Contracting Frictions in Global Sourcing: Implications for Welfare

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Backdrop

Contracting frictions matter:

- for the pattern of trade (e.g., Levchenko 2007; Nunn 2007); and
- for the global sourcing of inputs (e.g., Antràs & Helpman 2004, 2008; Nunn & Trefler 2008, 2013; Bernard et al. 2010; Corcos et al. 2013; Defever & Toubal 2013).

We now have:

- Frameworks that spotlight how decisions over organizational mode – i.e., integration vs outsourcing – can help firms to cope with contracting frictions encountered in global sourcing.
- Supporting empirical evidence, using the *intrafirm trade share* (as a proxy for the propensity to integrate vs outsource).

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- Supporting empirical evidence, using the *intrafirm trade share* (as a proxy for the propensity to integrate vs outsource).

However: Less is known about *how much* considerations related to contracting frictions in global sourcing matter for aggregate outcomes such as welfare.

(Some exceptions: Fally & Hillberry 2018; Startz 2018; Boehm 2020; Boehm & Oberfield 2020)

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  - ... the severity of contracting frictions *(a la Grossman-Hart-Moore)*,

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    through two sets of parameters governing: (i) contractibility, and (ii) bargaining.
- Adopt a nested-Fréchet specification for the joint distribution of supplier productivities across sourcing modes,
  - ... which facilitates aggregation (c.f., Lind and Ramondo 2019)

This project: Analytical results

The model delivers:

- **Sourcing:** An EK type expression for sourcing shares by country-mode, but modified by the presence of contracting frictions
  - *Interpretation:* Contracting frictions retard the effective state of technology accessible to input-sourcing firms.
- **Gravity:** A modified gravity equation for bilateral trade flows by source country and organizational mode

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  - *Interpretation:* Contracting frictions retard the effective state of technology accessible to input-sourcing firms.
- **Gravity:** A modified gravity equation for bilateral trade flows by source country and organizational mode
- **Welfare:** A closed-form expression for welfare change, in response to shifts in trade costs or contracting frictions
  - Nests ACR (2012) and Costinot and Rodriguez-Clare (2014)
  - ... while highlighting new effects introduced by the presence of contracting frictions.

This project: Quantification

- **Estimation strategy**. Based on:

  (i) a structural estimating equation where the dependent variable is the intrafirm trade share; and

  (ii) functional forms to map the contractibility and bargaining parameters to observables, viz: country variables (rule of law) and industry characteristics (Nunn-contractibility, Rauch-specificity)

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- Allows us to evaluate welfare counterfactuals. E.g.:
  - a hypothetical removal of contracting frictions in global sourcing
  - an improvement in the institutional rule of law that alleviates input sourcing frictions for a particular country

Remarks

- Model has a good number of building blocks.
  - At the expense of over-simplifying: Think Grossman & Hart (1986), Antràs & Helpman (2008, with partial contractibility) meets quantitative trade.
  - Production structure is more of a "spider".

    (For GVC "snakes", see: Harms et al. 2012; Costinot et al. 2013; Antràs & Chor 2013; Alfaro et al. 2019; Antràs & de Gortari 2020; etc.)
- Rich counterfactual exercises, related to:
  - A global removal of contracting frictions *in input sourcing*: Average welfare gain of 9.8%.
  - Contracting frictions and the gains from trade.
  - An improvement in rule of law in China.

Roadmap for this talk

1. Motivation and Introduction
2. Model: Contracting Frictions and Global Sourcing meets Quantitative Trade
3. Taking the Model to the Data
4. Estimation and Counterfactuals
5. Concluding remarks and next steps

Contracting Frictions and Global Sourcing in a Quantitative Trade Model

Utility

J countries (indexed by j).

Representative consumer derives utility from final-good varieties (indexed by ω):

$$U_j = \left(\int_{\omega \in \Omega} c_j(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}}, \ \rho \in (0,1). \tag{1}$$

Assume a fixed measure of firms, N_j. Associate each ω with a final-good firm whose core productivity ϕ is an iid draw from $G_j(\phi)$.

We have:

$$q_j(\phi) = A_j p_j(\phi)^{-\frac{1}{1-\rho}},$$
$$R_j(\phi) = A_j^{1-\rho} q_j(\phi)^{\rho}.$$

where $A_j = E_j P_j^{\frac{\rho}{1-\rho}}$ depends on country-j aggregate expenditure, E_j, and the price index, P_j.

Final-good Production

- Each final-good variety is produced using inputs from K industries (a la CDK 2012), and assembled with domestic labor. (Final-goods are not traded.)

- Within each industry k, input varieties ℓ are sourced globally.

$$y_j(\phi) = \phi \left(\prod_{k=1}^{K} \left(X_j^k(\phi) \right)^{\eta^k} \right)^{1-\alpha} L_j(\phi)^{\alpha}, \text{ where} \tag{2}$$

$$X_j^k(\phi) = \left(\int_{\ell=0}^{1} \tilde{x}_j^k(\phi;\ell)^{\rho^k} d\ell \right)^{\frac{1}{\rho^k}}.$$

- $X_j^k(\phi)$: Composite industry-k input, from a unit measure of input varieties, $\tilde{x}_j^k(\phi;\ell)$. (c.f., Tintelnot 2017, Antràs et al. 2017)

- $L_j(\phi)$: Labor used in final assembly.

- Assume: $0 < \alpha < 1$; $0 < \eta^k < 1$; $\sum_k \eta^k = 1$; $0 < \rho < \rho^k < 1$.

Final-good Production

$$X_j^k(\phi) = \left(\int_{\ell=0}^{1} \tilde{x}_j^k(\phi;\ell)^{\rho^k} d\ell \right)^{\frac{1}{\rho^k}}$$

- Each input variety combines firm headquarter services, $h_j^k(\phi;\ell)$, and supplier inputs, $x_j^k(\phi;\ell)$, both of which are relationship-specific. (Antràs 2003)

$$\tilde{x}_j^k(\phi;\ell) = \left[h_j^k(\phi;\ell) \right]^{\alpha^k} \left[x_j^k(\phi;\ell) \right]^{1-\alpha^k}, \; 0 < \alpha^k < 1.$$

Final-good Production

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- $h_j^k(\phi;\ell)$ (resp., $x_j^k(\phi;\ell)$) comprises a measure $\mu_{hij}^k \in [0,1]$ (resp., $\mu_{xij}^k \in [0,1]$) of contractible tasks. (Acemoglu et al. 2007, Antràs and Helpman 2008)

$$h_j^k(\phi;\ell) = \exp\left\{ \int_{\iota=0}^{\mu_{hij}^k} \log h_j^k(\iota;\phi,\ell) d\iota + \int_{\iota=\mu_{hij}^k}^{1} \log h_j^k(\iota;\phi,\ell) d\iota \right\},$$

$$x_j^k(\phi;\ell) = \exp\left\{ \int_{\iota=0}^{\mu_{xij}^k} \log x_j^k(\iota;\phi,\ell) d\iota + \int_{\iota=\mu_{xij}^k}^{1} \log x_j^k(\iota;\phi,\ell) d\iota \right\}.$$

Input Sourcing and Bargaining

For each input variety, ℓ:

- Let source country be i and organizational mode be $\chi \in \{V, O\}$ (V: integration; O: outsourcing)
- $2J$ possible "sourcing modes", (i, χ)

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- Let source country be i and organizational mode be $\chi \in \{V, O\}$ (V: integration; O: outsourcing)
- $2J$ possible "sourcing modes", (i, χ)
- Firm obtains a set of $2J$ productivity draws associated with a given input variety ℓ for each of the possible sourcing modes
- In each sourcing country i, $\exists$ a large number of potential suppliers that can deliver the input using the (i, χ) productivity draw
- Based on these draws, firm decides on the optimal sourcing mode for each input variety ℓ.

Input Sourcing and Bargaining

For each input variety, ℓ:

- Let source country be i and organizational mode be $\chi \in \{V, O\}$
  (V: integration; O: outsourcing)
- $2J$ possible "sourcing modes", (i, χ)
- Contracts are incomplete: Payoffs determined in ex-post negotiation between the firm and each supplier.
- $\beta^k_{ij\chi}$: Generalized Nash bargaining share that accrues to the firm under sourcing mode (i, χ). Varies by:
  - Source (i) or destination (j). E.g.: Rule of law.
  - Industry (k). E.g.: Specificity.
- Assume: $0 < \beta^k_{ijO} < \beta^k_{ijV} < 1$, reflecting the firm's greater residual rights of control under integration (Grossman and Hart 1986)

Input Sourcing and Bargaining: Timing

- Firm obtains its productivity draws (for all input varieties and sourcing modes)

- Firm posts contracts for a supplier for each input variety ℓ, specifying:
  (i) an ex-ante participation fee;
  (ii) the sourcing mode over ℓ; and
  (iii) the investment levels for contractible tasks, $h_j^k(\iota; \phi, \ell)$ for $\iota \in [0, \mu_{hij}^k]$ and $x_j^k(\iota; \phi, \ell)$ for $\iota \in [0, \mu_{xij}^k]$, as well as for $L_j(\phi)$.

- Firm picks a supplier for each ℓ

- Supplier of ℓ chooses how much to invest in the non-contractible input services: $x_j^k(\iota; \phi, \ell)$ for $\iota \in (\mu_{xij}^k, 1]$,...

- Firm simultaneously chooses how much to invest in the non-contractible headquarter services: $h_j^k(\iota; \phi, \ell)$ for $\iota \in (\mu_{hij}^k, 1]$.

Input Sourcing and Bargaining: Timing

- Firm and each supplier bargain over the incremental revenue contributed by the input variety ℓ, taking the investment levels for other inputs as given
- Incremental revenue $r_j^k(\phi;\ell)$ computed following heuristic from Acemoglu et al. (2007): Details

$$r_j^k(\phi;\ell) = (1-\alpha)\frac{\rho\eta^k}{\rho^k} R_j(\phi) \left(\frac{\tilde{x}_j^k(\phi;\ell)}{X_j^k(\phi)} \right)^{\rho^k} . \tag{3}$$

Input Sourcing and Bargaining: Setup

Firm chooses $h_j^k(\iota; \phi, \ell)$ to maximize:

$$\beta_{ij\chi}^k r_j^k(\phi; \ell) - s_j \int_{\mu_{hij}^k}^{1} h_j^k(\iota; \phi, \ell) d\iota. \tag{4}$$

where the firm's costs are in units of capital.

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Supplier ℓ chooses $x_j^k(\iota;\phi,\ell)$ to maximize:

$$(1-\beta_{ij\chi}^k) r_j^k(\phi;\ell) - c_{ij\chi}^k(\phi;l) \int_{\mu_{xij}^k}^{1} x_j^k(\iota;\phi,l)d\iota, \tag{5}$$

where $c_{ij\chi}^k$ is incurred in units of labor:

$$c_{ij\chi}^k(\phi;\ell) = \frac{d_{ij}^k w_i}{Z_{ij\chi}^k(\phi;\ell)}. \tag{6}$$

- $d_{ij}^k \geq 1$: iceberg trade costs
- $Z_{ij\chi}^k(\phi;\ell)$: labor productivity

Input Sourcing and Bargaining: Setup

Nested Fréchet joint distribution of the $Z^k_{ij\chi}(\phi;\ell)$'s over the $2J$ possible sourcing modes.

- $Pr\left(Z^k_{1jV} \leq z^k_{1jV}, Z^k_{1jO} \leq z^k_{1jO}, \ldots, Z^k_{JjO} \leq z^k_{JjO}\right)$ is given by:

$$\exp\left\{-\sum_{i=1}^{J} T_i^k \left(\left(z^k_{ijV}\right)^{-\frac{\theta^k}{1-\lambda_i}} + \left(z^k_{ijO}\right)^{-\frac{\theta^k}{1-\lambda_i}}\right)^{1-\lambda_i}\right\}, \qquad (7)$$

where $\theta^k > 1$ and $0 < \lambda_i < 1$.

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Remarks:

- Analogue of the nested logit in discrete choice models.
- Relaxes the Independence of Irrelevant Alternatives (IIA) assumption inherent in Fréchet, ... by introducing a correlation parameter λ_i for "within-nest" draws.

  ($\lambda_i = 0$ for all i gets us back to iid Fréchet.)

Sourcing Decision

- Solve for $h_j^k(\iota;\phi,\ell)$ and $x_j^k(\iota;\phi,\ell)$ – the non-contractible input tasks levels – from FOCs of firm and supplier ℓ.
- Bearing in mind the ex-ante transfer, firm specifies the contractible levels of investment and the sourcing mode to maximize:

$$r_j^k(\phi;\ell) - s_j \int_0^1 h_j^k(\iota;\phi,\ell)d\iota - c_{ij\chi}^k(\phi;l) \int_0^1 x_j^k(\iota;\phi,\ell)d\iota.$$

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- Sourcing decision boils down to:

$$\arg\max_{(i,\chi)} \; \Xi_{ij\chi}^k Z_{ij\chi}^k,$$

where: Details

$$\Xi_{ij\chi}^k = \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\rho^k(1-\alpha^k)}} \left(\frac{1-\rho^k}{\rho^k}\right)^{\frac{1-\rho^k}{\rho^k(1-\alpha^k)}} \left(\frac{\alpha^k}{s_j}\right)^{\frac{\alpha^k}{1-\alpha^k}} \left(\frac{1-\alpha^k}{d_{ij}^k w_i}\right) B_{ij\chi}^k.$$

Sourcing Shares

Share of inputs sourced under mode (i, χ) is equal to $\pi_{ij}^{k}\pi_{\chi|ij}^{k}$.

- π_{ij}^{k}: Probability of sourcing from country i

$$\pi_{ij}^{k} = \frac{T_i^k (d_{ij}^k w_i)^{-\theta^k} (B_{ij}^k)^{\theta^k}}{\sum_{i'=1}^{J} T_{i'}^k (d_{i'j}^k w_{i'})^{-\theta^k} (B_{i'j}^k)^{\theta^k}} = \frac{T_i^k (d_{ij}^k w_i)^{-\theta^k} (B_{ij}^k)^{\theta^k}}{\Phi_j^k}, \tag{8}$$

where: $\Phi_j^k \equiv \sum_{i'=1}^{J} T_{i'}^k (d_{i'j}^k w_{i'})^{-\theta^k} (B_{i'j}^k)^{\theta^k}$ is the **sourcing capability**.

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- **Compare with EK2002:** Contracting frictions – the $(B_{ij}^k)^{\theta^k}$'s – distort the effective state of technology available to sourcing firms.

$$B_{ij}^k = \left(\frac{1}{2}\left[(B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}} + (B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}\right]\right)^{\frac{1-\lambda_i}{\theta^k}}, \text{ where} \tag{9}$$

$$B_{ij\chi}^k = \left(\zeta_{ij\chi}^k/\zeta_{ij}^k\right)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}} \left(\beta_{ij\chi}^k\right)^{\frac{\alpha^k\left(1-\mu_{hij}^k\right)}{\left(1-\alpha^k\right)}} \left(1-\beta_{ij\chi}^k\right)^{\left(1-\mu_{xij}^k\right)}.$$

Sourcing Shares

Share of inputs sourced under mode (i, χ) is equal to $\pi^k_{ij}\pi^k_{\chi|ij}$.

- π^k_{ij}: Probability of sourcing from country i

$$\pi^k_{ij} = \frac{T^k_i(d^k_{ij}w_i)^{-\theta^k}(B^k_{ij})^{\theta^k}}{\sum_{i'=1}^{J} T^k_{i'}(d^k_{i'j}w_{i'})^{-\theta^k}(B^k_{i'j})^{\theta^k}} = \frac{T^k_i(d^k_{ij}w_i)^{-\theta^k}(B^k_{ij})^{\theta^k}}{\Phi^k_j}, \tag{8}$$

where: $\Phi^k_j \equiv \sum_{i'=1}^{J} T^k_{i'}(d^k_{i'j}w_{i'})^{-\theta^k}(B^k_{i'j})^{\theta^k}$ is the **sourcing capability**.

- Quick Lemma: B^k_{ij} is increasing in μ^k_{hij} and μ^k_{xij}. $B^k_{ij} \le 1$, with equality iff $\mu^k_{hij} = \mu^k_{xij} = 1$ (i.e., full contractibility).

- *Interpretation:* B^k_{ij} terms capture the effect of contracting frictions on sourcing capability (of final-good firms in country j)

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- *Caveat 1:* The π_{ij}^k's are sourcing probabilities, not expenditure shares. (Hence: Further aggregation step required.)

- *Caveat 2:* If interested in firm-level heterogeneity in sourcing patterns, could readily go the AFT (2017) route.

Sourcing Shares

Share of inputs sourced under mode (i, χ) is equal to $\pi_{ij}^k \pi_{\chi|ij}^k$.

- $\pi_{\chi|ij}^k$: probability of sourcing under organizational mode χ conditional on selecting country i

$$\pi_{\chi|ij}^k = \frac{(B_{ij\chi}^k)^{\frac{\theta^k}{1-\lambda_i}}}{(B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}} + (B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}}.$$

- Conditional probability is a function of the $B_{ij\chi}^k$'s. (In particular: Does not depend on T_i^k, d_{ij}^k or w_i.)
- Quick Lemma: As in AH (2008), propensity to integrate ($\pi_{V|ij}^k$) is:
  - increasing in μ_{xij}^k (supplier contractibility); and
  - decreasing in μ_{hij}^k (hq contractibility).

From Firm-level Decisions to Aggregate Variables

(i) *Composite industry-k input,* $X_j^k(\phi)^{\rho^k} = \mathbb{E}_\ell\left[\tilde{x}_j^k(\phi; l)^{\rho^k}\right]$. Details

(ii) *Assembly labor,* $L_j(\phi)$. Details

Upshot: $X_j^k(\phi)$, $L_j(\phi)$, and hence $q_j(\phi)$ are linear in $R_j(\phi)$.

(iii) *Revenue,* $R_j(\phi)$*:* After aggregation,

$$R_j(\phi) \quad = \quad E_j\left(\phi/\bar{\phi}\right)^{\frac{\rho}{1-\rho}},$$

where $\bar{\phi} = \left(\int \phi^{\frac{\rho}{1-\rho}}\, dG_j(\phi)\right)^{\frac{1-\rho}{\rho}}$ and $E_j = \frac{w_j \bar{L}_j + s_j \bar{H}_j}{1 - \frac{1-\rho)}{1-\rho(1-\alpha)}\bar{\Upsilon}_j}$.

Welfare

Plugging the expression for $q_j(\phi)$ into the utility function:

$$
\begin{aligned}
U_j &= (N_j)^{\frac{1-\rho}{\rho}} \rho E_j \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(w_j)^\alpha} \left(\frac{\bar{\Upsilon}_j}{1-\rho(1-\alpha)} \right)^\alpha \bar{\phi} \\
&\quad \times \prod_{k=1}^{K} \left[\left(\frac{\alpha^k}{s_j} \right)^{\alpha^k} \left(\frac{1-\alpha^k}{w_j} \right)^{1-\alpha^k} \eta^k \left(\frac{\bar{\Gamma}}{\Upsilon_j^k} \right)^{\frac{1-\rho^k}{\rho^k}} \left(\frac{T_j^k}{\pi_{jj}^k} \right)^{\frac{1-\alpha^k}{\theta^k}} (B_{jj}^k)^{1-\alpha^k} \right]^{\eta^k(1-\alpha)}
\end{aligned}
$$

Let $\hat{X} \equiv X'/X$.

- Consider shocks to trade costs (the d_{ij}^k's), to contracting frictions (the μ_{hij}^k's, μ_{xij}^k's), or to the bargaining parameters (the $\beta_{ij\chi}^k$'s).

Welfare Change: Decomposition

$$\widehat{U_j} = \widehat{E}_j \left(\widehat{w_j}\right)^{-\alpha} \left(\prod_{k=1}^{K} \left[\left(\widehat{w_j}\right)^{-(1-\alpha^k)} \left(\widehat{s}_j\right)^{-\alpha^k} \right]^{\eta^k(1-\alpha)} \right) \left(\widehat{\bar{\Upsilon}_j}\right)^{\alpha}$$
$$\times \prod_{k=1}^{K} \left[\left(\widehat{\Upsilon_j^k}\right)^{-\frac{1-\rho^k}{\rho^k}} \left(\widehat{\pi_{jj}^k}\right)^{-\frac{1-\alpha^k}{\theta^k}} \left(\widehat{B_{jj}^k}\right)^{1-\alpha^k} \right]^{\eta^k(1-\alpha)}$$

(i) $\prod_{k=1}^{K} \left(\widehat{\pi_{jj}^k}\right)^{-\frac{1-\alpha^k}{\theta^k}\eta^k(1-\alpha)}$: As in ACR (2012) and CR (2014)

(ii) $\prod_{k=1}^{K} \left(\widehat{B_{jj}^k}\right)^{(1-\alpha^k)\eta^k(1-\alpha)}$: Contracting frictions' effect on sourcing capability

(iii) $\prod_{k=1}^{K} \left(\widehat{\Upsilon_j^k}\right)^{-\frac{1-\rho^k}{\rho^k}\eta^k(1-\alpha)}$: Holding sourcing capability constant, contracting frictions' effect on relationship-specific input investments

(iv) $\left(\widehat{\bar{\Upsilon}_j}\right)^{\alpha}$: Contracting frictions' effect on choice of assembly labor

(v) $\widehat{w}_j$, $\widehat{s}_j$, $\widehat{E}_j$ terms: Factor price effects (in general equilibrium)

Trade flows by sourcing mode

Assume trade flows observed are valued at costs, and thus equal to factor payments to suppliers. Aggregating over all suppliers by sourcing mode:

$$t_{ij\chi}^{k} = \frac{(1-\alpha)\rho\eta^{k}}{\rho^{k}} \frac{\Upsilon_{j}^{k}}{\Phi_{j}^{k}} E_{j}\rho^{k}(1-\alpha^{k}) T_{i}^{k}(w_{i})^{-\theta^{k}} \left(B_{ij}^{k}\right)^{-\frac{\theta^{k}\lambda_{i}}{1-\lambda_{i}}} \left(d_{ij}^{k}\right)^{-\theta^{k}}$$
$$\times \left(\mu_{xij}^{k} + (1-\mu_{xij}^{k})(1-\beta_{ij\chi}^{k})\frac{\zeta_{ij}^{k}}{\zeta_{ij\chi}^{k}}\right) \frac{1}{2} \left(B_{ij\chi}^{k}\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}. \tag{10}$$

A gravity-like decomposition of terms into:

- a destination-country-by-industry component
- a source-country-by-industry component
- bilateral trade costs
- a country-pair-by-industry-by-organizational-mode (χ) component

Trade flows by sourcing mode

Assume trade flows observed are valued at costs, and thus equal to factor payments to suppliers. Aggregating over all suppliers by sourcing mode:

$$t_{ij\chi}^k = \underbrace{\frac{(1-\alpha)\rho\eta^k}{\rho^k} \frac{\Upsilon_j^k}{\Phi_j^k} E_j \rho^k (1-\alpha^k) T_i^k (w_i)^{-\theta^k} \left(B_{ij}^k\right)^{-\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(d_{ij}^k\right)^{-\theta^k}}_{a_{ij}^k}$$

$$\times \underbrace{\left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) \frac{1}{2} \left(B_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}_{a_{ij\chi}^k}.$$

Taking the Model to the Data

Estimation: Framework

Empirical setting: U.S. Related Party Trade Database

- $\tilde{t}^k_{ij\chi}$: Observed value of industry-k trade from country i to j, under mode $\chi \in \{V, O\}$. (Stack both the import $j = US$ and export $i = US$ data.)
- Map k to NAICS 3-digit industries.

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- Map k to NAICS 3-digit industries.

Posit that trade flows from (10) are observed with error:

$$\tilde{t}^k_{ij\chi} = t^k_{ij\chi} \cdot \epsilon^k_{ij\chi} \;=\; a^k_{ij\chi} \cdot a^k_{ij} \cdot \epsilon^k_{ij\chi}, \tag{11}$$

where:

- a^k_{ij} collects terms that are specific to the country-pair-by-industry; Details
- $a^k_{ij\chi}$ collects terms that further vary by organizational mode; and
- $\epsilon^k_{ij\chi}$ is an iid Poisson noise term with unit mean (consistent with zeros)

Estimation: Framework (cont.)

Implies a moment condition:

$$E\left[\frac{\tilde{t}_{ijV}^k}{\tilde{t}_{ij}^k}\,\middle|\,\tilde{t}_{ij}^k\right] = \frac{a_{ijV}^k a_{ij}^k}{\sum_{\chi=\{V,O\}} a_{ij\chi}^k a_{ij}^k} = \frac{a_{ijV}^k}{\sum_{\chi=\{V,O\}} a_{ij\chi}^k}. \tag{12}$$

- Why? Under the Poisson assumption, $\tilde{t}_{ijV}^k/\tilde{t}_{ij}^k$ conditional on $\tilde{t}_{ij}^k$ follows a Bernoulli distribution with success probability: $a_{ijV}^k a_{ij}^k / \sum_{\chi=\{V,O\}} a_{ij\chi}^k a_{ij}^k$. (Reminiscent of Eaton, Kortum, Sotelo 2013.) Details
- Alternative foundation: Replacing a_{ij}^k with its quasi-maximum likelihood estimator yields the same moment condition (Santos Silva and Tenreyro 2006; Fally 2015) Details

Estimation: Framework (cont.)

Implies a moment condition:

$$E\left[\frac{\tilde{t}^k_{ijV}}{\tilde{t}^k_{ij}}\middle|\tilde{t}^k_{ij}\right] = \frac{a^k_{ijV}a^k_{ij}}{\sum_{\chi=\{V,O\}}a^k_{ij\chi}a^k_{ij}} = \frac{a^k_{ijV}}{\sum_{\chi=\{V,O\}}a^k_{ij\chi}}. \tag{12}$$

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- Alternative foundation: Replacing a^k_{ij} with its quasi-maximum likelihood estimator yields the same moment condition (Santos Silva and Tenreyro 2006; Fally 2015) Details

Upshot: A structural estimating equation in which:

- the intrafirm trade share is the dependent variable; and
- $a^k_{ij\chi}$ on the RHS is a function of parameters: $\beta^k_{ij\chi}$, μ^k_{hij}, μ^k_{xij}, α^k, θ^k, λ_i, ρ^k.

Estimation: Mapping the μ_{hij}^k's, μ_{xij}^k's to observables

- Since $\mu_{hij}^k, \mu_{xij}^k \in [0,1]$, adopt a logistic function specification:

$$\mu_{hij}^k = \frac{e^{\mathbf{h}(i,j,k)}}{1+e^{\mathbf{h}(i,j,k)}}, \quad \text{and} \quad \mu_{xij}^k = \frac{e^{\mathbf{x}(i,j,k)}}{1+e^{\mathbf{x}(i,j,k)}}.$$

  - $\mathbf{h}(i,j,k)$: full 2nd-order polynomial in HQContractibility^k, ROL$_i$, and ROL$_j$.
  - $\mathbf{x}(i,j,k)$: full 2nd-order polynomial in SSContractibility^k, ROL$_i$, and ROL$_j$.

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- Distinguishing between hq and supplier contractibility:
  - Order industries according to their capital-intensity
  - HQContractibility^k: Weighted-average contractibility of inputs from NAICS industries with above-median capital-intensity (c.f., Nunn 2007).
  - SSContractibility^k: Analogous weighted-average contractibility of inputs from NAICS input industries with below-median capital-intensity

- ROL: Country rule-of-law index from the World Governance Indicators.

Estimation: Mapping the $\beta^k_{ij\chi}$'s to observables

- For β^k_{ijO}, two approaches:
  - $\beta^k_{ijO} = \beta_O$;
  - β^k_{ijO} specified as a logistic function of $\mathbf{b}(k)$, a quadratic in industry markup (de Loecker et al. 2016)
- Set: $\beta^k_{ijV} = (1 - \delta^k_{ij})\beta^k_{ijO} + \delta^k_{ij}$, where $\delta^k_{ij} \in [0, 1]$ is the share of bilateral surplus over which the firm has residual rights of control.

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- Since $\delta^k_{ij} \in [0, 1]$:

$$\delta^k_{ij} = \frac{e^{\mathbf{d}(i,j,k)}}{1 + e^{\mathbf{d}(i,j,k)}},$$

  where $\mathbf{d}(i, j, k)$ is a full 2nd-order polynomial in Specificity^k, ROL$_i$, and ROL$_j$. Also include a BITS dummy.

  (Similar results with Doing Business country measure of insolvency recovery rate.)

- Specificity^k: Industry specificity (c.f., Rauch 1999)

Estimation: Mapping the α^k's to observables

- For the α^k's:

$$\alpha^k = \frac{e^{\mathbf{a}(k)}}{1 + e^{\mathbf{a}(k)}},$$

  where $\mathbf{a}(k)$ is a quadratic in $\log(K/L)^k$.

- $\log(K/L)^k$: Capital-labor ratio, NBER CES Dataset (c.f., Antràs 2003)

Estimation: Framework (cont.)

$$m(\Theta) = \mathbb{E}\left[\frac{\tilde{t}^k_{ij\chi}}{\sum_{\chi\in\{V,O\}} \tilde{t}^k_{ij\chi}} - \frac{a^k_{ij\chi}}{\sum_{\chi\in\{V,O\}} a^k_{ij\chi}} \middle| \mathbf{X}^k_{ij}\right] = 0. \tag{13}$$

$\mathbf{X}^k_{ij}$: observables that enter into $\mathbf{a}(k)$, $\mathbf{b}(k)$, $\mathbf{d}(i,j,k)$, $\mathbf{h}(i,j,k)$, and $\mathbf{x}(i,j,k)$.

Weighted non-linear least squares (NLLS): $\Theta^* = \text{argmin}_{\Theta} \left(m(\Theta)\right)^T \Omega \left(m(\Theta)\right)$

- Intrafirm trade share: Use average over 2001-2005.

  Top 50 U.S. trade partners (less HKG, SAU, VEN and IRQ).

Estimation: Framework (cont.)

$$m(\Theta) = \mathbb{E}\left[\frac{\tilde{t}^k_{ij\chi}}{\sum_{\chi\in\{V,O\}}\tilde{t}^k_{ij\chi}} - \frac{a^k_{ij\chi}}{\sum_{\chi\in\{V,O\}} a^k_{ij\chi}} \middle| \mathbf{X}^k_{ij}\right] = 0. \tag{13}$$

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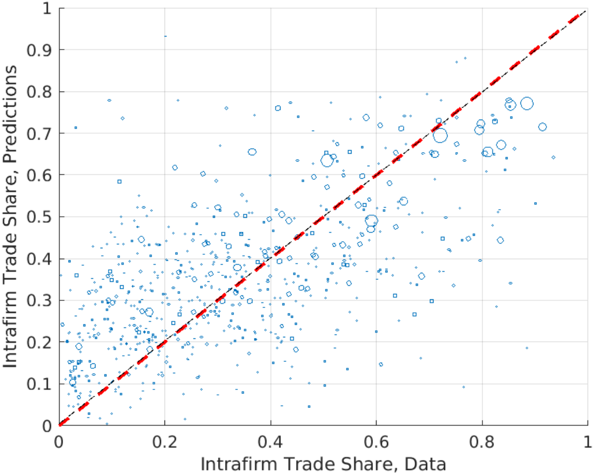
- Intrafirm trade share: Use average over 2001-2005.
  Top 50 U.S. trade partners (less HKG, SAU, VEN and IRQ).
- Pinned down externally: ρ^k (Soderbery 2015).
- Remaining parameters to be estimated: $\Theta = \{\theta^k, \lambda, \beta_O, \gamma_1, \ldots\}$.
- Algorithm: Levenberg-Marquardt (with theoretical restrictions)
  Standard errors: Gauss-Newton regressions

Predicted vs actual: Intrafirm trade shares



(b) Model v.s. Data

Point estimates

name	est.	se	95% CI
α^k constant	-0.632	0.019	[-0.67, -0.59]
α^k $\ln(K/L)$	0.436	0.019	[0.40, 0.47]
α^k $(\ln(K/L))^2$	-0.117	0.005	[-0.13, -0.11]
λ_{01}	0.905	0.004	[0.90, 0.91]
λ_{02}	0.869	0.005	[0.86, 0.88]
λ_{03}	0.776	0.010	[0.76, 0.80]
F-val	55.24	-	-

θ^k's, α^k's, ρ^k's
μ^k_{xij}'s, μ^k_{hij}'s, δ^k_{ij}'s

- Some normalizations: $\theta^1 = 4$
- Separate λ's for lower-middle, upper-middle, and high income countries:

  Estimates point to the relevance of within-"nest" correlation in productivity draws.

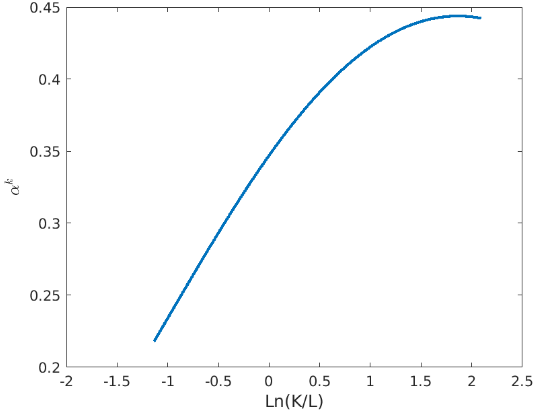
Point estimates

Identifying variation:

- θ^k's: Within-industry dispersion in the intrafirm trade share.
- λ_i's: Within-country group, cross-industry dispersion.
- μ^k_{hij}'s, μ^k_{xij}'s, δ^k_{ij}: Partial correlation between the intrafirm trade share and the relevant country and industry characteristics. More
- α^k's: Implied α^k's need to be sufficiently large, to rationalize high intrafirm trade share observations.

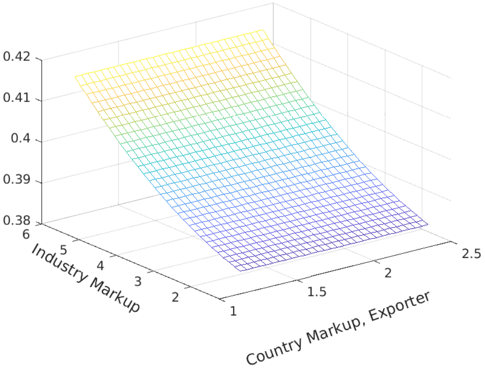
Illustrating the estimation results

- α^k: Increasing in the capital-labor ratio.



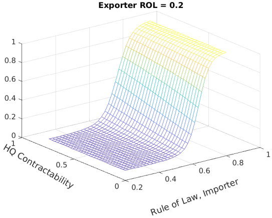
Illustrating the estimation results

- β_O: Increasing in the industry markup

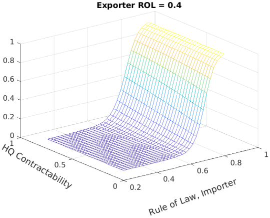


Illustrating the estimation results

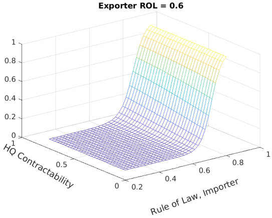
- For μ_{hij}^k and μ_{xij}^k: Both increasing in Importer ROL$_j$



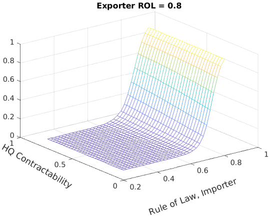
(e) Importer ROL



(f) Importer ROL



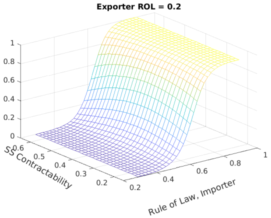
(g) Importer ROL



(h) Importer ROL

Illustrating the estimation results

- For μ^k_{hij} and μ^k_{xij}: Both increasing in Importer ROL$_j$



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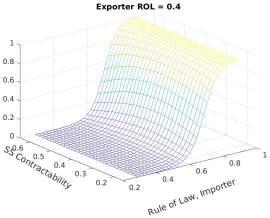

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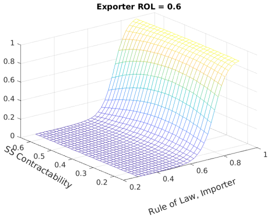
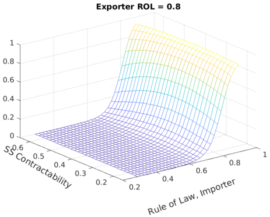


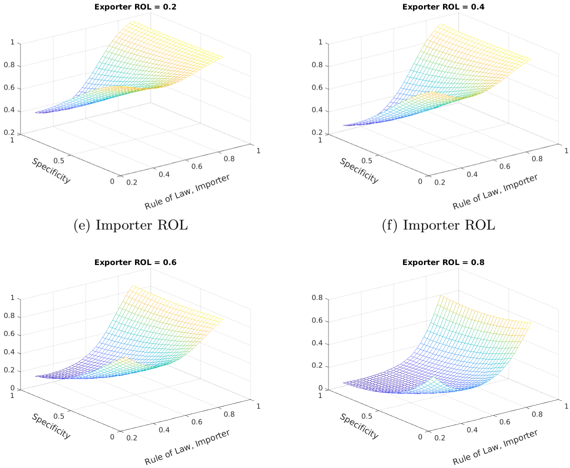
(g) Importer ROL



(h) Importer ROL

Illustrating the estimation results

- δ_{ij}^{k}: Firm's residual control rights weakest when industry specificity is high and importer ROL is low.



(e) Importer ROL

(f) Importer ROL

(g) Importer ROL

(h) Importer ROL

Closing the Model and Implementing Counterfactuals

- Factor-market clearing conditions in each country to close the model:
  - Labor endowment $\bar{L}_j$ equals the sum of factor demand from: (i) final-good assembly; and (ii) country-j input suppliers.
  - Capital endowment $\bar{H}_j$ equals the sum of factor demand from firms headquartered in country j
  - Firm profits accrue back to consumers via holdings in a domestic asset market. No international trade in assets; take deficits as fixed from the data.
- Counterfactual changes computed via a "hat algebra" system, following Dekle et al. (2008) ▸ Details

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  - Capital endowment $\bar{H}_j$ equals the sum of factor demand from firms headquartered in country j
  - Firm profits accrue back to consumers via holdings in a domestic asset market. No international trade in assets; take deficits as fixed from the data.

- To operationalize, need: (i) initial trade shares across countries (which we take from the ICIO); and (ii) calibrated/estimated model parameters
  - Perform a correction on trade shares to map them to the π_{ij}^k's in the model.
    Details
  - Upper-tier parameters: η^k, $\alpha = 0.2$ (targeting a labor share of 0.55), $\rho = 0.8$
  - Pin down $w_j\bar{L}_j$ and $s_j\bar{H}_j$ in the initial equilibrium, rather than take it from data.

Counterfactuals: Global improvement in μ_{hij}^k and μ_{xij}^k

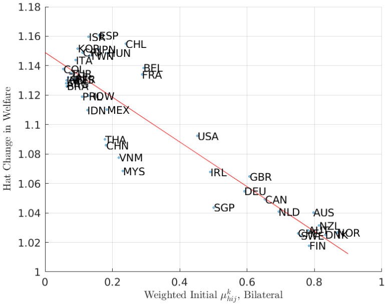
Shift all μ_{hij}^k's and μ_{xij}^k's to 1 globally, to get to the full contractibility world:

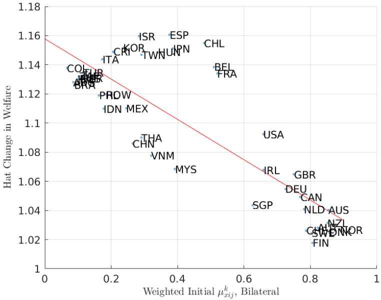
- Yields a mean country welfare increase of 9.8%
- Gains driven by the contracting frictions terms: $\widehat{B}_{jj}^k$, $\widehat{\Upsilon}_j^k$

	Total Effect	$\widehat{\pi}_{jj}^k$	$\widehat{B}_{jj}^k$	$\widehat{\Upsilon}_j^k$	$\widehat{\bar{\Upsilon}}_j$	Factor Price Effects
$\mu_{hij}^k = \mu_{xij}^k = 1$	0.098	-0.001	0.232	0.111	-0.059	-0.131
	(0.048)	(0.005)	(0.158)	(0.083)	(0.045)	(0.107)
$\mu_{xij}^k = 1$	0.091	-0.003	0.192	0.093	-0.052	-0.102
	(0.048)	(0.004)	(0.138)	(0.073)	(0.042)	(0.091)
$\mu_{hij}^k = 1$	0.061	-0.001	0.159	0.072	-0.043	-0.097
	(0.038)	(0.003)	(0.110)	(0.054)	(0.035)	(0.080)

Counterfactuals: Global improvement in μ_{hij}^k and μ_{xij}^k

- Greater welfare gains for countries with lower initial μ's.





Counterfactuals: Removing integration as an organizational mode

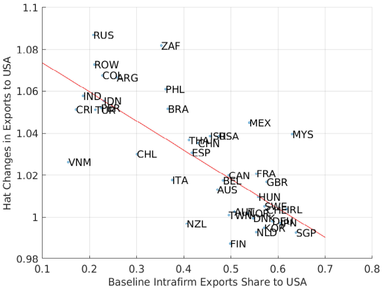
Set $\delta_{ij}^k = 1$ globally, so final-good firms never choose integration:

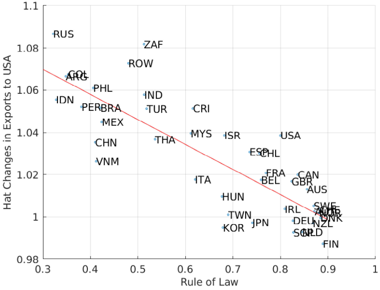
- Modest loss in average country welfare: -0.1%
- (Not directly comparable with the gains from MP: Garetto 2013, Ramondo and Rodriguez-Clare 2013, Ramondo 2014, Arkolakis et al. 2018, and Alvariez 2019)

	Total Effect	$\widehat{\pi}_{jj}^k$	$\widehat{B}_{jj}^k$	$\widehat{\Upsilon}_j^k$	$\widehat{\bar{\Upsilon}}_j$	Factor Price Effects
$\delta_{ij}^k = 1$	-0.001	-0.000	-0.003	0.000	-0.000	0.002
	(0.003)	(0.001)	(0.002)	(0.000)	(0.000)	(0.003)

Counterfactuals: Removing integration as an organizational mode

- Meaningful reallocations of trade flows across source countries (illustrated for $\delta_{ij}^{k} = 1$ for the USA as importer).
  - $\downarrow$ imports from high rule of law countries, previously conducted under intrafirm trade
  - $\uparrow$ imports from low rule of law countries, when outsourcing is the only available organizational mode

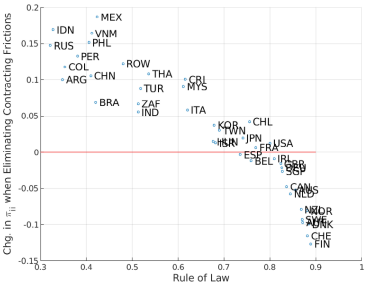


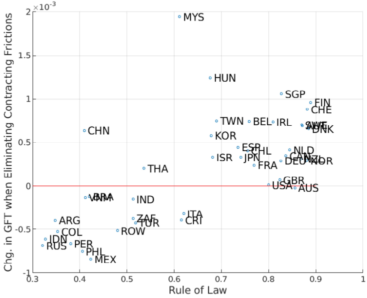


Counterfactuals: Gains from Trade and Contracting Frictions

When contracting frictions are removed (i.e., all μ^k_{hij}'s and μ^k_{xij}'s are set to 1):

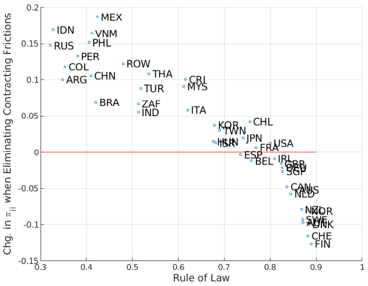
- the gains from trade increase for high ROL countries,
- while decreasing for low ROL countries.

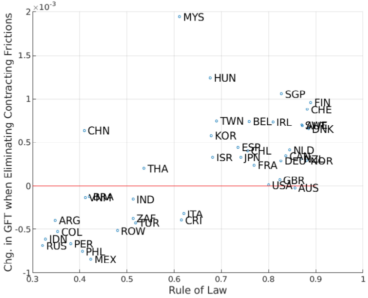




Counterfactuals: Gains from Trade and Contracting Frictions

- *Intuition:* Low ROL countries see large improvements in contractibility when these frictions are removed $\Rightarrow$ They end up sourcing more, particularly from themselves (i.e., $\pi_{jj} \uparrow$).
- This dampens the gains from trade.

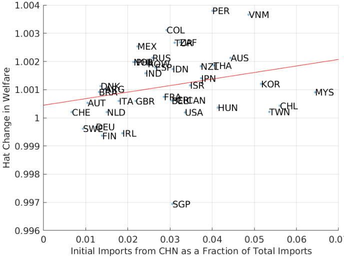


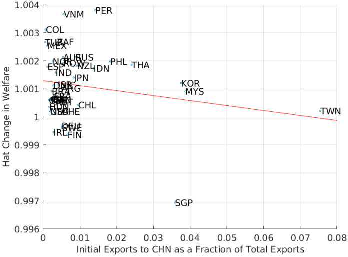


Counterfactuals: Improving CHN Rule of Law

Raise ROL in CHN to the world frontier (FIN):

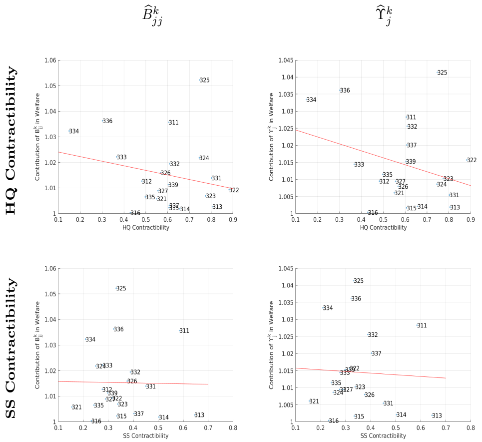
- CHN's welfare gain: $+8.3\%$
- All other countries gain too, with an average of $+0.11\%$
- Countries who see their imports with China rise more experience a larger welfare increase





Counterfactuals: Improving CHN Rule of Law

Raise ROL in CHN to the world frontier (FIN):



- Bigger contribution to aggregate welfare change in CHN from industries with lower hq contractibility.

Concluding Remarks

Wrapping Up

- Developed a bridge between: (i) models of contracting frictions with bilateral holdup; and (ii) the quantitative trade literature.

- The model delivers:
  - Tractable expressions for sourcing shares and a modified gains-from-trade formula, that reflect the effects of contracting frictions
  - A structural estimating equation for the intrafirm trade share

- Quantification allows us to address such issues as:
  - How much do contracting fictions in global sourcing impinge on country welfare?
  - How much would a country stand to gain from an improvement in institutions (e.g., related to the rule of law) that alleviates these frictions in input sourcing?

Supplementary Slides

Incremental revenue: Derivation

Details

Compute for discrete number of suppliers, L, each in charge of $\epsilon = 1/L$ inputs.

$$\widetilde{r}(\ell;\epsilon) \;=\; A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\times$$

$$\left\{\left[\left(\sum_{\ell'\neq\ell} x_j^k(\phi;\ell')^{\rho^k}\epsilon'\right)+x_j^k(\phi;\ell)^{\rho^k}\epsilon\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}} - \left[\left(\sum_{\ell'\neq\ell} x_j^k(\phi;\ell')^{\rho^k}\epsilon'\right)\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}}\right\}.$$

Approximate the term in the curly braces via a first-order Taylor expansion about $\epsilon = 0$. Then, evaluate the limit as $L \to \infty$.

$$\frac{\widetilde{r}(\ell;\epsilon)}{\epsilon} \;\approx\; A_j^{1-\rho}\phi^{\rho}L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\times$$

$$\left[\left(\sum_{\ell'\neq\ell} x_j^k(\phi;\ell')^{\rho^k}\epsilon'\right)+x_j^k(\phi;\ell)^{\rho^k}\epsilon\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}-1}\left(\frac{\eta^k(1-\alpha)\rho}{\rho^k}\right)x_j^k(\phi;\ell)^{\rho^k}$$

$$\Rightarrow \quad r_j^k(\phi;\ell) \;=\; \lim_{L\to\infty}\frac{\widetilde{r}(\epsilon)}{\epsilon} \;=\; (1-\alpha)\frac{\rho\eta^k}{\rho^k}R_j(\phi)\left(\frac{x_j^k(\phi;\ell)}{X_j^k(\phi)}\right)^{\rho^k}.$$

$\Xi^k_{ij\chi}$: Details ▸ Return

$$\Xi^k_{ij\chi} = \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right)^{\frac{1}{\rho^k(1-\alpha^k)}} \times \left(\frac{1-\rho^k}{\rho^k}\right)^{\frac{1-\rho^k}{\rho^k(1-\alpha^k)}} \left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)}}$$
$$\times \left(\frac{\alpha^k}{s_j}\right)^{\frac{\alpha^k}{1-\alpha^k}} \left(\frac{1-\alpha^k}{d^k_{ij} w_i}\right) \left(\beta^k_{ij\chi}\right)^{\frac{\alpha^k(1-\mu^k_{hij})}{1-\alpha^k}} \left(1-\beta^k_{ij\chi}\right)^{1-\mu^k_{xij}},$$

where:

$$\zeta^k_{ij\chi} = 1 - \rho^k\alpha^k\left(1-\mu^k_{hij}\right)\beta^k_{ij\chi} - \rho^k\left(1-\alpha^k\right)\left(1-\mu^k_{xij}\right)\left(1-\beta^k_{ij\chi}\right), \text{ and} \quad (14)$$
$$\zeta^k_{ij} = 1 - \rho^k\alpha^k\left(1-\mu^k_{hij}\right) - \rho^k\left(1-\alpha^k\right)\left(1-\mu^k_{xij}\right). \quad (15)$$

Note: $\zeta^k_{ij\chi} \geq \zeta^k_{ij}$, with equality if and only if $\mu^k_{hij} = \mu^k_{xij} = 1$ (i.e., in the full contractibility case).

$X_j^k(\phi)$: Details ▸ Return

(i) *Composite industry-k input*:

Assuming $\theta^k > \frac{(1-\alpha^k)\rho^k}{1-\rho^k}$, can be evaluated explicitly as:

$$\mathbb{E}_\ell\left[\tilde{Z}^k_{ij\chi}(\phi;\ell)^{\frac{(1-\alpha^k)\rho^k}{1-\rho^k}}\right] = \bar{\Gamma}^k \times \pi^k_{ij}\pi^k_{\chi|ij}\left(\Phi^k_j\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \frac{\left(d^k_{ij}w_i\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}}{\left(B^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}}.$$

where $\bar{\Gamma}^k \equiv \Gamma\left(1 - \frac{1}{\theta^k}\frac{(1-\alpha^k)\rho^k}{1-\rho^k}\right)$, and $\Gamma(\cdot)$ is the Gamma function.

$L_j(\phi)$: Details ▸ Return

(ii) *Full solution to the firm's problem.*

Firm's overall payoff (with ex-ante transfers):

$$F_j(\phi) = R_j(\phi) - \sum_{k=1}^{K} \int_{\ell=0}^{1} s_j h_j^k(\phi,\ell) d\ell - \sum_{k=1}^{K} \int_{\ell=0}^{1} c_{ij\chi}^k(\phi,\ell) x_j^k(\phi,\ell) d\ell - w_j L_j(\phi), \tag{16}$$

- From this, solve for $L_j(\phi)$ (final-good assembly labor)
- After simplification: $X_j^k(\phi)$, $L_j(\phi)$, and hence $q_j(\phi)$ are all linear functions of $R_j(\phi)$.

$L_j(\phi)$: Details

Return

(ii) *Full solution to the firm's problem.*

In particular:

$$
\begin{aligned}
X_j^k(\phi) &= (1-\alpha)\rho\eta^k R_j(\phi)\left(\frac{\alpha^k}{s_j}\right)^{\alpha^k}\left(1-\alpha^k\right)^{1-\alpha^k}\left(\Phi_j^k\right)^{\frac{1-\alpha^k}{\theta^k}}\left(\bar{\Gamma}^k\right)^{\frac{1-\rho^k}{\rho^k}}\left(\Upsilon_j^k\right)^{-\frac{1-\rho^k}{\rho^k}}, \\
L_j(\phi) &= \frac{\alpha\rho}{1-\rho(1-\alpha)}\frac{1}{w_j}\bar{\Upsilon}_j R_j(\phi).
\end{aligned}
$$

- Υ_j^k and $\bar{\Upsilon}_j$ depend on the underlying parameters, including the $\beta_{ij\chi}^k$ bargaining shares

  In particular, $\bar{\Upsilon}_j$ is the share of revenue that accrues to the firm (after accounting for payments to factors other than assembly labor)

Υ_j^k and $\bar{\Upsilon}_j$: Details

- $\bar{\Upsilon}_j$: Share of revenues that accrue to the firm (after accounting for the ex-ante transfer and payments to factors)

$$\Upsilon_j^k = \left(\sum_{i=1}^{J} \sum_{\chi \in \{V,O\}} \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \pi_{ij}^k \pi_{\chi|ij}^k \right)^{-1}, \text{ and}$$

$$\bar{\Upsilon}_j = 1 - (1-\alpha) \sum_{k=1}^{K} \frac{\rho \eta^k}{\rho^k} \left(1 - (1-\rho^k) \Upsilon_j^k \right).$$

- In the full-contractibility case ($\mu_{hij}^k = \mu_{xij}^k = 1$), we have: $\Upsilon_j^k = 1$ and $\bar{\Upsilon}_j = 1 - \rho(1-\alpha)$.

From Model to Data: Details ▸ Return

$$\tilde{t}^k_{ij\chi} = t^k_{ij\chi} \cdot \epsilon^k_{ij\chi} \;=\; a^k_{ij\chi} \cdot a^k_{ij} \cdot \epsilon^k_{ij\chi},$$

$$a^k_{ij} = (1-\alpha)\rho\eta^k \frac{\Upsilon^k_j}{\Phi^k_j} E_j \left(1-\alpha^k\right) T^k_i (w_i)^{-\theta^k} \left(B^k_{ij}\right)^{-\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(d^k_{ij}\right)^{-\theta^k} \frac{1}{2} \left(\frac{1}{\zeta^k_{ij}}\right)^{\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)} \frac{\theta^k}{1-\lambda_i}},$$

$$a^k_{ij\chi} = \left(\zeta^k_{ij\chi}\right)^{\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)} \frac{\theta^k}{1-\lambda_i}} \left(1-\beta^k_{ij\chi}\right)^{\frac{\theta^k}{1-\lambda_i}\left(1-\mu^k_{xij}\right)} \left(\beta^k_{ij\chi}\right)^{\left(1-\mu^k_{hij}\right)\frac{\alpha^k}{1-\alpha^k}\frac{\theta^k}{1-\lambda_i}}$$
$$\times \left(\mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi}) \frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)$$

Moment Condition: Details

Return

- $\tilde{t}_{ij}^{k}$ is the sum of two independent Poisson random variables, $\tilde{t}_{ijV}^{k} + \tilde{t}_{ijO}^{k} = \tilde{t}_{ij}^{k}$.
- Property: Conditional on the realized value of $\tilde{t}_{ij}^{k}$, the distribution of $\tilde{t}_{ijV}^{k}$ is a binomial distribution where:
  - $\tilde{t}_{ij}^{k}$ is the number of the trials; and
  - $a_{ijV}^{k} a_{ij}^{k} / \left(\sum_{\chi=\{V,O\}} a_{ij\chi}^{k} a_{ij}^{k} \right)$ is the success probability.
- It follows that the distribution of $\tilde{t}_{ijV}^{k} / \tilde{t}_{ij}^{k}$ conditional on $\tilde{t}_{ij}^{k}$, is Bernoulli with the same success probability.
- This yields the following moment condition for estimation; compare to (13):

$$E\left[\frac{\tilde{t}_{ijV}^{k}}{\tilde{t}_{ij}^{k}} \middle| \tilde{t}_{ij}^{k} \right] = \frac{a_{ijV}^{k} a_{ij}^{k}}{\sum_{\chi=\{V,O\}} a_{ij\chi}^{k} a_{ij}^{k}} = \frac{a_{ijV}^{k}}{\sum_{\chi=\{V,O\}} a_{ij\chi}^{k}}$$

Moment Condition: Quasi-MLE Approach

Return

Treat a_{ij}^k as a source-by-industry fixed effect. Writing down the quasi-maximum likelihood function, the FOC with respect to a_{ij}^k implies:

$$a_{ij}^k \sum_{\chi \in \{V,O\}} a_{ij\chi}^k = \sum_{\chi \in \{V,O\}} \tilde{t}_{ij\chi}^k$$

$$\Rightarrow \qquad a_{ij}^k = \frac{\sum_{\chi=V,O} \tilde{t}_{ij\chi}^k}{\sum_{\chi=V,O} a_{ij\chi}^k} \quad \text{(c.f., Fally 2015)}$$

Substituting this back into the expression for $\tilde{t}_{ij\chi}^k$, we have:

$$\frac{\tilde{t}_{ijV}^k}{\sum_{\chi \in \{V,O\}} \tilde{t}_{ij\chi}^k} = \frac{a_{ijV}^k}{\sum_{\chi \in \{V,O\}} a_{ij\chi}^k} \varepsilon_{ijV}^k. \tag{17}$$

Industry Parameters

Return

ID	NAICS3	Desc	α^k	θ^k	ρ^k	$1-\frac{(1-\alpha^k)\rho^k}{\theta^k(1-\rho^k)}$
1	311	Food Manufacturing ...	0.349	4.000	0.860	0.000
2	312	Beverage and Tobacco Prod ...	0.416	20.009	0.788	0.814
3	313	Textile Mills ...	0.363	5.105	0.878	0.103
4	314	Textile Product Mills ...	0.255	9.677	0.816	0.659
5	315	Apparel Manufacturing ...	0.242	9.384	0.884	0.384
6	316	Leather and Allied Produc ...	0.268	13.293	0.830	0.731
7	321	Wood Product Manufacturin ...	0.271	6.562	0.827	0.468
8	322	Paper Manufacturing ...	0.411	13.714	0.702	0.899
9	323	Printing and Related Supp ...	0.293	18.169	0.757	0.879
10	324	Petroleum and Coal Produc ...	0.444	12.418	0.930	0.405
11	325	Chemical Manufacturing ...	0.427	24.190	0.854	0.861
12	326	Plastics and Rubber Produ ...	0.328	8.422	0.916	0.134
13	327	Nonmetallic Mineral Produ ...	0.359	30.382	0.815	0.907
14	331	Primary Metal Manufacturi ...	0.413	23.407	0.933	0.652
15	332	Fabricated Metal Product ...	0.311	10.043	0.779	0.758
16	333	Machinery Manufacturing ...	0.342	23.598	0.889	0.777
17	334	Computer and Electronic P ...	0.396	23.289	0.816	0.885
18	335	Electrical Equipment Appl ...	0.328	1.533	0.695	-0.000
19	336	Transportation Equipment ...	0.368	28.847	0.825	0.897
20	337	Furniture and Related Pro ...	0.223	9.912	0.353	0.957
21	339	Miscellaneous Manufacturi ...	0.284	7.773	0.777	0.679
-	-	Mean	0.338	14.463	0.810	0.612

Estimates for bargaining parameters

▸ Return

name	est.	se	95% CI
$\gamma_1 : \mu_{xij}^k$ constant	-7.507	0.908	[-9.29, -5.73]
$\gamma_2 : \mu_{xij}^k$ $SSCont^k$	6.545	0.266	[6.02, 7.06]
$\gamma_3 : \mu_{xij}^k$ $\left(SSCont^k\right)^2$	-11.050	0.256	[-11.55, -10.55]
$\gamma_4 : \mu_{xij}^k$ ROL_i	-9.781	0.549	[-10.86, -8.70]
$\gamma_5 : \mu_{xij}^k$ $(ROL_i)^2$	2.765	0.439	[1.91, 3.62]
$\gamma_6 : \mu_{xij}^k$ ROL_j	5.615	2.445	[0.82, 10.41]
$\gamma_7 : \mu_{xij}^k$ $(ROL_j)^2$	13.218	1.567	[10.15, 16.29]
$\gamma_8 : \mu_{xij}^k$ $SSCont^k \times ROL_i$	-1.217	0.065	[-1.34, -1.09]
$\gamma_9 : \mu_{xij}^k$ $SSCont^k \times ROL_j$	2.813	0.308	[2.21, 3.42]
$\gamma_{11} : \mu_{hij}^k$ constant	-13.882	2.512	[-18.81, -8.96]
$\gamma_{12} : \mu_{hij}^k$ $HQCont^k$	6.728	0.725	[5.31, 8.15]
$\gamma_{13} : \mu_{hij}^k$ $\left(HQCont^k\right)^2$	-0.361	0.024	[-0.41, -0.31]
$\gamma_{14} : \mu_{hij}^k$ ROL_i	-11.258	0.332	[-11.91, -10.61]
$\gamma_{15} : \mu_{hij}^k$ $(ROL_i)^2$	6.342	0.275	[5.80, 6.88]
$\gamma_{16} : \mu_{hij}^k$ ROL_j	10.817	5.824	[-0.60, 22.23]
$\gamma_{17} : \mu_{hij}^k$ $(ROL_j)^2$	14.802	3.428	[8.08, 21.52]
$\gamma_{18} : \mu_{hij}^k$ $HQCont^k \times ROL_i$	0.074	0.034	[0.01, 0.14]
$\gamma_{19} : \mu_{hij}^k$ $HQCont^k \times ROL_j$	-7.147	0.913	[-8.94, -5.36]

Estimates for bargaining parameters

Return

name	est.	se	95% CI
γ_{21} : δ_{ij}^{k} constant	6.540	0.187	[6.17, 6.91]
γ_{22} : δ_{ij}^{k} $Speci^{k}$	-5.989	0.125	[-6.23, -5.74]
γ_{23} : δ_{ij}^{k} $\left(Speci^{k}\right)^{2}$	1.044	0.054	[0.94, 1.15]
γ_{24} : δ_{ij}^{k} ROL_{i}	0.477	0.566	[-0.63, 1.59]
γ_{25} : δ_{ij}^{k} $(ROL_{i})^{2}$	-5.214	0.426	[-6.05, -4.38]
γ_{26} : δ_{ij}^{k} ROL_{j}	-16.423	0.190	[-16.79, -16.05]
γ_{27} : δ_{ij}^{k} $(ROL_{j})^{2}$	15.095	0.179	[14.74, 15.45]
γ_{28} : δ_{i}^{k} $Speci^{k} \times ROL_{i}$	-0.055	0.043	[-0.14, 0.03]
γ_{29} : δ_{i}^{k} $Speci^{k} \times ROL_{j}$	5.028	0.057	[4.92, 5.14]
γ_{30} : δ_{i}^{k} $BITS$	0.019	0.002	[0.01, 0.02]
γ_{35} : β_{ijO}^{k} constant	-0.477	0.013	[-0.50, -0.45]
γ_{45} : β_{ijO}^{k} $Markup^{k}$	0.005	0.003	[-0.00, 0.01]
γ_{46} : β_{ijO}^{k} $\left(Markup^{k}\right)^{2}$	0.003	0.000	[0.00, 0.00]

Identifying variation

Return

LHS = Intrafirm Trade Share	Data		Model Prediction	
	(1)	(2)	(3)	(4)
Capital Intensity	0.219***		0.222***	
	(0.057)		(0.048)	
(Capital Intensity)^2	-0.047		-0.051	
	(0.032)		(0.029)	
Specificity	2.252**		1.948**	
	(1.026)		(0.864)	
(Specificity)^2	-0.765		-0.650	
	(0.718)		(0.596)	
Contractibility, hq	0.190		-0.306	
	(0.785)		(0.804)	
(Contractibility, hq)^2	-0.300		-0.287	
	(0.462)		(0.423)	
Contractibility, ss	4.269**		5.627***	
	(1.651)		(1.665)	
(Contractibility, ss)^2	-4.708**		-4.218**	
	(1.705)		(1.503)	
ROL, imp.	2.366**	2.331***	1.704	1.692*
	(0.836)	(0.759)	(0.995)	(0.935)
(ROL, imp.)^2	-0.262	-0.289	0.281	0.257
	(0.415)	(0.398)	(0.502)	(0.498)
ROL, exp.	0.008	-0.256	0.157	-0.031
	(0.938)	(1.016)	(0.661)	(0.741)
(ROL, exp.)^2	0.064	0.041	0.094	0.059
	(0.674)	(0.724)	(0.535)	(0.575)
BITs w/ U.S.	-0.006	-0.002	-0.001	0.002
	(0.045)	(0.041)	(0.020)	(0.019)
ROL, exp. × Specificity	-0.370	-0.167	-0.441***	-0.276***
	(0.265)	(0.222)	(0.129)	(0.089)
ROL, imp. × Specificity	-1.178***	-1.200***	-0.882**	-0.907**
	(0.308)	(0.279)	(0.397)	(0.382)
ROL, exp. × Contractibility, hq	1.193***	1.337***	1.162***	1.310***
	(0.308)	(0.282)	(0.196)	(0.136)
ROL, imp. × Contractibility, hq	-1.476***	-1.403***	-0.821	-0.762
	(0.469)	(0.400)	(0.631)	(0.592)
ROL, exp. × Contractibility, ss	-0.793	-0.687	-1.213**	-1.188***
	(0.686)	(0.524)	(0.464)	(0.307)
ROL, imp. × Contractibility, ss	-0.107	0.052	-1.912*	-1.822
	(0.652)	(0.597)	(1.102)	(1.133)
Constant	-2.077***		-1.948**	
	(0.678)		(0.702)	
N	1,926	1,926	1,926	1,926
R-squared	0.377	0.423	0.641	0.701
Fixed Effects	None	k	None	k

Hat algebra: Details ▸ Details

$$\left(\zeta_{ij}^{k}\right)' = 1 - \rho^{k} + \rho^{k}\alpha^{k}\left(\mu_{hij}^{k}\right)' + \rho^{k}\left(1-\alpha^{k}\right)\left(\mu_{xij}^{k}\right)' \tag{18}$$

$$\left(\zeta_{ij\chi}^{k}\right)' = 1 - \rho^{k}\alpha^{k}\left[1-\left(\mu_{hij}^{k}\right)'\right]\left(\beta_{ij\chi}^{k}\right)' - \rho^{k}\left(1-\alpha^{k}\right)\left(1-\left(\mu_{xij}^{k}\right)'\right)\left(1-\left(\beta_{ij\chi}^{k}\right)'\right). \tag{19}$$

$$\left(B_{ij\chi}^{k}\right)' = \left[1-\left(\beta_{ij\chi}^{k}\right)'\right]^{1-\left(\mu_{xij}^{k}\right)'}\left[\left(\beta_{ij\chi}^{k}\right)'\right]^{\left(1-\left(\mu_{hij}^{k}\right)'\right)\frac{\alpha^{k}}{1-\alpha^{k}}}\left[\frac{\left(\zeta_{ij\chi}^{k}\right)'}{\left(\zeta_{ij}^{k}\right)'}\right]^{\frac{\left(\zeta_{ij}^{k}\right)'}{\rho^{k}\left(1-\alpha^{k}\right)}} \tag{20}$$

$$\left(B_{ij}^{k}\right)' = \left(\frac{1}{2}\left[\left(\left(B_{ijV}^{k}\right)'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}} + \left(\left(B_{ijO}^{k}\right)'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}\right]\right)^{\frac{1-\lambda_{i}}{\theta^{k}}}. \tag{21}$$

Hat algebra: Details (cont.)

$$\left(\pi^k_{\chi|ij}\right)' = \frac{\left((B^k_{ij\chi})'\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left((B^k_{ijV})'\right)^{\frac{\theta^k}{1-\lambda_i}} + \left((B^k_{ijO})'\right)^{\frac{\theta^k}{1-\lambda_i}}}. \tag{22}$$

$$\widehat{\pi^k_{ij}} = \frac{\left(\widehat{d^k_{ij}}\widehat{w}_i\right)^{-\theta^k}\left(\widehat{B^k_{ij}}\right)^{\theta^k}}{\widehat{\Phi^k_j}} \tag{23}$$

$$\widehat{\Phi^k_j} \equiv \sum_{i=1}^{J} \pi^k_{ij}(\widehat{d^k_{ij}}\widehat{w}_i)^{-\theta^k}\left(\widehat{B^k_{ij}}\right)^{\theta^k} \tag{24}$$

Hat algebra: Details (cont.)

$$\left(\upsilon_{ij\chi}^{k}\right)' = \frac{\left(\pi_{ij}^{k}\right)'\left(\pi_{\chi|ij}^{k}\right)'}{\frac{\left(\varsigma_{ij\chi}^{k}\right)'}{\left(\varsigma_{ij}^{k}\right)'}} \tag{25}$$

$$\left(\Upsilon_{j}^{k}\right)' = \left\{\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\frac{\left(\pi_{ij}^{k}\right)'\left(\pi_{\chi|ij}^{k}\right)'}{\frac{\left(\varsigma_{ij\chi}^{k}\right)'}{\left(\varsigma_{ij}^{k}\right)'}}\right\}^{-1} = \left\{\sum_{i=1}^{J}\sum_{\chi=\{V,O\}}\left(\upsilon_{ij\chi}^{k}\right)'\right\}^{-1} \tag{26}$$

$$\left(\bar{\Upsilon}_{j}\right)' = 1-(1-\alpha)\sum_{k=1}^{K}\frac{\rho\eta^{k}}{\rho^{k}}\left[1-(1-\rho^{k})\left(\Upsilon_{j}^{k}\right)'\right]. \tag{27}$$

Hat algebra: Details (cont.)

$$\left(E_j\right)' = \frac{\widehat{w}_j w_j \bar{L}_j + \widehat{s}_j s_j \bar{H}_j + D_j}{1 - \frac{1-\rho}{1-\rho(1-\alpha)} \left(\bar{\Upsilon}_j\right)'} \tag{28}$$

$$\begin{aligned} \widehat{w_j} w_j \bar{L}_j = \rho\alpha \left(\frac{\bar{\Upsilon}_j}{1-\rho(1-\alpha)} \right)' \left(E_j\right)' + \rho\left(1-\alpha\right) \\ \times \sum_{k=1}^{K} \left(1-\alpha^k\right) \eta^k \sum_{m=1}^{J} \left(E_m\right)' \left(\Upsilon_m^k\right)' \sum_{\chi \in \{V,O\}} \left(\upsilon_{jm\chi}^k\right)' \left[\frac{\left(\mu_{xjm}^k\right)' \left(\varsigma_{jm\chi}^k\right)'}{\left(\varsigma_{jm}^k\right)'} + \left(1 - \left(\mu_{xjm}^k\right)'\right) \left[1 - \left(\beta_{jm\chi}^k\right)'\right] \right] \end{aligned} \tag{29}$$

$$\widehat{s_j^h} s_j^h \bar{H}_j = \rho(1-\alpha) \left(E_j\right)' \sum_{k=1}^{K} \alpha^k \eta^k \left(\Upsilon_j^k\right)' \sum_{i=1}^{J} \sum_{\chi=V,O} \left(\upsilon_{ij\chi}^k\right)' \left[\frac{\left(\mu_{hij}^k\right)' \left(\varsigma_{ij\chi}^k\right)'}{\left(\varsigma_{ij}^k\right)'} + \left(\beta_{ij\chi}^k\right)' \left(1 - \left(\mu_{hij}^k\right)'\right) \right] \tag{30}$$

Note: Data for E_j are from the ICIO. Value of $w_j \bar{L}_j$, $s_j \bar{H}_j$ and D_j inferred from the factor market clearing conditions and national expenditure identity in the initial equilibrium.

Hat algebra: Details (cont.)

The algorithm:

1. Given $(\mu^k_{hij})'$, $(\mu^k_{hij})'$ and $\left(\beta^k_{ij\chi}\right)'$, use equation (21) to solve for $(B^k_{ij\chi})'$.
2. Use equation (22) and $(B^k_{ij\chi})'$ to get $\left(\pi^k_{\chi|ij}\right)'$ and $\widehat{\pi^k_{\chi|ij}}$.
3. Guess a vector of $\widehat{w}_j$ and $\widehat{s}_j$.
4. Conditional on the guessed $\widehat{w}_j$ and $\widehat{s}_j$, use equation (24) to solve for $\widehat{\Phi^k_j}$.
5. Use $\widehat{\Phi^k_j}$ and equation (23) to solve for $\widehat{\pi^k_{ij}}$ and $\left(\pi^k_{ij}\right)'$.
6. With $\left(\pi^k_{ij}\right)'$, we can use equation (26) and (27) to get $\left(\Upsilon^k_m\right)'$ and $\left(\bar{\Upsilon}_m\right)'$.
7. With $\left(\bar{\Upsilon}_j\right)'$, use equation (28) to solve for $\left(E_j\right)'$.
8. With all the above information, invert equation (29) to get a new $\widetilde{w_j}$ Similarly, we can update the price of capital, $\widetilde{s_j}$ by inverting equation (30):
9. Update $\left(\widehat{w_j}, \widehat{s_j}\right)$ with $\left(\widetilde{w_j}, \widetilde{s_j}\right)$, and iterate from step 3 until convergence.

Trade Share Correction Term: Details

Back

Denote the correction term as σ_{ij}^k:

$$\sigma_{ij}^k = \left(B_{ij}^k\right)^{-\frac{\theta^k}{1-\lambda_i}} \sum_{\chi \in \{V,O\}} \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k) \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \right) \frac{1}{2} \left(B_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}}.$$

Applying this correction term to the observed bilateral trade flows, $\tilde{t}_{ij}^k$, in the data allows us to recover the model-implied sourcing probabilities, π_{ij}, since:

$$\frac{\tilde{t}_{ij}^k/\sigma_{ij}^k}{\sum_{i'=1}^J \tilde{t}_{i'j}^k/\sigma_{i'j}^k} = \frac{T_i^k \left(w_i d_{ij}^k\right)^{-\theta^k} \left(B_{ij}^k\right)^{-\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(B_{ij}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}{\sum_{i'=1}^J T_{i'}^k \left(w_{i'} d_{i'j}^k\right)^{-\theta^k} \left(B_{i'j}^k\right)^{-\frac{\theta^k \lambda_{i'}}{1-\lambda_{i'}}} \left(B_{i'j}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}} = \pi_{ij}^k.$$