Contagion in the CDS Market*

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Abstract

This paper analyzes counterparty exposures in the credit default swaps market and examines the impact of severe credit shocks on the demand for variation margin, which are the payments that counterparties make to offset price changes. We employ the Federal Reserve's Comprehensive Capital Analysis and Review (CCAR) shocks and estimate their impact on the value of CDS contracts and the variation margin owed. Large and sudden demands for variation margin may exceed a firm's ability to pay, leading some firms to delay or forego payments. These shortfalls can become amplified through the network of exposures. Of particular importance in cleared markets is the potential impact on the central counterparty clearing house. Although a central node according to conventional measures of network centrality, the CCP contributes less to contagion than do several peripheral firms that are large net sellers of CDS protection. During a credit shock these firms can suffer large shortfalls that lead to further shortfalls for their counterparties, amplifying the initial shock.

Keywords: Credit default swaps, stress testing, systemic risk, financial networks
JEL Classification Numbers: D85, G01, G17, L14

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1 Introduction

Credit default swaps are contracts that serve to manage credit risk. In exchange for a premium, the buyer of a CDS receives compensation from the seller upon the occurrence of a credit event that affects the value of the contract's underlying reference obligation. The reference obligation is usually a debt security issued by a corporation or sovereign government. The contract specifies the credit events that will trigger payment from the seller to the buyer, such as failure to pay, bankruptcy, and restructuring (IOSCO (2012)). A single name CDS provides protection on credit events of a corporate or sovereign entity, while an index CDS provides protection against credit events in a portfolio of single name CDS contracts.[1]

The financial crisis highlighted the potential risks posed by the credit default swaps market. As the crisis unfolded, insurance companies such as American International Group, Inc. (AIG) became liable for payments on the CDS contracts they had previously sold to banks and dealers as protection against credit defaults. Until the crisis, these protection sellers received a steady stream of payments from protection buyers and they rarely had to pay out claims. When the crisis hit, the sudden calls for collateral put great pressure on the sellers, which traditionally had a thin capital base due to their AAA-rated status. In particular, AIG had to be rescued by the U.S. Department of the Treasury in order to meet its margin obligations on CDS contracts to dealers[2], who in turn were threatened if the payments were not forthcoming.[3] This incident throws into high relief the potential for contagion that the CDS market poses for the financial system.

In this paper we analyze this contagion potential using detailed data provided to the Office of Financial Research (OFR) by the Depository Trust Clearing Corporation (DTCC). The data include all CDS transactions in which at least one of the counterparties or the reference entity is a U.S. entity. The data provide a detailed picture of counterparty exposures for a large segment of the CDS market, including exposures between banks, dealers, hedge funds, asset managers, and insurance companies. We are able to compute the value of the payment and premium legs of each CDS position as a function of spread, duration, and underlying reference entity, including both

[1] Markit (2012) provides a description of the effect of credit events on the cash flows and composition of credit default swap indices.

[2] see Figure 20.4 on p 377 of **?**

[3] The risk of nonpayment was exacerbated by the unwillingness of some dealers to face AIG, evidenced through their refusal of novation requests (Burrough (2008)). A novation is the transfer of a party's exposure from one counterparty to another.

single-name and reference indices, under a shock to the reference entities. We focus in particular on the change in value of each contract, and the resulting variation margin (VM) payment owed to each counterparty, under of the Federal Reserve's 2015 Comprehensive Capital Analysis and Review (CCAR) shock.[4] This shock was designed to test the robustness of the financial system under a large and sudden change in asset values. In the present context the shock implies a sudden decrease in the value of corporate and sovereign debt, which results in large demands for VM in the CDS market. Of particular interest is the VM owed by (and to) the 26 members of the CDS clearing house ICE Clear Credit, the principal central counterparty clearing house (CCP) in this market. These include many of the large US banks and broker-dealers as well as a number of foreign entities.[5]

VM payments must be made within a very short time frame – typically just a few hours. If the VM owed by a given firm exceeds the amount it is owed, the firm may experience short-term stress. This stress can be relieved by drawing on cash and cash equivalents held by the parent institution, but if the stress is large, those sources may prove to be inadequate. In that case, the firm may either delay payments or default completely, thus exacerbating the stress on its counterparties, which may lead to system-wide contagion. We develop a general model that allows us to estimate the extent to which these network exposures amplify the initial stress under a range of assumptions about how firms behave when they are subjected to stress.

The model also allows us to estimate the extent to which each individual firm contributes to stress amplification. It turns out that conventional notions of centrality do not capture the extent to which different firms contribute to contagion, a point emphasized by Glasserman and Young (2015; 2016). In particular, although the CCP is central according to various measures, such as eigenvector centrality, it contributes significantly less to contagion than do several of its members and nonmember firms. One might have thought that, since the CCP faces many firms, its role in contagion would be larger than either CCP members (dealers) or nonmembers (such as hedge funds and asset managers) that have positions with many fewer participants. The reason that

[4]Variation margin is the payment that a derivative counterparty makes to (or receives from) another in the event of loss (or gain) of the position. In the CDS market, variation margin is generally exchanged on a daily mark-to-market basis.

[5]The CME Group (CME) also acts as a CCP for some CDS contracts based on indices but it does not report its positions to the DTCC. This gap in the data does not pose a particular problem for our study because the CME represents a much smaller proportion of the CDS market than does ICE. As of March 2016, the CME cleared \$1.4 trillion in gross notional whereas ICE cleared \$52 trillion.

some member and nonmember firms contribute more than the CCP to contagion is that they are large net sellers of CDS protection, and under a credit shock they owe much more VM than they are owed. A shortfall in payments by these firms can cascade into the network of broker-dealers, causing some of them to suffer payment shortfalls. Under some scenarios, the contagion could bring down the CCP.

2 Related Literature

The financial crisis of 2007-09 has sparked a rapidly growing literature, both theoretical and empirical, on financial networks and systemic risk. Some of this work focuses specifically on the network structure of CDS markets. The potential for contagion in CDS markets was highlighted by Cont (2010), who emphasized the importance of adequate liquid reserves to cope with large and sudden demand for variation margin. This paper also analyzed the extent to which a CCP can mitigate contagion, a topic that was subsequently treated by Duffie and Zhu (2011), Cont and Kokholm (2014) and Cont and Minca (2016). Various authors have studied the structure of CDS exposures and the potential for contagion among European banks; see in particular Brunnermeier et al. (2013), Peltonen et al. (2014), Vuillemey and Peltonen (2015), and Clerc et al. (2013). Cont and Minca (2016) analyze the combined network of exposures in the CDS and interest rate swap markets together, and argue that central clearing in both markets can significantly reduce the probability and magnitude of illiquidity spirals.

Their work differs from the present paper in the methodologies used to study contagion, and in the focus on the European rather than the U.S. market. More recently, Ali et al. (2016) examine the network structure of the CDS market in the United Kingdom. These authors argue, as we do here, that the systemic importance of market participants is not fully captured by conventional measures of centrality; the size and structure of financial firms' balance sheets is crucial to understanding how much risk they pose to the system as a whole.

There has been less research on the CDS market in the United States, mainly due to limited data access. To get around this difficulty, Markose et al. (2012) attempt to reconstruct CDS bilateral positions and systemically important institutions using publicly reported balance sheet data. Markose (2012) and the BIS Macroeconomic Assessment Group on Derivatives (2013) employ similar techniques to reconstruct exposures in the derivatives market more generally. Our paper differs in that we are able to map the network of CDS exposures in the United States using detailed DTCC data as of a specific date (October 3, 2014). We do not look merely at notional exposures between participants as in the work mentioned above. Instead, we impose a specific shock constructed for the purpose of stress testing (the 2015 CCAR shock) and compute the resulting variation margin payment obligations between participants. We then analyze how much stress these demands create as they propagate through the network.[6]

Our results fit into a broader literature that relates network structure to the risk of contagion. A central theme of this literature is the extent to which interconnectedness increases or decreases systemic risk. Network connections can have a positive effect by diversifying the risk exposures of individual market participants, but they can also have a negative effect by creating channels through which shocks can spread. The tension between these two forces has been explored in a variety of papers: see among others Allen and Gale (2000); Freixas et al. (2000); Gai and Kapadia (2010); Gai et al. (2011); Haldane and May (2011); Blume et al. (2011); Cont et al. (2013); Elliott et al. (2014); Acemoglu et al. (2015). For general surveys of the literature see Bisias et al. (2012) and Glasserman and Young (2016).

Here we demonstrate, following Glasserman and Young (2015), that abstract measures of network structure, such as degree distribution and eigenvector centrality, are insufficient to identify the systemically important actors. Firm characteristics, choices, and behavior interact with the network structure of exposures to produce contagion. We illustrate this point concretely in the CDS market by showing that the most important contributors to contagion under stress are not the CCP or its members, but certain firms that are peripheral to the network but are very large and have highly unbalanced CDS positions. Under severe stress in the credit markets, the mismatch between incoming and outgoing payment demands at these firms may trigger contagion in the system as a whole, which can put severe stress on the CCP.

[6] A recent paper of Cetina et al. (2016) also uses DTCC data to study the effects of a CCAR shock, but it focuses on the direct impact of the shock on particular firms rather than on network spillover effects. Other papers have more broadly consider the systemic risk of large market participants on pricing and liquidity (Siriwardane (2015a), Shachar (2012)) and the potential pathways that CDS have in transmission risk (Gordy (2012)).

3 VM Payment Obligations Induced by the 2015 CCAR Shock

The main source of data for this study was provided to the OFR by DTCC at a weekly frequency. It provides both transactions and positions on all standardized and confirmed CDS involving U.S. entities since 2010. Positions represent the accumulation of all past transactions that contribute to a swap between counterparties. They include detailed information on the underlying reference entities, the notional amount bought and sold, the inception and termination dates, and other terms of the contract. We also use data from Markit to estimate single-name credit spreads for all reference entities in the positions we observe.

The Federal Reserve's 2015 CCAR shock prescribes a sudden widening of credit spreads for corporate, state, municipal, and sovereign debt according to their rating class (see Table 1). This widening changes the value of the premium and payment legs of the CDS contracts that reference these entities and these changes induce variation margin (VM) payment obligations between the counterparties. The methodology for estimating the VM payments is described in the Appendix; here we provide an overview of the results.

Table 1: The Impact of 2015 CCAR Severely Adverse Market Shocks on Credit Default Swaps

Corporate Credit							
Advanced Economies							
	AAA	AA	A	BBB	BB	B	<B or Not Rated
Spread Widening (%)	130.0	133.0	110.2	201.7	269.0	265.1	265.1
Emerging Markets							
	AAA	AA	A	BBB	BB	B	<B or Not Rated
Spread Widening (%)	191.6	217.2	242.8	277.5	401.9	436.4	465.8

State & Municipal Credit							
	AAA	AA	A	BBB	BB	B	<B or Not Rated
Spread Widening (bps)	12	17	37	158	236	315	393

Sovereign & Supra Credit
See FRB worksheet: CCAR-2015-Severely-Adverse-Market-Shocks-data.xlsx

Source: Federal Reserve Board (FRB (2016))

Figure 1 shows the network of payment obligations between the CCP and its 26 members.

It does not include indirect exposures between the CCP and nonmember clients, which will be considered below. The arrows show the direction of payment, and the width of the arrows indicate the relative size of the payments. The absolute amount of these payments are summarized in Table 2, where we have aggregated the members into groups of five or six to maintain anonymity. For example, the group A-E owes \$2.92 billion in VM and is owed \$1.29 billion. The difference (\$1.63 billion) must be covered by liquid assets held in the firms' treasuries or else they will default on their payments and the underlying contracts will be closed out.[7] The difference between the amount of VM that a firm owes and the amount that it is owed (if the difference is positive) is the *initial stress* induced by the shock.

Table 2: Variation Margin Payments and Initial Stress for CCP Member Firms (millions of dollars)

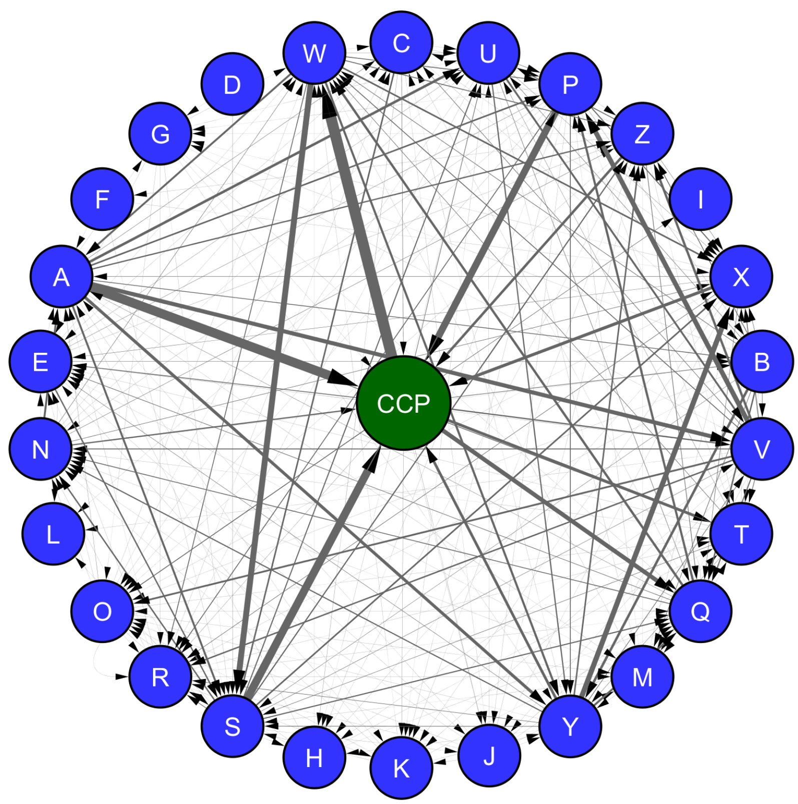
Firms	Variation Margin Owed By	Variation Margin Owed To	Initial Stress
A-E	2,917	1,287	1,630
F-J	35	46	8
K-O	439	571	27
P-T	5,135	4,399	1131
U-Z	7,614	8,449	532
CCP	8,602	8,602	-

Note: Initial stress is the difference between the VM a firm owes and the amount that it is owed, provided the difference is positive; otherwise the stress is zero. Firms are arranged in groups of five or six to maintain anonymity. Within each group some firms are under stress, hence the total stress for each group is positive.
Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

Thus far we have considered only payments between the CCP and its members, and between the members themselves. In addition, there are many nonmember firms that have positions directly with members, as well as positions with the CCP that are guaranteed by members. There are over 900 such firms, including a wide variety of hedge funds, asset managers, and insurance companies. Figure 2 provides an overview of how a sample of these nonmembers are connected to the core members and the CCP. The key features of this network are : (1) nonmembers tend to owe members rather than each other; (2) there is a high degree of heterogeneity among the nonmembers in terms

[7]Note that the VM owed by the CCP is the same as the VM owed to it; this is a consequence of the CCP's matched book. These amounts cannot be netted out, because they arise from contracts with different counterparties.

Figure 1: Variation Margin Payment Network after 2015 CCAR Shock

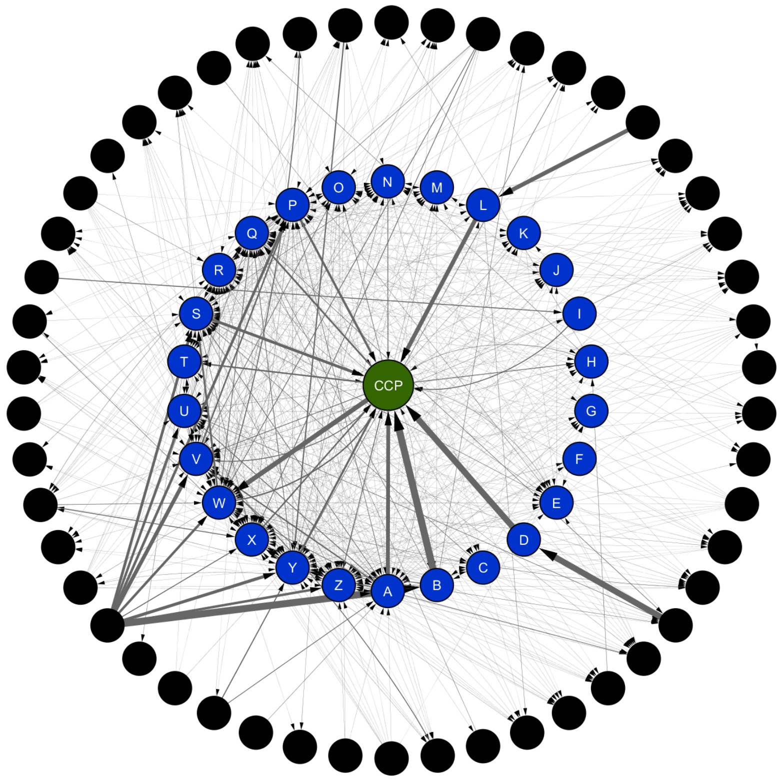


Note: The network diagram plots the central counterparty clearing house (CCP) (in green) and its members labeled A thru Z (in blue). The width of the arrows indicates the relative size of the net VM payment owed bilaterally between the counterparties.
Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

of VM owed; (3) some nonmembers experience substantially greater initial stress than any of the members. The latter point is highlighted in Table 3, which provides a summary of the incoming and outgoing VM balances for the top 26 nonmember firms, ordered by the size of the initial stress. (The results are aggregated in groups of five in order to preserve anonymity.)

Our premise is that higher levels of stress increase the likelihood that a firm does not make its VM payments in a timely way, or pays with illiquid assets whose values are not fully discounted, or does not pay at all. These payment deficiencies increase the stress on the firms' downstream counterparties, possibly leading them to reduce their payments too. The upshot is *network contagion*. In the following sections we propose a mathematical model that allows us to estimate the

Figure 2: Variation Margin Payment Network after 2015 CCAR Shock for CCP Members and a Subsample of Nonmember firms



Note: The network diagram plots the central counterparty clearing house (CCP) (in green), CCP members labeled A thru Z (in blue), and a sample of CCP nonmembers (in black). The width of each arrow indicates the relative size of the net VM payment owed bilaterally between counterparties.
Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

system-wide deficiency in payments that results from network contagion under different parametric assumptions about the firms' responses under stress. The model also allows an assessment of the extent to which individual firms contribute to contagion. A key finding is that some nonmember firms contribute much more to contagion than do any of the member firms or the CCP itself.

Table 3: Top 26 Nonmember Firms Ordered by Initial Stress (millions of dollars)

Firms	Variation Margin Owed By	Variation Margin Owed To	Initial Stress
I-V	10,296	830	9,466
VI-X	2,707	1,771	936
XI-XV	457	77	380
XVI-XX	1,254	967	287
XXI-XXVI	395	92	303

Note: Each row gives the variation margin owed by and to a given set of firms, which are ordered by size of initial stress. Firms are arranged in groups of five or six to maintain anonymity.
Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

4 Initial Margins

To conduct our analysis we need to specify the initial margins collected by the counterparties in CDS transactions. The role of initial margin (IM) is to cover deficiencies in VM payments by one's counterparty, including the cost of closing out the position in case of default. Initial margins collected from counterparties are held in segregated accounts and can only be used to cover losses induced by the counterparty's failure to pay. A portion of the IM is typically held in cash or cash equivalents, and the remainder is held in assets that can be liquidated on short notice but not necessarily at full value. The proportions held in cash and other assets are largely at the discretion of the collecting firm and thus affect its ability to cover deficiencies in incoming payments. In the remainder of the paper, we shall assume that IM is held entirely in cash equivalents, but it should be noted that this assumption may *underestimate* the potential for contagion.

It must be emphasized that the IM collected by a firm is *separate* from the liquidity buffer that it maintains to meet its *own* VM obligations. IM can only cover shortfalls in the incoming payments due from counterparties. The liquidity buffers of most CDS market participants are not publicly reported, and even if they were, the buffers can be tapped by other (non-CDS) activities within the firm that will probably be under stress at the same time.[8] For this reason we shall not attempt to estimate the size of the buffers at different firms. Instead, we shall treat the ability

[8] An exception is the CCP, which does report the size of the buffer that it can draw upon when the IM is depleted. This buffer consists mainly of the guarantee fund, which contained assets worth \$1.6 billion at the time of the study (ICE (2016a)).

of a firm to meet short-term payment demands as a parameter and explore the behavior of the system under different parametric settings. We shall consider two regimes regarding the payment of initial margin (IM). Up until September 2016, some types of bilateral CDS transactions did not require the posting of IM. In particular, dealers were not required to post IM with any other firms (except for the CCP), and commercial banks were required to post IM with other dealers but not with anyone else (except for the CCP). Other firms, such as asset managers and hedge funds, did not need to post IM in bilateral transactions with each other, but they did post IM in transactions with dealers, commercial banks, and the CCP (see Table 4).

Table 4: Initial Margin Payment Matrix

		Receiver				
	Participant:	CCP	Dealer	Commercial Bank	HF/Asset Manager	Other
Payer	CCP	No	No	No	No	No
	Dealer	Yes	No	No	No	No
	Commercial Bank	Yes	Yes	No	No	No
	HF/Asset Manager	Yes	Yes	Yes	No	No
	Other	Yes	Yes	Yes	No	No

Source: Authors' creation.

Beginning in September 2016, all counterparties to CDS transactions must post IM according to criteria established by the Basel Committee on Banking Supervision and Board of the International Organization of Securities Commissions (2015). The objective of the new policy is to create incentives for firms to clear CDS trades with a CCP rather than bilaterally.[9] In the context of this study, the effect is to increase the amount of IM in the system and to reduce the potential vulnerability of the system to contagion. In Section 6.3, we shall quantify this effect by comparing the propagation of losses under the two regimes.

To determine the amount of IM posted (where IM is required) we adopt a conventional portfolio-at-risk measure, namely a 99.5 percent VaR with a 10-day margin period of risk (BCBS and IOSCO (2015)). Specifically, we compute the change in value of a given portfolio over every 10-day window in the preceding 1,000 trading days, and the level of IM is set so that 99.5 percent of these changes are less than this amount. In contrast, this CCP (ICE Clear Credit) uses a

[9] As shown by Ghamami and Glasserman (2016), however, the policy may not provide a sufficient incentive to clear trades through the CCP instead of bilaterally.

10-day standard that is augmented by various upward adjustments (the details of which are not published). In this case, we estimate the IM that would be required to meet a 10-day 99.5 percent VaR bilaterally for each counterparty, and scale up the estimates by a common factor so that the total IM collected corresponds to the CCP's total reported IM at the end of 2014 (ICE (2016a)).

5 A Behavioral Model of Contagion under Stress

As shown in the preceding section a substantial shock such as CCAR leads to a situation where some firms owe substantially more VM than they expect to receive. These VM obligations are supposed to be satisfied within a very short time – typically within a few hours – and can put severe stress on the firms' trading desk. To relieve the stress, a firm may delay payments, post payments with collateral that is not fully marked-to-market, or default completely. Any of these 'behavioral' responses will increase the stress on the firm's downstream counterparties, amplifying the impact of the shock through the network of CDS exposures. In the next two sections, we describe a general way of modeling these effects, and then show how they play out for the 2015 CCAR trading shock.

The set-up is based on the framework of Glasserman and Young (2015, 2016), which in turn builds on the model of Eisenberg and Noe (2001). Given a shock x, we can represent the VM payment obligations by a matrix $\bar{P}(x) = (\bar{p}_{ij}(x))$, where $\bar{p}_{ij}(x)$ is the net amount of VM owed by node i to node j in the aftermath of the shock. Note both $\bar{p}_{ij}(x)$ and $\bar{p}_{ji}(x)$ are positive. Hereafter, we shall omit the dependence of $\bar{p}_{ij}$ on x. Let $\bar{p}_i = \sum_{j \neq i} \bar{p}_{ij}$ be the total payment obligations of i to all other nodes. We shall restrict attention to the nodes i such that $\bar{p}_i > 0$. The others do not transmit payment shortfalls; instead they act as shock absorbers. In the present context these firms are buyers of protection (not sellers) and under a shock they will have no VM obligations.

The *relative liability* of node i to node j is

$$a_{ij} = \bar{p}_{ij}/\bar{p}_i. \tag{1}$$

Note that for each i, $\sum_{j \neq i} a_{ij} \leq 1$; moreover $\sum_{j \neq i} a_{ij} < 1$ if node i owes payments to one or more absorbing nodes (which are not indexed). It follows that the matrix $A = (a_{ij})$ is row substochastic.

Consider a node i, other than the CCP, and let c_{ki}^{IM} denote the amount of initial margin it collects from counterparty k. The purpose of the IM is to cover the deficiency in VM payments. In particular if counterparty k fails to pay VM to i in a timely manner, the position will be closed out and the IM will be applied to any losses that are incurred between the time of the counterparty's default and the time it takes to close out the position.

Given a shock, let $p_{ki} \leq \bar{p}_{ki}$ denote the actual payment made by k to i. If $p_{ki} < \bar{p}_{ki}$ the difference will be made up out of the initial margin sitting in k's account at firm i provided that $\bar{p}_{ki} - p_{ki} \leq c_{ki}^{IM}$. If $\bar{p}_{ik} - p_{ki} > c_{ki}^{IM}$, however, then the difference $\bar{p}_{ik} - (p_{ki} + c_{ki}^{IM})$ must be borne by i. We define the *stress* at i, s_i, to be the amount by which i's payment obligations exceed the incoming payments from i's counterparties buttressed by the initial margins, that is,

$$s_i = \sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} \left((p_{ki} + c_{ki}^{IM}) \wedge \bar{p}_{ki} \right).^{10} \tag{2}$$

Note that when all of i's counterparties pay in full, that is $p_{ki} = \bar{p}_{ki}$ for all k, then there is no stress at i ($s_i = 0$).

Different firms will react to stress in ways that depend on such factors as their liquidity buffers, non-CDS positions, and general risk management policies. We do not have enough information to model these factors explicitly. Instead, we shall adopt a reduced-form approach in which we posit a response function for each firm i that maps the amount of stress, s_i, to an expected *deficiency* in payments

$$d_i = \bar{p}_i - p_i = f_i(s_i). \tag{3}$$

It is reasonable to assume that $f_i()$ is monotonic increasing in s_i, that is, the greater the stress the greater the expected deficiency in payments. A particularity transparent case arises when $f_i()$ is linear:

$$d_i = \bar{p}_i - p_i = \tau_i s_i \wedge \bar{p}_i, \quad \tau_i \geq 0. \tag{4}$$

The scalar τ_i is i's *transmission factor*. When τ_i is small the firm is able to absorb most of the stress, say by drawing on its own cash reserves. The larger the value of τ_i, the more the stress

[10] In general, $x \wedge y$ denotes the minimum of two real numbers x and y.

is passed on to i's counterparties in the form of reduced payments. When $\tau_i = 1$, the difference between i's VM obligations and incoming payments (buffered by IM) is transmitted in full to i's counterparties: in expectation, i absorbs none of the difference and passes it along to its downstream counterparties in the form of reduced payments.

The linear form can be used to approximate or place a bound on more complex responses to stress. To illustrate, let b_i denote the liquid reserves available at firm i to handle VM payment obligations. Let us view b_i as the realization of a random variable with density $g_i(b_i)$ on the non-negative real numbers. This random aspect of b_i may reflect unobserved heterogeneity in firm risk management practices or it could reflect short-term fluctuations in the quantity of a given firm's available reserves. Suppose that $g_i(b_i)$ is non-increasing, that is, smaller reserves are at least as likely as large reserves.[11] Given that firm i meets its VM obligations to the maximum extent possible (for a given level of b_i), the expected deficiency in its payments to counterparties is

$$d_i = E[s_i - b_i]_+ = s_i - E[b_i|b_i \leq s_i]P(b_i \leq s_i) + 0 \cdot f_i(s_i). \tag{5}$$

By assumption, $g_i(b_i)$ is non-increasing, hence $E[b_i|b_i \leq s_i] \leq s_i/2$. From this and expression (5) it follows that

$$d_i \geq s_i/2. \tag{6}$$

In other words, the stress transmission is bounded below by a linear function with transmission factor $\tau_i = 1/2$.

If a firm has no liquid reserves beyond the initial margin it collects from its counterparties, then $\tau_i = 1$. Furthermore it is entirely possible that $\tau_i > 1$. This would be the case if the firm defaults on its obligations completely, or if it pays out less than it has available due to precautionary hoarding.

Assuming that expected payment reductions are transmitted to i's counterparties in proportion to the size of its obligations, the payment from i to j can be expressed as

$$p_{ij} = [\bar{p}_{ij} - \tau_i a_{ij} s_i]_+ \,. \tag{7}$$

[11] As shown in Ghamami and Glasserman (2016), the amount of liquid reserves implied by regulatory capital requirements is typically very small compared to posted initial margin, suggesting that b_i will typically be small relative to realized levels of stress.

The transmission of stress by the CCP must be modeled somewhat differently due to the presence of the *guarantee fund*. This is a common pool of reserves that is funded by the various members and is available to cover residual losses when the initial margins are depleted. Let the CCP node have index 0, and let c_{k0}^{IM} be the initial margin collected by the CCP from each member $k \neq 0$. Let γ_0 be the total amount in the CCP's guarantee fund.[12] Then the difference between the CCP's VM obligations and the resources it has to meet them is

$$s_0 = \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} \left((p_{k0} + c_{k0}^{IM}) \wedge \bar{p}_{k0} \right) - \gamma_0 \right]_+ , \tag{8}$$

where k ranges over the members. Given the CCP's transmission factor τ_0, the actual payment from the CCP to each counterparty j is

$$p_{0j} = [\bar{p}_{0j} - \tau_0 a_{0j} s_0]_+ . \tag{9}$$

Given any vector $p \in \mathbb{R}^{2n+2}$ such that $0 \leq p_{ij} \leq \bar{p}_{ij}$ for all $0 \leq i, j \leq n$, let $\Phi(p)$ be the mapping defined by expressions 2 - 9, that is,

$$\forall i \neq 0, \quad \Phi(p)_{ij} = \left[\bar{p}_{ij} - \tau_i a_{ij} \left(\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} \left((p_{ki} + c_{ki}^{IM}) \wedge \bar{p}_{ki} \right) \right) \right]_+ , \tag{10}$$

$$\Phi(p)_{0j} = \left[\bar{p}_{0j} - \tau_0 a_{0j} \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} \left((p_{k0} + c_{k0}^{IM}) \wedge \bar{p}_{k0} \right) - \gamma_0 \right]_+ \right]_+ . \tag{11}$$

Φ is monotone non-decreasing, continuous, and bounded so by Tarski's Theorem it has at least one fixed point (Tarski et al. (1955)). In the empirical applications we specify values for the parameters $\tau_0, \tau_1 ..., \tau_n$, and then recursivelty compute a fixed point of this system by taking the limit of the sequence $p^1 = \Phi(\bar{p}), p^2 = \Phi(p^1), ...$

Such a fixed point can be interpreted as a consistent set of expectations about the incoming payments for each participant, given the behavior of all the participants in response to stress. This has the flavor of a rational expectations model in which the firms' responses are correctly

[12] If the guarantee fund is used up, a CCP may call on further resources, including members' capital contributions and special assessments, although it is unclear how much of these can be accessed on very short notice. These resources can be viewed as an addition to γ_0 but we do not model them explicitly. For a detailed description of ICE Clear Credit's waterfall structure see ICE (2016b).

conditioned given the responses of everyone else. Here however we do not attempt to rationalize specific values of the parameters τ_i; rather we treat them as given and study the behavior of the system under a range of possible values.

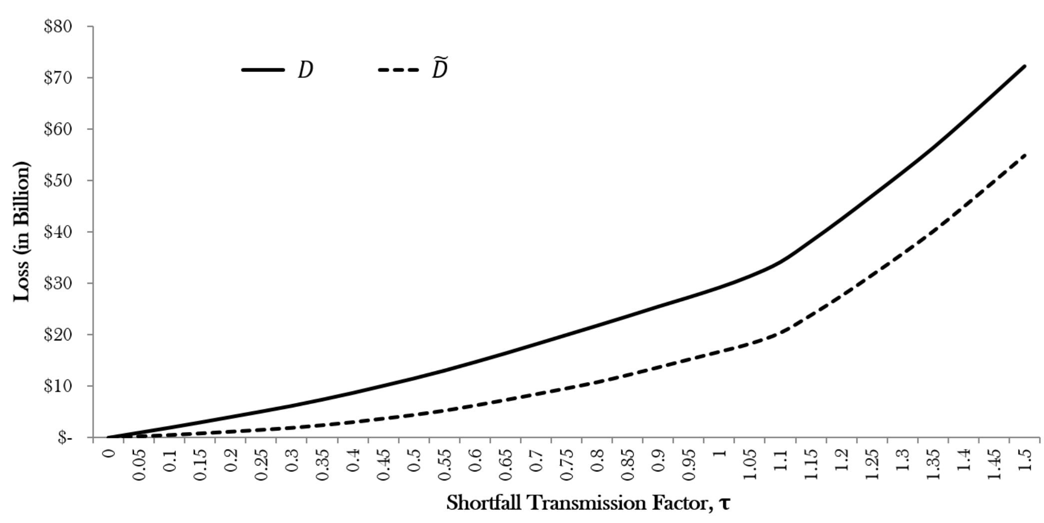
6 Empirical Results on Contagion

We now apply this framework to evaluate the potential amount of contagion in the CDS market that would be produced by a shock such as CCAR. The total *impact* of the shock is the total deficiency in VM payments it generates summed over all directed edges in the network. In our notation the total deficiency can be expressed as follows

$$D = D(\tau_0, ..., \tau_n; \bar{p}) = \sum_i d_i, \qquad (12)$$

where d_i defined in (4). For simplicity let us assume that $\tau_i = \tau_2 = ... = \tau_n = \tau$ for some scalar $\tau \geq 0$ and $\tau_0 = 1$ for the CCP. For each τ-value in the range [0, 1.5] we compute the payment deficiency $D = D(\tau)$; the results are shown in Figure 3.

Figure 3: Contagion Model Results for CCAR 2015 Shock



Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

A deficiency in payment from a given counterparty i to another counterparty j leads j to tap the

IM (if any) that it collected from i to deal with such contingencies. The *IM-adjusted deficiency* in i's payment to j is $[\bar{p}_{ij} - (p_{ij} + c_{ij}^{IM})]_+ = \tilde{d}_{ij}$, that is the shortfall after the IM posted by i and collected by j has been used up. The total IM-adjusted deficiency is $\tilde{D} = \sum_{0 \le i,j \le n} \bar{d}_{ij}$. Note that both D and $\tilde{D}$ depend on the parameter τ. Figure 3 plots $D(\tau)$ and $\tilde{D}(\tau)$ on the domain $0 \le \tau \le 1.5$. For each value of τ the difference $D(\tau) - \tilde{D}(\tau)$ is the total amount of IM used to offset payment deficiencies by counterparties, and $\tilde{D}(\tau)$ is the remaining loss that is neither absorbed by liquidity buffers nor by IM. The larger the transmission factor the greater the amount of contagion in the system. When τ is sufficiently large (in fact when $\tau > 0.95$), the guarantee fund at the CCP would be used up, and it would need to call on its members for additional funding. It must be emphasized that any IM that is used to cover payment deficiencies represents a loss to the firm that posted the IM, and it would have to be replaced if the contracts are renewed.

6.1 Initial versus Equilibrium Stress

To gain further insight into the contagion process, let us fix $\tau = 1$ for purposes of illustration. This corresponds to the situation where, at each node i, the difference between outgoing VM obligations and incoming payments (topped up by collected IM, if needed) is passed on by reducing i's payments to its counterparties. As noted earlier, the reduction could stem from payment delays or the posting of illiquid collateral. It can also be viewed as the expected reduction in payments that result from i completely defaulting with probability $s_i/\bar{p}_i$, and paying in full with probability $1 - (s_i/\bar{p}_i)$.[13]

To illustrate, suppose that i has outgoing VM obligations of \$2 billion and incoming payment obligations of \$1 billion.[14] The difference is the "initial stress" at i. If $\tau_i = 1$ it will pass on the difference to its counterparties, reducing its payments pro rata by 50 percent. Due to network contagion, however, the stress at i can be considerably larger than this initial estimate. The reason is that the incoming payments of \$1 billion may not be forthcoming due to stress at i's upstream counterparties. For example, in equilibrium there might be only \$500 million in incoming payments, in which case the actual stress would be \$1.5 billion (which is passed along to i's counterparties).

[13] More generally, if i defaults with probability q_i and pays in full with probability $1 - q_i$, then the expected loss transmitted to i's counterparties is $q_i\bar{p}_i$, and the associated transmission factor is $\tau_i = q_i\bar{p}_i/s_i$

[14] These numbers are not exaggerated: there are some nonmembers where the initial stress is even greater than \$1 billion.

In fact, the equilibrium stress for a firm can be large even though the initial stress is zero. The difference between initial stress and equilibrium stress is shown in Table 5 under the assumption that all $\tau_i = 1$.

Table 5: Stress Equilibrium when all $\tau_i = 1$

	Initial Stress	Equilibrium Stress
CCP	-	1,331
Members		
A-E	1,630	3,236
F-J	8	45
K-O	27	3,703
P-T	1131	6,978
U-Z	532	7,993
Nonmembers		
I-V	9,466	12,869
VI-X	936	1,195
XI-XV	380	484
XVI-XX	287	564
XXI-XXVI	303	364

Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

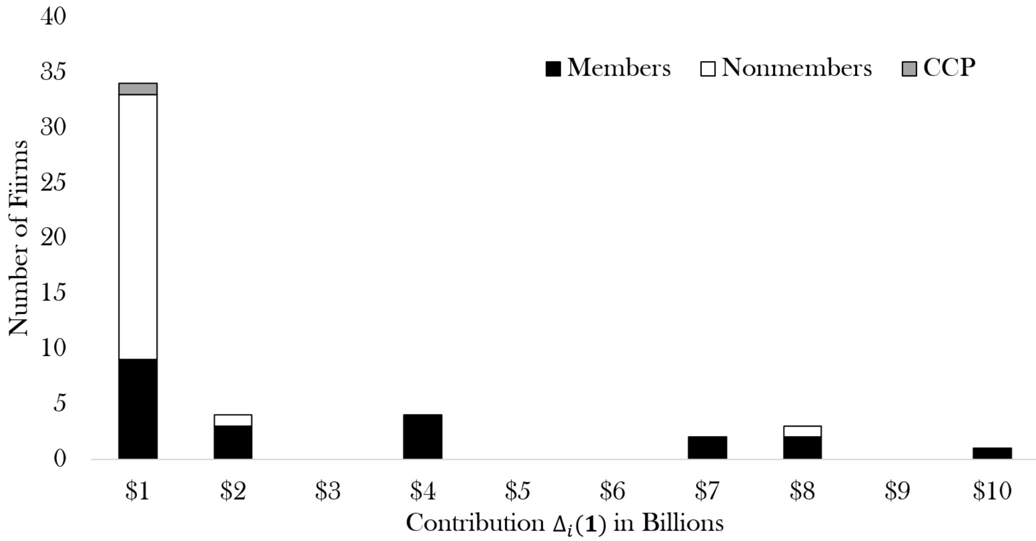
6.2 Contribution to Contagion by Individual Firms

How much does a given firm contribute to contagion? The answer depends on: i) the amount of stress (payment imbalances) at that firm; ii) the firm's τ-value, which measures how much of the payment imbalances it passes on to its counterparties; iii) the structure of the network of exposures. We can estimate the firm's marginal contribution to contagion by considering the reduction in total payment deficiencies D when we set the firm's τ-value equal to zero. In other words, for a given firm i and a given vector of values $(\tau_0, ..., \tau_n) = \boldsymbol{\tau}$, consider the difference

$$\Delta_i(\boldsymbol{\tau}) = D(\boldsymbol{\tau}) - D(\boldsymbol{\tau}'). \qquad (13)$$

where $\tau_j' = \tau_j$ for all $j \neq i$ and $\tau_i' = 0$. The $\boldsymbol{\tau}'$-scenario amounts to assuming that i can absorb any amount of stress by drawing on its treasury, hence it contributes nothing to network contagion. Figure 4 shows the distribution of values $\Delta_i(\boldsymbol{\tau})$ when initially all $\tau_i = 1$.

Figure 4: Distribution of Firms' Marginal Contributions to Contagion



Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

Note that there are several members (and one nonmember) that contribute over seven times as much to network contagion as does the CCP.

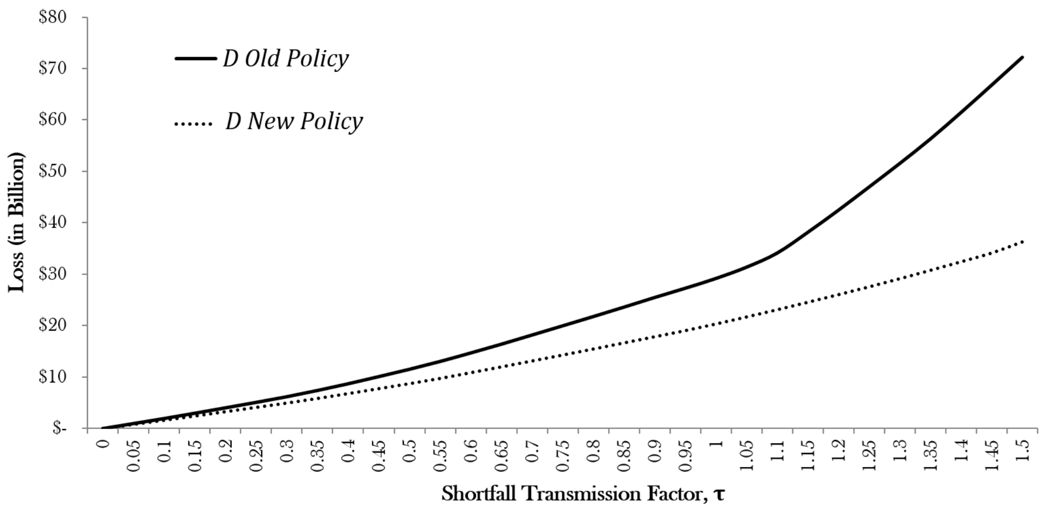
6.3 Comparison of Pre- and Post-2016 IM Regimes

The preceding analysis was conducted for the 2015 CCAR shock scenario and we assumed that IM was collected under the prevailing regime at the time of the shock (see Table 4). Starting in September 2016, IM must be posted by both counterparties in all non-centrally cleared CDS transactions using a 10-day margin period of risk (BCBS and IOSCO (2015)). Although this new regime is to be phased in and will undoubtedly change the network of exposures, we shall examine what would have happened if the new requirements had been in place when the CCAR shock was applied to the network of exposures as it existed at the time.

The results are shown in Figure 5, comparing the old and new policy regimes and show that there is a sizable reduction in the payment shortfalls throughout the system; moreover for values of $\tau \leq 1$ almost all of the shortfalls are covered by collected IM, thus reducing the amount of contagion.

Figure 6 shows the distribution of firms according to their marginal contributions to contagion

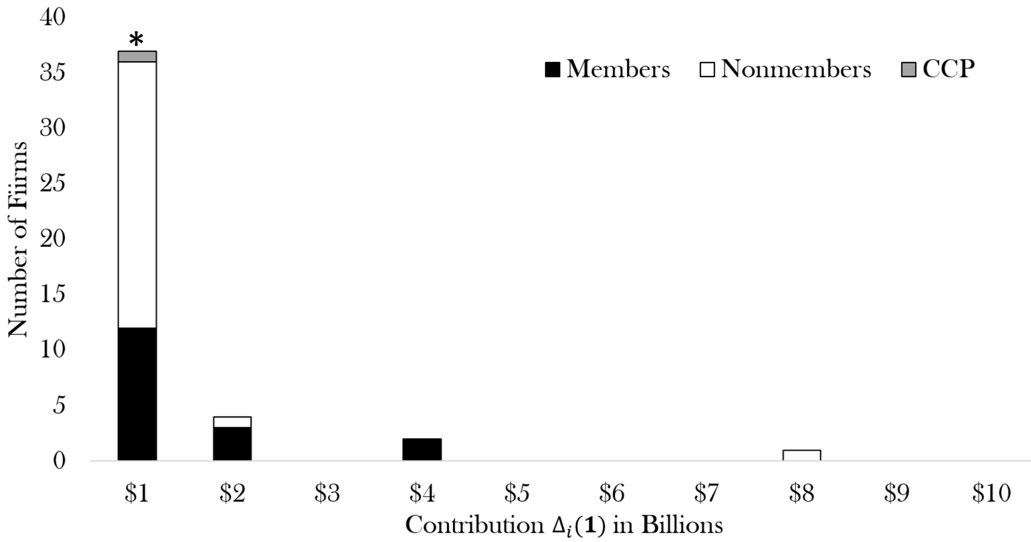
Figure 5: Policy Change Impact under CCAR 2015 Shock



Note: When $\tau > 1.5$, the central counterparty (CCP) is under severe stress and must call on its members for additional funding.
Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

when all $\tau_i = 1$. A comparison with Figure 4 shows that the distribution is shifted to the left: under the new regime there are only three firms that contribute more than \$4 billion, whereas in the previous regime there were 10. Nevertheless there is still one nonmember firm whose marginal contribution is very large (\$8 billion), which represents about 40 percent of total systemic losses. Although this firm is required to collect IM from its counterparties under the new regime, this does not offer much additional protection against losses transmitted *by* this firm to its counterparties, which are mainly members and the CCP that already collected IM under the previous regime. These results highlight the importance of requiring firms to hold strong liquidity buffers in addition to collecting adequate IM from their counterparties.

Figure 6: Policy Change Impact on Firms' Marginal Contributions to Contagion



Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

7 Conclusion

In this paper, we have analyzed the network of counterparty exposures in the CDS market. In contrast to much of the prior work on banking networks, the DTCC data provide a detailed picture of network exposures at different points in time. We are therefore able to track the potential effects of a shock, such as the CCAR shock specified by the Federal Reserve, as of a particular date. We estimate the impact of the shock on the value of both single-name and index positions, and by implication the variation margin (VM) owed between the contracting parties. A significant feature of this market is that demands for VM must be met over very short time horizons. When the demands for VM from a given firm exceed its initial margin and other ready sources of cash, the firm may fail to pay its counterparties promptly, and this shortfall can become amplified as it cascades through the network.

We examined the potential contribution to network contagion of the 26 members of the major CCP in this market (ICE Clear Credit), and also the potential contribution of the major nonmembers. We found that network exposures can significantly increase the amount of contagion when the transmission factor τ is greater than one. Furthermore there are many members (and some nonmembers) that contribute substantially more to contagion than does the CCP, in spite of the fact that they are peripheral in the network. Under the new policy in which all counterparties to bilateral CDS transactions must post IM, the total amount of contagion is substantially reduced, and so are the marginal contributions of individual firms. Nevertheless, the CCP still contributes substantially less to contagion than do some members (and some nonmembers). This finding contrasts with the prevailing view that CCP's constitute a major source of systemic risk. Our analysis suggests that more attention should be paid to firms that are very large and have highly unbalanced CDS positions, whose failure can trigger large systemic losses even when the CCP does not fail. It also highlights the key role of liquidity buffers in coping with large and sudden demands for variation margin that result from a credit shock.

Our study is limited to the analysis of a specific part of the derivatives market, and does not encompass the full range of shocks to which firms may be exposed. In particular, we have not included exposures to interest rate swaps, which form a substantially larger market (in notional terms) than the CDS market, but which is not part of our dataset. In this sense our analysis is somewhat conservative. Under a CCAR shock, firms may be subjected to simultaneous payment demands over multiple lines of business, increasing the stress on their resources. In spite of these limitations, the framework we propose is general and can be applied to many different settings where stresses are transmitted through the network of exposures.

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Appendix – Evaluating CDS Portfolios

In this section we describe the methodology for estimating the mark-to-market value of each counterparty's exposures at a given date for both single-name and index positions. We describe the bootstrap procedure to generate a schedule of hazard rates consistent with the market for all traded credit curves. We then describe the process of index disaggregation to single-name equivalents. Finally, we describe how we arrive at expressions for variation margin payments under stress.

Bootstrapping Credit Curves

We calibrate hazard rate schedules for all credit curves references by contracts we observe. At a minimum, this achieves consistency with market quotes on contracts with standard maturities. Further, this permits consistent valuation of contracts within our sample.

The CDS market quotes credit spreads for bearing risk through a range of standard terms: 1-year, 3-year, 5-year, 7-year, and 10-year, and sometimes longer maturities. Each additional term generates a hazard rate over a corresponding increment: the 3-year term generates a 1-3y increment, the 5-year term generates a 3-5y increment, and so on through 10 years. The bootstrapping technique we employ here generates a piecewise constant schedule of hazard rates. A CDS contract struck at inception at the market spread for a standard maturity, and valued using the schedule of hazard rates through that maturity, will have equal default and premium legs. Bootstrapping permits us to value any position whose remaining maturity at the time of stress may not correspond to market-quoted maturities.

Both CDS payment and premium legs are implicit functions of a hazard rate, λ, which enters the expression for the survival probability $S(0, t, \lambda)$ through its definition as $exp\left(-\int_t^u \lambda ds\right)$. $Z(0, t_i)$ is the risk-free discount factor through period i, which we compute from LIBOR rates from 1 through 6 months and swap rates from 9 months through 30 years. We assume CDS premia are paid on International Money Market (IMM) payment dates, consistent with market convention. Finally, we allow for the possibility CDS pay accrued premia on default to the protection seller, ie. that when a default occurs inter-period, CDS premia to the protection seller are pro-rated to the time of default. We assume $\alpha = 0.5$ (i.e., that any default occurs at the inter-period halfway point). In subsequent notation, $\Delta_{n,n+1}$ is the daycount fraction for the time interval (t_n, t_{n+1}). We use the ACT/365 convention standard in the CDS market.

We write the CDS payment and premium legs through maturity T:

$$V_{prem}^{0\to T}(\lambda,\ s_T) = s_T \sum_{i=1}^{N} Z(0,t_i)\Delta_i \left((1-\alpha)S(0,t_i) + \alpha S(0,t_{i-1})\right). \tag{14}$$

In any period, the payment leg derives value from the incremental default probability over that time. Given the relationship between default and survival, $S(t,u,\lambda) = 1 - P(t,u,\lambda)$, we can write the payment leg as follows:

$$V_{pay}^{0\to T}(\lambda) = (1-R)\sum_{i=1}^{N} Z(0,t_i)\left(P(0,t_i) - P(0,t_{i-1})\right). \tag{15}$$

λ^* is the solution that sets the CDS payment and premia legs to fair value (equality) at inception, ie. where $V_{contract}^{0\to T}(\lambda^*,\ s_T) = V_{pay}^{0\to T}(\lambda) - V_{prem}^{0\to T}(\lambda,\ s_T) = 0$.

The credit curve bootstrapping process establishes consistent hazard rates for all quoted contract maturities. In other words, for $T_{i+k} \geq T_i$, the bootstrap process establishes a vector of hazard rates $\boldsymbol{\lambda^*}$ such that $V_{contract}^{0\to T_{i+k}}(\boldsymbol{\lambda^*},\ s_{T_{i+k}}) = V_{contract}^{0\to T_i}(\lambda^*_{0,T_i},\ s_{T_i}) + V_{contract}^{T_i\to T_{i+k}}(\lambda^*_{T_i,T_{i+k}},\ s_{T_{i+k}})\ \forall\ k$.

We use the methodology from Luo (2005) to express the subsequent stage of the bootstrap technique. For any reference entity, we have a term structure of fair-value spreads $\Big\{T_1\colon s_{T_1},\ T_2\colon s_{T_2},$ ... $T_k\colon s_{T_k}\Big\}$. The bootstrap technique generates hazard rates $\Big\{\lambda^*_{0,T_1},\ \ \lambda^*_{T_1,T_2},\ \ ...\ \lambda^*_{T_{k-1},T_k}\Big\}$ that establish fair value for market standard maturities. Upon having computed the hazard rate from 0 (inception) through T_1, the second stage is to compute $\lambda^*_{T_1,T_2}$, given λ^*_{0,T_1}. More generally, any subsequent stage computes $\lambda^*_{T_{k-1},T_k}$, given $\boldsymbol{\lambda^*_{0,T_{k-1}}} = \Big\{\lambda^*_{0,T_1},\ \lambda^*_{T_1,T_2},\ ...\ \lambda^*_{T_{k-2},T_{k-1}}\Big\}$. These optimal values are related through the decomposition of the survival probability at any time t such that $T_k \leq t\ \leq\ T_m$:

$$P(0,t,\boldsymbol{\lambda_{0,T_m}}) = P(0,T_k,\lambda_{0,T_k})P(T_k,t,\lambda_{T_k,T_m}). \tag{16}$$

For brevity, we will refer to $P(0,t_l,\boldsymbol{\lambda_{0,T_l}})$ from here on as $P(t_l)$. We make a simplifying assumption relative to Luo (2005) that is consistent with Markit quotes: standard maturity dates fall upon IMM payment dates. Given this, let N_k be the payment period index that corresponds to a contract maturity of T_k. The conditional premia and payment legs are given as follows:

$$\begin{aligned} V_{prem}^{0\to T_m}(\lambda_{T_k,T_m},\ s_{T_m};\ \boldsymbol{\lambda^*_{0,T_k}}) = s_{T_m}\Bigg\{ & C(\boldsymbol{\lambda^*_{0,T_k}}) + D(\lambda_{T_k,T_m}) \\ & - \sum_{i=N_k+1}^{N_m} Z(0,t_i)\Delta_i\left(P(t_i) - P(t_{N_k}) - \alpha\frac{P(t_i)-P(t_{i-1})}{2}\right)\Bigg\} \end{aligned} \tag{17}$$

where

$$C(\boldsymbol{\lambda^*_{0,T_k}}) = \sum_{(f,g)=(0,1)}^{(k-1,k)} \sum_{i=N_f}^{N_g} Z(0,t_i)\Delta_i \left[(1-P(t_i)) + \alpha \frac{(P(t_i)-P(t_{i-1}))}{2} \right],$$

$$D(\lambda_{T_k,T_m}) = \sum_{i=N_k}^{N_m} Z(0,t_i)\Delta_i(1-P(t_{N_k})).$$

$$V_{pay}^{0\to T_m}(\lambda_{T_k,T_m};\ \boldsymbol{\lambda^*_{0,T_k}}) = A(\boldsymbol{\lambda^*_{0,T_k}}) + \sum_{i=N_k+1}^{N_m} (1-R)Z(0,t_i)\Big(P(t_i)-P(t_{i-1})\Big) \tag{18}$$

where

$$A(\boldsymbol{\lambda^*_{0,T_k}}) = \sum_{(f,g)=(0,1)}^{(k-1,k)} \sum_{i=N_f}^{N_g} (1-R)Z(0,t_i)\left(P(t_i)-P(t_{i-1})\right).$$

$P(t_i)$, and $P(t_{i-1})$ in Equations 17 and 18 are functions of λ_{T_k,T_m}, where $\lambda^*_{T_k,T_m}$ is the solution that sets the contract of maturity T_m to fair value, given hazard rate(s) observed over $(0,T_k)$, such that $V_{contract}^{0\to T_m}(\lambda_{T_k,T_m},\ s_{T_m};\ \boldsymbol{\lambda^*_{0,T_k}}) = 0$.

Once a credit curve is bootstrapped, we can value a contract of any maturity simply by identifying its last payment date index, computed hazard rate schedule, and market spread.

Portfolios of Single Name Equivalents

We disaggregated Markit credit indices to single-name constituents. For each position referencing a Markit credit index, we decompose the index using *Markit RED* data. This source provides the composition of the index at any point in time, taking into account index revisions and defaults. We employ the disaggregation technique described by Siriwardane (2015b) in Section 2.

Each Markit credit index is described by its series and version. A series may have one or more versions. An index series factor, f_i is defined for every version i as $f_i = 1 - \frac{D_{i-1}}{N}$, where D_{i-1} is the number of defaults for an index series version i in $1, 2, 3$ .... $D_0 = 0$, so $f_1 = 1$. The weight of a constituent within a version must be computed as of a valuation date of interest and is a function of the index composition as of the date the position was established (trade date). In general, the index composition at the trade date may not be its composition at inception. The current weight $w_i(u)$ for index version i of a constituent u whose inception index series weight is $w_0(u)$ is given as

$$w_i(u) = \frac{w_0(u)}{f_i}. \tag{19}$$

As an example, an index with 43 original constituents at inception would have a per-constituent weight of $w_1(u) = \frac{\frac{1}{43}}{1} = 0.0233$. Version 2 of the index would have a per-constituent weight of $w_2(u) = \frac{\frac{1}{43}}{0.953} = 0.0244$. The per-constituent weight is scaled by the notional value of the index position to arrive at the effective single-name notional equivalent. We perform all calculations in this paper on a firm's single-name equivalent notional CDS positions.

Estimating Variation Margin

Estimating the variation margin from the CCAR shock is a computationally and data-intensive task. We begin with a compendium of single-name and index positions and corresponding contractual information needed for marking-to-market each counterparty's individual exposures. At each CCAR valuation date, we generate profit & loss (P&L) using the approach outlined above. In the following section, we shall discuss how hazard rate curves are generated, and then document how positions are marked to market.

The change in value of exposures under stress follows from their valuation at baseline and revaluation after the market shock. At baseline, or under stress, an exposure is valued relative to the contract's inception. It is useful to incorporate the counterparty flows in the description of the net present value (NPV). For example, x sells protection to y. x is long the premium leg and short the payment leg; stated alternatively, x writes the payment leg, while y writes the premium leg. Incorporating counterparty flows, suppressing some earlier notation, we express $V_{prem}^{0\to T_m}(\lambda_m{}^*,\ s;\ \boldsymbol{\lambda_{m-1}}^*)$ as $V_{prem}^{x}(\boldsymbol{\lambda},\ s)$ and similarly the payment leg as $V_{pay}^{y}(\boldsymbol{\lambda})$. The hazard rate environment that exists at valuation date t_n is described as $\boldsymbol{\lambda^n}$. Analagously, the environment at t_n under stress is $\boldsymbol{\lambda^{shock}}$.

The NPV of a swap of $\$N$ notional at as-of-date t_n is defined:

$$NPV^{x\to y}(N, \boldsymbol{\lambda^n}, s) = N\left[V_{prem}^{x}(\boldsymbol{\lambda^n},\ s) - V_{pay}^{y}(\boldsymbol{\lambda^n})\right]. \tag{20}$$

Similarly, the swap at as-of-date t_n under stress is described by:

$$NPV^{x\to y}(N, \boldsymbol{\lambda^{shock}}, s) = N\left[V_{prem}^{x}(\boldsymbol{\lambda^{shock}},\ s) - V_{pay}^{y}(\boldsymbol{\lambda^{shock}})\right]. \tag{21}$$

Under CCAR, credit-risky securities are subject to a variety of shocks absolute and proportional to credit spreads, or proportional to market value. Shocks are prescribed across geographies, credit categories (e.g. municipal, state, and sovereign credit; and corporate credit), and ratings classes. These reflect the Federal Reserve's views on current risks for bank solvency. Using the CCAR trading shock stresses prescribed in Table 1 for 2015, we compute the NPV under stress as follows:

The stressed variation margin at any valuation is the change in NPV between baseline and stress:

$$\Delta MtM^{x \to y} = NPV^{x \to y}(N, \boldsymbol{\lambda^{shock}}, s) - NPV^{x \to y}(N, \boldsymbol{\lambda^{n}}, s). \tag{22}$$