Term Premium, Credit Risk Premium, and Monetary Policy*

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Abstract

We build and calibrate a New-Keynesian DSGE model with Epstein-Zin preferences and financial frictions in the shape of multi-period nominal defaultable debt, to fit U.S. data moments. We solve the model using higher-order perturbations and show that credit frictions can significantly increase the size and volatility of the nominal and real Treasury term premium through the interaction of preferences sensitive to long-run risk, and amplification of the economy’s response to TFP shocks. Our analysis suggests that introducing multi-period defaultable debt contracts helps fit the cyclical properties of macroeconomic and financial variables, including credit spreads, credit risk premia and leverage ratios together with the main features of the default-free term structure of interest rates. Model simulations show that unexpected monetary policy shocks have small effects on term and credit-risk premia dynamics, while the systematic component of monetary policy has sizable implications for the average and volatility of risk compensation. In particular, monetary policy that responds more to inflation fluctuations relative to output reduces the average and the volatility of nominal term premia, while increasing the average and volatility of credit risk premia, by affecting the mix of inflation volatility and debt-deflation risk in the economy.

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1 Introduction

Yields on nominal bonds with a long maturity are typically higher than short term interest rates. The empirical finance literature documents the existence of a sizable and volatile nominal term premium, defined as the excess yield that investors require to hold a long-term nominal bond instead of a series of shorter-term nominal bonds up to the same maturity. The term premium contributes to shape the dynamics of the yield spread.

Similarly, corporate bonds pay higher yields than Treasuries of the same maturities to compensate investors for solvency and liquidity risks.

A large literature on asset pricing has focused on building models that help explain the dynamics of default-free interest rates and estimate to what extent their movements can be ascribed to the evolution of term premia. Reduced form econometric models, for example, explain the cross-section and time series variation of nominal and real yield curve data by means of a combination of latent factors, and/or observed economic factors. Moreover, endowment economy models have been proposed to study term structure implications in micro-founded environments, and others have focused on the determination of credit risk and its compensation, with somewhat mixed success in fitting data.\(^1\)

It has proven hard to build general equilibrium models with a non-trivial role for a monetary authority that can fit both Treasury yield dynamics, corporate bond, and macroeconomic data. Nonetheless, this kind of exercise is critical to understand the structural interpretations of the term premium and of the credit risk premium and how and why the risk compensation embedded in yields varies over time in response to fundamental shocks that hit the economy. Treasury yield curve data, together with structural estimates of the term premium, can potentially reveal market expectations of the evolution of prices and macro aggregates, as well as on the future path of short-term rates. On the other hand corporate bond yield data, and the credit risk premium, are known to contain information about aggregate financial conditions and to be good predictors of business cycle fluctuations. These are all key ingredients in the conduct of monetary policy.

We present a model that can reproduce the dynamics of the default-free term structure of interest rates observed in the U.S. data and price long-term nominal corporate bonds jointly. We build a Neo-Keynesian dynamic stochastic general equilibrium (DSGE) model that features non-trivial inflation dynamics and a realistic role for monetary policy in setting short-term nominal interest rates. Our model is constructed in the spirit of Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005). One main difference of our set-up is that households are endowed with Epstein-Zin preferences (Kreps and Porteus (1978), Epstein and Zin (1989)) which exhibit a separation between the intertemporal elasticity of substitution and the coefficient of relative risk aversion.\(^1\)

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1We describe these models in more detail in the literature review in the next section.
These preferences are known to help generate sizable risk premia broadly across financial markets, including default-free term premia. Piazzesi and Schneider (2007), for example, document that in an endowment economy with Epstein-Zin preferences the negative correlation between news about future consumption growth and inflation commands a positive premium of plausible magnitude in long-term nominal bonds returns. Our model includes financial frictions that can significantly increase the size and volatility of the nominal and real Treasury term premium through the interaction of preferences sensitive to long-run risk, and amplification of the economy’s response to TFP shocks.

In our model, the process of capital accumulation must occur through entrepreneurs who optimally issue nominal multi-period defaultable debt to purchase capital. The entrepreneurs are hit by idiosyncratic shocks, whose distribution can vary over time, as in Christiano, Motto and Rostagno (2014). In response to such shocks entrepreneurs can optimally choose whether to default on their debt. An entrepreneur who defaults is liquidated and loses the firm value in its entirety. Higher leverage makes costly defaults more likely. On the other hand, higher leverage comes with an incentive in the form of tax deductions on debt payments. The costly leverage decision that accompanies capital accumulation constitutes the key financial friction in our model.

We calibrate the model to U.S. macroeconomic data, fitting standard variables of interest in the business cycle literature, as well as Treasury yields and corporate bond market variables. We solve the model using higher-order perturbation methods up to third order which allows us to observe non-zero and time-varying risk premia.

Under our calibration, we find the model fit to be good in terms of cyclical properties of macro variables. The model also performs well in replicating the level and volatility of nominal Treasury yields as well as credit market variables such as corporate credit spreads and leverage ratios. In addition, the model generates a sizable average level for the 10-year nominal term premium that is quantitatively consistent with estimates from the term structure models with multiple latent factors used more widely in the empirical finance literature. We find that incorporating credit frictions into the model increases the volatility of the term premium as well.

We also find that introducing multi-period debt contracts instead of one-period debt in DSGE models helps fit the cyclical properties of macroeconomic variables and credit variables, such as leverage ratios, corporate default rates, recovery rates, on top of credit spreads. Risk-sensitive preferences also allow our model with multi-period debt to generate positive and volatile average credit risk premium for our baseline model with multi-period debt, which is qualitatively consistent with the literature on the U.S. corporate bond market.

In our model, defaults rates are countercyclical due to the presence of risk shocks and are sensitive to inflation dynamics via the Fisher effect. Bond investors require risk compensation for the purchase of corporate bonds when negative inflation surprises can increase the real burden of existing corporate
debt and cause the default rate and expected credit losses to rise (a mechanism also known as debt deflation). We find that debt deflation risk can account for around 20\% of the total size of corporate spreads in our calibrated economy, by comparing our model with long-term debt contracts to a model economy with short-term debt contracts and a similar degree of credit frictions.

Our analysis suggests that transitory monetary policy shocks have limited effect on long-term rates and credit spreads while changes in the parameters that govern the systematic response of monetary policy affect both the size and the volatility of term and credit risk premia. In particular, a central bank that responds more to inflation fluctuations than to changes in output will affect the correlation between inflation and consumption growth, on top of reducing uncertainty around expected inflation. Both forces reduce the nominal term premium in the economy. On the other hand, a greater focus on output fluctuations, rather than inflation fluctuations, reduces the extent of credit cycles and lowers the unconditional correlation between consumption growth and expected losses on corporate bonds, compressing the credit risk premium share of corporate spreads.

The paper is structured so to offer a full characterization of the model in section 2. Section 4.2 follows with a description of the calibration strategy for the model parameters. Section 5 discusses the model fit of the data for macro, term structure and credit market variables, describes the impulse responses to the shocks in the economy (TFP shocks, risk shocks, inflation target shocks, and monetary policy shocks). This section also highlights the importance of the introduction of multi-period bonds in the model in matching credit market variables and as a amplifier of debt-deflation risk. The section concludes with a description of the response of the economy to a monetary policy shock and on the effects of differences in the systematic component of the monetary policy response on average and volatility of term premia and credit risk premia. Section 6 concludes.

**Literature Review**

There is a large body of work on modeling the term structure of default-free interest rates. A vast literature explores the relation between nominal interest rates and the macroeconomy. Early works directly relate current bond yields to past yields and macroeconomic variables using a vector autoregression approach (e.g., Estrella and Mishkin (1997), Evans and Marshall (1998) and Evans and Marshall (2007)). This literature has successfully established an empirical linkage between shocks to macroeconomic variables and changes in yields. More recently, a strand of studies have explored similar questions using macro-finance models with no-arbitrage restrictions (e.g., Ang and Piazzesi (2003), Ang, Piazzesi and Wei (2006), Diebold, Rudebusch and Boragan Aruoba (2006), Duffee (2006), Hördahl, Tristani and Vestin (2006), Mönch (2008), Diebold and Rudebusch (2005), Piazzesi (2005), Rudebusch and Wu (2008)).

Our paper builds more directly on the ‘consumption-based’ models of the term structure, seen as
early as in the work of Campbell (1986), Backus, Gregory and Zin (1989) and Den Haan (1995), and more recently in Wachter (2006), and Buraschi and Jiltsov (2007). These papers are based on endowment economies or simple production economies with a representative agent that has power or habit formation preferences. Alternatively, Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2012) study equilibrium term structure models based on endowment economies with Epstein-Zin preferences. Research on the term structure implications of DSGE models have started to pick up only recently and is still relatively limited, with examples including Gallmeyer et al. (2007), Hördahl, Tristani and Vestin (2008), Rudebusch and Swanson (2008), Bekaert, Cho and Moreno (2010), Van Binsbergen et al. (2010), Dew-Becker (2012), Kung (2012) and Rudebusch and Swanson (2012). Compared to these works, the most important contribution of this paper is to study the role of credit frictions on the dynamics of the term structure of default-free interest rates in a neo-Keynesian general equilibrium setting.

Our work is also closely related to a new literature that attempts to explain credit market variables such as corporate credit spreads and default probabilities jointly with business cycle dynamics, in general equilibrium. Such papers include Chen, Collin-Dufresne and Goldstein (2009), Chen (2010), Gilchrist, Sim and Zakrajsek (2010), Miao and Wang (2010), Gomes and Schmidt (2010), Bhamra, Fisher and Kuehn (2011) and Gourio (2013). In contrast to our research, none of these papers are monetary DSGE models with price and wage rigidities. The core form of credit frictions where entrepreneurs issue defaultable multi-period contracts is similar to Miao and Wang (2010). However, their model is a real business cycle model and this difference enables us to model yields of nominal debt contracts as well as the nominal term structure of interest rates and study their interaction with the macroeconomy.

This paper is related to the literature that explores the relations between financial frictions and macroeconomic dynamics and the ability of financial market frictions to amplify aggregate fluctuations. In this tradition Kiyotaki and Moore (1997) first analyzed the macroeconomic implications of the interaction of agency costs in credit contracts and endogenous fluctuations in the value of collateralizable assets, followed by Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) who first introduced similar frictions in dynamic general equilibrium models. A line of recent research has also incorporated similar financial frictions into medium-scale neo-Keynesian DSGE models and focused on the macro effects of shocks that hit the financial sector (see for example Jermann and Quadrini (2012), Christiano, Motto and Rostagno (2014) or Ajello (2016)). Our model instead extends the study of the amplification effect of credit market frictions to the pricing of the default-free yield curve in a DSGE setting where households have Epstein-Zin preferences.

In a recent paper, Swanson (2015) studies asset pricing facts, including the credit spread puzzle, in a model similar in spirit to Rudebusch and Swanson (2012). The simple framework makes use of the workhorse model without credit frictions to price synthetic risky assets under the assumption
of complete markets. While the research is instrumental to grasp clean intuition on the features that are important to generate sizable risk in a Neo-Keynesian DSGE model, our framework has the additional benefit of explicitly modeling and accounting for the role of endogenous interactions between credit and macro variables and their effect on term premia and credit risk premia.
2 The Model

In this section we describe a Neo-Keynesian DSGE model in the spirit of Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005), in which households are endowed with Epstein-Zin preferences (Kreps and Porteus (1978), Epstein and Zin (1989)) and entrepreneurs issue long-term corporate bonds to finance their capital accumulation activity (Miao and Wang (2010), Gomes, Jermann and Schmid (2016)).

2.1 Households

Agent $i$ maximizes recursive utility:

$$V_{i,t} = \frac{C_{i,t}^{1-\phi}}{1-\phi} - Z_t^{(1-\varphi)} \frac{L_{i,t}^{1+\chi}}{1+\chi} + \beta \{ E_t[V_{i,t+1}] \}^{\frac{1}{1-\alpha}}$$

where $C_{i,t}$ is consumption, $L_{i,t}$ is the amount of hours worked.\footnote{Note that we assume that the labor disutility shares the growth trend of the economy, $Z_t$, to simplify the scaling of the value function $V_{i,t}$, when solving for a stationary equilibrium.} Preference parameters pin down the intertemporal elasticity of substitution ($1/\phi$), the Frisch elasticity of labor supply ($1/\chi$) and govern the steady state labor supply ($\chi_0$), as well as the degree of relative risk aversion of the household ($\alpha$).\footnote{Epstein-Zin preferences allow us to separate the intertemporal elasticity of substitution parameter $\phi$ from risk aversion $\alpha$, and enable the model to fit both features of macro and asset prices data (See Jr. (2000) and Rudebusch and Swanson (2012) for detailed summaries of the implication of recursive preferences). Since the effective risk aversion of the household should take into account the endogenous labor supply decisions, we follow Swanson (2012) and calculate the household consumption-based coefficient of relative risk aversion (CRRA) in the model as:}

$$CRRA = \frac{\alpha}{\chi^{1+\psi}} + \frac{\psi}{1 + \frac{\psi}{\chi}}$$

The household’s budget constraint is:

$$P_t C_{i,t} + E_t [M_{t+1} W_{i,t+1}] = W_{i,t} L_{i,t} + \Pi_{i,t} + T_t$$

where $P_t$ is the price level of real aggregate consumption, $C_{i,t}$, while $W_{i,t}$ is the nominal wage that remunerates hours, $L_{i,t}$.\footnote{We will consider symmetric equilibria on the goods and labor market with Rotemberg adjustment costs in price and wage setting, so that every household receives the same dividend stream and wage bill in each period $t$, and the subscript $i$ can effectively be dropped. In the case of Calvo-style sticky wage- and price-setting mechanism (described in the appendix as an alternative way of introducing New Keynesian features in the model), under the assumption of complete markets households could trade in state-contingent claims to equalize asset holdings and consumption streams, effectively providing insurance against asymmetries in dividend payments and wage bills.} The household also purchases a portfolio of assets at nominal market value
\[ E_t [M_{t+1} \mathcal{W}_{i,t+1}], \] where \( \mathcal{W}_{i,t+1} \) is the pay-off delivered at a generic time \( t + 1 \). The portfolio \( \mathcal{W}_{i,t+1} \) contains equity and debt claims issued by entrepreneurs, as well as state-contingent securities and equity claims on final- and intermediate-good firms, traded in the model economy. Similarly, due to complete markets, we can include any asset in the portfolio \( \mathcal{W}_{i,t+1} \) and we will in fact price zero-net-supply government bonds of different maturities, using the unique real and nominal stochastic discount factors, \( M_{t+1}^R \) and \( M_{t+1} \):

\[
M_{t+1}^R = \beta \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\varphi} \left[ \frac{V_{t+s}}{E_t (V_{t+s}^{1-\alpha})} \right]^{-\alpha} \right\}
\]

\[
M_{t+1} = \frac{M_{t+1}^R}{\pi_{t+1}}
\]

where \( \pi_{t+1} = P_{t+1}/P_t \) is the inflation rate.

### 2.2 Final Goods Producers

We assume that perfectly competitive firms combine differentiated inputs \( Y_t(i) \) to produce a final consumption good \( Y_t \), according to the Dixit-Stiglitz technology:

\[
Y_t = \left[ \int_0^1 Y_t(i) \frac{1}{1+\theta_p} di \right]^{1+\theta_p}
\]

where \( \theta_p \) is the constant elasticity of substitution across inputs.

Final good producers maximize profits under perfect competition. As a result, the price of the final consumption good, \( P_t \), is then equal to a CES aggregator of the prices of intermediate inputs, \( P_t(i) \):

\[
P_t = \left[ \int_0^1 P_t(i) \frac{1}{1+\theta_p} di \right]^{\theta_p}
\]

and the demand for input \( Y_t(i) \) is:

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{\frac{1+\theta_p}{\theta_p}}
\]

### 2.3 Intermediate Goods Producers

Intermediate good firms are monopolists in the production of goods \( Y_t(i) \) and are endowed with a Cobb-Douglas production function:

\[
Y_t(i) = A_t K_t(i)^{\eta} (Z_t L_t(i))^{1-\eta}
\]
with constant return to scales in capital and labor inputs, \( K_t(i) \) and \( L_t(i) \). Aggregate productivity can be decomposed into a trend, \( Z_t \), with a constant growth rate \( \gamma \), and a stationary component, \( A_t \), that follows a process:

\[
\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_t^A
\]

with \( \varepsilon_t^A \sim N(0, \sigma_A) \).

As in Rotemberg (1982), intermediate good producers maximize their profits by choosing a price for their input, \( P_t(i) \), bearing an adjustment cost:

\[
\frac{\psi_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right)^2,
\]

Under this assumption, a re-optimizing firm with marginal cost, \( MC_t \), chooses its price \( P_t(i) \) in period \( t \) by maximizing:

\[
\max_{P_t(i)} \sum_{t=0}^{\infty} \beta^t E_t \left[ M_{t+s} \left( \left( \frac{P_t(i)}{P_t} - MC_t \right) Y_t(i) - \frac{\psi_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right)^2 \right) \right]
\]

subject to the input demand function in (1) and to cost-minimization. In a symmetric equilibrium the maximization problem gives rise to the Phillips curve:

\[
(-\theta_p) + (1 + \theta_p)MC_t - \psi_p \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} + E_t \left[ M_{t+1}^R \psi_p \left( \frac{\pi_t + 1}{\pi} - 1 \right) \frac{\pi_t + 1}{\pi} Y_{t+1} \right] = 0
\]

### 2.4 Entrepreneurs

We assume the economy is also populated by a continuum of ‘entrepreneurs’ indexed by \( j \in [0, 1] \), each of whom owns physical capital, and earns profits by renting it out to the intermediate goods producers. Entrepreneurs purchase capital from the capital goods producers by issuing nominal corporate bonds to the representative household. Per unit of bond issued, entrepreneurs are obliged to pay a unit of nominal cash flow at maturity. The entrepreneurs are identical ex-ante, until an idiosyncratic shock affects their ability to manage their capital stock. The idiosyncratic shock affects the time \( t \) value of entrepreneur \( j \). On the wake of this shock, an entrepreneur optimally chooses whether to default on the debt. In the case of default the entrepreneur is liquidated, and the equity holders lose the firm value in its entirety. Higher leverage increases the probability of the default state. On the other hand, corporate debt allows for tax deductions on interest payments, which creates an incentive for entrepreneurs to lever up their debt positions. This costly leverage decision that accompanies capital accumulation constitutes the key financial friction in our model.

We formulate the entrepreneur’s problem following a specification most similar to Miao and Wang (2010) (henceforth MW), but modify it in several directions. Most notably, we assume that debt
is denominated in nominal terms as in Gomes, Jermann and Schmid (2016) (henceforth GJS). The introduction of nominal debt is more realistic and potentially has important implications to our study since inflation dynamics is a critical component of term structure dynamics as well as a potential driver of credit risk in corporate bonds (see for example Kang and Pflueger (2015)). In particular, GSJ show how the interaction of unexpected shocks to inflation and long-term nominal debt contracts can vary the real burden of leverage and distort investment and production decisions. Along these lines, the presence of nominal and long-term corporate debt can affect the transmission channel of monetary policy.

An entrepreneur $j$ enters period $t$ with capital stock $\bar{K}_{t-1}^j$ and $B_{t-1}^j$ units of nominal debt outstanding. The capital stock is hit by an exogenous idiosyncratic shock $z_{jt}^j$, which we assume is i.i.d. across entrepreneurs. As is common in the literature, we assume the shock has a lognormal distribution with a mean of 1, and a standard deviation of $\sigma_{z,t}$. The volatility of the distribution of the idiosyncratic shocks is time-varying, as in Christiano, Motto and Rostagno (2014), and follows an exogenous process:

$$\log \sigma_{z,t} = (1 - \rho_{\sigma}) \log \sigma_z + \rho_{\sigma} \log \sigma_{z,t-1} + \sigma_{\sigma} \epsilon_t$$

with $\epsilon_t \sim N(0,1)$.

The entrepreneur rents out effective capital inputs $z_{jt}^j \bar{K}_{t-1}^j$ at a one-period rate of return $r_t^K$. This net profit is taxed at a tax rate of $\tau$. Each entrepreneur issues a single type of corporate bond, which has a random maturity structure. The bond matures every period with an exogenous probability $\lambda$, in which case it pays one unit of cash flow in nominal terms unless the entrepreneur decides to default on its debt. If it does not mature, the bond simply pays a nominal coupon $c$. We denote the nominal price of the bond at time $t$ as $q_t^i$. Despite its stylized formulation, this can be considered to be a parsimonious way of capturing bonds with maturities beyond one-period, since the average duration of a corporate bond will be $1/\lambda$. We assume that interest payments on the debt stock is tax deductible. Entrepreneurial real profit is, then,

$$\pi_{t,profit}^{j} = (1 - \tau) r_t^K z_{jt}^j \bar{K}_{t-1}^j + \tau \left( \lambda \left( 1 - q_t^i \right) + (1 - \lambda) c \right) \frac{B_{t-1}^j}{P_t}$$

where the first term on the right hand side is the after-tax revenue of renting capital net capital utilization costs, while the second term is the tax deduction on payments of interest accrued on each real unit maturing debt, $(1 - q_t^i) \frac{B_{t-1}^j}{P_t}$, and coupon payments, $c \frac{B_{t-1}^j}{P_t}$.

In addition to the rental profit and debt-tax shield, the equity value of the entrepreneur also consists of cash flows generated from net bond issuance and capital purchases as well as his continuation value. Note, since the entrepreneur will be better off by defaulting on its debt if his value is
zero, his value function is truncated below zero. Thus, the (cum-dividend) equity value function of entrepreneur \( j \) at time \( t \), after the realization of the idiosyncratic shock \( z_i^j \) is:

\[
V_t \left( \bar{K}_{t-1}^j, \tilde{B}_{t-1}^j, z_i^j \right) = \max \left\{ 0, \pi_t^{\text{profit}, j} - (\lambda + (1 - \lambda)c) \frac{\tilde{B}_{t-1}^j}{\exp(\pi_t)} + Q_t z_i^j \bar{K}_{t-1}^j (1 - \delta) + J_t \left( \bar{K}_{t-1}^j, \tilde{B}_{t-1}^j \right) \right\}
\]

where we have defined a new state variable \( \tilde{B}_{t-1}^j \equiv \frac{B_{t-1}^j}{\pi_t} \) as the real value of the outstanding debt.\(^5\) The second item in the max operator is the value of the entrepreneur if he does not choose to default, which consists of four terms. The first term includes the rental profit and debt-tax shield, the second term is the sum of the principal payment for maturing debt and coupon payment for existing debt. The third term is the value of the effective capital stock carried over from the previous period that gets depreciated at the rate \( \delta \) after being rented out.\(^6\) The final term \( J_t \) is defined as:

\[
J_t \left( \bar{K}_{t-1}^j, \tilde{B}_{t-1}^j \right) = \max_{\tilde{B}^j_t, \bar{K}^j_t} q_t^j \left( \tilde{B}^j_t - (1 - \lambda) \frac{\tilde{B}_{t-1}^j}{\exp(\pi_t)} \right) - Q_t \bar{K}^j_t + E_t M_{t+1}^R V_{t+1} \left( \bar{K}^j_t, \tilde{B}^j_t \right)
\]

The first term is the real value of newly purchased capital, and the final term is the entrepreneur’s continuation value where \( \bar{V}_t \left( \bar{K}_{t-1}^j, \tilde{B}_{t-1}^j \right) \equiv \int V_t \left( \bar{K}_{t-1}^j, \tilde{B}_{t-1}^j, z_i^j \right) d\Phi(z_i^j) \) is the expected value of equity at \( t \) before the realization of idiosyncratic shocks. The entrepreneur maximizes his value by optimally choosing the amount of capital to purchase (\( \bar{K}_t^j \)), the amount of bond issuance necessary to finance it (\( \tilde{B}_t^j \)), and whether not to default on existing debt.

The form of the value function in (2) implies that depending on the size of the idiosyncratic shock, the profits and wealth of an entrepreneur may not be sufficient to cover existing debt payment, thereby choosing to default. The default decision depends on how the realization of the idiosyncratic shock for entrepreneur \( j \) compares to an endogenous threshold value \( z_i^j = z_i^* \): the entrepreneur decides to default if \( z_i^j < z_i^* \) and continue operating if \( z_i^j \geq z_i^* \). From (2), \( z_i^* \) is naturally defined as the value of \( z_i^j \) which is an implicit solution to:

\[
\pi_t^{\text{profit}, j} - (\lambda + (1 - \lambda)c) \frac{\tilde{B}_{t-1}^j}{\exp(\pi_t)} + Q_t z_i^* \bar{K}_{t-1}^j (1 - \delta) + J_t \left( \bar{K}_{t-1}^j, \tilde{B}_{t-1}^j \right) = 0
\]

As shown in MW, the default threshold is not entrepreneur specific, and hence the superscript \( j \) is dropped.

\(^5\)Hence, the real market value of outstanding debt at the end of time \( t \) is \( q_t^j B_{t-1}^j = q_t^j \bar{B}_t \).

\(^6\)We assume that the idiosyncratic shock \( z_i^j \) affects entrepreneur \( j \)’s ability to manage (rent and, if necessary, liquidate) the capital stock. Alternatively, the shock \( z_i^j \) can be interpreted as a disturbance that affects the quality of the capital stock employed in period \( t \) by entrepreneur \( j \) and that dissipates at the end of the period.
When an entrepreneur decides on its leverage decision, he takes into account the fact that the bond issued will be priced according to the following Euler equation of the representative household:

$$\frac{q_t^j B_t^j}{P_t} = E_t M_{t+1}^R \left\{ (1 - 1_{z_{t+1}^j < z_{t+1}^*}) \left[ (\lambda + (1 - \lambda)(c + q_t^j)) \frac{B_t^j}{P_t} \right] + 1_{z_{t+1}^j < z_{t+1}^*} \left[ \pi_{t+1}^{\text{profit}, j} + \xi \left( Q_{t+1} z_{t+1}^j \tilde{K}_t^j (1 - \delta) + J_{t+1} \right) \right] \right\} \quad (5)$$

where $1_{z_{t+1}^j < z_{t+1}^*}$ is an indicator function that equals 1 if the entrepreneur defaults and 0 otherwise.

We assume that if an entrepreneur defaults, he is liquidated, and the bond holders collect all rental profits. Also, debt-holders are able to recoup a fraction $\xi \in [0, 1]$ of the entrepreneur’s wealth, $(1 - \delta) + J_{t+1}$. The combined cash-flow to the bond holders partially make up for the debt obligation they were originally entitled to. The Euler equation is the usual present value formula applied to corporate bonds, where the left hand side is the real value of debt entrepreneur $j$ has issued, while the right hand side is the expected payoff in the non-default and default states discounted according to the (real) stochastic discount factor of the household (the marginal investor in this economy). We assume that bondholders can only recover a fraction of the net worth of the entrepreneur (asset value minus debt). In the course of liquidation, a fraction $(1 - \xi)$ of the capital stock and of outstanding debt is lost, while the remainder fraction $\xi$ is transferred to a new-born entrepreneur who will expect to receive a new realization of the idiosyncratic shock in the next period.

By accounting for the Euler equation (5) as a constraint to the maximization problem, the entrepreneur understands that a higher leverage ratio will imply a higher probability of default for a given distribution of idiosyncratic shocks in the future, and consequently higher expected losses for bondholders and higher borrowing costs (i.e., lower $q_t^j$).

In terms of solving the entrepreneur’s problem, the fact that the model is homogeneous of degree one allows us to reduce the endogenous state space to a single dimension. We define a new state variable $\omega_t \equiv \frac{\tilde{K}_t}{\bar{K}_t}$, and using the relation $J_t \left( \tilde{K}_{t-1}, \tilde{B}_{t-1}^j \right) = J_t (1, \omega_{t-1}) \tilde{K}_{t-1}^j \equiv J_t (\omega_{t-1}) \bar{K}_{t-1}^j$, we can normalize the Bellman equation (3) by $\bar{K}_{t-1}^j$ as follows:

$$J_t (\omega_{t-1}) = \max_{\omega_t, \bar{K}_t} q_t^j \left( \omega_t \bar{K}_t - (1 - \lambda) \frac{\omega_{t-1}}{\exp(\pi_t)} \right) - Q_t \left( \bar{K}_t \right) + (1 - \tau) \tilde{K}_t E_t M_{t+1} \int_{z_{t+1}^*}^{\infty} (r_{t+1}^K + (1 - \delta)Q_{t+1}) (z_{t+1}^j - z_{t+1}^*) \phi(z_{t+1}^j) dz \quad (6)$$

where we define $\bar{K}_t \equiv \frac{K_t^j}{\bar{K}_{t-1}^j}$ and have substituted in the following expression for $\bar{V}_t \left( \tilde{K}_{t-1}^j, \tilde{B}_{t-1}^j \right)$:

$$\bar{V}_t (\omega_{t,j}) \equiv \int V_t (\omega_{t,j}) d\Phi(z_{t,j}^j) = (1 - \tau) \tilde{K}_t E_t M_{t+1} \int_{z_{t+1}^*}^{\infty} (r_{t+1}^K + (1 - \delta)Q_{t+1}) (z_{t+1}^j - z_{t+1}^*) \phi(z_{t+1}^j) dz \quad (7)$$
Note \( \Phi(z) \) and \( \phi(z) \) are the lognormal cumulative density function and the probability density function of \( z \), respectively. The entrepreneur’s decision problem at \( t \) boils down to solving (6) by optimally choosing \( \omega_t \) and \( \tilde{K}_t \) subject to constraints (4) and (5) that can be normalized accordingly.

The first constraint that defines the default threshold \( z_t^* \) is:

\[
(1 - \tau)r^K_t z_t^* + \tau \left( \lambda (1 - q^S_t) + (1 - \lambda)c \right) \frac{\omega_{t-1}}{\exp(\pi_t)} + (1 - \delta)z_t^* Q_t + J_t = \left( \lambda + (1 - \lambda)c \right) \frac{\omega_{t-1}}{\exp(\pi_t)} \tag{8}
\]

The second constraint is the normalized household Euler equation with respect to corporate bonds:

\[
q^S_t \omega_t = \left\{ \frac{\left[ (1 - \lambda) \left( 1 - \Phi(z^*_{t+1}) \right) q^S_{t+1} + \lambda + (1 - \lambda)c \right] \omega_t}{\exp(\pi_{t+1})} \right\} + \int_0^{z^*_{t+1}} \left[ \left( (1 - \tau)r^K_{t+1} + (1 - \delta)Q_{t+1} \right) (z^*_{t+1} - z^j_{t+1}) + (\xi - 1)(J_{t+1} + (1 - \delta)Q_{t+1} z^j_{t+1}) \right] d\Phi(z) \tag{9}
\]

Note that all entrepreneur specific variables are removed as a result of this scaling.\(^7\) We derive the equilibrium first order conditions from the normalized Bellman equation of the entrepreneur. Once we have the first-order conditions, we follow MW and differentiate them with respect to the leverage choice \( \omega_t \) to obtain expressions for the first derivative of the debt price \( q^S_t \) and of the default threshold \( z^*_{t+1} \) with respect to \( \omega_t \).\(^8\) The appendix contains the full set of equilibrium conditions.

### 2.5 Capital Producers

At the beginning of each period, capital producers buy the aggregate stock of old depreciated capital \((1 - \delta)\bar{K}^d_t \) from the entire population of entrepreneurs, where \( \bar{K}^d_t \) denotes the aggregate stock of capital at \( t \) after the idiosyncratic shocks have hit and defaults have occurred. Note \( \bar{K}_{t-1} \equiv \int \bar{K}^d_{t-1} dj \) is the aggregate capital stock at \( t \) before default. The capital producers buy an amount \( I_t \) of final goods, combine them with the old capital stock, and build new capital stock, \( \bar{K}_t \). Their profit maximization problem is:

\[
\max_{I_t} \sum_{s=0}^{\infty} \left\{ Q_{t+s} \left( \bar{K}_{t+s} - (1 - \delta)\bar{K}^d_{t+s-1} \right) - P_{t+s} I_{t+s} \right\}
\]

\(^7\)In rewriting the Euler equation (9), we subtract equation (8) that defines \( z^*_t \) (equal to zero) and rearrange terms accordingly. The same approach is used to simplify the expression (7) for \( V_t(\omega_{t,j}) \). See the appendix for more information on how to compute the equilibrium.

\(^8\)Gomes, Jermann and Schmid (2016) point out how MW assume that entrepreneurs are myopic, so that they fail to account that their present leverage choice influences future leverage, and through that, the current value of debt. Gomes, Jermann and Schmid (2016) then solve their model accounting for this intertemporal dependence, but report that the upgraded solution method implies crucial differences in the dynamics of the model when the economy does not feature nominal price and wage rigidities, while differences would be minimal in a model like ours. We plan on reporting results for both solution methods in our next draft.
subject to the physical capital accumulation technology:

$$\bar{K}_t = \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t + (1 - \delta) \bar{K}_{t-1}$$

where $\delta$ is the depreciation rate, and the function $S$ captures the presence of adjustment costs in the accumulation of capital. The steady-state properties of the function $S$ are standard: $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) > 0$, and characterize adjustment costs that are zero at the steady state growth rate of investment, while positive and convex at any other $\frac{I_t}{I_{t-1}}$.

2.6 Employment Agencies

Employment agencies hire differentiated labor inputs, $L_{i,t}$ from households at monopolistic wages $\tilde{W}_{i,t}$ and transform them into homogenous hours worked by means of the CES technology:

$$L_t = \left[ \int_0^1 L_{i,t} \frac{1}{1+\theta_w} \, di \right]^{1+\theta_w}$$

so that the demand of any differentiated labor input, $L_{i,t}$, is:

$$L_{i,t} = \left( \frac{W_{i,t}}{W_t} \right)^{-\frac{1+\theta_w}{\theta_w}} L_t$$

(10)

Household $i$ is the monopolistic supplier or labor inputs of kind $L_{i,t}$. In every period $t$ the households set wages that maximize their welfare. In similarity with the price-setting decision of intermediate firms, intertemporal adjustments in the monopolistic wage rate generate a cost:

$$\frac{\psi_w}{2} \left( \frac{W_{i,t}}{W_{i,t-1}} - 1 \right)^2$$

Households then re-optimize monopolistic wages $\tilde{W}_{i,t}$ by maximizing the difference between the real consumption value of its wage bill in every period $t + s$, $\tilde{W}_{i,t+s}L_{i,t+s}$, and the disutility induced by labor supply, $L_{i,t+s}$, minus the real adjustment cost:

$$E_t \left\{ \sum_{s=0}^{\infty} M_{t+s}^R \left[ \frac{\tilde{W}_{i,t+s}L_{i,t+s}}{P_{t+s}} - \chi_0 Z_{t+s}^{1-\alpha} \frac{L_{1,t+s}}{1+\chi} - \frac{\psi_w}{2} \left( \frac{W_{i,t+s}}{W_{i,t+s-1}^{1-\alpha}} - 1 \right)^2 \right] \right\}$$

subject to labor demand from employment agencies, (10).

In a symmetric equilibrium, the maximization problem gives rise to a standard wage Phillips curve:

$$-((\theta_w)u_t^L + w_t)L_t - \psi_w(\theta_w) \left( \frac{w_t}{w_{t-1}} - 1 \right) \frac{w_t}{w_{t-1}} + E_t M_{t+1}^R \psi_w \theta_w \left( \frac{w_{t+1}}{w_t} - 1 \right) \frac{w_{t+1}}{w_t}$$

where $w_t$ is the real hourly wage and $u_t^L$ is the per-period marginal disutility of hours worked.
2.7 Government

The fiscal authority raises taxes on effective capital returns, \( \tau (r^K_t u^*_t - a(u^*_t)) \bar{K}_{t-1} \). In each period \( t \), the government can issue zero-coupon bonds of different maturity \( n = (1, 2, ..., N) \), expressed in quarters, at value \( B_t^{(n)} \) that pay back a yield \( i_t^{(n)} \) at date \( t + n \).

Since the government can adjust lump-sum transfers, \( T_t \), fiscal policy is fully Ricardian and the fiscal authority runs a balanced budget:

\[
G_t + \sum_n i_t^{(n)} B_t^{(n)} + T_t = \tau (r^K_t u^*_t - a(u^*_t)) \bar{K}_{t-1} + \sum_n B_t^{(n)}
\]

where government bonds are in zero-net supply, \( B_t^{(n)} = 0 \) for every \( t \) and \( n \).

2.8 Monetary Authority

The monetary authority sets the nominal one-period default-free rate, \( i_t = i_t^{(n=1)} \), following a Taylor-type rule as in Rudebusch and Swanson (2012):

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) (r^{ss} + \pi_t + g_y(\tilde{Y}_t - \tilde{Y}^{ss}) + g_\pi (\pi_t - \pi^*_t)) + \varepsilon_{it}^{mp}
\]

The central bank sets the one-period nominal interest rate by responding to deviations of realized GDP, \( \tilde{Y}_t \), from its steady state value and inflation \( \pi_t \) from an inflation target \( \pi^*_t \). The interest rate rule is subject to a monetary policy shocks \( \varepsilon_{it}^{mp} \sim N(0, \sigma_{mp}) \) As in Rudebusch and Swanson (2012), and in line with empirical evidence in Grkaynak, Sack and Swanson (2005), we also assume that the inflation target \( \pi^*_t \) is time-varying and follows the stationary AR(1) process:

\[
\pi^*_t = (1 - \rho_{\pi^*}) \pi^{ss} + \rho_{\pi^*} \pi^*_{t-1} + \theta_{\pi^*} (\pi_t - \pi^*_t) + \varepsilon_{\pi^*_t}^{mp}
\]

where the target reverts to the non-stochastic steady-state level of inflation, \( \pi \), and responds to deviations of current inflation from the target \( (\pi_t - \pi^*_t) \) and to i.i.d. shocks \( \varepsilon_{\pi^*_t}^{mp} \sim N(0, \sigma_{\pi^*}) \).

2.9 Aggregation and Market Clearing

We derive the aggregate resource constraint from adding together the households, entrepreneurs and government’s budget constraints with the zero profit condition of final good producers, capital producers and employment agencies:

\[
C_t + I_t + G_t + \frac{\psi_P}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 + \frac{\psi_w}{2} \left( \frac{w_t}{w_{t-1}} - 1 \right)^2 = Y_t
\]

We also define GDP, \( \tilde{Y}_t \), as:

\[
C_t + I_t + G_t = \tilde{Y}_t
\]
Following the definition of $\bar{K}_t^d$ and the assumption that idiosyncratic shocks are i.i.d across entrepreneurs, the following relation holds between $\bar{K}_t^d$, the aggregate stock of capital at $t$ after the idiosyncratic shocks have hit and defaults have occurred, and $\bar{K}_{t-1}$, the aggregate stock of capital at $t$ before default:

$$\bar{K}_t^d = [(1 - \Phi(z_t^*)) + \Phi(z_t^*) \xi] \bar{K}_{t-1}.$$  

It is important to highlight that aggregate shocks dynamically affect the endogenous default threshold $z_t^*$, and as such have a direct effect on the extent of capital depreciation in our model. In particular shocks to the second moment of the idiosyncratic distribution of $z_{i,t}$, referred to as risk shocks, can affect the probability of default directly and cause the depreciation of the capital stock $\bar{K}_t^d$ to move countercyclically. When volatility of $z_{i,t}$ is on the rise, for example, defaults will increase, so that the capital stock is subject to an additional $\Phi(z_t^*) (1 - \xi)$ depreciation:

$$(1 - \delta) \bar{K}_t^d = (1 - \delta) \bar{K}_{t-1} - (1 - \delta) \Phi(z_t^*) (1 - \xi) \bar{K}_{t-1}.$$  

In addition, $\bar{K}_t^d$, $\bar{K}_t$ and $\tilde{K}_t$ satisfies the following equation:

$$\bar{K}_t = \bar{K}_t \bar{K}_t^d.$$  

### 3 Asset Pricing

#### 3.1 Term Structure of Default-free Interest Rates

A first contributions of our paper is to study the dynamics of the term structure of ‘default-free’ interest rates in our New Keynesian model with credit frictions. The equilibrium price of a $n$ period zero-coupon nominal bond that pays one dollar at maturity $P_t^{(n)}$ can be derived recursively using the nominal stochastic discount factor from the DSGE model:

$$P_t^{(n)} = E_t[M_{t+1}P_{t+1}^{(n-1)}].$$  

where $P_t^{(0)} = 1$ for $\forall t$. The continuously compounded yield to maturity of this bond follows directly from its price:

$$i_t^{(n)} = -\frac{1}{n} \ln P_t^{(n)}.$$  

Following the large existing literature on the term structure of interest rates, we further define the term premium of this bond as the difference between the yield and its ‘risk-neutral’ counterpart $i_t^{(n)Q}$.
\[ ntp_t^{(n)} \equiv i_t^{(n)} - i_t^{(n)Q} = \frac{1}{n} (\ln P_t^{(n)Q} - \ln P_t^{(n)}) \]

where the risk-neutral price of a \( n \) period zero-coupon nominal bond \( P_t^{(n)Q} \) can be derived similarly as:

\[ P_t^{(n)Q} = \exp(-i_t) E_t[P_{t+1}^{(n-1)Q}] \]

where again, \( P_t^{(0)Q} = 1 \ \forall t \). Note the yield to maturity and the term premium of a \( n \) period zero-coupon real bond can be derived analogously, by simply replacing the nominal stochastic discount factor \( (M_{t+1}) \) and the nominal one-period interest rate \( (i_t) \) used for discounting the risk-neutral prices with their real counterparts \( (M_{t+1}^R \text{ and } r_t) \). We will study the term structure dynamics of both the nominal and real term structure in the following sections.

### 3.2 Credit Spreads

We define the time \( t \) credit spread \( cs_t \) as:

\[ cs_t \equiv i_t^C - i_t^\lambda \]

where \( i_t^C \) denotes the (nominal) credit yield defined as:

\[ i_t^C \equiv \ln E_t \left[ \frac{\lambda + (1 - \lambda)(c + q_{t+1}^s)}{q_t^s} \right] \]

and \( i_t^\lambda \) denotes the (nominal) default-free yield with the same average maturity, pinned down by the parameter \( \lambda \), similarly defined as:

\[ i_t^\lambda \equiv \ln E_t \left[ \frac{\lambda + (1 - \lambda)q_{t+1}^{f,n}}{q_t^{f,n}} \right] \]

We define the (nominal) price of a default-free discount bond that matures with a probability of \( \lambda \) as \( q_t^{f,n} \). We can express this price recursively as:

\[ q_t^{f,n} = E_t \left[ M_{t+1} e^{-\pi t+1} \left( \lambda + (1 - \lambda)q_{t+1}^{f,n} \right) \right] \]

This particular definition closely follows MW, which is similar to the concept used in related work, such as Gomes and Schmidt (2010). It is worth pointing out that the prices of a zero-coupon default-free bond with an actual maturity of ten years \( (n = 40 \text{ in our quarterly model}) \), labeled as
$P_t^{(40)}$ in the previous section, will be similar but not identical to the price of a default-free bond with an *average* maturity of 10 years, $q_t^{f,40}$.  

We can further decompose the credit spread into the compensation for expected default and the credit risk premium. To compute the expectation component, we first define the (nominal) *risk-neutral credit yield* $i_t^{C,Q}$ as:

$$i_t^{C,Q} \equiv \ln E_t \left[ \frac{\lambda + (1 - \lambda)(c + q_{t+1}^{s,Q})}{q_t^{s,Q}} \right]$$

where $q_t^{s,Q}$ is the (nominal) price of a defaultable bond that matures with a probability of $\lambda$ evaluated under the default-free real discount rate, $r_t$. $q_t^{s,Q}$ is defined recursively as:

$$\frac{q_t^{s,Q} B_t^j}{P_t^j} = \exp(-r_t) E_t \left\{ (1 - 1_{z_t+1 < z_t}^j) \left[ \left( \lambda + (1 - \lambda)(c + q_{t+1}^{s,Q}) \right) \frac{B_t^j}{P_{t+1}^j} \right] + 1_{z_t+1 < z_t}^j \left[ e^{-\pi_{t+1}} + Q_{t+1}^j \xi_{t+1}^j K_{t+1}^j (1 - \delta) + \xi J_{t+1}^j \right] \right\}$$

We next define the (nominal) *risk-neutral long-term yield* $i_t^{\lambda,Q}$ as:

$$i_t^{\lambda,Q} \equiv \ln E_t \left[ \frac{\lambda + (1 - \lambda)q_{t+1}^{f,n,Q}}{q_t^{f,n,Q}} \right]$$

where $q_t^{f,n,Q}$ is the (nominal) price of a default-free discount bond that matures with a probability of $\lambda$ evaluated under the default-free discount rate. $q_t^{f,n,Q}$ is defined recursively as:

$$q_t^{f,n,Q} = \exp(-r_t) E_t \left[ e^{-\pi_{t+1}} \left( \lambda + (1 - \lambda)q_{t+1}^{f,n,Q} \right) \right]$$

Note the definitions for $q_t^{s,Q}$ and $q_t^{f,n,Q}$ are analogous to the definitions of the risk-neutral long-term yield in the previous subsection.

Finally, we define the *credit risk premium* $crp_t$ as:

$$crp_t \equiv cs_t - \left( i_t^{C,Q} - i_t^{\lambda,Q} \right)$$

where the term in the bracket that is subtracted from the credit spread equals the compensation for expected default.

Our model definition of credit risk premium is similar in spirit to the *Excess Bond Premium* (EBP) that Gilchrist and Zakrajsek (2011) derive from a panel of U.S. corporate bonds as the average price of bearing U.S. corporate credit risk, beyond the compensation that bondholders require for expected defaults. Gilchrist and Zakrajsek (2011) report that the EBP is volatile, markedly countercyclical and a significant predictor of business cycle conditions.

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9The definition used to calculate the default-free term structure results in section 3.1 is more consistent with that of the data, but we use the latter concept of long term default-free yields when we construct the credit spread since this concept is consistent with our definition of credit yields. As already mentioned, using either definition will not produce materially different results.
3.3 Solution

In the model presented, output, consumption, investment, capital, real wages and nominal debt fluctuate around a deterministic trend, since the technology factor $Z_t$ grows at a positive constant rate $\gamma$.

In order to solve the model, we first rewrite it in terms of stationary variables, by rescaling all trending variables by the level of technology $Z_t$. Once we have a stationary model, we can solve for its non-stochastic steady state and compute higher-order perturbations of the equilibrium conditions around it. Finally we solve the system of non-linear perturbed equilibrium conditions and obtain its state-space representation, using Dynare (a popular software package to solve and simulate DSGE models).

The interaction of Epstein-Zin preferences for early resolution of uncertainty with aggregate risk can potentially generate sizable compensation for holding financial assets with different maturities and risk-profile (government bonds, corporate bonds and equity, in this model). The mean and standard deviation of these risk premia is equal to zero in a log-linearized version of the model solution where certainty equivalence is at play. We hence revert to second-order perturbations to measure the average premia and third-order perturbations to obtain a measure of their second moments.

4 Data and Calibration

This section discusses the data series that we target in choosing the model parameters and the resulting calibration of the model.

4.1 Data

We choose the model parameters so that the ergodic moments of the perturbed model solution can match relevant moments of selected macroeconomic and financial variables. In particular we target second moments (standard deviations, autocorrelations and selected cross-correlations) of per-capita GDP growth, consumption growth, investment growth, hours worked, real wage growth, together with inflation and the nominal default-free rate reported in Table 3.\textsuperscript{10} In addition we target average and standard deviations of nominal Treasury yields at different maturities, obtained from the Board of Governors H.15 release and reported in table in Table 4. We report Grkaynak, Sack and Wright (2010) yields for Treasury Inflation Protected Securities (TIPS) as a proxy for real rates. We calibrate the parameters that govern the leverage decision of entrepreneurs so to match the properties of

\textsuperscript{10}All data series are from the USECON database in Haver. Real growth rates are computed by scaling nominal aggregate variables by the GDP deflator and working age population. We report statistics for total CPI inflation and refer to total CPE inflation statistics in the table caption, for direct comparison.
selected financial variables. We target corporate bond yield spreads against Treasuries at different maturities (obtained from the Bank of America Merrill Lynch U.S. Bond Yields database), as well as the average share of long-term debt over total assets (from the Flow of Funds data), and the annual year future cumulative default rate for Baa-rated bonds (from Moody’s annual reports, as in Chen, Collin-Dufresne and Goldstein (2009)), reported in Table 5.

We calibrate our model to U.S. data from 1983:Q1 to 2007:Q4, from the aftermath of Volker’s monetary experiment to the onset of the Great Recession. This choice minimizes concerns on possible structural breaks and regime changes in the U.S. economy that model simulations would not be able to capture, such as the shift in the conduct of monetary policy after 1981, or the binding effective lower bound on the federal funds rate and the adoption of unconventional monetary policy after 2008. The model fit is discussed at length in Section 5.

4.2 Calibration

Table 1 contains a summary of calibrated parameters.

**Steady State Values**

We set parameters to be compatible with Rudebusch and Swanson (2012) ‘best-fit’ calibration, as it has proven to offer a reasonably good fit to both macro moments and yield curve moments in a DSGE model that shares many features with ours. The adoption of minimal differences in calibrated parameters prove necessary to allow our medium-scale model with additional frictions to fit data moments, such as investment dynamics and financial variables, over a slightly different sample period. We verify that these changes in parameters still generate a sizable and volatile term premium in their model and report its statistics in table 6.

As in Rudebusch and Swanson (2012), we set the growth rate of total factor productivity at $\gamma = 0.025\%$. We match the ergodic mean of model inflation with the observed average annual rate between 2% and 3%, compatibly with total CPI and PCE data over the sample period, by setting its non-stochastic steady state value $\pi_{ss} = 10\%$. We calibrate government spending as a percentage of GDP to match its historical average in the data, $\frac{G_{ss}}{Y_{ss}} = 17\%$.

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11Rudebusch and Swanson (2012) compare model statistics to data from 1961 - 2007, as opposed to 1983 - 2007 as in our paper
12The ergodic mean of the higher order inflation policy function matches a 3% average annual inflation observed in our reference sample. It is important to highlight that the central bank in our model targets a slow-moving average of past and current inflation rates as in Rudebusch and Swanson (2012) and Grkaynak, Sack and Wright (2010). The steady state inflation parameter is hence not a relevant one to define the monetary policy stance in our model.
Preference Parameters

We set the effective discount rate $\beta = 0.99$, to target an average real default-free interest rate of around 2%. We set the Frisch elasticity equal to 0.25. This relatively low value limits the extent to which labor supply responds to changes in real wage and allows us to fit the volatility of hours worked in the data. This, together with the presence of sticky wages, prevents households from extensively using the labor margin to smooth out their consumption profile. We adjust the scaling parameter $\chi_0$ so to match a steady-state labor supply equal to $\frac{1}{3}$. Finally we set the coefficient of relative risk aversion (CRRA) to 120.\textsuperscript{13}

The main difference between our calibration and Rudebusch and Swanson (2012) lies in the value of the intertemporal elasticity of substitution (IES) parameter. We set the IES equal to 0.25 instead of 0.11, to match the volatility of consumption growth observed in the data.\textsuperscript{14}

Technology parameters

We assume that the capital stock depreciates at a rate $\delta = 0.02$. We set the output elasticity to labor inputs $\eta$ equal to $\frac{2}{3}$. The curvature parameter of the investment technology, $S''$, is equal to 0.03, much lower than most empirical estimates in the DSGE literature (Justiniano, Primiceri and Tambalotti (2010b)): in our model financial frictions act to smooth out the fluctuations of aggregate investment and to increase their persistence in place of adjustment costs, along the intuition of Carlstrom and Fuerst (1997). A higher adjustment cost parameter would significantly reduce the volatility of investment to empirically implausible levels.

We choose the price and wage mark-ups to equal $\theta_p = \theta_w = 0.20$, and the Rotemberg price and wage adjustment cost parameters for intermediate producers, $\psi_p = \psi_w = 80$.\textsuperscript{15}

\textsuperscript{13}As detailed by Rudebusch and Swanson (2012), the literature recognizes that the size of consumption risk in standard DSGE models is very small, and thus a high risk aversion parameter is required in those contexts to match asset prices facts ( (n.d.)). Such high coefficient of relative risk aversion can be justified, in models like Rudebusch and Swanson (2012) and ours, in light of findings by Barillas, Hansen and Sargent (2009) who show that high risk aversion in an Epstein-Zin specification is isomorphic to a model with low risk aversion and a moderate degree of uncertainty about the economic environment (in the form of parameter and/or model uncertainty). In addition, Malloy, Moskowitz and Vissing-Jrgensen (2009) show that the consumption of stockholders is more volatile than the consumption of nonstockholders. As a result, the required level of risk aversion in a model like ours in which the representative agent is the marginal investor will be higher than the level of risk aversion required in a model that recognized that asset holders are subject to higher consumption risk than households that do not hold assets.

\textsuperscript{14}The IES governs the elasticity of consumption to expected changes in real interest rates. Some studies advocate for a stronger response of consumption to real interest rate changes, and hence a higher IES (Weil (1989), Bansal and Yaron (2005)). The value we adopt is closer to the empirical analysis of Hall (1988) that estimates consumption to be resilient to expected variations in the real rate, and consequently argues in favor of small IES values (0.2, or negative). Our results are robust to lower IES values.

\textsuperscript{15}We verify that our results hold under the alternative modeling assumption of Calvo-style nominal frictions with
Taylor Rule

As discussed in section 5.4, we adopt the functional form of the Taylor rule proposed by Rudebusch and Swanson (2012). Parameters are calibrated for the model to match the relative volatilities of the nominal default-free rate, inflation and output growth observed in the data. The autoregressive component for the nominal interest rate is set to $\rho_i = .75$, while $g_\pi$ is equal to .5 (implying a response coefficient to deviations of the inflation rate from its target equal to 1.5). The response coefficient to deviations of output from its steady state level is set to $g_y = .2$.

Credit Friction Parameters

The parameters that govern financial frictions are calibrated to match the model-implied ergodic first and second moments of selected financial variables with those observed in the data.

We calibrate the parameters that govern the features of the credit contract (the average bond duration $1/\lambda$, the steady state level of debt with respect to the capital stock $\omega_{ss}$, the liquidation cost $1 - \xi$, and the tax benefit of debt $\tau$) so to match: 1) an average spread (relative to a Treasury yield) for a generic ten-year corporate bonds rated between the investment grade (BBB and higher on the S&P scale, with spreads on average of \approx 1.10%) and the high yields (BB and lower on the S&P scale, with spreads on average of \approx 4.10%); 2) a 60% average share of debt over equity (henceforth called leverage ratio) reported in the Flow of Funds tables for the U.S. corporate sector; 3) an average recovery rate as a percentage of the face value of the bond in case of default equal to 60% (2016). All parameters choices are detailed in Table 5.

The average maturity of corporate debt in the U.S. is around 7 years (see SIFMA (2017)). We round this number up and choose $\lambda = 1/40$, so that the average duration of corporate debt in our quarterly model is 10 years. Results are robust to lower and empirically plausible choices of $\lambda$. The importance of modeling long-term debt instead of relying on one-period contracts as in most of the DSGE literature will be discussed in section 5.3.

The structure of the entrepreneur’s balance sheet in our model is determined by a trade-off between costs and benefits of issuing corporate debt with respect to equity financing. Issuing debt can lead to costly defaults for entrepreneurs as well as for bond holders. Investors ask for a credit spread over the yield of a non-defaultable (government) bond, as a compensation for potentials future default losses on their bond purchases. Debt, however, also generates benefits for entrepreneurs in the form of a tax shield on interest payments. We set the corporate tax rate $\tau$ equal to 5% to obtain an average spread of around 2.8%. We also pick the steady state ratio of the stocks of debt and capital in the model, $\omega_{ss}$ to match the average leverage ratio observed in the Flow of Funds data of around 60%. Finally we set the liquidation cost $(1 - \xi)$ equal to 0.73 to match the empirical recovery adjustments every three quarters as in Rudebusch and Swanson (2012). Results are available upon request.
rate on defaulting corporate bonds of approximately 50% of their face value. The implied annual default rate in the model implied by the choice of the three parameters is equal to 2.3%, compared to 2% in historical corporate bond data, as reported in (2016).

Stochastic Processes

The natural logarithm of Total Factor Productivity (TFP) is assumed to follow an AR(1) process around a constant growth trend. The autoregressive coefficient is set to $\rho_a = 0.98$, while the standard deviation of the i.i.d. shocks is calibrated to 0.6% ($\sigma_a = 0.006$).

The Taylor rule is subject to i.i.d. shocks with a standard deviation of 15 basis points ($\sigma_{mp} = 0.0015$).

Finally, we follow Rudebusch and Swanson (2012) in choosing the function form and parameter values for the process that governs the evolution of the central bank’s inflation target, $\pi^*$. The target follows a very persistent AR(1) process, with an autoregressive coefficient $\rho_{\pi^*}$ set to 0.995. We assume the target to be resilient to temporary deviations of realized inflation, by picking a low value for $\theta_{\pi^*} = 0.01$. The target is also subject to i.i.d. shocks with a standard deviation of 5 basis points ($\sigma_{\pi^*} = 0.0005$).

Finally, we assume that the variance of the distribution of idiosyncratic revenue shocks, $z^j_t$ is time-varying and subject to exogenous disturbances, similar in spirit to the risk shocks described in Christiano, Motto and Rostagno (2014). In particular, we assume that the standard deviation of the log-normal distribution of the idiosyncratic shock $z^j_t, \sigma_{z,t}$, follows an AR(1) process with $\rho_\sigma = 0.98$ and standard deviation $\sigma_\sigma = 0.06$.

5 Results

This section reports the main results. We first describe the empirical fit of the calibrated model in terms of selected moments of macro and financial data. In this context we discuss the variance decompositions for the macro and financial observables and show that TFP shocks explain a large share of aggregate fluctuations in our calibration, followed by risk shocks. We pay particular attention to the implications of the model for the Treasury yield curve and the nominal term premium for 10-year default-free bonds, as well as for the real term structure of interest rates. We find that the model is able to produce reasonable moments that describe the main features of U.S. business cycles of macro variables. The model also performs well in replicating the level and volatility of yields and selected financial variables observed in the data. In addition, the model generates a sizable average level and significant time variation for the 10-year nominal term premium.

We then study what frictions in the model generate the observed term premium. In particular we discuss the role of credit frictions (and to a minor extent, wage rigidities) in our model with flexible
capital accumulation in generating a sizable and volatile term premium, and draw a comparison with a model like Rudebusch and Swanson (2012) with firm-specific fixed capital.

We then discuss the empirical implications of introducing long-term debt contracts in the model in comparison with one-period contracts that are common in the DSGE literature (Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2014)). In particular we study the empirical fit of the model with one-period contracts for financial variables and credit spreads, and compare it to our baseline case.

We also show that modeling long-term debt in the model has stark implications for the propagation of unexpected inflation shocks and for credit risk premia. We show the impulse responses to a shock to the monetary authority’s long-run inflation target, $\pi^*_t$, and show how unexpected shocks to the long-run inflation level can propagate by changing the real value of nominal long-term contracts.

We conclude by showing that transient monetary policy shocks have limited effect on term and credit premia in the model, in contrast with empirical evidence by Gertler and Karadi (2015). We find, however, that the systematic response of the monetary policy authority to fluctuations in output and inflation, have important implications for the magnitude and volatility of term premia and credit risk premia.

5.1 Model Fit and Dynamic Properties

5.1.1 Variance Decomposition

Table 2 displays the unconditional variance decomposition of selected variables through the lens of the model. Aggregate TFP shocks are the main drivers of economic fluctuations in our calibration, explaining more than 40% of GDP growth fluctuations, followed by risk shocks, responsible for around 35%. It is worth noting that while both TFP and risk shocks play an important role in explaining the dynamics of real aggregates, inflation and of the nominal term structure of Treasury rates, risk shocks are almost the exclusive driver of fluctuations in credit spreads, leverage and default probabilities, a common feature in this literature (see for example Christiano, Motto and Rostagno (2014) and Ajello (2016)). In section 5.2 we show that the features of the response of the cost of credit, the default rates and inflation to unexpected risk shocks have crucial implications for term and credit risk in the economy.

5.1.2 Relevant Moments

In this section we evaluate the model fit in terms of its ability to replicate dynamic properties of selected variables. We compute empirical moments for the data over a sample of quarterly observations from 1982:Q1 to 2007:Q4, when not differently specified. We compare empirical moments
with theoretical ergodic moments computed from a second-order perturbation of the model solution around its non-stochastic steady state. Since ergodic second moments of risk premium variables are zero at second-order, we report volatilities of the term premiums and credit risk premium computed from a third-order perturbation of the model solution (Rudebusch and Swanson (2012)). Second moments of risk premia are average statistics computed over short-sample simulations of the third-order perturbation to the model solution.

Table 3 reports second moments for macroeconomic variables of common interest in the empirical DSGE literature (see, among others, Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005), Justiniano, Primiceri and Tambalotti (2010a)). These include the quarterly growth rates of consumption, $\Delta c = 100 \times (\ln(C_t) - \ln(C_{t-1}))$, the growth rate of investment, GDP, real wages, $\Delta i$, $\Delta \bar{y}$, $\Delta w$, as well as the log of hours worked, $l$, and the quarterly rate of inflation, $\pi$.

The model generates unconditional volatilities of the macro variables that are in line with the data. It closely fits the volatility of quarterly consumption, investment and output growth and inflation, $\Delta c$, $\Delta i$, $\Delta \bar{y}$ and $\pi_t$. Despite calibrating wage and price rigidity parameters in our model to values that are in line with the empirical macro literature, the model generates smoother real wage growth than in the data and hence too volatile quarterly changes in hours worked, $\Delta l$. We will show in section 5.1.3 that wage stickiness offers only a minor contribution in amplifying risk in the economy, and our results are robust to a wide range of values of $\phi_w$ and implied wage growth and hours worked volatilities.

In an economy driven in large part by TFP shocks, business cycles feature a small unconditional cross-correlation between consumption grown and inflation in the data and in the model, $-0.07$, as reported in the bottom panel of table 3. Negative aggregate supply shocks generate a decrease in consumption growth together with persistent increase in inflation. Household perceive higher inflation today as a signal that the central bank will intervene and increase short-term nominal interest rates. In an attempt to reign in inflation, tighter monetary policy is hence expected to keep consumption growth low in the future. Under this scenario long-term nominal bonds need to offer a premium over short term bonds to be held in equilibrium: the persistent nature of inflation in fact erodes the realized pay-offs of long-term nominal bonds exactly when consumption growth is expected to be low. This mechanism is similar to the one described by Piazzesi and Schneider (2007) in the context of an endowment economy with EZ preferences.

Our model simulations confirm the intuition. In table 4 we report the ergodic means and standard deviations of the 1-quarter and 10-year nominal and real rates, $i^{(1)}$, $i^{(40)}$, $r^{(1)}$, $r^{(40)}$, as well as the nominal and real 10-year term premia, $ntp^{(40)}$ and $rtp^{(40)}$. The table also reports data counterparts for the same variables, when directly measurable, as well as estimates (in parentheses) of real rates and term premia obtained from the 3-factor affine term structure model of Kim and Wright (2005).

We find that our model with credit frictions is able to generate a sizable term premium for 10-year
nominal bonds of 177 basis points, comparable in size to the empirical estimate of 190 basis points from the term structure model, together with an upward-sloping average yield curve. The volatilities of nominal interest rates are in line with the data decreasing with the bond maturity, while we find the second moment of the nominal term premium to be lower than its estimated factor-model counterpart.\footnote{Introducing stochastic volatility in the TFP process following Caldara et al. (2012) increases the volatility of the term and credit risk premia without sacrificing the macro fit. Results are available upon request.}

Our model also has predictions for the real term structure of interest rates. In the right panel of table 4, we compare the first and second moments of the model implied 1-quarter and 10-year real rates with their data counterparts. Since the sample period of TIPS yields is fairly restricted (1999:Q1-2012:Q1), we report together estimates from a 3-factor real term structure model that covers a longer sample period (1990:Q1-2012:Q1). We find that the model implied real term structure is fairly flat, with a small negative real premium paid over long-term TIPS. Many equilibrium term structure models report a flat or downward sloping real term structure, due to the positively autocorrelated consumption growth dynamics that make long-term default-free real bonds an effective risk hedging tool for future consumption uncertainty.\footnote{See, for example, Bansal and Yaron (2005), or Piazzesi and Schneider (2007).}

Finally table 5 compares data and model moments for the financial variables: annual default rates ($dp^{(4)}$), corporate spreads for investment grade and high-yield bonds of 10 years of maturity ($cs^{(40)}$, IG and HY respectively), the average share of credit risk premium over credit spreads of the same maturity ($crp^{(40)}/cs^{(40)}$), and the leverage ratio ($lv$). Our model fares relatively well in matching the size of a generic corporate spread of 2.9%, in the ballpark of a 10-year bond with average credit rating versus the 10-year Treasury yield as well as a standard deviation of around 2%. The model also captures the empirical magnitude of mean and standard deviation of the leverage ratio, defined as credit market debt as a percentage of the market value of corporate equities in the Flow of Funds tables. Consistently with the data, the credit variables from the model are countercyclical, suggesting that recessions in the model are times when corporate defaults increase, together with corporate spreads, while the market value of corporate assets drops enough to increase the leverage ratio.

Importantly, the model attributes to credit risk premia a share of around 40% of corporate spreads, in line with empirical estimates by Elton et al. (2002), while at the same time keeping the average size and volatilities of default probability close to the levels reported in Moody’s historical analysis of the corporate bond market (2% average annual default probability and recovery rates in case of default of around 60% of the face value of the bond). We view the ability of our model to closely match empirical financial moments and to generate a sizable credit risk premium as an important feature, since a large body of literature reports that large credit spreads and small default
probabilities observed in the data are hard to reconcile in simple models of default risk, a conundrum that is often referred to as the credit spread puzzle.\footnote{See, for example, Huang and Huang (2002), Gieseke et al. (2011) or Gilchrist, Sim and Zakrajsek (2010). Potential solutions discussed in the literature such as the presence of heterogeneous firms with entry and exit decisions (Gomes and Schmidt (2010)) or countercyclical default boundaries (Chen, Collin-Dufresne and Goldstein (2009)) typically abstract from endogenous labor decisions or inflation dynamics, and therefore it is not clear how they will survive in the type of DSGE models we study.}

In our model economy the solution to the puzzle is a product of three main features: 1) the presence of long-term debt; 2) the nature of risk shocks, that effectively alter the volatility of capital depreciation by means of corporate defaults that can negatively affect the productive capacity of the economy, and 3) a high coefficient of relative risk aversion.

In section 5.3, we describe how long-term corporate debt contracts price deflation risk (Kang and Pflueger (2015)) by comparing our baseline model to a calibration with one-period debt and a similar degree of credit frictions (measured as similar implied average credit spreads). We find that the economy with one-period credit contracts will only produce about half of the average positive premium present in our baseline calibration. Moreover, in section 5.2 we describe the impulse responses of the economy to a negative risk shock and show how the implied time-varying volatility of capital depreciation can produce a negative comovement between consumption growth and expected default losses on corporate bonds that amplifies credit risk in the model.

\section*{5.1.3 The Role of Frictions}

In this section, we discuss our results in more detail by analyzing the importance of various frictions embedded in the model to understand the features of the term structure. In table 6, we compare some relevant first and second order moments of both the macroeconomic and term structure variables in the data with four model specifications (1) Our model (AT), with credit frictions, sticky wages and flexible capital, (2) AT without credit frictions (Flex. K+Sw), (3) AT without credit frictions and sticky wages (Flex. K), and (4) AT without credit frictions and sticky wages, but with firm-specific fixed capital (Fix. K). The latter specification closely follows Rudebusch and Swanson (2012), which interprets the framework as a model with firm-specific capital. We consider their work to be a useful benchmark in the literature, since their model is successful in matching some basic features of macroeconomic and term structure dynamics.\footnote{For example, Altig et al. (2011) claim that their model with ‘firm-specific’ capital can account for inflation inertia in the data with a more realistic cross-sectional distribution of prices and output across firms.} All models share the same degree of price-stickiness and are calibrated using the same pertinent subset of parameters listed in table 1, as well as aggregate shocks.\footnote{One notable exception are risk shocks, that can only affect the AT model by construction. Results hold also in the absence of risk shocks.}
Columns two and three show how credit frictions affect macroeconomic and term structure dynamics, by comparing our model, AT in column 2, with a DSGE model with sticky wages and capital accumulation but with no credit frictions in the spirit of Christiano, Eichenbaum and Evans (2005), in column 3.

We find that credit frictions amplify TFP-driven fluctuations in quantities and prices, creating higher volatility for variables such as consumption and investment growth. This is visible by comparing the impulse responses to a negative TFP shock for a baseline model and a model with a twice as large tax benefit of debt ($\tau = 0.10$). Figure 3 plots responses for levels of de-trended GDP, consumption, investment and inflation to a positive one standard deviation shock to TFP.\(^{21}\)

On the wake of a negative TFP shock, both economies exhibit a supply-driven recession, where consumption, falls and inflation rises. More importantly, the impulse responses show that in the presence of a higher degree of credit frictions, an additional amplification of the shock arises as credit conditions tighten in the long horizon. As shown in figure 3, a negative TFP shock reduces revenues of entrepreneurs as well as the price of capital hence increasing default probabilities leverage and corporate credit spreads above their ergodic means, after an initial drop.\(^{22}\)

Table 6 also reports comparisons of term structure moments. The increased degree of risk of the economy with credit frictions generate precautionary saving motives which drive down the levels of both real and nominal yields. However as discussed in section 5.1, the amplification in the covariance between news about consumption growth and long-run inflation increases the average nominal term premium by as much as 40%, from 126 to 177 basis points. Credit frictions have also an amplification effect on the volatility of the term premium. This is visible in figure 3 that reports impulse responses to a negative TFP shocks of the nominal term and credit risk premia in our baseline model, compared to a model with larger credit frictions (modeled as higher tax benefit of debt $\tau = 0.10$). For both models, term premia behave countercyclically over the business cycle, in line with the literature, see for example Cochrane and Piazzesi (2005).\(^{23}\) Moreover, the responses of the default-free term premia are more pronounced in the presence of credit frictions.

\(^{21}\)We show the results for de-trended levels here since the quantitative results are more stark compared to growth rates. The impulse responses for growth rates (not shown) exhibit a qualitatively similar amplification.

\(^{22}\)The quantitatively significant drop of default rates and credit spreads at the onset of a TFP-driven recession can be ascribed to the effect that unexpected inflation has on the real burden of corporate debt. The Fisher effect is at play here. As noted in the literature (Christiano, Motto and Rostagno (2014)), responses of macroeconomic variables to positive supply shocks can be dampened by the increase in the real burden of debt caused by the contemporaneous drop in inflation, the so-called ‘Fisher effect’. It turns out that for our calibration, this effect is not large enough to offset the amplification on consumption dynamics. See also section 5.3 for the role of long-term debt in amplifying debt-deflation risk.

\(^{23}\)A large body of literature documents the presence of countercyclical risk premia in financial markets at large. For additional evidence see for example Gilchrist and Zakrajsek (2011) for the U.S. corporate bond market, and Campbell and Cochrane (1999) for the equity market.
The fourth column removes sticky wages from the model without credit frictions. As wages are
allowed to adjust flexibly following shocks to the economy, households can optimize their hours worked
decision to smooth out their consumption stream. Using the labor margin to reduce consumption
risk naturally reduces the precautionary saving motives of households, raises the level of interest
rates and lowers the term premium of long-term bonds from 126 to 117 (see also Uhlig (2007) for a
similar argument in a model environment with habits in consumption). Although a higher degree of
wage and price rigidities (higher $\phi_w$ and/or $\phi_p$) can further increase the average term premium, we
find that such contribution is minor and comes at the expense of counterfactual inflation and wage
growth dynamics.\textsuperscript{24}

The final column reports the results using a specification with no credit frictions, with flexible
wages and where the (detrended) capital stock is fixed, as in Rudebusch and Swanson (2012). Ass-
suming that the capital stock is fixed has the obvious counterfactual implication that the volatility
of investment is zero. This also directly translates into a drop in the volatility of GDP growth. On
the other hand, since the representative agent cannot insure against underlying economic shocks
through investment activity, consumption and labor growth volatility must increase. In particular,
the relatively sharp increase of consumption growth volatility, which implies a larger market price of
risk, leads to a higher average term premium both in terms of nominal and real bonds. Interest rate
volatility across the yield curve increases broadly as well.

Overall, the comparison between the different models in table 6 suggests that a sizable term
premia in a model with fixed capital may come at the expense of unrealistic investment dynamics.
It is then useful to explore alternative channels which may help understand the large and volatile
term premia widely documented in the empirical finance literature. We confirm that sticky wages
can reduce the extent of self-insurance through the labor margin and increase risk premia, but can
generate implausible labor market dynamics.

5.2 Risk Shocks and Risk Premia

Figure 2 plots the impulse responses of selected variables to an increase in the standard deviation of
the distribution of idiosyncratic shocks, $z_{j,t}$.

Higher idiosyncratic risk in the economy raises the rate of default of entrepreneurs, increases
credit spreads, and makes investment drop on impact. Higher defaults, however, also increase the
rate at which physical capital depreciate, as described in section 2.9. The reduction in physical
capital that comes with the wave of defaults has two main consequences.

\textsuperscript{24}Based on our experience with the model calibration so far, it is hard to find a specification that can generate sizable
term premia with reasonable macroeconomic dynamics, without allowing for hours worked that are counterfactually
too volatile and wage growth that is instead too smooth with respect to the data. Relying on even stickier wages
would only exacerbate this problem, as already highlighted in section 5.1.2.
First, the larger rate of capital depreciation increases the marginal product of capital and creates an incentive for entrepreneurs to replenish their capital stocks, after the initial fall in investment. In anticipation of the increase in investment, households reduce their consumption on impact. The risk shock, coupled with the described countercyclical capital depreciation, are a source of credit risk premium in our baseline economy, as they lower consumption growth at times when corporate default losses are high. Similarly, the plots show that impulse responses of credit risk premia are large and countercyclical, accounting for around 40% of the total response of credit spreads.

Second, the higher marginal product of capital increases the marginal cost of intermediate good producers and hence has mild but persistent inflationary effects, after a rapid initial fall in inflation. Risk shocks, coupled with countercyclical capital depreciation can then also sustain the negative correlation between consumption growth and inflation that requires long-term Treasury bonds to pay sizable term premium in our model.

While aggregate investment tends to be a leading indicator in U.S. business cycles, the size of the rapid recovery in capital accumulation shown in our impulse responses may be hard to reconcile with empirical episodes of credit crunches. However, the assumption that the capital depreciation rate can be positively correlated with corporate defaults and that defaults could pose constraints to the productive capacity of the economy can be justified in light of empirical work on costly capital reallocation (see, for example, Remy and Shapiro (2001)). Moreover, the mild inflation that follows a risk shock in the model could help explain the ‘missing disinflation’ that the U.S. economy experienced during the Great Recession.

Our assumption also shares some qualitative similarities with disaster risk in capital depreciation with a time-varying probability adopted by Gourio (2013) and with the assumption of countercyclical recovery rates adopted by Swanson (2015) in a model without endogenous credit frictions and synthetic corporate bonds.

5.3 The Importance of Modeling Multiple Period Debt

A unique feature of our framework is that we model multiple-period debt contracts. This is in contrast to a bulk of the literature which introduces financial frictions into an otherwise standard macroeconomic model. In these models, debt contracts generally mature after one period, which in a typical calibration, corresponds to a quarter or possibly a year. To understand the implications of modeling defaultable debt with longer maturities, we compare our benchmark calibration in section 4.2 with one in which debt contract issued by the entrepreneurs matures after one period (quarter). This is achieved by setting \( \lambda = 1 \) while holding other parameters fixed at the baseline value.

Table 6 compares the credit variable moments for our baseline model with a 10-year credit contract \( (\lambda = 1/40) \) in the second column to a model with a 1-quarter contract \( (\lambda = 1) \) in the third column.
keeping all other model parameters fixed, and to a model with a 1-quarter contract that can closely match the magnitude of credit spreads of our baseline model specification ($\lambda = 1$ best fit), in the fourth column. The comparison between column two and three shows that reducing the average maturities without altering other model parameters reduces the extent of credit frictions greatly. Default probabilities on one-period contracts under the baseline distribution of idiosyncratic shocks are very low (0.02% per annum compared to around 2% per annum in the baseline and in the data), recovery rates are close to 100%, and average credit spread are just a few basis points over the risk free rate. The ergodic mean of leverage consequently increases to a share of debt over equity of 200% equilibrium. The volatility of financial variables is also much reduced compared to the baseline and to the data: the economy with one-period debt is close to a model without credit frictions.

Confronting columns two and four allows a side-to-side comparison between our baseline model and a model with one-period debt that can match the credit spread (i.e. the amount of credit frictions) in our baseline economy and in the data. The calibration in column four is achieved by setting $\omega = 0.47$, $1 - \xi = 0.151$ and $\tau = 0.29$, effectively raising the implied variance of the distribution of idiosyncratic shocks $z_{j,t}$, on top of setting $\lambda = 1$. In the best-fit short-term maturity model while average corporate spreads and the term premium are aligned with our baseline, the average default rate is around two times higher than in our baseline (at 4.2%), while the average recovery rate on corporate bonds is higher (around 87% of the face value of the bond, compared to 60% in our baseline). For a more direct comparison, we also report the average expected loss to a bondholder on a fictitious one-period bond with a face value of 1$ that is expected to default with probability $dp$ and to pay back $rcv$ in case of default. We express such loss in cents on a dollar. While in our baseline model, the expected loss on the 1$ bond is around 0.80 cents to produce a spread of 2.9%, the best-fit model with one-period contracts requires an expected loss of 1.35 cents to generate a lower spread of around 2.5%. The volatility of financial variables in the one-period best-fit model is also much higher than the multi-period model, due to the higher standard deviation of the idiosyncratic shocks implied by the calibration, while the volatility of macro variables (consumption growth and inflation, reported at the bottom of the table) and the average term premium are only slightly higher than in our baseline.\(^{25}\)

In the literature, it is somewhat conventional to rely on the one-period specification, and attempt to calibrate the model-implied credit spreads to data on credit spreads of longer maturities. However, we find that in order to calibrate the model to fit the observed long-term credit spreads with a one-period debt model, the underlying economy must exhibit a higher degree of risk. As we increase

\[^{25}\text{While ideally we would like to compare the average default rate across the two models, keeping average corporate spreads and recovery rates the same across the two specifications, this has proven hard to achieve due to the highly non-linear nature of the model solution and of the ergodic mean of financial variables, that appear to be very sensitive to changes in model parameters in the one-period version of the model.}\]
the variance of the idiosyncratic productivity shock to the entrepreneur to generate higher average credit spreads, we also incur in the unwanted consequence of increasing the default rates and average expected losses on the bonds, period by period. In this sense, allowing for multiple-period debt contracts is not only a more realistic representation of the aggregate credit market, but also improves the fit of the model.

This is evident, for example, by looking at the decomposition of credit spreads defined in section 3.2. In table 6 we report the average share of credit spreads attributable to credit risk premium implied by the two model specifications. In our model with multi-period debt we observe a ratio of around 40%, which is largely consistent with the literature on the U.S. corporate bond market.\footnote{For discussion on such evidence, see for example, Elton et al. (2002), Chen, Collin-Dufresne and Goldstein (2009), Gilchrist and Zakrajsek (2011), Gourio (2013).} In our model, defaults are countercyclical and positively correlated with inflation, so that bond investors require additional compensation for the purchase of corporate bonds. On the other hand, there is virtually no credit risk premium for the one-period specification obtained simply by setting $\lambda = 1$, while we are able to produce only a share of around 20% in the best-fit one-period debt economy, despite the increased volatility in idiosyncratic risk.

The intuition of why a model with one-period debt can only produce about half of the credit risk premia of a model with long-term debt can in part be ascribed to the different responses of the model economies to unexpected changes in inflation. Gomes, Jermann and Schmid (2016) describe how the presence of long-term debt in a model similar to ours implies that unexpected shocks to inflation can have large sizable effects on the economy, by changing the real value of debt denominated in nominal terms (the so-called Fisher effect). We confirm their finding, as shown in figure 6, that depict the response to the baseline and the (best fit) short-term debt economies to a one-standard deviation shock to the inflation target. In both economies the inflation rate increases on impact by around 10 basis points. In the model with long-term debt (the blue lines), the unexpected and persistent increase in inflation lowers the real value of the stock of long-term corporate bonds, reducing the leverage ratio, corporate defaults and credit spreads, while recovery rates increase. Unexpected upticks in inflation boost investment and output, while the reduced expected losses on corporate bonds allow consumption to rise. Unexpected inflation makes corporate bonds less risky and lowers credit risk premia, contributing to about 25% of the total decrease in corporate spreads in this simulation, albeit just by one basis point in absolute terms.

In particular, in a model with long-term debt, shocks that generate a positive comovement between inflation dynamics and consumption growth can potentially make corporate bonds more risky by means of the Fisher effect. An economic downturn accompanied by deflation can be amplified by a wave of defaults caused by a higher debt burden. Investors will price deflation risk in corporate bond spreads and premia will rise. Viceversa, shocks that generate a negative comovement between...
inflation dynamics and consumption growth, such as TFP shocks, will produce recessions in which inflation that can mitigate corporate default losses and reduce credit risks.\footnote{See Kang and Pflueger (2015) for cross-country empirical evidence on the effect of inflation volatility and cyclicality on corporate bond spreads.}

By comparison, in an economy with short-term contracts (the red lines in figure 6), optimal leverage decisions are renegotiated period by period and unexpected inflation does not reduce the real burden of entrepreneur’s liabilities on impact. The impulse responses show that, raising the inflation target will create an incentive for households to command entrepreneur to increase investment, by means of leverage, expecting a future rise in consumption. Default rates rise slightly, and recovery rates fall at first and then overshoot their ergodic mean. Credit spreads go up slightly as a result of extra borrowing, and fall below their ergodic mean, as the economy picks up. Debt deflation risk is absent from this economy and consequently the share of corporate spreads attributable to credit risk premia is lower than in our baseline, as shown in table 6.

5.4 Monetary Policy and Risk Premia

5.4.1 Unexpected Monetary Policy Shocks

Figure 4 shows the impulse responses of the model to a one-standard-deviation positive monetary policy shock. The shock increases the short-term nominal interest rate, $i$, by around 10 basis points. The interest rate then converges back to its ergodic mean fairly quickly. Tighter monetary policy translates into higher nominal interest rates at all maturities, $i$ and $i^{(40)}$, the impact being lower for yields of longer maturity. Real rates (not shown) also rise. As a result, consumption, investment and output drop while inflation falls, as in a typical demand shock. In credit markets, lower demand and weaker asset prices increases the leverage ratio, the default rates, the corporate bond spread, and credit risk premia. On the wake of a positive monetary policy shock, credit risk premia and nominal term premia increase but their responses are one order of magnitude smaller than the change in the credit spread and the 10-year Treasury yield respectively.

5.4.2 Systematic Monetary Policy

Empirical work by Gertler and Karadi (2015) suggests that changes in long-term rates and credit spreads following monetary policy surprises can be fully attributed to changes in term premia and credit risk premia. On the other hand, Nakamura and Jon (n.d.) suggest that changes in long-term rates that follow unexpected monetary policy shocks can be attributed in large part to changes in the expected path of the short term rate.

In our model, monetary policy shocks have limited effect on long-term rates and credit spreads, and their responses can be almost fully attributed to changes in the expected path of the short
term rate and expected default losses, not to variation in risk premia. This is not surprising, since unexpected shocks to the monetary policy rule do not have persistent effects on the economy and would not be priced by households with preferences sensitive to long-run risk.

Our findings suggest, however, that the systematic component of monetary policy affects both the size and the volatility of term and credit risk premia. To show this, we compute the ergodic mean of term premia and credit risk premia as a function of the coefficients on inflation and output fluctuations, $\phi_\pi$ and $\phi_y$, in the Taylor rule (2.8). Figure 7 shows the ergodic mean of the term premium (left) and of the credit risk premium (right) as a function of different pairs of coefficients of the central bank’s monetary policy rule. The average term premium on 10-year bonds (left-side panel) is decreasing in the Taylor rule response coefficient to inflation fluctuations $\phi_\pi$, while it is increasing in the response coefficient to output fluctuations, $\phi_y$. The opposite result holds instead for the average credit risk premium, on the right-side panel. Changes in average premia closely mirror the behavior of the correlations between consumption growth, inflation and the recovery rate on corporate bonds in the economy, in figure 8.

A central bank that responds more to inflation fluctuations than to changes in output will affect the correlation between inflation and consumption growth, on top of reducing uncertainty around expected inflation. Both forces reduce the nominal term premium in the economy. On the other hand, a greater focus on output fluctuations, rather than inflation fluctuations, reduces the extent of credit cycles and lowers the unconditional correlation between consumption growth and expected losses on corporate bonds, compressing the credit risk premium share of corporate spreads.

To conclude, figures 9 and 10 show the effect of increasing the Taylor rule coefficient on inflation $\phi_\pi$ from 0.5 to 1 on the impulse responses to a one-standard deviation negative TFP shock and to a one standard-deviation negative risk shock.

While qualitatively similar, the plots show that, following a TFP shock, a higher response coefficient on inflation stabilization compresses the volatility of both term premia and credit risk premia. A stronger monetary policy response to inflation not only reduces the correlation between inflation and consumption growth, but also lowers the volatility of the response in inflation, and hence reduces the Fisher effect that can relieve entrepreneurs of some of the burden of their stock of outstanding debt. The amount of unexpected inflation brought by the TFP shock will be smaller in the high $\phi_\pi$ economy (yellow lines), relative to our baseline (blue lines). Unexpected inflation will still reduce the real burden of corporate debt and compress spreads and the credit risk premium, but to a smaller extent than in our baseline economy. TFP risk commands a negative credit risk premium in our baseline, due to the sizable Fisher effect that makes corporate a hedge during TFP recessions. However, a stronger monetary policy reaction to inflation can make corporate bonds more risky and increase the average magnitude of risk compensation that investors require to bear TFP risk.

A similar story holds in the case of risk shocks, during which a stronger monetary policy response
to inflation will increase debt-deflation risk and increase the average and volatility of credit risk premium.

6 Conclusions

We model a Neo-Keynesian economy with Epstein-Zin preferences, financial frictions and long-term nominal defaultable debt. We calibrate the model and solve it using higher-order perturbation methods. We show that credit frictions can significantly increase the size and volatility of the nominal and real Treasury term premium through the interaction of preferences sensitive to long-run risk, and amplification of the economy’s response to TFP shocks. We also find that introducing multi-period debt contracts instead of one-period debt in DSGE models helps fit the cyclical properties of macroeconomic variables, credit variables together with the main features of the term structure.
References


Moody’s Analytics Default & Recovery Database. 2016. “Moody’s Analytics Default & Recovery Database.”

Nakamura, Emi, and Steinsson Jon. n.d..


Swanson, Eric. 2015. “A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt.”


## Appendix - Tables and Figures

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<thead>
<tr>
<th>Parameters</th>
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<tr>
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<td>invest. adj.</td>
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</tr>
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<th>Value</th>
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<tr>
<td>$1 - \xi$</td>
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<td>Taylor inflation</td>
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<td>$g_y$</td>
<td>Taylor output</td>
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<td>$\rho_a$</td>
<td>TFP shock persist.</td>
<td>0.98</td>
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<td>0.995</td>
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<td>lr $\pi$ shock to dev $\pi_{ss}$</td>
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<td>lr $\pi$ stdev.</td>
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<td>$\sigma_\sigma$</td>
<td>risk shock stdev.</td>
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Table 1: **Calibrated Model Parameters:** This table contains the calibrated parameter values used to generate our results for the baseline specification.
Figure 1: Impulse responses to a positive TFP shock (credit variables): GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread, Credit Risk Premium.
Figure 2: **Impulse responses to a positive TFP shock (credit variables):** GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread, Credit Risk Premium.
Figure 3: **Impulse responses to a negative TFP shock (selected variables):** The blue line shows impulse responses for model AT, while the red line shows response for the model with a stronger tax incentive on debt, $\tau$ (higher credit frictions).
Figure 4: **IRs to a Monetary Policy Shock, Selected Variables:** GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread, Credit Risk Premium.
Figure 5: **IRs to a Monetary Policy Shock, Selected Variables:** GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread, Credit Risk Premium.
Impulse Responses to a positive $\pi^* \uparrow$ shock - Baseline vs. Short Term Debt

Figure 6: IRFs to a Inflation Target Shock in baseline vs. one-period contract (best-fit) model. Selected Variables: GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread (10 year or 1 year), Credit Risk Premium (1 year, 10 year)
Figure 7: Term (left) and Credit Risk (right) Premia as a function of the coefficients in the Taylor Rule. x-axis = 1+φ_π, y-axis = φ_y.
Figure 8: Unconditional correlations between consumption growth and 1) Inflation (left) and 2) Bond Recovery Rates (right) as a function of the coefficients in the Taylor Rule. x-axis = 1+\( \phi_\pi \), y-axis = \( \phi_y \).
Figure 9: IRs to a TFP Shock, baseline vs. high $\phi_\pi$, Selected Variables: GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread, Credit Risk Premium.
Impulse Responses to a Negative RISK shock - Baseline vs. High $\phi_\pi$

Figure 10: IRs to a risk Shock, baseline vs. high $\phi_\pi$, Selected Variables: GDP, Investment, Consumption, Inflation, Federal Funds Rate, 10-year Treasury bond, 10-year Term Premium, Default Probability, Recovery Rate, Leverage, Credit Spread, Credit Risk Premium.
Table 2: **Unconditional Variance Decomposition**: This table reports the unconditional variance decomposition of selected model variables. Values are in percentages.

<table>
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<th>Variable</th>
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<th>MP</th>
<th>Risk</th>
<th>IT</th>
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<td>$\Delta c$</td>
<td>64.77</td>
<td>0.11</td>
<td>35.03</td>
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<td>$\Delta i$</td>
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<td>19.65</td>
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<td>$\Delta \tilde{y}$</td>
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<td>12.41</td>
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<td>$\Delta w$</td>
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<td>1.89</td>
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<td>$dp^4$</td>
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<td>0.00</td>
<td>98.59</td>
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### Table 3: Macro Moments

This table contains summary statistics for macro variables. It reports mean and standard deviation of consumption (non-durable, durable, and services), investment, GDP, hours worked and real wage growth, and of the inflation rate (total CPI for all Urban consumers) from the USECON database in Haver. Variable labels are respectively $\Delta c$, $\Delta i$, $\Delta \bar{y}$, $\Delta l$, $\Delta w$, $\pi$. Mean and standard deviation of the total PCE index are respectively 2.8% and 1.6%. The table also reports the auto-correlations of these variables, as well as selected cross-correlations.

<table>
<thead>
<tr>
<th>Moments</th>
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<th>Model</th>
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<td>$\sigma [\Delta i]$</td>
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<td>$\sigma [\Delta \bar{y}]$</td>
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<td>$\sigma [l]$</td>
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<td>$\sigma [\pi]$</td>
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<td>$\rho [\Delta \bar{y}, \Delta \bar{y}_{-1}]$</td>
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<td>$\rho [\Delta w, \Delta w_{-1}]$</td>
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<td>0.73</td>
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<tr>
<td>$\rho [\Delta i, \Delta \bar{y}]$</td>
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<td>0.93</td>
</tr>
<tr>
<td>$\rho [l, \Delta \bar{y}]$</td>
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<td>$\rho [\pi, \Delta l]$</td>
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<tr>
<td>$\sigma \left[ ntp^{(40)} \right]$</td>
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Table 4: **Term Structure Moments**: This table contains summary statistics for the nominal and real term structure of interest rates. It reports data and model-implied mean and standard deviation of short- and long-term nominal and real interest rates $i^{(1)}$, $i^{(40)}$, and $r^{(1)}$, $r^{(40)}$. The table also reports term structure model estimates of mean and standard deviation of nominal and real term premia (numbers in parentheses), $ntp^{(40)}$ and $rtp^{(40)}$ and compares them to predictions from our model.
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<td>$\mu[dp^4]$</td>
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<td>2.91%</td>
</tr>
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<td>–</td>
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<tr>
<td>$\mu[lv]$</td>
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<td>62.66%</td>
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<tr>
<td>$\mu[rcv]$</td>
<td>$\approx$ 60%$^1$</td>
<td>66.40%</td>
</tr>
<tr>
<td><strong>Standard Deviations</strong></td>
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<td>$\sigma[cs_{HY}]$</td>
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<tr>
<td>$\sigma[crp^{HY}]$</td>
<td>0.47%$^3$</td>
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<tr>
<td>$\sigma[lv]$</td>
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<td>23.56%</td>
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<td>$\rho[lv, \Delta \tilde{y}]$</td>
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Table 5: **Credit Variable Moments**: This table contains summary statistics for the credit variables in the model. It reports data and model-implied mean, standard deviation and correlations with GDP growth, $\tilde{y}$, of default rates, $dp^4$, credit spreads for investment grade (IG) and high yield (HY) bonds, $cs^{(40)}$, leverage ratio, $lv$ computed as credit market debt as a percentage of the market value of corporate equities, and recovery rates. 1) Moodys Annual Reports, 2) Elton et al. (2001), 3) Gilchrist and Zakrajsek (2012) EBP data, 4) Gomes and Schmidt (2010) (sample: 1951-2009).
### Table 6: Comparison across different model specifications

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<th>Fix. K</th>
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<td>$\sigma[\Delta \bar{y}]$</td>
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<td>1.95</td>
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<td>$\sigma[\bar{y}]$</td>
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<td>0.31</td>
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<td>2.09</td>
<td>2.36</td>
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<td>$\sigma[l]$</td>
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<td>2.33</td>
<td>1.85</td>
<td>1.81</td>
<td>2.00</td>
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<td><strong>Term Structure</strong></td>
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<td>$\mu[ntp^{(40)}]$</td>
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<td>1.26</td>
<td>1.17</td>
<td>2.67</td>
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<tr>
<td>$\mu[rtp^{(40)}]$</td>
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<td>0.56</td>
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<tr>
<td>$\sigma[i^{(1)}]$</td>
<td>2.58</td>
<td>2.64</td>
<td>2.39</td>
<td>2.34</td>
<td>2.89</td>
</tr>
<tr>
<td>$\sigma[i^{(40)}]$</td>
<td>2.41</td>
<td>2.42</td>
<td>2.21</td>
<td>2.18</td>
<td>2.12</td>
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<tr>
<td>$\sigma[ntp^{(40)}]$</td>
<td>1.43</td>
<td>0.31</td>
<td>0.18</td>
<td>0.04</td>
<td>0.08</td>
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<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*parenthesis indicates estimates

This table compares relevant first and second order moments of both the macroeconomic and term structure variables in the data with four model specifications (1) Our model (AT), with credit frictions, sticky wages and flexible capital, (2) AT without credit frictions (Flex. K+Sw), (3) AT without credit frictions and sticky wages (Flex. K), and (4) AT without credit frictions and sticky wages, but with firm-specific fixed capital (Fix. K).
Table 7: Credit Variable Moments - Comparison of long and short maturities: This table contains summary statistics for the credit variables in the baseline model and compares them to a model with debt maturity equal to one quarter, keeping all other parameters fixed to the baseline, and to a model with one-period debt and modified credit friction parameters that closely matches credit spreads in the baseline. It reports model-implied means of annual default rates, $dp^4$, credit spreads, $cs$, leverage ratio, $lv$, recovery rate $rcv$, and credit risk premium as a share of spreads, $crp^{(40)}/cs^{(40)}$, and term premium $tp$, when debt maturity is set to 10 years, $\lambda = \frac{1}{40}$, and one quarter, $\lambda = 1$. The table also reports the model-implied expected loss per dollar of debt, computed as $dp^4 \times (1 - rcv)$.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Long Mat. $\lambda = \frac{1}{40}$</th>
<th>Short Mat. $\lambda = 1$</th>
<th>Short Mat. (best fit) $\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu[cs]$</td>
<td>2.91%</td>
<td>0.014%</td>
<td>2.52%</td>
</tr>
<tr>
<td>$\mu[dp^4]$</td>
<td>2.25%</td>
<td>0.02%</td>
<td>4.2%</td>
</tr>
<tr>
<td>$\mu[lv]$</td>
<td>60%</td>
<td>206%</td>
<td>55%</td>
</tr>
<tr>
<td>$\mu[rcv]$</td>
<td>66.40%</td>
<td>97%</td>
<td>87.45%</td>
</tr>
<tr>
<td>Standard Deviations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma[cs]$</td>
<td>2.26%</td>
<td>0.01%</td>
<td>4.11%</td>
</tr>
<tr>
<td>$\sigma[dp^4]$</td>
<td>1.56%</td>
<td>0.02%</td>
<td>13.72%</td>
</tr>
<tr>
<td>$\sigma[rcv]$</td>
<td>3.34%</td>
<td>0.98%</td>
<td>6.41%</td>
</tr>
<tr>
<td>Expected Loss per $1$ of Debt</td>
<td>$\mu[dp^4 \times (1 - rcv)]$</td>
<td>0.80 \text{¢}</td>
<td>0 \text{¢}</td>
</tr>
<tr>
<td>Credit Risk Premium as Ratio of Spread</td>
<td>$\mu[crp^{(40)}/cs^{(40)}]$</td>
<td>38.10%</td>
<td>0%</td>
</tr>
<tr>
<td>Term Premium and Macro Volatility</td>
<td>$\mu[tp^{(40)}]$</td>
<td>1.77%</td>
<td>1.21%</td>
</tr>
<tr>
<td>$\sigma[\Delta c]$</td>
<td>0.42</td>
<td>0.33</td>
<td>0.60</td>
</tr>
<tr>
<td>$\sigma[\pi]$</td>
<td>2.27</td>
<td>1.95</td>
<td>2.27</td>
</tr>
</tbody>
</table>