The Aggregate Consequences of Tax Evasion*

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February 12, 2018

Preliminary Draft!

Abstract

There is a sizeable overall tax gap in the U.S., albeit tax noncompliance differs sharply across income types. While only small percentages of wages and salaries are underreported, the estimated misreporting rate of self-employment business income is substantial. This paper studies how tax evasion in the self-employment sector affects aggregate outcomes and inequality. To this end, we develop a dynamic general equilibrium model with incomplete markets in which heterogeneous agents choose between being a worker and being self-employed. Self-employed agents may hide a share of their business income but are confronted with the probability of being detected by the tax authority. Our model replicates important quantitative features of U.S. data, in particular, the misreporting rate, wealth inequality, and the firm size distribution. Our quantitative findings suggest that tax evasion induces self-employed businesses to stay small. In the aggregate, tax evasion increases the size but decreases the productivity of the self-employment sector. Moreover, it increases aggregate savings and reduces wealth inequality. We show that tax revenues follow a Laffer curve in the size of the tax evasion penalty.

JEL Classifications: H24, H25, H26, C63, E62, E65.
Keywords: Tax evasion, Self-Employment, Wealth inequality, Tax policy.

*We are grateful to Leo Kaas, Dirk Krueger, Moritz Kuhn, Marco Maffezzoli and Nawid Siassi for insightful comments.

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1 Introduction

The evasion of individual income taxes is substantial in the United States. The Internal Revenue Service (IRS) estimates that the lost tax revenue due to underreported income is \$197 billion in 2001, which is 18% of the actual income tax liability (U.S. Department of the Treasury 2009). Tax evasion is concentrated among the self-employed businesses. While only 1% of wages and salaries are not reported, this figure rises to 57% of self-employed income (Johns and Slemrod 2010). Self-employed businesses constitute an important component of the U.S. economy. They account for 39% of the assets and 21% of the income in the economy.[1]

What are the aggregate consequences of tax evasion in the self-employment sector in the U.S.? Does evading taxes by small businesses matter for aggregate outcomes, inequality and welfare? What are the channels through which such effects operate? What are the implications for tax policy and enforcement?

To answer these questions we develop a dynamic general equilibrium model with incomplete markets and occupational choice and analyze how imperfect tax enforcement affects aggregate outcomes, wealth inequality, and welfare. In our model environment, infinitely lived agents face idiosyncratic and persistent shocks to their labor productivity and their entrepreneurial talent. They pay progressive income taxes and each period choose between being a worker or a self-employed entrepreneur. Workers supply inelastically their effective time endowment to corporate firms which operate with a constant returns to scale technology. These firms use competitively labor and capital and produce a consumption good. Workers cannot evade taxes and make consumption and saving decisions. Self-employed entrepreneurs use a decreasing returns to scale technology in capital to produce the consumption good. They may hide a share of their business income but are confronted with the probability of being detected by the tax authorities and punished by paying the evaded taxes and a proportional fine. Self-employed entrepreneurs optimally determine the size of their firms by choosing capital, taking into account that detection becomes more likely as their firm grows. In doing so, they face borrowing constraints proportional to the amount of their own savings.

We calibrate our model to the U.S. economy at the start of the 2000s. First, we use the Panel Survey of Income Dynamics (PSID) data to estimate the labor productivity process for workers and parametric functions for the progressive income taxes paid by workers and entrepreneurs. Second, we set the parameter values related to production, the talent of running a self-employed business, and tax evasion by matching a selected number of data targets via a method-of-moments estimation. In particular, our model targets the capital-output ratio and the interest rate of the U.S. economy, the share of

[1] These numbers are derived from the Panel Study of Income Dynamics (PSID). For more details on the data work, see Appendix A.

self-employed business owners, their assets and income, and the annual exit rate from self-employment. Importantly, the parameters related to tax evasion are set to match the average misreporting rate of income as well as the cross-sectional misreporting rates conditional on the level of income.

The model replicates the empirical distributions of income and wealth even though they are not explicitly targeted. Another non-targeted dimension which the model successfully matches is the size distribution of self-employed businesses. The overall excellent fit of the model with respect to this broad set of empirical facts for the U.S. economy gives us confidence to use the model for a quantitative analysis.

In our quantitative analysis we study the impact of tax evasion by comparing our benchmark economy with a counterfactual economy in which taxes are perfectly enforceable. The optimal decision rules highlight three important channels through which tax evasion may affect aggregate outcomes. *(i)* The *subsidy channel*: Tax evasion acts like a subsidy and stimulates asset accumulation, allowing higher investment in business capital. *(ii)* The *selection channel*: The opportunity to evade taxes induces less talented agents to run self-employed businesses. *(iii)* The *detection channel*: Self-employed business owners have incentives to keep their businesses small to stay under the tax authorities radar and reduce the chances of being audited.

The three channels are reflected in the aggregate outcomes of our stationary equilibrium. The opportunity to evade taxes increases the share of self-employed business but reduces the average productivity of the self-employment sector. Moreover, tax evasion increases the share of small businesses, which is crucial for replicating the empirical firm size distribution. Furthermore, the economy with tax evasion is characterized by higher aggregate savings and higher aggregate output than the counterfactual economy with perfect tax enforcement. The increase in the aggregate capital stock lowers the interest rate and raises the wage which generates a lower wealth inequality.

Next, we study the implications for tax enforcement and tax policy. First, we vary the fine that detected evaders have to pay to the tax authorities. It turns out that tax revenues follow a Laffer curve in the size of the fine. This hump-shaped pattern is generated by two opposing forces. On the one hand, a higher fine allows the government to collect more revenues. On the other hand, a higher fine makes misreporting more risky and reduces the share of self-employed businesses. This, in turn, reduces aggregate output such that the lower tax base decreases tax revenues. Our quantitative findings suggest that the fine that maximizes tax revenues is roughly three times larger than the existing civil fraud penalty of 75 % on missing taxes in the U.S.

Second, we vary the average tax burden for workers and self-employed businesses by scaling their respective non-linear tax functions. The resulting Laffer curves show that the elasticity of self-employed taxable income is much higher than the elasticity of taxable labor income in the presence of tax evasion. Thus, the explicit modeling of the tax evasion has direct quantitative implications for the assessment of tax policy.

Finally, we look at the welfare implications of tax evasion. Perfect enforcement leads to average welfare in terms of benchmark consumption equivalence to decrease by 2.3%. Most of this loss is incurred by workers. However, in the absence of tax evasion, tax revenues increase by around 1.6% of GDP. If these additional benefits are distributed back to the households, perfect tax enforcement brings about small welfare gains of around 0.1%-0.4%.

The rest of the paper is organized as follows. The next subsection discusses the related literature. In Section 2 we provide further details on the technology of tax evasion in the United States. Section 3 presents the model. Section 4 explains the calibration procedure and shows the model fit and some features of the benchmark economy. Section 5.2 discusses the counterfactual experiments. The last section concludes.

1.1 Related Literature

The economic theory of the technology and practices of tax evasion was initiated by the seminal work of Allingham and Sandmo (1972). They present a stylized model of tax evasion by a risk-averse agent who faces the probability of getting caught and penalized by the tax authorities. Andreoni (1992) extends this framework to a two-period model with income uncertainty and borrowing constraints. [2] We take this classic modeling approach to tax evasion and incorporate it in modern heterogeneous agent macro model of income and wealth inequality.

Our paper is related to a couple of other studies which explore the aggregate consequences of tax evasion in heterogeneous agent macroeconomic models of incomplete markets. Maffezzoli (2011) looks the distributional effects of income tax evasion in a heterogeneous agent framework with incomplete markets. His model, similarly to ours, successfully replicates the cross-sectional pattern of tax evasion which increases in true individual income levels. The results point out moving from a progressive taxation to a proportional tax rate reduces the amount of evaded taxes and improves government revenues. In contrast to his model, our framework explicitly accounts for the role of self-employed businesses in tax evasion. This allows us to quantitatively document the consequences of tax evasion for capital accumulation and aggregate productivity.

Another related study is Dessy and Pallage (2003). Their two-period heterogeneous agent model features formal and informal sectors of production with taxes financing the provision of productive public infrastructure. While the study outlines the differential role of tax evasion for aggregate productivity and inequality in poor and rich countries, it does not attempt to quantitatively explore the role of tax evasion for the aggregates.

[2] Other notable extensions of the static theory are presented by Yitzhaki (1974) and Pencavel (1979) who allow for a more general penalization structure and introduce labor supply choice, respectively. For a detailed summary of the literature, see Slemrod and Yitzhaki (2002) and Slemrod (2007).

Our work builds on the existing quantitative models of incomplete markets with entrepreneurs facing borrowing constraints. The seminal works of Quadrini (2000) and Cagetti and De Nardi (2006) paved the way for generating adequate distributions of wealth in macroeconomic environments due to the savings behavior of entrepreneurs. Kitao (2008), on the other hand, explores the productive and welfare effects of capital taxation in a similar framework and shows that these effects vary depending on whether entrepreneurial or non-entrepreneurial capital is taxed. We complement these works by introducing the possibility of tax evasion for self-employed businesses and by exploring its role for aggregate economic outcomes and welfare.

Finally, we also contribute to the macroeconomic literature of occupational choice and informality featuring two-sector models with formal and informal production. Amaral and Quintin (2006) emphasize the fact that informal sectors in developing countries feature less-skilled workers than formal sectors. Antunes and Cavalcanti (2007) argue that the variation in regulation costs and financial contracts enforcement account for the cross-country differences in informality. In a similar spirit, Kuehn (2014) explains the variation of informality across OECD countries through differences in taxes and government quality. Ordonez (2014), like us, emphasizes the role of imperfect tax enforcement for aggregate output and productivity for the case of Mexico. We extend these models to an environment with richer heterogeneity and more realistic self-employed business sector which allows us to conduct a more elaborate quantitative analysis on the role of tax evasion for the U.S. economy.

2 Tax Evasion in the United States

The Internal Revenue Code contains three primary obligations on taxpayers: *(i)* to file timely returns, *(ii)* to report accurately on those returns, and *(iii)* to pay the required tax voluntarily and on time. Thus, non-compliance takes three forms: *(i)* underreporting (not reporting full liability on a timely-filed return), (ii) underpayment (not paying the full amount of tax reported on a timely-filed return), and, *(iii)* nonfiling (not filing the required returns on time). Given the scope of this paper, we concentrate our attention to the first component of noncompliance, namely, underreporting.

Individual Income Tax Evasion and Its Distribution. The underreporting tax gap is defined as the amount of tax liability which is not reported voluntarily by taxpayers who file tax returns on time. The IRS estimates that in 2001 underreporting activities with respect to individual income lead to a underreporting tax gap of $197 billion (U.S. Department of the Treasury 2009). This amounts to around 2% of the U.S. GDP in this year. The estimate is based on the data collected through the National Research Program (NRP) Individual Income Tax Reporting Compliance Study for the 2001 tax year. The

NRP analyzes approximately 46,000 randomly selected individual income tax returns.[3]

Only 1% of the of wages and salaries and 4% of taxable interest and dividends are misreported to the IRS. In contrast, 57% of self-employment business income is not reported. Thus, it is reasonable to assume that tax evasion of individual income happens almost exclusively in the group of self-employed businesses. Johns and Slemrod (2010) analyze the micro data from the NRP in order to assess the distribution of income tax noncompliance for the fiscal year of 2001. In their analysis tax payers are grouped according to percentiles of their true income, that is, the gross income they should have reported if not evading. According to their calculations, 11% of true income is misreported to the IRS. However, the misreporting rate varies with income levels. True income levels in the first decile of income are not misreported at all. Income in all other deciles below the median are misreported at a steady rate of around 5%. Around 7-8% of income in the four deciles above the median are hidden. Finally, tax evasion reaches its highest in the top decile where more than 15% of income is misreported.

Detecting and Punishing Tax Evasion. The IRS had around 13,000 revenue and tax agents in 2002 whose main responsibility is detecting tax evasion (Dubin 2004). The individual income tax examination coverage, that is, the audit rate was 1.27% in 1997. This was followed by a continued decline in the following years and at the start of the 2000s the audit rate fell below 1% (TIGTA 2002).
Legally it is very demanding to prove that a taxpayer knowingly committed a fraudulent act when evading taxes. Therefore, the IRS performs very few criminal investigations and more often pursues civil charges for evasion. Accuracy-related penalties vary between 20%-40% of the missing taxes, while the civil fraud penalty is fixed at 75% (U.S. Department of the Treasury 2016).

3 The Model

The model builds on the seminal contributions of Quadrini (2000) and Cagetti and De Nardi (2006) who introduce entrepreneurs in macroeconomic models of wealth inequality, but it differs from them in thee key aspects. First, we introduce income tax evasion following the classic paper by Allingham and Sandmo (1972). Second, we allow for non-linear taxes which describe the existing tax code more accurately. Third, in the light of the empirical facts on tax evasion, we concentrate our attention on the self-employed businesses and not on the general category of entrepreneurs.

Our model economy includes households, firms and government. Households are infinitely lived. Each time period corresponds to one year. Households receive a pair of idiosyncratic realizations of their working and entrepreneurial abilities each period. Based

[3]The estimated underreporting gap excludes unpaid taxes due purely illegal activities.

on these realizations and their stock of savings, they decide whether to form a self-employed business or to supply their work to a labor market. As in Aiyagari (1994), asset markets are incomplete, that is, households cannot insure against shocks to working or business ability. In addition, there is another source of market imperfection - borrowing of self-employed businesses is subject to collateral constraints.

3.1 Preferences and Endowments

Households maximize the expected sum of discounted utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\beta \in (0,1)$ is the time discount factor and c_t is consumption in period t. The utility function u is defined as $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ denotes the relative risk aversion coefficient. For simplicity we assume that labor is inelastically supplied. Each household is endowed with a working ability $\varepsilon \in E$ and a business ability $\theta \in \Theta$, where ε and θ are drawn from a finite-state Markov process with transition probability given by $F(\varepsilon', \theta' | \varepsilon, \theta)$.

3.2 Technology

The economy consists of two sectors of production. The single consumption good is produced either in small self-employed businesses or in a large corporate sector with a representative firm. Actors in both sectors are price takers.

Self-employed businesses run a single project which combines their business ability θ and capital k according to a production function,

$$f(k) = \theta k^v,$$

where $0 < v < 1$. The production function exhibits decreasing returns to scale capturing the *span of control* idea introduced by Lucas (1978): self-employed business skills gradually deteriorate as the size of the firm increases. The self-employed can save at a risk-free rate r and use their own wealth to finance capital used in the project. In addition to using their own assets, they are also allowed to borrow from a financial intermediary at a rate r. However, financial contracts are not perfectly enforceable, and thus, borrowing is limited up to a constant share of the assets self-employed businesses can pledge as collateral, $k \leq \lambda a$, where $\lambda \geq 1$.[4] The two polar cases of $\lambda = 1$ and $\lambda = \infty$ capture the two extremes of financial autarky and perfect credit markets, respectively.

[4] To be more precise we could explicitly introduce an intermediation sector which processes deposits from workers and self-employed and gives loans to self-employed businesses at the same interest rate because there is no cost of intermediation and no possibility of default. When giving loans to self-employed businesses, the financial intermediary requires a collateral.

The corporate firm operates according to a constant returns to scale technology,

$$F(K_C, N_C) = K_C^{\alpha} N_C^{1-\alpha},$$

where $0 < \alpha < 1$. The corporate firm rents capital from households at rate r and labor services from workers paying a wage w. Capital in both sectors depreciates at a rate $\delta \in (0,1)$. Profit maximization in the corporate sector implies that input prices are set according to their marginal productivity,

$$r = \alpha K_C^{\alpha-1} N_C^{1-\alpha} - \delta \tag{1}$$

and

$$w = (1-\alpha) K_C^{\alpha} N_C^{-\alpha}. \tag{2}$$

3.3 Government and Taxation

The government raises tax revenues to finance wasteful public spending G. Both workers and self-employed are subject to a nonlinear personal income tax $T^i(\cdot)$ meant to approximate the actual tax code for the U.S. by capturing not only the statutory tax rates but also available deductions, exemptions and tax credits. We allow the tax schedules to be different for workers and self-employed. In particular, following Gouveia and Strauss (1999), we assume that each agent of type $i = \{W, E\}$, where W stands for worker and E stands for self-employed, has to pay tax liabilities given by the following tax function,[5]

$$T^i(y) = a_0^i \left[y - \left(y^{-a_1^i} + a_2^i \right)^{-1/a_1^i} \right]. \tag{3}$$

Note that for $a_1 > 0$ we have a progressive tax system since the average tax rate,

$$\frac{T^i(y)}{y} = a_0^i \left[1 - \left(1 + a_2^i y^{a_1^i} \right)^{-1/a_1^i} \right],$$

is increasing with income y.[6]

The crucial element of our modeling exercise is the introduction of imperfect tax enforcement. Whereas workers cannot evade taxes, self-employed agents may hide a share $\phi \in [0,1]$ of their business income.[7] The government, knowing that the self-employed evade taxes, can monitor through audits and perfectly verify the individual tax returns.

[5] Guner et al. (2014) show that this tax function is very flexible and provides an excellent approximation to the effective U.S. tax schedule. This functional form has been used extensively in the quantitative macroeconomic and public finance literature. Notable examples are Conesa and Krueger (2006), Kitao (2008) and Cagetti and De Nardi (2009).

[6] In addition, the degree of tax progressivity is increasing with a_1. If $a_1 \to 0$, then taxes are proportional,$T^i(y) \to a_0 y$.

[7] We assume that interest income generated by savings cannot be underreported, for both workers and self-employed.

Let $p(k)$, with $p'(k) > 0$, be the probability that a self-employed tax return is subject to monitoring. The key assumption here is that the probability of being audited depends positively on the size of the business, capturing the idea that larger firms are more visible to the tax authorities. If the self-employed agent is audited and underreporting is detected, a fine $s > 1$ proportional to the amount of the underreported taxes is issued. In essence, the self-employed needs to pay back the hidden taxes and an additional proportional penalty. For simplicity we assume that the auditing efforts of the tax authorities are costless.

3.4 Household Problem

Timing of events. The sequence of events in this economy unfolds as follows. At the beginning of each period, the idiosyncratic shocks ε and θ for working ability and entrepreneurial ability are realized. After observing these shocks, and conditional on the value of assets a inherited from the previous period, an individual chooses whether to be a worker or a self-employed for the current period. Workers make optimal decisions regarding consumption and savings and pay income taxes to the government. On the other hand, self-employed decide how much to invest (i.e. they choose k, taking the collateral constraint into account) and how much to evade (i.e. they choose ϕ). After business decisions are taken, detection and auditing by the government takes place. After observing if they are detected or not, self-employed agents make consumption and savings decisions. Note that the optimal consumption and saving choice of the self-employed is contingent on detection.

The optimization problem of an agent can be cast in a recursive formulation, with the individual states being the assets level a and the current abilities ε and θ. If we let V^W and V^E denote respectively the values of being a worker or an entrepreneur, the beginning-of-the-period value function is given by:

$$V(a,\varepsilon,\theta) = \max\left\{V^W(a,\varepsilon,\theta), V^E(a,\varepsilon,\theta)\right\} \tag{4}$$

where $o(a,\varepsilon,\theta)$ denotes the occupational choice associated with problem (4),

$$o(a,\varepsilon,\theta) = \begin{cases} 1 & \text{, if } V^E \geq V^W(a,\varepsilon,\theta) \\ 0 & \text{, otherwise} \end{cases}.$$

Workers. The worker's problem can be written as

$$V^W(a,\varepsilon,\theta) = \max_{c,a'} \{u(c) + \beta E[V(a',\varepsilon',\theta')|\varepsilon,\theta]\}$$

subject to

$$y_W = w\epsilon + ra \tag{5}$$

$$c + a' \leq y_W + a - T^W(y_W) \tag{6}$$

$$a' \geq 0 \tag{7}$$

where $T^W(\cdot)$ is the nonlinear tax schedule defined in Section 3.3. Each worker supplies her total amount of time to the corporate sector as employed labor, earning a wage w for each productivity units ε. Equation (5) represents the worker's taxable income which consists of labor income $w\epsilon$ and income from financial assets ra. Equation (6) simply states that all available resources net of taxes are split between consumption and savings. The last constraint summarizes the assumption that workers cannot borrow.[8] Crucially, employed workers are not allowed to misreport their true income to the tax authority.

Self-Employed. The decisions of a self-employed agent over production and tax evasion amount to choosing the operational capital k and the share of business income ϕ which is not reported to the tax authorities. In doing so, the agent takes into account the probability of an audit by the government which is conditional on the amount of capital invested in the business. The beginning-of-the-period value function is given by

$$V^E(a,\varepsilon,\theta) = \max_{k,\phi\in[0,1]} \left\{p(k)\, V_d^E(a,\varepsilon,\theta,k,\phi) + (1-p(k))\, V_n^E(a,\varepsilon,\theta,k,\phi)\right\} \tag{8}$$

subject to

$$0 \leq k \leq \lambda a, \tag{9}$$

where (9) is the collateral constraint. The value function for the case of detection is given by

$$V_d^E(a,\varepsilon,\theta,k,\phi) = \max_{c,a'} \{u(c) + \beta E[V(a',\varepsilon',\theta')\,|\varepsilon,\theta]\} \tag{10}$$

subject to

$$\pi = \theta k^v - (\delta + r)k \tag{11}$$

$$y_E = \pi + ra \tag{12}$$

$$c + a' \leq y_E + a - T^E((1-\phi)\pi + ra) - s\left[T^E(\pi + ra) - T^E((1-\phi)\pi + ra)\right] \tag{13}$$

$$a' \geq 0. \tag{14}$$

Equations (11) and (12) define respectively the profits from business activity and taxable income, which includes both capital profits π and financial income from savings ra. The budget constraint is given by equation (13) and states that available resources, net of taxes and fines, are allocated between consumption and savings. In this case the self-employed is audited by the government and has to pay a fine whenever the last term in (13) is positive. Notice that self-employed agents may hide a fraction ϕ of their business income π but they report truthfully interest income ra.

The value function for the case of non-detection is defined as

$$V_n^{SE}(a,\varepsilon,\theta,k,\phi) = \max_{c,a'} \{u(c) + \beta E[V(a',\varepsilon',\theta')\,|\varepsilon,\theta]\} \tag{15}$$

[8] More generally, equation (7) can be replaced by $a' \geq -\underline{a}$ where $\underline{a} \geq 0$ is an ad hoc borrowing limit.

subject to

$$\pi = \theta k^v - (\delta + r)k$$

$$\begin{aligned} y_E &= \pi + ra \\ c + a' &\leq y_E + a - T^E\left(\left(1-\phi\right)\pi + ra\right) \\ a' &\geq 0 \end{aligned} \tag{16}$$

The optimization problem in (15) is very similar to (10) with the only difference coming from the flow budget constraint which now does not show any tax evasion penalties.

After solving the above problems, we get the following policy functions which we summarize here for later reference:

- $o\left(a, \varepsilon, \theta\right)$ is the policy function for the occupational choice;
- $g^W\left(a, \varepsilon, \theta\right)$ is the policy function for asset holdings a' if the agent is a worker;
- $g_d^E\left(a, \varepsilon, \theta\right)$ is the policy function for asset holdings a' if the agent is self-employed and is detected;
- $g_n^E\left(a, \varepsilon, \theta\right)$ is the policy function for asset holdings a' if the agent is self-employed and is not detected;
- $k\left(a, \varepsilon, \theta\right)$ is the policy function for business capital of self-employed;
- $\phi\left(a, \varepsilon, \theta\right) \in [0, 1]$ is the policy function for tax evasion.

3.5 Equilibrium

The stationary equilibrium in the economy is characterized by a stationary distribution of agents over assets and ability realizations when the optimal behavior of agents and firms is taken into account. First, define the functions

$$\mathbf{1}^W\left(a', a, \varepsilon, \theta\right) = \begin{cases} 1 & \text{, if } g^W\left(a, \epsilon, \theta\right) = a' \\ 0 & \text{, otherwise} \end{cases},$$

$$\mathbf{1}_d^E\left(a', a, \varepsilon, \theta\right) = \begin{cases} 1 & \text{, if } g_d^E\left(a, \varepsilon, \theta\right) = a' \\ 0 & \text{, otherwise} \end{cases},$$

$$\mathbf{1}_n^E\left(a', a, \varepsilon, \theta\right) = \begin{cases} 1 & \text{, if } g_n^E\left(a, \varepsilon, \theta\right) = a' \\ 0 & \text{, otherwise} \end{cases}.$$

These functions take the value of one if the current realizations of the state variables $\{a, \varepsilon, \theta\}$ are associated with a future realization of the asset position a' according to the policy functions for workers and self-employed. Second, redefine the probability of detection as a function of the state variables using the policy function for business capital, $p_k(a,\varepsilon,\theta) = p(k(a,\varepsilon,\theta))$. Then, the stationary distribution μ is defined as

$$\begin{aligned}\mu(a',\epsilon',\theta') = \int [&(1-o(a,\varepsilon,\theta))\,\mathbf{1}^W(a',a,\varepsilon,\theta)\,F(\varepsilon',\theta'|\varepsilon,\theta) \\ &+ o(a,\varepsilon,\theta)\,p_k(a,\varepsilon,\theta)\,\mathbf{1}_d^E(a',a,\varepsilon,\theta) \\ &+ o(a,\varepsilon,\theta)\,(1-p_k(a,\varepsilon,\theta))\,\mathbf{1}_{nd}^E(a',a,\varepsilon,\theta)]F(\varepsilon',\theta'|\varepsilon,\theta)d\mu(a,\epsilon,\theta)\,.\end{aligned} \tag{17}$$

The first row of equation (17) counts these agents who decide to be workers and reach future states $\{a',\epsilon',\theta'\}$ given that they are at current states $\{a,\varepsilon,\theta\}$. The second and third lines represent the flow of self-employed who transit between current states $\{a,\varepsilon,\theta\}$ and future states $\{a',\epsilon',\theta'\}$ depending on whether they are detected or not by the tax authorities.

In a competitive stationary equilibrium workers, self-employed businesses and the corporate firm solve their problems, all markets clear and the distribution over the state variables that govern the behavior of households is stationary over time. Let the vector $x = (a,\varepsilon,\theta)$ contain the state variables which summarize all the information necessary to solve the household problems in in the economy. Specifically, a stationary competitive equilibrium consists of value functions $V(x)$, $V^W(x)$ and $V^E(x)$, policy functions for the household $o(x)$, $g^W(x)$, $g_d^E(x)$, $g_{nd}^E(x)$, $k(x)$and $\phi(x)$, input prices r and w, government income tax functions $T^W(\cdot)$and $T^E(\cdot)$, and a probability distribution $\mu(x)$ such that:

1. Given input prices $\{r,w\}$ and tax functions $\{T^W(\cdot),T^E(\cdot)\}$, the value functions $\{V(x),V^W(x),V^E(x)\}$ and the policy functions $\{g^W(x),g_d^E(x),g_{nd}^E(x),k(x),\phi(x)\}$ solve problems (8), (10) and (15).

2. Prices $\{r,w\}$ satisfy the optimization conditions of corporate firms, (1) and (2).

3. The government budget constraint is satisfied:

$$\begin{aligned} G \;=\; \int &[(1-o(x))T^W(y_W(x)) + o(x)\,T^E((1-\phi(x))\,\pi(x)+ra) \\ &+o(x)\,p(k(x))\left[T^E(\pi(x)+ra) - T^E((1-\phi(x))\,\pi(x)+ra)\right]s]d\mu(x)\,. \end{aligned} \tag{18}$$

   In the above equation, the first and the second term on the right represent taxes paid by workers and self-employed, respectively. The last term represents the fines collected by the government when punishing tax evaders.

4. The capital and labor markets clear. Capital demand (by corporate sector and by self-employed businesses) is equal to capital supply:

$$K_C + \int o(x)\,k(x)\,d\mu(x) = \int a d\mu(x)\,.$$

Labor demand is equal to labor supply:

$$N_C = \int (1 - o(x))\varepsilon d\mu (x) .$$

By Walras' law the goods market clearing condition holds in equilibrium as well. Then, total output is equal to the sum of total consumption, investment and government spending. Total output can be also defined as the sum of aggregate production in the self-employed sector of the economy and aggregate output in the corporate sector. In particular,

$$Y = \int o(x) \theta k (x)^{\nu} d\mu (x) + (K_C)^{\alpha} (N_C)^{1-\alpha} .$$

5. The distribution $\mu (x)$ is stationary as implied by (17).

We describe the algorithm for the numerical solution of the stationary equilibrium in Appendix B.

4 Fitting the Model to the Data

We choose parameters in our model in order to replicate important quantitative features of the U.S. economy. In particular, the focus is on matching: *(i)* the share of self-employed households and their income and assets, and*, (ii)* the overall misreporting rate of self-employed business income and the misreporting rates across quintiles of true self-employed income.

We use the PSID for the years 1990-2003 to estimate the data moments related to *(i)*. For the wealth data targets we use wealth supplements which are available for the years 1994, 1999, 2001 and 2003. We consider all households with a male household head of age 25-65 who has worked at least 260 hours during the year. We follow Heathcote et al. (2017) and drop observations if: *(a)* there is no information on the age for either the household head or his spouse, *(b)* either the head or the spouse has positive labor income but zero annual hours, and *(c)* either the head or spouse has an hourly wage which is less than half of the corresponding federal minimum wage in that year. The data targets related to taxable income misreporting are taken from Johns and Slemrod (2010). For more details on our data work, we refer the reader to Appendix A.

The rest of this section is organized as follows: first we present parameters that are fixed outside the model, then we discuss internally calibrated parameters which are set so that the model matches the selected data targets. Finally, we report the model fit along several dimensions of the targeted and non-targeted data.

4.1 Externally calibrated parameters

Personal income tax

As explained in Section 3.3 we specify the income tax function separately for workers and self-employed, using the functional form of Gouveia and Strauss (1999),

$$T^i(y) = a_0^i \left[y - \left(y^{-a_1^i} + a_2^i \right)^{-1/a_1^i} \right], \tag{19}$$

where $i = \{W, E\}$. The parameters $\{a_0^i, a_1^i, a_2^i\}$ are estimated on the PSID data for the period 1990-2003. In our estimation we use average federal taxes levied on the household pre-government income which is defined as the sum of head's and wife's labor and asset income plus private transfers. The PSID does not feature information on paid taxes.[9] Therefore, we compute an estimate of income taxes using the NBER's TAXSIM program (see Heathcote et al. (2010) for a similar approach). Details on the estimation of the tax parameters are provided in Appendix A.3. Finally, the parameters a_2^W and a_2^E are re-scaled in order to balance the government budget in equilibrium.[10]

Working ability process

We estimate a stochastic process for working ability following two steps, as it is standard in the literature (see e.g. Guvenen (2009) or Heathcote et al. (2017)). First we regress labor earnings on observable household characteristics such as education, race and experience in order to obtain a measure of labor income residuals ϵ_{it}. Second, we model this stochastic part as a first order auto-regressive process:

$$\epsilon_{i,t+1} = (1 - \rho_\epsilon)\mu_\epsilon + \rho_\epsilon \epsilon_{i,t} + u_{i,t+1}, \tag{20}$$

where $u_{i,t+1} \sim N(0, \sigma_\epsilon^2)$. We estimate this process for workers and obtain a persistence parameter $\rho_\epsilon = 0.89$, whereas the dispersion parameter is $\sigma_\epsilon = 0.22$. The parameter governing the mean of the process, μ_ϵ, is normalized to one for the calibration. We approximate the stochastic process in (20) with a discrete Markov chain following the procedure described in Tauchen and Hussey (1991). More details can be found in Appendix A.2.

[9]Information on total taxes paid by survey respondents is available until 1990, but the survey question was discontinued in later releases.

[10]We re-scale both a_2^W and a_2^E by a constant factor χ, $a_2^{\prime i} = a_2^i \chi^{a_1^i}$ for $i = \{W, E\}$. The parameter χ is set externally to match a ratio of total income taxes to GDP of 15.2% as in Maffezzoli (2011). More details on this are provided in the following section.

Further parameters

We fix the coefficient of relative risk aversion σ to 2 which is standard in the macroeconomic literature. The parameter α represents the corporate capital share and is set to 0.38, which is the average corporate capital share for 1990-2007 (Karabarbounis and Neiman 2014). The choice for the parameter in the collateral constraint (9) is more delicate. When the borrowing constraint is binding, $\lambda = \frac{k}{a}$. Therefore, λ controls the maximum amount of leverage in the self-employed sector. Since we cannot observe the share of business capital from external sources in our data, we set λ to 1.2 as in Diaz-Gimenez et al. (1992).We perform robustness analysis with respect to this parameter in Appendix B. All externally set parameter values are reported in Table 1.

Table 1: Externally calibrated parameters

Parameter	Description	Value	Source/Target
σ	Elasticity of substitution	2	Standard value
α	Corp. capital share	0.38	Karabarbounis and Neiman (2014)
λ	Leverage ratio	1.2	Diaz-Gimenez et al. (1992)
Working ability			
ρ_ε	Persistence	0.89	micro data - PSID
σ_ε	Standard deviation	0.21	micro data - PSID
μ_ε	Unconditional mean	1	$E(\varepsilon) = 1$
Tax functions			
a_0^W	workers	0.32	micro data - PSID
a_1^W	workers	0.76	micro data - PSID
a_2^W	workers	1.23	micro data - PSID
a_0^E	self-employed	0.26	micro data - PSID
a_1^E	self-employed	1.40	micro data - PSID
a_2^E	self-employed	6.76	micro data - PSID

4.2 Internally calibrated parameters

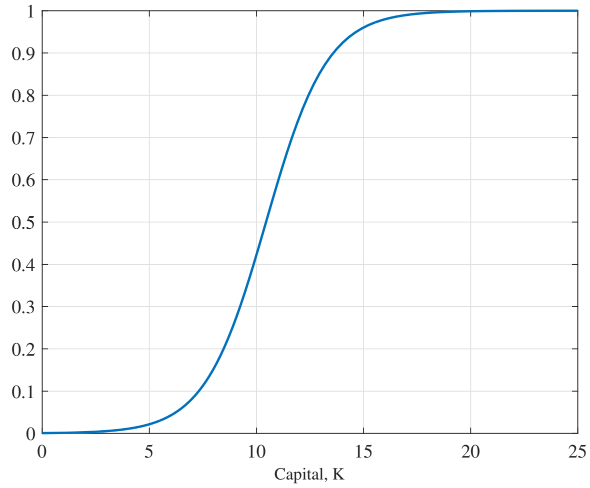
The business ability is assumed to follow a first order auto-regressive process:

$$\theta_{t+1} = (1-\rho_\theta)\mu_\theta + \rho_\theta\theta_t + \varepsilon_{\theta,t+1}, \text{ where } \varepsilon_\theta \sim \text{N}(0,\sigma_\theta^2). \tag{21}$$

The probability of tax fraud detection is a logistic function of business capital. In particular, we assume that

$$p(k) = \frac{1}{1 + p_1 \exp(-p_2 k)}, \tag{22}$$

Figure 1: Probability of auditing



with $p_1 > 0$ and $p_2 > 0$.[11]

We think it is reasonable to assume that the probability of being audited increases with the size of a business unit. There is some empirical evidence that government agencies target larger establishments when it comes to audits, and hence businesses reduce their scale of operation to remain undetected (see Lewis 2005 and Ordonez 2014). Figure 1 shows the function $p(k)$ evaluated at the estimated parameters.

We need to assign values to the following parameters: the household discount factor β, capital depreciation δ, the span of control for self-employed businesses v, the three parameters for the business ability process (ρ_θ, σ_θ and μ_θ), the fine on tax evasion s and the two parameters for the auditing probability, $p_1 > 0$ and $p_2 > 0$. Additionally, we need to pin down the scaling factor χ for the parameters controlling government revenue level in the tax functions (19). A number of data targets are considered which are sensitive to variations in the parameters. It is well-understood that all the model parameters affect all the targets in some way, but we can nonetheless outline which data moment is most informative about a certain parameter. The interest rate and the capital-output ratio identify the discount factor β and the capital depreciation δ. The parameter v controls the share of income that goes to self-employed. The persistence ρ_θ in the stochastic process for the business ability is identified mainly by the annual exit rate from self-employment: a higher persistence of the ability process implies that self-employed change their occupation less frequently. The standard deviation σ_θ crucially affects the strength of the precautionary saving motive by self-employed, and thus, the share of assets owned by them. The last parameter in (21), the unconditional mean level of ability μ_θ determines the

[11] We choose the logistic function for its flexibility. The parameter p_1 affects the vertical intercept of the function, $p(0) = 1/(1 + p_1)$. The parameter p_2 affects the inflexion point of the function. A higher p_2 shifts the inflexion to the left. We have experimented with other functional forms, namely, *(i)* a constant, and, *(ii)* an increasing and concave function.

share of self-employed in the population. When setting the penalty for tax evasion s, we target the overall taxable income misreporting rate in the U.S. economy. The parameters p_1 and p_2 of the probability function $p(k)$ are set to match the relationship between tax evasion and income. More precisely, we target the taxable income misreporting rate over quintiles of true household income. Finally, we need to determine the value of the scaling factor χ which adjusts the parameters a_2^W and a_2^E of the tax functions, so that that income tax revenue is an appropriate fraction of GDP.

To summarize, we set the 10 parameters $\Theta = \{\beta, \delta, v, \rho_\theta, \sigma_\theta, \mu_\theta, s, p_1, p_2, \chi\}$ by matching the following data targets:

1. *Share of self-employed, shares of total income and assets in possession of self-employed and their annual exit rate. These targets are derived from the PSID. [4 targets].*

2. *Capital-output ratio (NIPA) and an interest rate of 4% [2 targets].*

3. *Overall taxable income misreporting rate and taxable income misreporting rates in each quintile of income (Johns and Slemrod 2010) [6 targets].*[12]

4. *Tax revenue to GDP of 15.2% (Maffezzoli 2011) [1 target].*

In doing so, we use an overidentified method of moments approach. We minimize the squared difference between the 13 model moments and their counterparts in the U.S. data. We compute the difference of the model moments $\hat{m}_i(\Theta)$ from the data moments m_i as $d_i(\Theta) = m_i - \hat{m}_i(\Theta)$. Let $D(\Theta) = (d_1(\Theta), ..., d_{13}(\Theta))$ be the vector of differences between the model moments and the data moments. Then, the minimization problem is given by

$$\hat{\Theta} = \min_{\Theta} D(\Theta)'\mathcal{W}D(\Theta) \ ,$$

where $\mathcal{W}$ is the identity matrix. The recovered values for the internally set parameters are presented in Table 2. It is worth mentioning that the recovered value of the tax evasion fine $s = 1.75$ is equivalent to the existing penalty for civil fraud of 75% (U.S. Department of the Treasury (2016)).

[12] We use the data provided by Johns and Slemrod (2010), Table 2 for the targets related to tax evasion. Note that the overall misreporting rate is an independent target from the quintile misreporting rates. In essence, the overall misreporting rate is the share of misreported overall true taxable income. Therefore, by themselves, the income quintile misreporting rates are not sufficient to compute the overall misreporting rate. What is needed for such a computation are the misreporting rates in each quintile and the share of true taxable income out of total taxable income in each quintile.

Table 2: Internally calibrated parameters

Parameter	Description	Value	Target
Preferences			
β	Discount factor	0.935	4% interest rate
Production			
δ	Capital depreciation	0.11	Capital-output ratio
v	Span of control	0.62	Share of income, self-employed
Self-employed ability			
ρ_θ	Persistence	0.935	Exit rate, self-employed
σ_θ	Standard deviation	0.77	Share of assets, self-employed
μ_θ	Unconditional mean	-1.29	Share, self-employed
Tax evasion detection			
s	Fine	1.75	Misreporting rate
p_1	Parameter of $p(k)$	1500	Tax evasion by income (quintiles)
p_2	Parameter of $p(k)$	0.7	Tax evasion by income (quintiles)
Tax functions rescale			
χ	Rescaling parameter	1.4	Tax revenue to GDP

4.3 Model Fit

In this section we compare the outcomes generated by the model to the corresponding statistics for the U.S. economy, both targeted and non-targeted. A good fit of the model along dimensions that are not explicitly targeted in the parameterization process reinforces our confidence in the validity of our approach when it comes to counterfactual analysis.

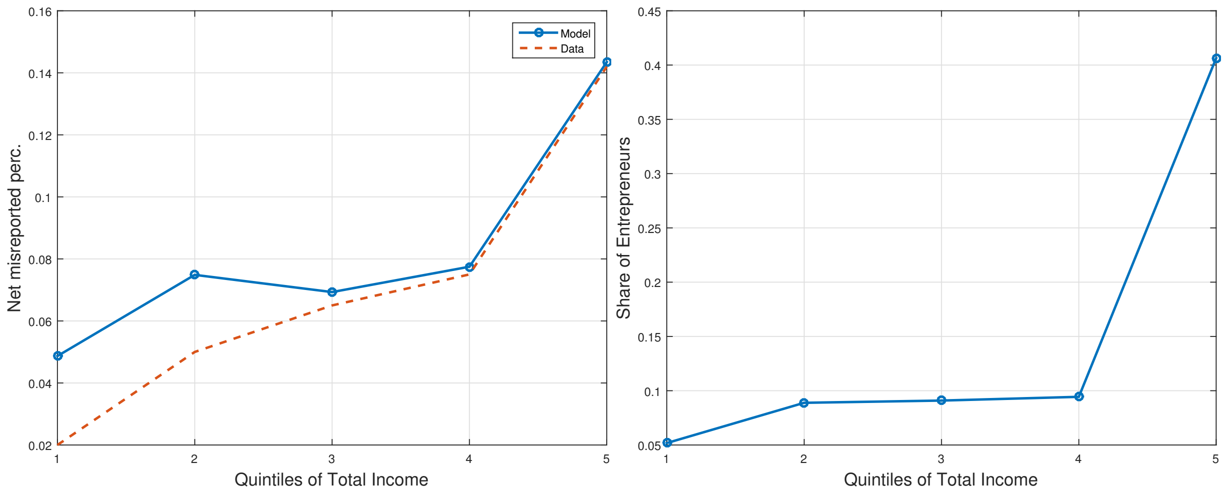
Table 3 shows the model fit in terms of the first set of targeted moments of the U.S. data. The interest rate and the capital-output ratio are replicated exactly. The model generates all basic targets on the share of self-employed, their income, assets and exit rate as in the data. The average misreporting rate for taxable income is as in data. Finally, tax revenue from income taxation is matched too as part of the budget balancing condition for the government.

The other targeted moments relate to the patterns of misreporting of taxable income by quintiles of true household income. Figure 2a reports the data facts and the the model outcomes of misreporting by income level. The model is able to match the increasing pattern of tax evasion with income. This happens because for lower income deciles the share of workers is higher and workers cannot evade (see Figure 2b that plots the share of self-employed for each income quintile). For higher income deciles there are more self-employed who can potentially evade. The overall effect is however non-trivial because richer self-employed tend to evade less due to the probability of auditing which rises in the size of the business.

Table 3: Basic Model Targets

Moments	Data	Model
Interest rate	0.04	0.04
Capital-output ratio	2.65	2.62
Share of self-employed	0.147	0.147
Share of assets, self-employed	0.39	0.43
Share of income, self-employed	0.210	0.238
Exit rate, self-employed	0.157	0.159
Misreporting rate	0.11	0.103%
Taxes/GDP	0.150	0.152

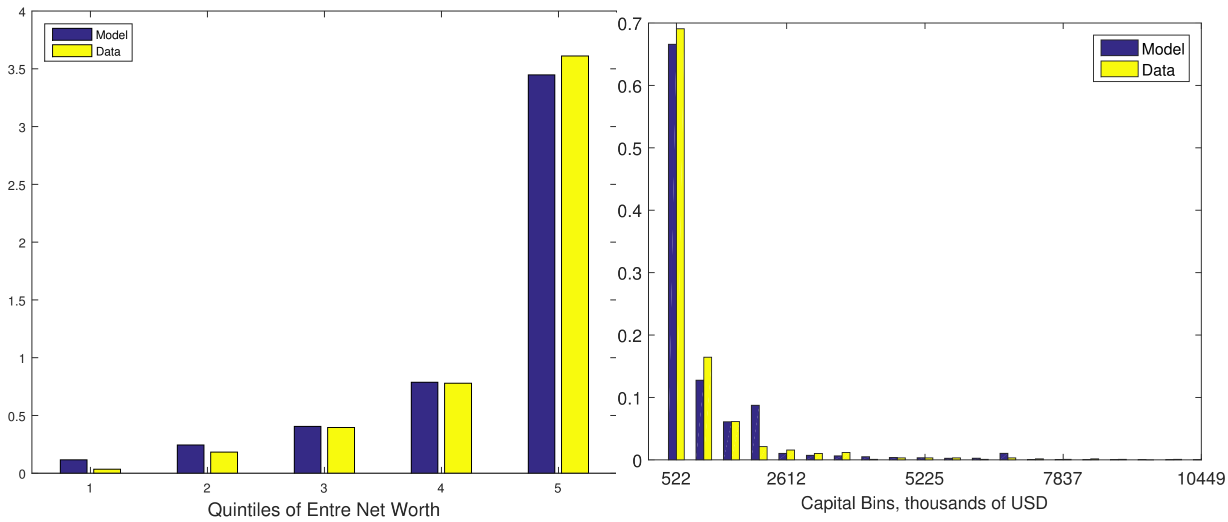
Figure 2: Model Targets



(a) Tax Evasion and Income

(b) Share of Self-Employed by Income

Figure 3: Distribution of Self-Employed Businesses



(a) Average Net Worth Normalized by Mean

(b) Size of Self-Employed Businesses

We report selected moments of the wealth and income aggregate distribution in the benchmark economy in Table 4. Even though we do not target the Gini coefficient, the mean-to-median ratio and the other measures of concentration of wealth and income, the model fits very closely the data in all these dimensions. The model replicates very well both the bottom and the top of the wealth and income distributions. Figure 3a shows the average of self-employed net wealth for different quintiles of income, while Figure 3b reports the model fit in terms of business capital distribution.[13]. The fit of the model in terms of quintiles of net wealth and firm size is quite good. In particular, the model is able to reproduce the fact that around 70 percent of firms are concentrated in the first bin of the size distribution (with capital less than 522,000 U.S. dollars).

Table 4: Wealth and Income Distribution

	Gini	Mean/Median	Bottom 40	Top 20	Top 10	Top 1
Wealth						
Model	73.5	2.90	3.26	76.38	63.32	21.53
U.S. Data	71.1	3.10	2.71	75.64	60.56	26.53
Income						
Model	36.6	1.34	19.84	45.03	31.71	10.69
U.S. Data	35.2	1.23	19.32	42.77	28.27	7.60

[13]Self-employed firm size and net wealth are strictly related due to the collateral constraint $k \leq \lambda a$.

5 The Impact of Tax Evasion

To understand the impact of tax evasion, we provide a comparison between our benchmark economy and a counterfactual economy in which taxes are perfectly enforced.[14] In a first step, to understand the mechanisms, we study how tax evasion affects the optimal decision rules. In a second step, we analyze the impact of tax evasion on aggregate outcomes.

5.1 Understanding the Mechanism

In this section we dig deeper into the economic mechanisms of our model. In our two-sector model setup with imperfect credit markets, the agents' occupational choice, depicted in Figure 4 depends both on business ability θ (relative to the ability of working as an employee) and wealth a.[15] For a given level of ability θ agents become self-employed as long as they hold sufficient wealth. Talented agents who receive a high ability realization may be credit-constrained if they are not rich enough, and thus they would not be able to run their business at a profitable scale. In general, there exists a wealth threshold $a^*(\varepsilon, \theta)$, (weakly) decreasing with business ability θ, such that agents with $a < a^*(\varepsilon, \theta)$ become workers and those with $a \geq a^*(\varepsilon, \theta)$ become self-employed.

In Figure 4 the solid line represents the threshold for our benchmark economy while the dashed line refers to the counterfactual economy with perfect tax enforcement. Tax evasion distorts the occupational choice at the margin, because it makes self-employment more attractive. With tax evasion the share of self-employed in the economy is higher because a group of low ability agents (those between the solid and the dotted line in Figure 4) find it profitable to form self-employed businesses. This suggests that the average business ability θ conditional on being self-employed is lower with tax evasion. This mechanism, through which tax evasion affects occupational choice and therefore the aggregates in the economy, is dubbed the *selection channel*.

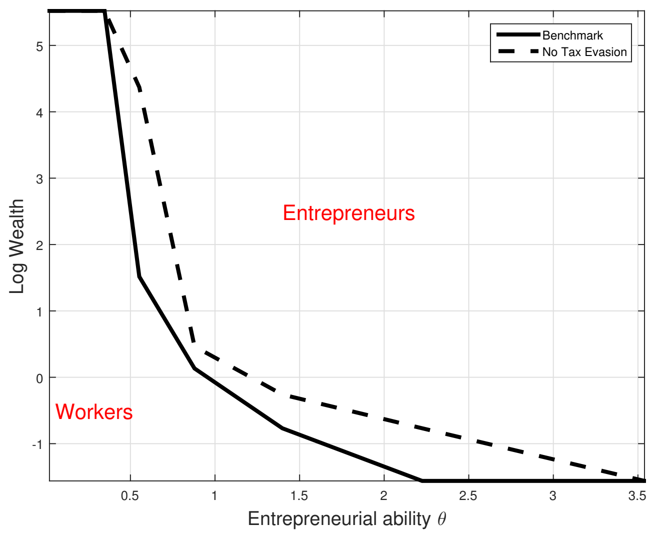
In Figure 5 we show the policy function for savings as a function of asset holdings of the workers and the self-employed. The blue solid line refers to the benchmark economy while the dashed red line refers to the counterfactual economy in which taxes are perfectly enforced. Tax evasion reduces the tax burden of self-employed agents and act as a subsidy that facilitate higher savings. We refer to this as the *subsidy channel*.

Figure 6 shows the optimal decision rule for business capital as a function of asset holdings of the self-employed. The blue solid line shows the decision rule for capital in the benchmark economy and the dashed red line refers to the counterfactual economy without tax evasion. For high level of assets (over 25) the collateral constraint $k \leq \lambda a$ is not binding and therefore the optimal choice for capital is a flat line (it depends only on θ).

[14] The technology of tax evasion is not present in this new economy. All the other parameters are fixed at their benchmark levels. The interest rate r is adjusted to clear the market for capital.

[15] We fix the working ability ϵ at the median in Figures 4 and 6.

Figure 4: Tax Evasion and Occupational Choice



For lower values of assets, instead, the financing constraint binds and the self-employed are not able to run their projects at the optimal scale: in such a case their optimal capital choice does depend upon wealth.

Interestingly, tax evasion creates a distortion in capital accumulation at low-to-medium values of assets. Indeed, the presence of a kink in $k(a, \varepsilon, \theta)$ (see the blue solid line) shows that wealth-constrained self-employed choose a sub-optimal level of capital in order to avoid a sharp increase in the probability of being audited. The intuition goes as follows. For low values of capital, $p(k)$ is flat so that there is no distortion on capital accumulation: the optimal choice for k is increasing. Then, as k approaches the inflexion point in $p(k)$, agents keep $k(a, \varepsilon, \theta)$ flat to avoid a sharp increase in the probability of detection. For larger k ($k > 15$), however, they stop evading (e.g. $\phi = 0$) and can thus freely increase their choice of capital, until the first best is reached. Under perfect tax enforcement, the kink in $k(a, \varepsilon, \theta)$ disappears (see the red dotted line). We refer to this effect of tax evasion as the *detection channel*.

Figure 6 also shows that self-employed businesses with high θ and non-binding collateral constraint utilize more capital in production than under perfect tax enforcement. In this case tax evasion acts as a subsidy for the self-employed who are not detected by the tax authorities. As shown before, this *subsidy channel* stimulates asset accumulation and increases the business capital for all self-employed. Business units which face the collateral constraint have more assets and thus higher capital.

Figure 5: Tax Evasion and Savings of Self-Employed

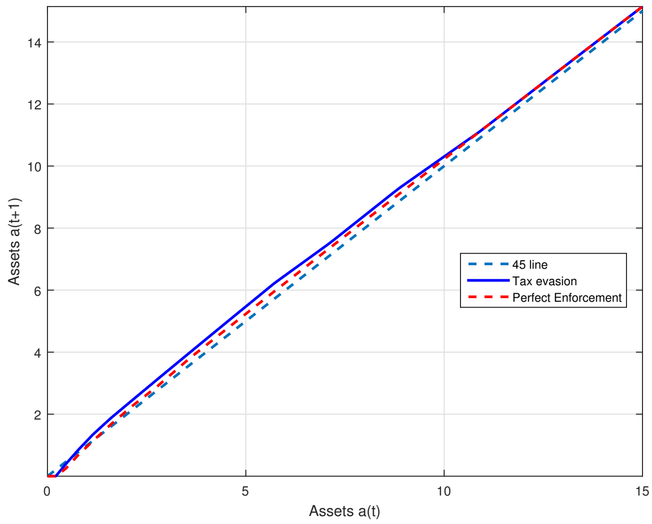
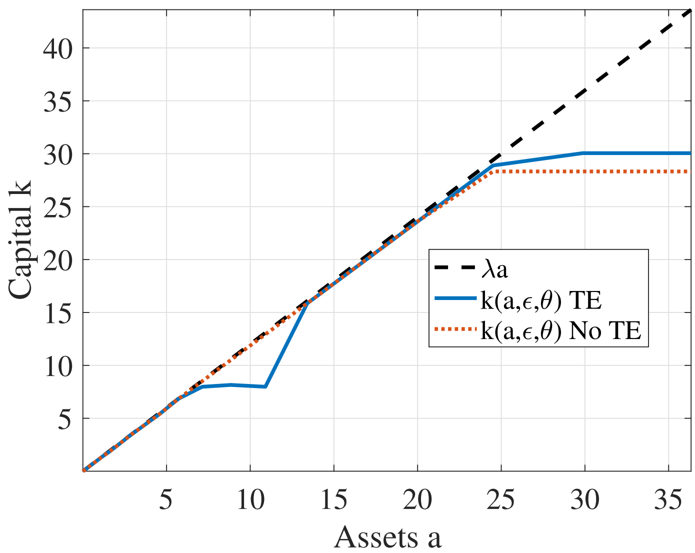


Figure 6: Tax Evasion and Business Capital



5.2 Aggregate Effects of Tax Evasion

Table 5 presents selected aggregate statistics for the benchmark economy and the counterfactual economy in which taxes are perfectly enforceable.

In our benchmark economy the share of self-employed agents is about 4 percentage points larger than in the economy with perfect tax enforcement. At the same time, the average business ability is lower highlighting the *selection channel*: the opportunity to evade taxes induces less talented agents to run self-employed businesses.

There are two opposing forces affecting capital in the self-employment sector. On the one hand, the *subsidy channel* stimulates asset accumulation and allows higher investment in business capital. On the other hand, the *detection channel* provides incentives to keep self-employed businesses small to stay under the tax authorities radar and reduce the chances of being audited. Our quantitative findings suggest that the business capital decision of a self-employed business owner is critically affected by the *detection channel*: In the economy with tax evasion the mean value of business capital of a self-employed agent, $E(k|E)$, is lower than in the economy with perfect tax enforcement. In the aggregate, however, business capital K^E increases when tax evasion is allowed due to the higher share of self-employed businesses in the economy. As a consequence, the output of the self-employed sector with tax evasion is higher too. The impact of tax evasion on the firm size distribution is shown in Figure 7. When tax evasion is not allowed, there are less firms in the smallest bin (from $0 to $522,000). Note that our benchmark economy with tax evasion provides a better description of the empirical firm size distribution than our counterfactual economy with perfect tax enforcement.

Since the opportunity to evade taxes increases the share of self-employed, fewer households become workers and aggregate labor in the corporate sector decreases. The increase in labor productivity is reflected in a higher real wage. The benchmark economy with tax evasion is characterized by a higher aggregate capital stock than the counterfactual economy such that the interest rate increases. Both, the higher wage and the lower interest rate contribute to a lower wealth inequality measured by the Gini coefficient of the household wealth distribution.

Tax evasion reduces tax revenues by 1.6 percentage points of GDP. This figure is close to the empirical estimate of the U.S. tax gap of 2% of GDP (U.S. Department of the Treasury 2009).

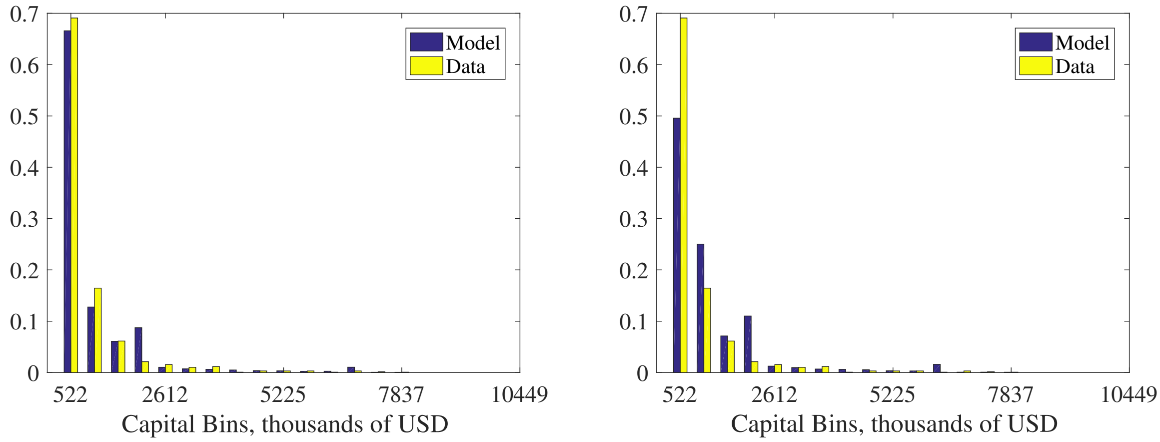
5.3 Decomposition

In our discussion so far, we highlighted three channels through which tax evasion affects the aggregate outcomes in the economy: *(i)* the *subsidy channel*, *(ii)* the *selection channel* and *(iii)* the *detection channel*. In this section, we seek to evaluate the quantitative importance of each of the three channels. We start our analysis for the counterfactual economy with

Table 5: Aggregate Effects of Tax Evasion

	Tax Evasion	Perfect Tax Enforcement
Sector of self-employed		
Share of self-employed	0.147	0.105
$E(\theta\|E)$	0.93	1.02
$E(k\|E)$	12.86	14.65
K^E	1.88	1.54
Y^E	0.68	0.56
Corporate sector		
K^C	3.84	3.82
N^C	0.85	0.89
Y^C	1.51	1.54
Prices		
r	3.97	4.34
w	1.10	1.08
Tax revenues		
T/Y	0.150	0.166
Wealth inequality		
Gini	73.50	75.24

Figure 7: Tax Evasion and the Distribution of Self-Employed Businesses



(a) Benchmark Economy

(b) Perfect Enforcement Economy

perfect tax enforcement presented before. Then, we move to a tax evasion economy in a partial equilibrium fashion, that is, we keep the wage and the interest rates at the values of the perfect enforcement economy. This way, we can document the changes in the aggregate economic outcomes solely due to the presence of tax evasion. Then, through a series of additional counterfactual experiments we deduce the strength of each of the channels for these changes.

We rely on the (imperfect) assumption that the overall change in an aggregate outcome i, Δ^i, is the sum of the changes due to each of the three channels,

$$\Delta^i = \Delta^i_{\text{subsidy}} + \Delta^i_{\text{selection}} + \Delta^i_{\text{detection}}.$$

Denote the policy functions for the occupational choice and for business capital in the economy with perfect tax enforcement with $\tilde{o}(x)$ and $\tilde{k}(x)$, respectively. To isolate the the effect of the subsidy channel of tax evasion, $\Delta^i_{\text{subsidy}}$, we impose exogenously the policy functions $o(x) = \tilde{o}(x)$ and $k(x) = \tilde{k}(x)$ in the economy with tax evasion and measure outcome i. Thus, we allow for tax evasion but the decisions on occupational choice and business capital are fixed (and the selection and detection channels are shut down). Now tax evasion affects outcome i only through the savings behavior of agents.

Next, fixing only the occupational choice $o(x) = \tilde{o}(x)$ in the partial equilibrium tax evasion economy shuts down the selection channel and delivers $\Delta^i_{\text{subsidy}} + \Delta^i_{\text{detection}}$. Finally, fixing only the choice of business capital $k(x) = \tilde{k}(x)$ shuts down the detection channel and delivers $\Delta^i_{\text{subsidy}} + \Delta^i_{\text{selection}}$. Given the total effect Δ^i and the derived $\Delta^i_{\text{subsidy}}$, we can recover $\Delta^i_{\text{detection}}$ and $\Delta^i_{\text{selection}}$.

Table 6 summarizes the main findings of our decomposition exercise. We consider the following aggregate outcomes: *(i)* the share of self-employed, *(ii)* the aggregate capital in the self-employed sector K^E, *(iii)* the aggregate capital in the corporate sector K^C, *and (iv)* the Gini coefficient of wealth. First, we list for comparative purposes the changes in these variables when we move from an economy with perfect tax enforcement to an economy with tax evasion allowing the wage and interest rates to adjust (*general equilibrium*). Then, the second row of the table, we list the corresponding changes with fixed prices of capital and labor (*partial equilibrium*). It is evident that when the wage and interest rates are not allowed to adjust, tax evasion has a larger positive effect on the self-employment rate and the capital stock in both sectors. Moreover, wealth inequality is more strongly reduced in partial equilibrium.

In the subsequent rows of Table 6, we list the results of our decomposition in partial equilibrium. The results point out that the change in the share of self-employed businesses (due to tax evasion) is mainly determined by the *selection channel*. The rise in self-employed business capital (when tax evasion is allowed) is driven by both the *selection* and *subsidy channels*. The *detection channel* reduces self-employed business capital but quantitatively this effect is less pronounced. The quantitative effect of tax evasion on wealth inequality is mainly driven by the *subsidy channel*.

Table 6: Decomposition of Aggregate Effects

Δ^i:	Share of self-employed in pp.	K^E in %	K^C in %	Gini
General equilibrium	+4.14	+22.40	+0.53	−2.31
Partial equilibrium	+4.59	+26.66	+13.81	−3.71
-subsidy channel	+0.81	+15.48	+12.28	−2.89
-selection channel	+3.56	+13.95	−1.75	−0.55
-detection channel	+0.08	−2.56	+2.77	−0.28

6 Tax Evasion, Fiscal Policy, and Welfare

In this section, in a first step, we analyze the impact of tax evasion on tax revenues and discuss the implications for fiscal policy. In a second step, we analyze the welfare effects of tax evasion.

6.1 The Fine

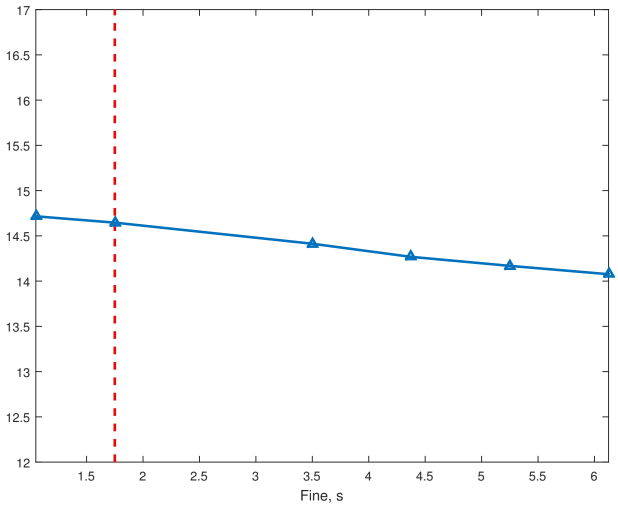
In this subsection, we study how the fine on tax evasion affects tax revenues and aggregate outcomes. In Figure 8 we vary the fine s and display how the share of self-employment and the average productivity in the self-employment sector as well as aggregate capital and output change. In addition, we plot the collected tax revenues and the overall misreported rate of self-employed income as the fine s changes.

When the level of the fine for tax evasion rises, the share of self-employed businesses decreases (Figure 8a). This goes hand in hand with an increase in the average productivity of the self-employment sector (Figure8b). The reason is intuitive: if tax evasion is punished with a higher fine, less talented agents leave the sector of self-employment as misreporting becomes too risky. The smaller size of the self-employment sector, however, decreases aggregate capital and aggregate output.

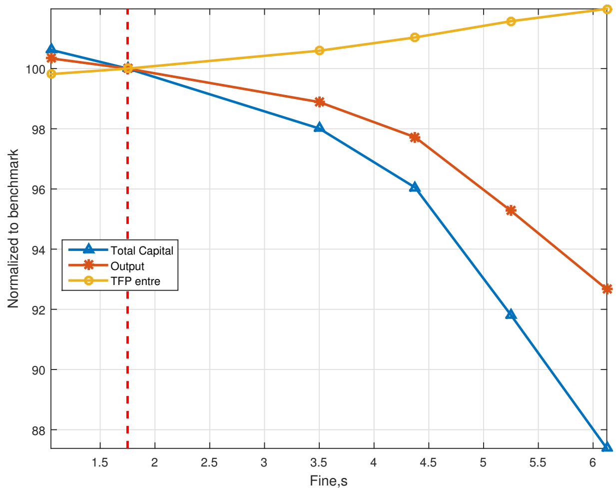
Our findings suggest that tax revenues are hump-shaped with respect to s. This Laffer-like pattern of tax revenues in Figure 8c is generated by two opposing forces. On the one hand, a higher fine allows government to collect higher revenues. On the other hand, a higher fine decreases the share of self-employed businesses as well as aggregate output. This reduces the tax base and therefore lowers tax revenues. Our findings suggest that a fine around 3.5 maximizes tax revenues. If the fine s is chosen beyond this value, the adverse effects on aggregate output dominate.

The overall misreporting rate of self-employed income is U-shaped with respect to s (Figure 8d). This pattern is characterized by opposing forces as well. If the fine increases, misreporting becomes more risky such that less taxes are evaded. On the other hand, when the fine is high (above 3.5), self–employed businesses can strategically decrease their business capital so that the probability of detection is very low. In this case, they start evading larger shares of their income which produces an the increasing pattern of the misreporting rate for fines above 3.5. If the fine is very high (well above 6, which is not shown), the misreporting rate starts decreasing again (because now it is too risky to evade even when the probability of detection is negligible) and the economy converges to the no-tax evasion economy.

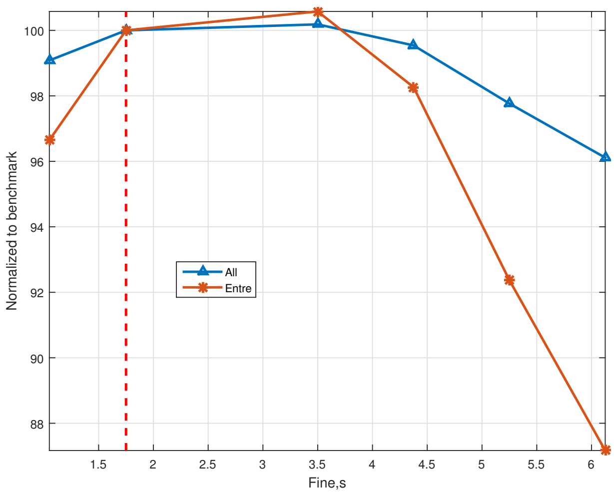
Figure 8: The Impact of the Fine



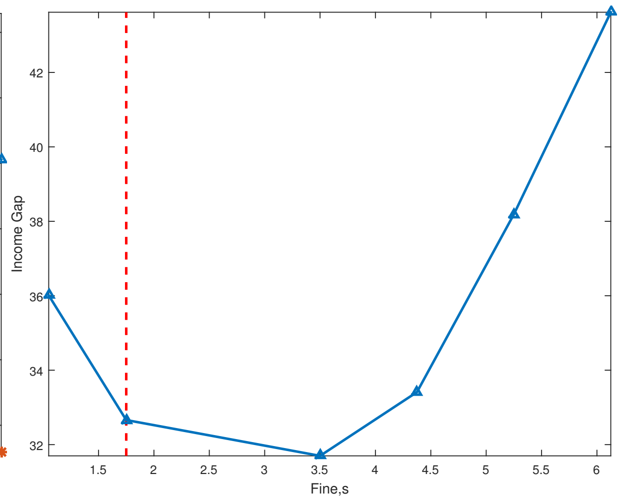
(a) Share of Self-Employed



(b) Aggregate Outcomes



(c) Tax Revenues



(d) Misreporting Rate of Self-Employed

6.2 The Tax Scheme

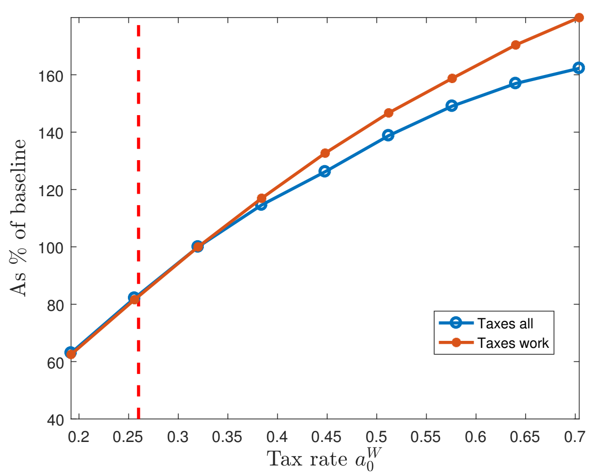
In this section we analyze how the tax scheme determines tax revenues, the size of the self-employment sector, the misreporting rate, and aggregate outcomes. In particular, we shift the tax scheme by increasing the tax parameters a_0^W and a_0^E proportionally, see the tax function (19).

In Figure 9a the red line shows total tax revenues while the blue line refers to tax revenues collected on workers. Within the considered range, raising the level of taxes increases both types of tax revenues. In contrast, Figure 9b reveals that the tax revenues collected on self-employed businesses (blue line) follow a hump-shaped pattern. This Laffer-like behavior is driven by the increasing misreporting rate (red line). Furthermore, the tax revenues collected on workers increase much stronger than the tax revenues on entrepreneurs suggesting that the elasticity of taxable income for self-employed businesses is high.

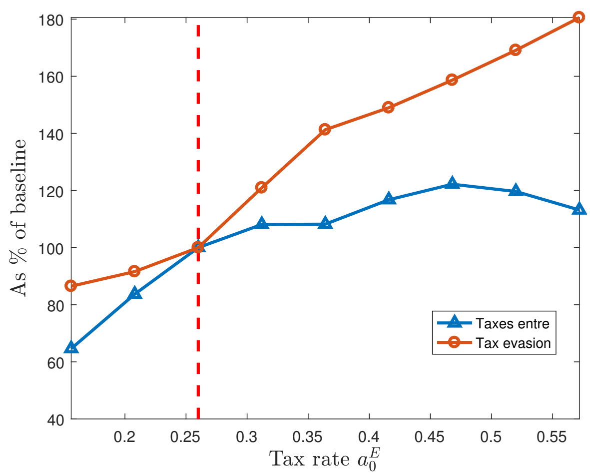
In Figure 10 we present the impact of increasing taxes on aggregate capital, aggregate output, and the average productivity of the self-employment sector. In line with our previous findings, higher taxes induce more agents to become self-employed such that the average business ability in the self-employment sector decreases. A lower productivity and higher distortionary taxes decrease aggregate capital and output in the economy.

To dig deeper into our results, we perform the same experiment for our counterfactual economy in which taxes are perfectly enforceable. From Figure 11 we see that as the tax rate increases, the share of self-employed businesses decreases in the economy with perfect tax enforcement while the opposite is true in the economy with tax evasion. The reason is quite intuitive: with tax evasion, self-employed agents can protect themselves against increases in taxes by evading more.

Figure 9: Tax Scheme - Tax revenues and Income Misreporting



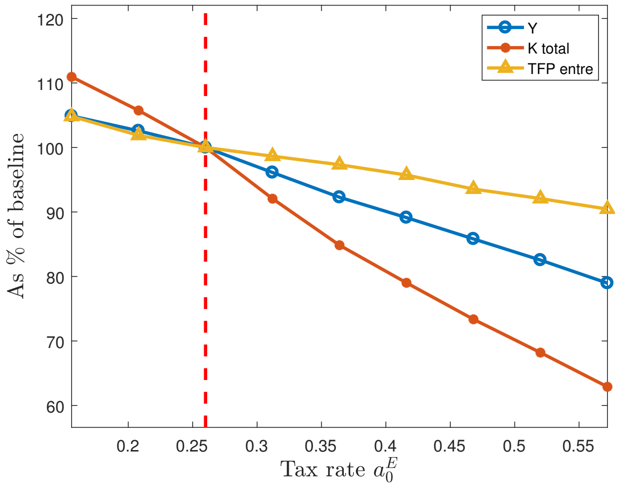
(a) Total



(b) Self-employed

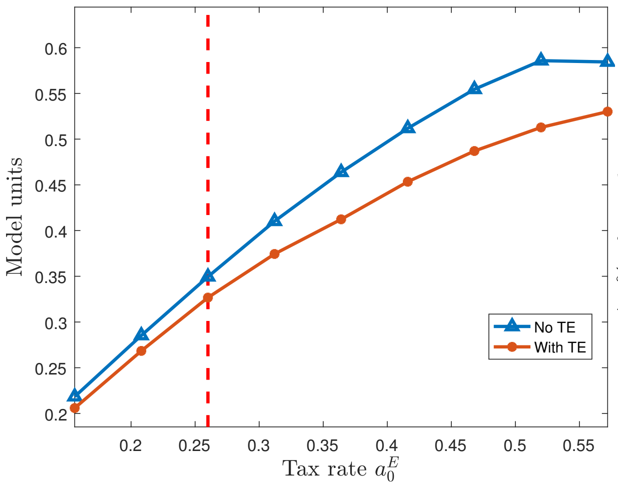
Notes: We vary a_0^W and a_0^E in the range $0.19 - 0.70$ and $0.15 - 0.57$, respectively. On the horizontal axis only a_0^E is shown. The red dotted line denotes the baseline values.

Figure 10: Tax Scheme - Capital, Output and Productivity of Self-Employed Businesses

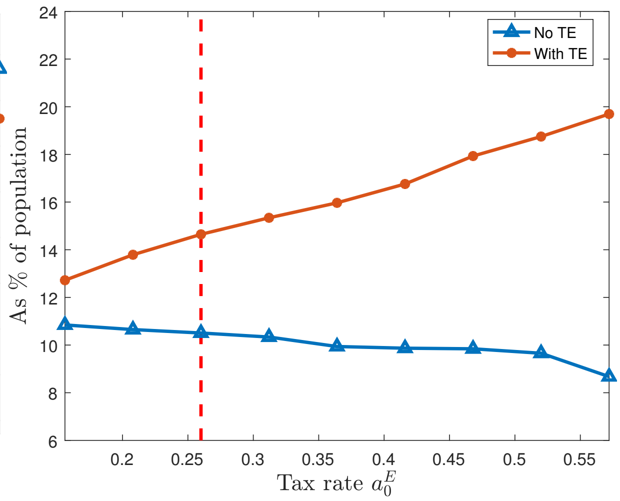


Notes: We vary a_0^W and a_0^E in the range $0.19 - 0.70$ and $0.15 - 0.57$, respectively. On the horizontal axis only a_0^E is shown. The red dotted line denotes the baseline values.

Figure 11: Tax Scheme - Tax Evasion versus Full Tax Enforcement



(a) Total



(b) Normalized to benchmark

Notes: We vary a_0^W and a_0^E in the range $0.19 - 0.70$ and $0.15 - 0.57$, respectively. On the horizontal axis only a_0^E is shown. The red dotted line denotes the baseline values.

6.3 Welfare

In our welfare analysis we calculate the welfare effects of eliminating tax evasion in terms of consumption equivalent variations, i.e., we calculate the consumption gain or loss of moving from our benchmark economy with tax evasion to an economy in which taxes are perfectly enforced.[16] Thereby, we compare the stationary equilibria and abstract from

[16] Appendix B.5 provides technical details.

transitional dynamics. Since the elimination of tax evasion increases tax revenues, we distinguish three fiscal policy scenarios. In the first scenario, the additional tax revenues are not redistributed to the agents in the economy. In the second scenario, the additional tax revenues are redistributed via lump-sum transfers. In the third case, the additional tax revenues are redistributed by reducing taxes. In particular, we decrease the tax level by re-scaling proportionately down the terms (a_2^W, a_2^W) in the non-linear tax functions (3).

In Table 7 we summarize our findings. Eliminating tax evasion without redistributing the additional tax revenues has a negative effect on welfare, which is not surprising since we have seen before that aggregate capital and output fall if taxes are perfectly enforceable. Interestingly, the reduction in welfare is more pronounced for workers than for self-employed because workers are particularly hurt by the reduction of their wages.

Table 7: Tax Evasion and Welfare

	Redistribution of tax revenues		
	none	lump-sum	tax reduction
All	−2.3	+0.1	+0.4
Self-employed	−0.8	+1.3	+1.1
Workers	−2.6	−0.1	+0.3

Tax evasion leads to a concentration of fiscal pressure on a subset of agents: workers and business owners who do not evade. It is therefore a meaningful exercise to assume that additional tax revenues are redistributed. Without redistribution the elimination of tax evasion leads to a reduction in overall welfare. Such result is, however, reversed when the gains from full tax enforcement are redistributed to the agents. The welfare ranking is as follows:

$$\mathcal{W}(\text{No tax evasion}) < \mathcal{W}(\text{Tax evasion}) < \mathcal{W}(\text{No tax evasion with redistribution})$$

If the government redistributes the additional tax revenues in a lump-sum fashion, total welfare increases compared to the benchmark economy, even though workers are still slightly worse off due to the decrease in wages. If redistribution is accomplished by slashing the level of nonlinear taxes, everyone gains from the elimination of tax evasion. Indeed, overall welfare in the economy increases by 0.4 percent, with self-employed business owners gaining the most. The reason is that eliminating tax evasion allows the government to reduce the discretionary effect of taxes.

7 Conclusions

In this paper we incorporate tax evasion technology in a heterogenous agent macroeconomic model of income and wealth inequality. Tax evasion in the U.S. in terms of underreporting income is concentrated among the self-employed businesses. Our model features such self-employed businesses who unlike the large corporate firms operate according to a decreasing returns to scale technology and can evade taxes on their business income. The results presented here show that tax evasion have non-trivial quantitative consequences for the aggregate economic outcomes in the U.S. This finding has implications for the design of optimal tax policy: macroeconomic models which abstract from tax evasion might deliver biased policy recommendations.

References

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 1994, *109* (3), 659–684. 3

Allingham, Michael G. and Agnar Sandmo, "Income Tax Evasion: A Theoretical Analysis," *Journal of Public Economics*, November 1972, *1* (3-4), 323–338. 1.1, 3

Amaral, Pedro S. and Erwan Quintin, "A Competitive Model of the Informal Sector," *Journal of Monetary Economics*, October 2006, *53* (7), 1541–1553. 1.1

Andreoni, James, "IRS as Loan Shark Tax Compliance with Borrowing Constraints," *Journal of Public Economics*, 1992, *49* (1), 35–46. 1.1

Antunes, Antonio R. and Tiago V. de V. Cavalcanti, "Start up Costs, Limited Enforcement, and the Hidden Economy," *European Economic Review*, January 2007, *51* (1), 203–224. 1.1

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, October 2006, *114* (5), 835–870. 1.1, 3

_ and _ , "Estate Taxation, Entrepreneurship, and Wealth," *American Economic Review*, March 2009, *99* (1), 85–111. 5

Conesa, Juan Carlos and Dirk Krueger, "On the Optimal Progressivity of the Income Tax Code," *Journal of Monetary Economics*, October 2006, *53* (7), 1425–1450. 5

Dessy, Sylvain and Stephane Pallage, "Taxes, Inequality and the Size of the Informal Sector," *Journal of Development Economics*, 2003, *70* (1), 225–233. 1.1

Diaz-Gimenez, Javier, Edward C. Prescott, Terry Fitzgerald, and Fernando Alvarez, "Banking in Computable General Equilibrium Economies," *Journal of Economic Dynamics and Control*, 1992, *16* (3-4), 533–559. 4.1, **??**

Dubin, Jeffrey, "Criminal Investigation Enforcement Activities and Taxpayer Noncompliance," Technical Report, 2004 IRS Research Conference June 2004. 2

Gouveia, Miguel and Robert P. Strauss, "Effective Tax Functions for the US Individual Income Tax: 1966-89," *Proceedings. Annual Conference on Taxation and Minutes of the Annual Meeting of the National Tax Association*, 1999, *92*, 155–165. 3.3, 4.1

Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura, "Income Taxation of U.S. Households: Facts and Parametric Estimates," *Review of Economic Dynamics*, October 2014, *17* (4), 559–581. 5

Guvenen, Fatih, "An Empirical Investigation of Labor Income Processes," *Review of Economic Dynamics*, January 2009, *12* (1), 58–79. 4.1

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante, "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006," *Review of Economic Dynamics*, January 2010, *13* (1), 15–51. 4.1

_ , Kjetil Storesletten, and Giovanni L. Violante, "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*, 2017, *132* (4), 1693–1754. 4, 4.1

Johns, Andrew and Joel Slemrod, "The Distribution of Income Tax Noncompliance," *National Tax Journal*, September 2010, *63* (3), 397–418. 1, 2, 4, 3, 12, B.2, 12

Karabarbounis, Loukas and Brent Neiman, "The Global Decline of the Labor Share," *The Quarterly Journal of Economics*, 2014, *129* (1), 61–103. 4.1, **??**

Kitao, Sagiri, "Entrepreneurship, Taxation and Capital Investment," *Review of Economic Dynamics*, January 2008, *11* (1), 44–69. 1.1, 5

Kuehn, Zoe, "Tax Rates, Governance, and the Informal Economy in High-Income Countries," *Economic Inquiry*, 2014, *52* (1), 405–430. 1.1

Lewis, William W., *The Power of Productivity: Wealth, Poverty, and the Threat to Global Stability*, University of Chicago Press, 2005. 4.2

Lucas, Robert E., "On the Size Distribution of Business Firms," *Bell Journal of Economics*, Autumn 1978, *9* (2), 508–523. 3.2

Maffezzoli, Marco, "Tax Evasion under Market Incompleteness," Working Papers 378, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University 2011. 1.1, 10, 4

Ordonez, Julio Cesar Leal, "Tax Collection, the Informal Sector, and Productivity," *Review of Economic Dynamics*, April 2014, *17* (2), 262–286. 1.1, 4.2

Pencavel, John H., "A Note on Income Tax Evasion, Labor Supply, and Nonlinear Tax Schedules," *Journal of Public Economics*, 1979, *12* (1), 115–124. 2

Quadrini, Vincenzo, "Entrepreneurship, Saving and Social Mobility," *Review of Economic Dynamics*, January 2000, *3* (1), 1–40. 1.1, 3

Slemrod, Joel, "Cheating Ourselves: The Economics of Tax Evasion," *Journal of Economic Perspectives*, Winter 2007, *21* (1), 25–48. 2

_ **and Shlomo Yitzhaki**, "Tax Avoidance, Evasion, and Administration," in A. J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, Vol. 3 of *Handbook of Public Economics*, Elsevier, 2002, chapter 22, pp. 1423–1470. 2

Tauchen, George and Robert Hussey, "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, March 1991, *59* (2), 371–396. 4.1

U.S. Department of the Treasury, "Update on Reducing the Federal Tax Gap and Improving Voluntary Compliance," Technical Report, Internal Revenue Service July 2009. 1, 2, 5.2

_ , "Investment Income and Expenses (Including Capital Gains and Losses)," Technical Report, Internal Revenue Service 2016. 2, 4.2

U.S. Department of the Treasury Inspector General for Tax Administration, "Management Advisory Report: The Internal Revenue Service's Response to the Falling Level of Income Tax Examinationsand Its Potential Impact on Voluntary Compliance," Technical Report Reference Number: 2002-30-092, Internal Revenue Service June 2002. 2

Yitzhaki, Shlomo, "Income Tax Evasion: A Theoretical Analysis," *Journal of Public Economics*, May 1974, *3* (2), 201–202. 2

A Appendix: Data

A.1 Data description

Here we provide more details regarding data (PSID) and definition of entrep etc.

For our calibration, we estimate the moments from Panel Study of Income Dynamics (PSID). We use a sample from 1990-2003 to estimate most of the relevant moments, however, for the wealth targets we use wealth supplements which are available for 1994, 1999-2003 biannually. The questions in the survey refer to the previous period(year).
We create our sample which includes variables related to the occupation and characteristics of the households and merge it with the Sample A of Heathcote et al.(2010) which contains information on household tax liabilities. This allows us to estimate tax functions for self-employed and workers separately. Heathcote et al.(2010) apply basic data cleaning by dropping records if 1) there is no information on age for either the head or spouse, 2) either the head or spouse has positive labor income but zero annual hours, or 3) either the head or spouse has an hourly wage less than half of the corresponding federal minimum wage in that year. In addition, we select all households where the head of the household is male, age 25-65 and who has worked at least 260 hours during the year.
Traditionally, the entrepreneurial literature distinguishes between two definitions of entrepreneurs (Quadrini, 2000). According to the first definition, entrepreneurs are families that own a business or have a financial interest in some business enterprise. This definition is based on the PSID variable "Whether Business" which is based on the following interview question: "Did you (Head) or anyone else in the family own a business at any time during the previous year or have a financial interest in any business enterprise?". If the answer is positive this household is recorded as an entrepreneur and if negative this household is thought of as 'a worker'. According to the second definition, entrepreneurs are families in which the head is self-employed in his or her main job and the precise interview question is: "In your main job, are you (head) self-employed or do you work for someone else". Unlike the previous survey question which allows only a binary answer (yes/no), this one specifies the occupation of the head and allows to identify a household directly as: a self-employed, an employee, both a self-employed and an employee, or an unemployed.
In our study, we opt for the second definition. First, this definition is more consistent with the data on tax evasion since underreported self-employed business income refers to those who are self-employed. Second, this definition is based on the survey question which refers to the head of the family and we base our analysis on the heads of the households. Third, the answer to the first question can be positive if the household has 'a financial interest in any business enterprise' and it would not reflect the occupation of the household which we have in mind in the model. Moreover, second survey question gives us more information on the occupation of the head of the household and allows to clearly distinguish between self-employed and workers. Based on the second survey question we define as an

Table 8: Summary Statistics for Alternative Definitions

Variable	Self-Employed	Business Owners
fraction of entre.	14.70%	20.11%
share of entre. income	21.04%	27.98%
assets owned by entre.	39.11%	46.15%
ratio of median assets (entre. to work.)	4.02	3.65
exit rate entre.	15.73%	24.43%
Obs	22647	22704

Note: entre. stays for 'entrepreneurs', whereas work. for 'workers'.

entrepreneurial household a household where the head is self-employed, a 'worker' household where the head is an employee or 'both a self-employed and an employee'[17] and we drop 'unemployed' from the sample. As the result, there are 14.7% of self-employed households in our sample. Some important summary statistics for the alternative definitions of entrepreneurs are presented in Table 1.

A.2 Estimating labor income process

In our income process estimation, we follow closely the procedure described by Heathcote et al.(2010). Since our model unit is a household, we focus on the log household labor income. We concentrate on the residual dispersion for log labor household income residuals obtained from a standard Mincerian regression(6), run separately year by year.

$$\text{lninc} = \alpha_0 + \beta_0 \text{educ} + \beta_1 \text{potexp} + \beta_2 \text{potexp}^2, \tag{23}$$

where lninc is log labor household income, educ stands for the years of education, and potexp for the years of potential experience. The latter is calculated as the difference between the age and years of education less 6, potexp = age − educ − 6, where 6 is the typical age for entering the elementary school. Hence, the one who is 40 years old, with 16 years of education can potentially have 18 years of working experience. We assume that the error term follows a first order Markov process of the form:

$$\epsilon_{t+1} = \mu_\epsilon + \rho_\epsilon \epsilon_t + \varepsilon_\epsilon, \tag{24}$$

We estimate the income process for the workers and get that the persistence of the process ρ_ϵ is 0.89, whereas the dispersion is $\sigma_\epsilon = 0.22$. The mean of the process, μ_ϵ is normalized to one.

Since we estimate the ϵ-process on the subsample of the workers only, and in the model

[17] There are 0.7% of such households, hence either dropping those households or including them to either of the group does not change the main moments.

Table 9: Exit Rates

Year	% stayed W	N Workers	Exit Rate W	% stayed E	N Entre	Exit Rate E
1989	0.97	1,572.00	0.03	0.89	278.00	0.11
1990	0.96	1,522.00	0.04	0.90	278.00	0.10
1991	0.96	1,424.00	0.04	0.80	235.00	0.20
1993	0.97	1,709.00	0.03	0.85	260.00	0.15
1994	0.98	1,715.00	0.02	0.82	252.00	0.18
1995	0.97	1,728.00	0.03	0.85	241.00	0.15
1996	0.96	1,609.00	0.04	0.83	219.00	0.17
1998	0.96	1,704.00	0.04	0.82	245.00	0.18
2000	0.96	1,844.00	0.04	0.81	224.00	0.19
Exit Rate Ave.						15.73%

Note

N stands for 'number'; W - 'workers'; E - 'entrepreneurs'.

this process is not conditional on the occupation, we make sure that in the model $E(\epsilon|o = worker)$ is not significantly different from $E(\epsilon)$.

A.3 Estimating tax Functions

Write text here.

A.4 Estimating entry and exit rates

The exit rates are calculated as follows: First, we sort individuals by their id number and consider two consecutive years. Then, we calculate how many of those remained workers from one year to another and divide by the initial number of workers. This gives us the share of people who stayed workers. In the same fashion, we calculate a share of those who stayed self-employed. Exit rates are calculated as one minus the share of those who stayed a worker/self-employed. Finally, we calculate a weighted sum of year-by-year exit rates to get an average number we use for calibration. We get that on average, per year, around 15.73% of those who were self-employed exited self-employment. This number is comparable with the number reported by Quadrini(2000) 13.6%. Table 4 shows year-by-year exit rates for workers and self-employed.

A.5 Estimating income and wealth inequality

Here we may show additional tables that we don't report in the main text. We base our estimates of asset distribution on the PSID variable 'Wealth' which includes: 1) 'market' value of farm/business net of debt, 2) money in checking and savings accounts, money market funds certificates or deposit, government savings bonds or treasury bills, 3) other

Table 10: Wealth Summary Statistics

	Gini	Mean/Median	Bottom 40	Top 20	Top 10	Top 1
All	71.1	3.10	2.71	75.64	60.56	26.53
Entre	68.4	2.61	4.35	72.17	57.23	21.13
Workers	67.0	2.61	3.17	71.86	55.21	21.14

real estate than your main home, 4) shares of stock in publicly held corporations, mutual funds, etc., 5) value of home equity. In the data, we recode negative asset positions with zeros to stay consistent with the model. Some important summary statistics about the asset distribution in the data are summarized in Table 5.

B Appendix: Model

B.1 Solution Algorithm

We summarize here the main steps to compute the stationary equilibrium formally defined in Section 3.5.

1. Make a guess for the interest rate r^0.

2. Compute the capital-labor ratio in the corporate sector $k_C = \frac{K_C}{N_C}$, which satisfies the following:
$$r^0 = \alpha A \left(k_C\right)^{\alpha - 1} - \delta$$

3. Compute wage w as follows:
$$w^0 = \left(1 - \alpha\right) A \left(k_C\right)^{\alpha}$$

4. Given r^0 and w^0, solve the individual optimization problem described in section (3.4) and get the relevant policy functions. Given the high non-linearity of the problem we use value function iteration with interpolation on a discrete grid.

5. Compute the invariant distribution μ using the policy functions and the exogenous Markov chains for the shocks ε, θ, as we described in section (3.5), equation (**??**).

6. Using the distribution μ and the policy functions, compute the aggregate conditions and get new values for K_C, N_C. In particular, do the following:
$$K_C = \int_x a d\mu\left(x\right) - \int_{\{x:o(x)=SE\}} k\left(x\right) d\mu\left(x\right)$$

$$N_C = \int_{\{x:o(x)=W\}} \varepsilon d\mu(x)$$

7. Excess demand function can be defined as:

$$ED(r^0) = k_C - \frac{K_C}{N_C}$$

Notice that finding a root of $ED(r)$ is equivalent to finding a fixed point of the capital-labor ratio in the corporate sector.

8. If $ED(r^0) = 0$, stop and exit the loop over r. Otherwise, if $ED(r^0) > 0$ set a new guess $r^1 > r^0$. If instead $ED(r^0) < 0$ set $r^1 < r^0$. Go back to step 2.

9. Stop if either $ED(r)$ or $|r^0 - r^1|$are sufficiently close to zero.

Observations:

- *In practice we noticed that it is faster to update the interest rate using bisection (see Matlab routine* `fzero`*)*
- *In some rare instances, the algorithm converges in term of the interest rate but not in term of the excess demand, i.e.* $|r^0 - r^1| < tol$ *but* $ED(r)$ *is not close to zero. This happens if the function* ED *is not monotonically decreasing in* r.

B.2 How we compute the tax gap in the model

Let $x = (a, \varepsilon, \theta)$ be the state vector. Furthermore, we define the following objects:

$$unpaid(x) \equiv T^E(\pi(x) + ra) - T^E((1 - \phi(x))\pi(x) + ra)$$

$$true\_tax(x) \equiv T^E(\pi(x) + ra)$$

where $\phi \in [0, 1]$ is the fraction of hidden income and π is business income[18].

Given these definitions, we compute the aggregate tax gap in the model economy as follows:

$$TG = \frac{\sum_x unpaid(x)\mu^E(x)}{\sum_x true\_tax(x)\mu^E(x)} \tag{25}$$

The data counterpart of this is 57% (see Johns and Slemrod (2010), Table 4). Are we actually sure that this figure reported in Johns and Slemrod (2010) is the ratio of under-reported tax to true tax liability, as oppsed to income misreported divided by true income (AGI)?

We compute the tax gap for each true income decile (based on overall income) following **two methods**. In the **first** method we define the tax gap in income decile $i = 1, .., 10$ as

[18]Taxable income for entrepreneurs is defined as $y_E = \pi + ra$.

the ratio between total unpaid taxes and total due taxes for all individuals whose income belongs to decile i :

$$TG_i = \frac{\sum_{\{x:y(x)\in I_i\}} unpaid(x)\mu^E(x)}{\sum_{\{x:y(x)\in I_i\}} true\_tax(x)\mu^E(x)} \tag{26}$$

It can be verified that the if we take the weighted sum of (26) we get back the aggregate tax gap in (25):

$$\sum_{i=1}^{10} TG_i \cdot \left(\frac{\sum_{\{x:y(x)\in I_i\}} true\_tax(x)\mu^E(x)}{\sum_x true\_tax(x)\mu^E(x)} \right) = TG$$

The above equation clarifies that the aggregate tax gap is **not** the simple average of the tax gap for each deciles: deciles that account for a higher tax liability receive more weight in the aggregation.

In the **second** method we define the tax gap in income decile $i = 1, .., 10$ as the average of the tax gap for each individual whose income belongs to decile i :

$$TG_i = \frac{\sum_{\{x:y(x)\in I_i\}} \left(\frac{unpaid(x)}{true\_tax(x)}\right) \mu^E(x)}{\sum_{\{x:y(x)\in I_i\}} \mu^E(x)} \tag{27}$$

Equation (27) can be interpreted as a conditional expected value, i.e. $E(tax\_gap\,|x \in I_i)$. Clearly, if we take the unweighted average of (27) we obtain what we label as *"average tax gap"*:

$$\sum_{i=1}^{10} TG_i \cdot \sum_{\{x:y(x)\in I_i\}} \mu^E(x) = \frac{1}{10}\sum_{i=1}^{10} TG_i = \sum_x \left(\frac{unpaid(x)}{true\_tax(x)}\right) \mu^E(x)$$

Slemrod and his coauthors sometimes report another measure of tax non-compliance: they define the net misreporting percentage (NMP) as the sum of underreported income divided by the absolute value of the corresponding true income. The results are reported in Johns and Slemrod (2010), Table 2, pag.404, reproduced here in Figure (12). The first column shows the NMP by income, whereas the second columns shows the NMP by taxes. The NMPs in each column are computed for each decile of true income. Of course, income misreporting percentages are equal to tax misreporting percentages under a proportional tax scheme[19], but with income tax progressivity they may differ substantially, and they do in Slemrod's data. In particular, whereas NMP by income rises with income, NMP by taxes *declines* with income, albeit non monotonically.

In terms of tax evasion targets for our paper, we can match the overall NMP of income (11 percent) and maybe also the NMP by deciles of AGI. We have a pretty good chance to

[19] If true income is y, undeclared income is $e \in [0, y]$, then unpaid taxes are $T(y) - T(y-e)$ and the tax gap is

$$\frac{T(y) - T(y-e)}{T(y)}$$

Clearly, if the tax schedule T is linear the tax gap above is equal to $\frac{e}{y}$ which is the "income gap", i.e. unreported income divided by true income.

match the increasing pattern since for lower income deciles the share of workers is higher and workers cannot evade. For higher income deciles there are more self-employed who can evade. The overall effect is however non-trivial in the sense that richer self-employed tend to evade less due to the probability of auditing increasin in size. So we have an extensive margin and an intensive margin as well.

To compute the NMP by income we follow a similar procedure as in (25). First, we compute the aggregate NMP as follows:

$$
\begin{aligned}
IG &= \frac{\text{Total undeclared income}}{\text{Total true income}} = \frac{\sum_x \left[\pi(x) + ra - (1-\phi(x))\,\pi(x) - ra\right]\mu^E(x)}{\sum_x \left[\pi(x) + ra\right]\mu^E(x)} = \\
&= \frac{\sum_x \phi(x)\pi(x)\,\mu^E(x)}{\sum_x \left[\pi(x) + ra\right]\mu^E(x)}.
\end{aligned}
$$

Please observe that this delivers a number between 0 and something less than 1 due to asset income. Second, we compute the disaggregated NMP by income deciles and/or quintiles, following the above approach.

Table 2

Net Misreporting Percentages by True AGI, Tax Year 2001

True AGI	NMP for AGI	NMP for Tax after Refundable Credits
Bottom 10%	−1	71
10% –20%	4	56
20%–30%	5	38
30%–40%	5	27
40%–50%	6	21
50%–60%	7	20
60%–70%	7	16
70%–80%	8	16
80%–90%	8	14
90%–95%	11	17
95%–99%	18	21
99.0%–99.5%	19	20
Top 0.5%	15	15
Total	11	18

Source: National Research Program data.

Figure 12: Table 2 from Johns and Slemrod (2010)

B.3 Computation of stationary distribution: details

Keep in mind that if we compute the policy function for assets using interpolation, an additional complication arises. Almost surely, the optimal choice does not happen to be one of the grid points (even if we use a finer grid for the distribution). Suppose $a^* \equiv g(a, \epsilon, \theta)$ falls between grid a_J and a_{J+1}, where $J = 1, 2, .., N_a$. If this is the case, we can proceed in two different ways. The simpler one is to force the agents to choose the closest grid point. Another way is that we let the agents draw a lottery and the proportion

$$\frac{a_{J+1} - a^*}{a_{J+1} - a_J}$$

are forced to choose a_J and the rest are forced to choose a_{J+1}. This method is described in greater detail in Heer and Maussner (2008, algorithm 7.2.3).

B.4 Entry and Exit rates in the model

Entry and Exit. Calculating the exit rate of entrepreneurs requires to track down each entrepreneur who changes occupation status from one period to the next. Hence, we need first to define the following transition operator:

$$\begin{aligned} T(a, \varepsilon, \theta, a', \varepsilon', \theta') &= (1 - o(a, \varepsilon, \theta))\, \mathbf{1}^W (a', a, \varepsilon, \theta) \Pr(\epsilon', \theta' | \epsilon, \theta) \\ &\quad + o(a, \varepsilon, \theta)\, p_k(a, \varepsilon, \theta)\, \mathbf{1}_d^{SE} (a', a, \varepsilon, \theta) \Pr(\epsilon', \theta' | \epsilon, \theta) \\ &\quad + o(a, \varepsilon, \theta)\, (1 - p_k(a, \varepsilon, \theta))\, \mathbf{1}_{nd}^{SE} (a', a, \varepsilon, \theta) \Pr(\epsilon', \theta' | \epsilon, \theta) \end{aligned}$$

which expresses the probability of moving from state (a, ε, θ) today to state a', ε', θ tomorrow. The number of exiting entrepreneurs is then given by:

$$\mathbf{e}_{SE} = \int_A \int_E \int_\Theta \int_A \int_E \int_\Theta o(a, \varepsilon, \theta)\, (1 - o(a', \varepsilon', \theta'))\, T(a, \varepsilon, \theta, a', \varepsilon', \theta')\, d\mu(a, \varepsilon, \theta)$$

where $o \in \{0, 1\}$ is the policy function for the occupational choice ($0 =$ worker, $1 =$ entrepreneur). If we define the state vector as $x = (a, \varepsilon, \theta)$, then the above equation can be written more compactly as

$$\mathbf{e}_{SE} = \int_x \int_{x'} o(x)(1 - o(x'))T(x, x')d\mu(x)$$

Recall that the number of entrepreneurs out of all households in the economy is given by

$$\mathbf{e}_{SE} = \int_A \int_E \int_\Theta o(a, \varepsilon, \theta)d\mu(x).$$

Therefore the *exit rate from entrepreneurship*, i.e. the share of entrepreneurs who become workers, can be computed as:

$$\mathbf{E}_{SE} = \frac{\mathbf{e}_{SE}}{\mathbf{s}_{SE}}.$$

Likewise, the number of workers who become entrepreneurs [ARE WE SURE ABOUT THIS?], is:

$$\mathbf{e}_W = \int_x \int_{x'} o(x)\left(1 - o(x')\right) T(x, x')\, d\mu(x)$$

and we can compute the *exit rate from working* as

$$\mathbf{E}_W = \frac{\mathbf{e}_W}{\mathbf{s}_W}$$

where $\mathbf{s}_W$ is the number of workers out of the population. Please notice that in a stationary distribution $\mathbf{e}_{SE} = \mathbf{e}_W$ but of course $\mathbf{E}_{SE}$ and $\mathbf{E}_W$ will in general differ. We can summarize the transitions between the different occupational status with the help of the following 2×2 transition matrix:

$t\backslash t+1$	W	SE
W	$1-\mathbf{E}_W$	$\mathbf{E}_W$
SE	$\mathbf{E}_{SE}$	$1-\mathbf{E}_{SE}$

B.5 Welfare analysis: details

Let us define the state space as $x = (a, \epsilon, \theta)$. This is how we compute the total welfare in the economy:

$$W = W^{SE} \times \#entre + W^{WORK} \times (1 - \#entre) \tag{28}$$

where

$$W^{SE} = \int V^{SE}(x)\, d\mu^{SE}(x) = \frac{\int V^{SE}(x)\, o(x)\, d\mu(x)}{\#entre} \tag{29}$$

and

$$W^{W} = \int V^{W}(x)\, d\mu^{W}(x) = \frac{\int V^{W}(x)\left(1 - o(x)\right) d\mu(x)}{1 - \#entre} \tag{30}$$

Remember that $o(x) = 1$ if SE and 0 if WORKER.

We have now to compute the consumption equivalent variation of moving from the tax evasion economy (TE) to a full tax enforcement economy (NTE). Assuming we have the total welfare in these two economies, W^{TE} and W^{NTE}, the CEV can be computed in the following way:

$$CEV(a, \epsilon, \theta) = \left(\frac{W^{NTE}(a, \epsilon, \theta)}{W^{TE}(a, \epsilon, \theta)}\right)^{\frac{1}{1-\sigma}} - 1$$

The interpretation is the following: $CEV(a, \epsilon, \theta)$ is the consumption gain of moving from TE economy to the NTE economy in the steady state for an agent who is born with type (a, ϵ, θ). We can also compute the *expected* CEV as

$$CEV = \left(\frac{\int W^{NTE}(a, \epsilon, \theta)\, d\mu}{\int W^{TE}(a, \epsilon, \theta)\, d\mu}\right)^{\frac{1}{1-\sigma}} - 1$$