The Organization of Bank Affiliates; A Theoretical Perspective on Risk and Efficiency*

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Abstract

We analyze theoretically banks choice of organization and leverage in branches or subsidiaries in the presence of economic and financial synergies, government bailouts and bankruptcy costs. We compare with stand-alone banks. The social efficiency of banks' choices are analyzed as well, taking into account financial returns to scale and distortions caused by banks' exploitation of limited liability if there is government bailout. Leverage choice depends on the trade-off between benefits of limited liability and bankruptcy costs. The choice of subsidiary vs branch or stand-alone organization depends on the trade-off between organizational synergies and coinsurance. We explore policy implications.
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1 Introduction

Firms face organizational choices with several economic and legal dimensions when they operate cross-border and across activities that are technologically separable. These organizational choices for banks have come to the forefront after the financial crisis in proposals for living wills, separation of different kinds of activities in different legal entities, "ring-fencing" of branches of foreign banks

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and requirements in some countries that foreign banks operate in subsidiaries, to mention a few.[1]

The legal dimension of the problem is the choice between operating an affiliate as a subsidiary or as a branch. A third alternative is to divest the affiliate as a stand-alone bank. The economic dimension to the choice of organization is to extract value from operational and financial synergies between the affiliates. For example, a subsidiary is a legally separate entity, which relative to a stand-alone bank may take advantage of a parent bank's specific knowledge or other strategic assets. A branch, on the other hand, is not a separate legal entity. It may offer greater opportunities to exploit operational synergies. Financially, the different organizations offer different opportunities to exploit financial synergies including the reduction of expected default costs, benefits of limited liability and potential government bailouts. These sources of financial synergies are the main focus of the analysis below.

We analyze theoretically value and risk effects of a bank's choice among branch, subsidiary and stand-alone organizations in the presence of default costs, a positive probability of bailout of a bank's creditors and limited liability of separate legal entities. A distinguishing feature of the different organizations is the extent to which there is coinsurance between two entities, which affects the ability of the bank to exploit financial synergies. There is clearly no coinsurance between two stand-alone banks. Two branches of the same bank have strong coinsurance since they support or rescue each other until the whole bank becomes insolvent.[2]

Subsidiaries may not be legally obliged to offer coinsurance, but they may decide to do so because of regulatory interventions or in view of other interests. For instance, they may do so in order to preserve brand-name value.[3]

Operational synergies are taken into account as well but they are not the main focus of the analysis.

The distinguishing characteristic of a bank relative to any corporation is, first, that there is a positive probability of a bailout by the government when a legal entity in its jurisdiction is insolvent. Second, we take into account that

[1] There are a number of different proposals for separation of traditional commercial banking from other financial activities. The so-called Volcker-, Vickers- and Liikanen proposals are reviewed in Blundell-Wignall and Roulet (2013). With respect to cross-border banking New Zealand requires foreign banks to operate in subsidiaries that should be operationally separable from the parent within 24 hours if the parent bank must be closed down (Mayes, 2013). In the US branches of foreign banks must be 'ringfenced' in the sense that the branches have separate capital buffers. In the EU banks are permitted to operate across the EU in branches under home country control and supervision but, in reality, most cross-border banking in the EU takes place in subsidiaries.

[2] We do not consider the possibility that a bank may spin off a loss-making part of the bank instead of rescuing it because losses at the time of default are unanticipated. Thus, at the time of default it is too late to spin off an entity. If the spin-off conditional on losses (no internal rescue) is planned from the beginning, then the bank would consist of two stand-alone banks in our terminology.

[3] As an example of the importance of reputation effects Gorton and Souleles (2006) show that prices paid by investors in credit card asset backed securities reflect issuers ability to bail out the asset backed securities.

banks may have economies of scale or enjoy tax relief in the production of deposit services. We do not take a stance on their existence.

With these characteristics of a bank there are four potential sources of financial synergies in the model, which can be influenced by the organizational choice. First, the organizational choice i.e. the bank's policy with respect to rescue of affiliates affects default costs. Second, banks can take advantage of limited liability in subsidiaries by abandoning rescue when the whole bank is threatened. Third, the benefits of a positive probability of bailout by the government depend on the choice of organization. A fourth potential source of financial synergies arises when the benefits from returns to scale depend on the way the deposits are allocated between affiliates.

The analysis of risk and efficiency in bank organizations in this paper is related to several strands of literature. One strand is the analysis of financial synergies arising as a result of the merger of two firms. Leland (2007) and Banal-Estanol, Ottaviani and Winton (2012) show that when two firms are merged the new firm cannot take advantage of limited liability but it can benefit from reduced default costs. Leland also considers tax-effects of the endogenous choice between debt and equity. Banal-Estanol *et al.*. restricts the analysis to debt financing and the effects of the merger of two firms on default costs, while the merged firm re-optimizes leverage in Leland. In the latter case, the merged firm always benefits from reduced default costs while a 'contamination effect' within the merged organization with fixed leverage can cause the default costs to rise above the default costs of the separately financed firms.

Luciano and Nicodano (2014) expands on the analysis in Leland (2007) and considers that a parent plus a subsidiary, which can be rescued by the parent, can economize on default costs relative to the merged firm. They focus on the organizational choice as a trade-off between default costs and tax-savings from debt financing. We build on Luciano and Nicodano taking into account the probability of bailouts of banks and potential cash flows due to returns to scale of deposits.

A second strand of literature analyzes risk-taking incentives in financial conglomerates and effects of capital regulation. Freixas, Loranth and Morrison (2007) compares risk-taking in an integrated entity subject to one liability constraint with a holding company structure within which divisions that can fail independently are subject to separate and possibly different capital constraint. The paper also considers stand-alone financial institutions. The holding-company structure can shift risk by means of asset transfers while it financially works as stand-alone entities. Freixas *et al.* conclude that the holding company in the presence of deposit insurance for one entity is efficient if capital requirements can be raised to incentivize asset transfers to entities subject to market discipline. This effect can outweigh the disadvantage of higher default costs in individual entities. In our model subsidiaries within a holding-company structure are able to save on default costs by means of limited liability. Our model also differs in that all or both entities have a probability of bailout.

A third group of papers analyzes risk-shifting in financial conglomerates, wherein activities with different risk are conducted either in separately financed subsidiaries or within a jointly financed firm. Most of these papers like Chemmanur and John (1996), Harr and Rønde (2006) and Kahn and Winton (2004) introduce agency and governance problems associated with different organizational structures. The closest to our model is Kahn and Winton, which emphasizes moral hazard incentives to shift risk to debt-holders who cannot observe the riskiness of the different activities. Since the debt-holders know that they do not have risk-information, the financial institution can reduce its cost of funding by separating the financing of high-risk and low-risk activities into different entities with different leverage. In the model below we do not explicitly introduce differences in risk between activities but risk-shifting between entities can occur in response to benefits from economoies of scale and from differences in bailout probabilities and default costs.

Agency problems also play a role in models of capital regulation and leverage when monitoring efforts are endogenized. Acharya, Mehran and Thakor (2013) analyze how capital regulation affect both risk-shifting and monitoring incentives. Although the theoretical model and the issues addressed in Acharya *et al.* are different from our model, there are similarities with respect to the effects of capital requirements with different bailout probabilities. Their focus lies on the optimal design of capital regulation while we emphasize how the impact of capital regulation may depend on bank organization.

Freixas and Ma (2013) show that the endogenization of leverage may upend the common proposition that reduced competition increases financial stability. Allen, Carletti and Marquez (2011) employs an agency framework wherein commitment to monitoring is linked to leverage and competition. We analyze optimal leverage under competitive conditions with an endogenous interest rate on the bank's deposits. Potential implications of leverage choices of different organizations for systemic risk are discussed as one aspect of financial stability.

There is a literature on efficiency properties of interbank insurance, which is comparable to our coinsurance within banks with branches and subsidiaries. Castiglionesi and Wagner (2012) analyses efficiency properties of interbank insurance against liquidity problems. Their analysis applies on insurance against insufficient capital in an entity as well. As noted, we associate branches and subsidiaries with different coinsurance arrangements. For subsidiaries we distinguish between the case when a parent bank offers unilateral insurance for capital deficiency of a subsidiary and the case of mutual insurance. The latter case can be thought of as a bank holding company owning two subsidiaries. In both cases the insurance of any subsidiary is constrained by the ability of the other entity to rescue the insolvent subsidiary without causing the insolvency of the whole bank. In branch organizations there is unlimited coinsurance or rescue as long as there is capital in the bank as a whole. Either all entities fail or none. No coinsurance is represented by stand-alone entities in the analysis.

The presence and effect of rescues, or internal insurance within banking groups, have been studied empirically by Bradley and Jones (2008), as well as by Ashcraft (2004) for the US. Bradley and Jones note that "the Federal Reserve Board (FRB) extended its longstanding position that a bank holding company serve as a source-of-strength to its subsidiary banks beyond the application process. Under the expanded policy, the FRB required that a holding company stand ready to provide troubled subsidiaries with both financial and managerial assistance in times of stress".

Flows of funds within cross-border banks have been documented empirically by Cetorelli and Goldberg (2012). Jeon *et al.* (2013) have documented the transmission of shocks generated by such flows. The choice of organization for affiliates has been examined empirically in Cerutti *et al.* (2007).

Dell'Ariccia and Marquez (2010) analyzed theoretically the choice between branches and subsidiaries in banking. They model the choice as a trade-off between benefits of limited liability for a bank with a subsidiary in the presence of economic risk and protection against political risk in a branch organization. Asset risk in different affiliates in the model below is described by imperfectly correlated return distributions. We also differ from Dell'Ariccia and Marquez in that we endogenize both leverage and each affiliate's interest rate.

The analytical and numerical results we derive with respect to value and risk effects of organizational choice from a private and social point of view have bearing on a range of policy issues in the emerging global framework for banking regulation, supervision and crisis management. For example, risk and efficiency effects of capital requirements may depend on the organization of banks domestically and internationally. Differences in bailout policies, distress resolution procedures and stringency of capital requirements may offer incentives for "regulatory arbitrage" that can be implemented through choice of organization. We also address the potential for contagion effects of bank failures in different organizations.

While we analyze how value and risk of different organizations depend on parameters for default costs and bailout expectations, it is possible that these parameters depend on the organization of banks, as well as their size and complexity. Carmassi and Herring (2010) emphasize this point. There is also a strand of related legal literature analyzing "asset partitioning" and its effect on value losses in default (Hansmann *et al.*, 2006). Ongena, Popov and Udell (2012) analyze empirically how cross-border banks shift risk between countries with different regulation. We return to these issues in the concluding Section.

We proceed as follows: the basic model for valuation of one stand-alone bank is set up in Section 2 to show the nature of the trade-off between the put option value of limited liability and bankruptcy costs. The likelihood of a government bailout of depositors is introduced as the basis for benefits of limited liability. Returns to scale in deposits are introduced as a third factor affecting value and leverage. In Section 3 the bank expands by adding an affiliate as a subsidiary or a branch. The financial synergies from internal rescues and reductions in bankruptcy costs depend on the coinsurance associated with different organizations. The subsidiary case with one-way internal rescue is formulated in Section 3.1. The decision problem for the bank expanding with a subsidiary with mutual internal rescue is developed in Section 3.2. The branch case follows in Section 3.3. Analytical results are derived in Section 4 for the private and social values of subsidiary, branch and stand-alone organizations under the assumptions that branches and subsidiaries are identical with respect to parameters, size and leverage. Values of optimally levered banks are analyzed in Section 5. To this end, we provide analytical results and then turn to numerical analysis assuming Gaussian log returns on loans. Optimal leverage for bank affiliates and stand-alone entities is then derived numerically. Parameters for bailouts, bankruptcy costs and returns to scale are varied in order to compare how the private and social values of the different organizations depend on these parameters. In Section 6 we turn to public policy issues by analysing value effects of capital requirements for different organizations. We also develop a proxy for potential contagion effects associated with the failure of banks with different organizations. The scope for regulatory arbitrage, i.e. taking advantage of differences in bailout probabilities and default costs across affiliates, through choice of organization is discussed briefly in Section 7. In the concluding Section 8 we summarize results and discuss policy implications.

2 The stand-alone bank

This section models a single or stand-alone (SA) bank using as a starting point the structural model of Merton (1974) and introducing default costs and the possibility of external bailout in default. The bank produces a fixed amount of loans at time 0. It may obtain leverage by issuing deposits with an endogenous, competitively determined interest rate. We argue that this creates an incentive to raise leverage so as to exploit bailout, and a disincentive determined by default costs. We then explain how financial returns to scale - i.e., revenues from additional banking activities, spurred by deposits - or tax provisions may create an additional incentive for banks to raise deposits.

For the sake of simplicity, consider two points in time only, $t = 0, T$, and classify bank liabilities into deposits and equity. Deposits represent customer as well as interbank net deposits, borrowing from the Central Bank and issued bonds. Equity represents capital and reserves. In order to model deposits in a simple way, we assume that they take the form of zero-coupon debt. They can be withdrawn at maturity[4] T. The face value of deposits is denoted by F. Deposits earn an interest rate which is implicitly determined by the fact that the (market) value of deposits at time 0, D_0, is the present value of the expected payoff to depositors at time T, which we determine below. Also the (market) value of equity at time 0, E_0, is determined as the present value of the expected payoffs to the equity holders at T.

To simplify, we label as "loans" all the bank assets.[5] We disregard interbank claims and consider as a unique entity proper loans and securities. In doing that we have in mind mainly commercial banks. The initial value of loans is denoted as $L_0 \in \mathbb{R}$. The final value of loans at time T is a non-negative random variable

[4] Structural models of the type we are going to build have proven to be quite resilient to the possibility of liabilities' repayment when the actual value of assets goes under a covenant level. This is why we do not introduce the hypothesis of a "bank run " when the value of deposits falls below a given threshold before T.

[5] We could equally well have chosen deposits as given and solved for the amount of loans.

- which we take to be continuous, for simplicity - denoted as $L(T)$.

Stakeholders' payoffs

At time T the bank collects the value of loans $L(T)$ and distributes it to depositors and equity holders.

Depositors are going to receive the face value of their deposits, F, either if this is greater or equal than the net loan value $L(T)$, or if it is smaller, $L(T) < F$, but the state intervenes. There is a probability π that the state bails out the bank. Bailout is independent from default.[6] When the asset value falls short of deposits then, bailout occurs with probability π. Default is assumed to be costly and the loan value to be distributed to depositors is net of default costs. For the sake of simplicity, we take default costs to be proportional to the loan value, $\alpha L(T)$. The value to be distributed to depositors becomes the sum of what equity holders pay to debt holders, what they receive from the government in case of bailout, less default costs :

$$\min(F, L(T)) + (F - L(T))\mathbf{1}_{\{L(T)<F,B\}} - \alpha L(T)\mathbf{1}_{\{L(T)<F,\bar{B}\}},$$

where $\mathbf{1}_{\{E\}}$ is the default indicator of event E, which is equal to one if and only if E occurs, B is the event of bailout, $\bar{B}$ the event of no bailout.

If we suppose that either agents are risk neutral or we work under the risk-neutral measure, the value of debt is the expected discounted value of the payoff from deposits:

$$D_0 = \exp(-rT)\times$$
$$\times \left\{\mathbb{E}\min(F, L(T)) + \pi\mathbb{E}\max(F - L(T), 0) - \alpha(1-\pi)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right]\right\}$$

Following Merton (1974), it is easy to argue that the first addendum is the difference between the face value of deposits discounted and a put (price) on loans, with strike F, the second addendum is (the price of) a put option on L, with strike F. Collecting the put terms, we have

$$D_0 = \exp(-rT)\left\{\begin{array}{c} F - (1-\pi)\mathbb{E}\max(0, F - L(T)) \\ -\alpha(1-\pi)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right] \end{array}\right\} \quad (1)$$

The payoffs to *equity holders* of the bank at T are

$$\max\left[L(T) - F, 0\right].$$

Equity holders are long a call on loans' net value, with strike F. The equity value at time 0, E_0, is then

$$E_0 = \exp(-rT)\mathbb{E}\max\left[L(T) - F, 0\right]. \quad (2)$$

The total value of the bank is the unlevered value, plus the bailout put minus default costs.

$$D_0 + E_0 =$$

[6] Freixas et al (2007) specifies two kinds of debt; insured deposits and non-insured loan funding. In the current environment implicit insurance of creditors of all types seems to be the rule rather than the exception. The implicit guarantees cannot be certain, however.

$$= L_0 + \exp(-rT)\pi\mathbb{E}\max(0, F - L(T)) - \alpha(1-\pi)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right]. \quad (3)$$

Throughout the paper bank managers choose the face value of deposits in order to maximize $D_0 + E_0$, which represents shareholders' wealth, since it is comprehensive of dividends (in E_0) and the deposits that equityholders cash in at time 0. Since bankruptcy costs and benefits from bailouts accrue to depositors, and therefore determine D_0, they are taken into account when choosing F.

If ever there are no default costs and no bailout, the bank value $D_0 + E_0$ reduces to the initial loan value L_0: an irrelevance property of the Modigliani-Miller type holds, since leverage does not affect the bank value.

Actually, the bank value specified so far is the *private* one. We are going to distinguish it from the *social value.* The difference between social and private value is simply the *bailout put*, since this value represents a redistribution effect from the government to the depositors. As a consequence, the social value is the asset value minus defaul costs, $L_0 - \alpha(1-\pi)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right]$. In Section 6 below we discuss a broader concept of social value taking into consideration that there are potential systemic consequences as well as other costs of bank insolvencies, whether they are bailed out or not.

We capture economies of scale in deposits or debt more generally by adding a cash flow component to the cash flows from loans. Thereby, the bank's total cash flows at T are expressed as:

$$L(T) + k(F - D_0),$$

where $k > 0$. These scale effects are assumed to be proportional to the total amount of interest paid on debt rather than to the face value or the market value of debt itself. We label these scale effects generated on the liability side as "financial economies of scale" or "economies of scale in debt." It is controversial whether there are such economies of scale in banking and other financial services except as a result of an interest tax shield. We do not take a stand on the existence of these financial economies of scale but work throughout the paper with the k=0 as well as $k > 0$. A higher k can be interpreted as greater ability to exploit economies of scale. This ability may depend on the organizational choice. We return to this issue in the next section.

In the presence of financial returns to scale, the bank value becomes

$$D_0 + E_0 =$$

$$\begin{aligned} = \quad & L_0 + k\exp(-rT)(F - D_0) + \exp(-rT)\pi\mathbb{E}\max(0, F - L(T) - k(F - D_0)) \\ & -\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[(L(T) + k(F - D_0))\mathbf{1}_{\{L(T)+k(F-D_0)<F\}}\right]. \end{aligned}$$

Even in this case one can distinguish the private from the social value.

Remark 1 *It should be clear from the payoffs to debt and equity - both in this case and the ones to follow - that we could equally well have taken deposits as given and solved for the amount of loans. Note also that the benefit of the bailout put is here revealed as a higher D_0 for a fixed F (lower deposit rate)* *while higher bankruptcy costs reduce D_0 (increase the deposit rate). A smaller (larger) difference between D_0 and Fimplies lower (higher) costs of leverage, which ultimately increases (decreases) the wealth of shareholders.*

3 A bank with two affiliates

We model a bank with two affiliates, namely a home bank and a subsidiary or a branch, by specifying its financial synergies, which are affected by cross-guarantees or co-insurance between the affiliates in case of default. We move from the merger model of Leland (2007) and the parent-subsidiary case in Luciano and Nicodano (2012, 2014). We modify these models in order to include a greater variety of policies with respect to cross-guarantees, which correspond to different organizations. We also introduce the possibility of state bailouts in case of insolvency and financial returns to scale. The bank can be thought of as a horizontally integrated cross-border bank or as a conglomerate. In both cases assets generate imperfectly correlated returns described by different distributions. Either way we call one entity the home bank and the other entity we call subsidiary or branch.

It is well-known that a merger of two imperfectly correlated activities within one firm can produce financial synergies by economizing on default costs. This type of synergy can be produced in a branch organization in our terminology. As noted by Banal-Estanol *et al.* (2012) there is also a negative side to merging the activities into a branch organization because there is "contamination" if one activity must come to the rescue of another loss-making activity, with the possible consequence that the whole merged firm defaults. This type of contamination can be reduced in a subsidiary organization wherein both entities enjoy limited liability. Therefore, the rescue of one subsidiary by another one can be interrupted if the latter would be threatened by the rescue costs.

In the following financial synergies exist not only as a result of default costs but also because of a positive probability of bailout by the state of any separate legal entity enjoying limited liability. If the home bank organizes its affiliate as a branch the two entities do not separately enjoy limited liability, they default together and they are bailed-out together. The two entities provide internal guarantees for each other as long as there is capital in the bank as a whole.

If the bank organizes the affiliated activity in a subsidiary both entities enjoy limited liability. They can default individually and they can be bailed out individually. We consider two types of subsidiary organization.[7] In one the home bank offers a unilateral insurance. It intervenes in order to recapitalize (rescue) the subsidiary if ever the latter is unable to pay back his depositors, provided that rescuing the subsidiary does not trigger its own default. Thus rescue is conditional on survivorship of the home bank. In the other type of

[7] We do not solve for an optimal rescue policy but compare risk and value effects in four organizations with different rescue policies. Subsidiary organizations without any internal rescue become like stand-alone banks in the analysis that follows. We state without proof that intermediate forms of rescue between those we specify will never be optimal

subsidiary organization there is a mutual insurance. Each affiliate rescues the other affiliate conditional on the survival of the rescuing entity. The organization with mutual insurance is particularly relevant for affiliates belonging to a bank holding company. The operation of a subsidiary in the two cases is described in sections 3.1 and 3.2 below. The operation of a branch is described in section 3.3 below.

We saw above that returns to scale are proportional to the amount of interest the bank pays, i.e. the difference between the face value and present value of deposits $(F - D_0)$. We capture different operational synergies in different organizations by allowing k to be higher in the organization with greater operational efficiency. Thus, operational synergies within an organization implies greater ability to exploit economies of scale on the financial side.[8] For example, we expect k_b to be higher than k_s as a result of greater operational synergies within a branch organization. The nature of returns to scale - as represented by the magnitude of k_s and k_b - proves to be important below. So we have the following cash flows:[9]

- for subsidiaries and their parents

$$L_s(T) + k_s(F_s - D_{0s}) \doteq M_s(T),$$

$$L_h(T) + k_s(F_h - D_{0h}) \doteq M_h(T),$$

- for branches

$$L_h(T) + k_b(F_h - D_{0h}) \doteq N_h(T),$$
$$L_b(T) + k_b(F_b - D_{0b}) \doteq N_b(T),$$

where $k_s \leq k_b$. The inequality is weak when subsidiaries have the synergies of branches.[10]

Later on, in order to compare the different structures, we set final loans in branches equal (in distribution) to loans in subsidiaries $L_b(T) = L_s(T)$. If not specified otherwise, but without loss of generality, apart from the fact that we exclude perfectly dependent loans, we also introduce the following assumption: (Assumption 1) the joint density of $L_h(T)$ and $L_b(T) = L_s(T)$ has non-null density over the whole positive orthant of $\mathbb{R}^2$.

The Gaussian distribution on loan log-returns introduced later satisfies this hypothesis, which is needed only in order to simplify the discussion and avoid having events of null probability.

[8] Although we do not explicitly introduce other sources of operational synergies the potential trade off between financial and operational synergies can be captured with our formulation.

[9] Recall that $\doteq$ means "equal by definition, renamed".

[10] In the real world we have a variety of situations, depending on legal provisions, judicial systems, regulation etc.

3.1 Home-country bank and a subsidiary, one-way rescue

In the set up specified above, consider now two banks, which specialize in being a home bank and its subsidiary. They remain two separate entities, but the home bank offers a insurance against default of the subsidiary. The insurance consists in providing the subsidiary with assets at T, if the affiliate is in default, conditional on not endangering the safety of the home bank. Thanks to *limited liability* contamination can be avoided in the subsidiary case. Thereby, default costs can be avoided in comparison with the branch case.

Stakeholders' payoffs

Subsidiary debt

Rescue of the subsidiary occurs if and only if the home bank is not in default or distress ($M_h(T) > F_h$), the subsidiary is in default - i.e. its asset value is below the default level ($M_s(T) < F_s$) and rescuing the subsidiary does not drive the home bank into default. Rescuing the subsidiary means that, using its surplus $M_h(T) - F_h$, the home bank pays that part of the subsidiary deposits that are not covered by its own assets, $F_s - M_s(T)$. The home-bank can do this without incurring into a default if its surplus is greater than the amount needed for rescue of the subsidiary, $M_h(T) - F_h > F_s - M_s(T)$. The rescue conditions are

$$\begin{cases} M_h(T) > F_h \\ M_s(T) < F_s \\ M_h(T) - F_h > F_s - M_s(T) \end{cases}$$

Since the first condition is always satisfied when the last is, the conditions can be reduced to the event

$$R \doteq \begin{cases} M_s(T) < F_s \\ M_h(T) - F_h > F_s - M_s(T) \end{cases} \tag{4}$$

For given face value of deposits and initial loans of the home bank and subsidiary, the payoff to depositors of the subsidiary is the pay-off to a stand alone bank, augmented by the conditional home-bank support $F_s - M_s(T)$ when rescue occurs. If rescue does not take place, because the magnitude of the home bank's cash flows is insufficient to cover the subsidiary's losses, the state bails out the subsidiary with probability π when

$$Q \doteq \begin{cases} M_s(T) < F_s \\ M_h(T) - F_h < F_s - M_s(T) \end{cases} \tag{5}$$

Since bailout comes after rescue by the home bank, the subsidiary debt before rescue and bailout is the one we obtained above, namely (1), with $\pi = 0$ and underlying M. Default costs are paid only if there is default, no rescue and no bailout. We have:

$$
\begin{aligned}
D_{0s} \quad = \quad & +\underbrace{\exp(-rT)\left[F_s - \mathbb{E}\max(0, F_s - M_s(T))\right]}_{\text{value without bailout and rescue}} + \\
& +\underbrace{\exp(-rT)\mathbb{E}\left\{\left[F_s - M_s(T)\right]\mathbf{1}_{\{R\}}\right\}}_{\text{rescue}} + \\
& +\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\mathbf{1}_{\{Q\}}\right\}}_{\text{government bailout}} \\
& \underbrace{-\exp(-rT)(1-\pi)\alpha\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default costs}}
\end{aligned} \tag{6}
$$

Home-bank equity

Since the assets for rescue come from the home bank, rescue diminishes the equity value of the home-bank in comparison with (2). The equity value of the home bank becomes

$$
\begin{aligned}
E_{0h} \quad = \quad & \exp(-rT)\mathbb{E}\max\left[M_h(T) - F_h, 0\right] \\
& -\exp(-rT)\mathbb{E}\left\{\left[F_s - M_s(T)\right]\mathbf{1}_{\{R\}}\right\},
\end{aligned}
$$

where the first term represents the home bank without a subsidiary.

Subsidiary equity, home-bank debt

The provision of a insurance from the home-bank to the subsidiary affects only the payoffs to the equity holders of the home-bank and the depositors of the subsidiary, since it entails a transfer from the former to the latter whenever it is in distress. It does not affect the payoffs to the equity holders of the subsidiary E_{0s} - which are described by

$$E_{0s} = \exp(-rT)\mathbb{E}\max\left[M_s(T) - F_s, 0\right]$$

- and to the depositors of the home bank D_{0h} - which are described by

$$D_{0h} = \exp(-rT)\times \tag{7}$$

$$\times\left[F_h - (1-\pi)\mathbb{E}\max(0, F_h - M_h(T)) - \alpha(1-\pi)\mathbb{E}\left[M_h(T)\mathbf{1}_{\left\{M_h(T) < F_h\right\}}\right]\right].$$

Bank optimization

The home bank and subsidiary-with-insurance have deposit and equity value which are interdependent. The home bank chooses how many deposits to raise directly and through its subsidiary in order to maximize the overall value:

$$\max_{F_h, F_s}\left(D_{0h} + E_{0h} + D_{0s} + E_{0s}\right). \tag{8}$$

After simplifying, we get the following expression for overall value:

$$\begin{aligned}
&M_{h0} + \underbrace{\pi \exp(-rT)\mathbb{E}\max(0, F_h - M_h(T))}_{\text{government bailout home}} + \\
&\underbrace{-\,\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{M_h(T)<F_h\}}\right]}_{\text{default cost home}} \\
&+M_{s0}+ \\
&\underbrace{+\pi\exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}} + \\
&\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}
\end{aligned} \tag{9}$$

where $M_{i0} = L_i(0) + \exp(-rT)\ k_s(F_i - D_{0i}), i = h, s.$

The home-bank plus subsidiary value is given by the sum of the asset values $M_h + M_s$ plus the government bailout put (which enters differently in the home and subsidiary case) minus their default costs. Rescue payments disappear because they are paid by one stakeholder (equity owners of the parent) to debt holders of the subsidiary. We summarize this as:

Lemma 2 *With one-way rescue, the home-bank plus subsidiary value is the sum of the asset values M_{h0} $+M_{s0}$ plus the bailout puts for each bank, minus their default costs. Default costs are paid by the home bank as in the single-bank case; they are paid by the subsidiary only if there are no rescue and no state bailout.*

3.2 Home-country bank and a subsidiary, mutual rescue

Assume now that, in the subsidiary case, rescue can be mutual. This means that rescue of the home bank by the subsidiary will take place when the the latter is not in default and is not endengered by rescue. Formally, when an event symmetric to R holds true:

$$R' \doteq \begin{cases} M_h(T) < F_h \\ M_s(T) - F_s > F_h - M_h(T) \end{cases}$$

This rescue does not take place, and there is room for bailout (which occurs with probability π) if the home bank is in default, but the subsidiary has not enough cash flows to rescue it:

$$Q' \doteq \begin{cases} M_h(T) < F_h \\ M_s(T) - F_s < F_h - M_h(T) \end{cases} \tag{10}$$

Appendix A specifies the payoffs to stakeholders for this case. The home bank can maximize the overall group value by choosing how many deposits to raise directly and through its subsidiary:

$$\max_{F_h, F_s} \left(D_{0h} + E_{0h} + D_{0s} + E_{0s}\right). \tag{11}$$

This simplifies to maximizing

$$M_{h0}+\underbrace{\pi\exp(-rT)\mathbb{E}\max\left\{0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout home}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home}}$$
$$+M_{s0}+$$
$$+\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s-M_s(T),0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}}$$
$$\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}} \tag{12}$$

The last formulation says that the home-bank plus subsidiary value is given again by the sum of the asset values $M_h + M_s$ plus the bailout puts (which enter in the same way in the home and subsidiary case, since bailout comes after potential rescue in both cases) minus their default costs. We can again summarize this as

Lemma 3 *With mutual rescue, the home-bank plus subsidiary value is the sum of the asset values, which include returns to scale, $M_{h0} + M_{s0}$, plus the bailout puts for each bank minus their default costs, which are paid only in the absence of rescue by the other group member and in the absence of state bailout.*

3.3 Home-country bank and a branch

Consider now the branch case. The returns to scale are the ones described by $N_h(T), N_b(T)$. With the same loans, $L_b = L_s$, the returns to scale may be greater than in the subsidiary case if greater operational synergies can be exploited in a branch organization. The branch case is different from the mutual subsidiary case also because there is no more limited liability of one bank versus the other, while there was in the subsidiary case. We expect this lack of limited liability to be a source of contamination that affects branches negatively relative to subsidiary organizations, exactly as the Sarig effect deprives mergers of value (see Sarig (1985), Leland (2007), Balan-Estanol *et al.* (2012)). Financial synergies work as follows. Rescue is mutual, but is not conditional on survivorship of the guarantor. This is a result of branches not preserving limited liability. So, support from the home bank to the branch is offered whenever.

$$R_b \doteq \begin{cases} N_h(T) > F_h \\ N_b(T) < F_b \end{cases}$$

while support in the other direction occurs when

$$R_b' \doteq \begin{cases} N_h(T) < F_h \\ N_b(T) > F_b \end{cases}$$

These events substitute R, R'. The transfer in the two events is respectively

$$\min\left(N_h(T) - F_h, F_b - N_b(T)\right),$$

and

$$\min\left(N_b(T) - F_b, F_h - N_h(T)\right).$$

It does not necessarily cover the difference between the face value of debt of the guaranteed company and its own cash flows. There is the possibility of government bailout when the whole bank is insolvent. This happens when either the home bank or its branch is insolvent and the other entity is either insolvent or is unable to rescue (since the minimum above is respectively $N_h(T) - F_h$, $N_b(T) - F_b$). In the branch case, insolvency for the whole bank organization is the only possibility, although analytically we treat the two entities as separate with equity assigned to them. Thereby, we are able to highlight conditions that determine the choice of branch vs subsidiary organizations..

Whenever the state intervenes, there has been joint insolvency of the branch and home bank. If the state does not bail out there are default costs. The event in which bailout of the branch occurs - which corresponds to Q for subsidiaries - is

$$Q_b \doteq \begin{cases} N_b(T) < F_b \\ N_h(T) - F_h < F_b - N_b(T) \end{cases} \tag{13}$$

The event in which bailout of the home bank occurs - which corresponds to Q' for subsidiaries - is

$$Q_b' \doteq \begin{cases} N_h(T) < F_h \\ F_h - N_h(T) > N_b(T) - F_b \end{cases} \tag{14}$$

The home bank can maximize the overall value by choosing how many deposits to raise directly and through its branch:

$$\max_{F_h, F_b} \left(D_{0h} + E_{0h} + D_{0b} + E_{0b}\right) \tag{15}$$

It can be shown (see Appendix A too) that the overall value is

$$\begin{gathered} N_{h0} + \underbrace{\pi \exp(-rT)\mathbb{E}\max\left\{0, F_h - N_h(T))\mathbf{1}_{\{Q_b'\}}\right\}}_{\text{government bailout home}} + \\ - \underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[\left[N_h(T) + \max(0, N_b(T) - F_b)\right]\mathbf{1}_{\{Q_b'\}}\right]}_{\text{default cost home}} + \\ + N_{b0} + \\ + \underbrace{\pi \exp(-rT)\mathbb{E}\left\{\max(F_b - N_b(T), 0)\mathbf{1}_{\{Q_b\}})\right\}}_{\text{government bailout branch}} + \\ \underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[\left[N_b(T) + \max(0, N_h(T) - F_h)\right]\mathbf{1}_{\{Q_b\}}\right]}_{\text{default cost branch}} \end{gathered} \tag{16}$$

Notice that the home-bank plus branch value is given again by the sum of the asset values $N_h + N_b$ plus the government bailout puts (which enter in the same way in the home and subsidiary mutual case, since for both it comes after rescue) minus their default costs. A lemma analogous to the mutual-rescue one holds.

4 Comparing organizations: exogenous leverage

We can compare analytically the different arrangements, when the level of deposits (in face value) and cash flows from loans (in distribution), as well as other parameters, are the same. In order to isolate the effect of financial synergies on firm scope, assume first that there are no returns to scale from debt, i.e. $k = 0$, then that there are benefits, $k > 0$. For a proof see Appendix B.

4.1 No returns to scale

If we compare the unilateral versus the mutual rescue case, we can conclude that

Proposition 4 *For any positive level of deposits of the home bank F_h, assume that cash flows from loans $L_s(T)$ are equally distributed but not perfectly correlated in the affiliates (Assumption 1 holds). Their face value of deposits F_s are positive and equal. Then unilateral conditional rescues in subsidiary-home-bank organizations are privately worse than mutual conditional rescues, if $\alpha > 0$ and $\pi = 0$, while they are better when $\pi = 1$ and $\alpha > 0$ or $\alpha = 0$. If $\alpha > 0$, there is a positive bailout probability π^* above which unilateral guarantees become better than mutual. Last, they are indifferent in a neighborhood of $\alpha = \pi = 0$.*

So, independently of how many deposits are collected in the affiliate, if there is no bailout but default is dissipative ($\alpha > 0$ and $\pi = 0$), it is better to provide mutual rescues in case one affiliate becomes insolvent, instead of leaving it alone. Mutual rescue saves on default costs in the absence of state bailouts. When there is external bailout with certainty or when default is not dissipative ($\pi = 1$ *or* $\alpha = 0$), mutual rescue is not a rational strategy. Above a given likelihood of state assistance (π), mutual rescue is not any more rational if default is dissipative ($\alpha > 0$). In this case, when the weight of rescuing is left to the external entity providing bailout, instead of being internalized, one way rescue is favored over mutual rescue. The situation is illustrated in Figure 1. In this and the Figures and Tables to come, Sowr stands for "subsidiary with one-way rescue", Smr stands for "subsidiary with mutual rescue", Sbr stands for "branch bank", while SA still denotes Stand-alone banks, as declared above.

Insert here Figure 1

If we compare the branch and the subsidiary with mutual rescue organizations, we get the following

Proposition 5 *Assume that cash flows from loans in the subsidiary and branch $L_b(T), L_s(T)$ are equally distributed and their face values of deposits are positive and equal ($F_s = F_b = F$) . Then the subsidiary organization (with mutual insurance) is privately preferable to the branch, for every $\alpha > 0$, when $0 \leq \pi < 1$; when $\alpha = 0$ or $\alpha > 0$ and $\pi = 1$, they are indifferent.*

The proposition states that in all cases of dissipative default costs and some degree of uncertainty about the bailout, the bank will prefer the subsidiary organization even when there is commitment to mutual rescue. The situation is illustrated in Figure 2.

Insert here Figure 2

The explanation is that there are higher default costs associated with the branch organization since each affiliate can draw the whole bank into default (Sarig effect). The subsidiaries with mutual rescue will abandon rescue of the other affiliate if rescue would draw both into default. Thus, there is a level of protection of each subsidiary's capital as a result of limited liability. When default costs do not exist ($\alpha = 0$), or there is bailout with certainty ($\pi = 1$), the subsidiary advantage vanishes.

Putting together propositions 4 and 5, it is easy to assess the following:

Corollary 6 *Under the assumptions of propositions 4 and 5, if $\alpha > 0$, there exists a bailout probability π^* above which the subsidiary organization with unilateral insurance is privately preferred to the subsidiary with mutual insurance, which in turn is preferred or equivalent to the branch organization.*

It can also be shown that

- If the correlation between returns on the loan portfolios of the affiliates is equal to one, the bank is indifferent between subsidiary, branch and stand alone organizations.
- Reducing the correlation between returns on the loan portfolios of the two affiliates increases the relative returns to scaleto the bank from subsidiary organization, because reduced correlation implies an increasing likelihood that at least one affiliate will benefit from limited liability. Reducing the correlation also increases the advantage of the branch organization relative to stand-alone banks.
- Increasing default costs when the bailout probability is positive - but stays below 1 - increases the relative advantage of the subsidiary organization. This observation is consistent with Proposition 5.
- Increasing the bailout probability when bankruptcy costs are dissipative reduces the relative advantage of the subsidiary organization. The explanation for this result is that the higher default costs associated with branches are reduced by bail-outs. This observation is also consistent with Proposition 5.

The comparison between the efficiency and the risk taking of two stand alone banks and the same banks (in terms of assets) once they become affiliate to a group - with a unilateral or mutual insurance - is quite straightforward too. In Appendix B we indeed prove that:

Proposition 7 *For any positive level of deposits in the two banks, organizing a bank as a stand-alone entity provides greater private value than organizing it as a subsidiary - with unilateral or mutual rescue - if default is not dissipative and there is a positive probability of bailout ($\alpha = 0$ and $\pi > 0$) or, for any level of default costs, if the probability of bailout is one ($\alpha \geq 0$, $\pi = 1$). The private value of the subsidiary organization is greater when default is dissipative but the probability of bailout is null ($\alpha > 0$ and $\pi = 0$). Under dissipative default costs, there is a positive bailout probability π^{**} above which stand alone banks are more valuable than unilaterally guaranteed subsidiaries. The stand alone organization becomes more valuable than both the unilaterally and the mutually guaranteed subsidiary and the branch (with the second more valuable than the third and fourth), when $\pi > \max(\pi^*, \pi^{**})$.*

A visual representation is given in Figure 3.

Insert here Figure 3

These preliminary results will be explored further in the next sections with endogenous leverage (risk-taking) as well.

We can now turn to social value, still supposing that returns to scale are null ($k = 0$) . For a proof see Appendix B.

Proposition 8 *Suppose that default is dissipative ($\alpha > 0$) and bailout does not occur with certainty ($\pi \neq 1$). With the same positive levels of deposits, the social value of the mutual organization is higher than both the unilateral and branch ones. If $\pi = 1$ or $\alpha = 0$ they are indifferent.*

Proposition 9 *Suppose that default is dissipative ($\alpha > 0$) and bailout does not occur with certainty ($\pi \neq 1$). For any positive level of deposits, organizing a bank as a stand-alone entity provides smaller social value than organizing it as a subsidiary, both with unilateral and mutual rescue. If $\pi = 1$ or $\alpha = 0$ they are indifferent.*

In comparison with Proposition 7 this result can be explained by the relatively high value of the bailout put for stand alone banks, which are never rescued internally.

The results of the last propositions are depicted in Figures 4 and 5 respectively.

Insert here Figures 4,5

4.2 Returns to scale and incentives to regulatory arbitrage

When returns to scale exist, i.e. $k > 0$, the comparison across organizations (unilateral, mutual and SA) is influenced by the convexity of the cash flows that they generate. This convexity can influence the bank's incentives to fund the two affiliates asymmetrically. The above comparison among organizations with exogenous and equal leverage has been explored for the case when $k > 0$ as well but we do not include the results here.[11] The greater impact of returns to scale on deposits occurs when leverage is endogenous since benefits from asymmetric leverage can only be realized when leverage is unconstrained. For this reason we explore the case of $k > 0$ in greater detail only in the next section.

We have assumed that operational synergies are the same in branches and subsidiaries. These synergies are captured by the magnitude of the parameter k in the expression for returns to scale. Intuitively, the subsidiary advantage decreases when returns to scale in the branch increase relative to those in subsidiaries, thanks to the relative magnitude of k_b and k_s. This result can also be shown.

Up to this point we have assumed that default costs and bailout probabilities are the same for the two affiliates. While the bank organized in branches face default and bailout as one entity, subsidiaries and stand-alone banks may face different parameters if the two affiliates are located in different legal jurisdictions or if their activities are different. For example, traditional commercial banks are more likely to be bailed out than investment banks. This difference is recognized, for example, in the Vickers-proposal for separation of commercial banking from investment banking into entities, which can default or be bailed out separately.

We have observed that the relative disadvantage of a branch organization increases with higher default costs and declines with higher bailout probability. If the branch bank is able to choose bailout probability and default costs by choice of jurisdiction or by integrating different activities, the disadvantage of the branch bank described in Figure 2 can be turned into an advantage. It can choose legal jurisdiction in a country with relatively low default costs and a high bailout probability or it can increase the bailout probability by not separating the affiliate with relatively high bailout probability.

We return to these issues in Section 7 taking into account endogenous leverage and financial economies of scale as well.

5 Comparing organizations: endogenous leverage

In order to compare the three organizational types when leverage is optimized we begin by assuming that there are no economies of scale in 5.1. Thereafter in 5.2 and 5.3 we conduct numerical analysis of optimized group values and leverage varying parameters for economies of scale, default costs and probability of bailouts.

[11] Results and proofs can be obtained from the authors upon request. The results in Section 4.1 do not change sufficiently to motivate their inclusion here.

5.1 No returns to scale

We can demonstrate that, when we put together two stand alone banks - which do not enjoy returns to scale but were optimally levered - and give them the possibility of creating a unilateral insurance, it is optimal to have debt diversity, i.e. to move deposits from the non-insured bank to the internally-insured bank. Let us simply assume that the two entities have the same default costs and bailout probabilities. For the sake of simplicity, let their loans be independent. The following proposition, which is proven in Appendix C, holds:

Proposition 10 *Let $k = 0$. Two banks with independent loans, which are optimally levered and decide to set up a conditional unilateral internal insurance, create debt diversity, at the margin.*

If the loans are dependent, with correlation smaller than one, the same result holds. Appendix C also observes that there is no incentive to have different face values of debt in the two affiliates in the absence of returns to scale whether the affiliates are organized as subsidiaries with mutual internal insurance (rescue) or as branches. It follows that under these assumptions Proposition 5 holds with the implication that the subsidiary organization with mutual rescue dominates the branch organization at optimized group values.

Another observation is that if $k = 0$ and there is perfect correlation between returns on assets, the subsidiary organization with mutual rescue, the branch organization and two stand alone banks become identical. In this case these organizations cannot exploit financial synergies if bailout probabilities and default cost rates are the same. "Regulatory arbitrage" in response to differences in these parameters are discussed in Section 7.

It is not hard to imagine that, whenever we add financial returns to scale ($k > 0$), they will increase the incentive to shift debt into one bank only (the guaranteed one) in the unilateral rescue case, while they will provide an incentive to create debt diversity in the mutual and branch case. So, debt diversity arises in the unilateral rescue as a result of guarantees and financial returns to scale, while it arises in mutual and branch organizations as a result of financial returns to scale only.

5.2 Including returns to scale

In order to fully include returns to scale and to study more closely not only the direction of their effects, but their likely magnitude, we need to specify a distribution for returns on loans G and to apply a numerical method for finding the face value of debt that maximizes the value to be distributed to the stakeholders of the bank. In each case representing a set of values for returns to scale, bailout probability, default costs and correlation between asset returns, the market values for debt and equity, the value of the potential bailout, expected default costs and consequently the private and social values of the bank are calculated. As before the social value is the private value minus the expected value of the bailout.

We assume that log returns on loans X, defined by $L(T) = L_0 \exp(X)$, are Gaussian with mean $\mu = (r - \sigma^2/2)T$ and variance $\sigma^2 T$. In the home bank with default cost and state-intervention case, default costs are

$$\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right] = \alpha(1-\pi)L_0 N(-d_1),$$

where N is the distribution function of the standard normal, and, as in the Black-Scholes formula,

$$d_1 = \frac{\ln(L_0/F) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}.$$

The bailout put is a plain vanilla Balck-Scholes put on the asset value, with strike equal to F:

$$\pi BS_p(L(T), F, T, \sigma, r) =$$
$$\pi\left[-L_0 N(-d_1) + F\exp(-rT)N(-d_2)\right],$$

where

$$d_2 = d_1 - \sigma\sqrt{T}.$$

The total value of the stand alone debt is

$$D_0 = F\exp(-rT) - (1-\pi)\left[-L_0 N(-d_1) + F\exp(-rT)N(-d_2)\right] - \alpha(1-\pi)L_0 N(-d_1)$$
$$= F\exp(-rT)\left[1 - (1-\pi)N(-d_2)\right] + (1-\pi)(1-\alpha)L_0 N(-d_1), \quad (17)$$

while the equity value at time 0, E_0, is the plain vanilla Black-Scholes call on the asset value, namely

$$E_0 = BS_c(L(T), F, T, \sigma, r) = L_0 N(d_1) - F\exp(-rT)N(d_2) \quad (18)$$

The total value of the bank is

$$D_0 + E_0 =$$

$$= F\exp(-rT) - (1-\pi)BS_p(L(T), F, T, \sigma, r) + BS_c(L(T), F, T, \sigma, r)$$
$$-\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right]$$

$$= L_0 + \pi BS_p(L(T), F, T, \sigma, r) - \alpha(1-\pi)\exp(-rT)\mathbb{E}\left[L(T)\mathbf{1}_{\{L(T)<F\}}\right],$$

or, equivalently:

$$D_0 + E_0 =$$
$$F\exp(-rT)\pi N(-d_2) + L_0\left[1 - \pi N(-d_1) - \alpha(1-\pi)N(-d_1)\right].$$

So, in the stand-alone case we have a closed formula for the objective of the bank's maximization. With more than one bank, the maximization problems to be solved are obtained by introducing Gaussian returns in expression (9) for the unilateral rescue case for subsidiaries, expression (**??**) for the mutual rescue case for subsidiaries and expression (16) for the branch case. In each case the initial value of the loans from each entity is the same, $L_0 = 100$. The time horizon is set to five years, $T = 5$, and the instantaneous riskless rate is conventionally set to 5%, the percentage volatility is $\sigma = 20\%$ per year. Up to this point, the parameters resemble the calibration of Leland (2007), which was calibrated to non-financial BBB firms. The key parameters that will be varied in this section are those representing cost savings due to returns to scale k, the probability of bailout π and default costs α.

5.3 Numerical analysis of financial synergies

In order to compare the effectiveness of financial synergies in subsidiaries and branches relative to stand-alone banks when deposits are optimally chosen, we assume that the cash-flow returns to scale (k) are the same for all organizations. We vary this parameter as well as default costs and probability of bailouts in order to compare the values of the different organizations from a financial-synergy point of view. In all cases the correlation between asset returns in the two affiliates is 0.2 but as long as the correlation is less than one the qualitative results are robust.

In Tables 1-3 the banks maximize the private group value. In the tables we present the (private) group values (GV), the social values (SV), default costs, the values of the bailout puts for the whole bank, and leverage for each affiliate for each organization. Within the tables, each panel represents a set of parameters.

Insert here Tables 1-3

In Table 1 we begin by comparing the organizations at relatively low values of the parameters describing financial returns to scale ($k = 5\%$) and the probability of bailout ($\pi = 5\%$) in both banks. The default cost is 20 % in Panel 1 and increased to 50 % in Panel 2. Thereafter we will vary the probability of bailout in Table 2. In Table 3 we set financial returns to scale (k) to zero.

In Table 1, Panel 1 with returns to scale at 5%, bailout probability at 5% and default costs at 20% we can see first that subsidiaries with unilateral as well as mutual rescue choose to shift most of the deposits to one subsidiary. As expected from our discussion of debt diversity without returns to scale, "debt diversity" exists for all $k > 0$. Returns to scale can be maximized by concentrating the leverage to one subsidiary. The asymmetry in the mutual rescue case implies that the bank does not take advantage of the possibility to reduce bankruptcy costs and the probability of bailouts is seemingly too small to be taken advantage of seriously in both affiliates. It can also be noted in the branch case that the bank is almost symmetric. The higher probability of joint default for the branch case in combination with a relatively small probability of bailout becomes a disincentive to push leverage as much as subsidiaries do. Symmetry shows up in the SA case too, since rescue cannot occur. Debt diversity in the unilateral and mutual case stands in contrast to the lack the symmetry in the branch and stand-alone cases as illustrated in Figure 6. Debt diversity in the unilateral and mutual rescue cases is illustrated in Figure 6, which also shows how debt diversity is not exploited in the branch and stand-alone cases

Insert here Figure 6

The group value (GV) is slightly higher in the mutual case because the group value of bailout puts is slightly greater. The social values (SV) are higher in both the unilateral rescue and the branch cases, however, primarily because the values of the group bailout puts are smaller. The differences between these cases are small because the parameter values for default costs and bailout probability are low. For the same reason, the joint value of two stand-alone banks is similar as well. The situation is illustrated in the extreme-left group of bars in Figure 7, which represents the private value of different organizations when $k = 5\%, \pi = 5\%, \alpha = 20\%$.

Insert here Figure 7

In Table 1 Panel 2 we push the default costs from 20% to 50%. The results are close to those we obtained in the previous case. The group values are slightly smaller than in the previous case. The mutual rescue case is marginally "better" privately than the other cases while the social values of the unilateral case and the branch case remain higher than the social value of the mutual case. The asymmetry of leverage in the subsidiary cases remains with higher default costs but debt diversity declines. The higher exposure of the branch to high bankruptcy costs actually induces it to keep leverage relatively low. Thereby it does not obtain exploit the combination of returns to scale and limited liability. The private as well as the social values of the stand-alone case remain below the other cases because it is unable to take advantage of financial synergies including debt diversity. The situation is illustrated in the second group of bars from the left in Figure 7.

In Table 2 we raise the economies of scale parameter from 5 to 15 percent while keeping the default costs at 50%. Comparing, first, Panel 1 in Table 2 with Panel 2 in Table 1 only the parameter for financial returns to scale (k) is increasing. The advantage of the two subsidiary cases relative to the branch and stand-alone cases increases because of the ability of the subsidiaries to take advantage of the higher returns to scale through debt diversity without increasing default costs as much as in the branch case. In other words, limited liability for each subsidiary enables the subsidiary organizations to take advantage of the returns to scale to a greater extent than the branch organization. As above, the stand-alone banks cannot benefit from the returns to scale by creating debt diversity. (See Figure 7, third group of bars from the left.)

In Table 2, Panels 1-3, we raise the probability of bailout from a low of 5% in Panel 1 to 10% in Panel 2 and to 40% in Panel 3 while parameters for returns to scale and default costs remain the same in the three panels. The increase of the bailout probability to 10 percent does not change the picture dramatically although leverage increases in the subsidiary and the branch cases. The asymmetry of leverage between subsidiaries remains strong. The ranking of the group values remains the same (see last group of bars in Figure 7). The mutual subsidiary, in particular, is able to extract value from the bailout put but the branch with its greater exposure to bankruptcy costs cannot. The social value of the unilateral rescue case is the highest, however. The benefit from the bailout put, which the subsidiaries with the mutual rescue can enjoy the most, is a purely private benefit.

The bailout probability is raised to 40 percent in Panel 3. Leverage in all the cases increases strongly. The numerical analysis stops when the level of deposits reaches 250. The mutual rescue case achieves the highest group value as well as the highest social value. To illustrate this circumstance, Figure 8 adds it to the private group values in Figure 7 (extreme-right group of bars).

Insert here Figure 8

The stand-alone banks actually obtains the highest benefit from the default put but their joint default costs are relatively high as well, since there is no internal rescue between them. It can also be seen that debt levels in the two affiliates are symmetric in all cases. It is noteworthy that the branch bank pushes leverage to the highest level in order to take advantage of the bailout put while facing higher default costs than the subsidiary banks. The stand-alone bank has the lowest leverage because of its higher exposure to default costs.

In Table 3 we set $k = 0$. Thus there are no financial returns to scale. Debt diversity disappears in this case except in the bank with unilateral rescue. In this panel the bailout probability remains at 40% and the default cost parameter remains at 50%. In comparison to Table 2, Panel 3, the group and social values decline but the mutual rescue case remains the "best" from a group value as well as a social value point of view. Qualitatively the results are very similar to those shown in Table 2, Panel 3.

A general conclusion so far is that the probability of the bailout put becomes an increasingly important consideration for the choice of organization as it increases while the relative importance of both the financial returns to scale and default costs decline. The weight of the latter declines as the probability of bailouts increases. The relative advantage of subsidiary organizations seems to increase as the probability of bailouts increases. Another observation is that the leverage of branch banks increases the most when the probability of bailout increases as a result of the reduced exposure to default costs.

We turn now the social values associated with the different organizations. The concept of social value in the comparisons in this section is narrow in the sense that there are no social costs associated with bailouts and no externalities. On the contrary, bailouts reduce the social and private costs associated with default. In the next section we discuss additional aspects of social value.[12]

Insert here Figures 9a and 9b

Figures 9a and 9b show the difference between private and social group values in the different cases described above. This difference is simply the value

[12] We do not present results for choice of leverage for social value maximization. They can be obtained from the authors upon request. The analysis of the difference between optimized private and social values leads to similar results.

of the group default puts in the previous tables. A relatively large value for the difference between private and social values indicates that there is a greater need for restriction of leverage to induce the bank to optimize social value. Figures 9a and 9b show that the differences between social and private values are relatively large for the mutual rescue case for the low bailout probabilities at 5 and 10 percent. The branch organization has the smallest difference because of its relatively greater sensitivity to default costs. The difference between social and private values increases dramatically for all organizations when the probability of bailout is increased to 40 percent. In relative terms the differences between the organizations become small.

As a result of the difference between social and private values the organization with the highest private value is not necessarily the organization with the highest social value. Tables 1 and 2 shows that for moderate levels of the bailout probability the privately optimal subsidiary organization with mutual insurance is inferior from a social point of view to the subsidiary organization with unilateral rescue as well as to the branch organization. Table 1 shows that the social values of the latter organizations are the same when the bailout probability is as low as 5 percent and the default costs are low as well. The branch loses out in terms of social value relative to the unilateral insurance when the default costs increase in Table 2. Then, as the probability of bailouts increases to 0.4 the bank with mutual guarantees has the highest social value again but the differences between the organizations are small in this case.

Last, we comment on the robustness of the analytical propositions derived in the previous sections with respect to the endogenization (or optimal choice) of deposits in this Section. A priori, debt diversity induced by unilateral guarantee could weaken those results, while it should not affect the case of mutual insuarnce and branches, at least for low financial returns to scale.

Proposition 4 stated that there is a probability of bailout above which the unilateral rescue organization provides greater group value than the mutual rescue organization. In this section we found that the mutual rescue organization is superior if the default costs are positive and there are financial returns to scale. It seems that the mutual rescue case is best able to optimize in the presence of such returns to scale.

In proposition 5 the branch organization is always inferior in terms of group value to the mutual rescue organization. This proposition holds when leverage is endogenized as well.

Contrary to Proposition 7 we do not find any case in this section when the stand-alone banks jointly provide greater group value than the other organizations. This is not surprising in the presence of financial returns to scale since the stand-alone banks cannot benefit from debt diversity and they cannot economize on default costs by means of rescues..

With respect to social values Proposition 8 states that the mutual rescue organization is superior to branch and unilateral rescue organizations. The results with respect to the difference between private and social values indicate that this proposition is not robust to endogenization of leverage. Although the results for social optimization are not presented here it can be stated that the unilateral and the mutual rescue organizations are equivalent from a social value point of view with all sets of parameters. The branch organization does not obtain the highest socially optimized value with any set of parameters.

Finally, Proposition 9 comparing social values of stand alone entities with both subsidiary organization is robust with respect to endogenization of leverage. The stand alone banks provide less social value if default costs are positive. This result is also intuitive since default costs offer opportunities for financial synergies in branches and subsidiaries.

5.4 Comparing organizations when operational synergies depend on organization

The results in the previous section imply that both types of subsidiary organizations offer greater private as well as social value as a result of financial synergies than both branch and stand alone organizations. Limited liability can be exploited in subsidiaries to a greater extent than in branches since the subsidiary organizations can avoid contamination between affiliates by interrupting rescue when the whole bank is threatened. Thus, as long the affiliates are subject to the same parameters for default costs and bailouts branch organizations offer greater value only if they enable the bank to exploit operational synergies to a greater extent than subsidiaries. Numerical results presented above could be reversed.

Almost by definition stand-alone banks are unable to exploit financial synergies. As a result, they are inferior to both branch and subsidiary organizations if only financial synergies are considered. This means that stand-alone banks create greater value only if there are negative operational synergies. It is not farfetched that many banks do not have expertise that can be applied in other countries or in other lines of financial services. If so expanding a bank into new markets is associated with costs that can be avoided in stand-alone banks.

In Section 7 we will also consider regulatory arbitrage in the sense that a bank can choose organization taking into account differences in default costs and bailout probabilities across affiliates

6 Bank organization, capital requirements and systemic risk

In this section we summarize the main conclusions from the numerical analysis of the role of financial synergies in bank organization before constraining leverage by introducing capital requirements in the numerical analysis of private and social values of the different organizations. Thereafter we discuss implications of broadening the concept of social costs relative to the narrow definition used so far. In particular, potential negative externalities of a bank's default are important considerations for policy makers.

One conclusion we can drawfrom the comparison between branch and subsidiary organizations is that the private group value of the branch bank is always lower than at least one of the subsidiary organizations when the parameters are the same for the three types of organization. Second, the difference between the group values of subsidiary and branch organizations increases with a higher probability of bailouts. Third, the ability of subsidiary organizations to take advantage of bailouts induces them to create high leverage and, therefore, a relatively high probability of default. The branch bank on the other hand is more strongly exposed to default costs that induces it to keep leverage lower. This result does not hold in the numerical case when the bailout probability is at its highest (Table 2, Panel 3 and Table 3). The numerical solutions in these cases were constrained by a maximum face value of debt in each affiliate.

Fourth, the subsidiary bank with mutual rescue is generally able to obtain higher group value than the subsidiary bank with one-way rescue. The former can exploit returns to scale to a greater extent and, especially, it can exploit the bailout put when the probability of bailouts is high. The same does not hold when we look at social values; under the assumption that banks maximize private value the social value of the bank with one-way rescue is greater except when the bailout probability is high (40%).

Fifth, unless the bailout probability is very high (Table 2, Panel 3) optimal deposits are asymmetric in the two affiliates. This happens both in the unilateral rescue case and in the mutual case as a result of returns to scale the face value of deposits, F. The asymmetry (debt diversity) occurs in the branch organization as well but to a smaller extent.[13]

It is clear that the probability of bailouts is a strong driver behind the choice of organization. One reason is that an increase in the bailout probability also reduces the probability that default costs will be incurred.

6.1 Capital requirements

In Section 4 we derived analytical results comparing the private values of the different organizations under the assumption that the face value of deposits was the same for all entities in the absence of economies of scale. Here we derive numerical solutions for optimized private values under the assumptions in Table 4 that the maximum face value of deposits is set in each affiliate and stand alone bank at the level obtained for the branch case when the returns to scale parameter is high (k=15%), default costs are 50% and the probability of bailouts is 10%. The face values of deposits for the branch bank's affiliates (49 and 73) are applied as the maximum face value for all affiliates and banks. Thus, we ask whether the unconstrained branch bank obtains greater value than the constrained subsidiary and stand-alone banks

Insert here Table 4

[13] Although the differences in group values seem to be small in several cases, it must be remembered that the values are maximized with a 5 year time horizon. The differences would be greater with a longer time horizon.

Table 4 shows that the subsidiary case with mutual rescue reaches the highest private group value followed by the branch case, which obtains a higher private value than the subsidiary with unilateral rescue. The home bank in the unilateral case remains unconstrained since it chooses a face value below the maximum permitted (42 instead of 49). The leverage in terms of market value is similar as well across the four cases.

Figure 10 compares the private values obtained in these cases with those obtained when no constraint on the maximum amount of debt was imposed (panel 2, Table 2). The constraint makes different organizations similar in value, since debt diversity cannot be exploited as much as in the unconstrained case. The ranking of the four organizations in terms of social value is the same as the ranking in terms of private value but the differences between the two subsidiary organizations and the branch organization become smaller since the value of the bailout put is relatively high for the mutual rescue case. The social value of two stand-alone banks falls the most relative to the private value as a result of the high value of the bailout put for the stand-alone organization.

Insert here Figure 10

There is one major difference between branch and subsidiary organizations when capital requirements are equal across all legal entities. Such a constraint implies that the subsidiary organization cannot fully exploit the non-linearity associated with returns to scale ($k > 0$). The branch organization can exploit these benefits, however, since the capital requirement is imposed on the whole bank over its affiliates. Therefore the branch bank can choose to raise deposits asymmetrically in the two entities.

Given the total constrained face value of deposits of the branch bank (122), which it can allocate as desired between the affiliates, it can be assumed that the subsidiary and the stand alone banks would be constrained to 61 for each affiliate. The group value of the branch bank gains relative to the other organizations under this capital requirement constraint (not shown). In this specific case the group value of the branch bank is very close to the value of the subsidiary bank with mutual rescue. It is clear we can create a scenario wherein the branch organization has the highest group value. The key assumptions are that there are (convex) returns to scale ($k > 0$) and that the equal capital requirements are imposed on the legal entity level.

6.2 Incorporating systemic risk; a preliminary view

We have so far we assumed that the social value of a bank is the private group value minus the value of the bailout put. This social valuation does not incorporate the possibility of contagion in case there is a default and it does not take into account that bailouts may have its own social costs beyond the pure fiscal costs. A more complete formulation for social value would include these considerations. Contagion implies that the social costs of default are higher than the private default costs and social costs of bailout would imply that a fraction of the value of the bailout put would remain a social cost. The fear of contagion is actually the main reason why there is an expectation of bailouts.

As a preliminary analysis of systemic risk of a bank's default we assume that the social cost of contagion from the default is proportional to the expected discounted loss to the bank's creditors. This loss is what may threaten other banks in the financial system. Thus, we evaluate the expected discounted loss that leverage induces. This expected loss is measured by the difference between the value of the bank's debt in the absence of default possibilities ($Fexp(-rT)$) and its actual no-arbitrage value, D_0. We present this difference for different bank organizations as a basis for evaluation of their contributions to systemic risk.

Table 5 shows the values of expected discounted losses for different organizations under the same assumptions about parameters as in Tables 1-4. In the subsidiary cases there is one column for each affiliate, as well as a column for the whole group, which can be compared with the columns for the branch organization and the stand-alone. There is only one column for the branch case since it can only default as one entity. To save space we include only one column for the stand-alone banks although two stand-alones can default separately. If one defaults the expected loss is half the number presented in the last column (2 total SA).

Insert here Table 5

We can observe that the expected discounted loss for all organizations declines as default costs increase and increases as the probability of bailout increases. This pattern is consistent with the pattern for values of default puts in Tables 1-3. The increases in the expected discounted losses are dramatic when the bailout probability increases from 10 percent to 40 percent. This is the reason why, in Figure 11, where we present graphically the group expected losses we exclude the cases of high bailout probability.

Insert here Figure 11

Table 5 shows that the expected discounted loss is always the lowest for the branch bank in a comparison of whole groups. This result is consistent with the observation that leverage tends to be relatively low in branch banks (Tables 1 and 2). The possible explanation is that high default costs discourages leverage in the branch organization, which has the highest incidence of defaults. However, the difference relative to (total) subsidiary organizations is relatively small when the bailout probability is high.

An important difference between subsidiary and stand-alone organizations, on the one hand, and the branch organization on the other, is that the branch organization defaults as a whole while the other organizations can default in parts. We can observe that when the bailout probability is high, there is an expected discounted loss for each subsidiary, which is about half of the total expected discounted loss for the group. The same consideration is valid for the stand-alone banks. Thus, the contagion risk is to some extent diversified in the subsidiary and stand-alone banks assuming that they operationally can default separately.

These observations have relevance in the current debate on 'too big to fail' banks and operational separability of subsidiaries within bank organizations. If subsidiaries are not operationally separable their contribution to systemic risk become higher or equal to systemic risk of branch organizations.

Subsidiary and branch organizations become similar in terms of systemic risk in cases of debt diversity as well. In Table 5 the subsidiary organizations concentrate their debt in one affiliate as a result of financial economies of scale in all cases when the bailout probability is 5 and 10 percent.. Figures 12, 13, 14 illustrate these phenomenon. Binding capital requirements negate the incentives of subsidiary banks to diversify debt and, thereby, they contribute to diversification of systemic risk in subsidiary organization (assuming separability in default).

Insert here Figures 12,13, 14

Finally, from a policy point of view capital requirements may be set with the objective of reducing risk of contagion from bank defaults. An analysis of optimal capital requirements is beyond the scope of this paper, however.

7 Regulatory arbitrage with differences in default costs and bailout probabilities

We noted in Section 4 that the branch organization may become superior in terms of group value if the bank can choose legal jurisdiction for all its affiliates in a country with relatively low default costs or relatively high bailout probability. Similarly, by incorporating financial activities with different bailout probabilities and default costs if they were separate legal entities, the integrated branch bank can influence the bailout probability and the default costs for the whole unit. A universal bank can be thought of as an integrated branch bank with a bailout probability dictated by the regulator's concern with systemically important parts of the bank. Default costs are influenced by insolvency law. Operationally separable subsidiaries with different activities, on the other hand, face different bailout probabilities and default costs.

Regulatory arbitrage can be said to occur if the bank chooses organizational structure to maximize its value taking into account bailout probability, default costs and leverage constraints. A complete analysis of regulatory arbitrage lies beyond the scope of this paper but a few consequences for regulatory arbitrage follows easily from the analysis above. In particular, the choice between a branch bank, subject to default and bailout for the whole organization, and a subsidiary bank, subject to separate defaults and bailouts, is likely to be influenced by differences in bailout probabilities (π), default costs (α), economies of scale (k) as well as leverage constraints. Maximizing group value implies that the bank takes advantage of a relatively high bailout probability and relatively low defaults cost while economies of scale creates incentives to concentrate leverage to one affiliate.

In section 4 we considered the case without economies of scale and equal leverage in each affiliate. It was noted that the branch bank can take advantage of selecting a jurisdiction with high bailout probability and/or low default costs or by incorporating activities with these characteristics within its organization.

Relaxing the constraint that leverage must be the same in the different affiliates in the absence of economies of scale ($k = 0$) implies that there are opportunities for regulatory arbitrage for subsidiary banks as well as branch banks. The branch bank can choose leverage for the whole organization based on the most favorable bailout probabilities in combination with default costs. The financially integrated subsidiary bank can choose to concentrate debt in the affiliate facing the most favorable combination of bailout probability and default costs. Thus debt diversity may arise in subsidiary banks as a result of differences in these institutional parameters even in the absence of economies of scale.

Allowing for economies of scale increases the advantage of the subsidiary organization further since it would benefit from the economies of scale as well as the regulatory arbitrage. These advantages of subsidiary organizations are negated by capital requirements maximizing the face value of debt within each affiliate.

We cannot make a general statement about subsidiary banks being superior to branch banks when there are differences across jurisdictions or activities in bailout probabilities and default costs, as well as economies of scale and capital regulation. The optimal organization depends on the specific combination of these factors.

We present in Table 6 the result of one set of numerical solutions for subsidiary and branch group values when the branch can choose between high default costs combined with high bailout probability (α/π=.50/.20) and low bailout probability combined with low default costs (α/π=.20/.05). We choose these combinations since a high bailout probability is likely to be associated with high default costs but other combinations are possible. There are low economies of scale (k=0.05) and capital requirement imposed on each legal entity is 50 for subsidiary banks, and 100 in total for the branch bank.

Insert here Table 6

Table 7 shows the chosen leverage in each affiliate. Only the individual affiliate of the branch bank is not constrained to $F = 50$. The two right hand columns show the branch bank subject to either high or low parameter values. Clearly the differences between the group values of different organizations are small; possibly because high bailout probabilities are value enhancing while high default costs have the opposite effect. The table shows that the branch bank with relatively low default costs and relatively low bailout probability has the highest value with the mutual rescue case close. This result depends very much on the specific parameters but the point is that regulatory arbitrage in the form of organizational choice is likely to occur in response to policy related parameters.

8 Summary, conclusions and policy issues

We have analyzed a bank's choice between subsidiaries and branches as organizational forms for affiliated entities and we have compared the different organizations in terms of private value, social value, leverage and potential contribution to systemic risk. Two stand-alone banks were considered, as well, as a reference organization that cannot exploit financial and operational synergies. The different organizations were compared with and without leverage constraints (capital requirements) over a range of values for default costs, bailout probabilities, financial economies of scale and correlation between affiliates' asset returns. We focused on financial synergies while operational synergies were considered as an exogenous factor modifying the valuation of different organizations.

An important aspect of the valuation model is that it permits endogenous determination of both the interest rate on bank debt and leverage. Analytical solutions were obtained when leverage was constrained while numerical solutions were derived when both leverage and the interest rate were endogenous.

A subsidiary is a separate legal entity with its own capital at risk while a branch is not a legal entity with its own capital. In the subsidiary organization the home bank can offer either one-way rescue to the extent it is not drawn into insolvency or the two affiliates can offer mutual rescue under the same condition. In the branch bank the rescue is unlimited conditional on the whole bank's survival. The two affiliates jointly are either solvent or insolvent.

There are four potential sources of financial synergies that differ across organizational forms. First, there are benefits from limited liability, which can be exploited by subsidiaries, in particular. Second, economizing on default costs is a source of value in both branch-and subsidiary organizations relative to two stand-alone banks but there is also a contamination effect within a branch organization. Third, exploiting expectations of bailouts provides private, but not social, value in each organizational form. Fourth, if there are economies of scale in deposits both private and social values can be enhanced by means of asymmetric leverage (debt diversity) within a multi-affiliate bank.

As a starting point for comparison of private values we began with conditions for indifference between organizational forms from a financial synergy point of view. In these cases, only operational synergies matter for the choice of organization. Perfect correlation between affiliates' returns on assets and no returns to scale imply indifference between the branch organization and the subsidiary organization with mutual rescue. In the presence of financial returns to scale subsidiary organizations can benefit relative to branch organizations from asymmetric leverage, however.

The conditions for indifference are clearly very restrictive even in the absence of operational (positive or negative) synergies, and equal default costs and probabilities of bailouts across affiliates. For positive default costs and a bailout probability less than one the stand-alone bank is never value maximizing. The branch organization economizes on default costs relative to two stand-alone banks but it faces higher default costs than the subsidiary organizations and it is less able to extract benefits from bailouts. Capital requirements reduce the advantage of subsidiary organizations and if there are economies of scale in deposits the branch organization may even become superior to the subsidiary organizations. With this exception higher operational synergies in branches than in subsidiaries are required for the branch organization to be valued higher than the subsidiary organizations.

The general superiority of the subsidiary organizations from a private valuation point of view stems from their ability to exploit limited liability separately. A subsidiary bank with mutual rescue as within a bank holding company is mostly but not always able to obtain higher group value than the subsidiary bank with one-way rescue. The former can exploit financial synergies to a greater extent and, especially, it can exploit the bailout put when the probability of bailouts is high. Leverage and, therefore, default risk is generally higher in subsidiary organizations than in branch organizations and higher with mutual rescue than with one-way rescue.

The valuation of the different organizations from a social point of view may become different if the risk of contagion from a bank's default is taken into account mainly because leverage is associated with systemic risk. We suggested that the discounted expected loss from default can be used as a measure of a bank's contribution to systemic risk. A preliminary analysis of the discounted expected loss associated with each organization as a measure of contagion risk indicated that the failure of a subsidiary organization is likely to create greater externalities of this kind if the bailout probability is not very high. On the other hand, when the bailout probability is high the subsidiary organization may offer a degree of diversification of the contagion risk if the subsidiaries can be separated as operational units in default.

If different affiliates face different bailout probabilities and default costs, the choice of organization can be viewed as a form of regulatory arbitrage. If leverage of subsidiaries is unconstrained, the subsidiary bank can choose to push leverage high where the probability of bailouts is high and default costs relatively low. On the other hand, if leverage is constrained by strict capital requirements the branch bank can gain an advantage because capital requirements are imposed on the sum of the branch affiliates and the bank can choose jurisdiction and, thereby, bailout probability and default costs. These consideration are valid for banks with cross-border activities as well as for banks with different activities in different affiliates.

A general observation is that capital requirements tend to negate the advantage of the subsidiary organization from a private point of view. Capital requirements also reduce the ability of subsidiary banks to benefit from regulatory arbitrage in the presence of differences in bailout probabilities and default costs. Instead the branch bank can take advantage of regulatory arbitrage.

Further research includes a number of items. First, as noted, we should optimize capital requirements from the social point of view including the risk of contagion. Such an analysis requires specification of costs of contagion as well as costs of bailouts, which here have been considered a pure transfer. Cost of violating capital requirements should also be made explicit.

Second, once default costs and bailout probabilities differ across affiliates optimal rescue policies (coinsurance) may differ from the ones considered above. Optimal rescue policies and optimal capital requirements are likely to depend on each other.

A third research agenda would be to compare theoretical predictions with banks' choice of organization across both countries and activities. This type of analysis would have to include understanding of operational synergies in different organizations as well. One aspect of operational synergies would be agency costs that may depend on organizational choice. For example, Freixas et al. (2007) incorporates asset-shifting between entities, which implies that banks can shift risk between countries in the face of regulatory differences.

A fourth issue is whether default costs and probability of bailout depend on the organization of a bank. For example, we observe that banks operate cross-border in legally separate subsidiaries but at the same time they seem to be operationally as integrated as branches. This observation seems to contradict our prediction that banks will choose to reduce default costs. The blurring of economic and legal organization has the effect of increasing default costs and preventing one subsidiary from being treated independently in default. It is possible that banks are induced to increase the probability of bailouts by creating complex organizational structures. High private and social default costs are likely to increase the probability of bailouts.[14]

Much of the regulatory agenda mentioned in the Introduction may actually be viewed as attempts to reduce default costs and the probability of bailouts. For example, strict legal and operational separation of traditional commercial banking from investment banking could serve to reduce default costs and reduce the likelihood than the joint entity would be bailed out. Ring-fencing of the capital of a branch may also serve to reduce the costs to the host country of default although it may not affect the bank's choice of leverage for the whole organization. Finally, separation and ring-fencing proposals may have consequences for the possibility of rescue among subsidiaries. To the extent such rescues are made impossible by national regulators, affiliates become more like stand-alone banks in our terminology.

[14] Carmassi and Herring (2013) notes this in the case of the Lehman Brothers insolvency in 2008.

References

Acharya, V., Mehran, H., and Thakor, A., Caught Between Scylla and Charybdis? Regulating Bank Leverage when there is Rent-seeking and Risk-Shifting, work in progress, NYU and NY Fed, 2013.

Allen, F., Carletti, E. and Marquez, R., Credit Market Competition and Capital Regulation, *The Review of Financial Studies*, 2011, 24, 983-1018.

Ashcraft, A. B., Are bank holding companies a source of strength to their banking subsidiaries?, 2004, Staff Reports 189, Federal Reserve Bank of New York.

Banal-Estanol, A., Ottaviani, M. and Winton, A., Separate or Joint Financing? Coinsurance, Risk Contamination and Optimal Conglomeration with Bankruptcy Costs, Working Paper , Universitat Pompeu Fabra, University of Minnesota and Kellogg School of Management, Jan. 2012.

Blundell-Wignall, A. and C. Roulet, Bank business models, capital rules and structural separation policies: An evidence-based critique of current policy trends. *Journal of Financial Economic Policy*, 2013, 5(4), 339-360.

Bradley C., and Jones, K. D., Loss Sharing Rules for Bank holding Companies: An Assessment of the Federal Reserve's Source-of-Strength Policy and the FDIC's Cross Guarantee Authority, *Financial Markets, Institutions & Instruments* , 2008, 17, 249-286.

Carmassi, J. and Herring, R., Living wills and cross-border resolution of systemically important banks, *Journal of Financial Economic Policy*, 2013, Forthcoming.

Castiglionesi, F. and Wagner, W., On the efficiency of bilateral bank insurance, *Journal of Financial Intermediation*, 2012, pp. 177-200.

Cerutti, E., Dell'Ariccia G., and Martinez Peria, M. S., How banks go abroad: branches or subsidiaries?, *Journal of Banking and Finance*, 2007, 31, 1669-1692.

Cetorelli, N., and Goldberg, L., Liquidity Management of US Global banks, New York FED Reports, 2012.

Chemmanur, T. and John, K., Optimal incorporation, structure of debt contracts and limited recourse project financing, *Journal of Financial Intermediation*, 1996, pp. 372-408.

Chiesa, G., Optimal Credit Risk Transfer, Monitored Finance, and Banks, *Journal of Financial Intermediation*, 2008, 17, 464-477.

Dell'Ariccia G., and Marquez, A., Risk and the Corporate Structure of Banks, *Journal of Finance*, 2010, 65, 1075-1096.

Freixas, X., Loranth, G. and Morrison, A. D., Regulating Financial Conglomerates, *Journal of Financial Intermediation*, 2007, 16, 479-514.

Freixas, X. and Ma, K., Banking Competition and Stability; The Role of Leverage, Working Paper , Universitat Pompeu Fabra, Febr. 2013.

Gorton, G. and Souleles, S.S., Special Purpose Vehicles and Securitization, in Carey, M. and Stulz, R.M.(eds), *The Risks of Financial Institutions*, 2006, University of Chicago Press.

Hansmann, H., Kraakman R. and Squire, R. Law and the Rise of the Firm, *Harvard Law Review*, 2006, 119, 1333-1403.

Harr, T. and Rønde, T., Regulation of Banking Groups, Working Paper, University of Copenhagen, 2006.

Herring, R., and Carmassi, J., The corporate structure of international financial conglomerates, ch. 8 in *The Oxford Handbook of Banking*, edited by A. Berger, P Molyneux and J. Wilson, Oxford University Press, 2010

Jeon, B.N., Olivero, N. P. and Wu, J., Multinational banking and the international transmission of financial shocks: Evidence from foreign bank subsidiaries, *Journal of Banking and Finance*, 2013, 37, 952-972.

Kahn, C. and Winton A., Moral Hazard and Optimal subsidiary Structure in Financial Institutions, *Journal of Finance*, 2004, 59, 2531-2575.

Leland H., Purely financial synergies and the optimal scope of the firm: implications for mergers, spin offs, and structured finance, *Journal of Finance,* 2007, 62, 765-807.

Luciano, E., and Nicodano, G., Default risk in business groups, Collegio Carlo Alberto Notebook 283/2012.

Luciano, E., and Nicodano, G., Intercorporate guarantees, leverage and taxes, *Review of Financial Studies*, 2014, Forthcoming.

Mayes, D. Achieving Plausible Separability for the Resolution of Cross-border Banks, *Journal of Financial Economic Policy*, 2013, 5(4),

Merton R.C., On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance*, 1974, 29, 449-70.

Ongena, S. Popov, A. and Udell, G. F., When Cat's Away the Mice Will Play; Does Regulation at Home Affect Bank Risk Abroad? Working Paper 1488, European Central Bank, Nov. 2012.

Sarig, O., On mergers, divestments, and options: a note, *Journal of Financial and Quantitative Analysis*, 1985, 20, 385-89.

Appendixes

A Payoffs to stakeholders in the mutual and branch case

This Appendix reports the payoffs to stakeholders of the subsidiary and its home bank, in the presence of a mutual rescue insurance, as well as payoffs to stakeholders of two branches..

Subsidiary, mutual rescue

The formulas in section 3.1 for the subsidiary debt and home-bank equity remain in force, since their payoffs are not affected (the corresponding agents neither give nor receive additional support).

Subsidiary debt becomes:

$$\begin{aligned} D_{0s} \quad = \quad & +\underbrace{\exp(-rT)\left[F_s - \mathbb{E}\max(0, F_s - M_s(T))\right]}_{\text{value without bailout and rescue}} + \\ & +\underbrace{\exp(-rT)\mathbb{E}\left\{\left[F_s - M_s(T)\right]\mathbf{1}_{\{R\}}\right\}}_{\text{rescue received}} + \\ & +\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\mathbf{1}_{\{Q\}}\right\}}_{\text{government bailout}} \\ & \underbrace{-\exp(-rT)(1-\pi)\alpha\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default costs}}. \end{aligned} \tag{19}$$

Home-bank equity becomes:

$$\begin{aligned} E_{0h} \quad = \quad & \exp(-rT)\mathbb{E}\max\left[M_h(T) - F_h, 0\right] \\ & -\exp(-rT)\mathbb{E}\left\{\left[F_s - M_s(T)\right]\mathbf{1}_{\{R\}}\right\}. \end{aligned}$$

As for the equity of the Subsidiary and the home-bank debt, they can be evaluated taking into consideration that rescue of the home bank is conditional on the subsidiary's survival. This is a result of subsidiaries preserving limited liability. The values of the affiliate equity and home-bank debt can be shown to be as follows:

$$\begin{aligned} E_{0s} \quad = \quad & \exp(-rT)\mathbb{E}\max\left[M_s(T) - F_s, 0\right] \\ & -\exp(-rT)\mathbb{E}\left\{\left[F_h - M_h(T)\right]\mathbf{1}_{\{R'\}}\right\}, \end{aligned}$$

$$
\begin{aligned}
D_{0h} \quad = \quad & +\underbrace{\exp(-rT)\left[F_h - \mathbb{E}\max(0, F_h - M_h(T))\right]}_{\text{value without bailout and rescue}} + \\
& +\underbrace{\exp(-rT)\mathbb{E}\left\{\left[F_h - M_h(T)\right]\mathbf{1}_{\{R'\}}\right\}}_{\text{rescue received}} + \\
& +\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_h - M_h(T), 0)\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout}} \\
& \underbrace{-\exp(-rT)(1-\pi)\alpha\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default costs}},
\end{aligned} \tag{20}
$$

where the events R', Q' are as defined in the main text and where we used the returns to scale of an affiliate which is organized as a subsidiary, on top of having the financial synergies of a subsidiary. When the affiliate is organized as a subsidiary, but has the returns to scale of a branch, we just put $k_s = k_b$.

Branch

Formulas in section 3.2 are unchanged whenever rescue or bailout occurs. They change when insolvency occurs, because insolvency is always joint, and default costs are paid on both the assets of the bank originally in default and the assets transferred from its home or affiliate (unless there is bailout). As a consequence, we have the following expression for the branch debt:

$$
\begin{aligned}
D_{0b} \quad = \quad & +\underbrace{\exp(-rT)\left[F_b - \mathbb{E}\max(0, F_b - N_b(T))\right]}_{\text{value without bailout and rescue}} + \\
& +\underbrace{\exp(-rT)\mathbb{E}\left\{\min\left(N_h(T) - F_h, F_b - N_b(T)\right)\mathbf{1}_{\{R_b\}}\right\}}_{\text{rescue received}} + \\
& +\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_b - N_b(T), 0)\mathbf{1}_{\{Q_b\}}\right\}}_{\text{government bailout}} + \\
& \underbrace{-\exp(-rT)(1-\pi)\alpha\mathbb{E}\left[\left[N_b(T) + \max(0, N_h(T) - F_h)\right]\mathbf{1}_{\{Q_b\}}\right]}_{\text{default costs}},
\end{aligned}
$$

the home-bank equity

$$
\begin{aligned}
E_{0h} \quad = \quad & \exp(-rT)\mathbb{E}\max\left[N_h(T) - F_h, 0\right] \\
& -\exp(-rT)\mathbb{E}\left\{\min\left(N_h(T) - F_h, F_b - N_b(T)\right)\mathbf{1}_{\{R_b\}}\right\},
\end{aligned}
$$

the branch equity

$$
\begin{aligned}
E_{0b} \quad = \quad & \exp(-rT)\mathbb{E}\max\left[N_b(T) - F_b, 0\right] \\
& -\exp(-rT)\mathbb{E}\left\{\min\left(N_b(T) - F_b, F_h - N_h(T)\right)\mathbf{1}_{\{R'_b\}}\right\},
\end{aligned}
$$

the home-bank debt

$$
\begin{aligned}
D_{0h} \quad = \quad & +\underbrace{\exp(-rT)\left[F_h - \mathbb{E}\max(0, F_h - N_h(T))\right]}_{\text{value without bailout and rescue}} + \\
& +\underbrace{\exp(-rT)\mathbb{E}\left\{\min\left(N_b(T) - F_b, F_h - N_h(T)\right)\mathbf{1}_{\{R'_b\}}\right\}}_{\text{rescue received}} + \\
& +\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_h - N_h(T), 0)\mathbf{1}_{\{Q'_b\}}\right\}}_{\text{government bailout}} + \\
& \underbrace{-\exp(-rT)(1-\pi)\alpha\mathbb{E}\left[\left[N_h(T) + \max(0, N_b(T) - F_b)\right]\mathbf{1}_{\{Q'_b\}}\right]}_{\text{default costs}}.
\end{aligned}
$$

B Comparing organizations' values with exogenous debt

This Appendix proves first Propositions 4 and 5. It then proves proposition **7**. There are no returns to scale ($k = 0$), so that $M_i = L_i, i = s, h$, both at time 0 and at time T, in all states of the world.

We start from Proposition 4.

Proof. Let us compare the values of the unilateral subsidiary arrangement and the mutual one, when the home bank and the affiliate have the same and positive level of deposits, cash flows of the affiliates are equally distributed and $k = 0$. They are respectively

$$
\begin{gathered}
M_{h0} + \underbrace{\pi\exp(-rT)\mathbb{E}\max(0, F_h - M_h(T))}_{\text{government bailout home}} - \underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{M_h(T)<F_h\}}\right]}_{\text{default cost home}} \\
+M_{s0}+ \\
+\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}} \\
\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}},
\end{gathered}
\tag{21}
$$

and

$$M_{h0}+\underbrace{\pi\exp(-rT)\mathbb{E}\max\left\{0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout home w/mutual support}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home w/mutual support}}$$
$$+M_{s0}+$$
$$+\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s-M_s(T),0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}}$$
$$\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}. \tag{22}$$

The former is smaller than the second if and only if the bailout put net of default costs in the home bank - with no support form the affiliate - is smaller than when the subsidiary intervenes, i.e.

$$\underbrace{\pi\exp(-rT)\mathbb{E}\max(0,F_h-M_h(T))}_{\text{government bailout home}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\left\{M_h(T)<F_h\right\}}\right]}_{\text{default cost home}}<$$
$$\underbrace{\pi\exp(-rT)\mathbb{E}\max\left\{(0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout home w/ mutual support}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home w/ mutual support}}. \tag{23}$$

The event Q' is not empty, under hp 1. Whenever $0<\pi<1$, this makes the expectation which represents bailout on the left hand side greater than on the right hand side; the same for default costs (in absolute value), if $\alpha>0, 0\leq\pi<1$. So, the difference on the left hand side can be greater, equal or smaller than the one in the right hand side. However, the overall inequality in (23) holds, for any positive value of α, if $\pi=0$, while the opposite inequality holds for $\pi=1$, any $\alpha>0$, or $\alpha=0$. The two sides are equal when $\alpha=\pi=0$ and in a neighbourhood of it. Since the direction of the inequality (23) changes when π goes from 0 to 1 and α stays positive, and both its left and right-hand side are continuous in π, there is a positive bailout probability, which we call π^*, above which mutual guarantees become worse than unilateral. This concludes the proof. ■

Consider now Proposition 5.

Proof. Since $k=0$ and cash flows from loans L_s and L_b are the same, $M_h=N_h$, $M_s=N_b$. Call the common value of the latter M. Notice also that the bailout events coincide for the two organizations, i.e. the sets Q,Q' coincide with Q_b,Q'_b.We keep the former notation. Let us compare the values of the mutual subsidiary arrangement and the branch one, which are respectively

$$M_{h0}+\underbrace{\pi\exp(-rT)\mathbb{E}\max\left[(0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right]}_{\text{government bailout home}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home}}$$
$$+M_0+$$
$$+\underbrace{\pi\exp(-rT)\mathbb{E}\left[\max(F-M(T),0)\mathbf{1}_{\{Q\}})\right]}_{\text{government bailout subsidiary}}$$
$$\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}},$$

and

$$\begin{gathered}
M_{h0}+\underbrace{\pi\exp(-rT)\mathbb{E}\max\left[(0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right]}_{\text{government bailout home}}+\\
-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[\left[M_h(T)+\max(0,M(T)-F)\right]\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home}}+\\
+M_0+\\
+\underbrace{\pi\exp(-rT)\mathbb{E}\left[\max(F-M(T),0)\mathbf{1}_{\{Q\}})\right]}_{\text{government bailout branch}}+\\
\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[\left[M(T)+\max(0,M_h(T)-F_h)\right]\mathbf{1}_{\{Q\}}\right]}_{\text{default cost branch}}
\end{gathered} \tag{24}$$

The former is greater than the latter if and only if

$$-\underbrace{\alpha(1-\pi)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home s}}$$
$$\underbrace{-(1-\pi)\alpha\mathbb{E}\left[M(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}>$$

$$-\underbrace{\alpha(1-\pi)\mathbb{E}\left[\left[M_h(T)+\max(0,M(T)-F)\right]\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home b}}+$$
$$\underbrace{-(1-\pi)\alpha\mathbb{E}\left[\left[M(T)+\max(0,M_h(T)-F_h)\right]\mathbf{1}_{\{Q\}}\right]}_{\text{default cost branch}}.$$

Default costs are paid only on the home or subsidiary cash flows in the subsidiary case, while they affect also the asset transfers from the affiliate ($\max(0,M(T)-F)$) or from the home bank ($\max(0,M_h(T)-F_h)$) in the branch organization. As a consequence, default costs in the subsidiary organization are smaller (in absolute value) than costs in the branch, for positive values of α and $0\le\pi<1$, and the previous inequality is satisfied; it follows that the subsidiary organization is more valuable than the branch. When $\alpha=0$, or $\pi=1,\alpha>0$, the two become indifferent. This concludes the proof. ■

Last, let us prove proposition 7**,** which compares two SA banks with a unilateral and mutual insurance, with no returns to scale from loans ($k=0$).

Proof. Let us compare the values of the stand alone and unilateral subsidiary arrangement, when the home bank has the same and positive level of deposits in both cases. If we already name home and subsidiary the two affiliates when they are stand-alone banks, the values of the two arrangements are respectively

$$\begin{aligned} M_{h0} + \underbrace{\pi \exp(-rT)\mathbb{E}\max(0, F_h - M_h(T))}_{\text{government bailout home as SA}} - \underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{M_h(T)<F_h\}}\right]}_{\text{default cost home as SA}} \\ +M_{s0}+ \\ +\underbrace{\pi \exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\right\}}_{\text{government bailout subsidiary as SA}} \\ \underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{M_s(T)<F_s\}}\right]}_{\text{default cost subsidiary as SA}}, \end{aligned} \tag{25}$$

and

$$\begin{aligned} M_{h0} + \underbrace{\pi \exp(-rT)\mathbb{E}\max(0, F_h - M_h(T))}_{\text{government bailout home}} - \underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{M_h(T)<F_h\}}\right]}_{\text{default cost home}} \\ +M_{s0}+ \\ +\underbrace{\pi \exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}} \\ \underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}. \end{aligned} \tag{26}$$

The difference in value between the SA arrangement and the unilaterally-guaranteed group is

$$\begin{aligned} +\underbrace{\pi \exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\right\}}_{\text{government bailout subsidiary as SA}} - \underbrace{\pi \exp(-rT)\mathbb{E}\left\{\max(F_s - M_s(T), 0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}} \\ \underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{M_s(T)<F_s\}}\right]}_{\text{default cost subsidiary as SA}} + \underbrace{(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}. \end{aligned}$$

Since the returns on loans satisfy hp 1, Q is not empty. This means that, in absolute value, both the bailout put and default costs are smaller in the unilateral case than in the SA one (notice that Q is a subset of $M_s(T) < F_s$). Since they show up with different signs, the trade-off between them depends on the parameters α and π. If $\alpha = 0$ and $\pi > 0$, only the first two terms in the above expression are non-null, and the value of the SA is greater than the value of a unilateral insurance. The same situation arises when $\pi = 1, \alpha \geq 0$. If $\pi = 0$ and $\alpha > 0$, only the last two terms in the above expression are non-null, and the value of the SA is smaller than the value of a unilateral insurance. Because of continuity of the above expression, it follows that there is a π^{**} above which the SA value becomes better than the unilateral one. Let us now compare SA and mutual, i.e.

$$
\begin{gathered}
M_{h0}+\underbrace{\pi\exp(-rT)\mathbb{E}\max(0,F_h-M_h(T))}_{\text{government bailout home as SA}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\left\{M_h(T)<F_h\right\}}\right]}_{\text{default cost home as SA}}\\
+M_{s0}+\\
+\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s-M_s(T),0)\right\}}_{\text{government bailout subsidiary as SA}}\\
\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\left\{M_s(T)<F_s\right\}}\right]}_{\text{default cost subsidiary as SA}},
\end{gathered}
\tag{27}
$$

versus

$$
\begin{gathered}
M_{h0}+\underbrace{\pi\exp(-rT)\mathbb{E}\max\left\{0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout home w/mutual support}}-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home w/mutual support}}\\
+M_{s0}+\\
+\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s-M_s(T),0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}}\\
\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}.
\end{gathered}
\tag{28}
$$

The difference is

$$
\begin{gathered}
+\underbrace{\pi\exp(-rT)\mathbb{E}\max(0,F_h-M_h(T))}_{\text{government bailout home as SA}}-\underbrace{\pi\exp(-rT)\mathbb{E}\max\left\{(0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout home w/mutual support}}\\
-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\left\{M_h(T)<F_h\right\}}\right]}_{\text{default cost home as SA}}+\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home w/mutual support}}\\
+\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s-M_s(T),0)\right\}}_{\text{government bailout subsidiary as SA}}-\underbrace{\pi\exp(-rT)\mathbb{E}\left\{\max(F_s-M_s(T),0)\mathbf{1}_{\{Q\}})\right\}}_{\text{government bailout subsidiary}}\\
\underbrace{-(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\left\{M_s(T)<F_s\right\}}\right]}_{\text{default cost subsidiary as SA}}+\underbrace{(1-\pi)\alpha\exp(-rT)\mathbb{E}\left[M_s(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default cost subsidiary}}.
\end{gathered}
\tag{29}
$$

Since returns on loans satisfy hp 1, the event Q' (in which the home is not rescued by its subsidiary) is not empty. Within each line the first addendum is greater than the second, in absolute value; as above, let us analyze the difference in value by changing α and π. If $\alpha = 0$ and $\pi > 0$, only the first and third line in the above expression are non-null, and the value of the SA is greater than the value of a mutual insurance. The same situation arises when $\pi = 1, \alpha \geq 0$. The difference between the unilateral and mutual arrangement is

$$
+\underbrace{\pi\exp(-rT)\mathbb{E}\max(0,F_h-M_h(T))}_{\text{government bailout home as SA}}-\underbrace{\pi\exp(-rT)\mathbb{E}\max\left\{(0,F_h-M_h(T))\mathbf{1}_{\{Q'\}}\right\}}_{\text{government bailout home w/mutual support}}>0,
$$

so that the SA arrangement is preferable to the unilateral, which in turn is better than the mutual (as we knew from Proposition 4). If $\pi = 0$ and $\alpha > 0$, only the second and fourth lines are non-null, and the value of the SA is smaller than the value of a mutual insurance. It was also smaller than the unilateral one in that case. The difference between the unilateral and mutual arrangement is

$$-\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{M_h(T)<F_h\}}\right]}_{\text{default cost home as SA}}+\underbrace{\alpha(1-\pi)\exp(-rT)\mathbb{E}\left[M_h(T)\mathbf{1}_{\{Q'\}}\right]}_{\text{default cost home w/mutual support}}<0,$$

so that the stand alone is smaller than the unilateral and the latter is smaller than the mutual (as we knew from proposition 4). Because of continuity of the above expressions, there is a π^{***} above which the SA value becomes better than the mutual one, whih is better than the branch. Using proposition 4 and the comparison between the unilateral and stand alone, such π^{***} is the maximum between π^* and π^{**}. This concludes the proof. ■

C Conditions for endogenous debt optimality and proof of proposition 10

Suppose $k = 0$. The stand alone problem for the choice of the optimal face value of debt can be restated as that of maximizing with respect to $F = b$ the following function, which is a restatement of $E_0 + D_0 - L_0$ in (3):

$$\pi b \int_0^b dG_X(x) + c \int_0^b x dG_X(x),$$

where G_X is the distribution function of the loans, whose support is assumed to be the positive real line, $c \doteq -(\pi + \alpha(1-\pi))$. The FOC for this maximization is the equality between the marginal increase in value due to bailout and marginal default costs in case bailout does not occur. These are the left and right-hand-side of the following equality:

$$\pi G_X(b) = \alpha(1-\pi) b g_X(b), \tag{30}$$

where g is the density corresponding to G. Let b^* be its solution.

Consider the case in which a unilateral insurance is provided from the home parent - for which the notation stays as above - to the subsidiary. Assuming that the loans of the two banks are independently distributed, the objective function of the group maximization, (9), becomes $L_{0h} + L_{0s} + F(a,b)$ where

$$F(a,b) \doteq \pi b \int_0^b dG_X(x) + c \int_0^b x dG_X(x) +$$

$$+\pi a \int_0^a \int_0^{h(y,a)} dG_Y(y) dG_X(x) + c \int_0^a \int_0^{h(y,a)} y dG_Y(y) dG_X(x), \tag{31}$$

G_Y is the distribution of the subsidiary's loans, a is the face value of its deposits and $h(y,a,b) \doteq -y+a+b$. We want to demonstrate Proposition 10. For the sake of simplicity, we demonstrate the proposition for independent loan distribution. The proof can be easily extended to the case of dependent loans, using conditional distributions, provided that correlation is less than perfect.

Proof. We assume that the default cost and bailout parameter is the same for both banks, and that at leat the home bank - the insurance provider - was optimally levered before setting up the insurance. In order to demonstrate that the banks would create debt diversity, and in particular would move deposits from the non-insured bank to the internally insured bank, at the margin, consider the differential of $F(a,b)$:

$$\frac{\partial F}{\partial a}da + \frac{\partial F}{\partial b}db.$$

We are going to prove that it will be worth moving a small amount of debt (in face value) from the home to the subsidiary by showing that, starting from $b=b^*$, $a=a^*$, a decrease in the former's debt, $db<0$ accompnied by a symmetric increase in the insured's debt, $da=-db>0$, makes the differential positive:

$$\begin{cases} \left[-\frac{\partial F}{\partial a}db + \frac{\partial F}{\partial b}db\right]_{a=a^*,b=b^*} > 0 \\ db < 0 \end{cases}$$

or

$$\left[\frac{\partial F}{\partial b} < \frac{\partial F}{\partial a}\right]_{a=a^*,b=b^*}. \tag{32}$$

Let us compute the derivatives:

$$\begin{aligned}
\frac{\partial F}{\partial a} &= \pi \int_0^a \int_0^{h(y,a,b)} dG_Y(y)dG_X(x) + \\
&\quad +\pi a g_Y(a) \int_0^{h(y,a,b)} dG_X(x) \\
&\quad +\pi a \int_0^a g_X(h(y,a,b))dG_Y(y) \\
&\quad +ca g_Y(a) \int_0^{h(y,a,b)} dG_X(x) \\
&\quad +c \int_0^a g_X(h(y,a,b))y dG_Y(y) \\
&= \pi \int_0^a \int_0^{h(y,a,b)} dG_Y(y)dG_X(x) + \\
&\quad -\alpha(1-\pi)a g_Y(a) G_X(h(y,a,b)) \\
&\quad +\pi a \int_0^a g_X(h(y,a,b))dG_Y(y) \\
&\quad +c \int_0^a g_X(h(y,a,b))y dG_Y(y),
\end{aligned}$$

$$\begin{aligned}\frac{\partial F}{\partial b} &= \pi G_X(b) - \alpha(1-\pi)bg_X(b) + \\ &+\pi a \int_0^a g_X(h(y,a,b))dG_Y(y) \\ &+c\int_0^a g_X(h(y,a,b))ydG_Y(y).\end{aligned}$$

Simplyfying and recalling that at b^* (30) holds, we get the following inequality, which is equivalent to (32):

$$\pi \int_0^a \int_0^{h(y,a,b)} dG_Y(y)dG_X(x) - \alpha(1-\pi)ag_Y(a)G_X(b) > 0. \tag{33}$$

Write (33) as

$$\pi \int_0^a G_X(h(y,a,b))dG_Y(y) - \alpha(1-\pi)ag_Y(a)G_X(b) > 0.$$

Since $y < a$, $h > b$ and $G_X(h(y,a,b)) > G_X(b)$ in the first integral. It follows that the left-hand side of (33) is greater than

$$\begin{aligned}& G_X(b)\pi\int_0^a dG_Y(y) - \alpha(1-\pi)ag_Y(a)G_X(b) \\ = \;& G_X(b)\left[\pi G_Y(a) - \alpha(1-\pi)ag_Y(a)\right].\end{aligned}$$

The last expression is equal to zero at $a = a^*$. As a consequence, (33) holds if the bank receiving insurance was initially at a^*. This completes the proof. ■

With a mutual guaranteee, the objective function is (12), which can be restated as $L_{0h} + L_{0s} + H(a,b)$

$$H(a,b) \doteq \pi b \int_0^b \int_0^{h(y,b)} dG_X(x)dG_Y(y) + c\int_0^b \int_0^{h(y,b)} xdG_X(x)dG_Y(y)$$

$$+\pi a \int_0^a \int_0^{h(y,a,b)} dG_Y(y)dG_X(x) + c\int_0^a \int_0^{h(y,a,b)} ydG_Y(y)dG_X(x).$$

It is clear from the last expression that the objective function for the mutual case is symmetric in a and b. Therefore, there is no reason to believe that - when the same default cost and bailout probability applies to both affiliates - we should have debt diversity. Also in this case, the same would hold for dependent loans. Similarly for branches.

D Figures

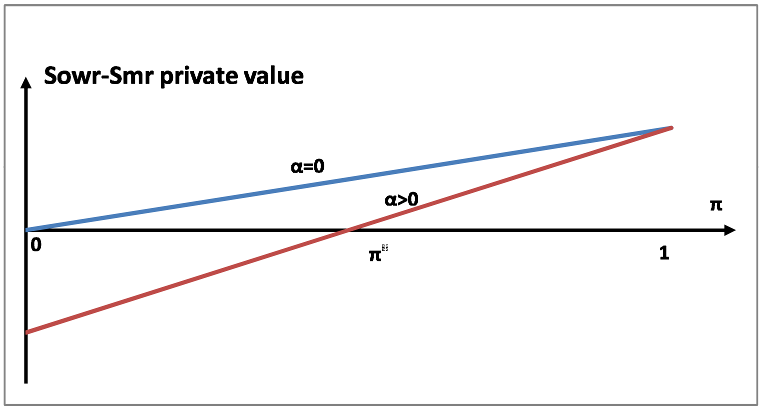


Figure 1: Difference between the private value of the two affiliates in the Sowr case and in the Smr case for α=0 (blue line) and α>0 (red line). The plot is based on Proposition 4.

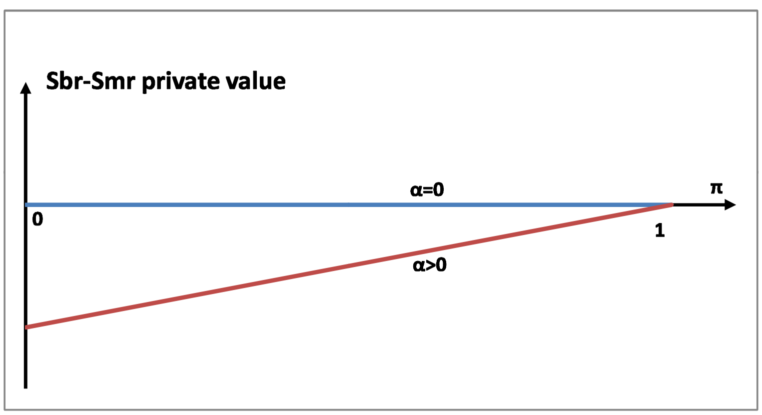


Figure 2: Difference between the private value of the two affiliates in the Sbr case and in the Smr case for α=0 (blue line) and α>0 (red line). The plot is based on Proposition 5.

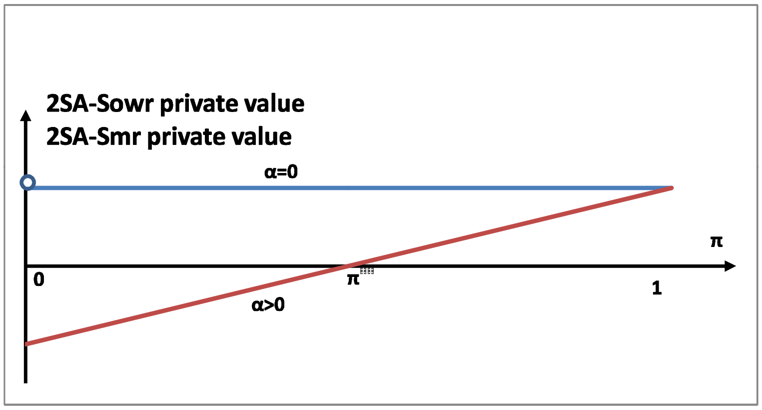


Figure 3: Difference between the private value of the two affiliates in the 2-SA case and in the Sowr or Smr case for α=0 (blue line) and α>0 (red line). The plot is based on Proposition 7. For α=π=0 the three structures are indifferent; the intersection of the vertical axis and the line for the α=0 case is excluded.

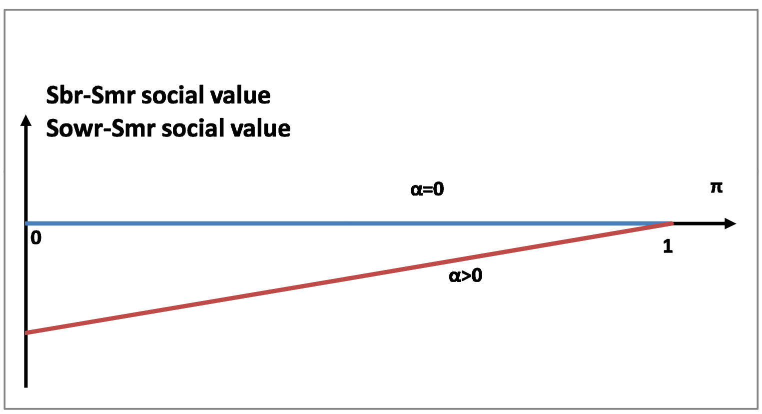


Figure 4: Difference between the social value of the two affiliates in the mutual rescue and branch or subsidiary case for α=0 (blue line) and α>0 (red line). The plot is based on Proposition 8.

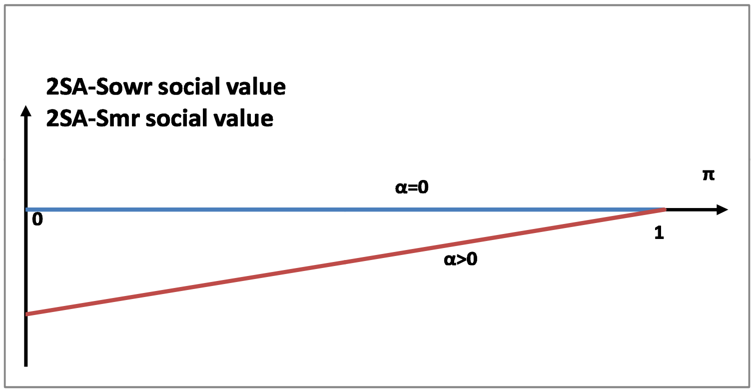


Figure 5: Difference between the social value of the two affiliates in the 2-SA and subsidiary case for α=0 (blue line) and α>0 (red line). The plot is based on Proposition 9.

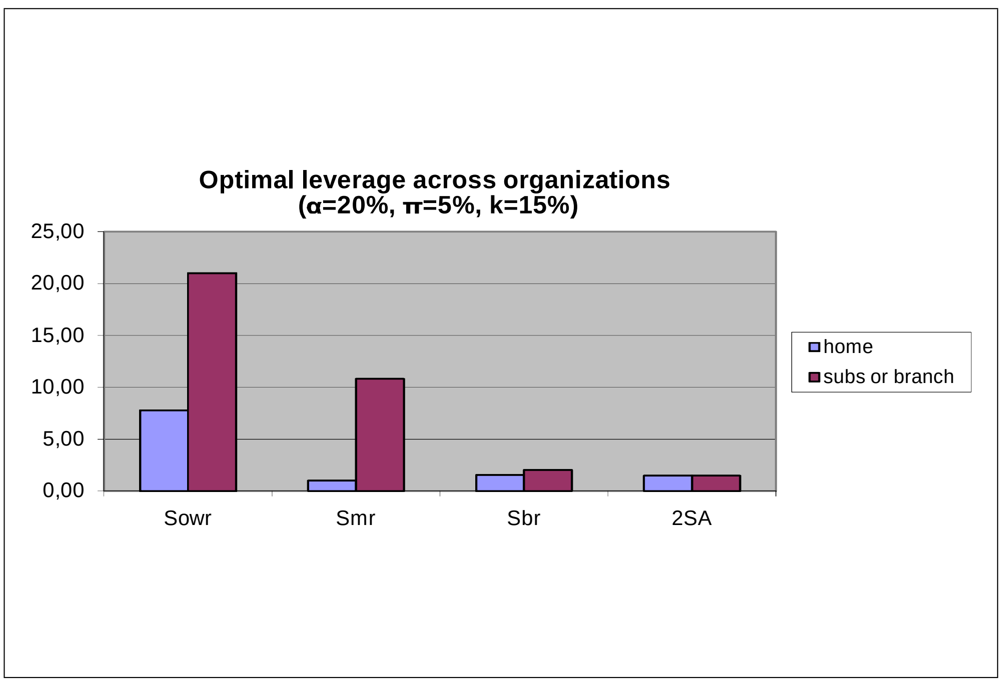


Figure 6: Optimal leverage across organizations.

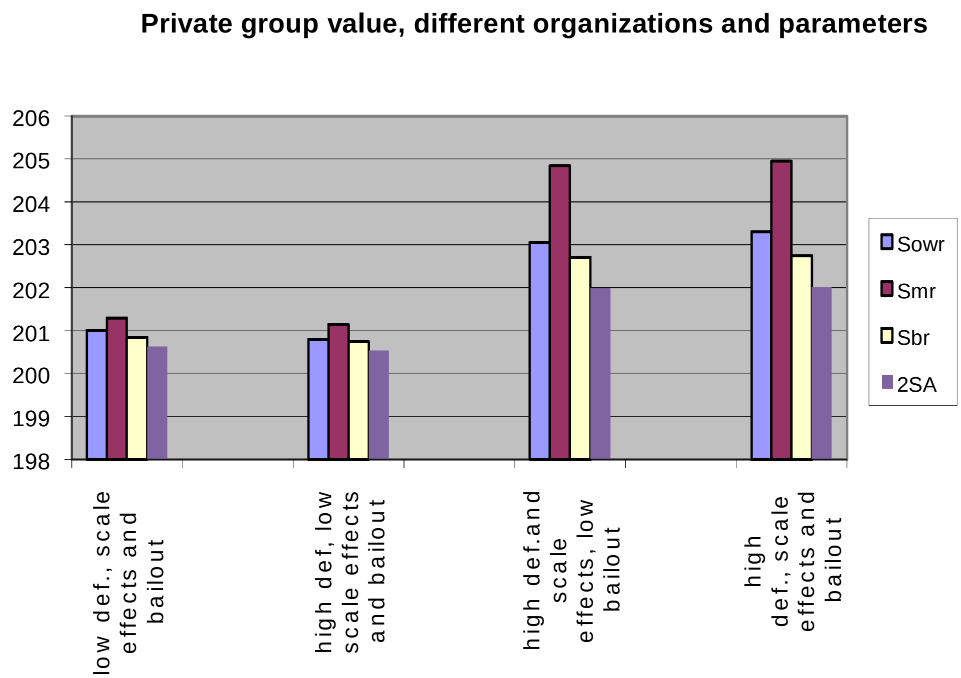


Figure 7: Private group value in different organizations, different parameter combinations, bailout probability smaller than 40%.

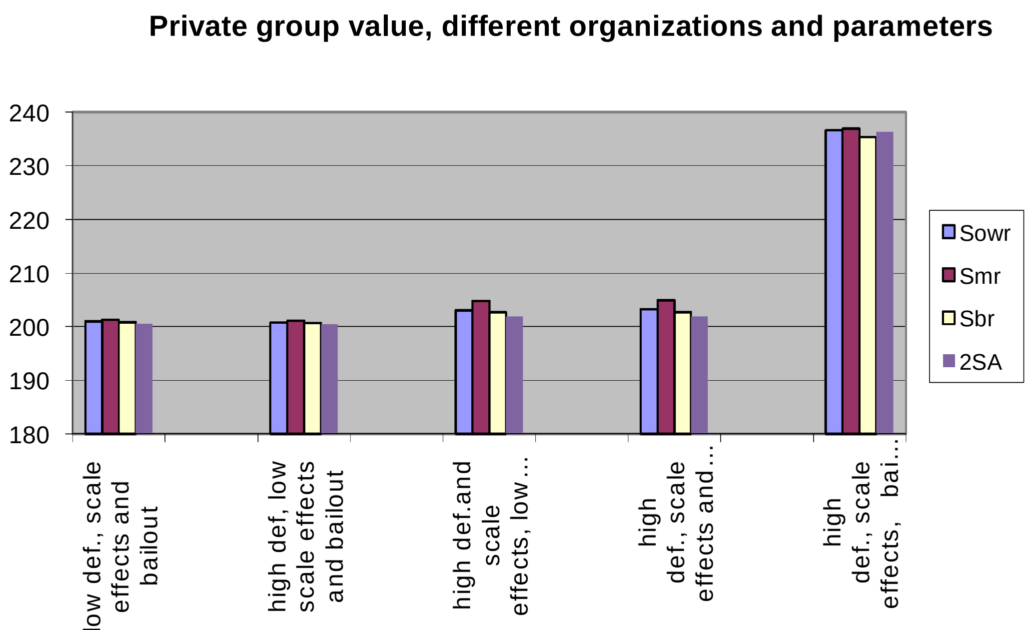
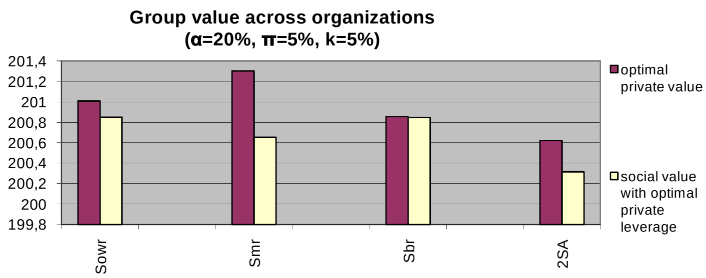


Figure 8: Private group value in different organizations, different parameter combinations, bailout probability smaller or equal to 40%.



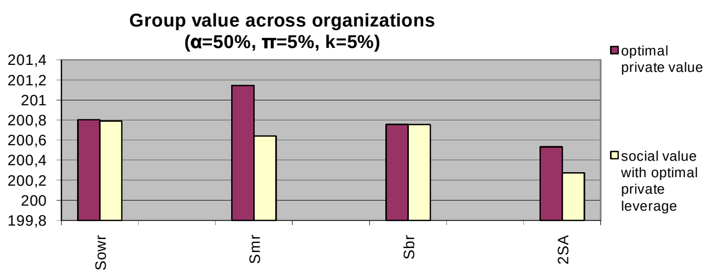


Figure 9a: Group value across organizations, private and social

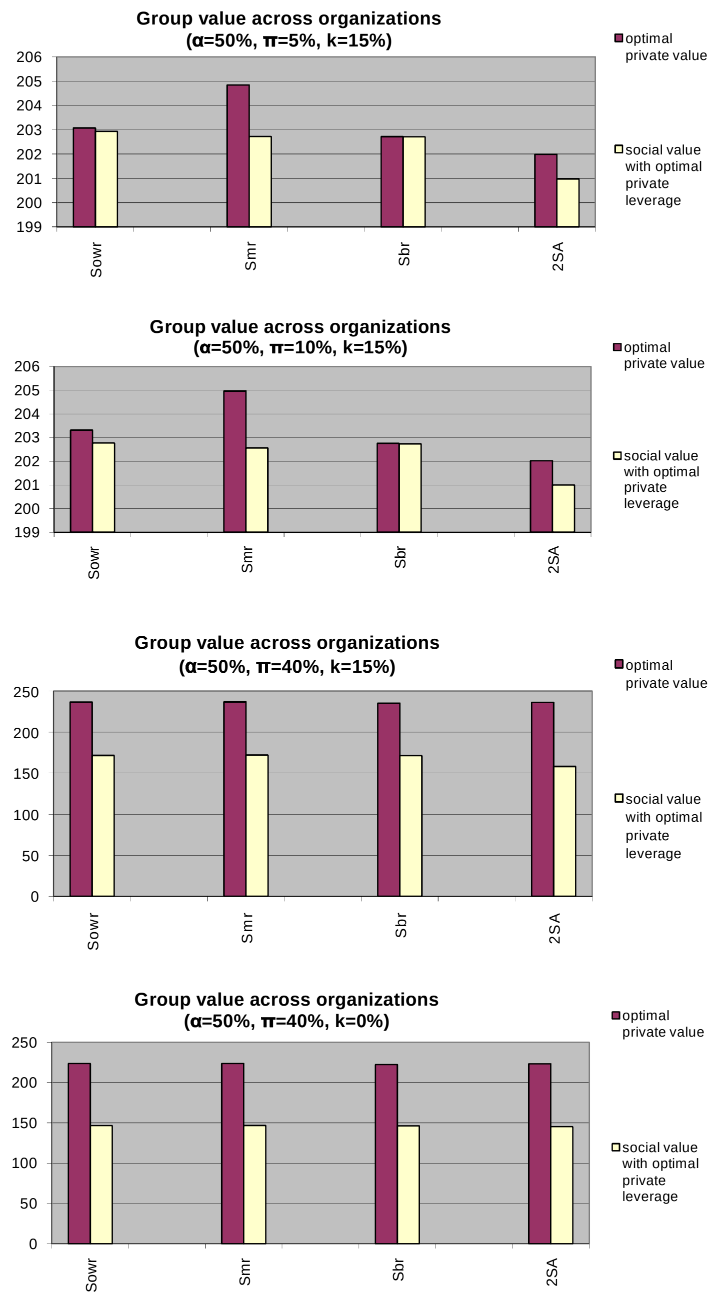


Figure 9b: Group value across organizations, private and social

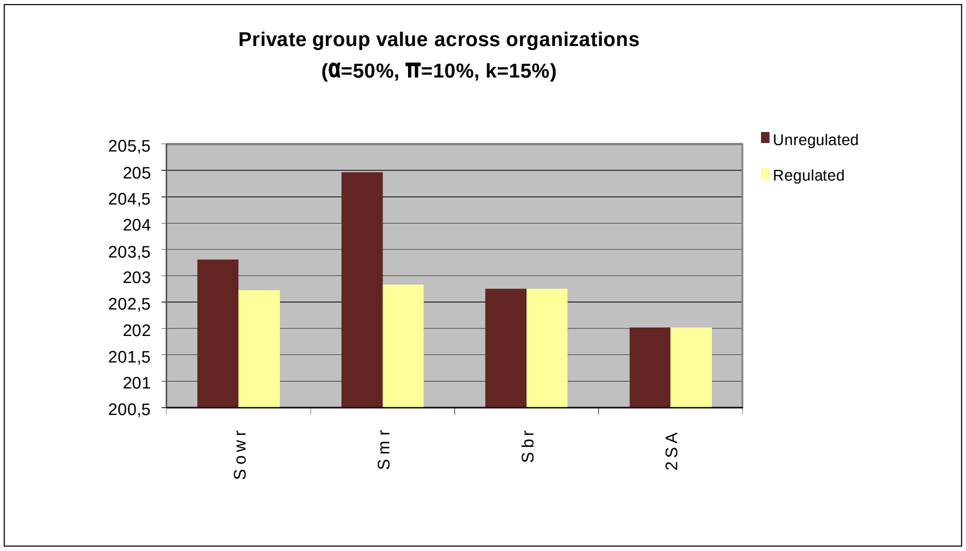


Figure 10: Private group value with and without capital requirements (regulated and unregulated case respectively), different organizations.

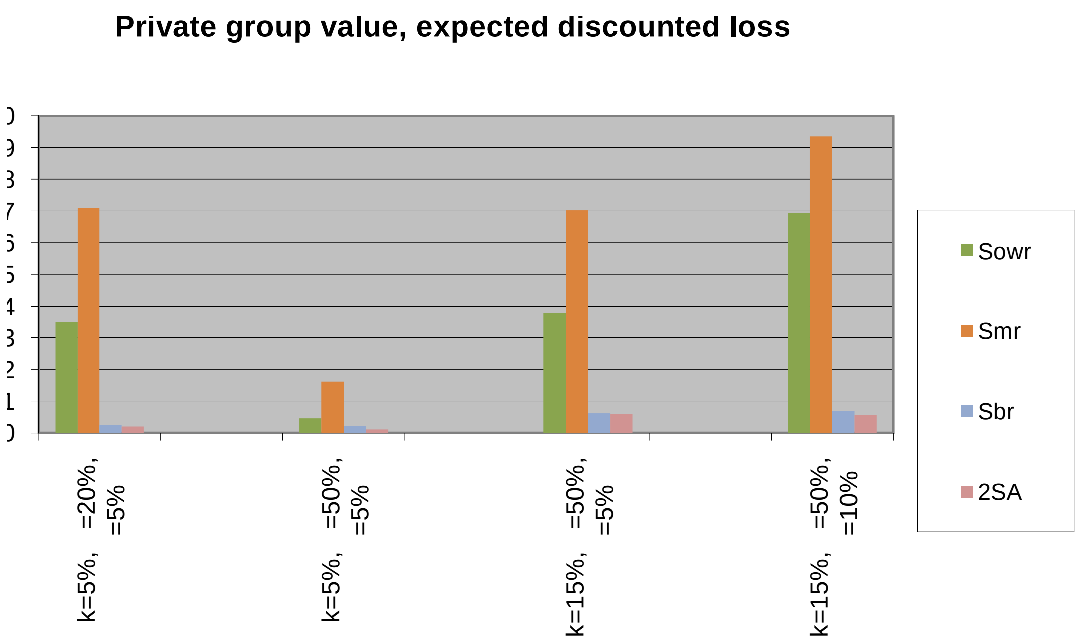


Figure 11: Private values, expected discounted loss

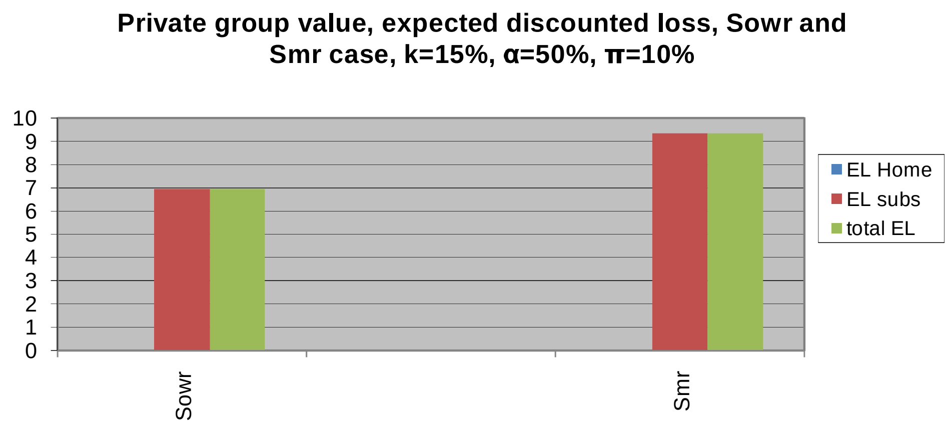


Figure 12: Private values, expected discounted loss

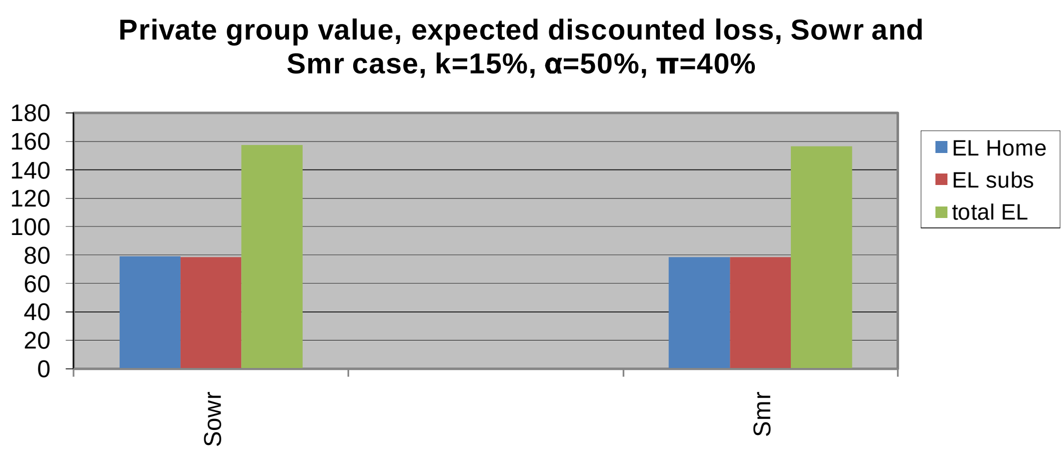


Figure 13: Private values, expected discounted loss

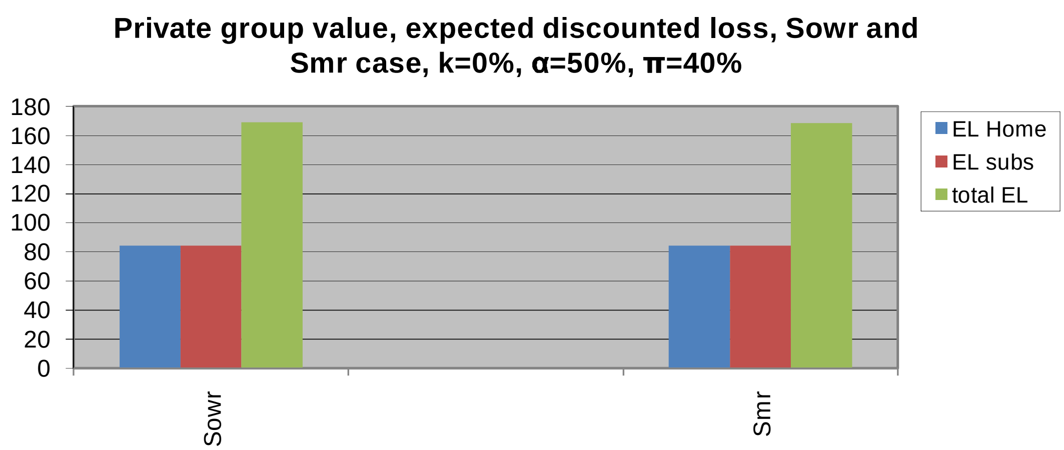


Figure 14: Private values, expected discounted loss

E Tables

Table 1 Private value maximization. k=5%, π=5%, corr=0.2
(low scale effects)

Panel 1: α=20%	Sowr		Smr		Sbr		2 SA
GV	201.0087		201.3030		200.8558		200.6208
Def. costs	0	0.48	0.03	0.959	0.030	0.104	0.1140
Group put	0.1591		0.6511		0.0066		0.3054
SV	200.8497		200.6519		200.8492		200.3154
Lev (V/E)	7.77	21.01	1.01	10.84	1.56	2.05	1.48
Panel 2: α=50%							
GV	200.8012		201.1456		200.7558		200.5310
Def.costs	0	0.138	0	0.475	0.025	0.069	0.0940
Group put	0.0173		0.5066		0.0017		0.2582
SV	200.7839		200.6390		200.7541		200.2726
Lev (V/E)	1	3.9	1.01	5.47	1.48	1.76	1.39

Table 2 Private value maximization. k=15%, α=50%, corr=0.2
(high scale effects)

Panel 1:π=5%	Sowr		Smr		Sbr		2 SA
GV	203.0677		204.8469		202.7187		201.9908
Def. costs	0	1.13	0	2.076	0.098	0.368	0.4654
Group put	0.1396		2.1231		0.0094		1.0132
SV	202.9295		202.7237		202.7094		200.9776
Lev (V/E)	1.01	7.52	2.56	9.89	1.59	2.18	1.54
Panel 2: π=10%							
GV	203.3115		204.9668		202.7511		202.0190
Def.costs	0	2.048	0	2.745	0.1088	0.3951	0.4410
Group put	0.5437		2.4069		0.0214		1.0192
SV	202.7678		202.5599		202.7297		200.9998
Lev (V/E)	1.01	10.13	1.01	11.82	1.61	2.21	1.54
Panel 3: π=40%							
GV	236.6403		236.9295		235.3064		236.3498
Def.costs	29.611	30.199		29.612	29.946	29.946	60.3982
Group put	65.1306		64.9372		64.0582		78.2300
SV	171.5096		171.9922		171.2482		158.1198
Lev (V/E)	44.74	56.39	56.94	56.94	124.67	124.67	44.30

Table 3 Private value maximization. k=0%, π=40%, corr=0.2,α=50%
(no scale effects)

	Sowr		Smr		Sbr		2 SA
GV	223.5606		223.7280		222.4624		223.3930
Def. costs	26.596	26.919	26.596	26.596	26.867	26.867	53.837
Group put	77.0753		76.9201		76.1969		78.1484
SV	146.4853		146.8080		146.2656		145.2446
Lev (V/E)	61.04	76.77	77.15	77.15	203.76	203.76	60.77

Table 4 Capital constrained. k=15%, π=10%, corr=0.2,α=50%
(high scale effects)

	Sowr		Smr		Sbr		2 SA
GV	202.7186		202.8282		202.7511		202.0190
Def. costs	0.0012	0.2250	0.1055	0.3397	0.1088	0.3951	0.4410
Group put	0.0259		0.0435		0.0214		1.0192
SV	202.6940		202.7951		202.7297		200.9998
Lev (V/E)	1.4838	2.2151	1.6114	2.2125	1.6126	2.2140	1.5428

Table 5 Expected loss for different parameters
$F \exp(-r\,T) - D0$

	uni, EL home	uni, EL subs	uni, total EL	mutual, EL home	mutual, EL subs	mutual, total EL	total EL branch	2*total SA
k=5%, α=20% π=5%	0	3.5	3.5	0	7.0953	7.0953	0.2592	0.2033
k=5%, α=50% π=5%	0	0.4663	0.4663	0	1.6285	1.6285	0.2268	0.1219
k=15%, α=50% π=5%	0	3.7833	3.7833	0	7.0280	7.0280	0.6398	0.6099
k=15%, α=50% π=10%	0	6.9423	6.9423	0	9.3571	9.3571	0.6940	0.5779
k=15%, α=50% π=40%	79.19	78.31	157.50	78.31	78.31	156.63	155.98	158.38
k=0% α=50% π=40%	84.84	84.28	169.12	84.28	84.28	168.57	168.02	169.68

Table 6. Face values of debts of Home (Fh) and affiliate (Fi), Group value (GV) and Social value (SV) with different default costs (α) and probability of bailout (π) for the two legal entities. Subsidiary entities are constrained to Face value=50, Branch bank to a total of 100. k=0.05;

	Sowr (α/π) Home=.5/.2 (α/π) Subs=(.2/.05)	Sowr (α/π) Home=.2/.05 (α/π) Subs=(.5/.2)	Smr (α/π)=.2/.05 and (.5/.20)	Sbr (α/π)= .2/.05	Sbr (α/π)= .5/.2
Fh/Fi, i=s,b	37/50	43/50	50/50	42/58	42/48
GV	200.700	200.729	200.807	200.820	200.778
SV	200.696	200.717	200.799	200.818	200.769