

# Safe assets, liquidity and monetary policy\*

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## Abstract

This paper studies monetary policy in models where multiple assets have different liquidity properties: safe and “pseudo-safe” assets coexist. A shock worsening the liquidity properties of the pseudo-safe assets raises interest-rate spreads and can cause a deep recession *cum* deflation. Expanding the central bank’s balance sheet fills the shortage of safe assets and counteracts the recession. Lowering the interest rate on reserves insulates market interest rates from the liquidity shock and improves risk sharing between borrowers and savers.

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# 1 Introduction

This paper studies monetary policy in models where multiple assets have different liquidity properties. There are some safe assets, like money, which can be perfect store of value and immediately resaleable, and other assets, labelled “pseudo-safe” assets, which are also perfect store of value but might instead have imperfect liquidity properties that can vary over time.

In fact, the recent US and European financial crises have shown sudden and dramatic changes in the quality of assets. Securities that had the reputation of safe assets with the property of being store of value and at the same time perfectly resaleable became illiquid and risky. This happened for mortgage-backed securities in the U.S. and for sovereign debt in the ongoing European crisis. A shortage of safe assets, if not immediately healed, can have effects not only on financial markets but also on the real economy. A deep recession occurred in the US followed by a slow and jobless recovery, Europe is currently experiencing a stagnation.

We look at these issues through the lens of monetary models where agents face a liquidity constraint: only some assets can be used to purchase goods, and to a different extent. In addition, the models feature a financial friction, as portfolio rebalancing takes place only after goods purchasing.

The liquidity properties of assets can change suddenly at the moment of purchasing goods and a deterioration of the quality of the pseudo-safe assets is able to bring about an adjustment in the real economy similar to that observed during the recent crisis. The overall shortage of liquidity implies a corresponding shortage of demand for goods since fewer assets remain available for goods purchasing. The mirror image of a disequilibrium in the financial market is a disequilibrium in the goods market, as often discussed in commentaries of the crisis, such as Lucas (2008) and De Long (2010), bringing out ideas that go back to Mill (1829). The consequent contraction in nominal spending can depress real activity in the presence of nominal rigidities. A deep recession and a deflation can easily emerge. In asset markets, the liquidity shock raises the premium required to hold pseudo-safe assets. The funding costs for intermediaries, which borrow in the pseudo-safe assets, increase and at the same time force them to charge higher interest rates on loans. Due to rising borrowing costs, debtors need to cut on their spending, amplifying the contraction in the real economy with important distributional effects between savers and borrowers.

Monetary policy has an important role in mitigating the adverse effects on the economy, mainly along two dimensions. The central bank can heal the shortage of safe assets and prevent the contraction in nominal spending by issuing more money, which remains a perfectly safe asset in circulation. The expansion of the central bank’s balance sheet is necessary to

maintain price stability. On the other side, monetary policy can insulate the interest rates on the pseudo-safe assets from the liquidity shock, by lowering the interest rate on reserves and therefore improving risk sharing between borrowers and savers. For a large shock, the zero-lower bound can be an important constraint along this dimension. A policy in which the interest rate on reserves is lowered, while the balance sheet of the central bank is not expanded, does not prevent the contraction in nominal spending. As well, a policy in which more liquidity is injected into the system, but the policy rate is not lowered, can only partially contain the rise in the liquidity premia and the distributional costs of the liquidity shock. It is important to note that the two policy prescriptions coming out of our model do not depend on the degree of nominal rigidities. In particular nominal spending is the key variable to stabilize in the face of a liquidity shock, as frequently discussed in the recent public debate.<sup>1</sup>

We present two simple models where pseudo-safe and safe assets coexist. In the first model, we describe the main elements of our framework through a single-agent endowment economy where the pseudo-safe assets are government bonds and the safe asset is money. The model is already rich of insights: a negative liquidity shock raises the interest rate on bonds, which can be offset by lowering the interest rate on reserves at the risk of hitting the zero-lower bound. A deflation can happen if the central bank's balance sheet is not expanded appropriately. In the second model, with heterogeneous agents, the pseudo-safe asset is an inside security, issued by intermediaries to finance lending in the economy.

Our approach to model liquidity is in line with that of Lagos (2010), where financial assets are valued for the degree to which they are useful in exchange for goods. In his model, agents are free to choose which assets to use as means of payment, between bonds and equity shares. However, he also restricts the analysis to cases in which bonds are assumed to be superior to equity shares for liquidity purposes.<sup>2</sup> The finance constraint in our model, through which goods and assets are exchanged, is of a simple form in line with the works of Lucas (1982), Svensson (1985) and Townsend (1987). The way we characterize a liquidity shock, as a change in the degree of resaleability of assets, is close to Kiyotaki and Moore (2012).<sup>3</sup> In their model, entrepreneurs face a borrowing constraint to finance investment and they need to use internal resources among which money and previous holdings of equity. Equity can be used only in part to finance investment, where the fraction available is known at the time when liquidity is needed. Instead, we model the exchange of assets for goods at the level of consumption. The shortcut taken here has the benefit of producing a highly

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<sup>1</sup>See, among others, Woodford (2012).

<sup>2</sup>In Aiyagari and Gertler (1991), instead, transaction costs in trading equities are responsible for a lower degree of liquidity of the latter with respect to bonds.

<sup>3</sup>Chiesa (2013) presents also a model in which liquidity holdings are an input to the investment process and assets have different degrees of pledgeability. Del Negro et al. (2012) estimate a quantitative version of Kiyotaki and Moore (2012) with nominal rigidities.

tractable model of the role of liquidity, which extends standard monetary models currently used for the analysis of monetary policy. Moreover, the partial resaleability of Lagos (2010) and Kiyotaki and Moore (2012) concerns a risky asset like equity and not risk-free assets as in our model. Finally, Trani (2012) is an example of an open-economy model in which multiple assets (equities) provide collateral services with time-varying properties specific to each asset, and therefore have different liquidity premia.

In monetary analysis, several works have introduced a transaction role for bonds, although an indirect one. In Canzoneri and Diba (2005), current income can also be used for liquidity purposes in a fraction that depends on the quantity of bonds held in the portfolio. In their model, bonds have indirect liquidity services since they enhance the fraction of income used to purchase goods. Woodford (1991) is an early example of a model in which current income has immediate liquidity value but bonds do not provide liquidity services. In our context, a liquidity constraint disciplines literally the exchange of assets for goods and the imperfect substitutability of pseudo-safe assets for money through a random factor. In Belongia and Ireland (2006, 2012), money and deposits are bundled together through a Dixit-Stiglitz aggregator, and can be used for liquidity purposes as in the work of Canzoneri et al. (2011) where bonds are instead imperfect substitutes for money. Canzoneri et al. (2008) consider instead a model in which bonds provide direct utility to the consumer. These latter works are not concerned with variation over time of the liquidity properties of assets.

Our analysis is complementary to a recent literature which has provided possible explanations of the macroeconomic adjustment following the recent crises. Compared to Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011), we focus on different forces for understanding financial crises.<sup>4</sup> The source of the shock in our model is to liquidity while they focus on a debt-deleveraging process which lowers the natural rate of interest.<sup>5</sup> As a consequence, in their model, the real interest rate should drop to negative values.<sup>6</sup> Instead, the optimal macroeconomic stabilization of a liquidity shock requires to keep real interest rates unchanged at the pre-crisis level. Despite this difference, we share the conclusion that staying at the zero-lower bound is optimal, but for different reasons. In our context, zero-lower bound policies are the way to insulate the market real rates from the liquidity shock and achieve a better risk-sharing of consumption between borrowers and savers. In the literature, instead, they are necessary to accommodate as much as possible the required fall in the real rate and to avoid a deep contraction in aggregate output.

In contrast, a liquidity shock requires more than just keeping the policy rate at the zero-

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<sup>4</sup>Justiniano, Primiceri and Tambalotti (2013) analyzes the quantitative implications of such models.

<sup>5</sup>It might be the case that a combination of both shocks can explain the mechanism of adjustment of the financial crisis with perhaps a different time frame where liquidity shocks hit earlier and deleveraging follows.

<sup>6</sup>This is also shown in Eggertsson and Woodford (2003).

lower bound in order to counteract the recession. An expansion of central bank balance sheet is required. In particular, its relevance is a direct consequence of the key role that assets play for liquidity purposes, and of the ensuing leverage that the central bank can exploit to heal shortages of liquidity by swapping money for pseudo-safe assets.<sup>7</sup> Therefore, we are able to rationalize as optimal policies the main monetary policy actions adopted during the recent crisis such as balance-sheet and zero-lower-bound policies. Conditionally on a liquidity shock, our work sheds some light on the optimal tapering of the monetary stimulus and the exit from zero-lower bound policies.

The paper is structured as follows. Section 2 presents the framework in which liquidity and pseudo-safe assets are introduced. It discusses a simple monetary model with flexible prices. In Section 3 the propagation of liquidity shocks is analyzed depending on alternative monetary policy regimes. Section 4 analyzes a monetary model with heterogenous agents (savers and borrowers) where an inside asset plays the role of a pseudo-safe security. Section 5 uses the general model to study the macroeconomic implications of a liquidity shock and the role of monetary policy. Section 6 concludes.

## 2 A simple model with pseudo-safe assets

We model liquidity as the resaleability of an asset in exchange for consumption goods. Several assets can be brought to buy goods, but they have different liquidity properties which can be discovered only at the time of purchasing. In the goods market, the portfolio of assets cannot be rebalanced nor new assets can be traded. The following liquidity constraint applies

$$\sum_{j=1}^N \gamma_t(j)(1 + i_{t-1}(j))B_{t-1}(j) \geq P_t C_t \quad (1)$$

where  $N$  assets are available and  $B_{t-1}(j)$  is the value of asset  $j$ , in units of currency, held in the agent's portfolio. At the time of purchasing goods, each security matures already a predetermined nominal interest rate, specific to the asset and given by  $(1 + i_{t-1}(j))$ ;  $P_t$  is the nominal price index while  $C_t$  is real consumption. Securities differ for their liquidity properties which are only known when they are exchanged for goods:  $\gamma_t(j)$  indicates literally the fraction of assets held from previous period that can be used to purchase goods, with  $0 \leq \gamma_t(j) \leq 1$ . Assets can be ordered from the worst to the best in terms of liquidity properties assuming that  $\gamma_t(j)$  is a non-decreasing function of  $j$ . In this set of assets, money may have

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<sup>7</sup>Curdia and Woodford (2010, 2011), Gertler and Kyotaki (2011), Gertler and Karadi (2011) are also related models for analyzing unconventional monetary policy but in contexts in which the relevant shocks have a financial nature instead of being directly related to the liquidity properties of assets.

the role of the best security for liquidity purposes, meaning  $\gamma_t = 1$ . This happens in a fully credible fiat-money system where the liabilities of the central bank are completely resaleable and trusted.

Liquidity in this model can have a dual interpretation. On the one hand, it can simply capture the degree of “acceptance” of an asset in exchange for goods. We could think of a consumer who goes to the goods market and discovers that, among all the securities that he has carried along, only a fraction is accepted to buy goods. On the other hand, it could simply refer to the fraction of securities which can be fully mobilized and exchanged for goods. In line with this interpretation, it can capture the intrinsic liquidity of the asset or a sort of delay in payment. To this end, we can think of these assets as the corresponding liabilities of some other agent, not modeled, that can be liquidated only in part at the exact time in which the creditor needs to purchase goods. There is a subtle difference between the two interpretations. In the first case, liquidity is a property that the “market” (seen from the perspective of who is offering goods) attributes to the asset. This property might have to do with the trust in the security as a medium of exchange. In the second case, it is an intrinsic property of the asset, although it can vary over time. Mixed interpretations could be given since the distinction is really subtle: indeed illiquidity at the origin can also be correlated with a low degree of acceptance of the asset at destination or viceversa.

In any case, all the securities traded in this model are “risk free”, meaning that they are perfect store of value;  $\gamma_t$  captures just liquidity risk, and not credit risk. By this virtue, all securities are remunerated at their specific predetermined nominal interest rate. The remaining fraction  $(1 - \gamma_t(j))$  – which cannot be used as liquidity – remains in the financial account becoming immediately available just after goods purchasing.

Money, the security with  $\gamma_t = 1$ , is the safe asset. Here safeness has a double meaning. First, it captures the property of an asset as a perfect store of value. In this model all assets share this property because each is remunerated at its specific risk-free nominal interest rate.<sup>8</sup> On top of this, the safe asset is fully liquid because it can always be accepted or mobilized to purchase goods. The other assets, with  $\gamma_t(j) < 1$ , are imperfect substitutes as means of exchange and can be labelled “pseudo”-safe assets.

Following goods purchasing, the financial market opens and consumers reallocate their portfolio according to the following constraint

$$\sum_{j=1}^N B_t(j) = \sum_{j=1}^N (1 - \gamma_t(j))(1 + i_{t-1}(j))B_{t-1}(j) + P_t Y_t + T_t + \left[ \sum_{j=1}^N \gamma_t(j)(1 + i_{t-1}(j))B_{t-1}(j) - P_t C_t \right]$$

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<sup>8</sup>We could easily amend this assumption by allowing for default risk. However, the purpose of this paper is to analyze the effects of the change in the liquidity properties of assets which do not necessarily materialize in a credit event.

where  $Y_t$  is exogenous output and  $T_t$  are transfers from the central bank or government. In the above constraint it is clear that the assets which are not carried in the goods market or are unspent remain in the financial account.

Given the above general framework, we start our analysis from a simple model in which there are two outside assets, money and government bonds, which can provide liquidity services. Later we consider also a model with inside assets.

Consider a closed economy with a representative agent maximizing the expected discounted value of utility

$$E_{t_0} \sum_{t=t_0}^{+\infty} \beta^{t-t_0} U(C_t) \quad (2)$$

where  $E_t$  is the conditional expectation operator,  $\beta$  is the intertemporal discount factor with  $0 < \beta < 1$ ;  $U(\cdot)$  is the utility flow which is a function of current consumption,  $C$ , and has standard properties.

At the end of a generic period  $t - 1$ , the representative agent invests  $M_{t-1}$  in money and  $B_{t-1}$  in bonds. At the beginning of the next period  $t$ , money and bonds mature their nominal interest rates, given respectively by  $(1 + i_{t-1}^m)$  and  $(1 + i_{t-1})$ , which are both predetermined. At this time, both assets can be used to purchase goods according to the following liquidity constraint

$$(1 + i_{t-1}^m)M_{t-1} + \gamma_t(1 + i_{t-1})B_{t-1} \geq P_t C_t, \quad (3)$$

where  $P_t$  is the price level. Since  $\gamma_t$  lies in the interval  $[0, 1]$ , bonds are an imperfect substitute for money for purchasing purposes. As discussed above,  $\gamma_t$  is a measure of the degree of saleability of bonds for goods when liquidity is needed.

After the goods market closes, the representative agent receives income and transfers from the government and, together with the unspent money and bonds, reallocates its overall wealth into new money and bonds to be carried over in the next period. The representative agent adjusts his portfolio through the following constraint

$$M_t + B_t \leq (1 - \gamma_t)(1 + i_{t-1})B_{t-1} + P_t Y_t + T_t + [(1 + i_{t-1}^m)M_{t-1} + \gamma_t(1 + i_{t-1})B_{t-1} - P_t C_t]$$

where  $M_t$  and  $B_t$  denote the holdings of money and bonds to carry in the next-period goods market. When the asset market opens the representative household receives the endowment  $Y_t$  which is also a random variable and transfers from the government,  $T_t$ . Since the endowment and the transfer are given to the agent after the goods market closes, they both have to be turned into either money or bond holdings, to be used for transactions purposes in the next period. The term in the square bracket on the second line captures the residual holdings of assets after goods purchases. It should be noted that the fraction  $(1 - \gamma_t)$  of bonds, which

cannot be used for transaction purposes, still remains in the financial account and is available for asset trading when the financial markets open. The above constraint simplifies to

$$P_t C_t + M_t + B_t \leq (1 + i_{t-1})B_{t-1} + (1 + i_{t-1}^m)M_{t-1} + P_t Y_t + T_t. \quad (4)$$

The representative agent maximizes the expected utility (2) under the constraints (3) and (4), and subject to an appropriate borrowing-limit condition, by choosing consumption,  $C_t$ , asset holdings ( $M_t, B_t$ ). Given Lagrange multipliers  $\psi_t$  and  $\lambda_t$  attached to the constraints (3) and (4) the following first-order condition holds with respect to consumption

$$\frac{U_c(C_t)}{P_t} = \psi_t + \lambda_t \quad (5)$$

showing that the liquidity constraint creates a wedge between the marginal utility of nominal consumption and that of nominal wealth – the latter being captured by  $\lambda_t$ . This wedge depends on the multiplier  $\psi_t$  on the liquidity constraint. Optimality conditions with respect to money and bonds imply respectively

$$\lambda_t = \beta(1 + i_t^m)E_t(\psi_{t+1} + \lambda_{t+1}), \quad (6)$$

$$\lambda_t = \beta(1 + i_t)E_t(\gamma_{t+1}\psi_{t+1} + \lambda_{t+1}). \quad (7)$$

A unit of currency carried from period  $t$  and invested in money delivers a return  $(1 + i_t^m)$  which can be used at time  $t + 1$  to purchase goods or for the remaining part to contribute to next period wealth. Instead, a unit of wealth invested in bonds is remunerated at  $(1 + i_t)$  but provides liquidity services only for the fraction  $\gamma_{t+1}$ . It should be noted that equations (6)–(7) show already that when  $\gamma_{t+1} = 1$  interest rates on money and bonds are equalized because the two assets become perfect substitutes as a means of payment. This happens also when the liquidity constraint is never binding, i.e. when  $\psi_t = 0$ .

To see this formally, simplify the first-order conditions to

$$\frac{i_t - i_t^m}{1 + i_t} E_t \left\{ \frac{U_c(C_{t+1})}{P_{t+1}} \right\} = E_t \{ (1 - \gamma_{t+1})\psi_{t+1} \}, \quad (8)$$

$$\psi_t = \frac{U_c(C_t)}{P_t} - \beta(1 + i_t^m)E_t \left\{ \frac{U_c(C_{t+1})}{P_{t+1}} \right\}. \quad (9)$$

In general, since  $\psi_{t+1}$  and  $\gamma_{t+1}$  are non-negative and  $\gamma_{t+1}$  is bounded above by 1, money has a lower return than bonds,  $i_t^m \leq i_t$ , which depends on  $\psi_{t+1}$  and  $\gamma_{t+1}$  and their covariance. For given  $\psi_{t+1}$ , when the liquidity properties of bonds improve, the interest rate on bonds

falls closer to that of money. Moreover, the premium on bonds will be high when their liquidity properties, measured by  $\gamma_{t+1}$ , correlate inversely with the marginal utility of liquidity, represented by  $\psi_{t+1}$ .

Money and bonds are supplied by the central bank and government, respectively. Their integrated budget constraint can be written as

$$M_t^s + B_t^s = (1 + i_{t-1})B_{t-1}^s + (1 + i_{t-1}^m)M_{t-1}^s + T_t.$$

Equilibrium in asset markets implies

$$M_t = M_t^s,$$

$$B_t = B_t^s,$$

while in goods market

$$Y_t = C_t.$$

We solve for the equilibrium allocation of this model. The following set of equations

$$(1 + i_{t-1}^m)M_{t-1}^s + \gamma_t(1 + i_{t-1})B_{t-1}^s \geq P_t Y_t, \quad (10)$$

$$\frac{i_t - i_t^m}{1 + i_t} E_t \left\{ \frac{U_c(Y_{t+1})}{P_{t+1}} \right\} = E_t \{(1 - \gamma_{t+1})\psi_{t+1}\} \quad (11)$$

$$\psi_t = \frac{U_c(Y_t)}{P_t} - \beta(1 + i_t^m)E_t \left\{ \frac{U_c(Y_{t+1})}{P_{t+1}} \right\} \quad (12)$$

characterizes the equilibrium of prices, interest rates and the Lagrange multiplier  $\psi_t$  for given exogenous processes  $\{Y_t, \gamma_t\}$  considering that  $\psi_t \geq 0$ . When  $\psi_t > 0$ , constraint (10) holds with equality. We further assume that there exists a technology through which the representative agent can store currency unaltered across periods so that the zero-lower bound on the nominal interest on money (or reserves) applies.<sup>9</sup> The following inequalities hold  $i_t \geq i_t^m \geq 0$ .

The equilibrium conditions have six unknowns  $\{M_t^s, B_t^s, i_t, i_t^m, P_t, \psi_t\}$  which leave room for the choice of three policy instruments. Considering an exogenous path of the supply of bonds, we are left with two dimensions along which to choose monetary policy. It should be noted that it is not necessary to specify policy in terms of money aggregates in our model. We could choose the two instruments of policy as  $i_t$  and  $i_t^m$ , for example.

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<sup>9</sup>In our model  $M_t^s$  are the liabilities of the central bank and  $i_t^m$  is the interest rate paid by the central bank on such liabilities. We call it interest rate on money or reserves interchangeably. Note that an hypothetical corridor system is zero in our model so that the interest rate on reserves coincides at the same time with the policy rate and the interest rate on the marginal lending facility.

### 3 Liquidity shocks and monetary policy

We study the effects of a liquidity shock which worsens the quality of the pseudo-safe assets. At time 0 it is learnt that the liquidity properties of bonds temporarily deteriorate – meaning a fall in  $\gamma$  starting from period 1 – and return back to normal levels in each period with a constant probability  $\xi$ . *Ex-post*, the shock lasts  $T$  periods until period  $T + 1$ .<sup>10</sup>

There are clearly no real effects of the shock because in the simple model of the previous section prices are fully flexible. However, the way prices and interest rates react to the shock can be already meaningful to intuit what will happen in more complicated models.

The specification of monetary policy is important for the results. We consider a benchmark policy in which the monetary policymaker is completely “passive”. This policymaker keeps the interest rate on reserves unchanged and at the same time does not alter the path of money growth with respect to the previous trend. More broadly we can think of a policymaker that does not react at all to the shock either with conventional policy, through the policy rate, nor with unconventional policy, through the balance sheet.

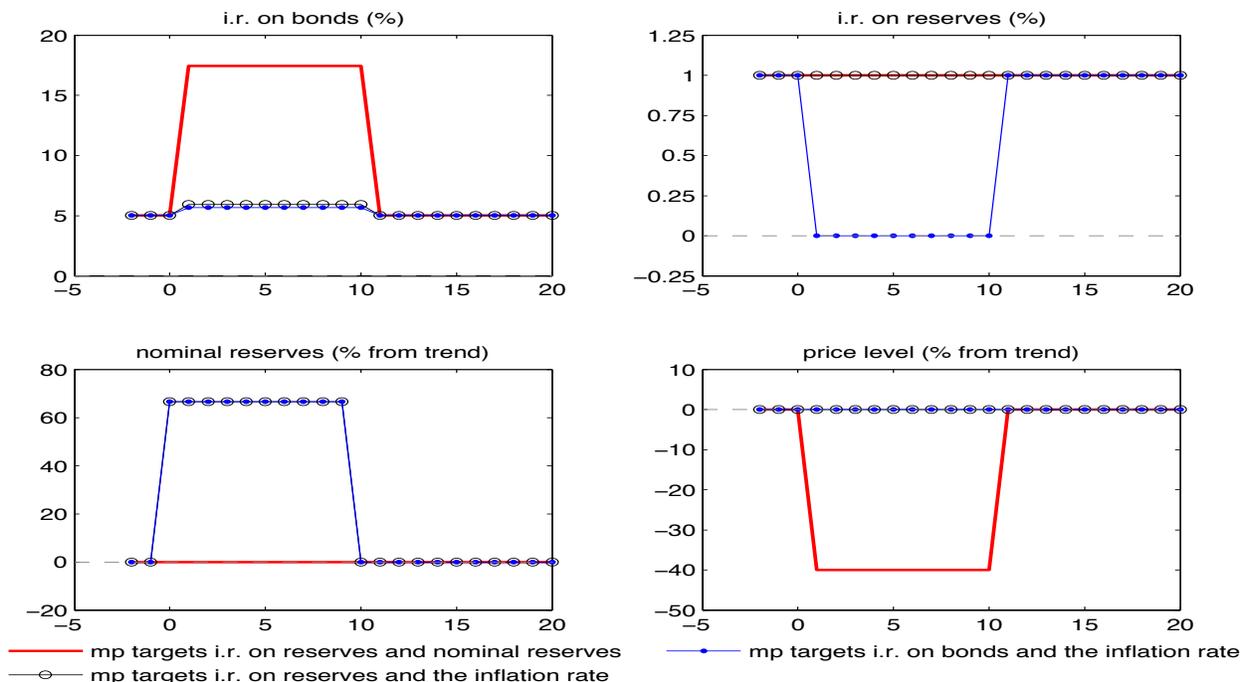
In this context, the liquidity shock has two effects. The liquidity properties of bonds deteriorate and this is immediately reflected into a fall in their price and a rise in their yield. To hold bonds, consumers ask for a higher return to compensate for the worsening in their quality. On the other side, there is a shortage of liquidity because the pseudo-safe assets have now a lower acceptance rate in exchange for goods. The overall shortage of assets as means of payment implies a shortage of demand of consumption goods. Since prices are flexible, they fall to keep the goods market in equilibrium. These effects are shown in Figure 1 by the bold solid line. The calibration implies that before the shock the interest rate on bonds is about 5% at annual rates and the interest rate on money is at 1%; money, prices and the supply of bonds grow at 2% at annual rates. We study the effects of a full deterioration in the quality of pseudo-safe assets which brings  $\gamma$  from 20% to 0 for 10 quarters. This shock leads to an increase in the spread between pseudo-safe and safe assets of about 13%. The price level falls substantially with respect to previous trend through a deep deflation.<sup>11</sup>

We compare the benchmark policy with two other policies in which the policymaker seeks

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<sup>10</sup>We assume that the realization of the shock is known one-period in advance by the monetary policy maker, to allow the latter to have the possibility to stabilize it completely. This is because in our model it is the money supply of previous period that influences the current price level. In this way, a feasible policy is one in which there is complete stabilization of inflation – or the price level – around the target.

<sup>11</sup>The following quarterly calibration is used:  $\beta = .99$ . The initial ratio of  $M/(PY)$  is set at  $\bar{m} = 0.15 \cdot 4$ , while that of  $B/(PY)$  at  $\bar{b} = .5 \cdot 4$ , as implied by the US post-WWII average of the velocity of M1 and of the debt-to-GDP ratio. Such calibration implies that the steady-state share of bonds providing liquidity services consistent with the constraint (10) is about 20%, and the steady-state annualized interest rate on bonds about 5%. In the initial equilibrium, the interest rate on money is set at 1% at annual rates while the growth rates of money, prices and bonds are all 2% at annual rates.



**Figure 1:** Response of selected variables to a 20% fall in the liquidity properties of bonds. Solid line: passive monetary policy. Circled line: monetary policy targets interest rate on reserves and the inflation rate. Dotted line: monetary policy targets interest rate on bonds and the inflation rate. The probability that in each period the shock returns back to steady state is  $\xi = 10\%$ ; the shock actually returns back after 10 quarters.

to stabilize inflation rate at the 2% target. The two policies differ because in one – the circled line in Figure 1 – the interest rate on reserves is kept unchanged at the initial level while in the other – the dotted line in Figure 1 – the policymaker tries to insulate the interest rate on bonds from the shock.

We have seen that the excess demand of liquidity and the corresponding excess supply of goods translate into a fall in the price level. To keep instead prices on their target, the excess demand of liquidity should be filled by assets with a high degree of acceptance in exchange for goods. To this end, the growth of money – the only safe asset in circulation – should increase substantially with respect to the previous target. The effectiveness is shown in Figure 1: the balance sheet is expanded by 70% and prices are stabilized. The expansion should last until the liquidity properties of bonds return back to the initial level. However, this policy does not prevent the spillovers of the liquidity shock into a higher interest rate on bonds. The expectation of a stable inflation throughout the period, though, mutes the response of the interest rate on bonds, and the latter therefore rises less than in the case of passive policy. To completely offset this surge, the monetary policymaker can in principle lower the interest rate on reserves, up to the point in which the zero-lower bound becomes relevant. If the shock is large in enough, as in Figure 1, the constraint is binding and the

interest rate on bonds still rises, although by a lower amount.<sup>12</sup>

At this point, it is useful to comment on the comparison between our analysis and that of Poole (1970). In Poole (1970), the driving shock is on money *demand*, and an interest-rate targeting policy fully stabilizes the economy because money supply endogenously adjusts. In our framework, instead, the driving shock is on the *supply* of liquidity, which includes money and also other assets. Given the overall demand of liquidity, a shock on the quality of pseudo-safe asset endogenously shifts the demand of money. However, in our framework and unlike in Poole (1970), targeting the interest rate (either on reserves or on bonds) is not sufficient to endogenously increase the money supply and fully absorb the shock. Monetary policy should specify two instruments of policy rather than one. A deliberate expansion in money supply is also needed on top of stabilizing the interest rate on bonds.

The simple model of this section does not have welfare implications because agents get utility from consumption, which is always equal to output in equilibrium. However, two important results already emerge from the analysis. First, to prevent prices from falling with respect to the target, the shortage of liquidity should be offset by issuing more safe assets. Second, a negative liquidity shock induces an upward pressure on the interest rate on bonds. The monetary policymaker can lean against it by cutting the interest rate on reserves. The extent to which it can be successful, however, depends on whether the liquidity shock is strong enough to drive the interest rate on reserves to the zero-lower bound.

## 4 A model with an inside pseudo-safe asset

Building on the insights of the previous simple model, we now present a more articulated framework in which money coexists with an inside security that plays the role of the pseudo-safe asset. To the end of better characterizing the propagation mechanism of a liquidity crisis like the recent one, we model a heterogenous-agent economy where consumers are divided between savers and borrowers. Financial intermediaries channel liquidity by issuing the pseudo-safe asset (deposit) to savers in order to lend to borrowers. The real effects of a liquidity shock are analyzed in a model featuring also price rigidities.

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<sup>12</sup>In the experiment of this section, we have focused on a monetary policy that acts directly through injection of liquidity into the system. However, since bonds and money are not perfect substitutes for liquidity purposes, the monetary policymaker can also operate by expanding money supply to buy pseudo-safe assets. In this case, the consumers' holdings of bonds,  $B_t$ , would fall during the experiment.

## 4.1 Households

Consider a closed-economy model with two types of agents: borrowers, denoted with “ $b$ ”, and savers, with “ $s$ ”. There is a mass  $\chi$  of savers and  $(1 - \chi)$  of borrowers. Utility is given by

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} [U(C_T^j) - V(L_T^j)] \quad (13)$$

for  $j = b, s$  where  $E_t$  denotes the standard conditional expectation operator and  $\beta$  is the discount factor, with  $0 < \beta < 1$ .  $C$  is a consumption bundle

$$C \equiv \left[ \int_0^1 C(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where  $C(i)$  is the consumption of a generic good  $i$  produced in the economy and  $\theta$  is the intratemporal elasticity of substitution with  $\theta > 1$ ;  $L^j$  is hours worked of quality of labor  $j$ .

At the beginning of period  $t$  the goods market opens and the following constraint limits the purchase of goods

$$(1 + i_{t-1}^m)M_{t-1}^j + \gamma_t(1 + i_{t-1}^d)I_{t-1}^j B_{t-1}^j \geq P_t C_t^j, \quad (14)$$

for each  $j = b, s$  where  $B_{t-1}^j$  represents the per-capita holdings of the inside security and  $I_t^j$  is an indicator function which takes the unit value only when  $B_{t-1}^j$  is positive – in which case it pays off  $(1 + i_{t-1}^d)$  – and zero otherwise. When in positive holdings, the inside asset takes the form of a deposit issued by the intermediary and it is a substitute of money to a certain degree, where  $\gamma_t$  is the quality value that the market attaches to it for its liquidity properties. All other variables have been previously defined.

When the goods market closes, the asset market opens and agents adjust their portfolios according to

$$M_t^j + B_t^j \leq (1 - \gamma_t)(1 + i_{t-1}^d)I_{t-1}^j B_{t-1}^j + (1 + i_{t-1}^b)(1 - I_{t-1}^j)B_{t-1}^j + W_t^j L_t^j + \Psi_t^j + \Upsilon_t^j + T_t^j + [(1 + i_{t-1}^m)M_{t-1}^j + \gamma_t(1 + i_{t-1}^d)I_{t-1}^j B_{t-1}^j - P_t C_t^j] \quad (15)$$

where  $(1 + i_{t-1}^b)$  is the nominal interest on borrowing, i.e. when  $B_{t-1}^j$  is negative and therefore  $I_{t-1}^j = 0$ ;  $W_t^j$  denotes the nominal wage which is specific to labor of agent  $j = b, s$ ;  $\Psi_t^j$  are profits obtained from goods production while  $\Upsilon_t^j$  are the profits of the intermediary sector.

Since we have assumed that agents share the same discount factor  $\beta$  in their preferences, we can start from an initial steady state featuring a non degenerate distribution of wealth and identify as “savers” the agents with positive holdings of the inside security,  $B_t^s > 0$ , and

as “borrowers” those with negative ones,  $B_t^b < 0$ . Furthermore, we make this steady-state distribution of wealth unique – and therefore assure convergence to it after the shock – by appropriately modeling the activity of the financial intermediaries. Finally we verify that borrowers and savers do not switch their portfolio positions during the time in which the liquidity shock hits the economy.

In models like Curdia and Woodford (2010), savers and borrowers have different preferences, which is the reason why a sector of financial intermediaries is meaningful. Instead, in our model, this role comes naturally because the inside asset has a different use for the two agents. Both borrowers and savers can transfer wealth intertemporally using the inside security, but only savers hold it in a positive amount and use it for liquidity purposes, as shown in (14). Intermediaries can raise liquidity from savers by paying the interest rate  $(1 + i_t^d)$  on their deposits and use it to lend to the borrowers at a higher rate,  $(1 + i_t^b)$ . Positive margins of intermediation, and credit spreads, emerge endogenously in our model.

Agents choose consumption and hours worked to maximize utility (13) under (14) and (15) taking into account standard borrowing-limit constraints. First-order conditions of the two optimization problems are symmetric with respect to consumption, money and labor

$$U_c(C_t^j) = (\psi_t^j + \lambda_t^j)P_t \quad (16)$$

$$\lambda_t^j = \beta(1 + i_t^m)E_t(\psi_{t+1}^j + \lambda_{t+1}^j), \quad (17)$$

$$V_l(L_t^j) = \lambda_t^j W_t^j \quad (18)$$

for  $j = b, s$ , where  $\psi_t^j$  and  $\lambda_t^j$  are the respective Lagrange multipliers of constraints (14) and (15). The first-order condition of the savers with respect to deposit holdings implies

$$\lambda_t^s = \beta(1 + i_t^d)E_t(\gamma_{t+1}\psi_{t+1}^s + \lambda_{t+1}^s), \quad (19)$$

while that of the borrowers with respect to loans<sup>13</sup>

$$\lambda_t^b = \beta(1 + i_t^b)E_t\lambda_{t+1}^b. \quad (20)$$

We can combine more compactly the above first-order conditions to obtain the interest-rate spread between deposits and money

$$\frac{i_t^d - i_t^m}{1 + i_t^d} E_t \left\{ \frac{U_c(C_{t+1}^s)}{P_{t+1}} \right\} = E_t \left\{ (1 - \gamma_{t+1})\varphi_{t+1}^s \frac{U_c(C_{t+1}^s)}{P_{t+1}} \right\}, \quad (21)$$

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<sup>13</sup>In writing the intertemporal first-order conditions of savers and borrowers we have already accounted for the fact that in equilibrium  $B_t^s > 0$  and  $B_t^b < 0$ .

and between loans and money

$$\frac{i_t^b - i_t^m}{1 + i_t^b} E_t \left\{ \frac{U_c(C_{t+1}^b)}{P_{t+1}} \right\} = E_t \left\{ \varphi_{t+1}^b \frac{U_c(C_{t+1}^b)}{P_{t+1}} \right\}. \quad (22)$$

Deposit and loan rates are in general higher than the interest rate on money (or reserves) insofar as the variables  $\varphi_{t+1}^s$  and  $\varphi_{t+1}^b$  are non-zero in some contingency, where

$$\varphi_t^j = 1 - \beta(1 + i_t^m) E_t \left\{ \frac{U_c(C_{t+1}^j)}{U_c(C_t^j)} \frac{P_t}{P_{t+1}} \right\}, \quad (23)$$

and we have used the following definitions  $\varphi_t^j \equiv \psi_t^j P_t / U_c(C_t^j)$  for  $j = b, s$ . The liquidity shock  $\gamma_t$  affects the interest-rate spread between deposits and money. When the liquidity properties of deposits improve, the interest rate on deposit falls closer to that on money.

Finally, we can write in a more compact way the marginal rate of substitution between leisure and consumption through the following conditions

$$\frac{V_l(L_t^j)}{U_c(C_t^j)} = (1 - \varphi_t^j) \frac{W_t^j}{P_t}, \quad (24)$$

for  $j = b, s$ . In this model, the liquidity constraint implies a financial friction which is captured by the variables  $\varphi_t^j$ . This friction creates also a wedge between the real wage and the marginal rate of substitution between leisure and consumption, as shown in (24).<sup>14</sup>

## 4.2 Financial intermediaries

Financial intermediaries channel liquidity from savers to borrowers and have positive margins of intermediation because deposits have a liquidity value, as they can be used by savers in exchange for goods. The overall level of deposits is  $D_t = \chi B_t^s$ , that of loans is  $A_t = -(1 - \chi) B_t^b$ . The intermediaries' balance sheet in each period implies  $A_t = D_t$ .

In period  $t$  profits of intermediation in real terms are

$$\frac{\Upsilon_t}{P_{t-1}} = (1 + i_{t-1}^b) a_{t-1} - (1 + i_{t-1}^d) d_{t-1} - k \cdot \phi \left( \frac{(1 + i_{t-1}^d) d_{t-1}}{(1 + \bar{r}^d) \bar{d}} \right)$$

which depend on the volume of lending and deposit supplied in the previous period, where  $a_t = A_t/P_t$  and  $d_t = D_t/P_t$ . As in Belongia and Ireland (2006, 2012) and Curdia and Woodford (2010), we also assume that financial intermediaries face a cost of increasing their borrowing capacity above a certain threshold. The cost is given by the function  $\phi(\cdot)$  with the

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<sup>14</sup>This is consistent with the cash-in-advance constraint model of Cooley and Hansen (1989).

properties  $\phi(1) > 0$ ,  $\phi'(1) = 1$  and  $\phi''(1) > 0$  where  $\phi'(\cdot)$  and  $\phi''(\cdot)$  are respectively the first and second derivatives of  $\phi(\cdot)$ . The variable  $\bar{d}$  defines the steady-state level of deposits and  $k$  is an appropriate scaling factor given by  $k = (1 + \bar{r}^b)\bar{\delta}\bar{d}$  where  $\bar{\delta}$  is the steady-state spread between borrowing and lending rates defined by  $(1 + \bar{\delta}) \equiv (1 + \bar{r}^b)/(1 + \bar{r}^d)$  which is positive because of the different liquidity properties of deposits and loans in the steady state. The function  $\phi(\cdot)$  captures the costs of enlarging the deposit capacity of the intermediaries, which can be related to the managerial costs of increasing the volume of deposits to their customers, but also to the macroeconomic risk that too much borrowing of the intermediaries can create on the overall quality of their deposits. This is the reason why we assume that the cost function depends on the overall payment that intermediaries have to deliver in each period to savers. The marginal cost of raising deposits is positive and increasing with the overall expected payment. In our model, the cost function  $\phi(\cdot)$  and its properties are important to determine the steady-state distribution of wealth between savers and borrowers and to assure converge to it after the shock. We assume that the costs  $k \cdot \phi(\cdot)$  are paid directly to the savers as are the profits of intermediation  $\Upsilon_t$ , which are known in period  $t - 1$  and delivered in period  $t$ .<sup>15</sup>

In a competitive market, intermediaries set the spread between borrowing and lending rates to maximize profits

$$1 + \delta_t \equiv \frac{(1 + i_t^b)}{(1 + i_t^d)} = \left[ 1 + \bar{\delta}\phi' \left( \frac{(1 + i_t^d) d_t}{(1 + \bar{r}^d) \bar{d}} \right) \right]. \quad (25)$$

The spread  $\delta_t$  is in general increasing in the overall repayments due to depositors and is consistent with its steady-state value since  $\phi'(1) = 1$ . Given the formulation of the cost function, an increase in the deposit rate moves more than proportionally the loans rate and raises the spread between lending and deposit rates.

### 4.3 Firms

We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function  $Y(i) = L(i)$  is linear in a bundle of labor which is a Cobb-Douglas index of the two types of labor:  $L(i) = (L^s(i))^\chi (L^b(i))^{1-\chi}$ . Given this technology, labor compensation for each type of worker is equal to total compensation  $W_j L_j = WL$  where the aggregate wage index is appropriately given by  $W = (W^s)^\chi (W^b)^{1-\chi}$ .

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<sup>15</sup>Quantitative results can be affected but not overturned by different assumptions on the distribution of intermediation profits, as we will discuss later.

Each firm faces a demand of the form  $Y(i) = (P(i)/P)^{-\theta}Y$  where aggregate output is

$$Y_t = \chi C_t^s + (1 - \chi)C_t^b. \quad (26)$$

Firms are subject to price rigidities as in Calvo's model: in each period a fraction of measure  $(1 - \alpha)$  of firms with  $0 < \alpha < 1$  is allowed to change its price, while the remaining fraction  $\alpha$  of firms indexes their previously-adjusted price to the inflation-target rate  $\bar{\Pi}$ . Adjusting firms choose prices to maximize the presented discounted value of the profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left[ \bar{\Pi}^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - (1 - \tau) \frac{W_T}{P_T} Y_T(i) \right]$$

where  $\Lambda_T$  is the stochastic discount factor used to evaluate profits at a generic time  $T$ , which is a linear combination of the marginal utilities of consumption of the two agents,  $\Lambda_T = \beta^{T-t}[\chi U_c(C_T^s) + (1 - \chi)U_c(C_T^b)]$  and  $\tau$  is an employment subsidy. The first-order condition of the optimal pricing problem implies

$$\frac{P_t^*}{P_t} = (1 - \tau) \frac{\theta}{\theta - 1} \frac{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left( \frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left( \frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{1-\theta} Y_T \right\}} \quad (27)$$

where we have set  $P_t(i) = P_t^*$  since all firms adjusting their prices will fix it at the same price. Calvo's model further implies the following law of motion for the general price index

$$P_t^{1-\theta} = (1 - \alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \bar{\Pi}^{1-\theta}, \quad (28)$$

through which we can write the aggregate supply equation

$$(1 - \alpha \bar{\Pi}_t^{\theta-1} \bar{\Pi}^{1-\theta})^{\frac{1}{1-\theta}} = (1 - \tau) \frac{\theta}{\theta - 1} \frac{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left( \frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left( \frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{1-\theta} Y_T \right\}}. \quad (29)$$

We assume that the utility flow from consumption is exponential  $u(C^j) = 1 - \exp(-vC^j)$  for some positive parameter  $v$  while the disutility of working is isoelastic  $v(L^j) = (L^j)^{1+\eta}/(1+\eta)$ . These are convenient assumptions for aggregation purposes and to keep tractability. These features can be easily discovered by taking a weighted average of (24), for  $j = s, b$ , with

weights  $\chi$  and  $1 - \chi$  respectively obtaining<sup>16</sup>

$$\frac{(Y_t \Delta_t)^\eta}{v \exp(-v Y_t)} = \frac{W_t}{P_t} (1 - \varphi_t^s)^\chi (1 - \varphi_t^b)^{1-\chi}, \quad (30)$$

where aggregate output and labor are related through  $Y_t \Delta_t = L_t$  and  $\Delta_t$  is an index of price dispersion defined by

$$\Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di,$$

which evolves as

$$\Delta_t \equiv \alpha (\Pi_t^\theta \bar{\Pi}^{-\theta}) \Delta_{t-1} + (1 - \alpha) \left( \frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{\theta}{\theta-1}}. \quad (31)$$

#### 4.4 Government budget constraint and monetary policy

To complete the characterization of the model we specify the consolidated budget constraint of government and central bank. We assume that there are no government bonds or public spending. The consolidated budget constraint simply reads as

$$M_t = (1 + i_{t-1}^m) M_{t-1} + \chi T_t^s + (1 - \chi) T_t^b + \tau W_t L_t. \quad (32)$$

It is clear that with heterogenous agents the distribution of transfers matters for the equilibrium allocation. We assume that each agent receives transfers corresponding to its holdings of money after subtracting a proportional share of the employment subsidy,

$$T_t^s = M_t^s - (1 + i_{t-1}^m) M_{t-1}^s - \tau W_t L_t, \quad (33)$$

$$T_t^b = M_t^b - (1 + i_{t-1}^m) M_{t-1}^b - \tau W_t L_t. \quad (34)$$

#### 4.5 Equilibrium in goods and asset markets

We consider equilibria in which the constraint (14) for  $j = b, s$  is binding. In this case, they imply

$$(1 + i_{t-1}^m) M_{t-1}^b = P_t C_t^b \quad (35)$$

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<sup>16</sup>In deriving (30), the assumption of a Cobb-Douglas production technology is also critical. It should be noted that another implication of our specification of preferences and production technology is that the steady-state level of output implied by (30) does not depend on the distribution of wealth. Indeed, in a steady state in which  $\Pi_t = \bar{\Pi}$  prices will be set as a mark-up over wages and moreover  $\bar{\varphi}^s = \bar{\varphi}^b$  in a way that the steady-state output implied in (30) is invariant to the distribution of wealth across agents.

$$(1 + i_{t-1}^m)M_{t-1}^s + \gamma_t(1 + i_{t-1}^d)B_{t-1}^s = P_t C_t^s. \quad (36)$$

In equilibrium money supply is equal to money demand

$$M_t = \chi M_t^s + (1 - \chi)M_t^b, \quad (37)$$

while financial market equilibrium requires

$$(1 - \chi)B_t^b + \chi B_t^s = 0. \quad (38)$$

Goods market equilibrium is given by (26).

## 4.6 Equilibrium conditions

We collect now the equations that characterize the equilibrium of the model. On the demand side, there are equations (21) and (22), and (23) for each  $j = b, s$ . Lending and borrowing interest rates are connected through equation (25). The two liquidity constraints (35) and (36) can be written in real terms as

$$(1 + i_{t-1}^m) \frac{m_{t-1}^b}{\Pi_t} = C_t^b \quad (39)$$

and

$$(1 + i_{t-1}^m) \frac{m_{t-1}^s}{\Pi_t} + \gamma_t(1 + i_{t-1}^d) \frac{(1 - \chi) b_{t-1}}{\chi \Pi_t} = C_t^s \quad (40)$$

where  $m_t^b \equiv M_t^b/P_t$ ,  $m_t^s \equiv M_t^s/P_t$  while  $b_t$  denotes the real debt of the borrowers given by  $b_t \equiv -B_t^b/P_t$ . To obtain (40), we have also used (38).

In real terms equation (41) implies

$$\chi m_t^s + (1 - \chi) m_t^b = m_t, \quad (41)$$

where  $m_t$  denotes aggregate real money balances, defined as  $m_t \equiv M_t/P_t$ .

The flow budget constraint of the borrowers can be simplified, using (34), to

$$b_t = (1 + i_{t-1}^b) \frac{b_{t-1}}{\Pi_t} + C_t^b - Y_t, \quad (42)$$

where we have used the fact that the Cobb-Douglas technology implies that  $W_t^b L_t^b = W_t L_t$  together with the assumption  $\Psi_t = 0$ .<sup>17</sup>

<sup>17</sup>If all the profits of intermediation were rebated to the borrowers, it would be easy to see that the relevant interest rate in (42) is the deposit rate,  $i_t^d$ , instead of the loan rate,  $i_t^b$ .

On the aggregate supply side, there is equation (29) together with (30) and (31) and the relationship  $Y_t \Delta_t = L_t$ .

The set of equations (21), (22), (23) for each  $j = b, s$  together with (25), (26), (29), (30), (31), (39), (40), (41), (42), describe the equilibrium conditions of the model. There are 13 equations in the following 15 unknowns  $Y_t, C_t^b, C_t^s, i_t^b, i_t^d, i_t^m, \Delta_t, W_t/P_t, P_t, b_t, \varphi_t^s, \varphi_t^b, m_t^s, m_t^b, m_t$  leaving the possibility to specify two instruments of policy.

## 5 Liquidity shocks and optimal monetary policy

We repeat the experiment of a shock that worsens the liquidity properties of the pseudo-safe asset. The model has now a richer transmission mechanism and there is also a propagation of the shock to the real economy, because of the redistributive effects between borrowers and savers and of nominal rigidities. We compare alternative monetary regimes with the Ramsey policy that maximizes the weighted sum of the utility of the consumers belonging to the economy:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} [\tilde{\chi}(U(C_T^s) - V(L_T^s)) + (1 - \tilde{\chi})(U(C_T^b) - V(L_T^b))], \quad (43)$$

given the equilibrium conditions of the model, in which  $\tilde{\chi}$  and  $(1 - \tilde{\chi})$  are the relative weights, respectively, of savers and borrowers in the objective function.

To get intuition about the underlining trade-offs, we can derive a simple quadratic loss function corresponding to just a second-order approximation of (43) under some relatively minor restrictions. In particular, we use assumptions such that the steady state resulting from the Ramsey problem coincides with the efficient steady-state allocation of consumption and labor. This efficient allocation solves the maximization of (43) under the resource constraint

$$\chi C_t^s + (1 - \chi) C_t^b = Y_t = (L_t^s)^\chi (L_t^b)^{1-\chi}.$$

In particular, as discussed in the Appendix, the first-order conditions of this problem imply

$$\frac{\tilde{\chi}}{1 - \tilde{\chi}} \frac{U_c(\bar{C}^s)}{U_c(\bar{C}^b)} = \frac{\chi}{1 - \chi} \quad (44)$$

$$\frac{V_l(\bar{L}^j)}{U_c(\bar{C}^j)} = \frac{\bar{Y}}{\bar{L}^j} \quad (45)$$

where the latter holds for each  $j = b, s$ . This is clearly the best allocation that can be achieved in this model, for given weights  $\tilde{\chi}$  and  $(1 - \tilde{\chi})$ . If the Ramsey policymaker could implement this allocation in the steady state, this would correspond to the Ramsey optimal policy when

there are no stochastic disturbances.

We show that the combination of policies:  $\Pi_t = \bar{\Pi}$  and  $i_t^m = \bar{i}^m$  where  $\bar{\Pi} \geq \beta$  and  $0 \leq \bar{i}^m \leq \bar{\Pi}/\beta - 1$  can indeed implement the first best, under two minor restrictions.<sup>18</sup> The first restriction requires that the employment subsidy to firms is set at  $\tau = (\bar{\mu} + \bar{\varphi})/(1 + \bar{\mu})$  where  $\bar{\mu} \equiv (\theta - 1)^{-1}$  and  $\bar{\varphi} = \bar{\varphi}^s = \bar{\varphi}^b$  are, respectively, the steady-state net markup and level of the financial friction. Indeed, a policy in which  $\Pi_t = \bar{\Pi}$  implies in equation (29) that

$$\frac{\bar{W}}{\bar{P}} = \frac{1}{(1 - \tau)(1 + \bar{\mu})}$$

which can be used in the steady-state version of (24) to get

$$\frac{V_l(\bar{L}^j)}{U_c(\bar{C}^j)} = \frac{1 - \bar{\varphi}}{(1 - \tau)(1 + \bar{\mu})} \frac{\bar{Y}}{\bar{L}^j}, \quad (46)$$

for each  $j = b, s$ . With the chosen subsidy  $\tau = (\bar{\mu} + \bar{\varphi})/(1 + \bar{\mu})$ , equation (46) is equivalent to (45).

To implement (44), note that  $\Pi_t = \bar{\Pi}$  and  $i_t^m = \bar{i}^m$  imply that

$$\bar{C}^s = \frac{1 - \chi}{\chi} \left( \frac{1 - \beta}{\beta} \right) \bar{b} + \bar{Y} \quad (47)$$

$$\bar{C}^b = - \left( \frac{1 - \beta}{\beta} \right) \bar{b} + \bar{Y} \quad (48)$$

where we have used the steady-state version of (20), (38) and (42). Given that  $\bar{b} > 0$ , it follows that  $\bar{C}^s > \bar{C}^b$ . To be consistent with (44), we add the second restriction, that the weight  $\tilde{\chi}$  is appropriately chosen to make the steady-state distribution of wealth efficient. In particular,  $\tilde{\chi} > \chi$ .

A corollary of these restrictions and implementation exercise is that the Ramsey policymaker can implement the first best for any choice of  $\bar{i}^m$  in the interval  $0 \leq \bar{i}^m \leq \bar{\Pi}/\beta - 1$  implying, from the steady-state version of (23), that the Lagrange multiplier  $\bar{\varphi}$  can fall in the interval  $0 \leq \bar{\varphi} \leq 1 - \beta/\bar{\Pi}$ . The Friedman's rule, which should command to completely eliminate the financial friction and set  $\bar{\varphi} = 0$ , is completely irrelevant for welfare in such a defined steady state. We can indeed choose a steady-state value of  $\bar{\varphi}$  different from zero and get the same first-best allocation of consumption and labor and, at the same time, be consistent with the optimal choice of a Ramsey policymaker.<sup>19</sup> A positive value of the multiplier  $\bar{\varphi}$  is

<sup>18</sup>It should be recalled that we can specify two policy instruments in our model.

<sup>19</sup>Note that our framework nests standard results on the Friedman's rule. In the case in which  $\bar{\mu} = 0$ ,  $\tau = 0$  and prices are flexible, it is optimal to set  $\bar{\varphi} = 0$ . However, this does not necessarily implies a rate of deflation equal to the time discount factor, since  $\bar{i}^m$  is also a policy variable and therefore  $\bar{\Pi} = \beta(1 + \bar{i}^m)$ .

convenient to make the financial friction non-negligible in a first-order approximation of our model. To further justify this choice, it is worth noting that in heterogenous-agent stochastic models with incomplete markets the Friedman's rule is not achievable. In our model, indeed, it requires to set  $\varphi_t^s$  and  $\varphi_t^b$  simultaneously to zero at all times, but this is not feasible unless  $U_c(C_{t+1}^s)/U_c(C_t^s) = U_c(C_{t+1}^b)/U_c(C_t^b)$  in all contingencies as shown in (23).<sup>20</sup>

We show in the Appendix that under these two assumptions, a second-order approximation of (43) delivers the following simple quadratic loss function

$$\frac{1}{2} E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[ \hat{Y}_T^2 + \chi(1-\chi)\lambda_c(\hat{C}_T^s - \hat{C}_T^b)^2 + \chi(1-\chi)\lambda_l(\hat{L}_T^s - \hat{L}_T^b)^2 + \lambda_\pi(\pi_T - \bar{\pi})^2 \right] \right\}$$

where in general hat variables denote deviations from the steady state, while  $\hat{C}_t^j \equiv (C_t^j - \bar{C}^j)/\bar{Y}$  for each  $j = b, s$ ,  $\pi_t = \ln P_t/P_{t-1}$  and  $\bar{\pi} = \ln \bar{\Pi}$ . The positive coefficients  $\lambda_c$ ,  $\lambda_l$  and  $\lambda_\pi$  are all defined in the Appendix.

The loss function contains some familiar terms to the literature. The only shock of the model is to liquidity, which is an inefficient shock, therefore deviations of output with respect to the efficient steady state are penalized appropriately. Inflation is also costly when it deviates from the trend to which price setters index prices, implying inefficient fluctuations of relative prices among goods produced according to the same technology. The other two terms in the loss function instead depend on the additional features that the heterogeneity of agents brings into the model. Since risk sharing of consumption and labor is efficient in the chosen steady state, departures from this allocation cause losses for aggregate welfare. In particular, the labor risk-sharing term can be further simplified noting that in a first-order approximation

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta}(\hat{C}_t^s - \hat{C}_t^b) - \frac{1}{(1-\bar{\varphi})(1+\eta)}(\varphi_t^s - \varphi_t^b),$$

where  $\rho = v\bar{Y}$ . In standard models without financial frictions, the labor risk-sharing argument is proportional to the consumption risk-sharing term. Here, instead, it is also relevant to consider the influence of the financial distortions across agents. It should be noted that in a first-order approximation of (23) we get

$$\varphi_t^j = \bar{\varphi} + (1-\bar{\varphi})E_t \left[ (\pi_{t+1} - \bar{\pi}) - \hat{v}_t^m + \rho\Delta\hat{C}_{t+1}^j \right]$$

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Only if  $\bar{v}^m = 0$ , as done in the literature,  $\bar{\Pi} = \beta$ .

<sup>20</sup>See Woodford (1990) for a general discussion.

for  $j = b, s$  and therefore we can simplify the labor risk-sharing term to

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1 + \eta} E_t(\hat{C}_{t+1}^s - \hat{C}_{t+1}^b). \quad (49)$$

Because of the financial friction, labor effort at time  $t$  is producing income which is only liquid to purchase goods in the next period.<sup>21</sup> As shown by the first-order condition (24), using (23), the consumers are optimally choosing labor given current wages and future prices taking into account their expectations of future consumption. It follows that the cross-agent difference in labor is proportional to the difference in the one-period ahead expectation of consumption. Equivalently, we can write the loss function as

$$\frac{1}{2} E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[ \hat{Y}_T^2 + \chi(1 - \chi) \lambda_c \left( \hat{C}_T^R \right)^2 + \chi(1 - \chi) \tilde{\lambda}_l \left( E_T \hat{C}_{T+1}^R \right)^2 + \lambda_\pi (\pi_T - \bar{\pi})^2 \right] \right\} \quad (50)$$

for some  $\tilde{\lambda}_l$ , where  $\hat{C}_T^R \equiv \hat{C}_T^s - \hat{C}_T^b$ . We compute the optimal policy under commitment by minimizing this loss function with respect to the log-linear approximation of the equilibrium conditions.

For the numerical exercise, the model is calibrated (quarterly) as follows. We set  $\beta = 0.99$  and  $\bar{\pi} = 0.02/4$ , to imply a steady-state annualized nominal interest rate on bonds of about 6%, while the steady-state interest rate on reserves is calibrated at  $\bar{r}^m = 0.01/4$ . We use the average velocity of M1 for the U.S. economy during the Great Moderation (1984-2007) to calibrate the steady-state money to GDP ratio:  $M/PY = 0.125 \cdot 4$ . To calibrate the ratio of households deposits to GDP, we use the average, over the same period, of M2 net of M1:  $\chi B^s/PY = 0.375 \cdot 4$ . The economy-wide liquidity constraint and the steady-state version of (21) imply that about one third of assets provides liquidity services (i.e.  $\bar{\gamma} = 0.33$ ) and an annualized nominal interest rate on bank deposits of about 4.4%. We then calibrate the share of savers in the economy to 62.5% so that the equilibrium in the bond market (38) is consistent with an initial debt to income ratio of about 100%, as in Eggertsson and Krugman (2012). We follow the latter also in setting the elasticity of the credit spread to the stock of real debt to 0.049.<sup>22</sup> A given value for the initial debt to income ratio, then, pins down a unique initial distribution of wealth: indeed, equations (47) and (48) yield the distribution of personal consumption, which in turn implies the distribution of money holdings, through equations (39) and (40). Finally, the relative risk-aversion coefficient is set to  $\rho = 1$ , the inverse of the Frisch-elasticity of labor supply to  $\eta = 2$ , the parameter  $\alpha$  capturing the

<sup>21</sup>It should be noted that (49) is valid also when  $\bar{\varphi} = 0$  and that it captures the impossibility to achieve the Friedman's rule in the stochastic equilibrium ( $\varphi_t^s = \varphi_t^b$ ) unless  $\Delta \hat{C}_{t+1}^b = \Delta \hat{C}_{t+1}^s$  at all times.

<sup>22</sup>This corresponds to the parameter  $\phi$  defined in the Appendix.

degree of nominal rigidity in the model implies an average duration of consumer prices of four quarters ( $\alpha = 0.75$ ).

In the simulations below, we assume that the liquidity index  $\hat{\gamma}_t \equiv \frac{\gamma_t - \bar{\gamma}}{1 - \bar{\gamma}}$  follows an autoregressive process of the kind  $\hat{\gamma}_t = \rho_\gamma \hat{\gamma}_{t-1} + \varepsilon_\gamma$  and analyze the dynamic effect of a 20% negative shock which gradually reverts back to mean, with half life of about four quarters ( $\rho_\gamma = 0.85$ ).

## 5.1 Optimal unconventional policies following a liquidity shock

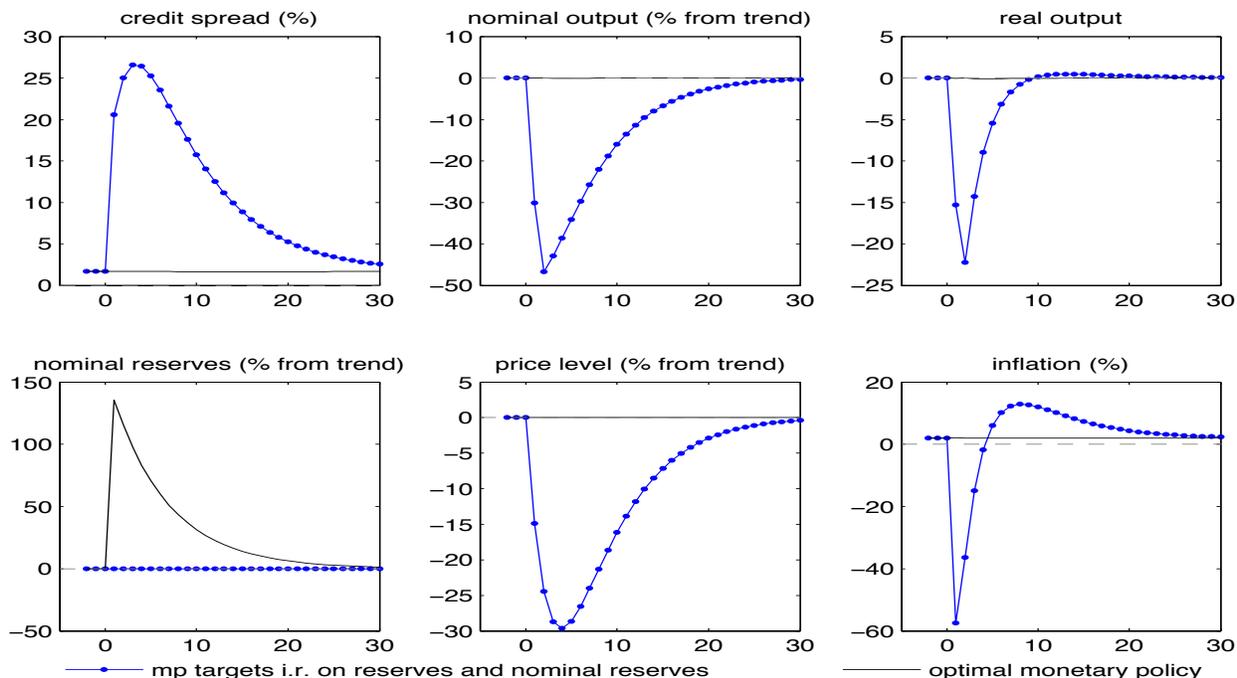
In Figures 2 and 3 we compare the optimal policy (solid line) with a passive monetary policy in which the interest rate on reserves and the growth rate of the nominal money supply are kept constant at the levels before the shock hits (dotted line).

There are two important policy implications on what monetary policy should do when facing a liquidity shock. Inject more liquidity in the form of money, as shown in Figure 2, and lower the interest rate on reserves up to the zero lower bound, as shown in Figure 3. Although it is in general hard to isolate the effects of the two channels in the general equilibrium of the model, we argue that the injection of liquidity avoids the deflation and the contraction in real activity, while lowering the interest rate on reserves helps to achieve a better risk sharing of the shock between savers and borrowers.

The transmission of the liquidity shock can be understood in a simple way through two main mechanisms.

First, the liquidity shock creates, at an aggregate level, a shortage in the supply of the assets available for goods exchange, because pseudo-safe assets have partially lost their qualities. An excess supply of goods is the corresponding disequilibrium in the goods market to that in asset market due to the shortage of safe assets. Nominal spending falls, and the split between prices and real output depends on the degree of price rigidities. This is what happens under the passive policy: real output drops widely, as shown in the figure, with a contraction of over 20% while prices fall by about 30% compared to their trend, through a deep deflation. The figure clearly illustrates the dramatic effects of a liquidity shock when the monetary policymaker is completely helpless. Under optimal policy, instead, the contraction in real output is very mild as well as the response of inflation and prices. The key change in policy, that leads to a near stabilization of output and prices, is the increase in the growth rate of money, as shown in Figure 2, which goes up to a path about 140% above the previous trend. A substantial expansion in the central bank's balance sheet and an increase in the supply of safe assets are required to optimally absorb the shock. Interestingly, the expansion should last for as long as the liquidity conditions are deteriorated, slowly returning back to the initial path as the liquidity properties of assets go back to normal.

The second mechanism of propagation works through asset prices. The liquidity shock

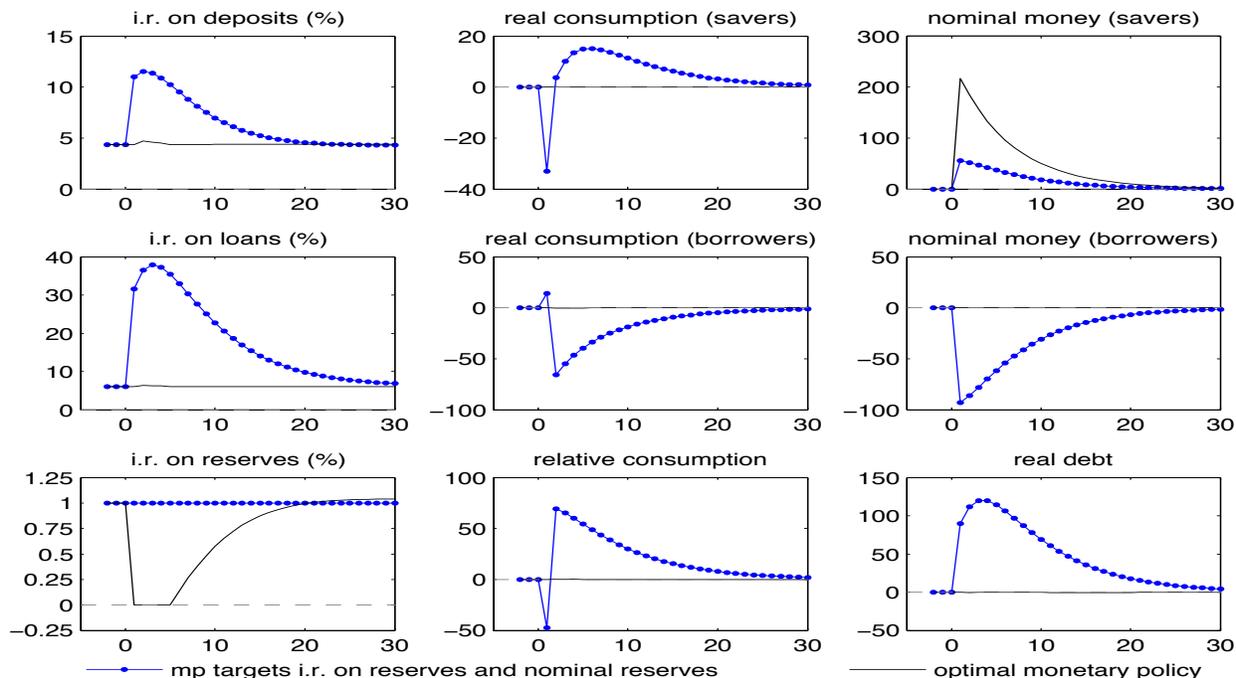


**Figure 2:** Responses of selected variables to a 20% fall in the liquidity properties of bonds: passive monetary regime vs optimal policy. Dotted line: monetary policy keeps both the money growth and the interest rate on reserves  $i_t^m$  constant (passive regime). Solid line: optimal monetary policy. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

requires a higher premium to hold pseudo-safe assets. This in turn increases the cost of funding for intermediaries which need to raise, more than proportionally, the interest rate on loans. Under the passive monetary policy, spreads and interest rates increase as shown in Figure 3. Under optimal policy, all market interest rates are instead insulated from the shock and stable around the previous steady-state levels. The important change in policy that helps explaining this result is the reduction in the interest rate on reserves up to the zero bound and for a quite long horizon. To intuit this result, Figure 3 displays the consumption of savers and borrowers, respectively, and their difference. As shown in the loss function (50), imperfect consumption risk-sharing is costly in this model. Under the passive monetary policy, the rise in the real interest rates has important wealth and redistributive effects between borrowers and savers. Borrowers are hit in a significant way by the increase in the real rate on loans, so that they have to cut on their consumption.<sup>23</sup> Their real debt even rises, mostly because of the deflation. Savers instead benefit from the increase in the real return on their savings and can raise their consumption by holding more safe assets at the expenses of borrowers.<sup>24</sup> Under

<sup>23</sup>This is also the case if the profits of financial intermediary are all distributed to the borrowers. In this case, the relevant interest rate is the deposit rate.

<sup>24</sup>On impact savers' consumption falls, as the liquidity value of their accumulated pseudo-safe assets shrinks and they are unable to reallocate their portfolio.

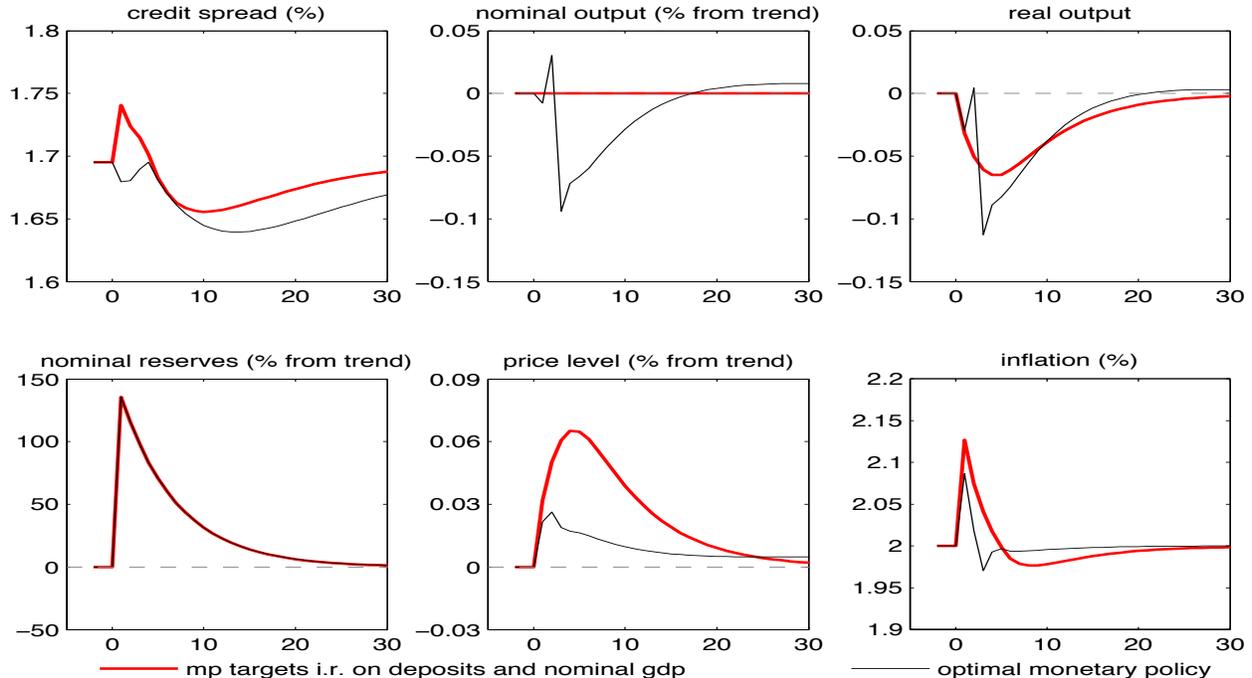


**Figure 3:** Responses of selected variables to a 20% fall in the liquidity properties of bonds: passive monetary regime vs optimal policy. Dotted line: monetary policy keeps both the money growth and the interest rate on reserves  $i_t^m$  constant (passive regime). Solid line: optimal monetary policy. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

optimal policy, instead, the interest rate on reserves falls and this stabilizes all other market interest rates.<sup>25</sup> The redistributive channel, which was strong under the passive policy, is now muted. Borrowers face approximately the same real interest rate as before the shock, and do not have to cut consumption. Savers can increase their money holdings to replace the deteriorated pseudo-safe assets without crowding out money holdings of borrowers. This is because the central bank injects more liquidity.

We can now appreciate some important differences between our model and those of the recent literature on financial crisis and unconventional policies. First, in models like Eggertsson and Krugman (2012), a deleveraging shock on borrowers' debt is responsible of the drop in the natural rate of interest which requires a parallel fall in the real interest rate. As a consequence, savers need to raise their consumption to compensate for the fall of the deleveragers. In our model the shock is to liquidity and the real rate does not move much under optimal policy. Consumption of savers and borrowers should remain at the levels before the shock hits. Second, in Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), the zero-lower bound on nominal interest rate is a constraint to achieve the optimal stabilization

<sup>25</sup>It is worth noticing that the expansion of the central bank's balance sheet, by stabilizing output and inflation, also contributes to more stable nominal interest rates, as the latter depend also on the expected paths of the former.



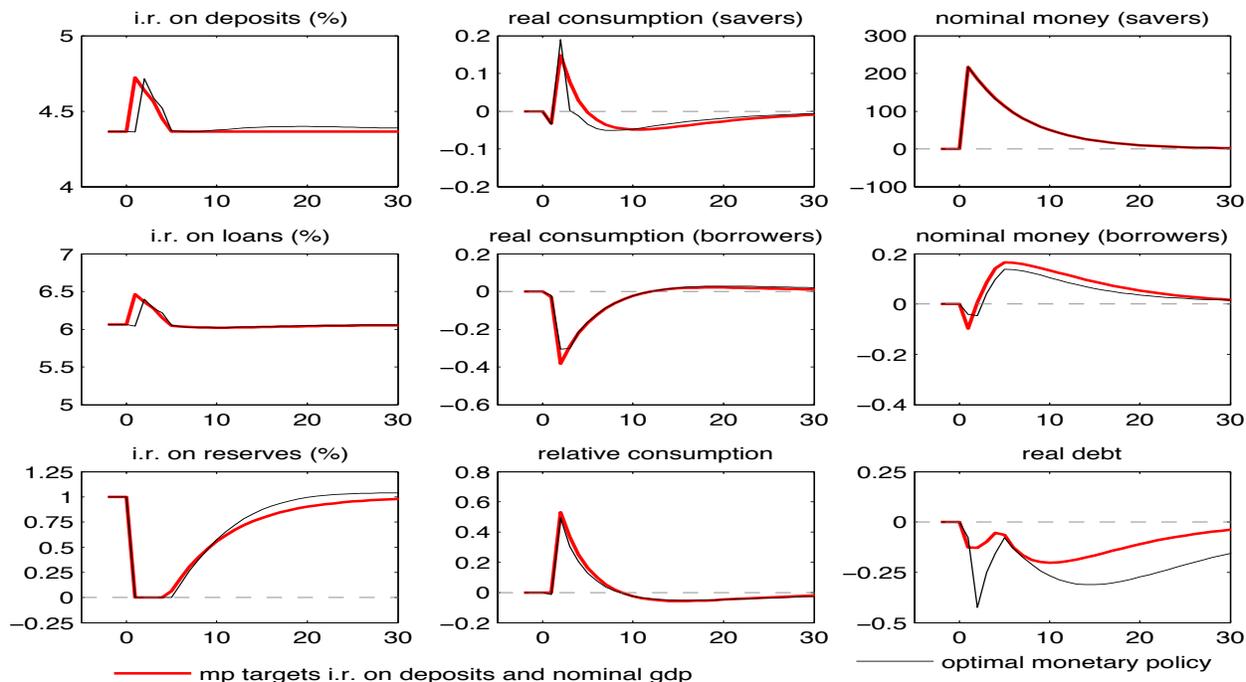
**Figure 4:** Responses of selected variables to a 20% fall in the liquidity properties of bonds: active monetary regime vs optimal policy. Bold solid line: monetary policy implements nominal-gdp targeting and seeks stabilization of  $i_t^d$ . Thin solid line: optimal monetary policy. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

of aggregate objectives like output and inflation. In our model, instead, balance-sheet policies can take care of the aggregate objectives.<sup>26</sup> The zero-lower bound is only a constraint to achieve a better risk-sharing of the shock between borrowers and savers.

## 5.2 The optimality of nominal-GDP targeting at the zero bound

We further investigate the features of the optimal policy and compare it with other simple policies that can approximate it, as shown in Figures 4 and 5. It should be noted that the scale of these figures is different from the previous ones and enables us to appreciate more the variation of the variables of interest. We display the optimal policy in contrast with a simple policy in which nominal GDP is stabilized in each quarter and the interest rate on reserves is lowered in order to insulate the interest rate on deposits from the liquidity shock. Under the calibration considered, the shock is too large to successfully stabilize the interest rate on deposits in all periods because the zero-lower bound becomes a relevant constraint for

<sup>26</sup>As it will be shown later, in our model, in contrast to Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), an inflation targeting policy or a nominal-GDP targeting policy can be implemented in all periods if the central bank is willing to commit to it, but this necessarily requires an expansion in the balance sheet exactly for the periods in which the liquidity properties are deteriorated. The zero-lower bound is not necessarily a constraint for these policies.



**Figure 5:** Responses of selected variables to a 20% fall in the liquidity properties of bonds: active monetary regime vs optimal policy. Bold solid line: monetary policy implements nominal-gdp targeting and seeks stabilization of  $i_t^d$ . Thin solid line: optimal monetary policy. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

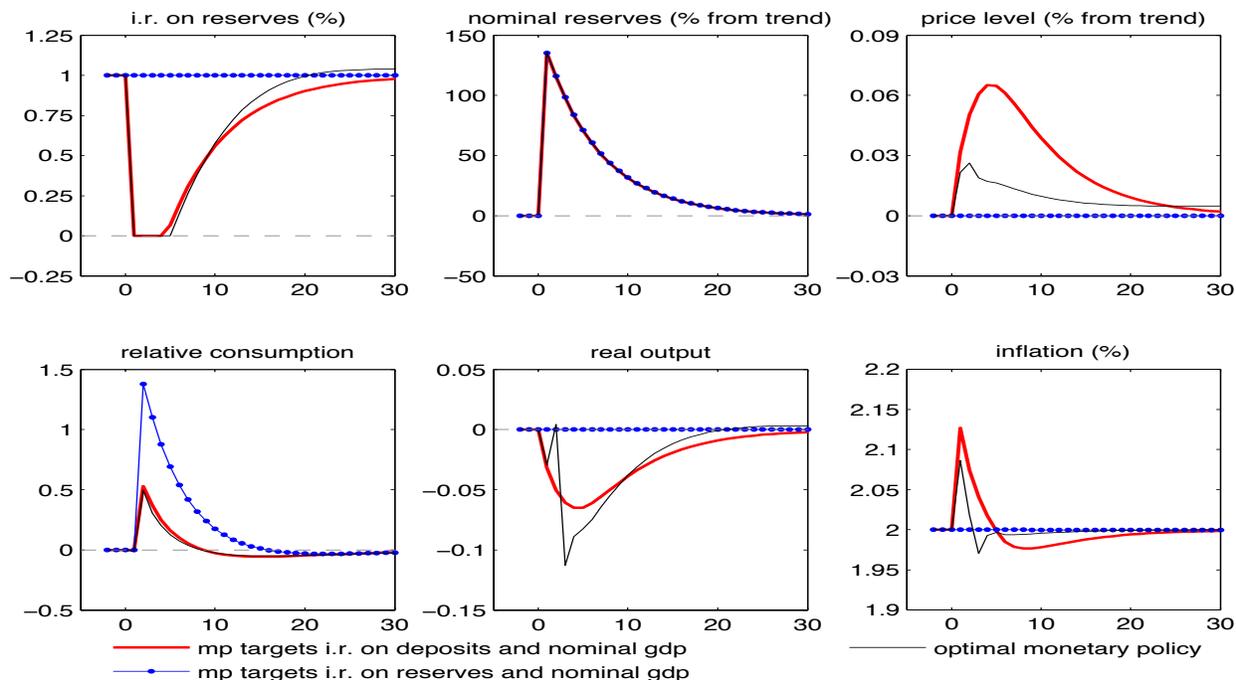
some quarters. This simple targeting regime can approximate quite well the optimal policy along the objectives of the loss function (50): the deviations are in general small and imply negligible losses in terms of welfare, unlike the case of passive monetary policy.<sup>27</sup>

We can further study the extent to which lowering the interest rates on reserves up to the zero-lower bound is an important tool for sterilizing the shock. Consider a policy in which nominal GDP is stabilized in each period but where now the interest rate on reserve is held constant. In Figure 6, we add this policy in comparison with the previous one and the optimal policy. When the interest rate on reserves is unchanged, a policy of targeting nominal GDP can stabilize completely inflation at the target but also real output. However, consumption dispersion across agents substantially rises. There is a trade-off between aggregate targets (output and inflation) on the one side, and cross-sectional ones (consumption dispersion) on the other.<sup>28</sup> As we conjectured, therefore, it is the cut in the interest rate on reserves that allows the central bank to improve the risk sharing of the liquidity shock, albeit at the cost of slightly more volatile output and inflation.<sup>29</sup>

<sup>27</sup>A result not shown in the graph is that a combined policy of strict inflation targeting with a stable interest rate on deposits gets also very close to optimal policy, but performs slightly worse than nominal-GDP targeting along all three dimensions relevant for welfare (relative consumption, real output and inflation).

<sup>28</sup>The same implication clearly follows if monetary policy targets inflation instead of nominal GDP.

<sup>29</sup>Note, however, that expansionary balance-sheet policies, even when interest rates on reserves are kept



**Figure 6:** Responses of selected variables to a 20% fall in the liquidity properties of bonds: alternative active monetary regimes vs optimal policy. Bold solid line: monetary policy implements nominal-gdp targeting and seeks stabilization of  $i_t^d$ . Dotted line: same as before but now monetary policy keeps  $i_t^m$  constant. Solid line: optimal monetary policy. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

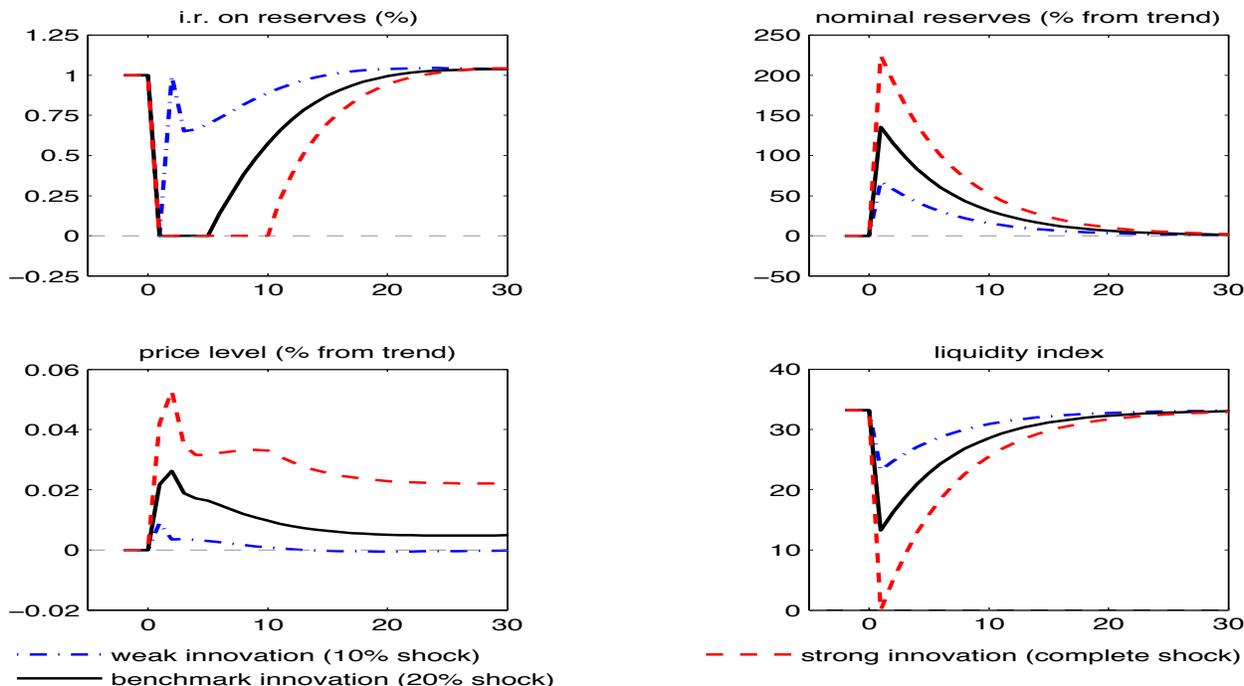
Finally, Figure 4 shows another interesting feature of optimal policy, i.e. that the long-run price level remains above the initial trend. A policy of nominal-GDP targeting, instead, implies long-run convergence of the price level to the initial path. In this respect, a policy that commits to raise the nominal-GDP target permanently after the shock will perform better.<sup>30</sup> In the next section, we also investigate the nature of this long-run price divergence under optimal policy.

### 5.3 Monetary policy tapering and exit from the zero bound

We now turn to analyze how the unconventional policy responses depend on the magnitude and persistence of the liquidity shock. Figure 7 displays the responses of the interest rate on reserves, the detrended level of reserves and the price level to negative liquidity shocks of different sizes, displayed in the bottom-right panel. We consider the benchmark shock of the previous sections in comparison with a weaker and a stronger shock. The latter is such that it completely deteriorates the liquidity properties of the pseudo-safe assets on impact.

constant, mitigate the rise in the deposit and loan rates, in contrast to the completely passive policy of Figures 3 and 4.

<sup>30</sup>We do not show this in the graph, but overall the improvements are marginal.

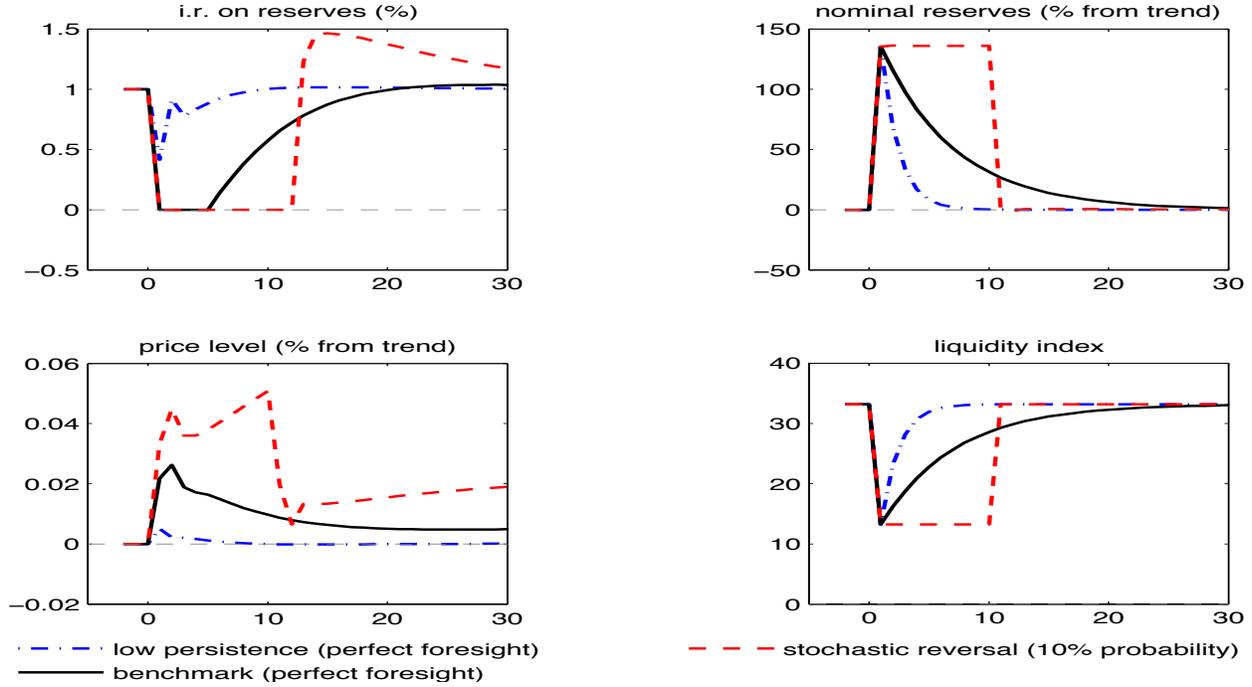


**Figure 7:** Responses of selected variables to negative liquidity shocks of different sizes, under the optimal monetary policy. Dash-dotted line: weak shock,  $\gamma$  falls down to 23%. Solid line: benchmark shock,  $\gamma$  falls down to 13%. Dashed line: strong shock,  $\gamma$  falls down to 0. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

The required expansion in the central bank’s balance sheet is larger the stronger the shock while the tapering of the stimulus is guided by the improvement in the quality of the pseudo-safe assets, as it goes back to normal. The stay at the zero-lower bound is longer, the stronger is the magnitude of the shock. When the liquidity shock is weak enough, the interest rate on deposits can be insulated from the shock without bringing the interest rate on reserves to zero. Interestingly, a prolonged stay at the zero lower bound implies that the long-run price level remains well above the initial trend even after the shock has faded away.<sup>31</sup> This is in line with other models of optimal policy under the zero-bound constraint like Eggertsson and Woodford (2003).

Figure 8 looks deeper into this analysis by studying alternative assumptions on the properties of persistence of the liquidity shock. The dash-dotted and solid lines show the cases of autoregressive processes with a half life of 1 and 4 quarters, respectively; the dashed line shows the case of a Markov-switching process for which, in every period, the liquidity properties go back to normal with probability  $\xi = 10\%$  and stay at the lower level with probability  $1 - \xi$ . In all cases, the liquidity properties fall initially by the same amount ( $-20\%$ ), but

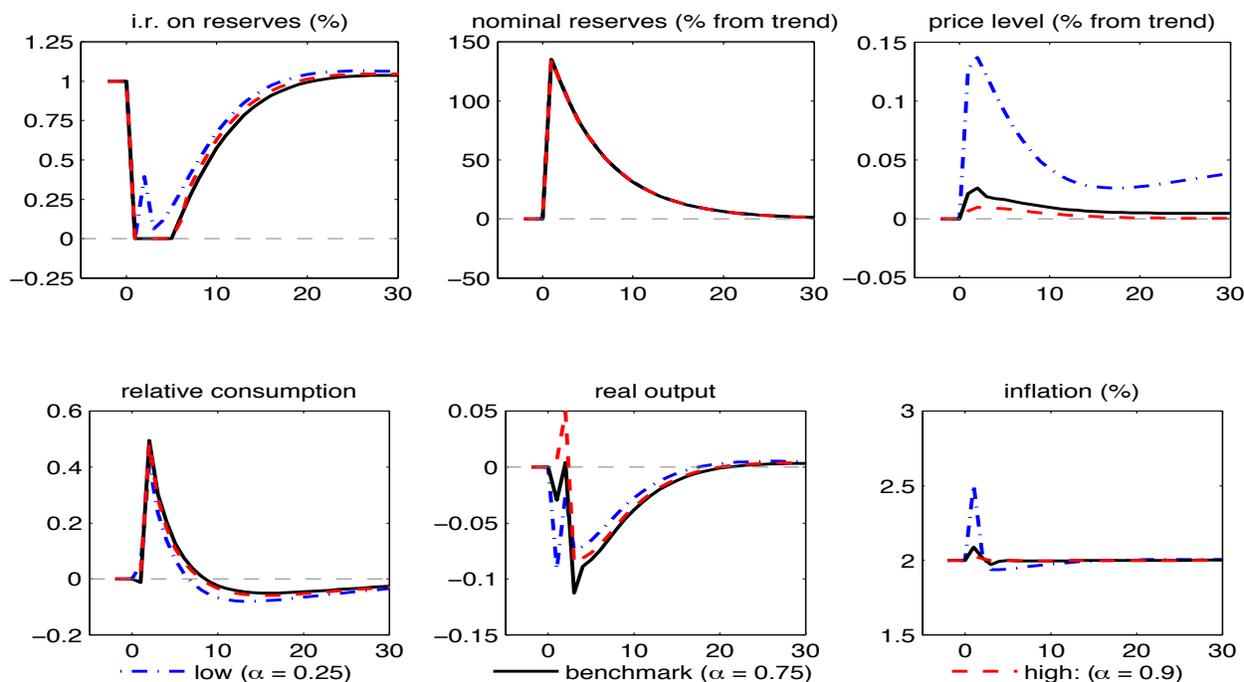
<sup>31</sup>If nominal interest rates could turn negative, the price level would increase more, the stronger the shock, but it would always converge back to the initial trend right after the shock has reversed to zero, regardless of the strength.



**Figure 8:** Responses of selected variables to negative liquidity shocks of same size but different persistence, under the optimal monetary policy. Dash-dotted line: low persistence, half-life of 1 quarter ( $\rho_\gamma = 0.5$ ). Solid line: benchmark case, half-life of about 4 quarters ( $\rho_\gamma = 0.85$ ). Dashed line: 20% shock with 10% probability that in each period the shock returns back to steady state; the shock actually returns back after 10 quarters.

then return to normal conditions with different speed. As the figure shows, the expansion of the central bank’s balance sheet is, on impact, the same across the three specifications, but the shape of the tapering is inherited from the properties of persistence of the shock. In the case of the Markov-switching process, in particular, the monetary policymaker keeps nominal reserves on the new path and suddenly brings them back to the old one as soon as the liquidity properties are back to normal.

It is interesting to note that the same link between zero-lower bound policies and commitment to a higher future price level holds in this case as well. Indeed, if the half life of the shock is short enough, stabilization of the interest rate on deposits can be achieved without bringing the policy rate down to the zero bound and without committing to a higher future price-level path. On the contrary, the higher the half life of the shock, the longer the required stay at the zero bound, and the higher the long-run price-level path to commit to. Under the Markov-switching process, the central bank is required to commit to keep the interest rate on reserves at the zero bound longer than the duration of the shock, and to immediately overshoot its long-run level afterwards.



**Figure 9:** Responses of selected variables to a 20% fall in the liquidity properties of bonds under the optimal monetary policy: the role of price stickiness. Dash-dotted line: prices are reset every 13 weeks. Solid line: prices are reset every 4 quarters. Dashed line: prices are reset every 10 quarters. The half-life of the liquidity shock is about 4 quarters ( $\rho_\gamma=0.85$ ).

## 5.4 The role of price rigidities

Finally, Figure 9 studies whether the degree of price stickiness plays a role in driving the results. In particular, the figure displays the response of the same variables as in Figure 7 to a 20% deterioration of the liquidity properties of the pseudo-safe asset, for different degrees of price stickiness. Interestingly, the figure shows that under the optimal monetary policy, the economy is required to stay at the zero-lower bound longer, the stickier are consumer prices. Differently from Figure 7, however, the stay at the zero-lower bound in this case is inversely related to the increase in the price-level path to which monetary policy has to commit in the distant future. On the one hand, indeed, more flexible consumer prices reduce the welfare costs of inflation, thereby allowing the central bank to focus more actively on the other stabilization objectives by committing strongly to increase the long-run price level. On the other hand, the stronger commitment to an increase of the future price level requires a shorter stay at the zero-lower bound, because more flexible consumer prices favor a quicker convergence to the new target path.

An additional important insight of Figure 9 is that the main policy implications of our model do not depend much on the degree of nominal rigidity. The size of the expansion in the balance sheet of the central bank, as well as the shape of the exit path, is independent

of the degree of price stickiness. Monetary policy should offset the shortage of *nominal* safe assets by supplying more money in order to stabilize *nominal* GDP in the economy.

The need to lower the interest-rate on reserves, hitting the zero-lower bound at least on impact, is also independent of the degree of price stickiness. Indeed, even under high price flexibility (the dash-dotted line in the figure) a liquidity shock has substantial real redistributive effects between savers and borrowers, which require a policy intervention to counteract them.

## 6 Conclusions

We have presented monetary models in which the main novelty is that financial assets can have different liquidity properties. In this framework, we studied the effects on the economy of a change in these properties for some assets, which we labelled pseudo-safe assets. The overall shortage of safe assets can produce significant effects on nominal spending, and thereby on aggregate prices and real activity, in a proportion that depends on the degree of nominal rigidities. A deep recession cum deflation can emerge for a reasonable parameterization. At the same time, in a model in which the pseudo-safe asset is a deposit security through which intermediaries finance their loans, a liquidity shock raises the funding costs of intermediaries which is passed through into higher loan rates. This shock has important distributional effects between borrowers and savers, with borrowers adversely hit by the rise in the loan rates.

The role of monetary policy is critical for the propagation of the shock. Two instruments can be used to minimize the welfare consequences of the shock both at the aggregate and distributional level. The monetary policymaker should offset the shortage of safe assets by issuing more liquidity in the form of money, which remains a safe asset in circulation in the model. This can be achieved by a policy of increasing the path of nominal reserves in the vein of Quantitative Easing, to stabilize the inflation rate around the target as well as nominal and real output. Moreover, the interest rate on reserves should be reduced in order to insulate the interest rate on the pseudo-safe assets and the interest-rate spreads. This policy improves the risk sharing of the liquidity shock between savers and borrowers and avoids a consumption recession, in particular for borrowers. For large shocks, the zero-lower bound becomes a constraint to this action.

Our work has contributed to an ongoing literature studying the cause and propagation of the financial crisis by analyzing liquidity in monetary models which have been frequently used for policy analysis before the crisis. A small departure from the standard framework is sufficient to produce an interesting transmission mechanism of a liquidity shock and capture

macroeconomic behavior and policy intervention close to what economies have experienced. We see our work as complementary to other approaches like Eggertsson and Krugman (2012) who have emphasized shocks to the natural rate in the form of deleveraging to explain the propagation mechanism of the crisis. However, under a deleveraging shock, a reduction in the real interest rate is important to mitigate the costs of the recession, as it is also in Eggertsson and Woodford (2003), while under a liquidity shock the real rates relevant for borrowers and savers should remain stable under optimal policy. Interestingly the policy rate in our model should go to the zero-lower bound, as in the literature, but for different reasons: in the literature it goes to the zero bound to achieve the aggregate targets and stabilize output and inflation, while in our case it does so to achieve the distributional targets and improve risk sharing.

There are some limitations of our framework which can constitute ground for further work and analysis. We have abstracted from credit risk and credit events, which can be also an important channel of transmission mechanism of the recent crisis. However, the research objective of this work is to identify clearly a liquidity shock and liquidity risk as drivers of the macroeconomic adjustment, and to shed some light on the transmission mechanism of such shock. The sector of intermediaries is quite rudimentary and could be further elaborated to endogenize the creation of pseudo-safe assets. In relation to this point, the degree of acceptance of assets in exchange of goods is exogenous as in Lagos (2010), but the literature spurred from the latter work has been trying to endogenize it through differences in the information set on the quality of assets between borrowers and savers. This might be an important qualification to add to our analysis which could change some policy implications. In this vein, it could be interesting to model the exchange of assets for goods through a bargaining process instead of market equilibrium conditions. These are clearly important issues, which we leave for future research.

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## Appendix A

In this appendix we present the log-linear approximation of the model of Section 2 which is used for the analysis of Section 3. The first-order approximation is taken around a deterministic steady state where we can combine equations (11) and (12) to imply

$$\frac{\bar{i} - \bar{i}^m}{1 + \bar{i}} = (1 - \bar{\gamma}) \left[ 1 - \frac{\beta(1 + \bar{i}^m)}{\bar{\Pi}} \right] \quad (51)$$

in which a bar denotes the steady-state value. A first-order approximation of equations (11) and (12) around the above steady state delivers

$$\hat{i}_t - \hat{i}_t^m = -\vartheta_1 E_t \hat{\gamma}_{t+1} + \vartheta_2 E_t (\hat{r}_{t+1} + (\hat{\pi}_{t+2} - \bar{\pi}) - \hat{i}_{t+1}^m) \quad (52)$$

where we have defined  $\hat{i}_t \equiv \ln(1 + i_t)/(1 + \bar{i})$ ,  $\hat{i}_t^m \equiv \ln(1 + i_t^m)/(1 + \bar{i}^m)$ ,  $\hat{\gamma}_{t+1} = (\gamma_{t+1} - \bar{\gamma})/(1 - \bar{\gamma})$ ,  $\pi_t = \ln P_t/P_{t-1}$  and the coefficients  $\vartheta_1$  and  $\vartheta_2$  are  $\vartheta_1 \equiv (\bar{i} - \bar{i}^m)/(1 + \bar{i}^m)$  and  $\vartheta_2 \equiv 1 - \bar{\gamma}(1 + \bar{i})/(1 + \bar{i}^m)$ . Notice that  $\vartheta_1 \geq 0$  and in particular  $\vartheta_1 = 0$  when the cash-in-advance constraint is not binding, while  $0 \leq \vartheta_2 \leq 1$ .<sup>32</sup>

In equation (52),  $\hat{r}_{t+1}$  captures the real interest rate that would apply in a model in which money and bonds are perfect substitutes and is defined as

$$\hat{r}_{t+1} = \rho E_{t+1} (\hat{Y}_{t+2} - \hat{Y}_{t+1})$$

where  $\rho \equiv -\bar{U}_{cc}\bar{Y}/\bar{U}_c$ ;  $\hat{Y}_t \equiv \ln Y_t/\bar{Y}$ . To complete the characterization of the equilibrium condition through a first-order approximation, we approximate the cash-in-advance constraint (10) to obtain

$$\hat{m}_t + s_b(\hat{b}_t + \vartheta_3 \hat{\gamma}_t) = 0 \quad (53)$$

where  $\hat{m}_t$  represents the log-deviations from the steady state of the ratio of money over nominal GDP defined as  $m_t \equiv \tilde{M}_{t-1}^s/(P_t Y_t)$  where  $\bar{m}$  is its steady-state value;  $\hat{b}_t$  is instead the log-deviations from the steady state of the ratio of bonds over nominal GDP, defined as  $b_t = \tilde{B}_{t-1}^s/(P_t Y_t)$  with steady-state value  $\bar{b}$ , while  $\vartheta_3 \equiv (1 - \bar{\gamma})/\bar{\gamma}$  and  $s_b \equiv \bar{\gamma}\bar{b}/\bar{m}$ . Moreover

$$\hat{m}_t = \hat{m}_{t-1} + \mu_{t-1}^m - \pi_t \quad (54)$$

$$\hat{b}_t = \hat{b}_{t-1} + \mu_{t-1}^b - \pi_t \quad (55)$$

where  $\mu_t^m$  and  $\mu_t^b$  are the rate of growth of money supply and bond supply from time  $t - 1$  to time  $t$ . Given exogenous processes  $\{\mu_t^b, \hat{\gamma}_t\}$  equations (52), (53), (54) and (55) determine the path of  $\{\hat{m}_t, \mu_t^m, \pi_t, \hat{b}_t, \hat{i}_t, \hat{i}_t^m\}$ . Accordingly, monetary policy should specify the path of two instruments of policy.

We describe now some analytical results which are used to produce Figures 1 and 2.

Under the regime in which the monetary policymaker is passive and keeps the interest rate on reserves and the growth of money constant at the rate followed before the shock hits,

<sup>32</sup>Indeed, equation (51) implies  $\bar{\gamma}(1 + \bar{i})/(1 + \bar{i}^m) = \bar{\gamma}/[\bar{\gamma} + (1 - \bar{\gamma})\bar{\Pi}^{-1}\beta(1 + \bar{i}^m)] \in [0, 1]$ , which in turn implies  $\vartheta_2 \in [0, 1]$ .

we have that the growth of money is given by

$$\bar{\mu}^m = (1 + s_b) \bar{\pi} - s_b \bar{\mu}^b. \quad (56)$$

while the inflation rates vary with the liquidity shock

$$\pi_t = \bar{\pi} + \frac{s_b}{1 + s_b} \vartheta_3 \Delta \hat{\gamma}_t. \quad (57)$$

and the path of the interest rate on bonds follows

$$\hat{i}_t = -\vartheta_1 E_t \hat{\gamma}_{t+1} + \vartheta_2 \vartheta_3 \frac{s_b}{1 + s_b} E_t \Delta \hat{\gamma}_{t+2}.$$

When the policymaker set inflation rate always at the steady state level of 2% at annual rate, as shown with the solid line in Figure 1, the path of money growth follows

$$\mu_{t-1}^m = (1 + s_b) \bar{\pi} - s_b (\bar{\mu}^b + \vartheta_3 \Delta \hat{\gamma}_t). \quad (58)$$

To keep inflation on target, the growth rate of money supply rises momentarily when the liquidity properties of bonds deteriorate, and falls to return to the previous path when liquidity conditions improve. A negative liquidity shock raises the interest rate on bonds which, under inflation targeting, follows

$$\hat{i}_t = -\vartheta_1 E_t \hat{\gamma}_{t+1}.$$

When, instead, the policymaker insulates the interest-rate on bonds from the shock, the interest rate on money follows

$$\hat{i}_t^m = \max \left( -\ln(1 + \bar{r}^m), \vartheta_1 \sum_{s=0}^{\infty} \vartheta_2^s E_t \hat{\gamma}_{t+1+s} \right)$$

which for the large shock discussed in the text can hit the zero-lower bound.

## Appendix B

We solve the model of Section 4 by taking a first-order approximation around the initial steady state. The Euler equations of the savers imply

$$\hat{i}_t^d - \hat{i}_t^m = -\vartheta_1^s E_t \hat{\gamma}_{t+1} + \vartheta_2^s E_t (\hat{r}_{t+1}^s + (\pi_{t+2} - \bar{\pi}) - \hat{i}_{t+1}^m) \quad (59)$$

where we introduce the following additional notation with respect to previous sections:  $\hat{i}_t^d \equiv \ln(1 + i_t^d)/(1 + \bar{r}^d)$ , and the coefficients  $\vartheta_1^s$  and  $\vartheta_2^s$  are defined as  $\vartheta_1^s \equiv (\bar{r}^d - \bar{r}^m)/(1 + \bar{r}^m)$  and  $\vartheta_2^s \equiv 1 - \bar{\gamma}(1 + \bar{r}^d)/(1 + \bar{r}^m)$ . The Euler equation of the borrowers read in a first-order approximation as

$$\hat{i}_t^b - \hat{i}_t^m = E_t (\hat{r}_{t+1}^b + (\pi_{t+2} - \bar{\pi}) - \hat{i}_{t+1}^m). \quad (60)$$

In both equations

$$\hat{r}_{t+1}^j = \rho E_{t+1} (\hat{C}_{t+2}^j - \hat{C}_{t+1}^j)$$

for each  $j = b, s$  where  $\rho \equiv v\bar{Y}$  while  $\bar{Y}$  is the steady-state output and we use the following definitions  $\hat{C}_t^j \equiv (C_t^j - \bar{C}^j)/\bar{Y}$  for each  $j = b, s$ .

Appropriately, goods market equilibrium (26) implies in a first-order approximation that

$$\hat{Y}_t = \chi \hat{C}_t^s + (1 - \chi) \hat{C}_t^b \quad (61)$$

where now  $\hat{Y}_t = (Y_t - \bar{Y})/\bar{Y}$ .

Finally in a first-order approximation the spread schedule (25) implies

$$\hat{i}_t^b = (1 + \phi \tilde{b}) \hat{i}_t^d + \phi \hat{b}_t \quad (62)$$

for some parameter  $\phi$  where  $\hat{b}_t \equiv (b_t - \bar{b})/\bar{Y}$  and  $\tilde{b} \equiv \bar{b}/\bar{Y}$ .

A first-order approximation of the flow budget constraint of the borrowers (42) implies that

$$\beta \hat{b}_t = \hat{b}_{t-1} + \tilde{b} \cdot \hat{i}_{t-1}^b - \tilde{b}(\pi_t - \bar{\pi}) + \beta \hat{C}_t^b - \beta \hat{Y}_t. \quad (63)$$

Euler equations (59) and (60) together with (61), (62) and (63) constitute the aggregate demand block of the model.

In a log-linear approximation, the supply block comes from approximating (29), (30) taking into account the definitions of  $\varphi_t^j$  for  $j = b, s$ . The following modified New-Keynesian Phillips curve is obtained

$$\pi_t - \bar{\pi} = \kappa(\eta + \rho) \hat{Y}_t + \kappa[\hat{r}_t + E_t(\pi_{t+1} - \bar{\pi}) - \hat{i}_t^m] + \beta E_t(\pi_{t+1} - \bar{\pi}) \quad (64)$$

where we have defined  $\kappa \equiv (1 - \alpha)(1 - \alpha\beta)/\alpha$  and now

$$\hat{r}_t = \rho E_t(\hat{Y}_{t+1} - \hat{Y}_t).$$

The New-Keynesian Phillips curve is augmented by a term reflecting the variations in the monetary frictions at the aggregate level.

Finally we take a first-order approximation of the equilibrium conditions for the money market obtaining

$$\hat{m}_{t-1}^s + \hat{i}_{t-1}^m + \vartheta_3 \left( \hat{b}_{t-1} + \tilde{b} \cdot \hat{i}_{t-1}^d + \vartheta_4 \hat{\gamma}_t \right) = \frac{1 + \vartheta_3 \tilde{b}}{\tilde{c}^s} \left( \hat{C}_t^s + \tilde{c}^s(\pi_t - \bar{\pi}) \right) \quad (65)$$

$$\hat{m}_{t-1}^b + \hat{i}_{t-1}^m = \frac{1}{\tilde{c}^b} \hat{C}_t^b + (\pi_t - \bar{\pi}), \quad (66)$$

where  $\vartheta_3 \equiv \bar{\gamma} \frac{1-\chi}{\chi} \frac{1+\tilde{v}^d}{1+\tilde{v}^m} (\tilde{m}^s)^{-1}$ ,  $\tilde{m}^s = \bar{m}^s/\bar{Y}$ ,  $\vartheta_4 \equiv \tilde{b}(1 - \bar{\gamma})/\bar{\gamma}$  and  $\tilde{c}^j = \bar{C}^j/\bar{Y}$  for each  $j$ . Real money balances follow

$$\hat{m}_t \equiv \chi \hat{m}_t^s + (1 - \chi) \hat{m}_t^b \quad (67)$$

$$\hat{m}_t = \hat{m}_{t-1} + \mu_t - \pi_t \quad (68)$$

and  $\mu_t$  is the nominal money-supply growth.

Equations (59), (60), (61), (62), (63) together with (64), (65), (66), (67), (68) and the definitions of  $\hat{r}_{t+1}^s$ ,  $\hat{r}_{t+1}^b$  and  $\hat{r}_{t+1}$  determine the equilibrium allocation for  $\pi_t$ ,  $\hat{C}_t^b$ ,  $\hat{C}_t^s$ ,  $\hat{Y}_t$ ,  $\hat{i}_t^b$ ,  $\hat{i}_t^d$ ,  $\hat{i}_t^m$ ,

$\hat{b}_t, \hat{m}_t^s, \hat{m}_t^b, \hat{m}_t, \mu_t$ , where two policy instruments should be specified.

## Appendix C

In this appendix, we show the derivations of the second-order approximation of the welfare (43). The approximation is taken with respect to an efficient steady state. This efficient steady state maximizes (43) under the resource constraint (26) considering that  $L = (L^s)^\chi(L^b)^{1-\chi}$ .

At the efficient steady state the following conditions hold

$$\begin{aligned}\tilde{\chi}\bar{U}_c^s &= \chi\bar{\lambda}; \\ (1-\tilde{\chi})\bar{U}_c^b &= (1-\chi)\bar{\lambda}; \\ \tilde{\chi}\bar{V}_l^s &= \chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s}; \\ (1-\tilde{\chi})\bar{V}_l^b &= (1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b}\end{aligned}$$

where all upper bars denote steady-state values and  $\bar{\lambda}$  is the steady-state value of the Lagrange multiplier associated with the constraint (26). Note that the above conditions imply  $\bar{U}_c^s/\bar{U}_c^b = \chi(1-\tilde{\chi})/[(1-\chi)\tilde{\chi}]$  so that an appropriately chosen  $\tilde{\chi}$  determines the efficient distribution of wealth in a consistent way with the steady state debt position of the borrowers in the model, given by  $\bar{b}$ . For the above efficient steady-state to be consistent with the steady-state of the model we need to offset the distortions of the model appropriately. Note that at the efficient steady state

$$\frac{\bar{V}_l^j}{\bar{U}_c^j} = \frac{\bar{Y}}{\bar{L}^j}$$

for each  $j = b, s$ . On the other side, the steady-state of the model, when inflation is at the target level, implies

$$\frac{\bar{V}_l^j}{\bar{U}_c^j} = \frac{\bar{Y}}{\bar{L}^j} \frac{\bar{W}}{\bar{P}} (1 - \bar{\varphi}),$$

for each  $j = b, s$  and

$$\frac{\bar{W}}{\bar{P}} = \frac{1}{(1-\tau)(1+\bar{\mu})}.$$

where  $\bar{\mu} \equiv 1/(\theta - 1)$  while

$$\bar{\varphi} = 1 - \frac{\beta(1 + \bar{v}^m)}{\bar{\Pi}}.$$

It is clear from the above equations that we just need to set the employment subsidy at the level

$$\tau = \frac{\bar{\mu} + \bar{\varphi}}{1 + \bar{\mu}}$$

in order to make the steady-state of the decentralized allocation efficient.

Having defined the efficient steady state, we take a second-order expansion of the utility

flow around it to obtain

$$U_t = \bar{U} + \tilde{\chi} \left[ \bar{U}_c^s (C_t^s - \bar{C}^s) + \frac{1}{2} \bar{U}_{cc}^s (C_t^s - \bar{C}^s)^2 \right] + (1 - \tilde{\chi}) \left[ \bar{U}_c^b (C_t^b - \bar{C}^b) + \frac{1}{2} \bar{U}_{cc}^b (C_t^b - \bar{C}^b)^2 \right] + \\ - \tilde{\chi} \left[ \bar{V}_l^s (L_t^s - \bar{L}^s) + \frac{1}{2} \bar{V}_{ll}^s (L_t^s - \bar{L}^s)^2 \right] - (1 - \tilde{\chi}) \left[ \bar{V}_l^b (L_t^b - \bar{L}^b) + \frac{1}{2} \bar{V}_{ll}^b (L_t^b - \bar{L}^b)^2 \right] + \mathcal{O}(\|\xi\|^3)$$

where an upper-bar variable denotes the efficient steady state while  $\mathcal{O}(\|\xi\|^3)$  collects terms in the expansion which are of order higher than the second. We can use the steady-state conditions to write the above equation as

$$U_t = \bar{U} + \chi \bar{\lambda} \left[ (C_t^s - \bar{C}^s) + \frac{1}{2} \frac{\bar{U}_{cc}^s}{\bar{U}_c^s} (C_t^s - \bar{C}^s)^2 \right] + (1 - \chi) \bar{\lambda} \left[ (C_t^b - \bar{C}^b) + \frac{1}{2} \frac{\bar{U}_{cc}^b}{\bar{U}_c^b} (C_t^b - \bar{C}^b)^2 \right] + \\ - \chi \bar{\lambda} \frac{\bar{Y}}{\bar{L}^s} \left[ (L_t^s - \bar{L}^s) + \frac{1}{2} \frac{\bar{V}_{ll}^s}{\bar{V}_l^s} (L_t^s - \bar{L}^s)^2 \right] - (1 - \chi) \bar{\lambda} \frac{\bar{Y}}{\bar{L}^b} \left[ (L_t^b - \bar{L}^b) + \frac{1}{2} \frac{\bar{V}_{ll}^b}{\bar{V}_l^b} (L_t^b - \bar{L}^b)^2 \right] + \mathcal{O}(\|\xi\|^3).$$

Note that for a generic variable  $X$ , we have

$$X_t = \bar{X} \left( 1 + \hat{X}_t + \frac{1}{2} \hat{X}_t^2 \right) + \mathcal{O}(\|\xi\|^3)$$

where  $\hat{X}_t \equiv \ln X_t / \bar{X}$  and moreover recall that

$$Y_t = \chi C_t^s + (1 - \chi) C_t^b,$$

implying that

$$\chi (C_t^s - \bar{C}^s) + (1 - \chi) (C_t^b - \bar{C}^b) = \bar{Y} \left[ \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right] + \mathcal{O}(\|\xi\|^3)$$

We can write the above approximation as

$$U_t = \bar{U} + \bar{\lambda} \bar{Y} \left[ \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right] - \frac{1}{2} \bar{\lambda} v \left[ \chi (C_t^s - \bar{C}^s)^2 + (1 - \chi) (C_t^b - \bar{C}^b)^2 \right] \\ - \chi \bar{\lambda} \bar{Y} \left[ \hat{L}_t^s + \frac{1}{2} (1 + \eta) (\hat{L}_t^s)^2 \right] - (1 - \chi) \bar{\lambda} \bar{Y} \left[ \hat{L}_t^b + \frac{1}{2} (1 + \eta) (\hat{L}_t^b)^2 \right] + \mathcal{O}(\|\xi\|^3), \quad (69)$$

where we have also used the fact that with the preference specification used  $\bar{U}_{cc}^s / \bar{U}_c^s = \bar{U}_{cc}^b / \bar{U}_c^b = -v$  and  $\bar{V}_{ll}^s \bar{L}^s / \bar{V}_l^s = \bar{V}_{ll}^b \bar{L}^b / \bar{V}_l^b = \eta$ .

Note that in equilibrium  $L_t = \Delta_t Y_t$  where  $L_t = (L^s)^\chi (L^b)^{1-\chi}$ . It follows that the following condition holds exactly

$$\hat{Y}_t = \chi \hat{L}_t^s + (1 - \chi) \hat{L}_t^b + \hat{\Delta}_t.$$

Using the above equation in (69), the latter can be simplified to

$$\begin{aligned} \frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} &= \frac{1}{2}\hat{Y}_t^2 - \frac{1}{2}\rho \left[ \chi(\hat{C}_t^s)^2 + (1-\chi)(\hat{C}_t^b)^2 \right] \\ &\quad - \frac{1}{2}(1+\eta) \left[ \chi(\hat{L}_t^s)^2 + (1-\chi)(\hat{L}_t^b)^2 \right] - \hat{\Delta}_t + \mathcal{O}(\|\xi\|^3), \end{aligned} \quad (70)$$

where  $\rho \equiv v\bar{Y}$  and we have used the definitions of  $\hat{C}_t^s$  and  $\hat{C}_t^b$ . Note that to a first-order approximation

$$\begin{aligned} \hat{C}_t^s &= \hat{Y}_t - (1-\chi)(\hat{C}_t^b - \hat{C}_t^s) + \mathcal{O}(\|\xi\|^2) \\ \hat{C}_t^b &= \hat{Y}_t + \chi(\hat{C}_t^b - \hat{C}_t^s) + \mathcal{O}(\|\xi\|^2) \\ \hat{L}_t^s &= \hat{Y}_t - (1-\chi)(\hat{L}_t^b - \hat{L}_t^s) + \mathcal{O}(\|\xi\|^2) \\ \hat{L}_t^b &= \hat{Y}_t + \chi(\hat{L}_t^b - \hat{L}_t^s) + \mathcal{O}(\|\xi\|^2) \end{aligned}$$

which can be used to simplify (70) to

$$\frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = -\frac{1}{2}(\rho + \eta)\hat{Y}_t^2 - \frac{1}{2}\chi(1-\chi)\rho(\hat{C}_t^s - \hat{C}_t^b)^2 - \frac{1}{2}\chi(1-\chi)(1+\eta)(\hat{L}_t^s - \hat{L}_t^b)^2 - \hat{\Delta}_t + \mathcal{O}(\|\xi\|^3).$$

Note that

$$\Delta_t = \alpha \left( \frac{\Pi_t}{\bar{\Pi}} \right)^\theta \Delta_{t-1} + (1-\alpha) \left( \frac{1 - \alpha \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}}$$

By taking a second-order approximation of  $\hat{\Delta}_t$ , as it is standard in the literature and integrating appropriately across time, we obtain that

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \theta \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{(\pi_t - \bar{\pi})^2}{2} + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3)$$

We can then obtain a second-order approximation of the utility of the consumers as

$$W_t = -\bar{\lambda}(\eta + \rho)\bar{Y} \cdot \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} Loss_t \right\} + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3)$$

where

$$Loss_t = \hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\hat{C}_t^s - \hat{C}_t^b)^2 + \chi(1-\chi)\lambda_l(\hat{L}_t^s - \hat{L}_t^b)^2 + \lambda_\pi(\pi_t - \bar{\pi})^2$$

where we have defined

$$\begin{aligned} \lambda_c &\equiv \frac{\rho}{\rho + \eta} \\ \lambda_l &\equiv \frac{1 + \eta}{\rho + \eta} \end{aligned}$$

$$\lambda_\pi \equiv \frac{\theta}{\kappa}.$$

Note finally that

$$\begin{aligned} \frac{(L_t^s)^{1+\eta}}{v \exp(-vC_t^s)} &= \frac{W_t}{P_t} \Delta_t Y_t (1 - \varphi_t^s) \\ \frac{(L_t^b)^{1+\eta}}{v \exp(-vC_t^b)} &= \frac{W_t}{P_t} \Delta_t Y_t (1 - \varphi_t^b) \end{aligned}$$

which imply in a log-linear approximation that

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta} (\hat{C}_t^s - \hat{C}_t^b) - \frac{\bar{\varphi}}{(1-\bar{\varphi})(1+\eta)} (\hat{\varphi}_t^s - \hat{\varphi}_t^b)$$

Moreover from log-linear approximations of (23), we get

$$\hat{\varphi}_t^j = \frac{1-\bar{\varphi}}{\bar{\varphi}} E_t \left[ (\pi_{t+1} - \bar{\pi}) - \hat{i}_t^m + \rho \Delta \hat{C}_{t+1}^j \right]$$

for  $j = b, s$  and therefore

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta} E_t (\hat{C}_{t+1}^s - \hat{C}_{t+1}^b),$$

which can be also used in the loss function to replace the term  $\hat{L}_t^s - \hat{L}_t^b$ .