that an external observer who only learns from the price would rarely put more than a 20% weight on the price when updating his beliefs to form a posterior over future payoffs. Since we have not included additional controls in the regressions besides quarter fixed effects, our results should be interpreted as the price informativeness for an external observer who exclusively observes prices and past payoffs. See the Online Appendix for the results including additional controls, which correspond to a notion of informativeness in which the external observer has a larger information set, as explained in Section 4.2.

Price Informativeness in the Cross Section By computing stock-specific measures of price informativeness, we are able to establish a new set of cross-sectional patterns relating price informativeness to stock characteristics. We focus on five stock characteristics that have been widely used to explain patterns in the cross section of stock returns — see, e.g., Bali, Engle and Murray (2016). These are i) size, measured as the natural log of stocks market capitalization, ii) value, measured as the ratio between a stock's book value and its market capitalization, iii) turnover, measured as the ratio between trading volume and shares outstanding, iv) idiosyncratic volatility, measured as the standard deviation — over a 30 month period — of the difference between the returns of a stock and the market return, and v) institutional ownership, measured as the proportion of shares held by institutional investors.

In Table 2, we report the estimates of panel regressions of relative price informativeness (in twentiles) on each of the five explanatory variables, using year fixed effects. The coefficients that we report can be interpreted as a weighted average of the slopes of running year-by-year regressions of price informativeness of a given explanatory variable (size, value, turnover, return volatility, institutional ownership). Figures OA-4 through OA-8 in the Online Appendix provide an alternative graphical illustration of our results. These figures show that the cross-sectional relations identified in Table 2 are stable over time. In the Online Appendix, we also show that these cross-sectional patterns remain valid using pooled measures of price informativeness over time.

Our cross-sectional analysis yields several robust patterns. First, we find a strong positive cross-sectional relation between a stock's size (market capitalization) and price informativeness; that is, large stocks have higher price informativeness. Second, we find a negative and weak cross-sectional relation between a stock's book-to-market ratio and price informativeness; that is, value stocks have lower price informativeness. Third, we find a strong positive cross-sectional relation between a stock's turnover and price informativeness; that is, stocks that trade fre-

Table 2: Cross-sectional results (1)

	Estimate	Std. Error	t-stat
Size	0.00214	0.000130	16.42
Value	-0.00092	0.000367	-2.52
Turnover	0.00030	0.000019	16.42
Idiosyncratic Volatility	0.00379	0.005241	0.72
Institutional Ownership	0.01436	0.001051	13.66

Note: Table 2 reports the estimates ($\widehat{a_1^c}$) of panel regressions of price informativeness on cross-sectional characteristics (in twentiles) with year fixed effects (ξ_t): $\tau_{\pi,t}^{R,b} = a_0^c + a_1^c c_t^b + \xi_t + \epsilon_{b,t}$, where $\tau_\pi^{R,b,t}$ denotes the average price informativeness per bin (twentile) in a given period, c_t^b denotes the value of the given characteristic per bin (twentile) in a given period, ξ_t denotes a year fixed effect, a_0^c and a_1^c are parameters, and $\epsilon_{b,t}$ is an error term. Figures OA-4 through OA-8 provide the graphical counterpart of the results in this table. Size is measured as the natural log of stock market capitalization, value is measured as the ratio between a stock's book value and its market capitalization, turnover is measured as the ratio between trading volume and shares outstanding, idiosyncratic volatility is measured as the standard deviation — over a 30 month period — of the difference between the returns of a stock and the market return, and institutional ownership is measured as the proportion of a stock held by institutional investors.

quently have higher price informativeness. Fourth, we find a positive but insignificant weak cross-sectional relation between a stock's idiosyncratic return volatility and price informativeness; that is, stocks whose returns are more volatile have higher price informativeness. Finally, we find a strong positive cross-sectional relation between a stock's institutional ownership share and price informativeness, that is, stocks owned mostly by institutional investors have higher price informativeness.

Figures 3 and 4 illustrate additional cross-sectional patterns of the behavior of informativeness by exchange, S&P 500 status, and sector. Instead of focusing on mean or median comparisons, we find it more informative to graphically compare the distributions of informativeness by characteristic after extracting year fixed effects. Even though the distributions of informativeness differ across characteristics, the relations seem less strong than those identified in Table 2. First, we compare across exchanges and find that stocks listed in the NYSE have higher median informativeness than those in the NASDAQ, which appear to be more informative than those listed in the AMEX. Second, we study whether price informativeness varies among stocks that belong to the S&P500 and those that do not. Consistent with our findings on size, we find that stocks outside of the S&P have lower price informativeness on average. Finally, we study the behavior of price informativeness across sectors. We find that the median price informativeness is highest in the wholesale/retail and the finance/insurance sectors, and lowest in the service sector.

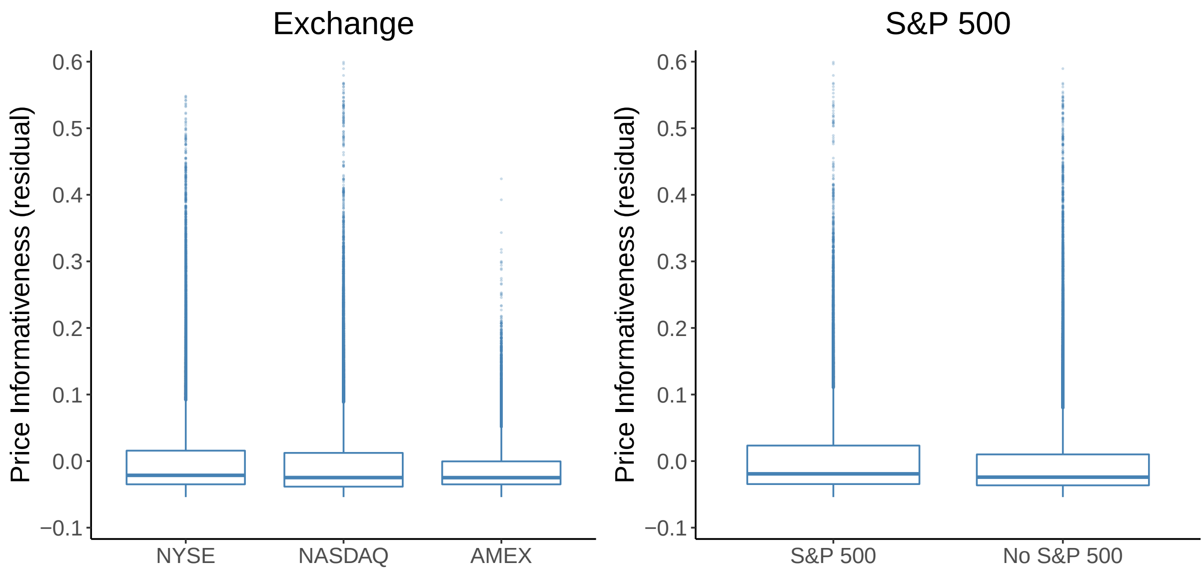

Figure 3: Cross-sectional results (2)

Note: The left panel in Figure 3 shows a box plot by exchange of the residuals of a regression of relative price informativeness on year fixed effects. The left panel in Figure 3 shows a box plot by S&P 500 status of the residuals of a regression of relative price informativeness on year fixed effects. The solid middle line represents the median. The top and bottom of the box represent the 75th and 25th percentiles. The whiskers extend up to 1.5 times the interquartile range.

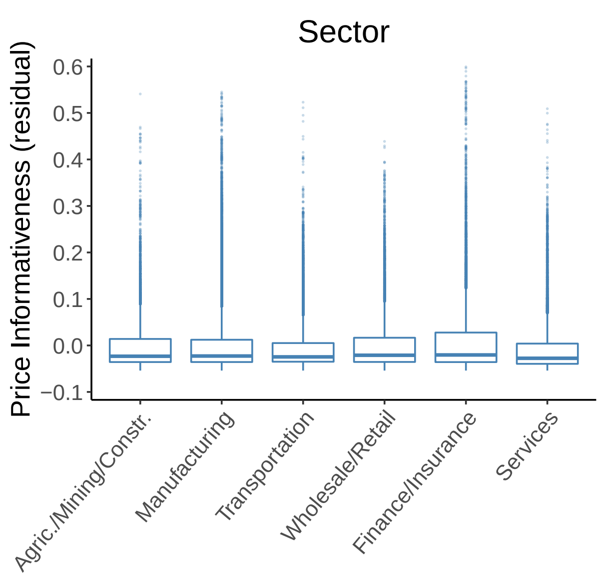

Figure 4: Cross-sectional results (3)

Note: Figure 4 shows a box plot by one-digit SIC industry code of the residuals of a regression of relative price informativeness on year fixed effects. The solid middle line represents the median. The top and bottom of the box represent the 75th and 25th percentiles. The whiskers extend up to 1.5 times the interquartile range.

Price Informativeness over Time We also study how the distribution of stock-specific price informativeness evolves over time.[9] Table 1 includes a large amount of information about the time evolution of the distribution of informativeness. To better illustrate the results, we show the behavior of the median, mean, and standard deviation of the cross-sectional distribution of informativeness between 1980 and 2017 graphically in Figure 5.

We find that both the median and the mean of the distribution of informativeness feature increasing trends starting in the mid-1980s. The median moves from roughly 1.5% to 2% between 1986 and 2017, while the mean moves from roughly 3.5% in 1986 to roughly 5% by 2017. The large and dispersed estimates of mean and median informativeness before 1985 are due to smaller sample sizes in that period. For that reason, we emphasize the steadily increasing trend that starts in the mid-1980s. We also find that the standard deviation of informativeness has a positive long-run trend in our sample. In this case, there is a large spike in the cross-sectional standard deviation of informativeness around the global financial crisis of 2008 — other measures of dispersion have a similar behavior.

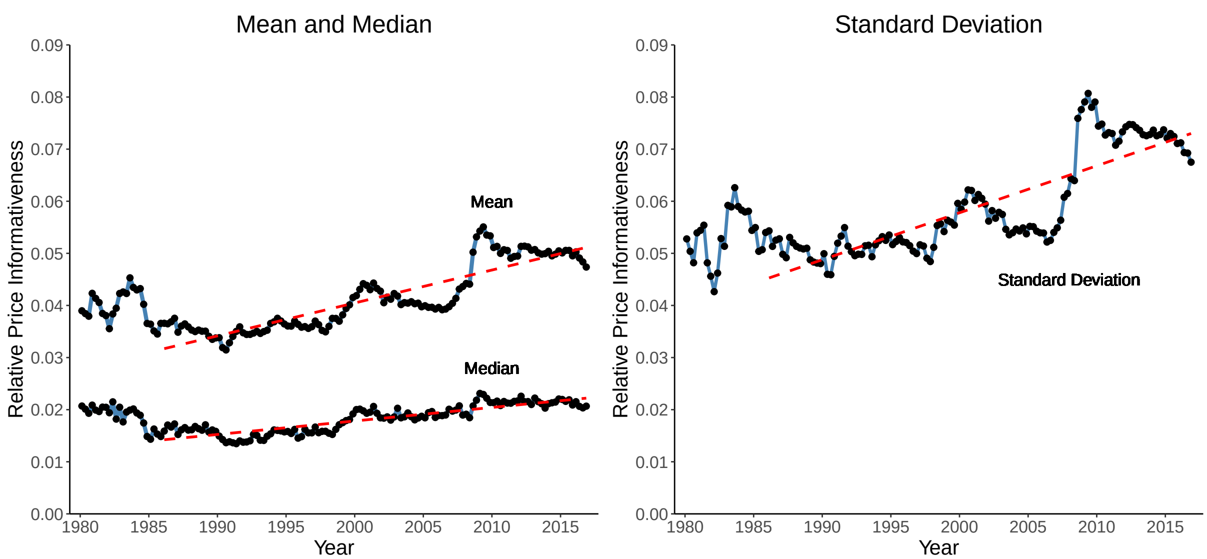

Figure 5: Price informativeness over time

Note: The left panel in Figure 5 shows the time-series evolution of the cross-sectional mean and median relative price informativeness. The right panel in Figure 5 shows the time-series evolution of the cross-sectional standard deviation of price informativeness. The red dashed lines show linear trends starting in 1986. In both panels, the dots correspond to the average within a quarter of the price informativeness measures computed using quarterly data.

[9]To keep the paper focused, we exclusively study the behavior of the panel of stock-specific price informativeness measures. There is scope to apply our approach to aggregate data in order to generate a time-series of aggregate price informativeness. There is also scope to explore the time series evolution of informativeness after grouping stocks by characteristics.

6 Economic Interpretation of Empirical Findings

Finally, we interpret the empirical findings presented in Section 5 through the lens of the general framework developed in Section 2 and the structural models developed in Section 3. If one were merely interested in knowing the precision of the signal contained in asset prices about future payoffs, our empirical results directly conclude that such signal is more precise for large, high turnover, and high institutional ownership stocks, and has become more precise on average over the last few decades.

However, one may be interested in translating these empirical patterns of informativeness to particular elements of a model. Within the general framework developed in Section 2, we can combine Equations (4) and (5) to express relative price informativeness as

$$\tau_\pi^R = \frac{1}{1 + \left(\frac{\phi_n}{\phi_1}\right)^2 \frac{\tau_u}{\tau_{\Delta n}}}. \tag{22}$$

From Equation (22), we can conclude that stocks with high informativeness are those with a high value of ϕ_1 (sensitivity of the asset price to the future payoff) relative to ϕ_n (sensitivity of the asset price to its non-payoff relevant component) and/or a high value of τ_u^{-1} (variance of the innovation to the payoff) relative to $\tau_{\Delta n}^{-1}$ (variance of the non-payoff relevant component, i.e., noise). Therefore, our empirical results imply that the (either of the) ratios $\frac{\phi_1}{\phi_n}$ and $\frac{\tau_{\Delta n}^{-1}}{\tau_u^{-1}}$ must be higher for large, high turnover, and high institutional ownership stocks, and that (either of) such ratios must have increased on average over the last few decades. Equation (22) clearly highlights that price informativeness captures the signal-to-noise ratio in asset prices, but not the sources of noise or information independently.

The models developed in Section 3 allow us to go one step further by relating our empirical findings on informativeness to deeper primitives. In all three models, the ratio $\frac{\phi_1}{\phi_n}$ corresponds to $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$, which denotes the ratio of the aggregate demand sensitivities to information and noise, respectively. Consequently, across all three models, higher price informativeness can be interpreted as either a higher $\frac{\overline{\alpha_s}}{\overline{\alpha_n}}$ and/or a higher $\frac{\tau_{\Delta n}}{\tau_u}$. While τ_u is a primitive in all three models, $\overline{\alpha_s}$, $\overline{\alpha_n}$, and, in some cases, $\tau_{\Delta n}$, are equilibrium objects, as we explain below.

The first model considered in Section 3, in which noise arises from investors' sentiment, provides the clearest connection between relative price informativeness and model primitives in the context of a fully structural model. In this model, the aggregate demand sensitivity to information relative to noise is exactly given by the ratio of the precision of investors' private signals (τ_s) about the future payoff relative to the precision of the innovation (τ_u), that is, $\frac{\overline{\alpha_s}}{\overline{\alpha_n}} = \frac{\tau_s}{\tau_u}$. In this model, the noise embedded in the price is only coming from the investors' sentiment and $\tau_{\Delta n}$ is also a primitive of the model. Therefore, price informativeness can be expressed as the following combination of primitives:

$$\tau_\pi^R = \frac{1}{1 + \left(\frac{\tau_u}{\tau_s}\right)^2 \frac{\tau_u}{\tau_{\Delta n}}}. \tag{23}$$

Equation (23) implies that price informativeness is increasing in the precision of investors' private signals (τ_s), decreasing in the volatility of the payoff innovation (τ_u^{-1}), and decreasing in the volatility of aggregate noise ($\tau_{\Delta n}^{-1}$). Through the lens of this model, one interpretation of our empirical results is that investors have more precise private information about stocks with higher market capitalization and high turnover. It is conceivable that investors acquire more private information about stocks with higher market capitalization and high turnover because they can benefit from such information at a larger scale. However, this conclusion is not obvious, since one may conjecture that larger firms attract the attention of more unsophisticated traders, which would make the prices of those stocks noisy and uninformative, or that high turnover stocks feature a large number of noise traders, thus engendering low price informativeness. Similarly, our time series empirical findings are consistent with an increase in the average precision of private information relative to noise over the last few decades. Similar arguments can be given for the other characteristics, e.g., value, institutional ownership, idiosyncratic volatility.

In the second model considered in Section 3, which features a representative agent, the relative aggregate demand sensitivity to noise and information is also given $\frac{\overline{\alpha_s}}{\overline{\alpha_n}} = \frac{\tau_s}{\tau_u}$, but in this case the precision of the noise embedded in the price $\tau_{\Delta n}$ is endogenous, which makes the connection between informativeness and primitives less direct. As in the model with sentiment, we show that price informativeness is increasing in the precision of investors' private signals (τ_s) and decreasing in the volatility of the innovation (τ_u^{-1}). All else equal, it is also the case that price informativeness is decreasing in the volatility of aggregate noise ($\tau_{\Delta n}^{-1}$). Therefore, the interpretation of the results is almost identical to the interpretation of the model with sentiment as noise. We should note that if we had allowed for time varying risk aversion — as discussed on page 17 and shown in the Appendix — the movements in discount rates could be interpreted as changes in the (endogenous) volatility of $\tau_{\Delta n}$.

Finally, the model with informed, uninformed, and noise traders delivers similar implications to the model with sentiment. In this last model, price informativeness is increasing in the fraction of informed investors (λ) and decreasing in the volatility of noise trading (τ_N). Through the lens of this model, our empirical results can be interpreted as concluding that large, high turnover, and high institutional ownership stocks feature a higher share of informed investors relative to noise traders, and that the share of informed investors has increased over the last few decades in relation to the volatility of noise trading.

7 Conclusion

We have shown that the outcomes of regressions of changes in asset prices on changes in asset payoffs can be combined to recover exact measures of price informativeness within a large class of linear/linearized models. Empirically, we compute a panel of stock-specific measures of price informativeness and find that the median and mean levels of price informativeness fluctuates around levels of 2% and 4%, respectively. These values, which can be interpreted as the weight that an external observer who only learns from the price puts on the price signal when forming a posterior belief about future payoffs, measure the precision of the public signal contained in prices about future payoffs. Cross-sectionally, we find that price informativeness is higher for stocks with higher market capitalization, that trade more frequently, and that have a higher institutional share. Over time, we find that mean and median price informativeness have steadily increased since the mid-1980s. We discuss alternative scenarios that may generate these patterns.

Our identification results open the door to answering a broad set of questions. Empirically, there is scope to explore further the relation between price informativeness measures and other characteristics in the cross section or over time. It also seems worthwhile to document and explain the behavior of price informativeness in different contexts, perhaps internationally or in different markets. Theoretically, our results can be used to discipline theories of information and learning in financial markets.[10] There is scope to export our approach to identification to other environments in which structurally recovering the informativeness of endogenous signals is important, for instance, auctions, macroeconomic environments, or labor markets. Finally, we hope that our approach encourages further research on formally identifying price informativeness in richer models, such as those with feedback effects or significant non-linearities.

[10]For instance, our results open the door to testing the results in Kacperczyk, Nosal and Sundaresan (2020), who theoretically characterize the relation between institutional ownership and price informativeness, or Dávila and Parlatore (2020), who theoretically characterize the relation between trading costs and price informativeness.

References

Albagli, Elias, Christian Hellwig, and Aleh Tsyvinski. 2015. "A Theory of Asset Prices based on Heterogeneous Information." *NBER Working Paper.*

Angeletos, George-Marios, and Alessandro Pavan. 2007. "Efficient Use of Information and Social Value of Information." *Econometrica*, 75(4): 1103–1142.

Angeletos, George-Marios, and Alessandro Pavan. 2009. "Policy with Dispersed Information." *Journal of the European Economic Association*, 7(1): 11–60.

Bai, Jennie, Thomas Philippon, and Alexi Savov. 2016. "Have Financial Markets Become More Informative?" *Journal of Financial Economics*, 122(3): 625–654.

Bali, Turan G., Robert F. Engle, and Scott Murray. 2016. *Empirical Asset Pricing: The Cross Section of Stock Returns.* John Wiley & Sons.

Blackwell, David. 1953. "Equivalent Comparisons of Experiments." *The Annals of Mathematical Statistics*, 24(2): 265–272.

Bond, Philip, Alex Edmans, and Itay Goldstein. 2012. "The Real Effects of Financial Markets." *Annual Review of Financial Economics*, 4(1): 339–360.

Campbell, John Y. 2017. *Financial Decisions and Markets: A Course in Asset Pricing.* Princeton University Press.

Campbell, John Y., and Robert J. Shiller. 1988. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *The Review of Financial Studies*, 1(3): 195–228.

Chen, Qi, Itay Goldstein, and Wei Jiang. 2006. "Price Informativeness and Investment Sensitivity to Stock Price." *The Review of Financial Studies*, 20(3): 619–650.

Cochrane, John. 2005. *Asset Pricing: Revised.* Princeton University Press.

Dávila, Eduardo, and Cecilia Parlatore. 2020. "Trading Costs and Informational Efficiency." *Journal of Finance (Forthcoming).*

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J Waldmann. 1990. "Noise Trader Risk in Financial Markets." *Journal of Political Economy*, 98(4): 703–738.

Diamond, Douglas W., and Robert E. Verrecchia. 1981. "Information Aggregation in a Noisy Rational Expectations Economy." *Journal of Financial Economics*, 9(3): 221–235.

Durnev, Art, Randall Morck, and Bernard Yeung. 2004. "Value-Enhancing Capital Budgeting and Firm-Specific Stock Return Variation." *Journal of Finance*, 59(1): 65–105.

Easley, David, Maureen O'Hara, and Joseph Paperman. 1998. "Financial Analysts and Information-Based Trade." *Journal of Financial Markets*, 1: 175–201.

Farboodi, Maryam, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran.

2020. “Where Has All the Data Gone?” *NBER Working Paper.*

Grossman, Sandford J., and Joseph E. Stiglitz. 1980. “On the Impossibility of Informationally Efficient Markets.” *American Economic Review*, 393–408.

Hayek, F.A. 1945. “The Use of Knowledge in Society.” *American Economic Review*, 35(4): 519–530.

Hellwig, Martin F. 1980. “On the Aggregation of Information in Competitive Markets.” *Journal of Economic Theory*, 22(3): 477–498.

Hou, Kewei, and Tobias J. Moskowitz. 2005. “Market Frictions, Price Delay, and the Cross-Section of Expected Returns.” *The Review of Financial Studies*, 18: 981–1020.

Hou, Kewei, Lin Peng, and Wei Xiong. 2013. “Is R^2 a Measure of Market Inefficiency?” *Working Paper.*

Kacperczyk, Marcin, Jaromir Nosal, and Savitar Sundaresan. 2020. “Market Power and Price Informativeness.” *Working Paper.*

Kacperczyk, Marcin, Savitar Sundaresan, and Tianyu Wang. 2020. “Do Foreign Institutional Investors Improve Price Efficiency?” *Working Paper.*

Morck, Randall, Bernard Yeung, and Wayne Yu. 2000. “The Information Content of Stock Markets: Why do Emerging Markets have Synchronous Stock Price Movements?” *Journal of Financial Economics*, 58(1-2): 215–260.

Roll, Richard. 1988. “R2.” *The Journal of Finance*, 43(3): 541–566.

Stambaugh, Robert F. 1999. “Predictive Regressions.” *Journal of Financial Economics*, 54(3): 375–421.

Veldkamp, Laura. 2011. *Information Choice in Macroeconomics and Finance.* Princeton University Press.

Vives, Xavier. 2008. *Information and Learning in Markets: the Impact of Market Microstructure.* Princeton University Press.

Weller, Brian M. 2018. “Does Algorithmic Trading Reduce Information Acquisition?” *The Review of Financial Studies*, 31(6): 2184–2226.

Wurgler, Jeffrey. 2000. “Financial Markets and the Allocation of Capital.” *Journal of Financial Economics*, 58(1-2): 187–214.

Appendix

A Proofs and Derivations

A.1 Section 2: Proofs and Derivations

Proof of Proposition 1. (Identifying price informativeness) For completeness, we reproduce here Regressions R1 and R2:

$$\Delta p_t = \overline{\beta} + \beta_0 \Delta x_t + \beta_1 \Delta x_{t+1} + e_t \tag{R1}$$

$$\Delta p_t = \overline{\zeta} + \zeta_0 \Delta x_t + e_t^{\zeta}. \tag{R2}$$

Note that the R-squareds of both regressions can be expressed as follows

$$R^2_{\Delta x, \Delta x'} = 1 - \frac{\mathbb{V}\mathrm{ar}\left(e_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta p_t\right)} \quad \text{and} \quad R^2_{\Delta x} = \frac{\mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta x_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta p_t\right)}.$$

After substituting Equation (1) in Equation (2), the following relation holds

$$\Delta p_t = \overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_n \mu_{\Delta n} + \left(\phi_0 + \rho \phi_1\right) \Delta x_t + \phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}. \tag{24}$$

By comparing Regression R2 with the structural Equation (24), it follows that $\overline{\zeta} = \overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_n \mu_{\Delta n}$, $\zeta_0 = \phi_0 + \rho \phi_1$, and $\varepsilon_t^{\zeta} = \phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}$. By comparing Regression R1 with the structural Equation (2), it follows that $\overline{\beta} = \overline{\phi}$, $\beta_0 = \phi_0$, $\beta_1 = \phi_1$, and $e_t = \phi_n \Delta n_t$.

From Equation (24), the following variance decomposition must hold

$$\mathbb{V}\mathrm{ar}\left(\Delta p_t\right) = \mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta x_t\right) + \mathbb{V}\mathrm{ar}\left(\phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}\right) = \mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta x_t\right) + \left(\phi_1\right)^2 \mathbb{V}\mathrm{ar}\left(u_t\right) + \mathbb{V}\mathrm{ar}\left(e_t\right),$$

which can be rearranged to express $\frac{\tau_\pi}{\tau_u}$ as follows

$$1 = \underbrace{\frac{\mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta x_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta p_t\right)}}_{R^2_{\Delta x}} + \underbrace{\frac{\mathbb{V}\mathrm{ar}\left(e_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta p_t\right)}}_{1 - R^2_{\Delta x, \Delta x'}} \left(\underbrace{\frac{\left(\phi_1\right)^2}{\mathbb{V}\mathrm{ar}\left(e_t\right)} \mathbb{V}\mathrm{ar}\left(u_t\right)}_{\frac{\tau_\pi}{\tau_u}} + 1 \right) \Rightarrow \frac{\tau_\pi}{\tau_u} = \frac{R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x, \Delta x'}}.$$

Therefore, relative price informativeness can be written as $\tau_\pi^R = \frac{\tau_\pi}{\tau_\pi + \tau_u} = \frac{1}{1 + \frac{1}{\frac{\tau_\pi}{\tau_u}}} = \frac{R^2_{\Delta x, \Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}}$.

Given the assumptions on u_t and Δn_t, it is straightforward to show that the OLS estimates of Regressions R1 and R2 are consistent, which implies that price informativeness can be consistently estimated as $\widehat{\tau_\pi^R} = \frac{\widehat{R^2_{\Delta x, \Delta x'}} - \widehat{R^2_{\Delta x}}}{1 - \widehat{R^2_{\Delta x}}}$. Formally, $\operatorname{plim}\left(\widehat{\tau_\pi^R}\right) = \operatorname{plim}\left(\frac{\widehat{R^2_{\Delta x, \Delta x'}} - \widehat{R^2_{\Delta x}}}{1 - \widehat{R^2_{\Delta x}}}\right) = \tau_\pi^R$.

A.2 Section 3: Proofs and Derivations

Portfolio Demand Approximation The optimality condition of an investor who maximizes Equation (10) subject to the wealth accumulation constraint in Equation (11) is given by

$$\mathbb{E}\left[U_i'\left(w_1^i\right)\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\middle|\mathcal{I}_t^i\right] = 0. \tag{25}$$

We approximate an investor's first-order condition in three steps.

First, we take a first-order Taylor expansion of an investor's future marginal utility $U'\left(w_1^i\right)$ around the current date t wealth level w_0^i. Formally, we approximate $U'\left(w_1^i\right)$ as follows

$$U'\left(w_1^i\right) \approx U'\left(w_0^i\right) + U''\left(w_0^i\right)\Delta w_1^i,$$

which allows us to express Equation (25) as

$$U'\left(w_0^i\right)\mathbb{E}_i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right] + U''\left(w_0^i\right)w_0^i\mathbb{E}_i\left[\left(R^f - 1 + \theta_t^i\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\right)\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\right] \approx 0.$$

Second, we impose that terms that involve the product of two or more net interest rates are negligible. In continuous time, these terms would be of order $(dt)^2$. Formally, it follows that

$$\left(R^f - 1\right)\mathbb{E}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right] \approx 0 \quad \text{and} \quad \left(\mathbb{E}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right]\right)^2 \approx 0,$$

which allows us to express Equation (25) as

$$U'\left(w_0^i\right)\mathbb{E}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right] + U''\left(w_0^i\right)w_0^i\theta_t^i\mathbb{V}\mathrm{ar}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t}\right] \approx 0.$$

Therefore, we can express an investor's risky portfolio share θ_t^i as

$$\theta_t^i \approx \frac{1}{\gamma^i}\frac{\mathbb{E}_t^i\left[\frac{X_{t+1}+P_{t+1}}{P_t} - R^f\right]}{\mathbb{V}\mathrm{ar}_t^i\left[\frac{X_{t+1}+P_{t+1}}{P_t}\right]}, \tag{26}$$

where $\gamma^i \equiv -\frac{w_0^i U_i''\left(w_0^i\right)}{U_i'\left(w_0^i\right)}$ denotes the coefficient of relative risk aversion. These coefficients are time invariant since we have assumed that the distribution of investor types is time invariant and the wealth distribution across time and investor type is i.i.d.

Third, as in Campbell and Shiller (1988), we take a log-linear approximation of returns around a predetermined dividend-price ratio. Formally, note that

$$\ln\left(\frac{X_{t+1}+P_{t+1}}{P_t}\right)=\ln\left(1+e^{p_{t+1}-x_{t+1}}\right)+\Delta x_{t+1}-\left(p_t-x_t\right),$$

where $y_t=\ln Y_t$ for any given variable Y_t. Following Campbell and Shiller (1988), we approximate the first term around a point $PX=e^{p-x}$, to find that

$$\begin{aligned}\ln\left(1+e^{p_{t+1}-x_{t+1}}\right)&\approx\ln\left(1+PX\right)+\frac{PX}{PX+1}\left(p_{t+1}-x_{t+1}-(p-x)\right).\\&=k_0+k_1\left(p_{t+1}-x_{t+1}\right),\end{aligned}$$

where $k_1\equiv\frac{PX}{PX+1}$ and $k_0\equiv\ln\left(1+PX\right)-k_1\left(p-x\right)$.

Therefore, starting from Equation (26), we can express an investor's risky portfolio share θ_t^i as

$$\theta_t^i\approx\frac{1}{\gamma^i}\frac{k_0+k_1\mathbb{E}_t^i\left[p_{t+1}-x_{t+1}\right]+\mathbb{E}_t^i\left[\Delta x_{t+1}\right]-\left(p_t-x_t\right)-r^f}{\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-x_{t+1}\right)+\Delta x_{t+1}\right]},$$

where we define $r^f\equiv\ln R^f$ and we used that $e^y\approx1+y$.

Forming expectations In order to characterize the equilibrium it is necessary to characterize investors' expectations. We conjecture and subsequently verify that $k_1\mathbb{E}_t^i\left[p_{t+1}-x_{t+1}\right]+\mathbb{E}_t^i\left[\Delta x_{t+1}\right]$ is linear in s_t^i, $\overline{n}_t^i$, and x_t and that $\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-x_{t+1}\right)+\Delta x_{t+1}\right]$ is a constant. Under this conjecture, θ_t^i is a linear function of s_t^i, x_t, and $\overline{n}_t^i$, given by

$$\theta_t^i\approx\alpha_x^ix_t+\alpha_s^is_t^i+\alpha_n^i\overline{n}_t^i-\alpha_p^ip_t+\psi^i.$$

These coefficients are time invariant since we have assumed that the distribution of investor types is time invariant and the wealth distribution across time and investor type is i.i.d.

This expression and the market clearing condition $\int\theta_t^iw_0^idi=Q$ imply that

$$p_t=\frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_t+\frac{\overline{\alpha_s}}{\overline{\alpha_p}}u_t+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_t+\frac{\overline{\psi}}{\overline{\alpha_p}},$$

where $\overline{\alpha_h}\equiv\int\alpha_h^iw_0^idi$ for $h=\{x,s,n,p\}$ and $\overline{\psi}\equiv\int\psi^iw_0^idi-Q$. As in Vives (2008), we make use of the Strong Law of Large Numbers, since the sequence of independent random variables $\left\{\alpha_s^iw_0^i\varepsilon_{st}^i,\alpha_n^iw_0^i\varepsilon_{\overline{n}t}^i\right\}$ has uniformly bounded variance and mean zero. This expression can also be written as

$$p_t=\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)x_t+\frac{\overline{\alpha_s}}{\overline{\alpha_p}}x_{t+1}+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_t+\frac{\overline{\psi}}{\overline{\alpha_p}}-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\mu_{\Delta x}.\tag{27}$$

Investors in the model learn from the price. The information contained in the price for an investor in the model is

$$\hat{\pi}_t = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p_t - \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_{\Delta n} + \frac{\overline{\psi}}{\overline{\alpha_p}}\right)\right),$$

which has a precision $\tau_{\hat{\pi}} \equiv \mathbb{V}\mathrm{ar}\left[\hat{\pi}_t | u_t, \{x_s\}_{s\leq t}, p_{t-1}\right]^{-1} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \tau_{\Delta n}$. Note that we denote by π_t the unbiased signal of u_t contained in the change in log prices Δp_t and by $\hat{\pi}_t$ the unbiased signal about u_t contained in the log price p_t.

Given the information set of the investor, $\mathbb{V}\mathrm{ar}\left[n_t | u_t, \{x_s\}_{s\leq t}, p_{t-1}\right] = \mathbb{V}\mathrm{ar}\left[\Delta n_t | u_t, \{x_s\}_{s\leq t}, p_{t-1}\right]$. Then,

$$\mathbb{E}_t^i\left[u_t\right] = \mathbb{E}\left[u_t | \mathcal{I}_t^i\right] = \frac{\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_{\hat{\pi}}\hat{\pi}_t}{\tau_s + \tau_u + \tau_{\hat{\pi}}} = \frac{\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_{\hat{\pi}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p_t - \frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_t - \frac{\overline{\alpha_n}}{\overline{\alpha_s}}\mu_{\Delta n} - \frac{\overline{\psi}}{\overline{\alpha_p}}\right)}{\tau_s + \tau_u + \tau_{\hat{\pi}}}$$

and $\mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] = (\tau_s + \tau_u + \tau_{\hat{\pi}})^{-1}$, where $\mathcal{I}_t^i = \left\{s_t^i, \overline{n}_t^i, \{p_s\}_{s\leq t}, \{x_s\}_{s\leq t}\right\}$. Note that these two expressions imply that our conjecture about θ_t^i is satisfied. To see this, note that

$$\begin{aligned}
k_1\mathbb{E}_t^i\left[p_{t+1} - x_{t+1}\right] + \mathbb{E}_t^i\left[\Delta x_{t+1}\right] &= k_1\mathbb{E}_t^i\left[\frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_{t+1} + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}u_{t+1} + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_{t+1} + \frac{\overline{\psi}}{\overline{\alpha_p}} - x_{t+1}\right] + \mu_{\Delta x} + \mathbb{E}_t^i\left[u_t\right] \\
&= k_1\left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1}\right)\left(\mu_{\Delta x} + \mathbb{E}_t^i\left[u_t\right]\right) + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mathbb{E}_t^i\left[n_t\right] + \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1\right)x_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_{\Delta n} + \frac{\overline{\psi}}{\overline{\alpha_p}}\right),
\end{aligned}$$

where we used that $\mathbb{E}_t^i\left[u_{t+1}\right] = 0$, that $\mathbb{E}_t^i\left[\varepsilon_{t+1}^{\Delta n}\right] = 0$, and that $\mathbb{E}_t^i\left[n_t\right]$ is linear in p_t and x_t. To see this, first note that n_{t-1} is known at time t since the information set of the investor includes all past prices and payoffs. Therefore, the prior mean of investor i about n_t is $\mu_{\Delta n} + n_{t-1}$. Second, the price p_t contains information about n_t. The unbiased signal about n_t contained in the price p_t is given by

$$\pi_t^n \equiv \frac{\overline{\alpha_p}}{\overline{\alpha_n}}\left(p_t - \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_t + \frac{\overline{\psi}}{\overline{\alpha_p}}\right)\right) = n_t + \frac{\overline{\alpha_s}}{\overline{\alpha_n}}u_t,$$

and its precision is given by $\tau_{\pi^n} \equiv \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\right)^2 \tau_u$. Then,

$$\mathbb{E}_t^i\left[n_t\right] = \frac{\tau_{\Delta n}\left(\mu_{\Delta n} + n_{t-1}\right) + \tau_{\pi^n}\frac{\overline{\alpha_p}}{\overline{\alpha_n}}\left(p_t - \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_t + \frac{\overline{\psi}}{\overline{\alpha_p}}\right)\right)}{\tau_{\Delta n} + \tau_{\pi^n}}$$

and $\mathbb{V}\mathrm{ar}_t^i\left[n_t\right] = (\tau_{\Delta n} + \tau_{\pi^n})^{-1}$. Moreover,

$$\mathbb{V}\mathrm{ar}_t^i\left[k_1\left(p_{t+1} - x_{t+1}\right) + \Delta x_{t+1}\right] = k_1^2\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1}\right)^2 (\tau_s + \tau_u + \tau_{\hat{\pi}})^{-1} + k_1^2\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \tau_u^{-1} + k_1^2\left(\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\right)^2 \tau_{\Delta n}^{-1}.$$

Using these expressions in the first-order condition and matching coefficients gives

$$\alpha_x^i = \frac{1}{\kappa_i} k_1 \left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1} \right) \left(1 - \frac{\tau_{\hat{\pi}} \frac{\overline{\alpha_x}}{\overline{\alpha_s}}}{\tau_s + \tau_u + \tau_{\hat{\pi}}} \right) - \frac{\tau_{\pi^n} \frac{\overline{\alpha_x}}{\overline{\alpha_p}}}{\tau_{\Delta n} + \tau_{\pi^n}} \right) \tag{28}$$

$$\alpha_s^i = \frac{1}{\kappa_i} k_1 \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1} \right) \frac{\tau_s}{\tau_s + \tau_u + \tau_{\hat{\pi}}} \tag{29}$$

$$\alpha_n^i = \frac{1}{\kappa_i} k_1 \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1} \right) \frac{\tau_u}{\tau_s + \tau_u + \tau_{\hat{\pi}}} \tag{30}$$

$$\alpha_p^i = \frac{1}{\kappa_i} \left(1 - k_1 \left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1} \right) \frac{\tau_{\hat{\pi}} \frac{\overline{\alpha_p}}{\overline{\alpha_s}}}{\tau_s + \tau_u + \tau_{\hat{\pi}}} - \frac{\tau_{\pi^n}}{\tau_{\Delta n} + \tau_{\pi^n}} \right) \right) \tag{31}$$

$$\psi^i = \frac{1}{\kappa_i} \left(\begin{array}{c} k_0 + k_1 \left(-\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1} \right) \left(\frac{\tau_{\pi} \frac{\overline{\alpha_p}}{\overline{\alpha_s}}}{\tau_s + \tau_u + \tau_{\hat{\pi}}} - \mu_{\Delta x} \right) + 1 \right) \left(\frac{\overline{\alpha_n}}{\overline{\alpha_p}} \mu_{\Delta n} + \frac{\overline{\psi}}{\overline{\alpha_p}} \right) \\ + k_1 \frac{\overline{\alpha_n}}{\overline{\alpha_p}} \frac{\tau_{\Delta n}(\mu_{\Delta n} + n_{t-1}) + \tau_{\pi^n} \frac{\overline{\alpha_p}}{\overline{\alpha_n}} \frac{\overline{\psi}}{\overline{\alpha_p}}}{\tau_{\Delta n} + \tau_{\pi^n}} - r^f \end{array} \right), \tag{32}$$

where $\kappa_i \equiv \gamma^i \mathbb{V}\mathrm{ar}_t^i \left[k_1 \left(p_{t+1} - x_{t+1} \right) + \Delta x_{t+1} \right]$.

Proof of Lemma 1 Iterating forward Equation (27) and taking differences, we find that

$$\Delta p_t = \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \right) \Delta x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \Delta x_{t+1} + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} \Delta n_t.$$

This maps to the price process in the general framework by setting $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\phi_1 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, and $\phi_n = \frac{\overline{\alpha_n}}{\overline{\alpha_p}}$.

Proof of Lemma 2 When investors are identical, the noise in their signal does not disappear from the price, and the price in (27) becomes

$$p_t = \left(\frac{\alpha_x}{\alpha_p} - \frac{\alpha_s}{\alpha_p} \right) x_t + \frac{\alpha_s}{\alpha_p} \left(x_{t+1} + \varepsilon_{st} \right) + \frac{\alpha_n}{\alpha_p} n_t + \frac{\psi - Q}{\alpha_p} - \frac{\alpha_s}{\alpha_p} \mu_{\Delta x},$$

where the demand coefficients are given by the system in Equations (28) through (32). Iterating backwards this price and taking differences we have

$$\Delta p_t = \left(\frac{\alpha_x}{\alpha_p} - \frac{\alpha_s}{\alpha_p} \right) \Delta x_t + \frac{\alpha_s}{\alpha_p} \Delta x_{t+1} + \frac{\alpha_n}{\alpha_p} \left(\Delta n_t + \frac{\alpha_s}{\alpha_n} \Delta \varepsilon_{st} \right).$$

Setting $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\alpha_p} - \frac{\overline{\alpha_s}}{\alpha_p}$, $\phi_1 = \frac{\overline{\alpha_s}}{\alpha_p}$, and $\phi_n = \frac{\overline{\alpha_n}}{\alpha_p}$ and where $\Delta \hat{n}_t \equiv \Delta \overline{n}_t + \frac{\alpha_s}{\alpha_n} \Delta \varepsilon_{st}$ maps to the process in the general framework.

Time-varying risk-aversion interpretation Note that from Equations (28) through (32) one can see that $\frac{\alpha_{ht}}{\alpha_{pt}} = \frac{\alpha_{ht-1}}{\alpha_{pt-1}} = \frac{\alpha_h}{\alpha_p}$ for all t and $h = x, s, n$, and that

$$\frac{\psi_t - Q}{\alpha_{pt}} - \frac{\psi_{t-1} - Q}{\alpha_{pt-1}} = \Delta\gamma_t \frac{\mathbb{V}\text{ar}\left[k_1 \left(p_{t+1} - x_{t+1}\right) + \Delta x_{t+1}\right]}{\left(1 - k_1 \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - 1 + \frac{1}{k_1}\right) \frac{\tau_{\hat{\pi}} \frac{\overline{\alpha_p}}{\overline{\alpha_s}}}{\tau_s + \tau_u + \tau_{\hat{\pi}}}\right)} Q = \frac{\Delta\gamma_t}{\gamma_t \alpha_{pt}} Q.$$

In this case, the price process is

$$\Delta p_t = \left(\frac{\alpha_x}{\alpha_p} - \frac{\alpha_s}{\alpha_p}\right) \Delta x_t + \frac{\alpha_s}{\alpha_p} \Delta x_{t+1} + \frac{\alpha_n}{\alpha_p} \left(\Delta n_t + \frac{\alpha_s}{\alpha_n} \Delta\varepsilon_{st} + \frac{\Delta\gamma_t}{\gamma_t \alpha_{nt}} Q\right),$$

and setting $\overline{\phi} = 0$, $\phi_0 = \frac{\alpha_x}{\alpha_p} - \frac{\alpha_s}{\alpha_p}$, $\phi_1 = \frac{\alpha_s}{\alpha_p}$, and $\phi_n = \frac{\alpha_n}{\alpha_p}$, where $\Delta\hat{n}_t \equiv \Delta\overline{n}_t + \frac{\alpha_s}{\alpha_n} \Delta\varepsilon_{st} + \frac{\Delta\gamma_t}{\gamma_t \alpha_{nt}} Q$, maps to the process in the general framework. In this case, the noise in the price can come from time-varying risk aversion.

Proof of Lemma 3 The case in which there are informed and uninformed investors and noise traders is a special case of the model in Section 3.1 with three types of agents. In that case, the demand for informed and uninformed investors is respectively given by

$$\begin{aligned}
\theta_t^I &\approx \alpha_x^I x_t + \alpha_s^I u_t - \alpha_p^I p_t + \psi^I \\
\theta_t^U &\approx \alpha_x^U x_t + \alpha_n^U \overline{n}_t^U - \alpha_p^U p_t + \psi^U,
\end{aligned}$$

and the demand of noise traders is given by δ. Market clearing and the SLLN imply that the equilibrium price in Equation (12).

Taking first differences for this price process we have

$$\Delta p_t \approx \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta\tilde{n}_t,$$

where the coefficients $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\phi_1 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, and $\phi_n = \frac{\overline{\alpha_n}}{\overline{\alpha_p}}$ are equilibrium outcomes and $\Delta\tilde{n}_t \equiv \Delta\overline{n}_t + \frac{1}{\overline{\alpha_n}} \Delta\delta_t$, which proves our claim.

A.3 Section 4: Proofs and Derivations

Proof of Proposition 2 (Identifying price informativeness when payoff and noise are correlated) When $\Delta n_t = \mu_{\Delta n} + \omega u_t + \varepsilon_t^{\Delta n}$, the price process in Equation (13) can be written as

$$\Delta p_t = \overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_n \mu_{\Delta n} + \left(\phi_0 + \rho\phi_1\right) \Delta x_t + \left(\phi_1 + \phi_n \omega\right) u_t + \phi_n \varepsilon_t^{\Delta n}. \tag{33}$$

Hence, the unbiased signal about the innovation u_t contained in the price change is

$$\pi_t = \frac{1}{(\phi_1 + \phi_n\omega)}\left(\Delta p_t - \left(\overline{\phi} + \phi_1\mu_{\Delta x} + \phi_n\mu_{\Delta n} + (\phi_0 + \rho\phi_1)\,\Delta x_t\right)\right) = u_t + \frac{\phi_n}{(\phi_1 + \phi_n\omega)}\varepsilon_t^{\Delta n},$$

and absolute price informativeness is given by $\tau_\pi \equiv \mathbb{V}\mathrm{ar}\left[\pi_t|\, u_t, \Delta x_t\right]^{-1} = \left(\frac{\phi_1+\phi_n\omega}{\phi_n}\right)^2 \tau_{\Delta n}$. The price process can also be written as a function of Δx_t and Δx_{t+1} as follows

$$\Delta p_t = \overline{\phi} - \phi_n\omega\mu_{\Delta x} + \phi_n\mu_{\Delta n} + (\phi_0 - \phi_n\omega\rho)\,\Delta x_t + (\phi_1 + \phi_n\omega)\,\Delta x_{t+1} + \phi_n\varepsilon_t^{\Delta n}.$$

Comparing this equation with Regression R1 it is easy to see that $\overline{\beta} = \overline{\phi} - \phi_n\omega\mu_{\Delta x} + \phi_n\mu_{\Delta n}$, $\beta_0 = \phi_0 - \phi_n\omega\rho$, $\beta_1 = \phi_1 + \omega\phi_n$, and $e_t = \phi_n\varepsilon_t^{\Delta n}$ with $e_t \perp \Delta x_t, \Delta x_{t+1}$. Moreover, comparing Equation (33) with Regression R2, it follows that $\overline{\zeta} = \overline{\phi} + \phi_1\mu_{\Delta x} + \phi_n\mu_{\Delta n}$, $\zeta_0 = \phi_0$, and $e_t^\zeta = (\phi_1 + \phi_n\omega)\,u_t + \phi_n\varepsilon_t^{\Delta n}$ with $e_t^\zeta \perp \Delta x_t$. Then, we can recover relative price informativeness in the same way as in the baseline model, as follows

$$\tau_\pi^R = \frac{R^2_{\Delta x,\Delta x'} - R^2_{\Delta x}}{1 - R^2_{\Delta x}},$$

where the steps to recover absolute price informativeness are the same as in the baseline model.

Proof of Proposition 3 (Identifying price informativeness with public signals) When investors have access to public signals, the equilibrium price process is given by Equation (16).

a) If the public signals are not available to the external observer, then the model with public information can be cast in terms of having noise correlated with the payoff by replacing the noise term in Equation (2) by

$$\phi_n'\Delta n_t' \equiv \phi_\chi \cdot \Delta\chi_t + \phi_n\Delta n_t = \phi_n\mu_{\Delta n} + \phi_\chi \cdot \overline{\omega}u_t + \phi_\chi\Delta\overline{\varepsilon}_t^\chi + \phi_n\varepsilon_t^{\Delta n}.$$

In this case, Proposition 2 holds.

b) If the public signals are available to the external observer, one can extend the identification results as follows. Using that $\Delta x_{t+1} = \mu_{\Delta x} + \rho\Delta x_t + u_t$ and that $\Delta n_t = \mu_{\Delta n} + \varepsilon_t^{\Delta n}$, we can write the price process as

$$\Delta p_t = \overline{\phi} + \phi_1\mu_{\Delta x} + \phi_n\mu_{\Delta n} + (\phi_0 + \phi_1\rho)\,\Delta x_t + \phi_1 u_t + \phi_\chi \cdot \Delta\chi_t + \phi_n\varepsilon_t^{\Delta n}. \tag{34}$$

Hence, the unbiased signal contained in the price for an external observer who has access to the public signals is

$$\pi'_t \equiv \frac{1}{\phi_1}\left(\Delta p_t - \left(\overline{\phi} + \phi_1\mu_{\Delta x} + \phi_n\mu_{\Delta n} + (\phi_0 + \phi_1\rho)\,\Delta x_t + \phi_\chi \cdot \Delta\chi_t\right)\right) = u_t + \frac{\phi_n}{\phi_1}\varepsilon_t^{\Delta n},$$

and price informativeness is $\tau_{\pi'} \equiv \mathbb{V}\mathrm{ar}\left[\pi'_t\right| u_t, \Delta x_t, \Delta\chi_t]^{-1} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n}$. Comparing the expression for the equilibrium price in Equation (16) with Regression R1-PS, we have that $\beta'_1 = \phi_1$ and $\mathbb{V}\mathrm{ar}\left[e'_t\right] = \phi_n^2 \tau_{\Delta n}^{-1}$. Therefore, absolute price informativeness can be recovered as $\tau_{\pi'} = \frac{(\beta'_1)^2}{\mathbb{V}\mathrm{ar}[e'_t]}$. Moreover, comparing Regression R2-PS with the expression for the price process in Equation (34) we have that $e_t^{\zeta\prime} \equiv \phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}$. Taking the variance on both sides of Regression R2-PS gives

$$1 = \frac{\mathbb{V}\mathrm{ar}\left[\zeta'_0 \Delta x_t + \zeta'_1 \cdot \Delta\chi_t\right]}{\mathbb{V}\mathrm{ar}\left[p_t\right]} + \frac{\mathbb{V}\mathrm{ar}\left[e'_t\right]}{\mathbb{V}\mathrm{ar}\left[p_t\right]}\left(\frac{\mathbb{V}\mathrm{ar}\left[\phi_1 u_t\right]}{\mathbb{V}\mathrm{ar}\left[e'_t\right]} + 1\right).$$

Noting that $\frac{\mathbb{V}\mathrm{ar}[e'_t]}{\mathbb{V}\mathrm{ar}[p_t]} = 1 - R^2_{\Delta x, \Delta x', \Delta\chi}$ and $\frac{\mathbb{V}\mathrm{ar}[\zeta'_0 \Delta x_t + \zeta'_1 \cdot \Delta\chi_t]}{\mathbb{V}\mathrm{ar}[p_t]} = R^2_{\Delta x, \Delta\chi}$ implies

$$\tau_{\pi'}^R \equiv \frac{\tau_{\pi'}}{\tau_u + \tau_{\pi'}} = \frac{R^2_{\Delta x, \Delta x', \Delta\chi} - R^2_{\Delta x, \Delta\chi}}{1 - R^2_{\Delta x, \Delta\chi}},$$

where $R^2_{\Delta x, \Delta x', \Delta\chi}$ and $R^2_{\Delta x, \Delta\chi}$ are the R-squareds of Regression R1-PS and Regression R2-PS.

Proof of Proposition 4 *(*Identifying price informativeness about learnable component of payoffs) When there is an unlearnable component of payoffs, the portfolio demand of an investor i can be approximated by

$$\theta_t^i \approx \alpha_x^i x_t + \alpha_s^i s_t^i + \alpha_n^i \overline{n}_t^i - \alpha_p^i p_t + \psi^i,$$

where the coefficients α_x^i, α_s^i, α_n^i, and α_p^i are positive scalars that represent the individual demand sensitivities to the contemporary payoff, the private signal, the private trading needs, and the asset price, respectively, and ψ^i can be positive or negative and incorporates the risk premium.

The market clearing condition and this approximation imply an equilibrium (log) price

$$p_t \approx \frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} u_t^L + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_t + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where $\overline{\alpha_h} \equiv \int \alpha_h^i w_0^i di$ for $h = \{x, s, n, p\}$ and $\overline{\psi} \equiv \int \psi^i w_0^i di - Q$. Using that the payoff process is $\Delta x_{t+1} = \mu_{\Delta x} + u_t^U + u_t^L$, the price can be written as

$$p_t = \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right) x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\left(x_{t+1} - u_t^U\right) + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_t + \frac{\overline{\psi} - \overline{\alpha_s}\mu_{\Delta x}}{\overline{\alpha_p}}.$$

Taking first differences, we find that

$$\Delta p_t = \overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_0 \Delta x_t + \phi_1 u_t^L + \phi_1 u_{t-1}^U + \phi_n \Delta n_t.$$

where $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\alpha_p} - \frac{\overline{\alpha_s}}{\alpha_p}$, $\phi_1 = \frac{\overline{\alpha_s}}{\alpha_p}$, and $\phi_n = \frac{\overline{\alpha_n}}{\alpha_p}$. Then, the unbiased signal about the change in the learnable component of the innovation contained in the price is

$$\pi_t^L \equiv \frac{1}{\phi_1} \left(\Delta p_t - \left(\overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_n \mu_{\Delta n} + \phi_0 \Delta x_t \right) \right) = u_t^L + u_{t-1}^U + \frac{\phi_n}{\phi_1} \varepsilon_t^{\Delta n},$$

and absolute and relative price informativeness are respectively given by

$$\tau_{\pi^L} \equiv \mathbb{V}\text{ar} \left[u_{t-1}^U + \frac{\phi_n}{\phi_1} \varepsilon_t^{\Delta n} \middle| u_t^L, \Delta x_t \right]^{-1} = \left(\left(\tau_u^L + \tau_u^U \right)^{-1} + \left(\frac{\phi_n}{\phi_1} \right)^2 \tau_{\Delta n}^{-1} \right)^{-1} \tag{35}$$

and

$$\tau_{\pi^L}^R = \frac{\tau_{\pi^L}}{\tau_{\pi^L} + \tau_u^L} = \left(1 + \frac{\tau_u^L}{\tau_u^L + \tau_u^U} + \left(\frac{\phi_n}{\phi_1} \right)^2 \left(\frac{\tau_{\Delta n}}{\tau_u^L} \right)^{-1} \right)^{-1}.$$

When there is an unlearnable component of the payoff, the error term in regression R1 is given by $e_t = \phi_n \varepsilon_t^{\Delta n} - \phi_1 \Delta u_t^U$ and the error term in Regression R2 is given by $e_t^\zeta = \phi_n \varepsilon_t^{\Delta n} + \phi_1 u_t^L + \phi_1 u_{t-1}^U$. Consequently,

$$\frac{\mathbb{V}\text{ar}\left(e_t^\zeta\right) - \mathbb{V}\text{ar}\left(e_t\right)}{\mathbb{V}\text{ar}\left(e_t^\zeta \middle| \Delta x_t\right)} = \frac{\mathbb{V}\text{ar}\left(\phi_1 u_t^L\right) - \mathbb{V}\text{ar}\left(\phi_1 u_t^U\right)}{\mathbb{V}\text{ar}\left(\phi_1 u_t^L\right) + \mathbb{V}\text{ar}\left(\phi_1 u_{t-1}^U + \phi_n \varepsilon_t^{\Delta n} \middle| \Delta x_t\right)} = \frac{\tau_{\pi^L}}{\tau_{\pi^L} + \tau_u^L} \left(1 - \frac{\mathbb{V}\text{ar}\left(u_t^U\right)}{\mathbb{V}\text{ar}\left(u_t^L\right)} \right).$$

Rewriting this expression, we have

$$\tau_{\pi^L}^R = \frac{\tau_{\pi^L}}{\tau_{\pi^L} + \tau_u^L} = \frac{\mathbb{V}\text{ar}\left(e_t^\zeta\right) - \mathbb{V}\text{ar}\left(e_t\right)}{\mathbb{V}\text{ar}\left(e_t^\zeta \middle| \Delta x_t\right)} \frac{1}{1 - \frac{\mathbb{V}\text{ar}\left(u_t^U\right)}{\mathbb{V}\text{ar}\left(u_t^L\right)}}. \tag{36}$$

Finally, using the fact that $\mathbb{V}\text{ar}\left(u_t\right) = \mathbb{V}\text{ar}\left(u_t^L\right) + \mathbb{V}\text{ar}\left(u_t^U\right)$, it follows that

$$\frac{\mathbb{V}\text{ar}\left(u_t^U\right)}{\mathbb{V}\text{ar}\left(u_t^L\right)} = \frac{1 - \frac{\mathbb{V}\text{ar}\left(e_t^\zeta\right) - \mathbb{V}\text{ar}\left(e_t\right)}{\phi_1^2 \mathbb{V}\text{ar}\left(u_t\right)}}{1 + \frac{\mathbb{V}\text{ar}\left(e_t^\zeta\right) - \mathbb{V}\text{ar}\left(e_t\right)}{\phi_1^2 \mathbb{V}\text{ar}\left(u_t\right)}},$$

where $\mathbb{V}\text{ar}\left(u_t\right)$ is the variance of the residuals in a regression of Δx_{t+1} on Δx_t, which proves our results. Note that if $\mathbb{V}\text{ar}\left(u_t^L\right) = \mathbb{V}\text{ar}\left(u_t^U\right)$, the expression in Equation (36) is not well defined. In this case, we can recover relative price informativeness as $\tau_{\pi^L}^R = \frac{\frac{\phi_1^2}{2} \mathbb{V}\text{ar}\left(u_t\right)}{\mathbb{V}\text{ar}\left(e_t^\zeta \middle| \Delta x_t\right)}$.

Online Appendix

For Online Publication Only

Section A of this Online Appendix relates our results to existing work, in particular the literature on predictability and the work that focuses on forecasting price efficiency, including the work of Bai, Philippon and Savov (2016).

Section B of this Online Appendix describes in more detail the data used for the empirical implementation of the results in Section 5.

Section C of this Online Appendix reports additional empirical results. First, we include a series of figures that give more insight into the cross-sectional results presented in Table 2. Second, we present the results of the model after including a set of controls (public signals). Finally, we present cross-sectional relations using a single price informativeness estimate per stock obtained using the largest possible window of time-series data.

Section D of this Online Appendix includes a simulation of the model with learnable and unlearnable payoff innovations studied in Section 4.3.

A Relation to Existing Work

A.1 Predictability vs. Informativeness

As explained in the paper, price informativeness and price/return predictability are conceptually different notions. In this section, we provide a simple illustration of the source of the bias that would arise if one were to run Regression R3.[11] We simulate the model of Section 2 using the following parameters, chosen purely to illustrate existence of the bias: $\mu_{\Delta x} = \mu_n = \rho = \overline{\phi} = \phi_0 = 0$, $\tau_u = \tau_n = 1$, $\phi_1 = 1$, and $\phi_n = 1$.

Figure OA-1 shows the distribution of the OLS estimates of β_1 in Regression R1 (equivalently, ϕ_1 in Equation (2)) when simulating the model for different time periods. As shown in Proposition 1, the estimates of ϕ_1 are consistent. Figure OA-1 also shows the distribution of the OLS estimates of $\varphi_1 = \frac{1}{\beta_1}$ in Regression R3. As described in the text, the estimates of φ_1 are downward biased and inconsistent.

Figure OA-2 provides an alternative illustration of the bias in this scenario. The left panel in Figure OA-2 shows a simulation of Regression R1 with 200 observations. The estimated slope of this regression provides a consistent estimate of the estimate of ϕ_1. The right panel in Figure OA-2 shows a simulation of Regression R3 for the exact same simulation. The estimated slope

[11] Note that the bias identified here, which arises due to the presence of noise, is different from Stambaugh (1999) bias, which arises whenever the predictive variable is persistent.

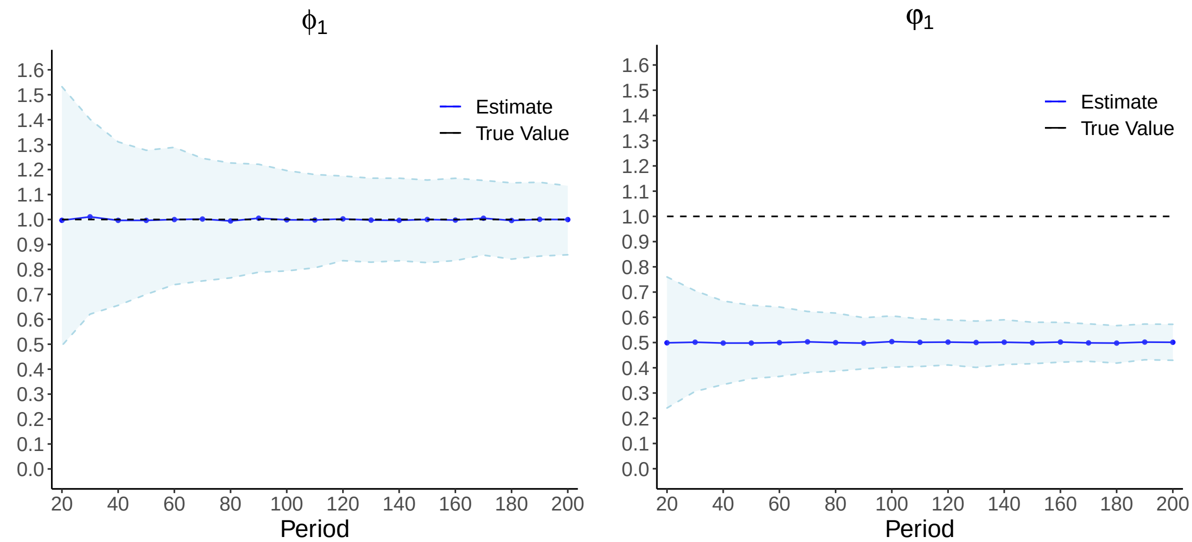

Figure OA-1: Predictability vs. Informativeness: biased estimates (1)

Note: The left panel in Figure OA-1 shows the distribution of estimates ϕ_1 and its true value when estimating Regression R1 via OLS. The right panel in Figure OA-1 shows the distribution of estimates $\frac{1}{\varphi_1}$ and its true value when running the reverse (predictive) Regression R3. In both panels, for every number of periods between 20 to 200, we simulate the model $N = 1000$ times, and report the mean estimate and the 5% and 95% estimates. The dashed black lines respectively show the true values of ϕ_1 and φ_1. The solid blue lines show the mean of the OLS estimates in each case, while the shaded area includes estimates within the 5% and 95% percentile.

of this regression provides a biased estimate of φ_1 in Regression R3, as shown theoretically in the text.

A.2 Forecasting Price Efficiency

In this section, we compare forecasting price efficiency (FPE), i.e., the unconditional variance of the expected value of the payoff conditional on the price, with price informativeness. While higher price informativeness will lead to higher FPE, higher FPE may not reflect an increase in price informativeness. More specifically, we show in Equation (38) below that FPE confounds changes in the volatility of the payoff with changes in the ability of markets to aggregate dispersed information. Hence, FPE can increase because price informativeness increases or because the payoff becomes more volatile and harder to predict. Alternatively, absolute price informativeness is the precision of the unbiased signal about the payoff contained in the price. This precision is a direct measure of the ability of financial markets to aggregate dispersed information and it is independent of the volatility of the payoff.

To illustrate this point, we explicitly re-derive the model in Bai, Philippon and Savov (2016) (BPS) as a special case of our general framework.[12] Consistent with our approach, we abstract

[12] As described in the introduction, BPS estimate FPE running cross-sectional regressions at specific points in time and report the time-series evolution of their cross-sectional estimates of FPE. This approach implicitly

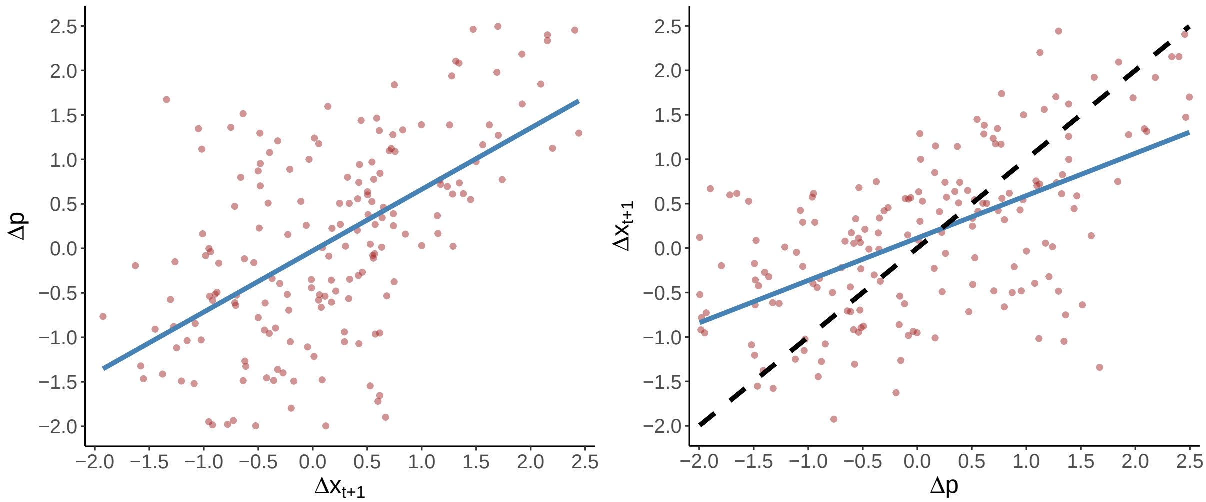

Figure OA-2: Predictability vs. Informativeness: biased estimates (2)

Note: The left panel in Figure OA-2 shows a simulation of Regression R1 with 200 observations. The estimated slope of this regression — shown by the blue solid line — provides a consistent estimate of the estimate of ϕ_1. The right panel in Figure OA-2 shows a simulation of Regression R3 for the exact same simulation. The estimated slope of this regression provides a biased estimate of φ_1 in Regression R3. The OLS regression is represented with a solid blue line in both panels. The dashed black line in the right panel shows the correct relation between both variables.

from investment decisions, and exclusively focus on the role of financial markets aggregating information. We first describe the environment in BPS using our notation to show how it is nested in our general specification. Then, we show that while FPE is relevant for welfare, it does not disentangle the ability of markets to aggregate information from how volatile are asset payoffs.

Environment There are two periods, $t = 0, 1$. There is one asset with a payoff $u \sim N\left(\overline{u}, \tau_u^{-1}\right)$. There are $i = 1, ..., I$ informed traders who choose their demand q_{1i} to maximize mean-variance preferences with imperfect information about u. The asset payoff u is not observable. However, informed investors observe a private signal

$$s = u + \varepsilon_s$$

and a public signal

$$\chi = u + \varepsilon_\chi,$$

assumes that the data generating process (including the distribution of payoffs, signals, and noise) is the same for all stocks at a given point in time — Figure OA-3 shows that this assumption is clearly not true in the data. We instead recover a panel of stock-specific measures of price informativeness by using rolling regressions. While they run predictive regressions of future payoffs on current market values, we show that in order to have consistent estimates of price informativeness, one must regress price changes on future payoffs — see the previous subsection.

where $\varepsilon_s \sim N\left(0, \tau_s^{-1}\right)$, $\varepsilon_\chi \sim N\left(0, \tau_\chi^{-1}\right)$, and $\varepsilon_s \perp \varepsilon_\pi$. Note that all informed investors observe the same set of signals. There are N noise traders whose total demand is random and given by $n \sim N\left(0, \tau_n^{-1}\right)$.

The informed traders' problem is

$$\max_{\theta_{1i}} \left(\mathbb{E}\left[u|s,\chi\right] - p\right)\theta_{1i} - \frac{\gamma}{2}\mathbb{V}\mathrm{ar}\left[u|s,\chi\right]\theta_{1i}^2 + p\theta_{0i},$$

which leads to the following demand curve:

$$\theta_{1i} = \frac{\mathbb{E}\left[u|s,\chi\right] - p}{\gamma\mathbb{V}\mathrm{ar}\left[u|s,\chi\right]},$$

where

$$\mathbb{E}\left[u|s,\chi\right] = \frac{\tau_u\overline{u} + \tau_s s + \tau_\chi\chi}{\tau_u + \tau_s + \tau_\chi} \quad \text{and} \quad \mathbb{V}\mathrm{ar}\left[u|s,\chi\right] = \frac{1}{\tau_u + \tau_s + \tau_\chi}.$$

Since all informed investors share the same information set, there is no learning from the price.

In an equilibrium in linear strategies, demands for informed traders and uninformed traders are respectively given by

$$\theta_{1i}^I = \alpha_s^I s + \alpha_\chi^I\chi + \alpha_n^I n - \alpha_p^I p + \psi^I$$
$$\theta_{1i}^U = \alpha_s^U s + \alpha_\chi^U\chi + \alpha_n^U n - \alpha_p^U p + \psi^U.$$

Matching coefficients we have that

$$\alpha_s^I = \frac{\tau_s}{\gamma}, \quad \alpha_\chi^I = \frac{\tau_\chi}{\gamma}, \quad \alpha_n^I = 0, \quad \alpha_p^I = \frac{1}{\gamma}\left(\tau_u + \tau_s + \tau_\chi\right), \quad \psi^I = \frac{\tau_u}{\gamma}\overline{u} - \theta_{0i},$$

and $\alpha_s^U = \alpha_\chi^U = \alpha_p^U = \psi^U = 0$, and $\alpha_n^U = \frac{1}{N}$.

Market clearing implies

$$\sum_{i=1}^{I}\theta_{si}^I + n = Q,$$

which is the same as

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}s + \frac{\overline{\alpha_\chi}}{\overline{\alpha_p}}\chi + \frac{\overline{\psi}}{\overline{\alpha_p}} + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}n,$$

where $\overline{\alpha_s} = I\alpha_s^I + N\alpha_s^N$. $\overline{\alpha_\chi} = I\alpha_\chi^I + N\alpha_\chi^N$, $\overline{\alpha_p} = I\alpha_p^I + N\alpha_p^N$, $\overline{\alpha_n} = I\alpha_n^I + N\alpha_n^N$, and $\overline{\psi} = I\psi^I + N\psi^N - Q$.

Price informativeness and forecasting price efficiency Observing the price is equivalent to observing the signal π,

$$\pi = \frac{\overline{\alpha_p}}{\overline{\alpha_s} + \overline{\alpha_\chi}} \left(p - \frac{\overline{\psi}}{\overline{\alpha_p}} \right) = u + \frac{\overline{\alpha_s}}{\overline{\alpha_s} + \overline{\alpha_\chi}} \varepsilon_s + \frac{\overline{\alpha_\chi}}{\overline{\alpha_s} + \overline{\alpha_\chi}} \varepsilon_\chi + \frac{\overline{\alpha_n}}{\overline{\alpha_s} + \overline{\alpha_\chi}} n,$$

where

$$\pi | u \sim N\left(u, \tau_\pi^{-1}\right),$$

with

$$\tau_\pi = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_s} + \overline{\alpha_\chi}} \right)^2 \tau_s^{-1} + \left(\frac{\overline{\alpha_\chi}}{\overline{\alpha_s} + \overline{\alpha_\chi}} \right)^2 \tau_\chi^{-1} + \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s} + \overline{\alpha_\chi}} \right)^2 \tau_n^{-1}. \tag{37}$$

The precision of the price signal π, τ_π in Equation (37), corresponds to our measure of absolute price informativeness when there is a finite number of investors. There are two differences with respect to the baseline model presented in the main text. First, there are multiple sources of aggregate noise: the error of the private signal, ε_s; the error of the public signal, ε_χ; and the demand of noise traders, n. Second, price informativeness is modulated by $\overline{\alpha_s} + \overline{\alpha_\chi}$ instead of by $\overline{\alpha_s}$ because there are two sources of external information about the innovation to the payoff u.

A Bayesian external observer who only observes the price learns from the price in the following way:

$$\mathbb{E}\left[u | \, \pi \right] = \frac{\tau_u \overline{u} + \tau_\pi \pi}{\tau_u + \tau_\pi}.$$

Forecasting price efficiency (FPE) is then given by

$$\mathcal{V}_{\text{FPE}} = \mathbb{V}\text{ar}\left(\mathbb{E}\left[u | \, \pi \right] \right) = \left(\frac{\tau_\pi}{\tau_u + \tau_\pi} \right)^2 \left(\tau_u^{-1} + \tau_\pi^{-1} \right) = \frac{\tau_\pi^R}{\tau_u} \tag{38}$$

The expression for FPE in Equation (38) is the predicted variance of cash flows u from prices. From this equation, it is easy to see that FPE confounds two effects. FPE can increase due to changes in the ability of prices to aggregate information, τ_π^R, or due to changes in the variability of the payoff, τ_u^{-1}. Hence, conditional on the variance of the payoff remaining constant, FPE and price informativeness will co-move. However, without controlling for changes in payoff volatility, one cannot make any inferences about price informativeness by looking at FPE.

Relation to Bai, Philippon and Savov (2016) There are three significant differences between our approach and theirs. First, they aim to measure the informational content of prices using forecasting price efficiency (FPE), a concept introduced in Bond, Edmans and

Goldstein (2012). As we formally show in this paper, FPE does not separately identify price informativeness from payoff volatility. That is, FPE can be high because asset payoffs have low volatility or because asset prices are very informative. Second, they estimate FPE running cross-sectional regressions at specific points in time. This approach implicitly assumes that the data generating process (including the distributions of payoffs, signals, and noise) is the same for all stocks at a given point in time — an assumption that is easily falsifiable, as we show in this paper. We instead recover a panel of stock-specific measures of price informativeness by using rolling regressions. Finally, while they run predictive regressions of future payoffs on current market values, we show that one must regress price changes (endogenous) on future payoffs (exogenous) to recover consistent estimates of price informativeness.

B Detailed Data Description

This section describes in more detail the data used for the empirical implementation of the results in Section 5. See https://github.com/edavila/identifying_price_informativeness for additional details and replicating files.

We obtain stock market price data from the Center for Research in Security Prices (CRSP) for the time period between January 1, 1950 and December 31, 2019. First, we import monthly price data from the Monthly Stock File (msf) for ordinary common shares (shrcd = 10 or 11). Second, we import delisting prices and other delisting information from the monthly delisting file (msedelist). Third, we import market returns from the monthly stock indicators file (msi). Lastly, we import the start and/or end date(s) of when a stock has been part of the S&P 500 from dsp500list. We restrict our sample to securities listed on the NYSE, AMEX or the NASDAQ (exchcd = 1, 2, or 3). We compute market capitalization by multiplying the stock price by the number of shares outstanding. For companies with multiple securities, we sum the market cap for all the company's securities and keep only the permno with the highest market capitalization. We define turnover as the ratio between trading volume and shares outstanding.

From FRED, we obtain monthly time series for Personal Consumption Expenditure (PCEPI), 1-Year and 10-Year Treasury Rates (GS1, GS10), Unemployment Rate (UNRATE), Personal Consumption Expenditures (PCE), and Personal Income (PI).

We import firm performance data from both the COMPUSTAT Fundamentals Annual Data & the Fundamentals Quarterly Data in the standard, consolidated, industrial format for domestic firms (INDFMT = 'INDL' and DATAFMT = 'STD' and CONSOL = 'C' and POPSRC = 'D') for observations between January 1, 1950 and December 31, 2019 . For future linking with CRSP, we also import GVKEYs and permnos from the CRSP/COMPUSTAT Merged (CCM) database, keeping the following linktypes: "LU," "LC," or "LS," and for which the issue marker is primary (linkprim = "P" or "C"). For both the annual and quarterly data, we only keep observations where the observation date is between the beginning and end of the period for which the CCM link is valid.

We form book value (book) for annual[quarterly] data as shareholders equity (seq[q]) + deferred assets plus investment tax credit (txditc[q]) - pstk[q] (preferred stock). To deal with missing values, we replace seqq with common equity plus preferred equity (ceq[q] + pstk[q]) if an observation of the former but not the latter is unavailable, and if both of are unavailable, we replace with total assets minus total liability (at[q] - lt[q]). If an observation for txditc[q] and/or pstk[q] are missing, we replace it with 0. We respectively use oiadpq and ebit as our payoff measures in the quarterly and annual datasets. We merge for both the annual and quarterly datasets, where in the shifted specifications, we shift CRSP respectively one quarter and one month back. After merging the COMPUSTAT and CRSP datasets using the timing describing in the text, we use PCEPI to deflate all nominal variables. We also discard stocks with non-finite prices and whose payoff is always 0 or NA. We winsorize payoff and price values at the 2.5th and 97.5th percentile to reduce the impact of outliers. We compute growth rates of payoffs as follows: when the lagged payoff is positive, the growth rate is defined as $\text{payoff}/\text{payoff}_{t-1} - 1$. When the lagged payoff is negative, the growth rate is defined as $\text{payoff}_t/|\text{payoff}_{t-1}| + 1$. We disregard the observations (less than 0.1%) for which the lagged payoff is exactly zero.

Figure OA-3 illustrates the distribution of stock-specific standard deviation of quarterly earnings' growth rates in our sample of stocks with more than 40 observations. As one would expect, the volatility of earnings across stocks varies widely in the cross section.

C Empirical Implementation: Additional Results

In this section, we report additional empirical results. First, we include a series of figures that give more insight into the cross-sectional results presented in Table 2. Second, we present the results of the model after including a set of controls (public signals). Third, we present cross-sectional relations using a single price informativeness estimate per stock obtained using the largest possible window of time-series data. Finally, we present the results using annual observations, instead of quarterly.

C.1 Cross-sectional Relation: Graphical Illustration

Figures OA-4 through OA-8 are the counterparts of the cross-sectional results presented in Table 2. Each figure shows scatter plots of cross-sectional regressions of relative price informativeness (in twentiles) on each of the five variables considered: size, value, turnover, return volatility, and institutional ownership, for each of the years between 1981 and 2016. These figures make clear

Figure OA-3: Cross-sectional standard deviation of earnings' growth rates

Note: Figure OA-3 shows a relative-frequency histogram of the distribution across stocks of the time-series standard deviation of earnings growth rates. This histogram features 6,803 stocks. For reference, the median and mean of the average growth rate of earnings across the stocks represented in this figure are, respectively, 0.11 and 0.56. Section A.2 explains why this evidence invalidates the use of cross-sectional regressions.

that the positive relationships between price informativeness and size, turnover, and institutional ownership are robust across time.

C.2 Public Signals/Additional Controls

While in the body of the paper we report measures of price informativeness that do not include additional controls — public signals, in the language of Section 4.2 — in this section we report the results once we include several controls. Formally, the results reported here are the outcome of running the following two regressions

$$\begin{aligned}
\Delta p_t^j &= \overline{\beta}^j + \beta_0^j \Delta x_t^j + \beta_1^j \Delta x_{t+1}^j + \beta^c \cdot \Delta w_t^{j,q} + d_t^{j,q} + \varepsilon_t^j \quad &&\Rightarrow R^{2,j}_{\Delta x, \Delta x'} \\
\Delta p_t^j &= \overline{\zeta}^j + \zeta_0^j \Delta x_t^j + \beta^c \cdot \Delta w_t^{j,q} \qquad\qquad\quad + d_t^{j,q} + \hat{\varepsilon}_t^j \quad &&\Rightarrow R^{2,j}_{\Delta x} ,
\end{aligned}$$

where $w_t^{j,q}$ denotes a given set of controls/public signal. The results reported here use the following aggregate variables as controls for all stocks: i) changes in the one- and ten-year treasury rates, ii) changes in unemployment rates, iii) change in log consumption, and iv) changes in log income. We obtain similar results using stock-specific controls as well.

Figure OA-9, which is the counterpart of Figure 2, shows a relative-frequency histogram of price informativeness for a representative time period, the last quarter of 2015. Figure OA-10, which is the counterpart of Figure 5, shows the time-series evolution of the cross-sectional mean, median, and standard deviation of relative price informativeness. Table OA-1 and Figures OA-11

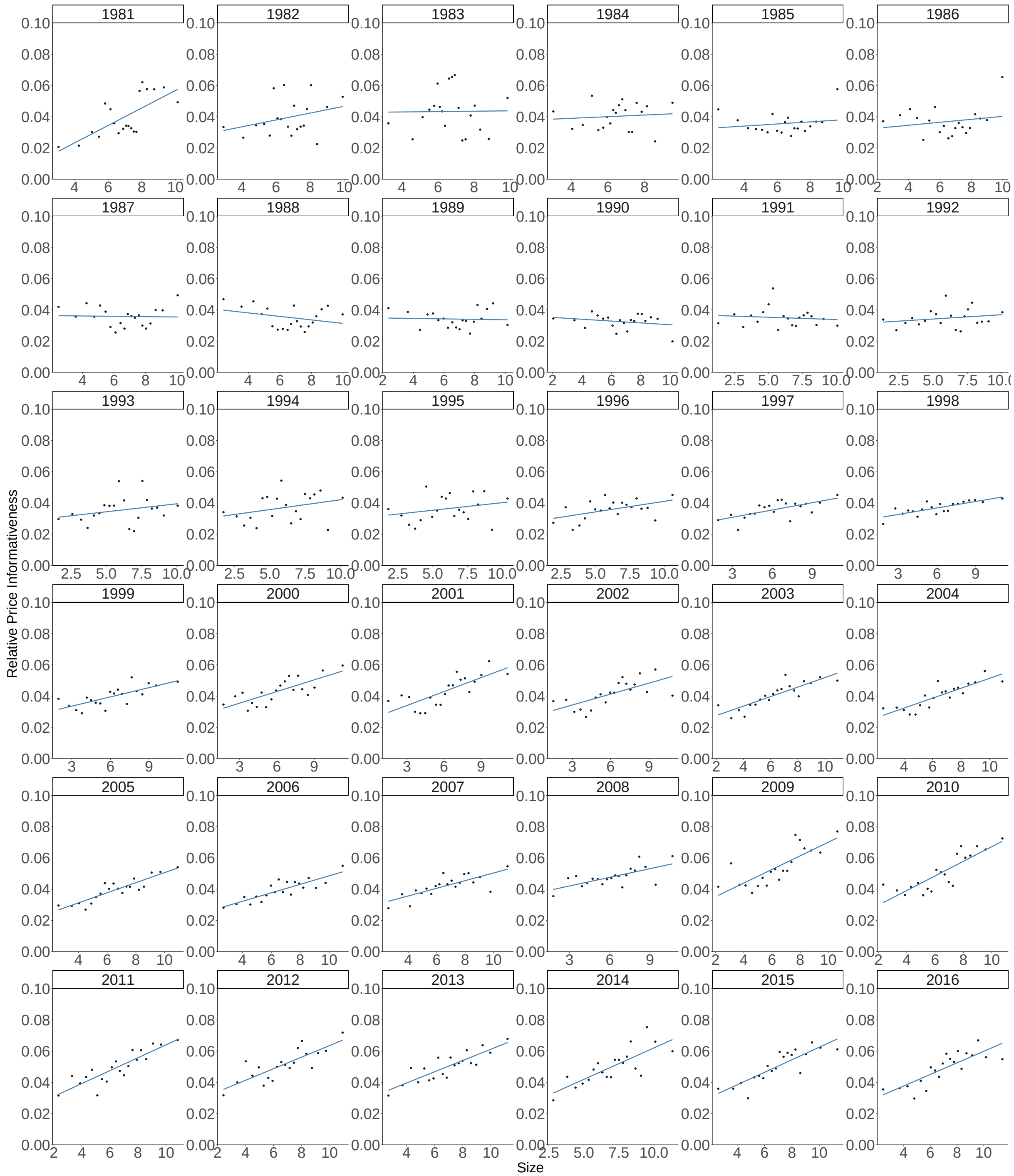

Figure OA-4: Price informativeness and size

Note: Figure OA-4 shows year-by-year cross-sectional regressions of relative price informativeness (in twentiles) on size, defined as the log of market capitalization — see e.g. Bali, Engle and Murray (2016). The estimate reported in Table 2 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

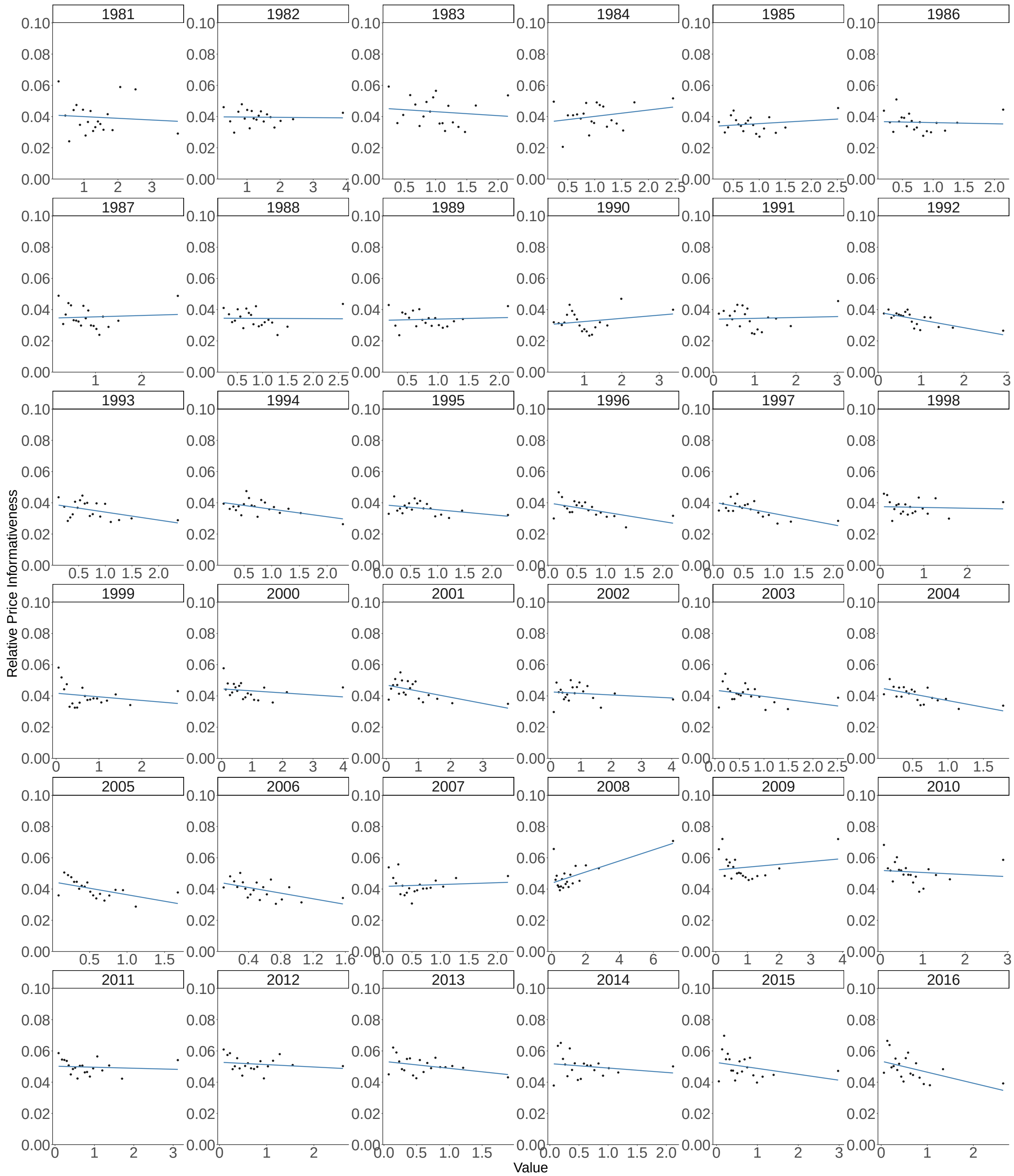

Figure OA-5: Price informativeness and value

Note: Figure OA-5 shows year-by-year cross-sectional regressions of relative price informativeness (in twentiles) on value, defined as the ratio between a stock's book value and its market capitalization. The estimate reported in Table 2 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

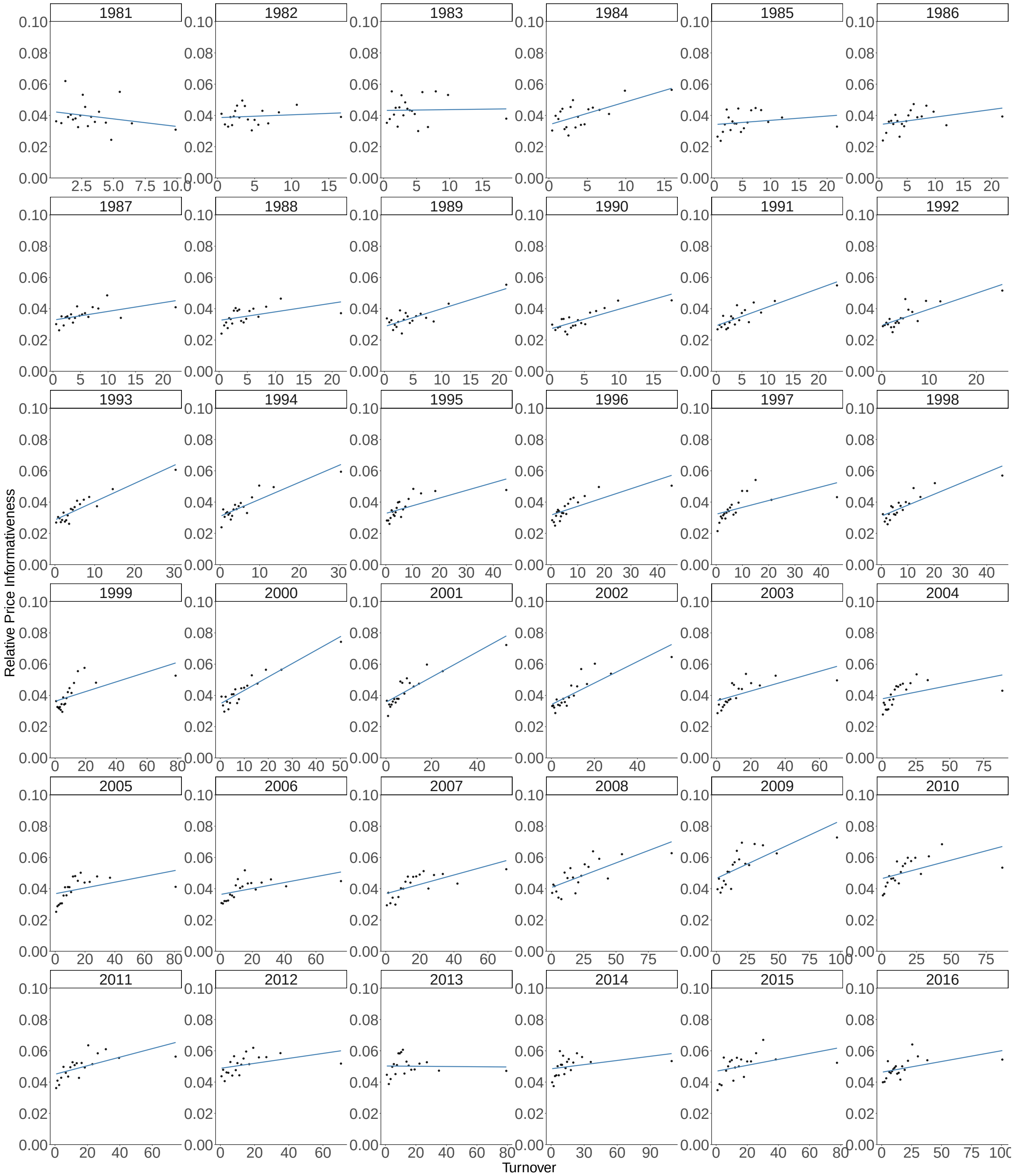

Figure OA-6: Price informativeness and turnover

Note: Figure OA-6 shows year-by-year cross-sectional regressions of relative price informativeness (in twentiles) on turnover, defined as the ratio between trading volume and shares outstanding. The estimate reported in Table 2 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

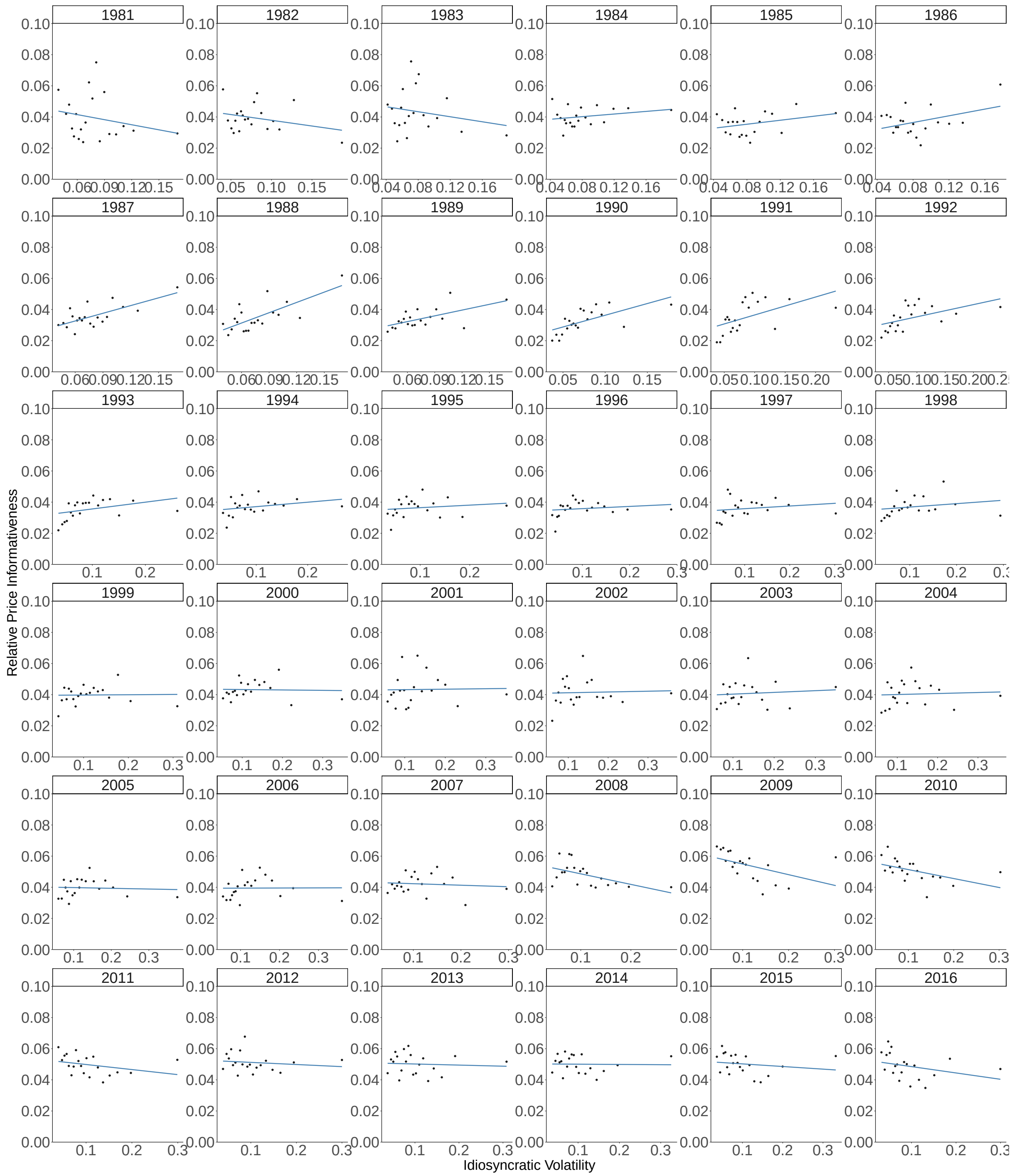

Figure OA-7: Price informativeness and idiosyncratic return volatility

Note: Figure OA-7 shows year-by-year cross-sectional regressions of relative price informativeness (in twentiles) on idiosyncratic volatility, define as the standard deviation over a 30 month period of the difference between the returns of a stock and the market return. The estimate reported in Table 2 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

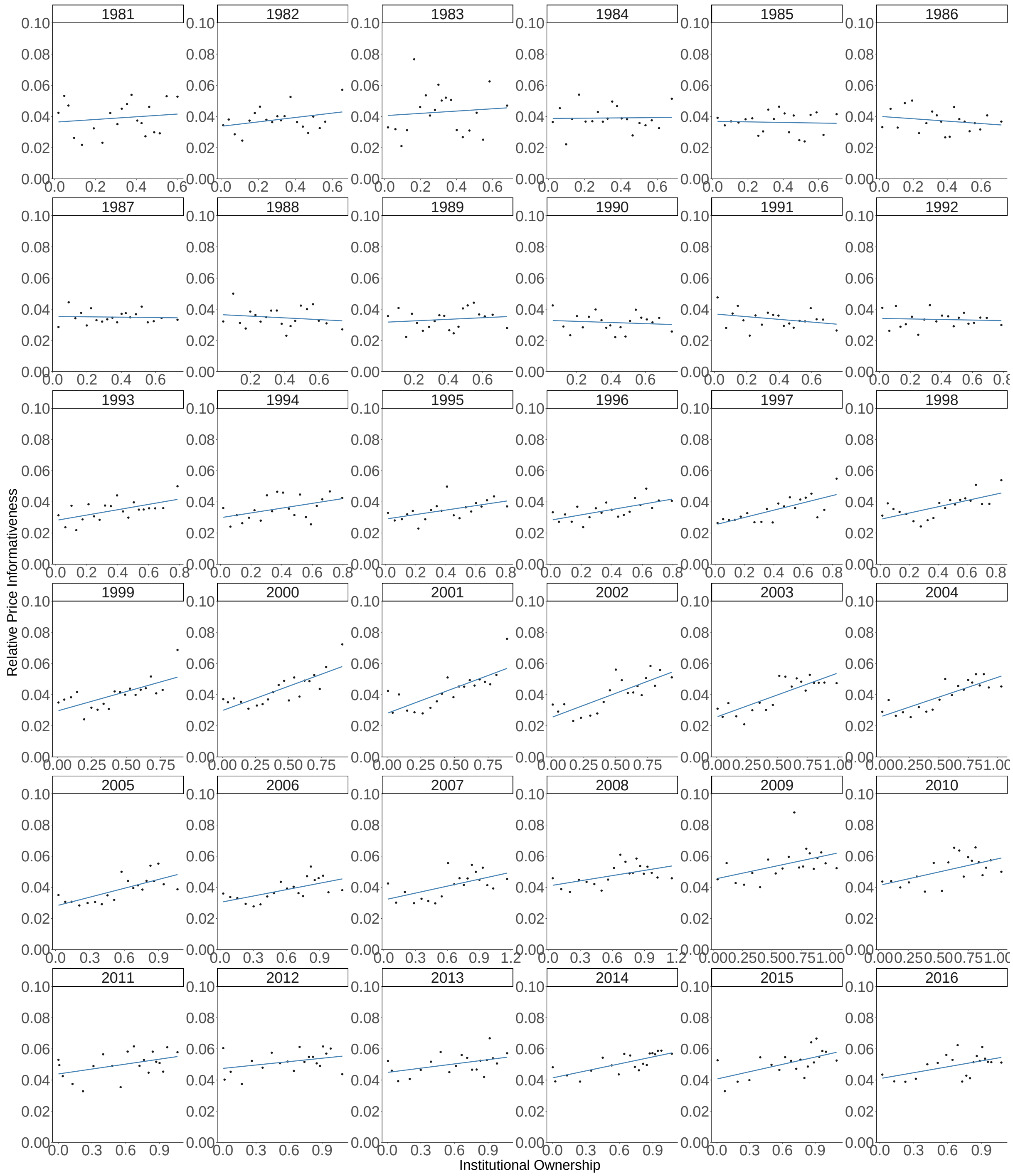

Figure OA-8: Price informativeness and institutional ownership

Note: Figure OA-8 shows year-by-year cross-sectional regressions of relative price informativeness (in twentiles) on institutional ownership, defined as the proportion of a stock held by institutional investors. The estimate reported in Table 2 can be interpreted as a weighted averaged of the year-by-year slope coefficient illustrated here.

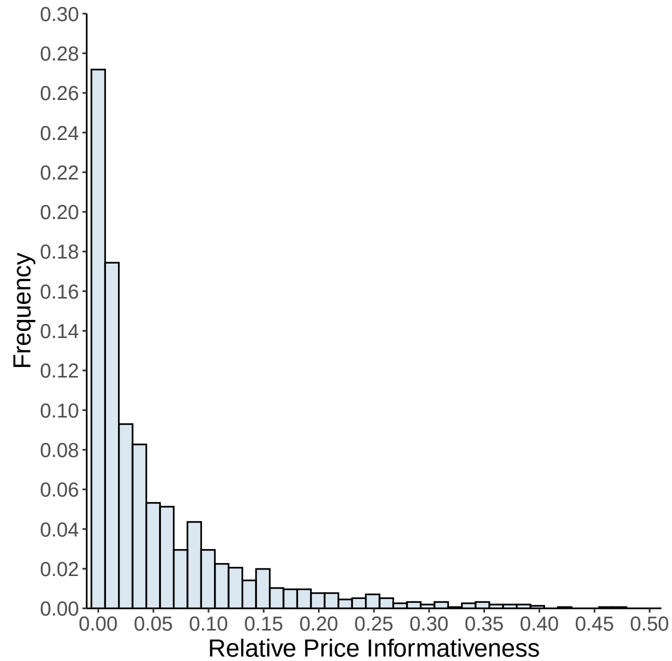

Figure OA-9: Price informativeness: relative-frequency histogram, public signals

Note: Figure OA-9 shows a relative-frequency histogram of price informativeness for a representative time period, the last quarter of 2015. Note that informativeness is computed over a rolling window of 40 quarters prior.

and OA-12, which are the counterparts of Table 2 and Figures 3 and 4, show the cross-sectional properties of the distribution of price informativeness across stocks.

C.3 Price Informativeness in the Cross Section: Full Sample Results

In the body of the paper, we compute price informativeness using rolling windows of 40 quarters. Alternatively, we can compute a single measure of price informativeness for each stock using the largest possible window of time-series data available for each. In this case, we obtain a single price informativeness measure per stock. The upside of this approach is that it uses all the available information for a given stock. The main drawback of this approach is that the recovered informativeness measures rely on observations from different time periods. Figure OA-13 shows the cross-sectional results established in Table 2 remain valid in this case.

D Simulation: Learnable and Unlearnable Payoff

In this section, we simulate the model with an unlearnable payoff component studied in Section 4.3. This simulation is helpful to understand the sign and magnitude of the potential bias caused by estimating the model via OLS and using an incorrect identification procedure. We use the following parameters: $\tau_u^L = \tau_u^U = 4$, $\tau_{\Delta n} = 0.15$, $\mu_{\Delta x} = \mu_{\Delta n} = \overline{\phi} = 0$, and $\phi_0 = \phi_n = \phi_1 = 1$.

Table OA-1: Cross-sectional results, public signals

	Estimate	Std. Error	t-stat
Size	0.00219	0.000144	15.25
Value	-0.00203	0.000443	-4.58
Turnover	0.00028	0.000021	13.55
Idiosyncratic Volatility	-0.00065	0.005998	-0.11
Institutional Ownership	0.01417	0.001200	11.80

Note: Table OA-1 reports the estimates ($\widehat{a_1^c}$) of panel regressions of price informativeness on cross-sectional characteristics (in twentiles) with year fixed effects (ξ_t): $\tau_{\pi,t}^{R,b} = a_0^c + a_1^c c_t^b + \xi_t + \epsilon_{b,t}$, where $\tau_{\pi}^{R,b,t}$ denotes the average price informative per bin (twentile) in a given period, c_t^b denotes the value of the given characteristic per bin (twentile) in a given period, ξ_t denotes a year fixed effect, a_0^c and a_1^c are parameters, and $\epsilon_{b,t}$ is an error term. Figures OA-4 through OA-8 provide the graphical counterpart of the results in this table. Size is measured as the natural log of stock market capitalization, value is measured as the ratio between a stock's book value and its market capitalization, turnover is measured as the ratio between trading volume and shares outstanding, idiosyncratic volatility is measured as the standard deviation — over a 30 month period — of the difference between the returns of a stock and the market return, and institutional ownership is measured as the proportion of a stock held by institutional investors.

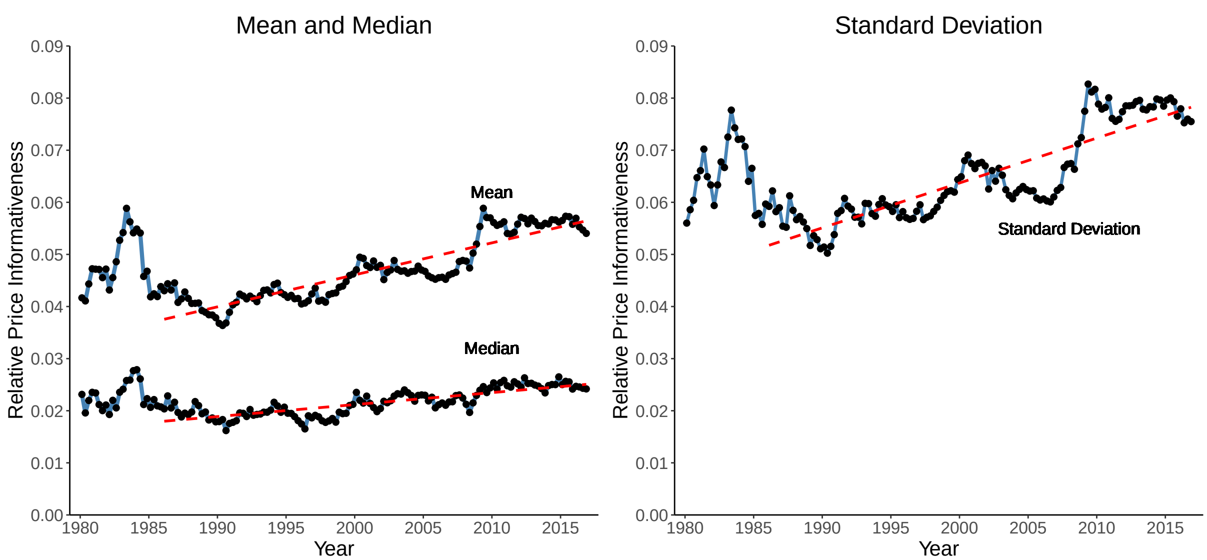

Figure OA-10: Price informativeness over time, public signals

Note: The left panel in Figure OA-10 shows the time-series evolution of the cross-sectional mean and median relative price informativeness. The right panel in Figure OA-10 shows the time-series evolution of the cross-sectional standard deviation of price informativeness. The red dashed lines show linear trends starting in 1986.

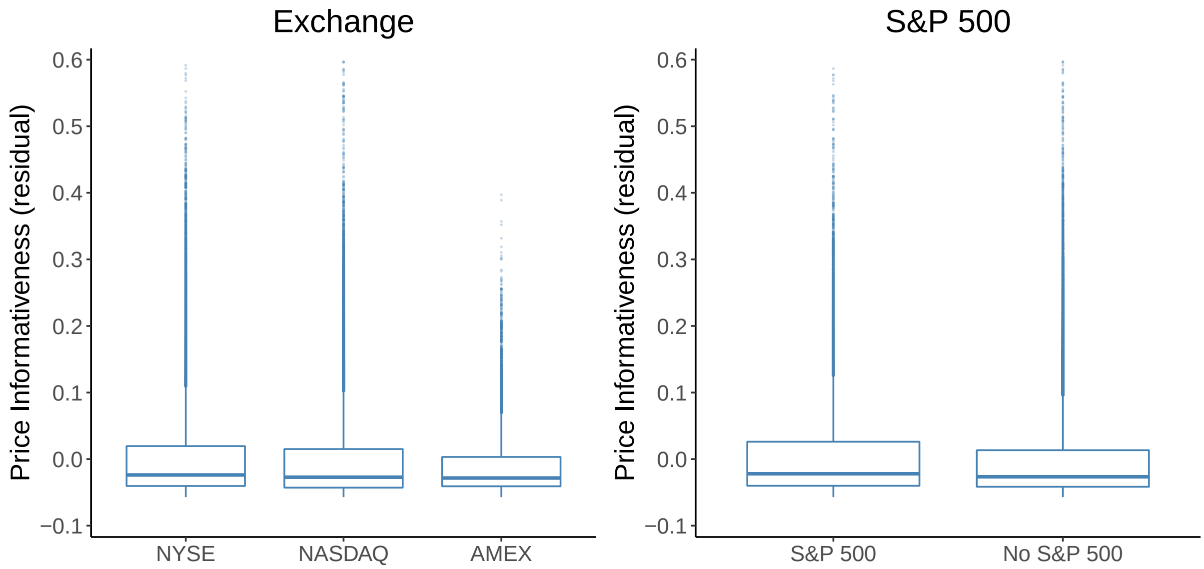

Figure OA-11: Cross-sectional results, public signals

Note: The left panel in Figure OA-11 shows a box plot by exchange of the residuals of a regression of relative price informativeness on year fixed effects. The left panel in Figure OA-11 shows a box plot by S&P 500 status of the residuals of a regression of relative price informativeness on year fixed effects. The solid middle line represents the median. The top and bottom of the box represent the 75th and 25th percentiles. The whiskers extend up to 1.5 times the interquartile range.

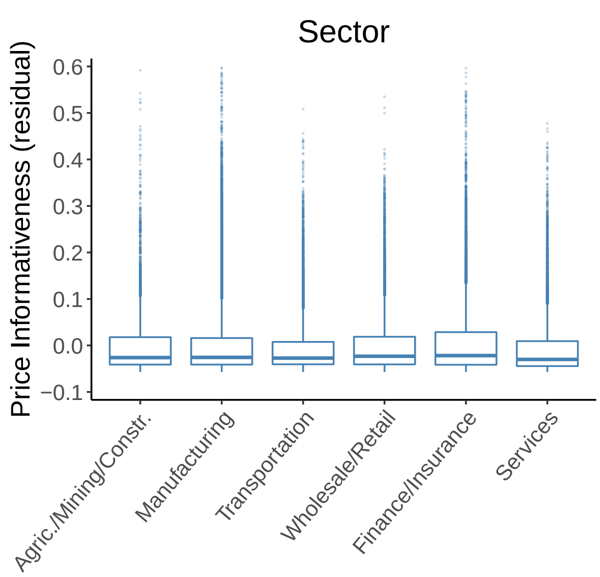

Figure OA-12: Cross-sectional results, public signals

Note: Figure OA-12 shows a box plot by one-digit SIC industry code of the residuals of a regression of relative price informativeness on year fixed effects. The solid middle line represents the median. The top and bottom of the box represent the 75th and 25th percentiles. The whiskers extend up to 1.5 times the interquartile range.

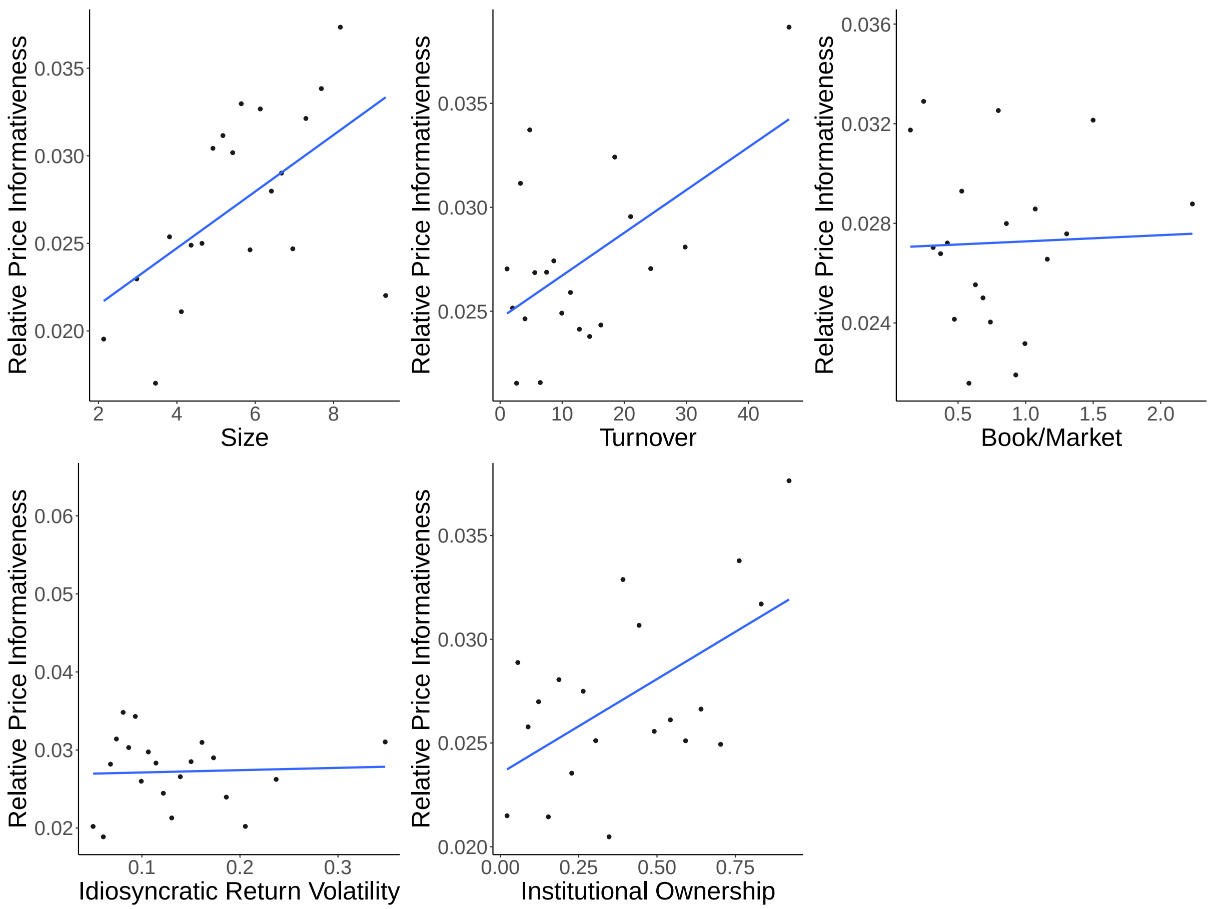

Figure OA-13: Price informativeness: cross-sectional results, largest window

Note: Figure OA-13 shows cross-sectional regressions of relative price informativeness (in twentiles) on size, value, turnover, return volatility, and institutional ownership. The estimates of price informativeness in this figure are computed using the largest possible window for each stock.

These parameters are chosen so that i) 50% of the payoff innovation is learnable and 50 is unlearnable, and ii) the true value of price informativeness, computed from the theoretical expression in Equation (43), is 3.5%.

Figure OA-14 shows the distribution of two different estimates of relative price informativeness in an environment in which the innovation to the asset payoff has a learnable and an unlearnable component, as described in Section 4.3. The solid red line shows the mean estimate of price informativeness estimated using Proposition 1 (OLS estimation of Regressions R1 and R2). The solid blue line shows the mean estimate of price informativeness estimated using Proposition 4 (IV estimation of Regressions R1 and R2, using u_t^L as instrument for Regression R2, and u_t^L and u_{t-1}^L as instruments for Regression R1). The shaded areas contain the distribution of estimates between the 5% and the 95% for each estimator. Note that while the OLS estimation of informativeness using Regressions R1 and R2 are weakly positive, it is possible to find negative estimates when using IV estimation.

In this simulation, the OLS estimates of price informativeness can be higher or lower than the true value. Interestingly, while the OLS estimates of price informativeness are not consistent — as the number of periods grows, the OLS estimates of informativeness tend to be lower than the true estimate —, in this specific scenario, the OLS estimates of informativeness using Regressions R1 and R2 are closer to the true estimate than the IV estimates using Regressions R1 and R2.

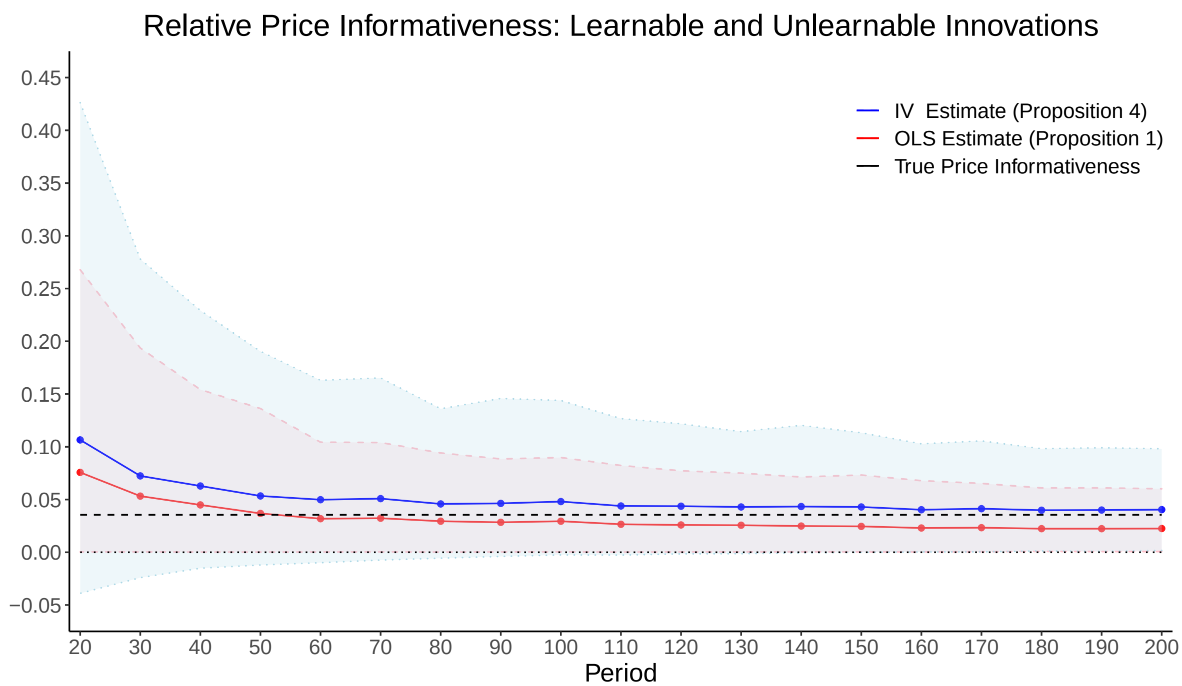

Figure OA-14: Price informativeness: estimates with learnable and unlearnable payoff innovations

Note: Figure OA-14 shows the distribution of two different estimates of relative price informativeness in an environment in which the innovation to the asset payoff has a learnable and an unlearnable component, as described in Section 4.3. We simulate the model of Section 4.3 with parameters: $\tau_u^L = \tau_u^U = 4$, $\tau_{\Delta n} = 0.15$, $\mu_{\Delta x} = \mu_{\Delta n} = \overline{\phi} = 0$, and $\phi_0 = \phi_n = \phi_1 = 1$. These parameters are chosen so that i) 50% of the payoff innovation is learnable and 50% is unlearnable and ii) the true value of price informativeness, computed as in Equation (43), is 3.5%. For every number of periods between 20 to 200, in intervals of 10, we simulate the model $N = 1000$ times, and report the mean estimate and the 5% and 95% estimates of price informativeness. The dashed black line shows the true value of relative price informativeness. The solid red line shows the mean estimate of price informativeness estimated using Proposition 1 (OLS estimation of Regressions R1 and R2). The solid blue line shows the mean estimate of price informativeness estimated using Proposition 4 (IV estimation of Regressions R1 and R2, using u_t^L as instrument for Regression R2, and u_t^L and u_{t-1}^L as instruments for Regression R1). The shaded areas contain the distribution of estimates between the 5% and the 95% for each estimator.

Supplemental Appendix

For Online Publication Only

Section A of this Supplemental Appendix is the counterpart of the analysis in Section 5 using annual data, instead of quarterly.

Section B of this Supplemental Appendix includes additional results, including an identification results for absolute price informativeness, the derivation of the Kalman gain for an external observer, and an extension of the model in which informativeness is defined in term of one-period ahead prices.

Section C of this Supplemental Appendix describes how to identify price informativeness starting from assumptions on asset demands and information. This is an intermediate approach between the approach in Section 2, which starts from a pricing equation, and the approach in Section 3, which studies fully microfounded models.

Section D of this Supplemental Appendix considers three alternative specifications. First, we extend our approximate results to the case in which the payoff follows a stationary AR(1) process. Second, we develop our identification results using an exact linear formulation for the price process under difference-stationary and stationary specifications for the payoff. Third, we also provide the respective CARA-Normal models to microfound these exact linear formulations.

A Empirical Implementation: Annual Observations

In this section, we report results when using annual data, instead of quarterly data. These results show that both the cross-sectional and time-series findings identified using quarterly observations remain true when using annual data.

Table SupApp-1: Price informativeness: year-by-year summary statistics, annual data

t	$Median$	$Mean$	SD	$Skew$	$Kurt$	$P5$	$P25$	$P75$	$P95$	n
1980	0.0643	0.1023	0.1088	1.3051	1.1713	0.0004	0.0150	0.1570	0.3356	280
1981	0.0859	0.1238	0.1174	1.0531	0.4677	0.0015	0.0246	0.1942	0.3565	288
1982	0.0903	0.1250	0.1226	1.2963	1.3625	0.0014	0.0228	0.1900	0.3836	430
1983	0.0775	0.1185	0.1222	1.5262	2.4278	0.0012	0.0242	0.1707	0.3716	452
1984	0.0713	0.1110	0.1181	1.5360	2.2401	0.0018	0.0201	0.1590	0.3739	453
1985	0.0632	0.1036	0.1077	1.4092	1.5899	0.0008	0.0209	0.1529	0.3347	451
1986	0.0556	0.0944	0.1008	1.3941	1.5216	0.0007	0.0161	0.1451	0.2968	471
1987	0.0593	0.0946	0.1017	1.4405	1.6748	0.0004	0.0169	0.1440	0.3142	472
1988	0.0484	0.0911	0.1063	1.6457	2.6934	0.0005	0.0152	0.1310	0.3252	483
1989	0.0511	0.0910	0.1046	1.6350	2.7352	0.0005	0.0136	0.1354	0.3052	508
1990	0.0509	0.0922	0.1050	1.6846	3.3961	0.0007	0.0136	0.1422	0.3087	514
1991	0.0513	0.0915	0.1033	1.5840	2.3096	0.0008	0.0142	0.1376	0.3186	529
1992	0.0527	0.0943	0.1062	1.5677	2.1055	0.0006	0.0153	0.1371	0.3363	680
1993	0.0569	0.0954	0.1061	1.5697	2.4399	0.0005	0.0150	0.1425	0.3168	791
1994	0.0564	0.0939	0.1047	1.5570	2.1203	0.0006	0.0155	0.1352	0.3243	798
1995	0.0564	0.0960	0.1058	1.4937	1.9063	0.0009	0.0162	0.1437	0.3270	786
1996	0.0573	0.0988	0.1103	1.6053	2.7019	0.0008	0.0149	0.1490	0.3344	752
1997	0.0600	0.0986	0.1086	1.6494	3.0178	0.0016	0.0156	0.1480	0.3237	724
1998	0.0611	0.1004	0.1126	1.6294	2.7944	0.0007	0.0132	0.1498	0.3413	687
1999	0.0560	0.1013	0.1175	1.7452	3.3078	0.0009	0.0145	0.1482	0.3538	660
2000	0.0651	0.1108	0.1263	1.5673	2.1201	0.0007	0.0169	0.1649	0.3923	649
2001	0.0620	0.1050	0.1181	1.5670	2.2149	0.0007	0.0141	0.1507	0.3737	643
2002	0.0575	0.0972	0.1119	1.6551	2.6571	0.0004	0.0121	0.1413	0.3464	639
2003	0.0682	0.1040	0.1156	1.5367	2.2066	0.0005	0.0134	0.1502	0.3455	654
2004	0.0663	0.1023	0.1138	1.5297	2.3824	0.0005	0.0131	0.1522	0.3282	753
2005	0.0609	0.0978	0.1103	1.4841	1.9502	0.0004	0.0123	0.1456	0.3355	753
2006	0.0606	0.1003	0.1114	1.4421	1.7090	0.0006	0.0133	0.1498	0.3385	762
2007	0.0634	0.1070	0.1205	1.5149	2.0227	0.0005	0.0137	0.1607	0.3698	774
2008	0.0757	0.1262	0.1379	1.3379	1.1922	0.0009	0.0166	0.1952	0.4037	775
2009	0.0794	0.1363	0.1479	1.3389	1.2611	0.0007	0.0210	0.2079	0.4471	747
2010	0.0719	0.1362	0.1522	1.3449	1.1752	0.0006	0.0193	0.2117	0.4528	747
2011	0.0707	0.1383	0.1533	1.2670	0.8677	0.0005	0.0178	0.2173	0.4632	765
2012	0.0729	0.1374	0.1544	1.3146	1.0753	0.0007	0.0158	0.2176	0.4518	797
2013	0.0711	0.1362	0.1530	1.3148	1.0972	0.0005	0.0169	0.2217	0.4478	913
2014	0.0767	0.1398	0.1544	1.2922	1.0374	0.0007	0.0177	0.2266	0.4482	950
2015	0.0800	0.1398	0.1523	1.2974	1.0991	0.0009	0.0177	0.2180	0.4546	967
2016	0.0808	0.1398	0.1543	1.3185	1.0735	0.0009	0.0179	0.2159	0.4681	988

Note: Table SupApp-1 reports year-by-year summary statistics on the panel of price informativeness measures recovered. It provides information on the median; mean; standard deviation; skewness; excess kurtosis; and 5th, 25th, 75th, and 95th percentiles of each yearly distribution, as well as the number of stocks in each year. Since our panel of price informativeness is quarterly, we average the measures of quarterly price informativeness at the yearly level before computing the summary statistics. We start reporting summary statistics in 1980, since we have informativeness measures for more than 100 stocks starting on that year. Informativeness in year t is computed over a rolling window of 40 quarters prior.

Table SupApp-2: Cross-sectional results, annual data

	Estimate	Std. Error	t-stat
Size	0.0085	0.000431	19.77
Value	-0.0124	0.001650	-7.51
Turnover	0.0011	0.000086	12.30
Idiosyncratic Volatility	-0.1777	0.019008	-9.35
Institutional Ownership	0.0718	0.004295	16.71

Note: Table SupApp-2 reports the estimates ($\widehat{a_1^c}$) of panel regressions of price informativeness on cross-sectional characteristics (in twentiles) with year fixed effects (ξ_t): $\tau_{\pi,t}^{R,b} = a_0^c + a_1^c c_t^b + \xi_t + \epsilon_{b,t}$, where $\tau_\pi^{R,b,t}$ denotes the average price informativeness per bin (twentile) in a given period, c_t^b denotes the value of the given characteristic per bin (twentile) in a given period, ξ_t denotes a year fixed effect, a_0^c and a_1^c are parameters, and $\epsilon_{b,t}$ is an error term. Size is measured as the natural log of stock market capitalization, value is measured as the ratio between a stock's book value and its market capitalization, turnover is measured as the between trading volume and shares outstanding, idiosyncratic volatility is measured as the standard deviation — over a 30 month period — of the difference between the returns of a stock and the market return, and institutional ownership is measured as the proportion of a stock held by institutional investors.

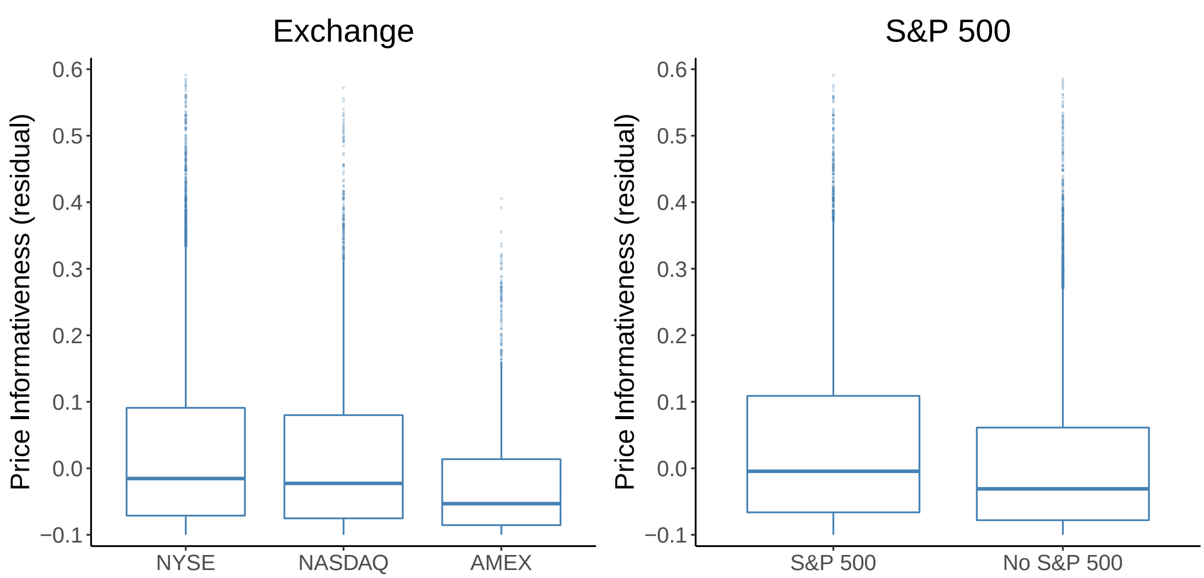

Figure SupApp-1: Cross-sectional results, annual data

Note: The left panel in Figure SupApp-1 shows a box plot by exchange of the residuals of a regression of relative price informativeness on year fixed effects. The left panel in Figure SupApp-1 shows a box plot by S&P 500 status of the residuals of a regression of relative price informativeness on year fixed effects. The solid middle line represents the median. The top and bottom of the box represent the 75th and 25th percentiles. The whiskers extend up to 1.5 times the interquartile range.

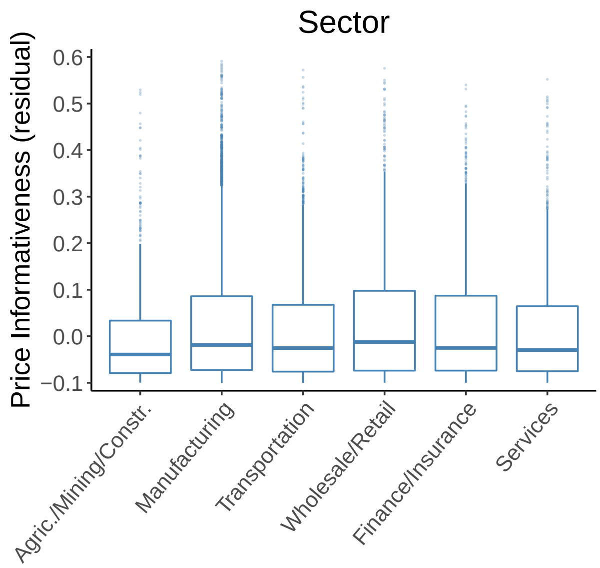

Figure SupApp-2: Cross-sectional results, annual data

Note: Figure SupApp-2 shows a box plot by one-digit SIC industry code of the residuals of a regression of relative price informativeness on year fixed effects. The solid middle line represents the median. The top and bottom of the box represent the 75th and 25th percentiles. The whiskers extend up to 1.5 times the interquartile range.

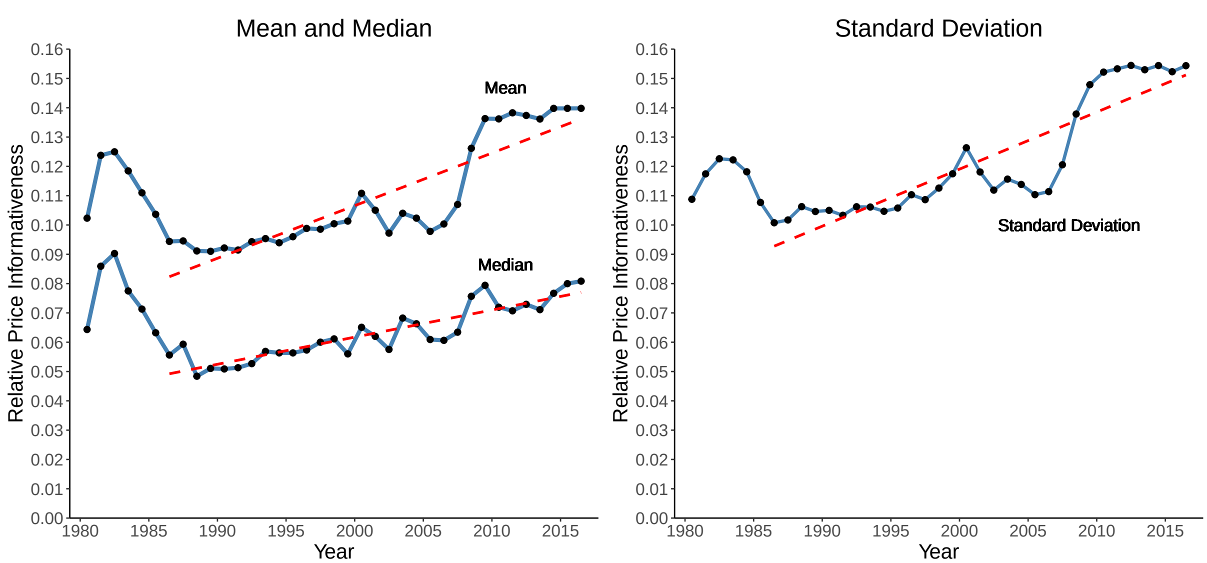

Figure SupApp-3: Price informativeness over time, annual data

Note: The left panel in Figure SupApp-3 shows the time-series evolution of the cross-sectional mean and median relative price informativeness. The right panel in Figure SupApp-3 shows the time-series evolution of the cross-sectional standard deviation of price informativeness. The red dashed lines show linear trends starting in 1986. In both panels, the dots correspond to the average within a year of the price informativeness measures computed using annual data.

B Additional Results

B.1 Absolute Price Informativeness

Proposition 5. (Identifying absolute price informativeness) *Let $\overline{\beta}$, β_0, and β_1 denote the coefficients of the following regression of log-price differences on realized and future log-payoff differences, then*

$$\Delta p_t = \overline{\beta} + \beta_0 \Delta x_t + \beta_1 \Delta x_{t+1} + e_t, \tag{R1}$$

where $\Delta p_t = p_t - p_{t-1}$ denotes the date t change in log-price, and $\Delta x_t = x_t - x_{t-1}$ and $\Delta x_{t+1} = x_{t+1} - x_t$ respectively denote the date t and $t+1$ log-payoff differences.

Proof. By comparing Regression R1 with the structural Equation (2), it follows that $\overline{\beta} = \overline{\phi} + \phi_n \mu_{\Delta n}$, $\beta_0 = \phi_0$, $\beta_1 = \phi_1$, and $e_t = \phi_n \varepsilon_t^{\Delta n}$. Consequently, $\sigma_e^2 = \mathbb{V}\mathrm{ar}\left[e_t\right] = \left(\phi_n\right)^2 \mathbb{V}\mathrm{ar}\left[\varepsilon_t^{\Delta n}\right] = \left(\phi_n\right)^2 \tau_{\Delta n}^{-1}$. Therefore, we can recover absolute price informativeness as follows

$$\tau_\pi = \frac{\left(\beta_1\right)^2}{\sigma_e^2} = \frac{\left(\phi_1\right)^2}{\left(\phi_n\right)^2 \tau_{\Delta n}^{-1}} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n}.$$

Given the assumptions on u_t and Δn_t, it is straightforward to show that the OLS estimates of Regressions R1 and R2 are consistent, which implies that price informativeness can be consistently estimated as $\widehat{\tau_\pi} = \frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}$. Formally, $\operatorname{plim}\left(\widehat{\tau_\pi}\right) = \operatorname{plim}\left(\frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}\right) = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n} = \tau_\pi$. $\square$

B.2 Kalman Gain for a Bayesian External Observer

The priors of a Bayesian external observer over u_t (allowing here for $\mu_u \neq 0$) and Δn_t are given by

$$u_t \sim N\left(\mu_u, \tau_u^{-1}\right) \quad \text{and} \quad \Delta n_t \sim N\left(\mu_{\Delta n}, \tau_{\Delta n}^{-1}\right).$$

The unbiased signal contained in the price about the innovation to the payoff is

$$\pi_t = u_t + \frac{\phi_n}{\phi_1}\left(\Delta n_t - \mu_{\Delta n}\right),$$

where

$$\pi_t | \, u_t \sim N\left(u_t, \tau_\pi^{-1}\right).$$

A standard application of Bayesian updating (Vives, 2008; Veldkamp, 2011) immediately implies that the posterior distribution of an external observer who makes use of the price as a signal about the innovation to the payoff is given by

$$\hat{\mu}_t \equiv \mathbb{E}\left[u_t | \, \pi_t\right] = \left(1 - K\right) \mu_t + K \pi_t \tag{39}$$

$$\hat{\tau}_u \equiv \mathbb{V}\mathrm{ar}\left[u_t | \, \pi_t\right] = \left(\tau_u + \tau_\pi\right)^{-1}, \tag{40}$$

where $K = \frac{\tau_\pi}{\tau_u + \tau_\pi}$ is the Kalman gain, which represents the weight that a Bayesian observer who only learns from the price puts on the information contained in the price. Equations (39) and (40) are equivalent to Equation (7) in the text.

To see that the reduction in uncertainty about the innovation to the payoff for the external observer is given by the Kalman gain, note that

$$1 - \frac{\mathbb{V}\text{ar}\left[u_t | \pi_t\right]}{\mathbb{V}\text{ar}\left[u_t\right]} = 1 - \frac{\left(\tau_u + \tau_\pi\right)^{-1}}{\tau_u^{-1}} = \frac{\tau_\pi}{\tau_u + \tau_\pi} = K.$$

B.3 Learning about Future Prices

In the body of the paper, we have looked at the information about the next period's payoff that is contained in the price. However, there are other measures that may be of interest to investors when making decisions. For example, an external observer may want to learn about the next period's price from the current price. In this subsection, we adapt our analysis to price informativeness about the next period's price.

We assume that the price process at date t is given by

$$\Delta p_t = \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta n_t. \tag{41}$$

Iterating this expression forward and using the process for the change in payoff we can express the current price difference in terms of the future price difference, as follows

$$\Delta p_t = \overline{\phi}^p + \phi_0 \Delta x_t + \phi_1^p \Delta p_{t+1} + e_t^p, \tag{42}$$

where $\overline{\phi}^p \equiv \overline{\phi} - \frac{\phi_1}{\phi_0 + \rho\phi_1}\left(\overline{\phi} + \phi_1 \mu_{\Delta x}\right)$, $\phi_1^p \equiv \frac{\phi_1}{\phi_0 + \rho\phi_1}$, and $e_t^p \equiv \phi_n \Delta n_t - \frac{\phi_1}{\phi_0 + \rho\phi_1}\left(\phi_1 u_{t+1} + \phi_n \Delta n_{t+1}\right)$. Hence, the unbiased signal contained in the price about the next period's price is

$$\tilde{\pi}_t = \frac{\Delta p_t - \left(\overline{\phi}^p + \phi_0 \Delta x_t + \mathbb{E}_t\left[e_t^p\right]\right)}{\phi_1^p} = \Delta p_{t+1} + \frac{1}{\phi_1^p}\left(e_t^p - \mathbb{E}_t\left[e_t^p\right]\right).$$

Then, absolute and relative price informativeness are respectively given by

$$\tau_{\tilde{\pi}} = \mathbb{V}\text{ar}\left[\tilde{\pi}_t | x_t, \Delta p_{t+1}\right]^{-1} = \mathbb{V}\text{ar}\left[\left.\frac{1}{\phi_1^p} e_t^p \right| x_t, \Delta p_{t+1}\right]^{-1} \quad \text{and} \quad \tau_{\tilde{\pi}}^R \equiv \frac{\tau_{\tilde{\pi}}}{\tau_{\tilde{\pi}} + \tau_{\Delta p}}. \tag{43}$$

From Equation (42) and the definition of e_t^p it follows that we cannot recover price informativeness from regressions of prices on future prices, since the error term of that regression would not be orthogonal to the future price and, hence, the estimated coefficients would be biased estimates of the structural parameters. However, using the structural mapping between the equilibrium price process and the coefficients in Regressions R1 and R2 we can recover price informativeness about the next period's price as the proposition below shows.

Proposition 6. (Identifying price informativeness about future price) *Relative price informativeness about the future price can be recovered and consistently estimated from Regressions R1 and R2 as follows:*

$$\tau_{\tilde{\pi}}^R = \frac{1}{1 + \left(1 - R_{\Delta x}^2 + \left(\frac{\zeta_0}{\beta_1}\right)^2 \left(1 - R_{\Delta x, \Delta x'}^2\right)\right)}.$$

While we focus our empirical implementation on the behavior of price informativeness about future payoffs, there is scope to study further the behavior of price informativeness about future prices. A similar approach can be used to identify price informativeness at different horizons.

B.3.1 Proof of Proposition 6 (Identifying price informativeness about future prices)

Iterating Equation (41) forward and substituting, we find that

$$\Delta p_{t+1} = \overline{\phi} + \phi_1 \Delta \mu_x + (\phi_0 + \rho\phi_1)\,\Delta x_{t+1} + \phi_1 u_{t+1} + \phi_n \Delta n_{t+1},$$

where we substituted for the process for Δx_{t+2}. This last equation implies that

$$\Delta x_{t+1} = \frac{\Delta p_{t+1} - \left(\overline{\phi} + \phi_1 \mu_{\Delta x} + \phi_1 u_{t+1} + \phi_n \Delta n_{t+1}\right)}{\phi_0 + \rho\phi_1}.$$

Using this expression in (41) we can express the current price difference in terms of the future price difference, as follows:

$$\Delta p_t = \overline{\phi} - \frac{\phi_1}{\phi_0 + \rho\phi_1}\left(\overline{\phi} + \phi_1 \mu_{\Delta x}\right) + \phi_0 \Delta x_t + \frac{\phi_1}{\phi_0 + \rho\phi_1}\Delta p_{t+1} - \frac{\phi_1}{\phi_0 + \rho\phi_1}\left(\phi_1 u_{t+1} + \phi_n \Delta n_{t+1}\right) + \phi_n \Delta n_t.$$

The unbiased signal contained in the price about the next period's price is

$$\tilde{\pi}_t = \Delta p_{t+1} - \left(\phi_1 u_{t+1} + \phi_n \varepsilon^{\Delta n}_{t+1}\right) + \frac{\phi_n}{\frac{\phi_1}{\phi + \rho\phi_1}}\varepsilon^{\Delta n}_t.$$

Absolute price informativeness is given by

$$\tau_{\tilde{\pi}} = \mathbb{V}\mathrm{ar}\left[\tilde{\pi}_t \middle| x_t, \Delta p_{t+1}\right]^{-1} = \left(\mathbb{V}\mathrm{ar}\left(\phi_1 u_{t+1} + \phi_n \varepsilon^{\Delta n}_{t+1}\right) + \left(\frac{\phi_0 + \rho\phi_1}{\phi_1}\right)^2 \mathbb{V}\mathrm{ar}\left(\phi_n \varepsilon^{\Delta n}_{t+1}\right)\right)^{-1}$$

and relative price informativeness is $\tau^R_{\tilde{\pi}} = \frac{\tau_{\tilde{\pi}}}{\tau_{\tilde{\pi}} + \tau_{\Delta p}}$. From the structural mapping of the coefficients in Regressions R1-PS and R2-PS, it follows that $e_t = \phi_n \varepsilon^{\Delta n}_t$ and $e^{\zeta}_t = \phi_1 u_{t+1} + \phi_n \varepsilon^{\Delta n}_t$. Moreover, since innovations to the noise are i.i.d., it follows that $\mathbb{V}\mathrm{ar}\left(\phi_1 u_{t+1} + \phi_n \varepsilon^{\Delta n}_{t+1}\right) = \mathbb{V}\mathrm{ar}\left(\phi_1 u_{t+1} + \phi_n \varepsilon^{\Delta n}_t\right)$. Moreover, $\beta_1 = \phi_1$ and $\zeta_0 = \phi_0 + \rho\phi_1$. Then

$$\tau_{\tilde{\pi}} = \left(\mathbb{V}\mathrm{ar}\left[e^{\zeta}_t\right] + \left(\frac{\zeta_0}{\beta_1}\right)^2 \mathbb{V}\mathrm{ar}\left[e_t\right]\right)^{-1}. \tag{44}$$

Using Equation (44), we have that

$$\frac{\tau_{\Delta p}}{\tau_{\tilde{\pi}}} = \frac{\mathbb{V}\mathrm{ar}\left[e^{\zeta}_t\right] + \left(\frac{\zeta_0}{\beta_1}\right)^2 \mathbb{V}\mathrm{ar}\left[e_t\right]}{\mathbb{V}\mathrm{ar}\left[\Delta p\right]}.$$

Then, since $\frac{\mathbb{V}\mathrm{ar}\left[e^{\zeta}_t\right]}{\mathbb{V}\mathrm{ar}[\Delta p]} = 1 - R^2_{\Delta x}$ and $\frac{\mathbb{V}\mathrm{ar}[e_t]}{\mathbb{V}\mathrm{ar}[\Delta p]} = 1 - R^2_{\Delta x, \Delta x'}$, it follows that

$$\tau^R_{\tilde{\pi}} = \frac{1}{1 + \left(1 - R^2_{\Delta x} + \left(\frac{\zeta_0}{\beta_1}\right)^2\left(1 - R^2_{\Delta x, \Delta x'}\right)\right)}.$$

C Asset Demand Model

C.1 Environment and Results

Here, we show that one can directly specify investors' asset demand and information structure, along with a payoff process, to identify price informativeness. This is an intermediate approach between using a fully specified model, as in Sections 3.1 to 3.3, and directly postulating an equilibrium pricing equation, as in Section 2.

Consider a discrete time environment with a continuum of investors, indexed by $i \in I$, who trade a risky asset in fixed supply at a (log) price p_t each date $t = 0, 1, \ldots, \infty$. Assume that the (log) payoff of the risky asset in period $t+1$, x_{t+1}, is given by the following stationary AR(1) process in differences:

$$\Delta x_{t+1} = \mu_{\Delta x} + \rho \Delta x_t + u_t,$$

where $\mu_{\Delta x}$ is a scalar, $|\rho| < 1$, and where the innovations to the payoff, u_t, have mean zero, finite variance, and are independently distributed. Investors trade in period t with imperfect information about the innovation to the payoff, u_t, which is realized at the beginning of the period and observed at the end of the period. When trading in period $t+1$, the contemporaneous payoff u_t has already been realized and is common knowledge to all investors.

Each period t, an investor i observes a private signal s_t^i of the innovation to the payoff u_t.[13] Investors have an additional motive for trading the risky asset that is orthogonal to the asset payoff. We denote by $\overline{n}_t^i$ investor i's additional trading motive in period t. These additional trading motives are private information of each investor and are random in the aggregate.

We rederive the main result of the paper under two assumptions. The first assumption imposes an additive informational structure and guarantees the existence of second moments, while the second assumption imposes a linear structure for investors' equilibrium asset demands. In general, linear demands can be interpreted as a first-order approximation to other forms of asset demands, so one may expect our results to approximately hold in a larger class of models. Both assumptions facilitate the aggregation of individual demands in order to yield a linear equilibrium pricing function.

Assumption 1. (Additive noise) *Each period t, every investor i receives an unbiased private signal s_t^i about the innovation to the payoff, u_t, of the form*

$$s_t^i = u_t + \varepsilon_{st}^i,$$

where ε_{st}^i for all t and all i, are random variables with mean zero and finite variances, whose realizations are independent across investors and over time. Each period t, every investor i has a private trading need $\overline{n}_t^i$, of the form

$$\overline{n}_t^i = n_t + \varepsilon_{\overline{n}t}^i,$$

where Δn_t is a random variable with finite mean, denoted by $\mu_{\Delta n}$, and finite variance, and where $\varepsilon_{\overline{n}t}^i$ for all t and all i, are random variables with mean zero and finite variances, whose realizations are independent across investors and over time.

Assumption 1 imposes restrictions on the noise structure in the signals about the innovation to the payoff u_t and on all other sources of investors' private trading needs by making them additive and

[13] Assuming that investors observe private signals about the payoff, x_{t+1}, or its innovation, u_t, is formally equivalent, since x_t is known to investors when trading in period t.

independent across investors. This assumption does not restrict the distribution of any random variable beyond the existence of finite first and second moments. Our second assumption describes the structure of the investors' demands for the risky asset θ_t^i.

Assumption 2. (Linear asset demands) *Investors' asset demands satisfy*

$$\theta_t^i = \alpha_s^i s_t^i + \alpha_x^i x_t + \alpha_n^i \overline{n}_t^i - \alpha_p^i p_t + \psi^i,$$

where α_s^i, α_x^i, α_n^i, α_p^i, and ψ^i are individual demand coefficients, determined in equilibrium.

Assumption 2 imposes a linear structure on the individual investors' net asset demand for the risky asset. More specifically, it imposes that an individual investor's demand is linear in his signal about the payoff and his private trading needs, as well as in the asset price p_t and the current payoff realization x_t. It also allows for an individual specific invariant component ψ^i. All the models explored in Section 3 are consistent with Assumptions 1 and 2.

Lemma 4. *The price process assumed in Equation* (2) *in the general framework in Section 2 can be obtained endogenously when Assumptions 1 and 2 are satisfied.*

Note that these assumptions allow for rich cross-sectional heterogeneity among investors. In particular, it accommodates heterogeneity in investors' risk aversion, in the precision of their information, and in the distribution of their idiosyncratic trading motives. As the applications above show, our assumptions can accommodate models with informed and uninformed traders, which can be mapped to environments in which one set of agents does not observe any private signal, and those with classic noise traders, which can be mapped to environments in which one set of agents trades fixed amounts regardless of the price or other features of the environment. Given that linear asset demands can be interpreted as an approximation to more general models, Lemma 4 implies that our results should be valid more broadly in an approximate sense.

C.2 Proof of Lemma 4

From Assumption 2 and market clearing it follows that

$$Q = \int \alpha_s^i s_t^i w_0^i di + \int \alpha_x^i w_0^i di x_t + \int \alpha_n^i \overline{n}_t^i w_0^i di - \int \alpha_p^i w_0^i di p_t + \int \psi^i w_0^i di.$$

Using Assumption 1, we have

$$Q = \int \alpha_s^i \left(u_t + \varepsilon_{st}^i\right) w_0^i di + \overline{\alpha_x} x_t + \int \alpha_n^i \left(n_t + \varepsilon_{nt}^i\right) w_0^i di - \overline{\alpha_p} p_t + \int \psi^i w_0^i di.$$

Using the Strong Law of Large Numbers, since the sequence of independent random variables $\left\{\alpha_h^i w_0^i \tau_h^{-1}\right\}$ has uniformly bounded variance and finite means for $h = \{s, n\}$, we have the following equilibrium price

$$p_t = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_t + \frac{\overline{\psi}}{\overline{\alpha_p}}.$$

Taking differences gives

$$\Delta p_t = \overline{\phi} + \phi_0 \Delta x_t + \phi_1 \Delta x_{t+1} + \phi_n \Delta n_t,$$

where $\overline{\phi} = 0$, $\phi_0 = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \rho \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\phi_1 = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, and $\phi_n = \frac{\overline{\alpha_n}}{\overline{\alpha_p}}$. This proves our result.

D Alternative Modeling Frameworks

Our identification results extend to any linear or log-linear setup. In this section, we illustrate how to extend our results in the context of three different specifications. First, we extend our approximate results to the case in which the payoff follows a stationary AR(1) process. Second, we develop our identification results using an exact linear formulation for the price process under difference-stationary and stationary specifications for the payoff. Third, we also provide the respective CARA-Normal models to microfound these exact linear formulations.

D.1 Log-Linear Model in Levels

General framework and identification

We consider a discrete time environment with dates $t = 0, 1, 2, \ldots, \infty$, in which investors trade a risky asset in fixed supply at a (log) price p_t at each date t. We assume that the (log) payoff of the risky asset at date $t+1$, x_{t+1}, follows a stationary AR(1) process

$$x_{t+1} = \mu_x + \rho x_t + u_t, \tag{45}$$

where μ_x is a scalar, $|\rho| < 1$, and where the innovations to the payoff, u_t, have mean zero, a finite variance denoted by $\mathbb{V}\text{ar}[u_t] = \sigma_u^2 = \tau_u^{-1}$, and are identically and independently distributed over time.[14] We assume that the equilibrium price is given by

$$p_t = \overline{\phi} + \phi_0 x_t + \phi_1 x_{t+1} + \phi_n n_t, \tag{46}$$

where $\overline{\phi}$, ϕ_0, ϕ_1, and ϕ_n are parameters and where n_t represents the aggregate component of investors' trading motives that are orthogonal to the asset payoff, given by $n_t = \mu_n + \varepsilon_t^n$, where $\mathbb{E}[\varepsilon_t^n] = 0$ and $\mathbb{V}\text{ar}[\varepsilon_t^n] = \sigma_n^2 = \tau_n^{-1}$. For simplicity, we assume that u_t and n_t are independent.

In this environment, the unbiased signal of the innovation to future payoffs u_t contained in the *price level*, which we denote by $\tilde{\pi}_t$, is given by

$$\tilde{\pi}_t \equiv \frac{p_t - \left(\overline{\phi} + \phi_1 \mu_x + \phi_n \mu_n + (\phi_0 + \rho \phi_1) x_t\right)}{\phi_1} = u_t + \frac{\phi_n}{\phi_1}(n_t - \mu_n)$$

and absolute and relative price informativeness are respectively given by

$$\tau_{\tilde{\pi}} \equiv \left(\mathbb{V}\text{ar}[\tilde{\pi}_t | x_{t+1}, x_t]\right)^{-1} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_n \quad \text{and} \quad \tau_{\tilde{\pi}}^R \equiv \frac{\tau_{\tilde{\pi}}}{\tau_{\tilde{\pi}} + \tau_u}.$$

Proposition 7. (Identifying price informativeness: log-linear case)

a) Absolute price informativeness. Let $\overline{\beta}$, β_0, and β_1 denote the coefficients of the following regression of log-prices on realized and future log-payoffs:

$$p_t = \overline{\beta} + \beta_0 x_t + \beta_1 x_{t+1} + e_t, \tag{R1-LL}$$

where p_t denotes the date t log-price, x_t and x_{t+1} respectively denote the dates t and $t+1$ log-payoff, and where $\sigma_e^2 = \mathbb{V}ar[e_t]$ denotes the variance of the error. Then, absolute price informativeness, τ_π, can be

[14] As in the body of the paper, we index the innovation to the date $t+1$ payoff u_t by t — instead of $t+1$ — because investors may be able to learn about it at date t.

recovered by

$$\tau_{\tilde{\pi}} = \frac{\beta_1^2}{\sigma_e^2}.$$

The OLS estimation of Regression R1-LL yields consistent estimates of β_1 and σ_e^2.

b) Relative Price Informativeness. Let $R^2_{x,x'}$ denote the R-squared of Regression R1-LL. Let R^2_x, ζ, and ζ_0 respectively denote the R-squared and the coefficients of the following regression of log-price on log-payoff,

$$p_t = \overline{\zeta} + \zeta_0 x_t + e_t^{\zeta}. \tag{R2-LL}$$

Then, relative price informativeness, $\tau^R_{\tilde{\pi}}$, can be recovered by

$$\tau^R_{\tilde{\pi}} = \frac{R^2_{x,x'} - R^2_x}{1 - R^2_x}.$$

The OLS estimation of Regressions R1-LL and R2-LL yields consistent estimates of $R^2_{x,x'}$ and R^2_x.

Proof. a) By comparing Regression R1-LL with the structural Equation (46), it follows that $\overline{\beta} = \overline{\phi} + \phi_n \mu_n$, $\beta_0 = \phi_0$, $\beta_1 = \phi_1$, and $e_t = \phi_n \varepsilon_t^n$. Consequently, $\sigma_e^2 = \mathbb{V}\mathrm{ar}[e_t] = (\phi_n)^2 \mathbb{V}\mathrm{ar}[\varepsilon_t^n] = (\phi_n)^2 \tau_n^{-1}$. Therefore, we can recover absolute price informativeness as follows:

$$\tau_{\tilde{\pi}} = \frac{(\beta_1)^2}{\sigma_e^2} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_n.$$

Given Equations (45) and (46), as well as the assumptions on u_t and n_t, it is straightforward to show that the OLS estimates of Regressions R1-LL and R2-LL are consistent, which implies that price informativeness can be consistently estimated as $\widehat{\tau_{\tilde{\pi}}} = \frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}$. Formally, $\mathrm{plim}\left(\widehat{\tau_{\tilde{\pi}}}\right) = \mathrm{plim}\left(\frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}\right) = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_n = \tau_\pi$.

b) Note that the R-squareds of Regressions R1-LL and R2-LL can be expressed as follows

$$R^2_{x,x'} = 1 - \frac{\mathbb{V}\mathrm{ar}(e_t)}{\mathbb{V}\mathrm{ar}(p_t)} \quad \text{and} \quad R^2_x = \frac{\mathbb{V}\mathrm{ar}(\zeta_0 x_t)}{\mathbb{V}\mathrm{ar}(p_t)}.$$

After substituting Equation (45) in Regression R1-LL, the following relation holds

$$p_t = \overline{\phi} + \phi_1 \mu_x + \phi_n \mu_n + (\phi_0 + \rho\phi_1) x_t + \phi_1 u_t + \phi_n \varepsilon_t^n. \tag{47}$$

By comparing Regression R2-LL with the structural Equation (47), it follows that $\overline{\zeta} = \overline{\phi} + \phi_1 \mu_x + \phi_n \mu_n$, $\zeta_0 = \phi_0 + \rho\phi_1$, and $\varepsilon_t^{\zeta} = \phi_1 u_t + \phi_n \varepsilon_t^n$.

From Equation (47), the following variance decomposition must hold

$$\begin{aligned}\mathbb{V}\mathrm{ar}(p_t) &= \mathbb{V}\mathrm{ar}(\zeta_0 x_t) + \mathbb{V}\mathrm{ar}(\phi_1 u_t + \phi_n \varepsilon_t^n) \\ &= \mathbb{V}\mathrm{ar}(\zeta_0 x_t) + (\phi_1)^2 \mathbb{V}\mathrm{ar}(u_t) + \mathbb{V}\mathrm{ar}(e_t),\end{aligned}$$

which can be rearranged to express $\frac{\tau_{\tilde{\pi}}}{\tau_u}$ as follows:

$$1 = \underbrace{\frac{\mathbb{V}\mathrm{ar}(\zeta_0 x_t)}{\mathbb{V}\mathrm{ar}(p_t)}}_{R^2_x} + \underbrace{\frac{\mathbb{V}\mathrm{ar}(e_t)}{\mathbb{V}\mathrm{ar}(p_t)}}_{1-R^2_{x,x'}} \left(\underbrace{\frac{(\phi_1)^2}{\mathbb{V}\mathrm{ar}(e_t)} \mathbb{V}\mathrm{ar}(u_t)}_{\frac{\tau_{\tilde{\pi}}}{\tau_u}} + 1 \right) \Rightarrow \frac{\tau_{\tilde{\pi}}}{\tau_u} = \frac{R^2_{x,x'} - R^2_x}{1 - R^2_{x,x'}}.$$

Therefore, relative price informativeness can be written as

$$\tau_{\tilde{\pi}}^{R}=\frac{\tau_{\tilde{\pi}}}{\tau_{\tilde{\pi}}+\tau_{u}}=\frac{1}{1+\frac{1}{\frac{\tau_{\tilde{\pi}}}{\tau_{u}}}}=\frac{R_{x,x'}^{2}-R_{x}^{2}}{1-R_{x}^{2}}.$$

$\square$

Microfoundation

Time is discrete, with dates denoted by $t=0,1,2,\ldots,\infty$. The economy is populated by a continuum of investors, indexed by $i\in I$, who live for two dates. An investor born at date t has well-behaved expected utility preferences over terminal wealth w_1^i, with flow utility given by $U_i\left(w_1^i\right)$, where $U_i'(\cdot)>0$ and $U_i''(\cdot)<0$.

There are two long-term assets in the economy: a risk-free asset in perfectly elastic supply, with gross return $R^f>1$, and a risky asset in fixed supply Q, whose date t (log) payoff is $x_t=\ln(X_t)$ and which trades at a (log) price $p_t=\ln(P_t)$. The process followed by x_t is given by

$$x_{t+1}=\mu_x+\rho x_t+u_t,$$

where $\Delta x_{t+1}=x_{t+1}-x_t$, μ_x is a scalar, $|\rho|<1$, and $x_0=\Delta x_0=0$. The realized payoff x_t is common knowledge to all investors before the price p_t is determined. The realized payoff at date $t+1$, x_{t+1}, is only revealed to investors at date $t+1$.

We assume that investors receive private signals about the innovation to the risky asset payoff. Formally, each investor receives a signal about the payoff innovation u_t given by

$$s_t^i=u_t+\varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i\sim N\left(0,\tau_s^{-1}\right),$$

where $\varepsilon_{st}^i\perp\varepsilon_{st}^j$ for all $i\neq j$, and $u_t\perp\varepsilon_{st}^i$ for all t and all i.

We also assume that investors also have private trading motives that arise from random heterogeneous priors that are random in the aggregate. Formally, each investor i born at date t has a prior over u_t given by

$$u_t\sim_i N\left(\overline{n}_t^i,\tau_u^{-1}\right),$$

where

$$\overline{n}_t^i=n_t+\varepsilon_{\overline{n}t}^i \quad \text{with} \quad \varepsilon_{\overline{n}t}^i\overset{\text{iid}}{\sim}N\left(0,\tau_{\overline{n}}^{-1}\right),$$

and

$$n_t=\mu_n+\varepsilon_t^n \quad \text{with} \quad \varepsilon_t^n\sim N\left(0,\tau_n^{-1}\right),$$

where μ_n is a scalar, and where $\varepsilon_t^n\perp\varepsilon_{\overline{n}t}^i$ for all t and all i. The variable n_t, which can be interpreted as the aggregate sentiment in the economy, is not observed and acts as a source of aggregate noise, preventing the asset price from being fully revealing.

Each investor i born at date t is endowed with wealth w_0^i, and optimally chooses a portfolio share in the risky asset, denoted by θ_t^i, to solve

$$\max_{\theta_t^i}\mathbb{E}_t^i\left[U_i\left(w_1^i\right)\right] \tag{48}$$

subject to a wealth accumulation constraint

$$w_1^i = \left(R^f + \theta_t^i \left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\right) w_0^i, \tag{49}$$

where the information set of an investor i in period t is given by $\mathcal{I}_t^i = \left\{s_t^i, \overline{n}_t^i, \{x_s\}_{s\leq t}, \{p_s\}_{s\leq t}\right\}$.

The optimality condition of an investor who maximizes Equation (48) subject to the wealth accumulation constraint in Equation (49) is given by

$$\mathbb{E}\left[U_i'\left(w_1^i\right)\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\middle| \mathcal{I}_t^i\right] = 0. \tag{50}$$

We approximate an investor's first-order condition in three steps.

First , we take a first-order Taylor expansion of an investor's future marginal utility $U'\left(w_1^i\right)$ around the current date t wealth level w_0^i. Formally, we approximate $U'\left(w_1^i\right)$ as follows

$$U'\left(w_1^i\right) \approx U'\left(w_0^i\right) + U''\left(w_0^i\right)\Delta w_1^i,$$

which allows us to express Equation (50) as

$$U'\left(w_0^i\right)\mathbb{E}_i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right] + U''\left(w_0^i\right) w_0^i \mathbb{E}_i\left[\left(\left(R^f - 1\right) + \theta_t^i\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\right)\left(\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right)\right] \approx 0.$$

Second, we impose that terms that involve the product of two or more net interest rates are negligible. In continuous time, these terms would be of order $(dt)^2$. Formally, it follows that

$$\left(R^f - 1\right)\mathbb{E}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right] \approx 0 \quad \text{and} \quad \left(\mathbb{E}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right]\right)^2 \approx 0,$$

which allows us to express Equation (50) as

$$U'\left(w_0^i\right)\mathbb{E}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t} - R^f\right] + U''\left(w_0^i\right) w_0^i \theta_t^i \mathbb{V}\mathrm{ar}_t^i\left[\frac{X_{t+1} + P_{t+1}}{P_t}\right] \approx 0.$$

Therefore, we can express an investor's risky portfolio share θ_t^i as

$$\theta_t^i \approx \frac{1}{\gamma^i}\frac{\mathbb{E}_t^i\left[\frac{X_{t+1}+P_{t+1}}{P_t} - R^f\right]}{\mathbb{V}\mathrm{ar}_t^i\left[\frac{X_{t+1}+P_{t+1}}{P_t}\right]}, \tag{51}$$

where $\gamma^i \equiv -\frac{w^i U''\left(w^i\right)}{U'\left(w^i\right)}$ denotes the coefficient of relative risk aversion.

Third, as in Campbell and Shiller (1988), we take a log-linear approximation of returns around a predetermined dividend-price ratio. Formally, note that

$$\frac{X_{t+1} + P_{t+1}}{P_t} = e^{\ln\left(\frac{\left(1+\frac{P_{t+1}}{X_{t+1}}\right)\frac{X_{t+1}}{X_t}}{\frac{P_t}{X_t}}\right)},$$

and

$$\ln\left(\frac{X_{t+1} + P_{t+1}}{P_t}\right) = \ln\left(1 + e^{p_{t+1} - x_{t+1}}\right) + \Delta x_{t+1} - \left(p_t - x_t\right),$$

where we define $r^f = \ln R^f$. Following Campbell and Shiller (1988), we approximate the first term around a point $PX = e^{p-x}$, to find that

$$\begin{aligned}\ln\left(1+e^{\ln P_{t+1}-\ln X_{t+1}}\right) &\approx \ln(1+PX) + \frac{PX}{PX+1}\left(p_{t+1}-x_{t+1}-p-x\right)\\ &= k_0 + k_1\left(p_{t+1}-x_{t+1}\right),\end{aligned}$$

where $k_1 = \frac{PX}{PX+1}$ and $k_0 = \ln(1+PX) - k_1(p-x)$.

Therefore, starting from Equation (51), we have that the risky asset demand of an investor i can be approximated as

$$\theta_t^i \approx \frac{1}{\gamma^i}\frac{k_0 + k_1\mathbb{E}_t^i\left[p_{t+1}-x_{t+1}\right] + \mathbb{E}_t^i\left[\Delta x_{t+1}\right] - (p_t - x_t) - r^f}{\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-x_{t+1}\right) + \Delta x_{t+1}|\mathcal{I}_t^i\right]}, \tag{52}$$

where we define $r^f \equiv \ln R^f$ and we used that $e^y \approx 1 + y$.

In order to characterize the equilibrium it is necessary to characterize investors' expectations. We conjecture and subsequently verify that $k_1\mathbb{E}_t^i\left[p_{t+1}-x_{t+1}\right] + \mathbb{E}_t^i\left[\Delta x_{t+1}\right]$ is linear in s_t^i, $\overline{n}_t^i$, and x_t and that $\mathbb{V}\mathrm{ar}\left[k_1\left(p_{t+1}-x_{t+1}\right) + \Delta x_{t+1}\right]$ is a constant, which we denote by V. Under this conjecture, θ_t^i is a linear function of s_t^i, x_t, and $\overline{n}_t^i$, and it is given by

$$\theta_t^i \approx \alpha_x^i x_t + \alpha_s^i s_t^i + \alpha_n^i \overline{n}_t^i - \alpha_p^i p_t + \psi^i.$$

Using this expression and the market clearing condition $\int \theta_t^i w_0^i di = Q$ implies

$$p_t = \frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_t + \frac{\overline{\psi}}{\overline{\alpha_p}}.$$

This expression can also be written as

$$p_t = \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\rho\right) x_t + \frac{\overline{\alpha_s}}{\overline{\alpha_p}} x_{t+1} + \frac{\overline{\alpha_n}}{\overline{\alpha_p}} n_t + \left(\frac{\overline{\psi}}{\overline{\alpha_p}} - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\mu_x\right).$$

Investors in the model learn from the price. The information contained in the price for an investor in the model is

$$\hat{\pi}_t = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p_t - \left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t + \frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_n - \frac{\overline{\psi}}{\overline{\alpha_p}}\right)\right)$$

which has a precision

$$\tau_{\hat{\pi}} \equiv \mathbb{V}\mathrm{ar}\left[\left.\hat{\pi}_t\right| u_t, x_t\right]^{-1} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \tau_n.$$

Then,

$$\mathbb{E}_t^i\left[u_t\right] = \mathbb{E}\left[u_t|s_t^i, \overline{n}_t^i, p_t\right] = \frac{\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_{\hat{\pi}}\hat{\pi}_t}{\tau_s + \tau_u + \tau_{\hat{\pi}}} = \frac{\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_{\hat{\pi}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p_t - \frac{\overline{\alpha_x}}{\overline{\alpha_p}} x_t - \frac{\overline{\alpha_n}}{\overline{\alpha_s}}\mu_n - \frac{\overline{\psi}}{\overline{\alpha_p}}\right)}{\tau_s + \tau_u + \tau_{\hat{\pi}}}$$

and

$$\mathbb{V}\mathrm{ar}\left[u_t|\mathcal{I}_t^i\right] = \left(\tau_s + \tau_u + \tau_{\hat{\pi}}\right)^{-1}.$$

Note that these two expressions imply that our conjectures above are satisfied. To see this note that

$$
\begin{aligned}
k_1\mathbb{E}_t^i\left[p_{t+1}-x_{t+1}\right]+\mathbb{E}_t^i\left[\Delta x_{t+1}\right] &= k_1\mathbb{E}_t^i\left[\frac{\overline{\alpha_x}}{\overline{\alpha_p}}x_{t+1}+\frac{\overline{\alpha_s}}{\overline{\alpha_p}}u_{t+1}+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_{t+1}+\frac{\overline{\psi}}{\overline{\alpha_p}}-x_{t+1}\right]+\mathbb{E}_t^i\left[\mu_x+(\rho-1)x_t+u_t\right] \\
&= k_1\left(\mathbb{E}_t^i\left[\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1\right)x_{t+1}+\frac{\overline{\alpha_s}}{\overline{\alpha_p}}u_{t+1}\right]+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_n+\frac{\overline{\psi}}{\overline{\alpha_p}}\right)+(\rho-1)x_t+\mu_x+\mathbb{E}_t^i\left[u_t\right] \\
&= k_1\left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1\right)\left(\mu_x+\mathbb{E}_t^i\left[u_t\right]\right)+\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1\right)\rho x_t+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_n+\frac{\overline{\psi}}{\overline{\alpha_p}}\right)+(\rho-1)x_t+\mu_x+\mathbb{E}_t^i\left[u_t\right] \\
&= k_1\left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\left(\mu_x+\mathbb{E}_t^i\left[u_t\right]\right)+\left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1\right)\rho+\frac{(\rho-1)}{k_1}\right)x_t+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_n+\frac{\overline{\psi}}{\overline{\alpha_p}}\right)
\end{aligned}
$$

where we used that $\mathbb{E}_t^i\left[u_{t+1}\right]=0$ and that $\mathbb{E}_t^i\left[\varepsilon_{t+1}^n\right]=0$. Moreover,

$$
\begin{aligned}
\mathbb{V}\text{ar}\left[k_1\left(p_{t+1}-x_{t+1}\right)+\Delta x_{t+1}|\mathcal{I}_t^i\right] &= \mathbb{V}\text{ar}\left[k_1\left(\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1\right)x_{t+1}+\frac{\overline{\alpha_s}}{\overline{\alpha_p}}u_{t+1}+\frac{\overline{\alpha_n}}{\overline{\alpha_p}}n_{t+1}\right)+u_t|\mathcal{I}_t^i\right] \\
&= k_1^2\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)^2\mathbb{V}\text{ar}\left[u_t|\mathcal{I}_t^i\right]+k_1^2\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\mathbb{V}\text{ar}\left[u_{t+1}|\mathcal{I}_t^i\right]+k_1^2\left(\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\right)^2\mathbb{V}\text{ar}\left[\varepsilon_t^n|\mathcal{I}_t^i\right] \\
&= k_1^2\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)^2\left(\tau_s+\tau_u+\tau_{\hat{\pi}}\right)^{-1}+k_1^2\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\tau_u^{-1}+k_1^2\left(\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\right)^2\tau_n^{-1}.
\end{aligned}
$$

Using these expressions in the first-order condition and matching coefficients gives

$$
\begin{aligned}
\alpha_x^i &= \frac{1}{\kappa_i}k_1\left(-\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\frac{\tau_\pi\frac{\overline{\alpha_x}}{\overline{\alpha_s}}}{\tau_s+\tau_u+\tau_{\hat{\pi}}}+\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\rho\right) \\
\alpha_s^i &= \frac{1}{\kappa_i}k_1\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\frac{\tau_s}{\tau_s+\tau_u+\tau_{\hat{\pi}}} \\
\alpha_n^i &= \frac{1}{\kappa_i}k_1\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\frac{\tau_u}{\tau_s+\tau_u+\tau_{\hat{\pi}}} \\
\alpha_p^i &= \frac{1}{\kappa_i}\left(k_1\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\frac{\tau_\pi\frac{\overline{\alpha_p}}{\overline{\alpha_s}}}{\tau_s+\tau_u+\tau_{\hat{\pi}}}-1\right) \\
\psi^i &= \frac{1}{\kappa_i}\left(k_0+k_1\left(-\left(\frac{\overline{\alpha_x}}{\overline{\alpha_p}}-1+\frac{1}{k_1}\right)\left(\frac{\tau_\pi\frac{\overline{\alpha_p}}{\overline{\alpha_s}}}{\tau_s+\tau_u+\tau_{\hat{\pi}}}-\mu_x\right)+1\right)+\left(\frac{\overline{\alpha_n}}{\overline{\alpha_p}}\mu_n+\frac{\overline{\psi}}{\overline{\alpha_p}}\right)-r^f\right)
\end{aligned}
$$

where $\kappa_i\equiv\gamma^i\mathbb{V}\text{ar}\left[k_1\left(p_{t+1}-x_{t+1}\right)+\Delta x_{t+1}|\mathcal{I}_t^i\right]$.

In this equilibrium, our guess in Equation (52) is verified and the equilibrium price is linear and can be expressed as in Equation (46).

D.2 An Exact CARA-Normal Formulation

In an earlier version of this paper, we developed our identification results using an exact linear formulation, motivated by the use of a CARA-Normal framework, which is the workhorse model in the learning literature, see e.g., Vives (2008) and Veldkamp (2011). In this section, we reproduce our identification results using these exact linear formulations in the case of difference-stationary and stationary linear payoffs, and we provide microfoundations in the context of CARA-Normal models.

D.2.1 Difference-stationary linear payoff

General framework and identification We consider a discrete time environment with dates $t = 0, 1, 2, \ldots, \infty$, in which investors trade a risky asset in fixed supply at a price P_t at each date t. We assume that the payoff of the risky asset at date $t+1$, X_{t+1}, follows a difference-stationary AR(1) process

$$\Delta X_{t+1} = \mu_{\Delta X} + \rho \Delta X_t + u_t, \tag{53}$$

where $\mu_{\Delta X}$ is a scalar, $|\rho| < 1$, and where the innovations to the payoff, u_t, have mean zero, a finite variance denoted by $\mathbb{V}\mathrm{ar}\left[u_t\right] = \sigma_u^2 = \tau_u^{-1}$, and are identically and independently distributed over time. We assume that the equilibrium price difference is given by

$$\Delta P_t = \overline{\phi} + \phi_0 \Delta X_t + \phi_1 \Delta X_{t+1} + \phi_n \Delta n_t, \tag{54}$$

where $\overline{\phi}$, ϕ_0, ϕ_1, and ϕ_n are parameters and where n_t represents the aggregate component of investors' trading motives that are orthogonal to the asset payoff, given by $\Delta n_t = \mu_{\Delta n} + \varepsilon_t^{\Delta n}$, where $\mathbb{E}\left[\varepsilon_t^{\Delta n}\right] = 0$ and $\mathbb{V}\mathrm{ar}\left[\varepsilon_t^{\Delta n}\right] = \sigma_n^2 = \tau_{\Delta n}^{-1}$. For simplicity, we assume that u_t and Δn_t are independent.

In this case, the unbiased signal of the innovation to the change in the future payoff u_t contained in the price, which we denote by Π_t, is given respectively by

$$\Pi_t \equiv \frac{\Delta P_t - \left(\overline{\phi} + \phi_1 \mu_{\Delta X} + \phi_n \mu_{\Delta n} + \left(\phi_0 + \rho \phi_1\right) \Delta X_t\right)}{\phi_1} = u_t + \frac{\phi_n}{\phi_1}\left(\Delta n_t - \mu_{\Delta n}\right)$$

and absolute and relative price informativeness are given by

$$\tau_\Pi \equiv \left(\mathbb{V}\mathrm{ar}\left[\Pi_t \middle| \Delta X_{t+1}, \Delta X_t\right]\right)^{-1} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n} \quad \text{and} \quad \tau_\Pi^R \equiv \frac{\tau_\Pi}{\tau_\Pi + \tau_u}.$$

Proposition 8. (Identifying price informativeness: difference-stationary linear case)

a) Absolute price informativeness. Let $\overline{\beta}$, β_0, and β_1 denote the coefficients of the following regression of prices on realized and future payoffs:

$$\Delta P_t = \overline{\beta} + \beta_0 \Delta X_t + \beta_1 \Delta X_{t+1} + e_t, \tag{R1-Linear-Diff}$$

where ΔP_t denotes the date t price change, ΔX_t and ΔX_{t+1} respectively denote the dates t and $t+1$ payoff change, and where $\sigma_e^2 = \mathbb{V}ar\left[e_t\right]$ denotes the variance of the error. Then, absolute price informativeness, τ_Π, can be recovered by

$$\tau_\Pi = \frac{\beta_1^2}{\sigma_e^2}.$$

The OLS estimation of Regression R1-Linear-Diff yields consistent estimates of β_1 and σ_e^2.

b) Relative Price Informativeness. Let $R^2_{\Delta X, \Delta X'}$ denote the R-squared of Regression R1-Linear-Diff. Let $R^2_{\Delta X}$, ζ, and ζ_0 respectively denote the R-squared and the coefficients of the following regression of price differences on payoff differences,

$$\Delta P_t = \overline{\zeta} + \zeta_0 \Delta X_t + e_t^\zeta. \tag{R2-Linear-Diff}$$

Then, relative price informativeness, τ_Π^R, can be recovered by

$$\tau_\Pi^R = \frac{R^2_{\Delta X, \Delta X'} - R^2_{\Delta X}}{1 - R^2_{\Delta X}}.$$

The OLS estimation of Regressions R1-Linear-Diff and R2-Linear-Diff yields consistent estimates of $R^2_{\Delta X, \Delta X'}$ and $R^2_{\Delta X}$.

Proof. a) By comparing Regression R1-Linear-Diff with the structural Equation (54), it follows that $\overline{\beta} = \overline{\phi} + \phi_n \mu_{\Delta n}$, $\beta_0 = \phi_0$, $\beta_1 = \phi_1$, and $e_t = \phi_n \varepsilon_t^{\Delta n}$. Consequently, $\sigma_e^2 = \mathbb{V}\mathrm{ar}\left[e_t\right] = \left(\phi_n\right)^2 \mathbb{V}\mathrm{ar}\left[\varepsilon_t^{\Delta n}\right] = \left(\phi_n\right)^2 \tau_{\Delta n}^{-1}$. Therefore, we can recover absolute price informativeness as follows:

$$\tau_\Pi = \frac{\left(\beta_1\right)^2}{\sigma_e^2} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n}.$$

Given Equations (53) and (54), as well as the assumptions on u_t and n_t, it is straightforward to show that the OLS estimates of Regressions R1-Linear-Diff and R2-Linear-Diff are consistent, which implies that price informativeness can be consistently estimated as $\widehat{\tau_\Pi} = \frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}$. Formally, $\mathrm{plim}\left(\widehat{\tau_\Pi}\right) = \mathrm{plim}\left(\frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}\right) = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_{\Delta n} = \tau_\Pi$.

b) Note that the R-squareds of Regressions R1-Linear-Diff and R2-Linear-Diff can be expressed as follows:

$$R^2_{\Delta X, \Delta X'} = 1 - \frac{\mathbb{V}\mathrm{ar}\left(e_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta P_t\right)} \quad \text{and} \quad R^2_{\Delta X} = \frac{\mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta X_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta P_t\right)}.$$

After substituting Equation (53) in Regression R1-Linear-Diff, the following relation holds:

$$\Delta P_t = \overline{\phi} + \phi_1 \mu_{\Delta X} + \phi_n \mu_{\Delta n} + \left(\phi_0 + \rho \phi_1\right) \Delta X_t + \phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}. \tag{55}$$

By comparing Regression R2-Linear-Diff with the structural Equation (55), it follows that $\overline{\zeta} = \overline{\phi} + \phi_1 \mu_{\Delta X} + \phi_n \mu_{\Delta n}$, $\zeta_0 = \phi_0 + \rho \phi_1$, and $\varepsilon_t^{\zeta} = \phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}$.

From Equation (55), the following variance decomposition must hold:

$$\begin{aligned} \mathbb{V}\mathrm{ar}\left(\Delta P_t\right) &= \mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta X_t\right) + \mathbb{V}\mathrm{ar}\left(\phi_1 u_t + \phi_n \varepsilon_t^{\Delta n}\right) \\ &= \mathbb{V}\mathrm{ar}\left(\zeta_0 \Delta X_t\right) + \left(\phi_1\right)^2 \mathbb{V}\mathrm{ar}\left(u_t\right) + \mathbb{V}\mathrm{ar}\left(e_t\right), \end{aligned}$$

which can be rearranged to express $\frac{\tau_\Pi}{\tau_u}$ as follows:

$$1 = \underbrace{\frac{\mathbb{V}\mathrm{ar}\left(\zeta_0 x_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta P_t\right)}}_{R^2_{\Delta X}} + \underbrace{\frac{\mathbb{V}\mathrm{ar}\left(e_t\right)}{\mathbb{V}\mathrm{ar}\left(\Delta P_t\right)}}_{1 - R^2_{\Delta X, \Delta X'}} \left(\underbrace{\frac{\left(\phi_1\right)^2}{\mathbb{V}\mathrm{ar}\left(e_t\right)} \mathbb{V}\mathrm{ar}\left(u_t\right)}_{\frac{\tau_\Pi}{\tau_u}} + 1 \right) \Rightarrow \frac{\tau_\Pi}{\tau_u} = \frac{R^2_{\Delta X, \Delta X'} - R^2_{\Delta X}}{1 - R^2_{\Delta X, \Delta X'}}.$$

Therefore, relative price informativeness can be written as

$$\tau_\Pi^R = \frac{\tau_\Pi}{\tau_\Pi + \tau_u} = \frac{1}{1 + \frac{1}{\frac{\tau_\Pi}{\tau_u}}} = \frac{R^2_{\Delta X, \Delta X'} - R^2_{\Delta X}}{1 - R^2_{\Delta X}}.$$

□

Microfoundation Time is discrete, with periods denoted by $t = 0, 1, 2, \ldots, \infty$. Each period t, there is a continuum of investors, indexed by $i \in I$. Each generation lives two periods and has exponential utility over its last period wealth. An investor born at time t has preferences given by

$$U_i\left(w_{t+1}\right) = -e^{-\gamma_i w_{t+1}},$$

where γ is the coefficient of absolute risk aversion and w_{t+1} is the investor's wealth in his final period. There are two long-term assets in the economy: A risk-free asset in perfectly elastic supply, with return $R > 1$, and a risky asset in fixed supply Q that trades at a price P_t in period t.[15] The process for the payoff of the risky asset each period t is given by

$$\Delta X_{t+1} = \mu_{\Delta X} + u_t,$$

where $\Delta X_t = X_t - X_{t-1}$,$\mu_{\Delta X}$ is a scalar and $X_0 = 0$. The payoff X_t is realized and becomes common knowledge at the end of period $t-1$. The innovation in the payoff process, u_t, and, hence, X_{t+1} are realized and observed at the end of period t. The innovations to the payoff are independently distributed over time.

To preserve tractability, we assume that investors' private trading needs arise from random heterogeneous priors — see Dávila and Parlatore (2020) for a thorough analysis of this formulation. Formally, each investor i in generation t has a prior over the innovation at time t given by

$$u_t \sim_i N\left(\overline{n}_t^i, \tau_u^{-1}\right),$$

where

$$\overline{n}_t^i = n_t + \varepsilon_{\overline{n}t}^i \quad \text{with} \quad \varepsilon_{\overline{n}t}^i \overset{\text{iid}}{\sim} N\left(0, \tau_{\overline{n}}^{-1}\right)$$

and $\Delta n_t = \mu_{\Delta n} + \varepsilon_t^{\Delta n}$ with $\varepsilon_t^{\Delta n} \sim N\left(0, \tau_{\Delta n}^{-1}\right)$. The term n_t can be interpreted as the aggregate sentiment in the economy, where $n_t \perp \varepsilon_{\overline{n}t}^i$ for all t and all i. The aggregate sentiment n_t is not observed and acts as a source of aggregate noise in the economy, preventing the price from being fully revealing. For simplicity we assume $n_t \perp u_{t+s}$ for all t and all s. Moreover, we assume investors think of their prior as the correct one and do not learn about the aggregate sentiment from it.[16]

Each investor i in generation t receives a signal about the innovation in the asset payoff u_t given by

$$s_t^i = u_t + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right)$$

and $\varepsilon_{st}^i \perp \varepsilon_{st}^j$ for all $i \neq j$, and $u_t \perp \varepsilon_{st}^i$ for all t and all i.

The asset demand submitted by investor i born in period t is given by the solution to the following problem

$$\max_{Q_t^i} \left(\mathbb{E}\left[X_{t+1} + R^{-1}p_{t+1} | \mathcal{I}_t^i\right] - P_t\right) Q_t^i - \frac{\gamma^i}{2} \mathbb{V}\text{ar}\left[X_{t+1} + R^{-1}P_{t+1} | \mathcal{I}_t^t\right] \left(Q_t^i\right)^2,$$

where $\mathcal{I}_t^i = \left\{X_t, s_t^i, \overline{n}_t^i, P_t\right\}$ is the information set of an investor i in period t.

The optimality condition for an investor i in period t satisfies

$$Q_t^i = \frac{\mathbb{E}\left[X_{t+1} + R^{-1}P_{t+1} | \mathcal{I}_t^i\right] - P_t}{\gamma^i \mathbb{V}\text{ar}\left[X_{t+1} + R^{-1}P_{t+1} | \mathcal{I}_t^i\right]}.$$

[15] To simplify notation, we denote the risk-free rate by R, instead of R^f as we did in the body of the paper.

[16] Dávila and Parlatore (2020) show that the equilibrium structure is preserved if this assumption is relaxed.

In a stationary equilibrium in linear strategies, we assume and subsequently verify that the equilibrium demand of investor i can be expressed as

$$Q_t^i = \alpha_X^i X_t + \alpha_s^i s_t^i + \alpha_n^i \overline{n}_t^i - \alpha_P^i P_t + \psi^i, \tag{56}$$

where α_θ^i, α_s^i, α_n^i, α_p^i, and ψ^i are individual equilibrium demand coefficients. Market clearing and the Strong Law of Large Numbers (SLLN) allows us to express the equilibrium price in period t as

$$P_t = \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_t + \frac{\overline{\alpha_s}}{\overline{\alpha_P}} u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_P}} n_t + \frac{\overline{\psi}}{\overline{\alpha_P}},$$

where we define cross-sectional averages $\overline{\alpha_X} = \int \alpha_X^i di$, $\overline{\alpha_s} = \int \alpha_s^i di$, $\overline{\alpha_P} = \int \alpha_P^i di$, and $\overline{\psi} = \int \psi^i di - Q$.

The unbiased signal of the innovation in the payoff contained in the price is

$$\Pi_t = \frac{\overline{\alpha_P}}{\overline{\alpha_s}} \left(P_t - \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \mu_{\Delta n} - \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_t - \frac{\overline{\psi}}{\overline{\alpha_P}} \right) = u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \left(n_t - \mu_{\Delta n} \right),$$

where

$$\Pi_t | X_{t+1}, X_t \sim N\left(u_t, \tau_\Pi^{-1}\right),$$

with price informativeness given by

$$\tau_\Pi = \left(\mathbb{V}\text{ar}\left[\Pi_t | X_{t+1}, X_t\right]\right)^{-1} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}}\right)^2 \tau_{\Delta n}.$$

Given our guesses for the demand functions and the linear structure of prices we have

$$X_{t+1} + R^{-1} P_{t+1} = X_{t+1} + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_{t+1} + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} u_{t+1} + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} n_{t+1} + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}},$$

$$\begin{aligned}
\mathbb{E}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^i\right] &= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right) \mathbb{E}\left[X_{t+1} | \mathcal{I}_t^i\right] + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \mathbb{E}\left[u_{t+1}\right] + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mathbb{E}\left[n_{t+1}\right] + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}} \\
&= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right) \left(X_t + \mathbb{E}\left[u_t | \mathcal{I}_t^i\right]\right) + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \mathbb{E}\left[u_{t+1}\right] + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mu_{\Delta n} + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}},
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{V}\text{ar}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^i\right] &= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[X_{t+1} | \mathcal{I}_t^i\right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[u_{t+1}\right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[n_{t+1}\right] \\
&= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[u_{t+1}\right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[n_{t+1}\right].
\end{aligned}$$

Moreover, given the Gaussian structure of the signals in the information set, Bayesian updating implies

$$\mathbb{E}\left[u_t | s_t^i, \overline{n}_t^i, P_t\right] = \frac{\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_\Pi \Pi_t}{\tau_s + \tau_u + \tau_\Pi} = \frac{\tau_s s_t^i + \tau_{\Delta n} \overline{n}_t^i + + \tau_\Pi \frac{\overline{\alpha_P}}{\overline{\alpha_s}} \left(P_t - \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \mu_{\Delta n} - \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_t - \frac{\overline{\psi}}{\overline{\alpha_P}}\right)}{\tau_s + \tau_u + \tau_\Pi},$$

and

$$\mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] = \mathbb{V}\text{ar}\left[t_t | s_t^i, \overline{n}_t^i, P_t\right] = \left(\tau_s + \tau_u + \tau_\Pi\right)^{-1}.$$

Then, the first-order condition is given by

$$Q_t^i = \frac{1}{\gamma^i} \frac{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)\left(X_t + \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right]\left(\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_\Pi \Pi\right)\right) + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \mathbb{E}\left[u_{t+1}\right] + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mu_{\Delta n} + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}} - P_t}{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_{it}\right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\mathrm{ar}\left[u_{t+1}\right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}}\right)^2 \tau_{\Delta n}^{-1}}.$$

Matching coefficients we have

$$\begin{aligned}
\alpha_s^i &= \frac{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)}{\kappa^i} \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_s \qquad (57)\\
\alpha_n^i &= \frac{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)}{\kappa^i} \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_\eta \\
\alpha_X^i &= \frac{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)}{\kappa^i} \left(1 - \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_\Pi \frac{\overline{\alpha_X}}{\overline{\alpha_s}}\right) \\
\alpha_P^i &= \frac{1}{\kappa^i} \left(1 - \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right) \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_\Pi \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right) \\
\psi^i &= -\frac{1}{\kappa^i} \left(\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right) \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_\Pi \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s}} \mu_{\Delta n} + \frac{\overline{\psi}}{\overline{\alpha_s}}\right) - R^{-1} \left(\frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mu_{\Delta n} + \frac{\overline{\psi}}{\overline{\alpha_P}}\right)\right),
\end{aligned}$$

where

$$\kappa^i \equiv \gamma^i \left(\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\mathrm{ar}\left[u_{t+1}\right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}}\right)^2 \tau_{\Delta n}^{-1}\right),$$

since $\mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i\right] = \left(\tau_s + \tau_u + \tau_\Pi\right)^{-1}$ for all i.

Then, an equilibrium in linear strategies exists if the system above has a solution. In this equilibrium, our guess in Equation (56) is verified and the equilibrium price is linear and can be expressed as in Equation (59).

Note that the if the investors are ex-ante identical, the demand sensitivities are the same for all i. Then, there exists a unique solution to the system in Equations (57) given by

$$\begin{aligned}
\alpha_s^i &= \frac{1}{\kappa} \frac{1}{1 - R^{-1}} \frac{\tau_s}{\tau_u + \tau_s + \tau_\Pi}, \quad \alpha_n^i = \frac{1}{\kappa} \frac{1}{1 - R^{-1} \rho} \frac{\tau_\eta}{\tau_u + \tau_s + \tau_\Pi} \\
\alpha_X^i &= \frac{1}{\kappa} \frac{\rho}{1 - R^{-1}} \frac{\tau_s}{\tau_s + \tau_\Pi}, \quad \alpha_P^i = \frac{1}{\kappa} \frac{\tau_s}{\tau_s + \tau_\Pi}, \quad \text{and} \\
\psi^i &= -\frac{\frac{1}{\kappa} \frac{1}{1 - R^{-1}} \left(\left(1 - R^{-1}\right) \tau_\Pi - R^{-1} \tau_s\right) \frac{\tau_u}{\tau_u + \tau_s + \tau_{\hat{\Pi}}} \mu_{\Delta n}}{1 + \left(1 - R^{-1}\right) \tau_\Pi - R^{-1} \tau_s},
\end{aligned}$$

where $\tau_\Pi = \left(\frac{\tau_s}{\tau_u}\right)^2 \tau_{\Delta n}$, and

$$\kappa = \gamma \left(\left(\frac{1}{1 - R^{-1}}\right)^2 \frac{1}{\tau_u + \tau_s + \tau_\Pi} + \left(R^{-1} \frac{1}{1 - R^{-1}} \frac{\tau_s + \tau_\Pi}{\tau_u + \tau_s + \tau_\Pi}\right)^2 \tau_u^{-1} + \left(\frac{R^{-1}}{1 - R^{-1}} \frac{\tau_s + \tau_\Pi}{\tau_u + \tau_s + \tau_\Pi} \frac{\tau_u}{\tau_s}\right)^2 \tau_{\Delta n}^{-1}\right).$$

D.2.2 Stationary linear payoff

General framework and identification Consider a discrete time environment with dates $t = 0, 1, 2, \ldots, \infty$, in which investors trade a risky asset in fixed supply at a price P_t at each date t. We assume that the payoff of the risky asset at date $t+1$, X_{t+1}, follows a stationary AR(1) process

$$X_{t+1} = \mu_X + \rho X_t + u_t, \tag{58}$$

where μ_X is a scalar, $|\rho| < 1$, and where the innovations to the payoff, u_t, have mean zero, a finite variance denoted by $\mathbb{V}\text{ar}\left[u_t\right] = \sigma_u^2 = \tau_u^{-1}$, and are identically and independently distributed over time. We assume that the equilibrium price is given by

$$P_t = \overline{\phi} + \phi_0 X_t + \phi_1 X_{t+1} + \phi_n n_t, \tag{59}$$

where $\overline{\phi}$, ϕ_0, ϕ_1, and ϕ_n are parameters and where n_t represents the aggregate component of investors' trading motives that are orthogonal to the asset payoff, given by $n_t = \mu_n + \varepsilon_t^n$, where $\mathbb{E}\left[\varepsilon_t^n\right] = 0$ and $\mathbb{V}\text{ar}\left[\varepsilon_t^n\right] = \sigma_n^2 = \tau_n^{-1}$. For simplicity, we assume that u_t and n_t are independent.

In this case, the unbiased signal of the innovation to the change in the future payoff u_t contained in the price, which we denote by Π_t, is given by

$$\hat{\Pi}_t \equiv \frac{P_t - \left(\overline{\phi} + \phi_1 \mu_X + \phi_n \mu_n + \left(\phi_0 + \rho \phi_1\right) X_t\right)}{\phi_1} = u_t + \frac{\phi_n}{\phi_1}\left(n_t - \mu_n\right)$$

and absolute and relative price informativeness are given respectively by

$$\tau_{\hat{\Pi}} \equiv \left(\mathbb{V}\text{ar}\left[\hat{\Pi}_t \middle| X_{t+1}, X_t\right]\right)^{-1} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_n \quad \text{and} \quad \tau_{\hat{\Pi}}^R \equiv \frac{\tau_{\hat{\Pi}}}{\tau_{\hat{\Pi}} + \tau_u}.$$

Proposition 9. (Identifying price informativeness: difference-stationary linear case)

a) Absolute price informativeness. Let $\overline{\beta}$, β_0, and β_1 denote the coefficients of the following regression of prices on realized and future payoffs,

$$P_t = \overline{\beta} + \beta_0 X_t + \beta_1 X_{t+1} + e_t, \tag{R1-Linear}$$

where P_t denotes the date t price, X_t and X_{t+1} respectively denote the dates t and $t+1$ payoff, and where $\sigma_e^2 = \mathbb{V}ar[e_t]$ denotes the variance of the error. Then, absolute price informativeness, $\tau_{\hat{\Pi}}$, can be recovered by

$$\tau_{\hat{\Pi}} = \frac{\beta_1^2}{\sigma_e^2}.$$

The OLS estimation of Regression R1-Linear yields consistent estimates of β_1 and σ_e^2.

b) Relative Price Informativeness. Let $R^2_{X,X'}$ denote the R-squared of Regression R1-Linear. Let R^2_X, ζ, and ζ_0 respectively denote the R-squared and the coefficients of the following regression of price differences on payoff differences,

$$\Delta P_t = \overline{\zeta} + \zeta_0 \Delta X_t + e_t^{\zeta}. \tag{R2-Linear}$$

Then, relative price informativeness, $\tau_{\hat{\Pi}}^R$, can be recovered by

$$\tau_{\hat{\Pi}}^R = \frac{R^2_{X,X'} - R^2_X}{1 - R^2_X}.$$

The OLS estimation of Regressions R1-Linear and R2-Linear yields consistent estimates of $R^2_{X,X'}$ and R^2_X.

Proof. a) By comparing Regression R1-Linear with the structural Equation (46), it follows that $\overline{\beta} =$

$\overline{\phi} + \phi_n \mu_n$, $\beta_0 = \phi_0$, $\beta_1 = \phi_1$, and $e_t = \phi_n \varepsilon_t^n$. Consequently, $\sigma_e^2 = \mathbb{V}\mathrm{ar}\left[e_t\right] = \left(\phi_n\right)^2 \mathbb{V}\mathrm{ar}\left[\varepsilon_t^n\right] = \left(\phi_n\right)^2 \tau_n^{-1}$. Therefore, we can recover absolute price informativeness as follows

$$\tau_{\hat{\Pi}} = \frac{\left(\beta_1\right)^2}{\sigma_e^2} = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_n.$$

Given Equations (58) and (59), as well as the assumptions on u_t and n_t, it is straightforward to show that the OLS estimates of Regressions R1-Linear and R2-Linear are consistent, which implies that price informativeness can be consistently estimated as $\widehat{\tau_{\hat{\Pi}}} = \frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}$. Formally, $\operatorname{plim}\left(\widehat{\tau_{\Pi}}\right) = \operatorname{plim}\left(\frac{\left(\widehat{\beta_1}\right)^2}{\widehat{\sigma_e^2}}\right) = \left(\frac{\phi_1}{\phi_n}\right)^2 \tau_n = \tau_{\hat{\Pi}}$.

b) Note that the R-squareds of Regressions R1-Linear and R2-Linear can be expressed as follows

$$R^2_{X,X'} = 1 - \frac{\mathbb{V}\mathrm{ar}\left(e_t\right)}{\mathbb{V}\mathrm{ar}\left(P_t\right)} \quad \text{and} \quad R^2_X = \frac{\mathbb{V}\mathrm{ar}\left(\zeta_0 X_t\right)}{\mathbb{V}\mathrm{ar}\left(P_t\right)}.$$

After substituting Equation (58) in Equation (59), the following relation holds

$$P_t = \overline{\phi} + \phi_1 \mu_X + \phi_n \mu_n + \left(\phi_0 + \rho \phi_1\right) X_t + \phi_1 u_t + \phi_n \varepsilon_t^n. \tag{60}$$

By comparing Regression R2-Linear with the structural Equation (60), it follows that $\overline{\zeta} = \overline{\phi} + \phi_1 \mu_X + \phi_n \mu_n$, $\zeta_0 = \phi_0 + \rho \phi_1$, and $\varepsilon_t^\zeta = \phi_1 u_t + \phi_n \varepsilon_t^n$.

From Equation (60), the following variance decomposition must hold

$$\begin{aligned}
\mathbb{V}\mathrm{ar}\left(P_t\right) &= \mathbb{V}\mathrm{ar}\left(\zeta_0 X_t\right) + \mathbb{V}\mathrm{ar}\left(\phi_1 u_t + \phi_n \varepsilon_t^n\right) \\
&= \mathbb{V}\mathrm{ar}\left(\zeta_0 X_t\right) + \left(\phi_1\right)^2 \mathbb{V}\mathrm{ar}\left(u_t\right) + \mathbb{V}\mathrm{ar}\left(e_t\right),
\end{aligned}$$

which can be rearranged to express $\frac{\tau_{\hat{\Pi}}}{\tau_u}$ as follows

$$1 = \underbrace{\frac{\mathbb{V}\mathrm{ar}\left(\zeta_0 x_t\right)}{\mathbb{V}\mathrm{ar}\left(P_t\right)}}_{R^2_X} + \underbrace{\frac{\mathbb{V}\mathrm{ar}\left(e_t\right)}{\mathbb{V}\mathrm{ar}\left(P_t\right)}}_{1 - R^2_{X,X'}} \left(\underbrace{\frac{\left(\phi_1\right)^2}{\mathbb{V}\mathrm{ar}\left(e_t\right)} \mathbb{V}\mathrm{ar}\left(u_t\right)}_{\frac{\tau_{\hat{\Pi}}}{\tau_u}} + 1 \right) \Rightarrow \frac{\tau_{\Pi}}{\tau_u} = \frac{R^2_{X,X'} - R^2_X}{1 - R^2_{X,X'}}.$$

Therefore, relative price informativeness can be written as

$$\tau^R_{\hat{\Pi}} = \frac{\tau_{\hat{\Pi}}}{\tau_{\hat{\Pi}} + \tau_u} = \frac{1}{1 + \frac{1}{\frac{\tau_{\hat{\Pi}}}{\tau_u}}} = \frac{R^2_{X,X'} - R^2_X}{1 - R^2_X}.$$

$\square$

Microfoundation Time is discrete, with periods denoted by $t = 0, 1, 2, \ldots, \infty$. Each period t, there is a continuum of investors, indexed by $i \in I$. Each generation lives two periods and has exponential utility over its last period wealth. An investor born at time t has preferences given by

$$U_i\left(w_{t+1}\right) = -e^{-\gamma^i w_{t+1}},$$

where γ is the coefficient of absolute risk aversion and w_{t+1} is the investor's wealth in his final period. There are two long-term assets in the economy: A risk-free asset in perfectly elastic supply, with return $R > 1$, and a risky asset in fixed supply Q that trades at a price P_t in period t. The payoff of the risky asset each period t is given by

$$X_{t+1} = \mu_X + \rho X_t + u_t,$$

where μ_X is a scalar, $|\rho| < 1$, and $X_0 = 0$. The payoff X_t is realized and becomes common knowledge at the end of period $t-1$. The innovation in the payoff, u_t, and, hence, X_{t+1} are realized and observed at the end of period t. The innovations to the payoff are independently distributed over time.

To preserve tractability, we assume that investors' private trading needs arise from random heterogeneous priors — see Dávila and Parlatore (2020) for a thorough analysis of this formulation. Formally, each investor i in generation t has a prior over the innovation at time t given by

$$u_t \sim_i N\left(\overline{n}_t^i, \tau_u^{-1}\right),$$

where

$$\overline{n}_t^i = n_t + \varepsilon_{\overline{n}t}^i \quad \text{with} \quad \varepsilon_{\overline{n}t}^i \overset{\text{iid}}{\sim} N\left(0, \tau_{\overline{n}}^{-1}\right),$$

and $n_t = \mu_n + \varepsilon_t^n$ with $\varepsilon_t^n \sim N\left(0, \tau_n^{-1}\right)$. Note that n_t can be interpreted as the aggregate sentiment in the economy, where $n_t \perp \varepsilon_{\overline{n}t}^i$ for all t and all i. The aggregate sentiment n_t is not observed and acts as a source of aggregate noise in the economy, preventing the price from being fully revealing. For simplicity we assume $n_t \perp u_{t+s}$ for all t and all s. Moreover, we assume investors think of their prior as the correct one and do not learn about the aggregate sentiment from it.[17]

Each investor i in generation t receives a signal about the innovation in the asset payoff u_t given by

$$s_t^i = u_t + \varepsilon_{st}^i \quad \text{with} \quad \varepsilon_{st}^i \sim N\left(0, \tau_s^{-1}\right),$$

and $\varepsilon_{st}^i \perp \varepsilon_{st}^j$ for all $i \neq j$, and $u_t \perp \varepsilon_{st}^i$ for all t and all i.

Definition. The asset demand submitted by investor i born in period t is given by the solution to the following problem:

$$\max_{Q_t^i} \left(\mathbb{E}\left[X_{t+1} + R^{-1} p_{t+1} | \mathcal{I}_t^i\right] - P_t\right) Q_t^i - \frac{\gamma^i}{2} \mathbb{V}\text{ar}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^t\right] \left(Q_t^i\right)^2,$$

where $\mathcal{I}_t^i = \left\{s_t^i, \overline{n}_t^i, \{X_s\}_{s \leq t}, \{P_s\}_{s \leq t}\right\}$ is the information set of an investor i in period t.

The optimality condition for an investor i in period t satisfies

$$Q_t^i = \frac{\mathbb{E}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^i\right] - P_t}{\gamma^i \mathbb{V}\text{ar}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^i\right]}.$$

In a stationary equilibrium in linear strategies, we assume and subsequently verify that the equilibrium demand of investor i can be expressed as

$$\Delta Q_t^i = \alpha_X^i X_t + \alpha_s^i s_t^i + \alpha_n^i \overline{n}_t^i - \alpha_P^i P_t + \psi^i, \tag{61}$$

where α_θ^i, α_s^i, α_n^i, α_p^i, and ψ^i are individual equilibrium demand coefficients. Market clearing and the

[17] Dávila and Parlatore (2020) show that the equilibrium structure is preserved if this assumption is relaxed.

Strong Law of Large Numbers allows us to express the equilibrium price in period t as

$$P_t = \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_t + \frac{\overline{\alpha_s}}{\overline{\alpha_P}} u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_P}} n_t + \frac{\overline{\psi}}{\overline{\alpha_P}},$$

where we define cross-sectional averages $\overline{\alpha_X} = \int \alpha_X^i di$, $\overline{\alpha_s} = \int \alpha_s^i di$, $\overline{\alpha_P} = \int \alpha_P^i di$, and $\overline{\psi} = \int \psi^i di - Q$.

The unbiased signal of the innovation in the payoff contained in the price is

$$\hat{\Pi}_t = \frac{\overline{\alpha_P}}{\overline{\alpha_s}} \left(P_t - \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \mu_n - \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_t - \frac{\overline{\psi}}{\overline{\alpha_P}} \right) = u_t + \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \left(n_t - \mu_n \right),$$

where

$$\hat{\Pi}_t | X_{t+1}, X_t \sim N\left(u_t, \tau_{\hat{\Pi}}^{-1} \right),$$

with price informativeness given by

$$\tau_{\hat{\Pi}} = \left(\mathbb{V}\mathrm{ar}\left[\hat{\Pi}_t | X_{t+1}, X_t \right] \right)^{-1} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_n}} \right)^2 \tau_n.$$

Given our guesses for the demand functions and the linear structure of prices we have

$$X_{t+1} + R^{-1} P_{t+1} = X_{t+1} + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_{t+1} + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} u_{t+1} + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} n_{t+1} + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}},$$

$$\begin{aligned} \mathbb{E}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^i \right] &= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} \right) \mathbb{E}\left[X_{t+1} | \mathcal{I}_t^i \right] + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \mathbb{E}\left[u_{t+1} \right] + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mathbb{E}\left[n_{t+1} \right] + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}} \\ &= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} \right) \left(\rho X_t + \mathbb{E}\left[u_t | \mathcal{I}_t^i \right] \right) + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \mathbb{E}\left[u_{t+1} \right] + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mu_n + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}}, \end{aligned}$$

and

$$\begin{aligned} \mathbb{V}\mathrm{ar}\left[X_{t+1} + R^{-1} P_{t+1} | \mathcal{I}_t^i \right] &= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[X_{t+1} | \mathcal{I}_t^i \right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[u_{t+1} \right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[n_{t+1} \right] \\ &= \left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i \right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[u_{t+1} \right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[n_{t+1} \right]. \end{aligned}$$

Moreover, given the Gaussian structure of the signals in the information set, Bayesian updating implies

$$\mathbb{E}\left[u_t | s_t^i, \overline{n}_t^i, P_t \right] = \frac{\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_{\hat{\Pi}} \hat{\Pi}_t}{\tau_s + \tau_u + \tau_{\hat{\Pi}}} = \frac{\tau_s s_t^i + \tau_n \overline{n}_t^i + + \tau_{\hat{\Pi}} \frac{\overline{\alpha_P}}{\overline{\alpha_s}} \left(P_t - \frac{\overline{\alpha_n}}{\overline{\alpha_s}} \mu_n - \frac{\overline{\alpha_X}}{\overline{\alpha_P}} X_t - \frac{\overline{\psi}}{\overline{\alpha_P}} \right)}{\tau_s + \tau_u + \tau_{\hat{\Pi}}},$$

and

$$\mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i \right] = \mathbb{V}\mathrm{ar}\left[u_t | s_t^i, \overline{n}_t^i, P_t \right] = \left(\tau_s + \tau_u + \tau_{\Pi} \right)^{-1}.$$

Then, the first-order condition is given by

$$Q_t^i = \frac{1}{\gamma^i} \frac{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} \right) \left(\rho X_t + \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_t^i \right] \left(\tau_s s_t^i + \tau_u \overline{n}_t^i + \tau_{\hat{\Pi}} \hat{\Pi}_t \right) \right) + R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \mathbb{E}\left[u_{t+1} \right] + R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \mu_n + R^{-1} \frac{\overline{\psi}}{\overline{\alpha_P}} - P_t}{\left(1 + R^{-1} \frac{\overline{\alpha_X}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[u_t | \mathcal{I}_{it} \right] + \left(R^{-1} \frac{\overline{\alpha_s}}{\overline{\alpha_P}} \right)^2 \mathbb{V}\mathrm{ar}\left[u_{t+1} \right] + \left(R^{-1} \frac{\overline{\alpha_n}}{\overline{\alpha_P}} \right)^2 \tau_n^{-1}}.$$

Matching coefficients we have

$$\alpha_s^i = \frac{\left(1 + R^{-1}\frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)}{\kappa^i} \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_s \tag{62}$$

$$\alpha_n^i = \frac{\left(1 + R^{-1}\frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)}{\kappa^i} \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_\eta$$

$$\alpha_X^i = \frac{\left(1 + R^{-1}\frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)}{\kappa^i} \left(\rho - \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_{\hat{\Pi}} \frac{\overline{\alpha_X}}{\overline{\alpha_s}}\right)$$

$$\alpha_P^i = \frac{1}{\kappa^i}\left(1 - \left(1 + R^{-1}\frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right) \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_{\hat{\Pi}} \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right)$$

$$\psi^i = -\frac{1}{\kappa^i}\left(\left(1 + R^{-1}\frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right) \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] \tau_{\hat{\Pi}} \left(\frac{\overline{\alpha_n}}{\overline{\alpha_s}}\mu_n + \frac{\overline{\psi}}{\overline{\alpha_s}}\right) - R^{-1}\left(\frac{\overline{\alpha_n}}{\overline{\alpha_P}}\mu_n + \frac{\overline{\psi}}{\overline{\alpha_P}}\right)\right),$$

where

$$\kappa^i \equiv \gamma^i \left(\left(1 + R^{-1}\frac{\overline{\alpha_X}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] + \left(R^{-1}\frac{\overline{\alpha_s}}{\overline{\alpha_P}}\right)^2 \mathbb{V}\text{ar}\left[u_{t+1}\right] + \left(R^{-1}\frac{\overline{\alpha_n}}{\overline{\alpha_P}}\right)^2 \tau_n^{-1}\right),$$

since $\mathbb{V}\text{ar}\left[u_t | \mathcal{I}_t^i\right] = \left(\tau_s + \tau_u + \tau_{\hat{\Pi}}\right)^{-1}$ for all i.

Then, an equilibrium in linear strategies exists if the system above has a solution. In this equilibrium, our guess in Equation (61) is verified and the equilibrium price is linear and can be expressed as in Equation (59).

Note that when investors are ex-ante identical, the demand sensitivities are the same for all i. Then, there exists a unique solution to the system in Equations (62), that is given by

$$\alpha_s^i = \frac{1}{\kappa}\frac{1}{1 - R^{-1}\rho}\frac{\tau_s}{\tau_u + \tau_s + \tau_{\hat{\Pi}}}, \quad \alpha_n^i = \frac{1}{\kappa}\frac{1}{1 - R^{-1}\rho}\frac{\tau_\eta}{\tau_u + \tau_s + \tau_{\hat{\Pi}}}$$

$$\alpha_X^i = \frac{1}{\kappa}\frac{\rho}{1 - R^{-1}\rho}\frac{\tau_s}{\tau_s + \tau_{\hat{\Pi}}}, \quad \alpha_P^i = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_{\hat{\Pi}}}, \quad \text{and}$$

$$\psi^i = -\frac{\frac{1}{\kappa}\frac{1}{1 - R^{-1}\rho}\left(\left(1 - R^{-1}\right)\tau_{\hat{\Pi}} - R^{-1}\tau_s\right)\frac{\tau_u}{\tau_u + \tau_s + \tau_{\hat{\Pi}}}\mu_n}{1 + \left(1 - R^{-1}\right)\tau_{\hat{\Pi}} - R^{-1}\tau_s},$$

where $\tau_{\hat{\Pi}} = \left(\frac{\tau_s}{\tau_u}\right)^2 \tau_n$ and

$$\kappa = \gamma\left(\left(\frac{1}{1 - R^{-1}\rho}\right)^2 \frac{1}{\tau_u + \tau_s + \tau_{\hat{\Pi}}} + \left(R^{-1}\frac{1}{1 - R^{-1}\rho}\frac{\tau_s + \tau_{\hat{\Pi}}}{\tau_u + \tau_s + \tau_{\hat{\Pi}}}\right)^2 \tau_u^{-1} + \left(\frac{R^{-1}}{1 - R^{-1}\rho}\frac{\tau_s + \tau_{\hat{\Pi}}}{\tau_u + \tau_s + \tau_{\hat{\Pi}}}\frac{\tau_u}{\tau_s}\right)^2 \tau_n^{-1}\right).$$