Bargaining Leads to Sorting: a Model of Bilateral On-the-Match Search*

Cristian Bartolucci and Ignacio Monzón†

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PRELIMINARY DRAFT

Abstract

We present a matching model where heterogeneous agents bargain over the gains from trade and are allowed to search on the match. Because of frictions, agents extract higher rents from more productive partners, generating an endogenous preference for high types. This preference generates positive assortative matching and arises even in cases with a submodular production function.

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†Collegio Carlo Alberto, Via Real Collegio 30, 10024 Moncalieri (TO), Italy. Emails: cristian.bartolucci@carloalberto.org and ignacio@carloalberto.org. Websites: http://www.carloalberto.org/people/bartolucci/ and http://www.carloalberto.org/people/monzon/

1. Introduction

Markets with two-sided heterogeneity are prevalent. In labor markets, firms and workers typically differ in their characteristics, quality and ability. The same is true in other markets, such as the marriage market and the market for CEOs. Now, is it true that workers of higher quality work for better firms? Do the best CEOs work for the best companies? Moreover, if in fact there is positive assortative matching, what drives it? Traditionally, positive assortative matching has been interpreted as evidence of complementarity in the production function.

In this paper we argue that, even in the absence of complementarity in production, frictions generate a simple and natural reason for positive assortative matching to arise: agents sort to avoid being held up. Once workers and firms meet in the labor market, their productivity is predetermined. In the presence of frictions, as it takes time to find alternative partners, there are gains from trade to be bargained. Agents of higher types may be held up by lower types because of the threat of match dissolution. To illustrate this mechanism, consider an example with only two types of agents: $x, y \in \{\ell, h\}$, with $\ell < h$. Assume agents produce $f(x, y) = x + y$ if matched and zero otherwise. When agents of the same type meet, they divide production equally, so agents earn their type. Now, say a low and a high agent meet. If agents only cared about today's payoff (that is, if they were infinitely impatient), the outside option would be zero, so ℓ would hold-up part of the production of h and both agents would earn $(\ell + h)/2$.

As the previous example illustrates, the hold-up problem may induce agents to prefer being matched with partners of high types. This, in turn, is a force leading to sorting. In fact, if the value of the match were exogenously increasing in partner type, perfect positive sorting would arise in a frictionless market. However, this is not necessarily true in a market with frictions. For example, in Shimer and Smith [2000] the value of a match is increasing in the partner's type when the production function is modular, but there is no positive assortative matching since no search is allowed after a match is formed. In this paper we show that if agents in both sides of the market are allowed to replace their partners, for example allowing on-the-match search, a valuation increasing in partner's type can lead to positive sorting.

We present a partnership model that builds on Shimer and Smith [2000]. The economy is populated by heterogeneous agents who can produce only in pairs. When two agents are matched, they produce a flow of divisible output and they bargain on how to split the production. We depart from Shimer and Smith [2000] in that we allow for bilateral on-the-match search. A matched agent who finds a new partner can choose to dissolve her current match and form a new one. After the match-to-match transition, she bargains with her new partner without the possibility of returning to the previous one. Our bargaining protocol prevents agents from exploiting the presence of multiple suitors to raise their payoffs.[1]

Job-to-job transitions are pervasive in most developed economies. According to conservative estimates, half of all new employment relationships result from job-to-job transitions (see Fallick and Fleischman [2004]). On the firm side, Albak and Sørensen [1998] and Burgess, Lane, and Stevens [2000] present empirical evidence of replacement hiring for Denmark and the U.S. (see Kiyotaki and Lagos [2007] for a discussion). Although the evidence of on-the-match search in the marriage market is less widespread, Stevenson and Wolfers [2007] and others show that remarriage is one of the main determinants of divorce.

It is natural then to allow agents to search on the match. However, it adds an extra layer of difficulty to the bargaining problem: the surplus from the match depends on the bargaining outcome. Patient agents face a trade-off between per period payoff and expected duration of the match: higher wages paid to a worker on the one side reduce the firm's per period profits, but on the other they decrease the likelihood that the worker quits. In fact, a higher wage may increase the value of the match for both worker and firm. As highlighted by Shimer [2006], the standard axiomatic Nash Bargaining is not applicable in this setup. Indeed, the outcome resulting from symmetric surplus splitting

[1] In some markets (like the one for academic economists) counteroffers are common practice. However, this is not the norm in most labor markets (see Mortensen [2005]). In Section 5.2 we discuss a simple case with renegotiation where frictions lead to positive sorting, also without productive complementarity.

may even be Pareto sub-optimal.

To see why surplus may be split unevenly, consider patient agents with $\ell \approx h$, and again a linear production function. If ℓ and h split the total surplus symmetrically, the low-type agent makes marginally more than ℓ per period but is dismissed when the high-type agent finds a high-type partner. Therefore it is more convenient for the low-type agent to receive a per period payoff of ℓ and get a larger expected duration of the match. The high type also benefits from that. Then, for $\ell \approx h$, the outcome from even surplus splitting is dominated. Thus, hold-up does not occur.

On the other side, when the difference in types' qualities is high, it is profitable for low types to give up match duration in exchange for higher per period payoffs. To see this, consider patient agents with $\ell \approx 0$. The high type only stays if he receives h, which leaves the low type with a payoff close to 0. The low type is better off holding-up the high agent, equalizing surplus.

We present a solution for axiomatic bargaining when both sides can leave the match if they find a preferred option. Bargaining sets do not satisfy Nash [1950]'s axioms. We show, however, that bargaining sets are compact under on-the-match search. We follow a modified version of Nash's axioms proposed by Kaneko [1980]. Kaneko shows that when bargaining sets are compact the solution is exactly as in Nash [1950]: it selects the outcome which maximizes the product of agents' individual surpluses.

We introduce first a model where agents are of one of two types: either low or high productivity. This simple two type model is rich enough to illustrate the trade-offs patient agents face when they can search on-the-match. We show that several different equilibria can arise, depending on the degree of complementarity in production, agents' patience and the degree of frictions in the market. Each possible equilibrium induces a pattern of sorting. We fully characterize this two type model.

In our main result in this part, we provide necessary and sufficient conditions for an endogenous preference for the high type to arise. We show that a preference for the high type leads to positive assortative matching in our model. We state, as function of the primitives, up to which degree of submodularity in production an equilibrium with positive assortative matching exists. Moreover, we show that for some primitives, an equilibrium with positive assortative matching is the only possible one, even with a submodular production function.

We then present the case with any (finite) number of types. A characterization of equilibria for all possible values of the primitives is out of reach in this context. However, our intuition extends to this case. We first show that as agents become impatient or frictions large, an equilibrium where agents endogenously prefer higher types arises, for any (strictly increasing) production function. Moreover, this is the only possible equilibrium. Thus, for any number of types positive assortative matching may arise, even with submodularity in production. Second, with parameter values in line with the literature, we provide numerical examples of equilibria where agents endogenously prefer higher types. Both with modular or slightly submodular production function there are equilibria where matching is positively assortative.

The relationship between frictions and hold-ups is well known in markets with one-sided heterogeneity (see for example Acemoglu and Shimer [1999] for the case of the firm's investment or Flinn and Mullins [2011] for the case of human capital investment). However, its implications on assortative matching in markets with two-sided heterogeneity have not been studied before. The literature on assortative matching is mostly focused on how complementarity in production affects the allocation of workers to firms. In Becker [1973]'s seminal partnership model, a supermodular production function is necessary and sufficient for positive assortative matching. Shimer and Smith [2000] show that, with frictions, supermodularity does not suffice for positive assortative matching to arise.[2]

The results in this paper go against the conventional wisdom that stronger frictions require stronger complementary in production for positively assortative matching to arise. Moreover, it invalidates the interpretation of sorting as evidence of complementary in production. Positive assortative matching may instead result from bargaining. This dis-

[2] Atakan [2006] and Eeckhout and Kircher [2011] model frictions differently than Shimer and Smith [2000]. However, supermodularity remains a necessary condition for positive assortative matching.

tinction is relevant for policy. To see this, consider the linear production function case. There, sorting results from bargaining. Thus, a program providing incentives for agents to look for the "right" partner may be welfare detrimental. A different distribution of matches does not change the aggregate production of the economy and moreover search is costly in terms of forgone output.

In the next section we present the model, describe bargaining sets with on the-match-search and we present our notion of equilibrium. In Section 3 we solve the two type case. We provide a full characterization of all equilibria in this simplified setting. Section 4 shows how our intuition extends to the case with any number of types. In Section 5 we present some extensions and variations to our model. Section 6 concludes.

2. The Model

We build on Shimer and Smith [2000]'s partnership model.[3] Consider a continuous time, infinite horizon stationary economy, populated by infinitely lived, risk neutral agents. There is a unit mass population of heterogeneous agents, denoted by their fixed type $x \in X$, where X is a finite list of all possible types. All types are present in equal proportion in the population.

We assume first that agents can be either of low productivity or of high productivity, so $X = \{\ell, h\}$, with $0 < \ell < h$. The production function is simple. Two ℓ agents produce $f(\ell, \ell) = 2\ell$, two h agents produce $f(h, h) = 2h$ and a ℓ-type with a h-type produce $f(h, \ell) = f(\ell, h) = F$. Parameter F captures the degree of complementarity in production. A modular production function has $F = \ell + h$, a supermodular one has $F < \ell + h$ and finally $F > \ell + h$ corresponds to the submodular case. We assume that high-productivity agents are always more productive than low-productivity agents, therefore $2\ell < F < 2h$. Unmatched agents produce zero. Agents discount the future at rate $r > 0$.

[3] Shimer and Smith [2000] is the current state-of-the-art framework to analyze markets with frictions and transferable utility. Most recent studies on topics related to assortative matching, such as Lopes de Melo [2013], Hagedorn, Law, and Manovskii [2012] and Lise, Meghir, and Robin [2013], take Shimer and Smith [2000] as a starting point.

In our model, each agent has to decide which partners to accept while matched to *each* possible partner; acceptance sets are conditional on the current partner. Match-to-match transitions are feasible, and thus dynamics become more complicated than in models without on-the-match search. A decision function $d(x, y, y') : \{\ell, h\} \times \{\ell, h\} \times \{\ell, h\} \rightarrow [0, 1]$ specifies the probability that an agent of type x matched to an agent of type y chooses, given the chance, to switch to a partner of type y'.

The steady state distribution $e(x, y) : \{\ell, h\} \times \{\emptyset, \ell, h\} \rightarrow \left[0, \frac{1}{2}\right]$ specifies the number $e(x, \emptyset)$ of unmatched x-type agents and the number $e(x, y)$ of x-type agents matched to agents of type $y \in \{\ell, h\}$. Since in the population there are as many low as high productivity agents, $\sum_{y \in \{\emptyset, \ell, h\}} e(x, y) = \frac{1}{2}$ for $x \in \{\ell, h\}$.

Transitions between states occur due to both exogenous destruction and match-to-match transitions. Our model is standard in that 1) matches are exogenously destroyed at rate δ and that 2) meetings occur at rate ρ. However, we allow *both* matched and unmatched agents to meet potential partners (who *also* themselves may be matched or unmatched).[4] For example, $e(\ell, \ell) e(h, h)$ ℓ-type agents matched to other ℓ-type agents meet h-type agents matched to other h-type agents at a rate ρ. Those meetings are not allowed in standard models, even in cases where both ℓ and h would be happy to dissolve their current matches and form a new match (ℓ, h).

Flow payoffs are deterministic, last for the duration of the match and are determined through bargaining, as discussed in the next subsection. Let $\pi(x, y) : \{\ell, h\} \times \{\ell, h\} \rightarrow [0, f(x, y)]$, with $\pi(x, y) + \pi(y, x) \leq f(x, y)$, be the allocation agent x receives when matched to agent y. Unmatched agents obtain a zero flow payoff.[5]

Let $q(x, y) : \{\ell, h\} \times \{\ell, h\} \rightarrow \mathbb{R}_+$ be the rate at which x finds a y who is willing to form a match with him: $q(x, y) \equiv \rho \left[e(y, \emptyset) + \sum_{x' \in \{\ell, h\}} e(y, x') \, d(y, x', x)\right]$. The value

[4] For simplicity, we assume that on-the-match search and search while unmatched are equally intensive. In equilibrium, payoffs while matched are strictly positive. Then, unmatched agents accept all partners. In Section 5.1 we allow search intensities to differ.

[5] One can always allow for a non-zero flow payoff for unmatched agents by modifying the production function accordingly.

$V(x,\varnothing)$ for an unmatched x-type agent is given by

$$[r+q(x,\ell)+q(x,h)]\,V(x,\varnothing)=0+q(x,\ell)\,V(x,\ell)+q(x,h)\,V(x,h),$$

where $V(x,y)$ is the value for a x-type agent matched to a y-type, which is given by

$$\left(r+\delta+\sum_{y'\in\{\ell,h\}}d(x,y,y')\,q(x,y')+\sum_{x'\in\{\ell,h\}}d(y,x,x')\,q(y,x')\right)V(x,y)=\pi(x,y)$$
$$+\left(\delta+\sum_{x'\in\{\ell,h\}}d(y,x,x')\,q(y,x')\right)V(x,\varnothing)+\sum_{y'\in\{\ell,h\}}d(x,y,y')\,q(x,y')\,V(x,y').$$

It is usually more convenient to work directly with the surplus agents obtain relative to being unmatched. Surplus $S(x,y):\{\ell,h\}\times\{\ell,h\}\to\mathbb{R}$ are given by

$$S(x,y)=\left(r+\delta+\sum_{x'\in\{\ell,h\}}d(y,x,x')\,q(y,x')\right)^{-1}\Bigg[\pi(x,y) \tag{1}$$
$$+\sum_{y'\in\{\ell,h\}}d(x,y,y')\,q(x,y')\,[S(x,y')-S(x,y)]-\sum_{y'\in\{\ell,h\}}q(x,y')\,S(x,y')\Bigg].$$

We distinguish individual surpluses $S(x,y)$ and $S(y,x)$ from the total surplus of the match $S(x,y)+S(y,x)$, since there can be cases where total surplus is not split symmetrically.

2.1 Timing and Bargaining

We propose the following timing. When a matched agent finds a potential partner, she observes his type, and before bargaining, she must decide which partner to stay with. If she chooses the new partner she must dissolve her current match. Therefore, when an agent bargains with her partner, she cannot exploit the existence of an alternative partner to improve her bargaining position. As a result, the outside option is always the value of the being unmatched.[6]

[6]The timing of match-to-match transitions follows Pissarides [1994] and several more recent papers (see Shimer [2006], Gautier, Teulings, and Van Vuuren [2010] and Bartolucci [2013]). In Section 5.2 we discuss a case with renegotiation where agents can make counteroffers à la Kiyotaki and Lagos [2007]. An equilibrium with endogenous preferences for higher types exists, even without complementarity in production.

When agents search on the match the bargaining set is non-standard, so we need to describe it carefully. Once agents x and y form a match, they bargain on how to split production. This allocation of production remains in place until the match breaks, either exogenously or endogenously. Whenever a matched agent finds a suitor offering a higher surplus, he leaves his partner. Agents cannot commit not to leave each other, and do not engage in renegotiation when an offer arrives.

When two agents bargain, they take all information from *other* matches as given. $S^* = \{S^*(x,y)\}_{(x,y)\in X\times X}$ and $q^* = \{q^*(x,y)\}_{(x,y)\in X\times X}$ summarize a given state of the economy.

A possible agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ between x and y specifies both an allocation $\widehat{\pi}$ and a decision function $\widehat{d}$. Let $\widehat{\pi} = (\widehat{\pi}_1, \widehat{\pi}_2)$ and $\widehat{d} = \left(\left\{\widehat{d}_1\left(y'\right)\right\}_{y'\in X}, \left\{\widehat{d}_2\left(x'\right)\right\}_{x'\in X}\right)$, so for instance $\widehat{d}_1\left(y'\right)$ denotes agent x's decision when faced with the possibility to match with a (willing) agent of type y'. Of course, $\widehat{\pi}_1 + \widehat{\pi}_2 \leq f(x,y)$. Taking market outcomes (S^*, q^*) as given, an agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ induces surplus pair $\widehat{S}^c = \left(\widehat{S}^c_1, \widehat{S}^c_2\right)$ with

$$\widehat{S}^c_1 = \left(r + \delta + \sum_{x'\in X} \widehat{d}_2\left(x'\right) q^*\left(y, x'\right)\right)^{-1} \left[\widehat{\pi}_1 + \sum_{y'\in X} \widehat{d}_1\left(y'\right) q^*\left(x, y'\right) \left[S^*\left(x, y'\right) - \widehat{S}^c_1\right] - \sum_{y'\in X} q^*\left(x, y'\right) S^*\left(x, y'\right)\right]$$

and $\widehat{S}^c_2$ defined accordingly.

Since there is no renegotiation or commitment, only consistent agreements can occur:

Definition 1. Consistent Agreements. *Fix market outcomes (S^*, q^*). An agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ is consistent if for all $y' \in X$,*

$$\widehat{d}_1\left(y'\right) \begin{cases} = 1 & \text{if } S^*(x,y') - \widehat{S}^c_1 > 0 \\ \in [0,1] & \text{if } S^*(x,y') - \widehat{S}^c_1 = 0 \\ = 0 & \text{if } S^*(x,y') - \widehat{S}^c_1 < 0 \end{cases}$$

and the same holds for $\widehat{d}_2\left(x'\right)$, for all $x' \in X$.

With this definition in hand, we can define our bargaining sets:

Definition 2. Bargaining Sets $\mathcal{S}$ under On-the-Match Search. *Fix market outcomes (S^*, q^*). Agents x and y bargain over*

$$\mathcal{S}_{xy} = \left\{ (S_1, S_2) : \exists \text{ consistent } c \text{ with } \widehat{S}_1^c = S_1 \text{ and } \widehat{S}_2^c = S_2 \right\}.$$

Bargaining sets under on-the-match search have two features that make finding a solution non-trivial. First, they may be non-convex, so Nash [1950]'s assumptions are not satisfied. Second, bargaining sets under on-the-match search may be non comprehensive.[7] Kaneko [1980] presents an extension of Nash [1950]'s model to allow for bargaining over non-convex (and non-comprehensive) sets. Kaneko's version of Nash's axioms permits set-valued decision functions.[8] For the purpose of this paper, let $\mathfrak{B}$ denote the class of all compact subsets S of $\mathbb{R}^2_+$. A decision correspondence ϕ assigns to each $S \in \mathfrak{B}$ a non-empty subset $\phi(S) \subset S$. Kaneko shows that a decision correspondence ϕ satisfies those axioms if and only if it is given by

$$\phi(S) = \left\{ \left(\overline{S}_1, \overline{S}_2\right) \in S : \overline{S}_1 \overline{S}_2 \geq S_1 S_2 \text{ for all } (S_1, S_2) \in S \right\}. \quad (2)$$

In the present model, bargaining sets $\mathcal{S}_{xy}$ are compact. See Appendix A.1 for details. So from now on we assume that $\phi(\cdot)$ as defined in (2) is the solution to the bargaining problem.

2.2 Equilibrium

We can now characterize an equilibrium in this economy.

Definition 3. Equilibrium with On-the-Match Search. *Take a pair of decision functions and allocations (d^*, π^*), its induced state of the economy (S^*, q^*) and its resulting bargaining sets $\{\mathcal{S}_{xy}\}_{(x,y) \in X \times X}$. We say (d^*, π^*) is an equilibrium if for all $(x, y) \in X \times X$,*

[7] $\mathcal{S}$ is comprehensive if $0 \leq x \leq y$ and $y \in \mathcal{S}$ implies $x \in \mathcal{S}$. Non-comprehensiveness makes the analysis in Zhou [1997] and others unapplicable in a setup with on-the-match search.

[8] Let us summarize the main differences between Nash's and Kaneko's axioms. First, Kaneko assumes strict Pareto Optimality, whereas Nash assumes a weak version. Second, the axiom of independence of irrelevant alternatives (IIA) is now: $T \subset S, \phi(S) \cap T \neq \varnothing \Rightarrow \phi(T) = \phi(S) \cap T$. This is consistent with Nash's IIA, but it is a fairly restrictive version. Third, Kaneko assumes a weak form of continuity in the choice correspondence ϕ.

1. *agreements are consistent,*[9]

2. *surpluses solve the bargaining problem:* $(S^*(x,y), S^*(y,x)) \in \phi\left(\mathcal{S}_{xy}\right)$, *and*

3. $S^*(x,y) > S^*(y,x)$ *only if there exists* $y' \neq y$ *with* $S^*(x,y) = S^*(x,y')$.

Before presenting our results, we provide a short discussion of our definition of equilibrium and its properties. First, equilibrium outcomes have some straightforward properties. For all matches, allocations exhaust production: $\pi(x,y) + \pi(y,x) = f(x,y)$. Moreover, agents only perform match-to-match transitions if they are strictly better off after the transition: $d\left(x,y,y'\right) = \mathbb{1}\left\{S^*(x,y') > S^*(x,y)\right\}$. These results are direct consequences of the assumption of Strict Pareto Optimality in bargaining. Second, our model is symmetric in that both sides come from the same population. Thus, by construction, a low firm matched to a high worker obtains the same surplus as a low worker matched to a high firm. Third, we focus on equilibria where behavior is a function of own type and partner's type. As a result, equilibrium outcomes with two agents of the same type are symmetric.

As pointed out by Shimer [2006], on-the-match search leads to some uninteresting multiplicity of equilibria. The third condition in our definition of equilibrium tackles this multiplicity. This condition guarantees that equilibria would survive if there was a small cost of transition. We elaborate on this point in Appendix A.2.

An equilibrium decision function d^* induces a steady state distribution of matches $e(x,y)$. We argue that this steady state distribution may be positively assortative due to bargaining.

Becker [1973]'s definition of positive assortative matching cannot be used in our model. As pointed out by Shimer and Smith [2000], when meeting agents takes time, individuals are willing to form matches with a group of partners rather than with singletons. Furthermore, the definition of assortative matching for markets with frictions proposed by Shimer and Smith [2000] does not apply in our model. When agents are allowed to search

[9] For each match, define agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ by $\widehat{d} = \left(d^*(x,y,y'), d^*(y,x,x')\right)$ and $\widehat{\pi} = (\pi^*(x,y), \pi^*(y,x))$.

on the match, acceptance sets are a function of the *current* partner type. Therefore we use the definition of positive assortative matching proposed by Lentz [2010].

Definition 4. Positive Assortative Matching. *Take any $x_1, x_2 \in X$ with $x_1 > x_2$. There is positive assortative matching if the distribution of partners of x_1 first order stochastically dominates the distribution of partners of x_2.*

3. Solution for the Two Type Case

The main insight of our paper is that frictions generate rents, and rent splitting may induce an endogenous preference for higher types. From now on, we refer to this endogenous preference as *hyperphily*, and we define it by $d^*(\ell, \ell, h) = d^*(h, \ell, h) = 1$ and $d^*(x, y, y') = 0$ for all other $x, y, y' \in \{\ell, h\}$. Therefore bargaining leads to sorting since hyperphily implies positive assortative matching.

Lemma 1. *In an equilibrium with hyperphily and two types, h's distribution of partners first order stochastically dominates ℓ's.*

See Appendix A.5.1 for the proof.

We now present necessary and sufficient conditions for the existence of an equilibrium with hyperphily. Then, we present a complete characterization of the model. We describe all possible equilibria and the conditions for their existence. This allows us to state necessary and sufficient conditions for hyperphily to be the unique equilibrium.

3.1 An Equilibrium with Hyperphily

Under hyperphily all agents strictly prefer agents of higher types. Let equilibrium allocations π^* be given by $\pi^*(\ell, \ell) = \ell, \pi^*(h, h) = h$ and let $\pi^*(\ell, h)$ be set so that $S^*(\ell, h) = S^*(h, \ell)$.

As explained in the previous section, our definition of equilibrium requires agents' transitions to be consistent with the surplus they obtain in each match. Moreover, we require that, *for each match*, no consistent agreement leads to a higher product of individual surpluses. Thus, the agreement between agents must be a global maximum in the bargaining set. This is a restrictive condition, which is not easy to check in general. We check each match step by step.

Pair (d^*, π^*) is consistent in an equilibrium with hyperphily if the resulting surpluses satisfy

$$S^*(h,h) > S^*(h,\ell) \quad \text{and} \quad S^*(\ell,h) > S^*(\ell,\ell)\,. \tag{3}$$

We discuss next when (d^*, π^*) solves the bargaining problem for each possible match.

Bargaining Solution in Match (ℓ, h)

Under hyperphily, no agent is indifferent between partners of different types. Therefore, given the third condition in the equilibrium definition, the total match surplus is split evenly. Then, if an agreement leading to a higher product of individual surpluses exists, it must also induce a larger *total* surplus. Now, a larger total surplus can only be reached if h chooses not to leave (since ℓ does not leave the match (ℓ, h) under hyperphily). Thus, to verify that no consistent agreement with a larger product of individual surpluses exists, it suffices to look at consistent agreements between ℓ and h where h does not leave. Let $\left(\widehat{S}^c_\ell, \widehat{S}^c_h\right)$ denote the surplus in some alternative agreement c. h does not leave for a high agent only if $\widehat{S}^c_h \geq S^*(h,h)$.

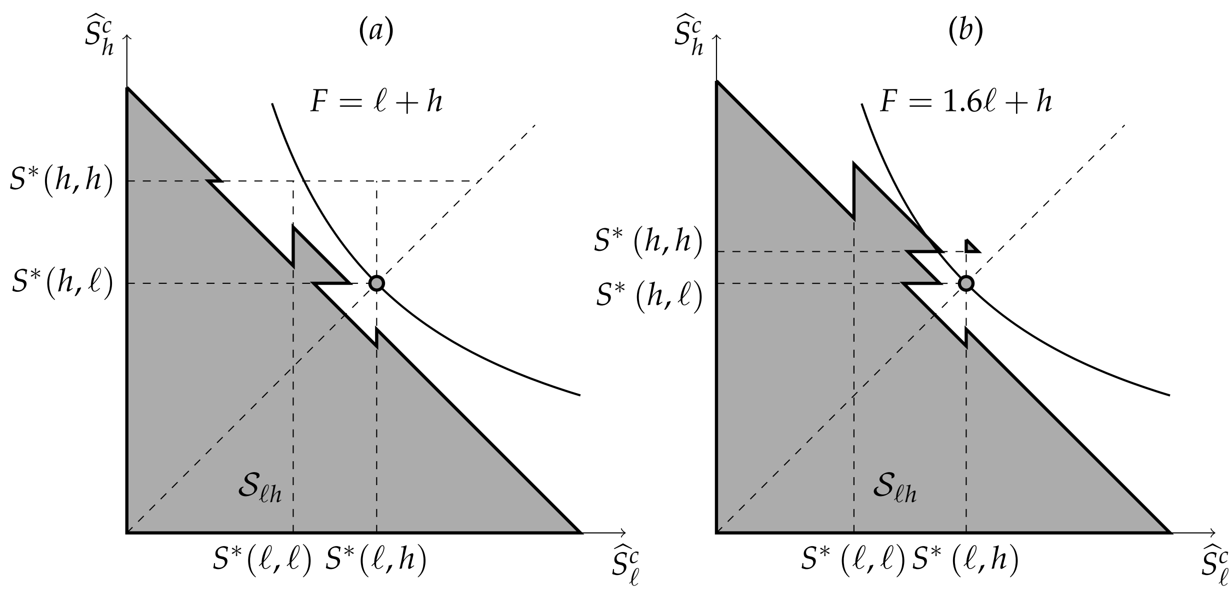
There are three possible kinds of agreements with h staying. Either ℓ always stays, or she leaves when she finds a new h, or she leaves when she finds either an ℓ or an h. In the first kind of agreement (c_1), both ℓ and h choose not to leave each other. In the second one (c_2), h always stays, but ℓ leaves when he finds a new h. In the third one (c_3), h always stays, but ℓ leaves when she finds *any* new partner. If the first kind of agreement exists, it makes both agents better off, so our original candidate is not an equilibrium. The second and third cases involve ℓ obtaining a lower surplus. However, given our definition of equilibrium, we need to check whether these cases lead to a higher product of individual surpluses. To sum up, (d^*, π^*) solves the bargaining problem in match (ℓ, h) if and only if Condition 1 holds.

CONDITION 1. *Let c_1, c_2 and c_3 be defined as stated. No allocation generates*

$$\widehat{S}_h^{c_1} \geq S^*(h,h) \text{ and } \widehat{S}_\ell^{c_1} \geq S^*(\ell,h), \text{ or}$$
$$\widehat{S}_h^{c_2} \geq S^*(h,h), \quad S^*(\ell,\ell) \leq \widehat{S}_\ell^{c_2} < S^*(\ell,h) \text{ and } \widehat{S}_\ell^{c_2}\widehat{S}_h^{c_2} > S^*(\ell,h)S^*(h,\ell), \text{ or}$$
$$\widehat{S}_h^{c_3} \geq S^*(h,h), \quad \widehat{S}_\ell^{c_3} < S^*(\ell,\ell) \text{ and } \widehat{S}_\ell^{c_3}\widehat{S}_h^{c_3} > S^*(\ell,h)S^*(h,\ell).$$

Panel *a* in Figure 1 presents bargaining set $\mathcal{S}_{\ell h}$ (the shaded area) under hyperphily and a modular production function. The curve depicted through $(S^*(\ell,h), S^*(h,\ell))$ indicates all points attaining product $S^*(\ell,h) \times S^*(h,\ell)$. As no element in the bargaining set attains a higher product of individual surpluses, hyperphily solves the bargaining problem. Note this occurs without complementarity in production and with patient agents.

Figure 1: Bargaining Sets $\mathcal{S}_{\ell h}$



Note: $\rho = 0.1$, $r = 0.1, \delta = 0.05, \ell = 1$ and $h = 2$.

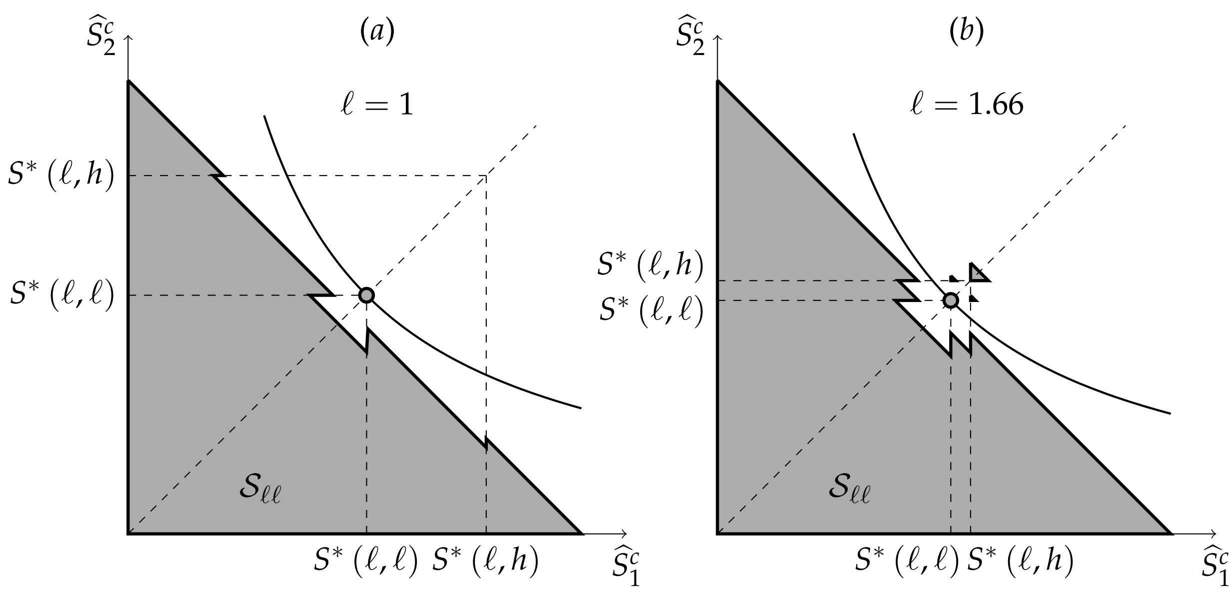
Now, when we make the production function sufficiently submodular, hyperphily is no longer an equilibrium, as shown in panel *b*. There, an alternative consistent agreement leads to a higher product of individual surplus *and* to a higher individual surplus for both agents. Agent ℓ receives *less* than half of a *larger* surplus in order to make her partner indifferent. Still, agent ℓ is better off. Therefore the first line of Condition 1 is violated.

In fact, in the example presented in panel b the second line of Condition 1 is also violated. An agreement that makes 1) h indifferent to a match with another h and 2) ℓ worse off than in a match to a different h is also consistent and leads to a larger product of individual surpluses.

Bargaining Solution in Match (ℓ, ℓ)

Panels a and b in Figure 2 present bargaining set $\mathcal{S}_{\ell\ell}$ with hyperphily and a modular production function. In panel b, types are closer: $\ell = 1.66$ and $h = 2$, whereas in panel a $\ell = 1$ and $h = 2$. It is easy to see that hyperphily solves the bargaining problem in panel a. In panel b, however, an alternative agreement with both ℓ agents choosing not to leave each other makes them better off, so hyperphily does not solve the bargaining problem.

Figure 2: Bargaining Sets $\mathcal{S}_{\ell\ell}$



Note: $\rho = 0.1$, $r = 0.1, \delta = 0.05, h = 2$ and $F = \ell + h$.

As in match (ℓ, h), there are three cases to consider. In the first (c_4), both agents choose not to leave each other (as in panel b). In the second (c_5), one ℓ agent never leaves while the second one leaves only when finding a willing h. In the third (c_6), one ℓ agent never leaves while the other one leaves when finding *any* willing partner. Let $\left(\widehat{S}_1^c, \widehat{S}_2^c\right)$ denote the surplus in an alternative contract c. To sum up, (d^*, π^*) solves the bargaining problem in match (ℓ, ℓ) if and only if Condition 2 holds.

CONDITION 2. *Let c_4, c_5 and c_6 be defined as stated. No allocation generates*

$$\widehat{S}_1^{c_4} \geq S^*(\ell, h), \text{ or}$$
$$\widehat{S}_1^{c_5} \geq S^*(\ell, h), \text{ and} \qquad S^*(\ell, \ell) \leq \widehat{S}_2^{c_5} < S^*(\ell, h), \text{ or}$$
$$\widehat{S}_1^{c_6} \geq S^*(\ell, h), \qquad \widehat{S}_2^{c_6} < S^*(\ell, \ell) \text{ and} \qquad \widehat{S}_1^{c_6}\widehat{S}_2^{c_6} > [S^*(\ell, \ell)]^2.$$

In fact, in the example presented in panel b, the second line in Condition 2 is also violated. An agreement that makes 1) one ℓ indifferent to a match with h and 2) the second ℓ at least as well off as before is also feasible.

Bargaining Solution in Match (h, h)

There is no endogenous destruction in match (h, h) and agents split the surplus evenly. Therefore, no consistent agreement leads to a higher product of individual surpluses.

Equilibrium with Hyperphily

Our first proposition summarizes the necessary and sufficient conditions for hyperphily.

PROPOSITION 1. EQUILIBRIUM WITH HYPERPHILY. *Conditions 1 and 2 are necessary and sufficient for hyperphily. In fact, Condition 1 defines an upper bound for F. This upper bound establishes the maximum degree of submodularity in the production function consistent with hyperphily. Moreover, Condition 2 defines a lower bound for F. This lower bound establishes the maximum degree of supermodularity in the production function consistent with hyperphily.*

Proof. Equation (3), and Conditions 1 and 2 generate 8 inequalities which determine when hyperphily can be an equilibrium. Whenever Conditions 1 and 2 are satisfied, then equation (3) also is. We express Conditions 1 and 2 as explicit functions of $(\ell, h, F, r, \rho, \delta)$. We present the details in Appendix A.6. ■

Figure 3 illustrates Conditions 1 and 2. In each panel, the shaded area represents the set of primitives such that hyperphily is an equilibrium. The upper bound is determined by Condition 1, and the lower bound is determined by Condition 2. Panels a, b, c and d present the set of values of F consistent with an equilibrium with hyperphily as a function of the matching rate ρ, the destruction rate δ, the discount rate r and the difference between $h - \ell$ respectively.

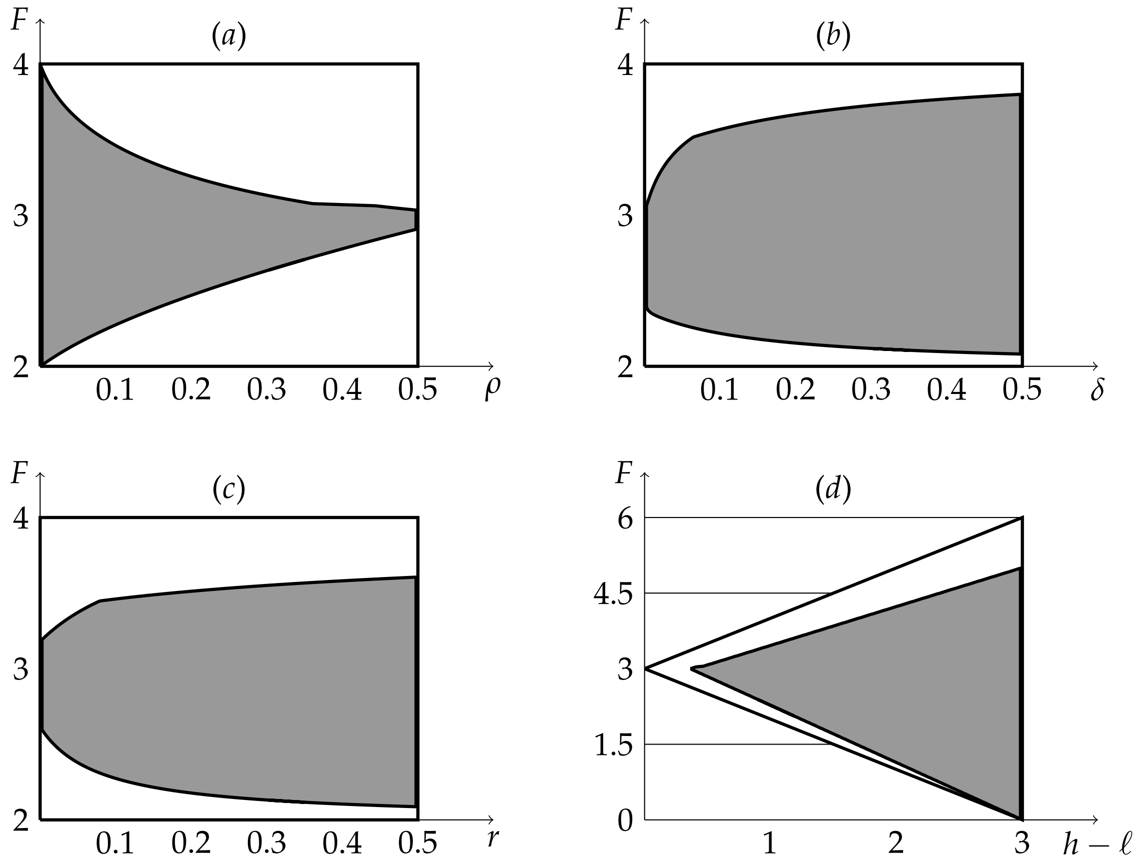
As we see in panel a, low values of ρ allow for hyperphily even when the production function is significantly submodular. As ρ decreases, the probability that h leaves the match (ℓ, h) becomes lower, so compensating him to make him stay becomes less attractive. In the limit as $\rho \to 0$, hyperphily is an equilibrium for all degrees of complementarity in the production function. On the other side, as $\rho \to \infty$, the duration of any match with voluntary destruction approaches zero. Thus, hyperphily cannot be an equilibrium.

As we see in panel b, low values for the destruction rate δ leave less room for hyperphily. When δ is low, there are few unmatched agents, so being unmatched becomes relatively less attractive. However, in the limit as $\delta \to 0$, there are equilibria with hyperphily even when the production function is submodular. On the other side, as δ increases, endogenous destruction becomes less relevant relative to exogenous destruction. Therefore the maximum degree of submodularity tolerated by hyperphily increases. As $\delta \to \infty$, the duration of every match goes to zero independently of the allocation of production, so hyperphily holds for every value of the other primitives.

Panel c illustrates the first intuition discussed in the Introduction. As agents become more impatient (higher r), complementarity in production becomes less important relative to rent splitting. In the limit as $r \to \infty$, hyperphily is an equilibrium for any degree of complementarity in the production function. When agents are patient, there are equilibria with hyperphily provided that the complementarity in production is not too strong.

Panel d illustrates the second intuition discussed in the Introduction. When the difference between types is close to zero, ℓ does not get much from extracting surplus from h. Thus, ℓ makes h indifferent, so he does not leave for another h. Agreement c_1 leads to a higher product of surpluses in match (ℓ, h). As $h - \ell$ increases, hyperphily becomes an equilibrium. Moreover, as ℓ approaches 0, hyperphily holds even for a significantly submodular production function.

Figure 3: Existence of Equilibrium with Hyperphily



In (a), $\ell = 1, h = 2, \delta = 0.05$ and $r = 0.1$.
In (b), $\ell = 1, h = 2, \rho = 0.1$ and $r = 0.1$.
In (c), $\ell = 1, h = 2, \delta = 0.05$ and $\rho = 0.1$.
In (d), $r = 0.1, \rho = 0.1, \delta = 0.05$ and $\ell + h = 3$ with $0 < \ell < 1.5 < h < 3$.

3.2 All Possible Equilibria

Depending on the value of the primitives, several different equilibria arise in our simple two type model. In principle, there could be nine different types of equilibria, each associated to a different vector d^*. Table 1 shows all of them. Solving for the conditions for each equilibrium involves going through the same process as already performed for hyperphily. First, we verify that transitions are consistent with surplus. Second, for each possible match, we verify that the equilibrium agreement solves the bargaining problem.

PROPOSITION 2. ALL EQUILIBRIA WITH TWO TYPES. *For each possible type of equilibrium* *in Table 1 there is a set* $(\ell, h, F, r, \rho, \delta)$ *such that the equilibrium holds.*[10]

Table 1: All Possible Equilibria in the Two Type Model

ℓ's decision	h's decision		
	$d^*(h,\ell,h)=1$ $d^*(h,h,\ell)=0$	$d^*(h,\ell,h)=0$ $d^*(h,h,\ell)=0$	$d^*(h,\ell,h)=0$ $d^*(h,h,\ell)=1$
$d^*(\ell,\ell,h)=1$ $d^*(\ell,h,\ell)=0$	Hyperphily	Weak Heterophily	Strict Heterophily
$d^*(\ell,\ell,h)=0$ $d^*(\ell,h,\ell)=0$	Weak Homophily	Indifference	*Impossible*
$d^*(\ell,\ell,h)=0$ $d^*(\ell,h,\ell)=1$	Strict Homophily	Weak Homophily	*Impossible*

We discuss now the main results regarding other equilibria. First, note that neither weak nor strict heterophily can be equilibria with a supermodular production function. To see this, note that $\pi^*(h,\ell) < h$ makes h strictly prefer another h. Similarly, $\pi^*(\ell,h) < \ell$ makes ℓ strictly prefer another ℓ. Then, $F \geq h + \ell$ is a necessary condition for both weak and strict heterophily. It is also straightforward to show that neither weak nor strict homophily can be equilibria with a submodular production function. Finally, only strict heterophily exists when h strictly prefers ℓ (see Appendix A.3 for details).

Only strict heterophily implies negative assortative matching. Thus, sorting negatively only occurs with a submodular production function. Positive assortative matching occurs both with homophily and hyperphily. Random sorting only happens if both h and ℓ are indifferent, which requires $\pi^*(\ell,h) = \ell$ and $\pi^*(h,\ell) = h$. Hence indifference, and therefore random sorting, can only happen if the production function is modular.

Figures 4 and 5 illustrate the set of primitives which lead to each possible equilibrium.

Equilibria with strict heterophily or strict homophily are rare, as shown in panels c of Figures 4 and 5. In strict heterophily h prefers a match with ℓ over a more produc-

[10]The sets are obtained analogously to those from Proposition 1. We present closed-form solutions for these sets (and how they are obtained) in an online appendix.

tive match with another h. This can happen when ℓ's outside option is lower than h's. Therefore, although the production of the match (ℓ, h) is smaller than the production of the match (h, h), the total surplus of the match (ℓ, h) is larger than the total surplus of the match (h, h). On the other hand, strict homophily requires ℓ to strictly prefer another ℓ, which is demanding given that the match (ℓ, h) is more productive. As in the case of strict heterophily, the agent prefers a less productive match because its total surplus is larger. When r or δ increase, or when ρ decreases, the outside option becomes less relevant and therefore strict homophily and strict heterophily require stronger complementarity in production.

If agents can search while matched, the match duration depends on the bargaining outcome. Symmetric surplus splitting might not solve the bargaining problem, as it occurs in the cases of weak heterophily and weak homophily. In these equilibria, one agent is indifferent between partner types and takes a larger fraction of the total surplus in the match (ℓ, h). Uneven surplus splitting produces a larger product of surpluses because it implies a longer duration of the match and a larger total surplus. These equilibria are more likely to exist when agents care more about endogenous destruction (when r or δ are low); or when it is easier to find partners (when ρ is large). This is shown in panels b and d of Figures 4 and 5

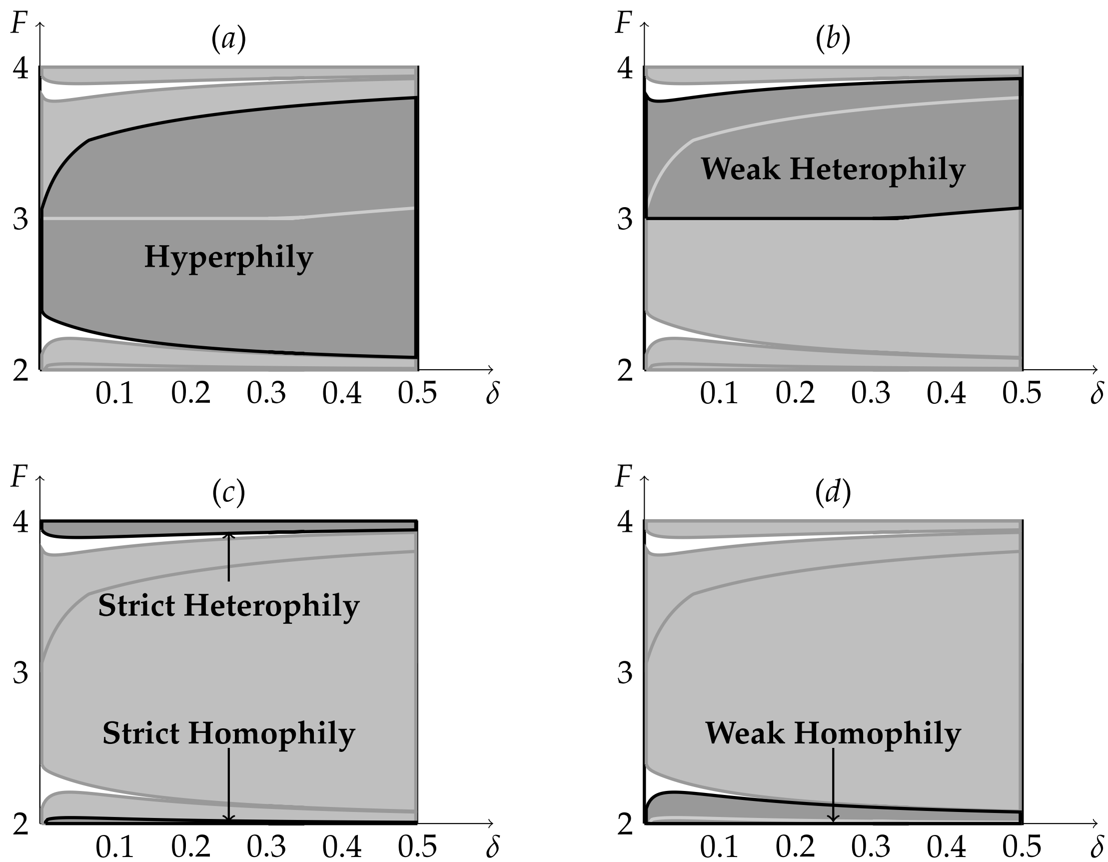
When δ and r increase, or when ρ decreases, the region where hyperphily is the unique equilibrium grows. To study the interplay of these parameters, consider the index of labor market frictions $\kappa \equiv \frac{\rho}{\delta}$.[11] When κ increases, the equilibrium in this economy does not necessarily becomes closer to the one described in Becker [1973]. When $\kappa \to \infty$ the distribution of matches converges to perfect sorting in an equilibrium with hyperphily. However, the existence of equilibrium with hyperphily depends on how κ grows. If κ is large because ρ is large, there is no room for an equilibrium featuring hyperphily. This is because meeting alternative partners becomes more likely and therefore an agreement in the match (ℓ, h) where h does not leave the match is more likely to be the solution of

[11] κ has been used as an index of frictions in a number of papers, see for example Ridder and van den Berg [2003]. A larger κ implies weaker frictions.

the bargaining problem. If κ is large because δ is small there can still be equilibria with hyperphily without a supermodular production function. If agents are patient enough, ℓ is willing to trade off a shorter duration of the match for a higher allocation.

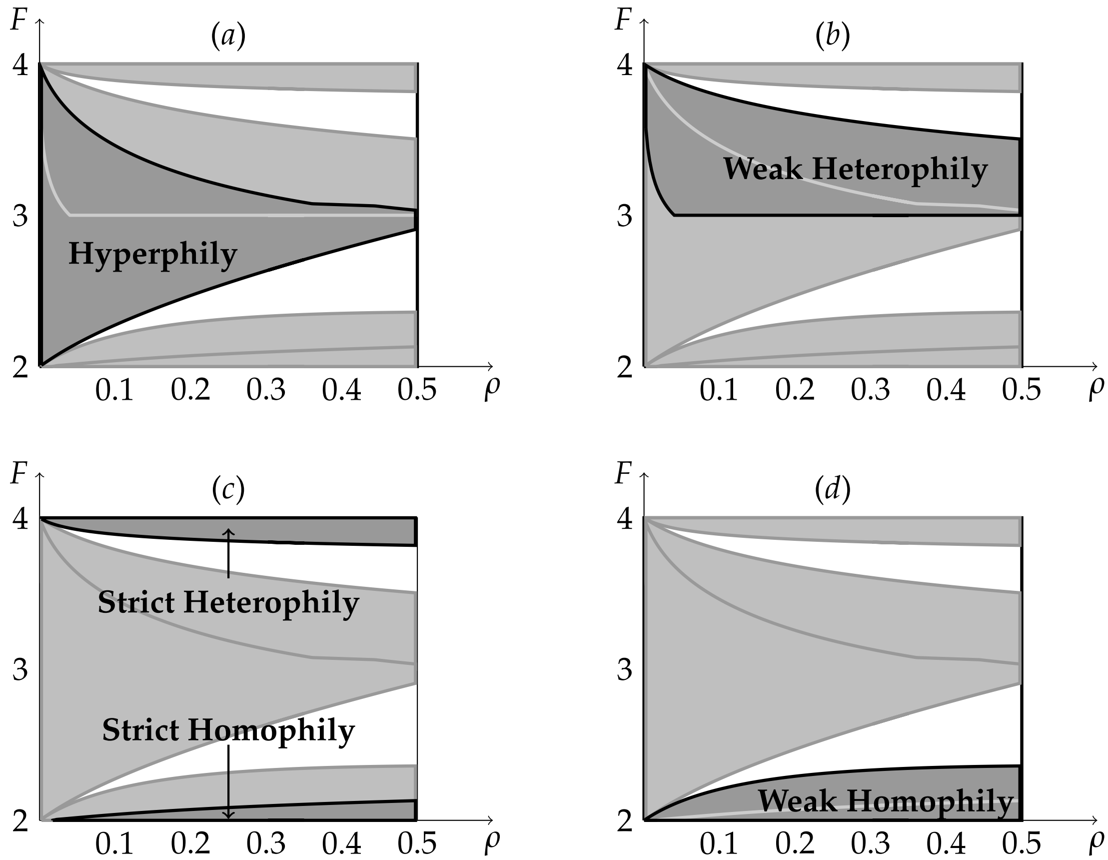
On the other hand, as κ decreases positive assortative matching becomes pervasive. This results goes against the conventional wisdom that stronger frictions require stronger complementarity in production for the equilibrium to be positively assortative. In the next section we show that also for a model with N types, if δ or r are high enough, or if ρ is low enough, hyperphily is the unique equilibrium. Better partners might have better outside options and thus be more likely to leave the match. However, when individuals care less about match duration, the larger production that comes from better partners shapes preferences for partner types.

Figure 4: The Impact of Destruction Rate δ



Note: $\ell = 1, h = 2, r = 0.1$ and $\rho = 0.1$.

Figure 5: The Impact of Search Intensity ρ



Note: $\ell = 1, h = 2, r = 0.1$ and $\delta = 0.05$.

4. The Case with N Types

In this section, we study equilibrium behavior when there is a large number of types. We apply the intuition described in the introduction and developed for 2 types to the case of N types. Now, with a large number of types one cannot characterize equilibrium behavior for all parameter values. We address this in two ways. First, we consider the case when the future is of limited importance for agents. This case encompasses impatient agents (high value for r), matches with high exogenous destruction rates (high value for δ), and low meeting rates (low values for ρ). We show that for low enough $\frac{\rho}{r+\delta}$, hyperphily is an equilibrium, and no other equilibrium exists.[12] Second, we present numerical examples.

We extend the model to allow for N types: $x \in X = \{1, 2, \ldots, N\}$. Types are in equal proportion in the population: $e(x, y) : X \times X \cup \{\emptyset\} \to \left[0, \frac{1}{N}\right]$ has $\sum_{y \in X \cup \{\emptyset\}} e(x, y) = \frac{1}{N}$. We assume the production function $f(x, y) : X \times X \to \mathbb{R}_+$ is strictly increasing in both variables. All other functions are modified appropriately to allow for N types.

4.1 When the Future is of Limited Importance

When agents care mostly about the present, it is current payoffs that matter the most. We show in Proposition 3 that this makes hyperphily an equilibrium, and in fact the only possible one. First, to show that only hyperphily can be an equilibrium, we argue that for low values of $\frac{\rho}{r+\delta}$, per period equilibrium payoffs are close to an equal split of production. We show then that when $\frac{\rho}{r+\delta}$ is low, surplus from the match depends mostly on current payoffs. Since payoffs reflect production and production is increasing in the partner's type, then surplus is increasing in partner type. Only hyperphily can be an equilibrium. Second, we argue that hyperphily is in fact an equilibrium. To show this, we compute individual surpluses under hyperphily, and show that for low values of $\frac{\rho}{r+\delta}$, surpluses depend mainly on production. This makes hyperphily consistent. Finally, we show that no alternative consistent agreement leads to a higher producct of individual surpluses.

Proposition 3. Hyperphily with N Types. *When $\frac{\rho}{r+\delta}$ is low enough, (d^*, π^*) is an equilibrium if and only if it is hyperphily.*

See Appendix A.4 for the proof.

4.2 Numerical Examples with N Types

In Section 3 we provide necessary and sufficient conditions for hyperphily in a model with two types. We then show in Section 4.1 that when there are N types, as the future

[12] We computed the steady state distribution under hyperphily for $1,000$ values of $\kappa^{-1} \in (0, 1)$ and $1,000$ values of $\kappa \in (0, 1)$ for $N = 10, 20$ and 100. In all cases there is positive assortative matching in the steady state distribution.

becomes less relevant hyperphily is the unique equilibrium. We provide now numerical examples in a model with N types for parameter values in line with the literature. We show that equilibria with hyperphily and positive assortative matching arise without complementarity in production.

We solve the model by a nested fixed point algorithm. We start from a flat distribution of matches and calculate value functions for all possible matches. These first value functions induce preferences over partner types which we use to update the steady state distribution of matches. With the updated steady state distribution, we update the value functions. We iterate this process until we find a fixed point for both the steady state distribution of matches and the value functions.

We search specifically for equilibria without indifference over partners. When no agent is indifferent, symmetric surplus splitting solves the bargaining problem in all matches. Once we find a candidate set of value functions and distribution of matches that solves the model, we check that the solution maximizes the product of surpluses in all matches. To do this, we take each match and evaluate all possible consistent agreements $c = \left(\widehat{d}, \widehat{\pi}\right)$, given our candidate. Our candidate solves the model if it maximizes the product of surpluses in every match.

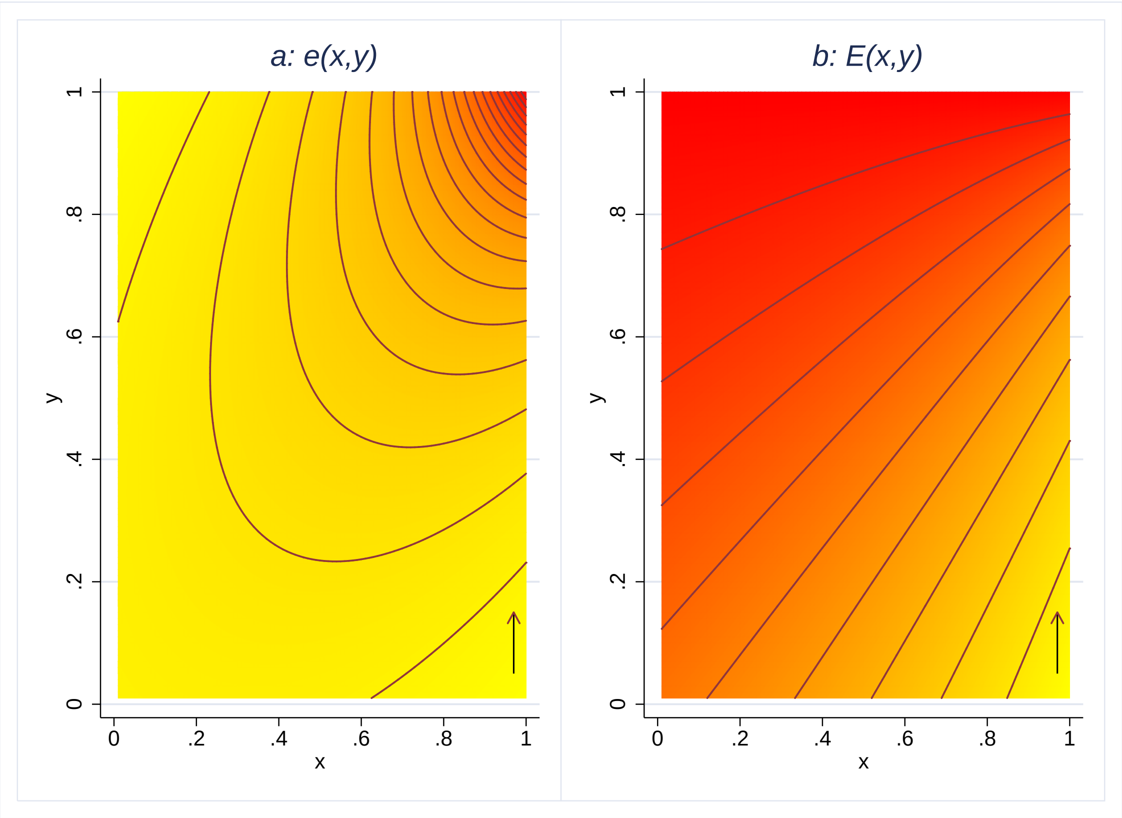
The following example shows hyperphily for a case with $N = 100$.

EXAMPLE 1. *Types are uniformly distributed in a* 100*-point grid between* 0 *and* 1*. Production is modular:* $f(x, y) = x + y$*. (*$\rho = 0.1, \delta = 0.05$ *and* $r = 0.1$*).*

In Example 1, $S^*(x, y)$ is increasing in partner type, so hyperphily is an equilibrium. Panel *a* in Figure 6 shows the probability distribution function $e(x, y)$ and panel *b* shows the cumulative density function $E(x, y) \equiv \sum_{\tilde{y} \in X \cup \{\varnothing\}, \tilde{y} < y} e(x, \tilde{y})$. Panel *a* shows that a high type agent is more likely to be matched to another high type than to a low type. The cumulative distribution is more informative of sorting patterns. A darker region in panel *b* denotes a larger $E(x, y)$. The contour lines of $E(x, y)$ are increasing in x, so the cumulative distribution of partners of x is decreasing in his own type, which implies positive assortative matching.

A similar result holds with a slightly submodular production function. However, if

Figure 6: Hyperphily Leads to Positive Assortative Matching.



either δ or r decrease enough or if ρ increases enough, symmetric surplus splitting does not maximize the product of surpluses in some matches. Therefore, positive assortative matching does not hold for the whole support of types. Take Example 1 and double the search intensity (so $\rho = 0.2$). Now, hyperphily does not maximize the product of individual surpluses in matches where $|x - y|$ is large. For a given set of parameters, if there is at least one match where the agreement from hyperphily does not solve the bargaining problem, then hyperphily is not an equilibrium. However, this does not imply that matching is not positively assortative. As in the case of two types with weak and strict homophily, there may be other equilibria with positive assortative matching. However, we do not characterize all possible equilibria in this case.

5. Extensions and Discussion

5.1 Different Search Intensities

We now relax the assumption that on-the-match search efficiency is equal to search efficiency out-of-the-match. Let ρ_0 denote the search intensity of an unmatched agent and let ρ_1 be the search efficiency of a matched one. The meeting rate is simply the product of the search intensities of those who meet. For example, there are $\rho_0 \rho_1 e(\ell, \varnothing) e(h, h)$ unmatched ℓ-type agents who meet h-type agents matched to other h-type agents. Similarly, there are $(\rho_0)^2 e(\ell, \varnothing) e(h, \varnothing)$ unmatched ℓ-type agents who meet unmatched h-type agents.

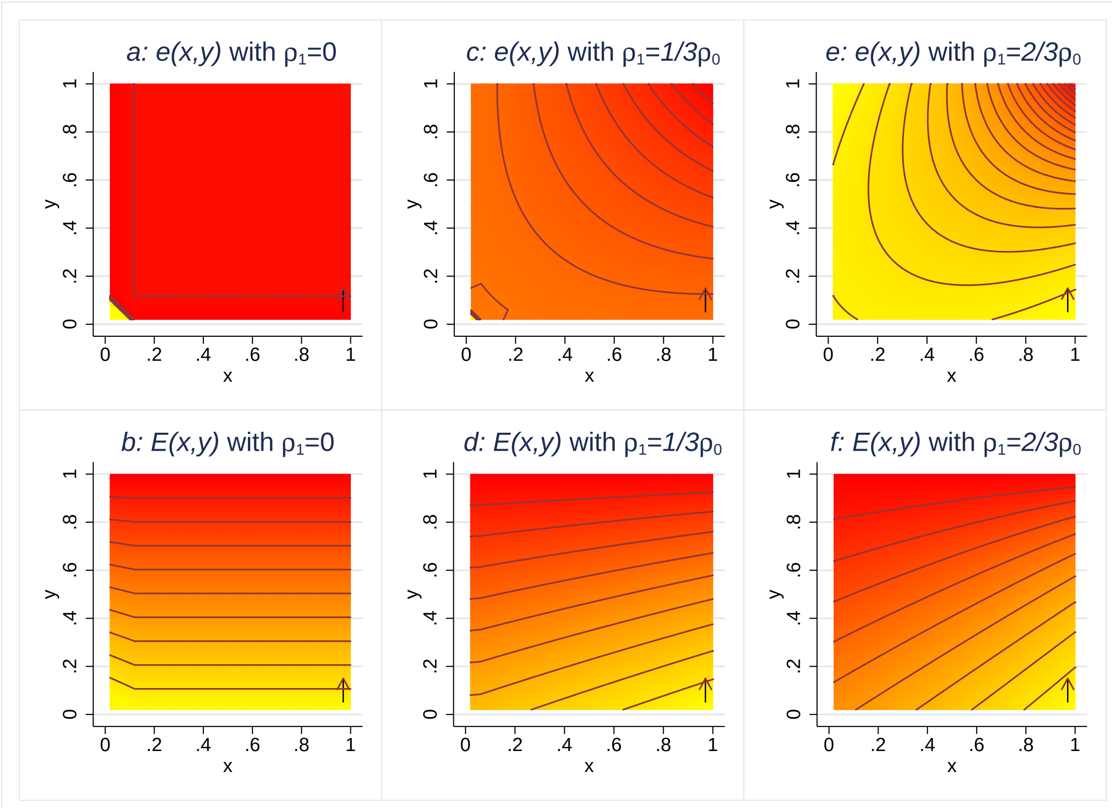
In Example 2, we discuss how different values of ρ_1 affect the equilibrium.

Example 2. *Types are uniformly distributed in a* 50*-point grid between* 0 *and* 1*. Production is modular:* $f(x, y) = x + y$*.* ($\delta = 0.05, r = 0.1$ *and* $\rho_0 = \sqrt{0.1}$)*. Consider three cases:* $(i) : \rho_1 = 0, (ii) : \rho_1 = \frac{1}{3}\rho_0$*, and* $(iii) : \rho_1 = \frac{2}{3}\rho_0$

Figure 7 illustrates the distribution of matches in Example 2. Panels *a* and *b* present the cumulative distribution and probability distribution of matches when search efficiency on-the-match is zero. With $\rho_1 = 0$ our model is identical to that in Shimer and Smith [2000]. The equilibrium features hyperphily but no positive assortative matching, since agents are not allowed to search on-the-match. When $\rho_1 = 0$ some low type unmatched agents do not accept other low type agents. They prefer to wait for partners of better quality from whom they can extract higher rents.

When we allow agents to search on-the-match but with low search efficiency, there is assortative matching for agents of high type, but low agents still prefer to wait unmatched for more profitable partners. Therefore matching is not positively assortative for low type agents. With the parameter values used in these simulations, for values of search efficiency on-the-match as low as two thirds of the search efficiency out-of-the-match, there is no difference in acceptance sets between unmatched agents of different types. All unmatched agents accept all partners, and since agents search on-the-match and prefer better partners, there is positive assortative matching in the whole support of types.

Figure 7: Different on-the-job and out-of-the-job Search Intensity.



5.2 An Example with Renegotiation

The main result in this paper is that bargaining can lead to hyperphily, even without productive complementarity. When agents are allowed to search on the match, hyperphily generates positive assortative matching. In our stylized model, agents are not allowed to renegotiate when they find alternative partners. However, the mechanism we highlight may also hold if agents are allowed to renegotiate. We present next an example with on-the-match search, renegotiation and no complementarity in production that features hyperphily and positive assortative matching.

It is not straightforward how to model renegotiation when both partners are allowed to search on the match because both partners' outside options may change. Kiyotaki and Lagos [2007] present a search model where both the firm and the worker search on-the-match. In their setting, contracts can be renegotiated if a partner has a credible threat. When an agent finds an alternative partner, her current partner and the poaching one compete *à la* Bertrand. Kiyotaki and Lagos do not study sorting since agents are homogeneous in their model. Matches are heterogeneous due to a fixed match-specific productivity shock.

For simplicity, consider a case with infinitely impatient agents and 2 types where allocation are bargained *à la* Kiyotaki and Lagos. As in Section 4.1, when the future is less relevant, the value of a match converges to its flow-payoff and the outside options are zero. If there is Bertrand competition between two possible suitors whose outside option is to be unmatched, the one with whom the output is larger wins. Since the production function is increasing in agents' type, the low type agent is left for the high type one. However, when matched agents meet other matched agents, some of the transitions that occur without renegotiation no longer happen. In particular, if an agent h matched to ℓ meets another h who is also matched to ℓ, both h renegotiate their contracts and no match is destroyed.[13] However, both h and ℓ still leave ℓ in case they find an unmatched h and therefore the steady-state distribution of partners of h stochastically dominates the distribution of partners of ℓ.[14]

6. Conclusion

In this paper we argue that frictions represent a natural reason for positive assortative matching. With frictions, finding alternative partners is time consuming. When two agents meet there are gains from trade to be bargained. Since better agents are more productive, we find conditions such that gains from trade are increasing in agents' types. Thus, an endogenous preference for better partners (hyperphily) arises. We show that if agents are allowed to search on-the-match, hyperphily shapes the distribution of matches generating positive assortative matching.

We present a partnership model that builds on Shimer and Smith [2000]. We depart

[13] See proposition 1 in Kiyotaki and Lagos.

[14] See Section A.5.2 in the Appendix.

from their model in that we allow for bilateral on-the-match search. On-the-match search is natural extension since job-to-job transitions are pervasive in the labor market. We first analyze the case where agents are of one of two types: either low or high productivity. We provide necessary and sufficient conditions for hyperphily and show that this preference leads to positive assortative matching. We highlight conditions such that positive assortative matching arises even with a modular or submodular production function. Moreover, when frictions are sufficiently large, this is the unique equilibrium. Our result extends to the case of any (finite) number of types. We show that as agents become impatient or frictions large, an equilibrium where agents endogenously prefer higher types exists and it is the only possible one.

Our result goes against the conventional wisdom that stronger frictions require stronger complementary in production for the equilibrium to be positively assortative. Moreover, it invalidates the interpretation of positive assortative matching as evidence of complementary in production.

A. Appendix

A.1 Bargaining Sets are Compact

Lemma 2. *Take any state of the economy (S^*, q^*). Then, bargaining sets $\mathcal{S}_{xy}$ under on-the-match search are compact.*

Proof. Take a sequence $\left\{\left(S_1^n, S_2^n\right)\right\}_{n=1}^{\infty} \in \mathcal{S}$ with $\lim_{n\to\infty}\left(S_1^n, S_2^n\right) = \left(\overline{S}_1, \overline{S}_2\right)$. Let $\left\{\left(\widehat{\pi}^n, \widehat{d}^n\right)\right\}_{n=1}^{\infty}$ denote a sequence of consistent agreements generating $\left\{\left(S_1^n, S_2^n\right)\right\}_{n=1}^{\infty}$. Since $\left\{\left(S_1^n, S_2^n\right)\right\}_{n=1}^{\infty}$ converges, there exists a convergent subsequence $\left\{\left(S_1^{n_m}, S_2^{n_m}\right)\right\}_{m=1}^{\infty}$ where for some i and for all m, $S_i^{n_m} > \overline{S}_i$ or $S_i^{n_m} < \overline{S}_i$. Without loss of generality, let $i = 1$. Along the subsequence, for large enough m, the consistent decision function is constant:[15], there exists M_1 such that $\widehat{d}_1^{m_n} = \overline{d}_1$ for all $m > M_1$. Moreover, say we picked our subsequence so that also either $S_2^{n_m} > \overline{S}_2$, or $S_2^{n_m} < \overline{S}_2$, or $S_2^{n_m} = \overline{S}_2$ for all m. In the first two cases, there exists M_2 such that $\widehat{d}_2^{m_n} = \overline{d}_2$ for all $m > M_2$. Then, along the

[15]To see this, assume $S_1^{n_m} > \overline{S}_1$. Then, there is M_1 large enough such that for $m > M_1$: $\min_{y'\in X}\left\{S^*(x,y') : S^*(x,y') > \overline{S}_1\right\} > S_1^{n_m} > \overline{S}_1$.

subsequence, for $m > \max\{M_1, M_2\}$, S_1^n and S_2^n are simply linear functions of $\pi_1^{n_m}$ and $\pi_2^{n_m}$. Since S^{n_m} converges, so does $\pi^{n_m} \to \overline{\pi}$. Now, since $\pi_1^{n_m} + \pi_2^{n_m} \leq f(x,y)$ for all m, then it is also true that $\overline{\pi}_1 + \overline{\pi}_2 \leq f(x,y)$. Then, $\bar{c} = \left(\overline{\pi}, \overline{d}\right)$ is consistent and generates $\left(\overline{S}_1, \overline{S}_2\right)$. We still need to consider the case $S_2^{n_m} = \overline{S}_2$ for all m. In this case, it may be the case that agent 2 is indifferent. If he is not, we are done. If he is, Let $\overline{d}_2$ denote the decision function when m is large enough and the indifference is resolved in favor of 1. $\overline{\pi}_2$ is fixed along the sequence, so let $\overline{\pi}_1 = f(x,y) - \overline{\pi}_2$. The surplus derived from this agreement has $S_2 = \overline{S}_2$. Assume along the sequence $S_1^{n_m} < \overline{S}_1$. Note $S_1 \geq \overline{S}_1$. Adjust π_1 and d_2 as needed. ■

A.2 Details on Multiplicity of Equilibria

In an online appendix we provide a simple example of how on-the-match search can lead to some uninteresting multiplicity of equilibrium. The third condition in our definition of equilibrium guarantees that equilibria are in the following sense.

Assume that agents have to pay a small cost $t > 0$ each time they quit their current partner to form a new match. Surplus from matches are then given by the following slightly modified version of (1):

$$S^*(x,y) = \left(r + \delta + \sum_{x' \in X} d^*(y,x,x')\, q^*(y,x')\right)^{-1} \Bigg[\pi^*(x,y) + \sum_{y' \in X} d^*(x,y,y')\, q^*(x,y') \left[S^*(x,y') - S^*(x,y) - t\right] - \sum_{y' \in X} q^*(x,y')\, S^*(x,y')\Bigg]$$

Take a pair (d^*, π^*) satisfying the first two conditions in our equilibrium definition. We show next that $S^*(x,y) > S^*(y,x) \Rightarrow \exists\, y' : S^*(x,y) = S^*(x,y') - t$ must be satisfied. Assume it is not. Then, there exists an alternative consistent agreement between x and y which leads to a higher product of individual surpluses. To build it, keep the decision function unchanged but pick $\tilde{\pi}(x,y) = \pi^*(x,y) - \varepsilon$ and $\tilde{\pi}(y,x) = \pi^*(y,x) + \varepsilon$. For small $\varepsilon > 0$, agent x does not change his behavior. Thus, the new pair $(d^*, \tilde{\pi})$ is consistent. Moreover, again for small $\varepsilon > 0$, the product of individual surpluses is larger. Then, the original pair (d^*, π^*) does not solve the bargaining problem.

A.3 Only Strict Heterophily with $d^*(h,h,\ell)=1$

Lemma 3. *$S^*(h,\ell) > S^*(h,h) \Rightarrow S^*(\ell,h) > S^*(\ell,\ell)$.*

Proof. First, since $S^*(h,\ell) > S^*(h,h)$, the third condition in the equilibrium definition guarantees $S^*(\ell,h) \geq S^*(h,\ell)$. Next, consider the following alternative agreement for (h,h): they never leave each other and they split production. Let $\widehat{S}$ denote the surplus resulting from that agreement. Then,

$$S^*(h,\ell) \geq \widehat{S} = (r+\delta)^{-1}\left[h - q^*(h,\ell)\, S^*(h,\ell) - q^*(h,h)S^*(h,h)\right]$$

We show our result by contradiction. Assume $S^*(\ell,\ell) \geq S^*(\ell,h)$. Note that $q^*(\ell,h) \geq q^*(h,h)$ and $q^*(\ell,\ell) \geq q^*(h,\ell)$, since both agents prefer low types (at least weakly). Then,

$$S^*(\ell,\ell) = (r+\delta)^{-1}\left[\ell - q^*(\ell,\ell)\, S^*(\ell,\ell) - q^*(\ell,h)S^*(\ell,h)\right] < \widehat{S}$$

To sum up, $\widehat{S} > S^*(\ell,\ell) \geq S^*(\ell,h) \geq S^*(h,\ell) \geq \widehat{S}$. That is our contradiction. ■

A.4 Details on Proposition 3

We show first that if (d^*,π^*) is an equilibrium, then it has to be hyperphily. To do this, let us build a simple agreement for match (x,y): they share production evenly and leave for any willing partner. $\widetilde{S} = \widetilde{S}(x,y) = \widetilde{S}(y,x)$ denotes the surplus from such agreement:

$$\begin{aligned}\widetilde{S} &= \left(r+\delta+\sum_{x'\in X} q^*(y,x') + \sum_{y'\in X} q^*(x,y')\right)^{-1} \frac{f(x,y)}{2} \\ &\geq \left(r+\delta+\sum_{x'\in X}\rho\frac{1}{N} + \sum_{y'\in X}\rho\frac{1}{N}\right)^{-1}\frac{f(x,y)}{2} \geq \frac{1}{r+\delta+2\rho}\frac{f(x,y)}{2}\end{aligned}$$

There exists a consistent agreement leading to surplus higher than $\widetilde{S}$ for both agents.[16] Moreover, surplus is bounded above – see (1): $(r+\delta)\,S^*(x,y) \leq \pi^*(x,y)$. Then,

$$\pi^*(x,y)\pi^*(y,x) \geq (r+\delta)^2\, S^*(x,y)S^*(y,x) \geq \left(\frac{r+\delta}{r+\delta+2\rho}\frac{f(x,y)}{2}\right)^2 \quad \text{which implies}$$

[16] The argument is straightforward. Consider the simple agreement described. Calculate x's surplus. See who would x actually optimally choose to leave for. Assume x behaves that way. Notice now y's surplus is weakly larger. Calculate y's best response now. At each step, neither x nor y can be worse off. So they leave each time for less people. Eventually, the process stops. That behavior is consistent.

$$(1-k)\,\frac{f(x,y)}{2} \le \pi^*(x,y) \le (1+k)\,\frac{f(x,y)}{2} \quad \text{with } k = \sqrt{1-\left(1+2\frac{\rho}{r+\delta}\right)^{-2}}. \tag{4}$$

Next, consider matches (x,y) and $(x,y+1)$. In the best possible case for x when matched to y, y never leaves him, and pays him the maximum value in (4). In the worst possible case when matched to $y+1$, $y+1$ always leaves x, and pays him the minimum value in (4). Let $\overline{X}(y) = \{y' \in X : d\,(x,y,y') = 1\}$ and $\underline{X}(y) = \{y' \in X : d\,(x,y,y') = 0\}$. Note that $X = \overline{X}(y) \cup \underline{X}(y)$ and $\overline{X}(y) \cap \underline{X}(y) = \varnothing$. Then $S^*(x,y)$ is bounded above as follows:

$$\begin{aligned}
(r+\delta)\,S^*(x,y) &\le \pi^*(x,y) - \left[\sum_{y'\in\overline{X}(y)} q^*(x,y')\right] S^*(x,y) - \sum_{y'\in\underline{X}(y)} q^*(x,y')\,S^*\left(x,y'\right) \\
&\le \frac{f(x,y)}{2}\,(1+k) - \left[\sum_{y'\in\overline{X}(y)} q^*(x,y')\right] S^*(x,y) - \sum_{y'\in\underline{X}(y)} q^*(x,y')\,S^*\left(x,y'\right)
\end{aligned}$$

Next, in the worst case for x, agent $y+1$ always leaves him, and pays him the minimum value in (4). Then $S^*(x,y+1)$ is bounded below as follows:

$$\begin{aligned}
(r+\delta)\,S^*(x,y+1) &\ge \pi^*\,(x,y+1) - \left[\sum_{x'\in X} q^*(y+1,x') + \sum_{y'\in\overline{X}(y)} q^*(x,y')\right] S^*(x,y+1) \\
&\quad - \sum_{y'\in\underline{X}(y)} q^*(x,y')\,S^*\left(x,y'\right) \\
&\ge (1-k)\frac{f\,(x,y+1)}{2} - \left[\rho + \sum_{y'\in\overline{X}(y)} q^*(x,y')\right] S^*(x,y+1) \\
&\quad - \sum_{y'\in\underline{X}(y)} q^*(x,y')\,S^*\left(x,y'\right) \\
&\ge \left(1-k-\frac{\rho}{r+\delta}(1+k)\right)\frac{f\,(x,y+1)}{2} \\
&\quad - \left[\sum_{y'\in\overline{X}(y)} q^*(x,y')\right] S^*(x,y+1) - \sum_{y'\in\underline{X}(y)} q^*(x,y')\,S^*\left(x,y'\right)
\end{aligned}$$

Then, whenever the following condition holds, $S^*(x,y+1) > S^*(x,y)$:

$$\left(1-k-\frac{\rho}{r+\delta}(1+k)\right)\frac{f\,(x,y+1)}{2} > \frac{f(x,y)}{2}\,(1+k)$$

$$\frac{1-k}{1+k}-\frac{\rho}{r+\delta}>\frac{f(x,y)}{f\left(x,y+1\right)}$$

And it is easy to show that $1+3\frac{\rho}{r+\delta}-4\sqrt{\frac{\rho}{r+\delta}\left(1+\frac{\rho}{r+\delta}\right)}=\frac{1-k}{1+k}-\frac{\rho}{r+\delta}$. Then, whenever the following condition holds, only hyperphily can be an equilibrium:

$$\max_{x,y}\frac{f(x,y)}{f\left(x,y+1\right)}\leq 1-\frac{\rho}{r+\delta}\left[4\sqrt{1+\left(\frac{\rho}{r+\delta}\right)^{-1}}-3\right] \tag{5}$$

.

We show next that hyperphily is an equilibrium. Hyperphily is characterized by $d^*(x,y,y')=1$ if and only if $y'>y$ and π^* such that $S^*(x,y)=S^*(y,x)$ for all (x,y). We need to show that for low enough $\frac{\rho}{r+\delta}$, (d^*,π^*) is an equilibrium.

First, under hyperphily, surpluses are given by:

$$\left(r+\delta+\sum_{x'>x}q^*\left(y,x'\right)+\sum_{y'>y}q^*\left(x,y'\right)\right)S^*(x,y)+\sum_{y'\leq y}q^*\left(x,y'\right)S^*\left(x,y'\right)=\pi^*\left(x,y\right)$$
$$\left(r+\delta+\sum_{y'>y}q^*\left(x,y'\right)+\sum_{x'>x}q^*\left(y,x'\right)\right)S^*(y,x)+\sum_{x'\leq x}q^*\left(y,x'\right)S^*\left(y,x'\right)=\pi^*\left(y,x\right)$$

$S^*(x,y)=S^*(y,x)$ and $\pi^*(x,y)+\pi^*(y,x)=f(x,y)$. Then, individual surpluses are given by

$$2\left(r+\delta+\sum_{x'>x}q^*\left(y,x'\right)+\sum_{y'>y}q^*\left(x,y'\right)\right)S^*(x,y)=f(x,y)-\sum_{y'\leq y}q^*\left(x,y'\right)S^*\left(x,y'\right)-\sum_{x'\leq x}q^*\left(y,x'\right)S^*\left(y,x'\right). \tag{6}$$

It is easy to find the following lower bound from (6):

$$2\left(r+\delta+\sum_{x'>x}q^*\left(y,x'\right)+\sum_{y'>y}q^*\left(x,y'\right)\right)S^*(x,y)\geq f(x,y)-\left(\sum_{x'\leq x}q^*\left(x,y'\right)+\sum_{y'\leq y}q^*\left(y,x'\right)\right)\frac{\overline{F}}{r+\delta}$$

Then,

$$2\left(r+\delta+2\rho\right)S^*(x,y)\geq f(x,y)-2\overline{F}\frac{\rho}{r+\delta}$$

Let $\overline{F} = \max_{x,y} f(x,y)$. From (6), we create lower and upper bounds for $S^*(x,y)$,

$$\frac{\frac{f(x,y)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho} \leq S^*(x,y) \leq \frac{\frac{f(x,y)}{2}}{r+\delta} \tag{7}$$

Consider matches (x,y) and $(x,y+1)$. Given (7),

$$S^*(x,y) \leq \frac{\frac{f(x,y)}{2}}{r+\delta}, \quad \text{and} \quad \frac{\frac{f(x,y+1)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho} \leq S^*(x,y+1)\,.$$

For $\frac{\rho}{r+\delta}$ small, $f(x,y) < \frac{f(x,y+1)-\overline{F}\frac{2\rho}{r+\delta}}{1+\frac{2\rho}{r+\delta}}$. Then $S^*(x,y+1) > S^*(x,y)$ and thus (d^*, π^*) is consistent. We need to verify next that no consistent agreement c leads to a higher product of surpluses. Any such agreement must have $\widehat{S}_1^c \geq S^*(x,y+1)$ or $\widehat{S}_2^c \geq S^*(y,x+1)$ or both.[17] Assume without loss of generality that $\widehat{S}_1^c \geq S^*(x,y+1)$. Next, note that for any agreement $\widehat{S}_1^c + \widehat{S}_2^c \leq \frac{f(x,y)}{r+\delta}$. Again, pick $\frac{\rho}{r+\delta}$ small, so $S^*(x,y+1) \geq \frac{\frac{f(x,y)}{2}}{r+\delta}$. Then the product of surpluses must be bounded:

$$\begin{aligned}\widehat{S}_1^c\widehat{S}_2^c &\leq \frac{\frac{f(x,y+1)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho}\left[\frac{f(x,y)}{r+\delta} - \frac{\frac{f(x,y+1)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho}\right] \\ &< \left[\frac{\frac{f(x,y)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho}\right]^2 \leq S^*(x,y)S^*(y,x)\end{aligned}$$

Where again the last inequality holds for small $\frac{\rho}{r+\delta}$. ■

A.5 Hyperphily and Positive Assortative Matching

Steady state conditions characterize the equilibrium distribution. Take match $(x,y) \in X \times X$. Inflow $I(x,y)$ and outflow $O(x,y)$ are given by:[18]

$$\begin{aligned}I(x,y) &= \rho\left(\sum_{\tilde{y}<y} e(x,\tilde{y})\right)\left(\sum_{\tilde{x}<x} e(\tilde{x},y)\right) \\ O(x,y) &= \delta e(x,y) + \rho e(x,y)\left[\sum_{\tilde{y}>y}\sum_{\tilde{x}<x} e(\tilde{x},\tilde{y}) + \sum_{\tilde{x}>x}\sum_{\tilde{y}<y} e(\tilde{x},\tilde{y})\right]\end{aligned}$$

[17] To see this, note that if $\widehat{S}_1^c < S^*(x,y+1)$ and $\widehat{S}_2^c < S^*(y,x+1)$ then neither agent leaves the other less often. Then $\widehat{S}_1^c + \widehat{S}_2^c \leq 2S^*(x,y)$.

[18] X does not include $\varnothing$. But $\sum_{\tilde{y}<y}$ does include it.

Let $\kappa = \frac{\rho}{\delta}$. In steady state, $I(x,y) = O(x,y)$, so

$$\left(\sum_{\tilde{y}<y} e\left(x,\tilde{y}\right)\right)\left(\sum_{\tilde{x}<x} e\left(\tilde{x},y\right)\right) = e(x,y)\left[\kappa^{-1} + \sum_{\tilde{y}>y}\sum_{\tilde{x}<x} e\left(\tilde{x},\tilde{y}\right) + \sum_{\tilde{x}>x}\sum_{\tilde{y}<y} e\left(\tilde{x},\tilde{y}\right)\right] \tag{8}$$

We are interested stochastic dominance. We focus then on $\sum_{\tilde{y}<y} e\left(x+1,\tilde{y}\right) - e\left(x,\tilde{y}\right)$. Equation 8 leads to

$$\left(\sum_{\tilde{y}<y} e\left(x+1,\tilde{y}\right) - \sum_{\tilde{y}<y} e\left(x,\tilde{y}\right)\right)\sum_{\tilde{x}<x} e\left(\tilde{x},y\right) = e\left(x,y\right)\left[\sum_{\tilde{y}>y} e\left(x,\tilde{y}\right) - 2\sum_{\tilde{y}<y} e\left(x+1,\tilde{y}\right)\right] \tag{9}$$
$$+ \left[e(x+1,y) - e(x,y)\right]\left[\kappa^{-1} + \sum_{\tilde{y}>y}\sum_{\tilde{x}<x+1} e\left(\tilde{x},\tilde{y}\right) + \sum_{\tilde{x}>x+1}\sum_{\tilde{y}<y} e\left(\tilde{x},\tilde{y}\right)\right].$$

Similarly, for unmatched agents $I\left(x,\varnothing\right) = O\left(x,\varnothing\right)$. Then,

$$\sum_{y\in X} e\left(x,y\right)\sum_{\tilde{x}>x}\sum_{\tilde{y}<y} e\left(\tilde{x},\tilde{y}\right) = -\frac{\kappa^{-1}}{N} + e\left(x,\varnothing\right)\left[\kappa^{-1} + \sum_{\tilde{x}\in X}\sum_{y<x} e\left(\tilde{x},y\right)\right] \tag{10}$$

A.5.1 The Two type Case

Proof. In an equilibrium with hyperphily and 2 types, h's distribution of partners first order stochastically dominates ℓ's if and only if $e\left(\ell,\varnothing\right) > e\left(h,\varnothing\right)$ and $e\left(\ell,\varnothing\right) + e\left(\ell,\ell\right) > e\left(h,\varnothing\right) + e\left(h,\ell\right)$.

In equation (9), let $x = \ell, x+1 = h$ and $y = h$. Then,

$$\left(\sum_{y\in\{\varnothing,\ell\}}\left[e\left(h,y\right) - e\left(\ell,y\right)\right]\right)\left[e\left(h,\varnothing\right) + \kappa^{-1}\right] = -2e\left(\ell,h\right)\sum_{y\in\{\varnothing,\ell\}} e\left(h,y\right)$$

Then, $\sum_{y\in\{\varnothing,\ell\}} e\left(h,y\right) - e\left(\ell,y\right) < 0$. Next, consider the steady state conditions for $e(h,\varnothing)$ and $e(\ell,\varnothing)$. Given (10), they are respectively given by:

$$0 = -\frac{\kappa^{-1}}{N} + e\left(h,\varnothing\right)\left[\kappa^{-1} + \sum_{\tilde{x}\in\{\ell,h\}}\sum_{y\in\{\varnothing,\ell\}} e\left(\tilde{x},y\right)\right] \quad \text{and}$$

$$\sum_{y\in\{\varnothing,\ell\}} e\left(\ell,y\right)e\left(h,\varnothing\right) = -\frac{\kappa^{-1}}{N} + e\left(\ell,\varnothing\right)\left[\kappa^{-1} + \sum_{\tilde{x}\in\{\ell,h\}} e\left(\tilde{x},\varnothing\right)\right].$$

Thus, $e\left(\ell,\varnothing\right) > e\left(h,\varnothing\right)$. ■

A.5.2 The Two Type Case with Renegotiation

Renegotiation prevents inefficient destruction: the sum of the values of the destroyed matches can never exceed the value of the newly created one. In the example presented in Section 5.2, individuals who meet unmatched agents switch partners as often as in an equilibrium with hyperphily in our model. On the other hand, since $2F > 2h$, an agent of type h matched to one of type ℓ does not break the match when finding a matched type h agent. Conversely, In our model, h leaves ℓ if she meets another h matched to ℓ. Therefore the steady state distribution of matches in a model with and without renegotiation differ.[19] In any case, it is still straightforward to show that h's distribution of partners first order stochastically dominates ℓ's.

We first show that $e(\ell,\varnothing) > e(h,\varnothing)$. The inflow to $e(h,\varnothing)$ is $\delta[1-e(h,\varnothing)]$ and its outflow is $\rho e(h,\varnothing)[e(\ell,\varnothing)+e(\ell,\ell)+e(h,\varnothing)+e(h,\ell)]$. The inflow to $e(\ell,\varnothing)$ is $\delta[1-e(\ell,\varnothing)]+\rho e(\ell,\ell)e(h,\varnothing)+\rho e(\ell,h)e(h,\varnothing)$. The outflow to $e(\ell,\varnothing)$ is $\rho e(\ell,\varnothing)[e(\ell,\varnothing)+e(h,\varnothing)]$. If $e(\ell,\varnothing) \leq e(h,\varnothing)$, the inflow to $e(\ell,\varnothing)$ is larger than the inflow to $e(h,\varnothing)$ but the outflow of $e(\ell,\varnothing)$ is smaller than the outflow of $e(h,\varnothing)$. Therefore $e(\ell,\varnothing) > e(h,\varnothing)$.

Second, we show that $e(\ell,\varnothing)+e(\ell,\ell) > e(h,\varnothing)+e(h,\ell)$. The inflow to $e(h,h)$ is $\rho[\frac{1}{2}-e(h,h)]e(h,\varnothing)+e(h,\varnothing)e(h,\ell)$. The outflow to $e(h,h)$ is $e(h,h)\delta$. The inflow to $e(h,\ell)$ is $\rho[\frac{1}{2}-e(\ell,h)]e(h,\varnothing)$. The outflow to $e(h,\ell)$ is $e(h,\ell)[\delta+\rho e(h,\varnothing)]$. If $e(h,h) \leq e(h,\ell)$, the inflow to $e(h,h)$ is larger than the inflow to $e(h,\ell)$ and the outflow of $e(h,h)$ is smaller than the outflow of $e(h,\ell)$. Therefore $e(h,\ell) < e(h,h)$.

A.6 Conditions for hyperphily

In an equilibrium with hyperphily, surplus are as follows:

$$S^*(h,\ell) = [r+\delta+q^*(h,\ell)+q^*(h,h)]^{-1}\,\pi^*(h,\ell)$$

$$S^*(h,h) = [r+\delta+q^*(h,h)]^{-1}\,[\pi^*(h,h)-q^*(h,\ell)\,S^*(h,\ell)]$$

$$S^*(\ell,h) = [r+\delta+q^*(\ell,h)+q^*(h,h)]^{-1}\,[\pi^*(\ell,h)-q^*(\ell,\ell)\,S^*(\ell,\ell)]$$

$$S^*(\ell,\ell) = [r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]^{-1}\,\pi^*(\ell,\ell)$$

[19] If only one side of the market searches on the match, the distribution of matches in a model with or without renegotiation is the same. This is because a matched agent can never meet another matched agent.

Allocations are $\pi^*(h,h) = h$, $\pi^*(\ell,\ell) = \ell$, and $\pi^*(h,\ell) = F - \pi^*(\ell,h)$. Surplus equalization $S^*(h,\ell) = S^*(\ell,h)$ requires:

$$\pi^*(\ell,h) = \frac{[r+\delta+q^*(h,h)+q^*(\ell,h)]\,F + \frac{r+\delta+q^*(h,h)+q^*(h,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)} q^*(\ell,\ell)\,\ell}{2\,(r+\delta+q^*(h,h)) + q^*(\ell,h) + q^*(h,\ell)\,.}$$

As discussed in Section 3.1, a pair (d^*, π^*) is consistent in an equilibrium with hyperphily whenever

$$G^1_{HYPER} \equiv S^*(h,h) - S^*(h,\ell) > 0 \quad \text{and} \quad G^2_{HYPER} \equiv S^*(\ell,h) - S^*(\ell,\ell) > 0.$$

Condition 1 in the main text includes the three kinds of agreements which may prevent (d^*, π^*) from solving the bargaining problem in the match (ℓ,h). We check next when these agreements do not hold. First, in agreement c_1, neither ℓ nor h leave each other, and h is made indifferent. ℓ obtains $\widehat{S}^{c_1}_\ell$. We need then $G^3_{HYPER} \equiv S^*(\ell,h) - \widehat{S}^{c_1}_\ell \geq 0$. In agreement c_2, $\widehat{S}^{c_2}_h = S^*(h,h)$ and ℓ only leaves when she finds an h, leading to surplus $\widehat{S}^{c_2}_\ell$. We need then $G^4_{HYPER} \equiv S^*(\ell,h)\,S^*(h,\ell) - \widehat{S}^{c_2}_\ell S^*(h,h) \geq 0$. Finally, Agreement c_3 also has $\widehat{S}^{c_2}_h = S^*(h,h)$, but now ℓ always leaves. We need $G^5_{HYPER} \equiv S^*(\ell,h)\,S^*(h,\ell) - \widehat{S}^{c_3}_\ell S^*(h,h) \geq 0$.

Condition 2 includes the three kinds of agreements which may prevent (d^*, π^*) from solving the bargaining problem in match (ℓ,ℓ). Let us check they do not hold. First, let $\widehat{S}^{c_4}_1$ be the surplus obtained by either agent in match (ℓ,ℓ) when they do not leave each other. We need that $G^6_{HYPER} \equiv S^*(\ell,h) - \widehat{S}^{c_4}_1 \geq 0$. Next, in agreement c_5 one agent ℓ obtains $S^*(\ell,h)$ and does not leave, whereas the other one leaves only when meeting agent h. We need then that $G^7_{HYPER} \equiv S^*(\ell,\ell) - \widehat{S}^{c_5}_1 \geq 0$. Finally, in agreement c_6, $\widehat{S}^{c_6}_1$ is the surplus obtained by a ℓ agent in match (ℓ,ℓ) when she always leaves and her partner is indifferent between this match and one with h. We require that $G^8_{HYPER} \equiv S^*(\ell,\ell)^2 - \widehat{S}^{c_6}_1 \widehat{S}^{c_6}_2 \geq 0$.

We start with consistency. Regarding G^1_{HYPER}, note that $S^*(h,h) > S^*(h,\ell)$ if and only if $h > \pi^*(h,\ell)$. We show later that G^3_{HYPER} implies this. Next, regarding G^2_{HYPER}, it holds whenever G^6_{HYPER} holds. To see this, note that $\widehat{S}^{c_4}_1 = S^*(\ell,\ell) + \frac{q^*(\ell,h)[2S^*(\ell,\ell)-S^*(\ell,h)]}{r+\delta}$. Then, whenever $S^*(\ell,\ell) - S^*(\ell,h) \geq 0$, also $\widehat{S}^{c_4}_1 - S^*(\ell,h) \geq 0$.

We characterize next each condition as a function of primitives. Let us start with match (ℓ,h). Let agreement c_1 for match (ℓ,h) be $\widehat{d}_\ell = \widehat{d}_h = 0$ and $\widehat{\pi}_h = h$. Note that for any lower $\widehat{\pi}_h$, h leaves. Next, any higher $\widehat{\pi}_h$ leads to a lower product of surpluses. It suffices then to focus on this agreement. We need to show that $G^3_{HYPER} \equiv S^*(\ell,h) - \widehat{S}^{c_1}_\ell \geq 0$.

$$\widehat{S}^{c_1}_\ell = (r+\delta)^{-1}\left[F - h - q^*(\ell,\ell)\,S^*(\ell,\ell) - q^*(\ell,h)\,S^*(\ell,h)\right] \quad \text{and}$$
$$S^*(\ell,h) = \frac{F - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}$$

Therefore, we need

$$F - h - \frac{q^*(\ell,\ell)\,\ell}{r+\delta+2q^*(\ell,h)+s} \leq (r+\delta+q^*(\ell,h))\,\frac{F - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}$$

Then,

$$F \leq h\left(1 + \frac{r+\delta+q^*(\ell,h)}{r+\delta+2q^*(h,h)+q^*(h,\ell)}\right) + \frac{q^*(\ell,\ell)\,\ell}{r+\delta+2q^*(\ell,h)+q^*(h,\ell)}$$

Finally, note that $S^*(\ell,h) - \widehat{S}^{c_1}_\ell = (r+\delta)^{-1}\left[\pi^*(\ell,h) - (F-h) - q^* S^*(\ell,h)\right]$. Condition G^3_{HYPER} implies the previous expression is positive. Thus $\pi^*(\ell,h) - (F-h) > 0 \Rightarrow F - \pi^*(h,\ell) - F + h > 0 \Rightarrow h > \pi^*(h,\ell)$. So G^1_{HYPER} holds.

Next, let agreement c_2 for match (ℓ,h) be $\widehat{d}_h = 0$, $\widehat{d}_\ell(\ell) = 0$, $\widehat{d}_\ell(h) = 1$ and $\widehat{\pi}_\ell$ such that $\widehat{S}^{c_2}_h = S^*(h,h)$. Note that for any lower $\widehat{\pi}_h$, h leaves. Next, any higher $\widehat{\pi}_h$ leads to a lower product of surpluses. It suffices then to focus on this agreement. We need to show that $G^4_{HYPER} \equiv S^*(\ell,h)\,S^*(h,\ell) - \widehat{S}^{c_2}_\ell \widehat{S}^{c_2}_h \geq 0$. We start by explicitly calculating $\widehat{\pi}_\ell$. We have:

$$\widehat{S}^{c_2}_h = (r+\delta+q^*(\ell,h))^{-1}\left(F - \widehat{\pi}(\ell,h) - q^*(h,h)S^*(h,h) - q^*(h,\ell)\,S^*(h,\ell)\right) = S^*(h,h) \tag{11}$$

$$\widehat{S}^{c_2}_\ell = (r+\delta+q^*(\ell,h))^{-1}\left(\widehat{\pi}(\ell,h) - q^*(\ell,\ell)\,S^*(\ell,\ell)\right)$$

Given (11),

$$\widehat{\pi}_\ell = F - [r+\delta+q^*(h,h)]^{-1}\left[(r+\delta+q^*(\ell,h)+q^*(h,h))\,h - \frac{q^*(h,\ell)\,q^*(\ell,h)\,(F-\pi(\ell,h))}{r+\delta+q^*(h,h)+q^*(h,\ell)}\right]$$

From now on, we work with $\pi^*(\ell,h) = A_1 + B_1 F$ and $\widehat{\pi}_\ell = A_2 + B_2 F$ with:

$$A_1 = \frac{r+\delta+q^*(h,h)+q^*(h,\ell)}{\left[r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)\right]\left[2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)\right]}q^*(\ell,\ell)\,\ell$$
$$B_1 = \frac{r+\delta+q^*(h,h)+q^*(\ell,h)}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}$$

$$A_2 = -\frac{r+\delta+q^*(\ell,h)+q^*(h,h)}{r+\delta+q^*(h,h)}h + \frac{q^*(h,\ell)\,q^*(\ell,h)\,A_1}{(r+\delta+q^*(h,h)+q^*(h,\ell))\,(r+\delta+q^*(h,h))}$$
$$B_2 = 1 + \frac{q^*(h,\ell)\,q^*(\ell,h)\,(1-B_1)}{(r+\delta+q^*(h,h))\,(r+\delta+q^*(h,h)+q^*(h,\ell))}$$

We first check that $\widehat{S}_\ell^{c_2} \geq S^*(\ell,\ell)$. This occurs whenever:

$$F \geq B_2^{-1}\left(\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\ell - A_2\right)$$

If this agreement is consistent we still have to check that F is large enough to make the product of surpluses larger:

$$S^*(h,\ell)\,S^*(\ell,h) \geq \widehat{S}_h^{c_2}\widehat{S}_\ell^{c_2}$$
$$[(1-B_1)F - A_1]^2 \geq C_1\left(B_2F + A_2 - \frac{q^*(\ell,\ell)\,\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right)$$
$$\times\left(h - q^*(h,\ell)\,\frac{(1-B_1)F - A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right) \quad \text{with}$$
$$C_1 = \frac{(r+\delta+q^*(h,h)+q^*(h,\ell))^2}{(r+\delta+q^*(\ell,h))(r+\delta+q^*(h,h))}.$$

The previous expression holds with equality for F given by:

$$\left[(1-B_1)^2 + C_1B_2\frac{q^*(h,\ell)\,(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]F^2$$
$$+\left[-2(1-B_1)A_1 - C_1B_2\left(h + \frac{q^*(h,\ell)\,A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\right]F$$
$$+\frac{C_1q^*(h,\ell)\,(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\left(A_2 - \frac{q^*(\ell,\ell)\,\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right)F$$
$$+A_1^2 - C_1\left(h + \frac{q^*(h,\ell)\,A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\left(A_2 - \frac{q^*(\ell,\ell)\,\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right) = 0$$

Since $(1-B_1)$, C_1 and B_2 are positive, G^4_{HYPER} is a convex function of F. There are two values of F that equalize the product of surpluses. In order to have an equilibrium with hyperphily, F has to be smaller than the lower root or larger than then higher one. Only the first of these two conditions is relevant. Note that there exist a $\widehat{F}$ such that $S^*(\ell,h) = S^*(h,h)$. For $F = \widehat{F}$ hyperphily is not an equilibrium because $\widehat{S}_\ell^{c_2}\widehat{S}_h^{c_2} > S^*(\ell,h)\,S^*(h,\ell)$.[20] F larger than the large root of $G^4_{HYPER} = 0$ requires that $F > \widehat{F}$. however, the consistency condition G^1_{HYPER} states than an equilibrium with hyperphily requires $F < \widehat{F}$. Therefore

[20] If $F = \widehat{F}$, $\widehat{S}_\ell^{c_2} > S^*(\ell,h)$. This is because it cannot be the case that $\widehat{S}_h^{c_2} + \widehat{S}_\ell^{c_2} \leq S^*(h,\ell) + S^*(\ell,h)$. If

if F^4_{HYPER} is the small root of $G^4_{HYPER} = 0$, an equilibrium with hyperphily requires:

$$F \leq \max\left[F^4_{HYPER}, B_2^{-1}\left(\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\ell - A_2\right)\right]$$

We move next to c_3. We need to show that $G^5_{HYPER} \equiv S^*(h,\ell)\,S^*(\ell,h) - \widehat{S}^{c_3}_h \widehat{S}^{c_3}_\ell \geq 0$ with $\widehat{S}^{c_3}_h = S^*(h,h)$ and

$$\widehat{S}^{c_3}_h = (r+\delta+q^*(\ell,h)+q^*(\ell,\ell))^{-1}\left(F - \widehat{\pi}_\ell - q^*(h,h)S^*(h,h) - q^*(h,\ell)\,S^*(h,\ell)\right)$$
$$\widehat{S}^{c_3}_\ell = (r+\delta+q^*(\ell,\ell)+q^*(\ell,h))^{-1}\widehat{\pi}_\ell$$

Then,

$$\widehat{\pi}_\ell = A_3 + B_3 F \qquad \text{with}$$
$$A_3 = \frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)+q^*(h,h)}{r+\delta+q^*(h,h)}h + \frac{q^*(h,\ell)\,(q^*(\ell,h)+q^*(\ell,\ell))A_1}{r+\delta+q^*(h,h)+q^*(h,\ell)}$$
$$B_3 = 1 + \frac{q^*(h,\ell)\,(q^*(\ell,h)+q^*(\ell,\ell))(1-B_1)}{[r+\delta+q^*(h,h)]\,[r+\delta+q^*(h,h)+q^*(h,\ell)]}$$

Condition G^5_{HYPER} holds if:

$$[(1-B_1)F - A_1]^2 \geq C_2\,(B_3F + A_3)\left(h - q^*(h,\ell)\,\frac{(1-B_1)F - A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right) \quad \text{with}$$
$$C_2 = \frac{(r+\delta+q^*(h,h)+q^*(h,\ell))^2}{(r+\delta+q^*(\ell,h)+q^*(\ell,\ell))(r+\delta+q^*(h,h))}.$$

Therefore:

$$G^5_{HYPER} = \left[(1-B_1)^2 + C_2 B_3 \frac{q^*(h,\ell)\,(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]F^2$$

$\widehat{S}^{c_2}_h + \widehat{S}^{c_2}_\ell \leq S^*(h,\ell) + S^*(\ell,h)$

$$\frac{\widehat{F} - q^*(\ell,h)\,\widehat{S}^{c_2}_\ell - (q^*(\ell,h)+q^*(h,h)+q^*(h,\ell))\,S^*(h,h) - q^*(\ell,\ell)\,S^*(\ell,\ell)}{r+\delta}$$
$$\leq \frac{\widehat{F} - (q^*(\ell,h)+2q^*(h,h)+q^*(h,\ell))\,S^*(h,h) - q^*(\ell,\ell)\,S^*(\ell,\ell)}{r+\delta}$$

Therefore $q^*(\ell,h)\,\widehat{S}^{c_2}_\ell \geq q^*(h,h)S^*(h,h)$. Since $q^*(\ell,h) < q^*(h,h)$:

$$\widehat{S}^{c_2}_\ell > S^*(h,h)$$
$$\widehat{S}^{c_2}_\ell + S^*(h,h) > S^*(h,h) + S^*(h,h)$$
$$\widehat{S}^{c_2}_\ell + \widehat{S}^{c_2}_h > S^*(h,\ell) + S^*(\ell,h)$$

which contradicts the first assumption.

$$
\begin{aligned}
&+\left[-2(1-B_1)A_1 - C_2B_3\left(h+\frac{q^*(h,\ell)\,A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)+\frac{q^*(h,\ell)\,(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}C_2A_3\right]F\\
&+A_1^2-\left(h+\frac{q^*(h,\ell)\,A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)C_2A_3\geq 0
\end{aligned}
$$

Since $(1-B_1)$, C_2 and B_3 are positive G^6_{HYPER} is a convex function of F. There are two values of F that equalize the product of the surplus. In order to have an equilibrium with hyperphily F has to be smaller than the small one or larger then the large one. Similarly to condition G^4_{HYPER}, if condition G^3_{HYPER} holds the only relevant condition in this case is that F has to be smaller than the small root of $G^5_{HYPER}=0$.

We move next to match (ℓ,ℓ). We start with contract c_4. We need to show that $G^6_{HYPER}\equiv S^*(\ell,h)-\widehat{S}^{c_4}_1\geq 0$, with

$$
\widehat{S}^{c_4}_1=(r+\delta)^{-1}\left[\ell-q^*(\ell,h)\,S^*(\ell,h)-q^*(\ell,\ell)\,S^*(\ell,\ell)\right]
$$

This occurs whenever

$$
F\geq\frac{\left[2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)\right]\left[r+\delta+2q^*(\ell,h)\right]+\left[r+\delta+q^*(\ell,h)\right]q^*(\ell,\ell)}{\left[r+\delta+q^*(\ell,h)\right]\left[r+\delta+q^*(\ell,\ell)+2q^*(\ell,h)\right]}\ell
$$

Next, consider agreement c_5. Agent ℓ indexed by 2 does not leave so $\widehat{S}^{c_5}_2=S^*(\ell,h)$. This requires

$$
\begin{aligned}
\widehat{\pi}_2=&\frac{r+\delta+2q^*(\ell,h)}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}F\\
&+\frac{q^*(\ell,\ell)\,\ell}{\left[2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)\right]\left[r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)\right]\left[r+\delta+q^*(\ell,h)+q^*(h,h)\right]}\\
&\times\Big[\left(r+\delta+2q^*(\ell,h)\right)\left(r+\delta+q^*(h,\ell)+q^*(h,h)\right)\\
&\qquad-\left[2\left(r+\delta+q^*(h,h)\right)+q^*(h,\ell)+q^*(\ell,h)\right]\left(q^*(\ell,h)-q^*(h,h)\right)\Big].
\end{aligned}
$$

We need to verify now that $G^7_{HYPER}\equiv S^*(\ell,\ell)-\widehat{S}^{c_5}_1\geq 0$ with:

$$
\widehat{S}^{c_5}_1=\frac{2\ell-\widehat{\pi}_2-q^*(\ell,\ell)\,S^*(\ell,\ell)}{r+\delta+q^*(\ell,h)}\leq S^*(\ell,\ell)
$$

If condition G^6_{HYPER} holds, we know that $\widehat{S}^{c_5}_1<S^*(\ell,h)$. We look for the maximum F that makes the agreement $(\widehat{S}^{c_5}_1,\widehat{S}^{c_5}_2)$ consistent:

$$
F\geq\frac{\left[2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)\right]\left(r+\delta+3q^*(\ell,h)+q^*(\ell,\ell)\right)\ell}{\left(r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)\right)\left(r+\delta+2q^*(\ell,h)\right)}
$$

$$-\frac{\left[r+\delta+q^*(\ell,h)+q^*(h,h)-\frac{(r+\delta+q^*(h,h)+q^*(h,\ell))(q^*(\ell,h)-q^*(h,h))}{r+\delta+q^*(\ell,h)+q^*(h,h)}\right]q^*(\ell,\ell)\,\ell}{(r+\delta+2q^*(\ell,h)+q^*(\ell,\ell))\,(r+\delta+2q^*(\ell,h))}$$

We finish hyperphily by checking agreement c_6. We need to verify that $G^8_{HYPER} \equiv S^*(\ell,\ell)^2 - \widehat{S}^{c_6}_1 \widehat{S}^{c_6}_2 \geq 0$ with

$$\widehat{S}^{c_6}_1 = \frac{2\ell - \widehat{\pi}_2}{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)} \text{ and}$$
$$\widehat{S}^{c_6}_2 = \frac{\widehat{\pi}_2 - q^*(\ell,h)\,S^*(\ell,h) - q^*(\ell,\ell)\,S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)}.$$

Note that

$$\widehat{S}^{c_6}_1 + \widehat{S}^{c_6}_2 = \frac{2\ell - q^*(\ell,h)\,S^*(\ell,h) - q^*(\ell,\ell)\,S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)} \quad \text{and}$$
$$2S^*(\ell,\ell) = \frac{2\ell}{r+\delta+q^*(\ell,\ell)+2q^*(\ell,h)} = \frac{2\ell - 2q^*(\ell,h)\,S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)}.$$

Since $S^*(\ell,\ell) < S^*(\ell,h)$ and $q^*(\ell,\ell) = \rho e(\ell,\varnothing) > \rho e(h,\varnothing) = q^*(\ell,h)$, then $2S^*(\ell,\ell) > \widehat{S}^{c_6}_1 + \widehat{S}^{c_6}_2$. Both ℓ agents equalize surplus in $S^*(\ell,\ell)$, and no agreement in the same segment of the frontier or in an interior segment of the frontier, can generate a larger product of surpluses. Therefore condition G^8_{HYPER} always holds.

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