Gender Gaps in the Labour Market and Aggregate Productivity*

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Abstract

This paper examines the quantitative effects of gender gaps in entrepreneurship and labour force participation on aggregate income and productivity. We calibrate and simulate an occupational choice model with heterogeneous agents in entrepreneurial ability and show that gender gaps in entrepreneurship affect aggregate productivity negatively, while gender gaps in labour force participation reduce income per capita. We find that gender gaps and their implied income losses are quite similar across income groups but differ importantly across geographical regions, with a total income loss of 27% in Middle East and North Africa and a 14% loss in Europe and Central Asia.

1 Introduction

Gender inequality is a pervasive feature in many countries, especially developing ones. Gaps between male and female outcomes and opportunities are present in several dimensions, including education, earnings, occupation, access to formal employment, access to entrepreneurship, access to productive inputs, political representation, or bargaining power inside the household. Dollar and Gatti (1999), for instance, calculate that, in 1990, only 5% of adult

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women had some level of secondary education in the poorest quartile of countries, half of the corresponding level for men. Although the gaps in employment and pay are closing faster in developing countries than they did in industrialized ones (Tzannatos 1999), the prevalence of gender inequality in the former group is still sizeable, especially in South Asia and the Middle East and North Africa (Klasen and Lamanna 2009). Moreover, women are under-represented among top positions in most countries: even in the most developed countries, the average incidence of females among employers is less than 30% (World Bank 2001).

Everything else equal, a better use of women's potential in the market is likely to result in greater macroeconomic efficiency. When they are free to choose occupation, for example, the most talented people - independently of their gender - typically organize production carried out by others, so they can spread their ability advantage over a larger scale. From this point of view, obstacles to women's access to entrepreneurship reduce the average ability of a country's entrepreneurs and affect negatively the way production is organized in the economy and, hence, its efficiency.

There are many empirical articles studying the two-way relationship between economic development and gender inequality, like Goldin (1990), Hill and King (1995), Dollar and Gatti (1999), Tzannatos (1999), Klasen (2002), or Klasen and Lamanna (2009). This literature has reached some consensus on the fact that there is a positive effect of economic growth on gender equality and a negative effect of gender inequality on economic development.[1] With respect to the theoretical literature, several studies focus on explaining the effects of economic growth on the gender gap, like Galor and Weil (1996), Fernandez (2012), or Duflo (2010). Other theoretical articles analyse the reverse effect, i.e. the impact of gender inequality on development. These theories are, in most cases, based on the fertility and children's human capital channels, like Galor and Weil (1996), Doepke and Tertilt (2009), Lagerlof (2003) or Blackden et al. (2006).[2] Galor and Weil (1996), for instance, argue that an increase in women's relative wage increases the cost of raising children, which lowers population growth, increases education levels and leads to higher labour productivity and higher future growth.

There has been, however, very little theoretical work on the female labour productivity

[1] One remarkable exception is the pioneer study of Barro and Lee (1994). In this paper and in different studies that followed (Barro and Lee 1996, Barro and Sala-i-Martin 2003) the authors find that when they include male and female primary and secondary schooling in their growth regressions the coefficient associated with female schooling is negative. They interpret this negative sign as a reflection of a large gap in schooling between genders which in turn is a proxy for backwardness.

[2] See, for example, Cuberes and Teignier (2011) for a comprehensive review of the empirical and theoretical literature on this topic.

channel, i.e. on the negative effects of gender inequality in the labour market on aggregate productivity. Intuitively, assuming that people's ability is distributed randomly, gender inequality in the labour market is expected to distort the allocation of productive resources and impact aggregate productivity negatively. Esteve-Volart (2009) is, to our knowledge, the only paper that highlights this channel. She presents a model of occupational choice and talent heterogeneity, and finds that labour market discrimination leads to lower average entrepreneurial talent, slower female human capital accumulation. This, in turn, has a negative impact on technology adoption and innovation and, so, it reduces economic growth. The model, however, is only used to derive qualitative results but not to perform numerical exercises.

Finally, on the quantitative side, Hsieh et al. (2001) estimates the contribution to U.S. economic growth from the changing occupational allocation of white women, black men, and black women between 1960 and 2008. The paper finds that the improved allocation of talent within the United States accounts for 17 to 20 percent of growth over this period. [3]

In this paper, we develop and calibrate a simple theoretical model illustrating the positive impact of gender equality on resource allocation and aggregate labour productivity. The model is then used to quantify the costs of gender inequality and the effects of the existing gender gaps across countries. We introduce gender inequality into a span-of-control framework based on Lucas (1978), in which agents are endowed with entrepreneurial talent drawn from a fixed distribution and the most talented ones choose to become entrepreneurs. Gender inequality is introduced in the model as an exogenous restriction on women's access to entrepreneurship and participation in the workforce. When women are excluded from entrepreneurship, a larger fraction of men become entrepreneurs and, as a result, the average talent of entrepreneurs decreases. Moreover, since all women are forced to work as employees, the supply of labour increases and the equilibrium wage rate decreases even further.

We parametrize and simulate the model to quantify the negative effects of gender inequality on firms distribution, average productivity and income per capita. We find that if all women are excluded from entrepreneurship, the density of firms at all sizes falls by 4.5%, average output per worker drops by more than 12% and wages fall by even more, while if all women are excluded from the labour force, income per capita falls by almost 40%.In the cross-country analysis, we find that gender gaps and their implied income losses are quite similar across income groups but differ importantly across geographical regions, with a 27% income loss in

[3] Rodriguez-Mora (2009), Pica and Rodriguez-Mora (2011) also study the effects of talent misallocation in different contexts.

the Middle East and North Africa, a 23% loss in South Asia, and a loss around 14% in Europe and Central Asia and 12% in Sub-Saharan Africa.

The rest of the paper is organized as follows. In Section 2, we present the theoretical model; Section 3 explains the simulation and numerical results; Section 4 discusses the quantitative implications of our model for a large set of countries, and, finally, Section 5 concludes.

2 Model

In this section, we present a simple static general equilibrium model of agents with heterogeneous entrepreneurial skills, as in Lucas (1978). Agents are endowed with a specific talent for managing, based on which they decide to work as either entrepreneurs or employees. The model assumes an underlying distribution of entrepreneurial talent in the population, and studies the resulting allocation of productive factors across entrepreneurs as well as the size distribution of firms.

2.1 Model Setup

The economy we consider has a continuum of agents indexed by their entrepreneurial talent x, drawn from a cumulative distribution Γ that takes values between B and $\bar{z}$.[4] It is a closed economy with a workforce of size N and with K units of capital. these two inputs are inelastically supplied in the market by consumers and then combined by firms to produce an homogeneous good.

At each period, agents rent the capital stock they own to firms in exchange for the rental rate r, and decide to become either firm workers, who earn the equilibrium wage rate w, or entrepreneurs, who earn the profits generated by the firm they manage.

An agent with entrepreneurial talent level x who manages n units of labour and k units of capital produces y units of output and earns profits $\pi(x) = y(x) - rk(x) - wn(x)$, where the price of the homogeneous good is normalized to one. As in Lucas (1978) and Buera and Shin (2011), the production function is given by

$$y(x) = x\left(k^{\alpha} n^{1-\alpha}\right)^{\eta}, \tag{1}$$

where $\alpha \in (0,1)$ and $\eta \in (0,1)$. The parameter η measures the "span of control" of entre-

[4]In the next section, we justify why we need to assume an upper bound in the talent distribution.

preneurs and, since it is lower than one, the entrepreneurial technology involves an element of diminishing returns.

2.2 Agents' optimization

Entrepreneurs choose the labour and capital they hire in order to maximize their current profits π. The first order conditions that characterize their optimization problem are given by

$$(1-\alpha)\,\eta x k(x)^{\alpha\eta} n(x)^{\eta(1-\alpha)-1} = w \tag{2}$$

$$\alpha\eta x k(x)^{\alpha\eta-1} n(x)^{\eta(1-\alpha)} = r. \tag{3}$$

Hence, at the optimum, all firms have the common capital-labour ratio (4):

$$\frac{k(x)}{n(x)} = \frac{\alpha}{1-\alpha}\frac{w}{r} \tag{4}$$

where $k\,(x)$ and $n\,(x)$ denote the optimal capital and labour levels for an entrepreneur with talent level x. Intuitively, a higher $\frac{w}{r}$ ratio implies a more intensive use of capital relative to labour. The solution values for $n(x)$ and $k(x)$ for a given firm can be obtained combining equations (2) and (4). Both $n(x)$ and $k(x)$ depend positively on the productivity level x, as equations (5) and (6) show:

$$n\,(x) = \left[x\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right]^{1/(1-\eta)} \tag{5}$$

$$k\,(x) = \left[x\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right]^{1/(1-\eta)} \tag{6}$$

Given this efficient allocation, agents choose their occupation to maximize their earnings. Thus, there is a cut-off talent level $z > 0$ such that if $x \leq z$ agents choose to work as employees, and if $x > z$ agents agents become entrepreneurs. At the cut-off level z, the agent is indifferent between the two occupations, so that $\pi\,(z) \equiv y\,(z) - wn\,(z) - rk\,(z) = w$, that is,

$$z\left(k\,(z)^{\alpha}\,n\,(z)^{1-\alpha}\right)^{\eta} - wn\,(z) - rk\,(z) = w. \tag{7}$$

If they become employees they obviously do not hire any capital or labour input, i.e. $k(x) = n(x) = 0 \; \forall x \leq z$.

2.3 Equilibrium and Aggregation

In equilibrium, the total demand of capital by entrepreneurs must be equal to the exogenously given aggregate capital endowment K, and, in the labour market, the total demand of workers must also be equal to the non-entrepreneurs workforce:

$$\int_{z}^{\bar{z}} k(x) d\Gamma(x) = K/N \tag{8}$$

$$\int_{z}^{\bar{z}} n(x) d\Gamma(x) = \Gamma(z), \tag{9}$$

where N denotes total work force, which is equal to total population in the benchmark case, and $k = \frac{K}{N}$ denotes the capital stock per capita.

Aggregate income per capita is equal to total production per capita,

$$\frac{Y}{N} = \int_{z}^{\bar{z}} x \left(k(x)^{\alpha} n(x)^{1-\alpha} \right)^{\eta} d\Gamma(x) \tag{10}$$

where Y denotes total output.

Plugging (5) and (6) into equation (10) we get total income per capita as a function of the talent distribution Γ and the equilibrium unknowns (z, w, r):

$$\frac{Y}{N} = \left[\eta \frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{r^{\alpha} w^{1-\alpha}} \right]^{\frac{\eta}{1-\eta}} \int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\Gamma(x) \tag{11}$$

A competitive equilibrium in this economy is a cut-off level z, a set of quantities $[n(x), k(x)]_{x>z}$ and prices (w, r) such that equations (4) - (9) are satisfied, that is agents choose their occupation optimally, entrepreneurs choose the amount of capital and labour to maximize their profits, and all markets clear.

As explained in Appendix A, the three equilibrium conditions in equations (7), (8) and

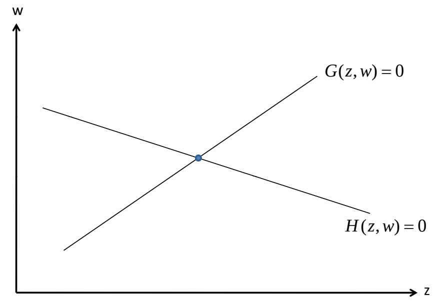


Figure 1: Graphical representation of the equilibrium

(9) can be summarized in the two equations $G(z,w)=0$ and $H(z,w)=0$:

$$H(z,w)=\Gamma\left(z\right)^{\frac{1-\eta(1-\alpha)}{1-\eta}}-\left[\eta\left(1-\alpha\right)\frac{k^{\alpha\eta}}{w}\right]^{\frac{1}{1-\eta}}\int_{z}^{\bar{z}}x^{\frac{1}{1-\eta}}d\Gamma(x)=0, \tag{12}$$

$$G(z,w)=w^{\frac{1}{1-\eta}}-\Phi z^{\frac{1}{1-\eta}}\left(\frac{\alpha}{1-\alpha}\frac{\Gamma\left(z\right)}{k}\right)^{\frac{-\alpha\eta}{1-\eta}}=0. \tag{13}$$

where Φ is a constant.

As appendix A explains, the equation $H(z,w)=0$ always has a negative slope in the (z,w) diagram, while the equation $G\left(z,w\right)=0$ has a positive slope for the relevant range of z.[5] Intuitively, $H\left(z,w\right)=0$ is downward sloping because a larger z implies a lower number of entrepreneurs and more workers and, therefore, a lower equilibrium wage to clear the worker's market. $G\left(z,w\right)=0$, on the other hand, is upward sloping because larger z implies a larger profit $\pi\left(z\right)$ and, therefore, a larger w is needed to get the occupational indifference at z. The intersection of the two equations defines the equilibrium, as Figure 1 shows.

2.4 Equilibrium with Gender Gaps

2.4.1 Gender Gaps in Entrepreneurship

We introduce gender inequality in entrepreneurship in our setup by imposing that only a randomly selected fraction $\theta\in(0,1)$ of the population is eligible to be an entrepreneur. That is, assuming that men and women have the exact same talent distribution and given that the percentage of women in the population is around 50%, if all women are excluded from

[5] Figure (11) in appendix C plots the two functions for the parameter values and the talent distribution used in the quantitative sections.

entrepreneurship, the parameter θ takes a value equal to 1/2. When a randomly selected fraction $1-\theta$ of the population is excluded from the pool of potential entrepreneurs, the talent distribution becomes

$$\widetilde{\Gamma} = \theta\Gamma, \tag{14}$$

and the labour market clearing condition becomes

$$\int_{z}^{\bar{z}} n(x)d\widetilde{\Gamma}(x) = (1-\theta) + \widetilde{\Gamma}(z). \tag{15}$$

In words, the supply of workers has now two components: those with skill below z, $\Gamma(z)$, and those with skill greater or equal than z who are not allowed to be entrepreneurs, $(1-\theta)(1-\Gamma(z))$. Therefore, the total labour labour supply is equal to

$$\Gamma(z) + (1-\theta)(1-\Gamma(z)) = 1-\theta+\theta\Gamma(z) = (1-\theta)+\widetilde{\Gamma}(z).$$

The capital market clearing condition is the same as before, except for the talent distribution of entrepreneurs, which is now $\widetilde{\Gamma} = \theta\Gamma$:

$$\int_{z}^{\bar{z}} k(x)d\widetilde{\Gamma}(x) = K/N. \tag{16}$$

As before, the three equilibrium conditions in equations (7), (15) and (16) can be summarized in the two equations $\widetilde{G}(z,w) = 0$ and $\widetilde{H}(z,w) = 0$:

$$\widetilde{H}(z,w) = 1-\theta+\tilde{\Gamma}(z) - \left(\frac{\eta(1-\alpha)}{w}\right)^{\frac{1}{1-\eta}} \left(\frac{1-\theta+\tilde{\Gamma}(z)}{k}\right)^{\frac{-\alpha\eta}{1-\eta}} \int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\tilde{\Gamma}(x) = 0, \tag{17}$$

$$\widetilde{G}(z,w) = w^{\frac{1}{1-\eta}} - \Phi z^{\frac{1}{1-\eta}} \left(\frac{\alpha}{1-\alpha}\frac{1-\theta+\tilde{\Gamma}(z)}{k}\right)^{\frac{-\alpha\eta}{1-\eta}} = 0. \tag{18}$$

As explained in Appendix B, $\widetilde{H}(z,w)$ is downward sloping in the (z,w) diagram and $\widetilde{G}(z,w)$ is upward sloping for the relevant range of z.

Effects of an increase in the gender gap. Graphically, an increase in gender inequality,

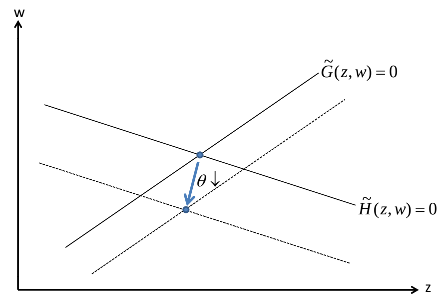


Figure 2: Graphical effects of entrepreneurial inequality

i.e. a decrease in θ, leads to a downward shift of both $\widetilde{G}(z,w)=0$ and $\widetilde{H}(z,w)=0$. As a result, the equilibrium wage w will be always lower under gender inequality, as well as the threshold level z given that the shift in $\widetilde{H}(z,w)=0$ is larger than the shift in $\widetilde{G}(z,w)=0$.[6] Figure (2) shows the equilibrium change in this case. Intuitively, an increase in gender inequality reduces the pool of workers eligible for entrepreneurship, which affects negatively the average productivity of entrepreneurs in equilibrium. Interestingly, the equilibrium is characterized by a lower number of less talented entrepreneurs who run larger firms, as we will see in the next section. This fall in aggregate productivity clearly affects the equilibrium wage rate negatively, which also decreases because of the rise in the supply of workers.

2.4.2 Gender Inequality in Labour Force Participation

Another type of gender inequality that can be introduced in the model is the exclusion of women from the work force, both as entrepreneurs and employees, as in Esteve-Volart (2009). If we keep the capital stock fixed, when a fraction of women does not supply labour to the market, output per worker mechanically increases. Income per capita, however, decreases. Formally, recalling that N denotes the total labour force and defining P as total population, we have

$$\frac{Y}{P}=\frac{N}{P}\int_{z}^{\bar{z}} x\left(k(x)^{\alpha}n(x)^{1-\alpha}\right)^{\eta}d\Gamma(x).$$

With this formulation, it is then possible to study the impact of reducing the employment-to-population ratio $n=N/P$ below 1.

[6]See Appendix B for a formal proof.

3 Model Simulation

3.1 Skill Distribution

To simulate the model, we use a Pareto function for the talent distribution, as in Lucas (1978) and Buera, Kaboski and Shin (2011).[7] We assume an upper bound $\bar{z}$ on talent to guarantee that the talent distribution is bounded.[8] Hence, the cumulative distribution of talent is

$$\Gamma(x) = \frac{1 - B^{\rho} x^{-\rho}}{1 - B^{\rho} \bar{z}^{-\rho}},\ 0 \leq x \leq \bar{z} \tag{19}$$

where $\rho,\ B > 0$, and the density function of talent is

$$\gamma(x) = \frac{\rho B^{\rho} x^{-\rho-1}}{1 - B^{\rho} \bar{z}^{-\rho}},\ ,\ 0 \leq x \leq \bar{z}. \tag{20}$$

Using equations (5) and (20), we can derive the density function of the firms' size,

$$\begin{aligned} s(n) &= \gamma\left(n^{-1}(x)\right) = \gamma\left(\frac{n^{1-\eta}}{\eta(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha\eta}\frac{w^{-\alpha\eta+1}}{r^{-\alpha\eta}}\right) \\ &= \frac{\rho B^{\rho}}{1 - B^{\rho}\bar{z}^{-\rho}} n^{-(1-\eta)(1+\rho)}\left(\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right)^{1+\rho}, \end{aligned} \tag{21}$$

which shows that the distribution of firms size is also Pareto and, if $(1-\eta)(1+\rho) = 1$, it satisfies Zipf's law. This law has been shown to fit the US firm size distribution remarkably well (see Axtell 2001 and Gabaix 2012).

3.2 Parameter Values

Table (1) shows the values used in the simulations for the different parameters of the model. The parameter B of the Pareto distribution is normalized to 1. The span-of-control parameter η is chosen equal to 0.8, following Buera and Shin (2011).[9] The value used for the parameter

[7] See Gabaix (2012) for a detailed summary of the applications of the Pareto distribution.

[8] If we want to use an unbounded Pareto distribution, i.e. $\bar{z} \to \infty$, we need to assume that $\rho > \frac{1}{1-\eta}$ so that the integral $\int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\Gamma(x)$ is defined. Note, however, that this would imply $1 + \rho > \frac{1}{(1-\eta)}$, which contradicts Zipf's Law for the distribution of firms' size.

[9] Buera and Shin (2011) calibrate η from the fraction of total income of the top five per cent of earners in the US population, which is 30%, given that top earners are are always entrepreneurs in the model and in most cases in the data.

Table 1: Parameter values

Parameter	Value	Explanation
B	1	Normalization
η	0.8	Buera and Shin (2011)
ρ	4	To satisfy Zipf's Law for firms distribtuion, $-(1-\eta)(-1-\rho) \approx 1$
α	0.375	To match capital share, $\alpha\eta$=0.3
$\bar{z}$	$7.8 * B$	To match average fraction of entrepreneurs in the data, which is 3.55%

ρ is set equal to 4 so that the talent distribution satisfies Zipf's Law.[10] The capital exponent parameter α is set to 0.375 in order to make $\alpha\eta$ equal to 30%, as in Buera and Shin (2011), since 30% is the value typically used for the aggregate income share of capital and we are considering the entrepreneurs' earnings as labour income. Finally, the talent upper bound $\bar{z}$ is chosen equal to 7.2 times B, to make the world-average share of entrepreneurs predicted by the model match the one observed in our data set.[11]

3.3 Quantitative effects of entrepreneurial gaps

To show the effects of gender inequality in entrepreneurship, we now compare the talent distribution and the firms size distribution when there is no gender inequality, $\theta = 1$, with the one in which all women are excluded from entrepreneurship, $\theta = 0.5$. Figure 3 shows that when 50% of the workforce are not eligible as entrepreneurs, the talent threshold z decreases and the entire talent distribution shifts to the left. In Figure 4, we can see that when the gender gap is positive, the entire firm distribution shifts to the left, since the talent of the smallest firms managers' talent drops and, at all talent levels, firms are now larger. Figure 5, on the other hand, shows that when the gender gap is positive, the density of firms decreases at all sizes.

Finally, Figure 6 shows the negative effects of gender inequality on average productivity

[10] This is similar to the value used by Buera, Kaboski and Shin (2011), 4.84, which is chosen to match the employment share of the largest 10 percent of establishments in the US.

[11] The world-average fraction of entrepreneurs in the data is estimated using data on the variable *Employers* from the International Labour Organization, Table 3 - Status in Employment (by sex), where the weights of each country are equal to the country's employment over total employment.

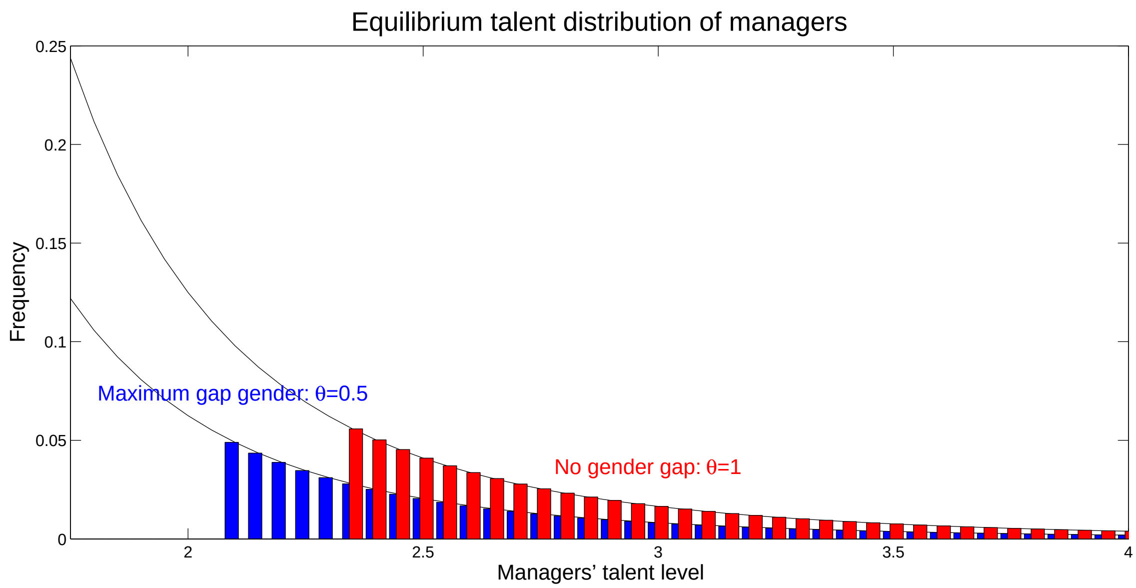


Figure 3: Talent distribution

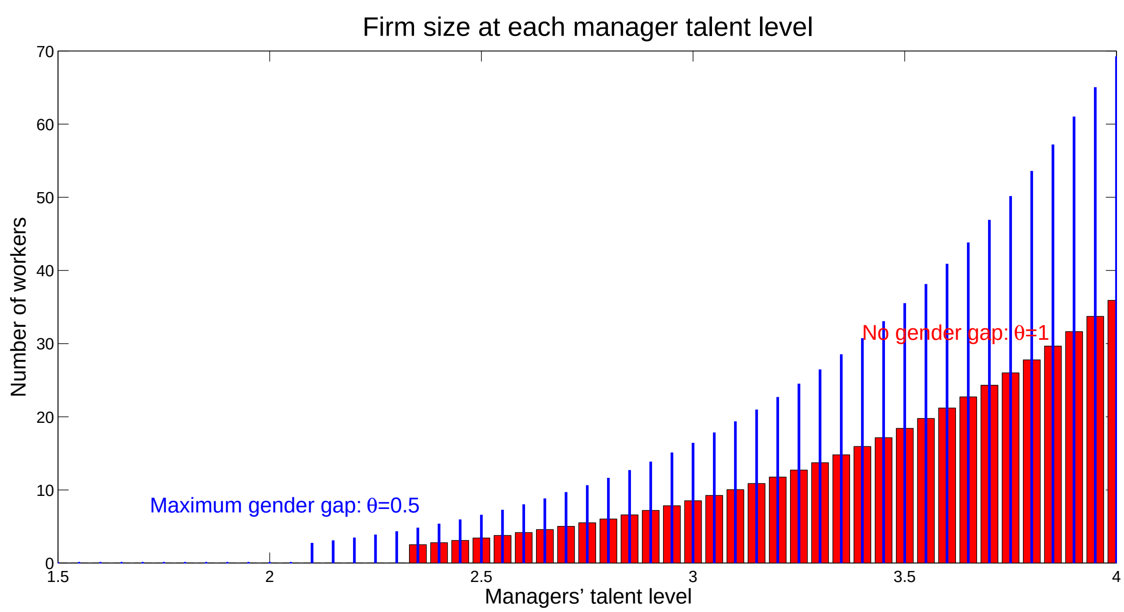


Figure 4: Firms Size

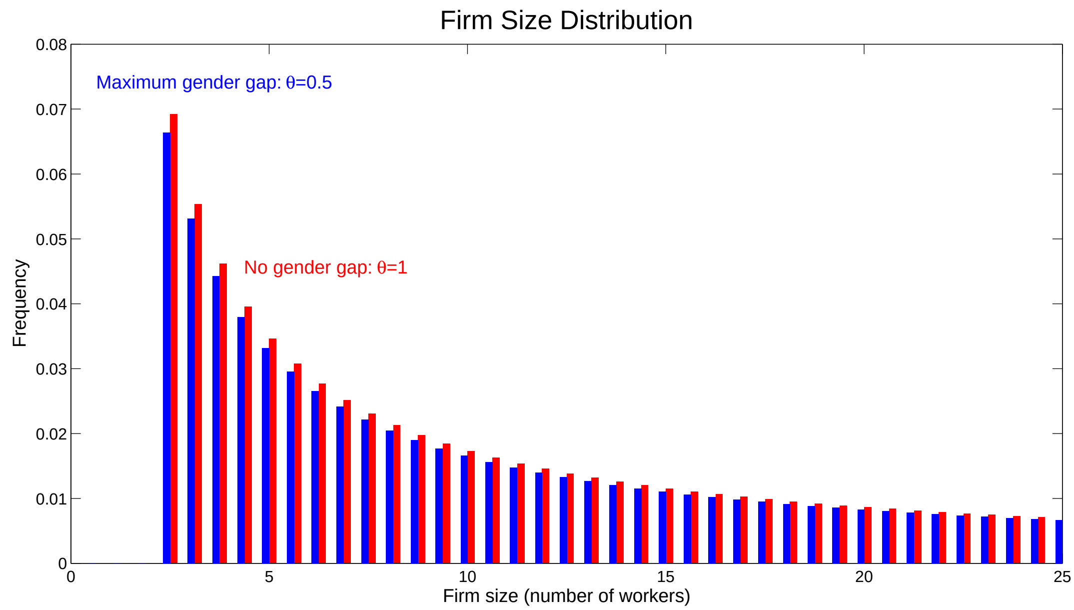


Figure 5: Firms Distribution

and workers' wages. The further one moves to the left on the horizontal axis, the higher is the percentage of population excluded from entrepreneurship, and the larger the loss in income per worker and wages with respect to the no gender inequality case. When the fraction of agents excluded from the workforce is 50%, output per worker is 88% of the one with no gender inequality because of the gender gap effect on productivity. In other words, if the gender gap is the highest possible one, the loss in output per worker is about 12%. The loss in worker wages, on the other hand, is slightly higher since there is a workers' supply increase effect on top of the productivity effect.

When interpreting the results, it is important to keep in mind that we are assuming that men and women have exactly the same talent distribution. This assumption may not be very accurate for some developing countries in which women have less education than men and, as a result, they are likely to be less skilled. These results, thus, are capturing the effects of gender inequality in access to entrepreneurship as well as access to those education programs that give the necessary skills to become entrepreneurs.

Also, it is important to note we are modelling the economy as if there was only one production sector, while in the real world there are many different sectors, probably with different parameters η, i.e. different returns to entrepreneurial ability. To the extent that gender gaps in entrepreneurship are stronger in some sectors than others, the economic losses

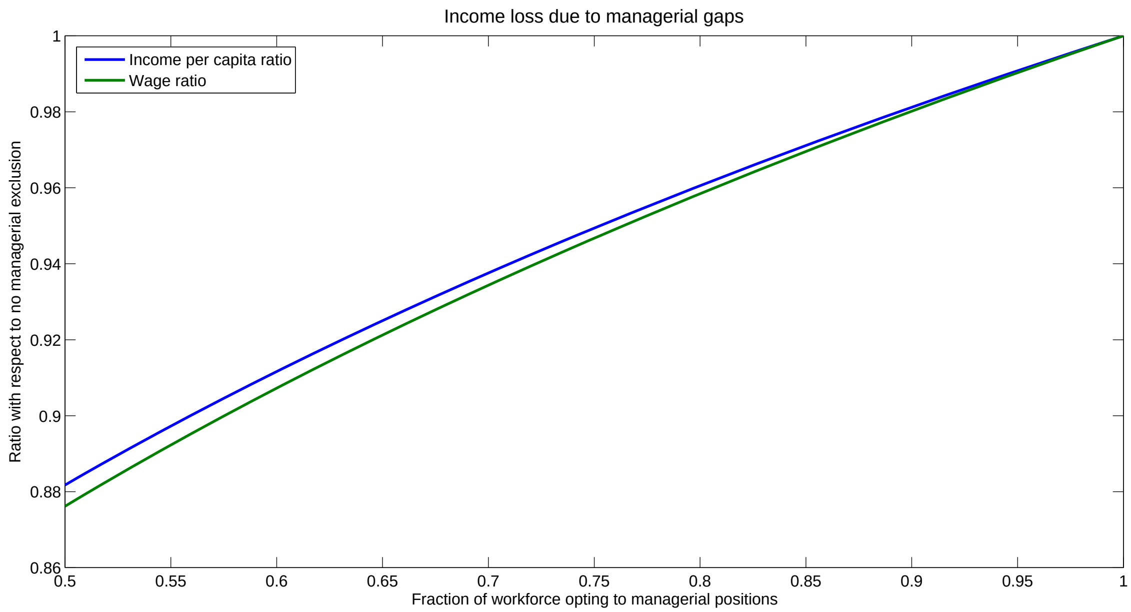


Figure 6: Effects of gender gaps in entrepreneurship

due to entrepreneurial gaps may be larger or smaller than computed here. If, for instance, entrepreneurial inequality is higher in sectors with a larger span of control of entrepreneurs, we would be underestimating the true effects of gender inequality.

3.4 Quantitative effects of labour force gaps

If we simulate the model with a fraction of the population excluded from the workplace, we get that output per worker increases because there are diminishing returns to scale to labour keeping the stock of capital constant. However, income per capita obviously decreases since fewer people actually work. As Figure 7) shows, the larger the gender gap in the labour force - i.e. the further we move to the left on the horizontal axis, the higher is the income loss. When 50% of the population are excluded from the labour force, for instance, the income per capita loss with respect to the no-exclusion case is almost 40%.

3.5 Robustness analysis

In this subsection we compare the parameter values used in previous subsections to other values considered in the literature, in order to check the sensitivity of our results to alternative specifications. Table (2) below summarizes the results.

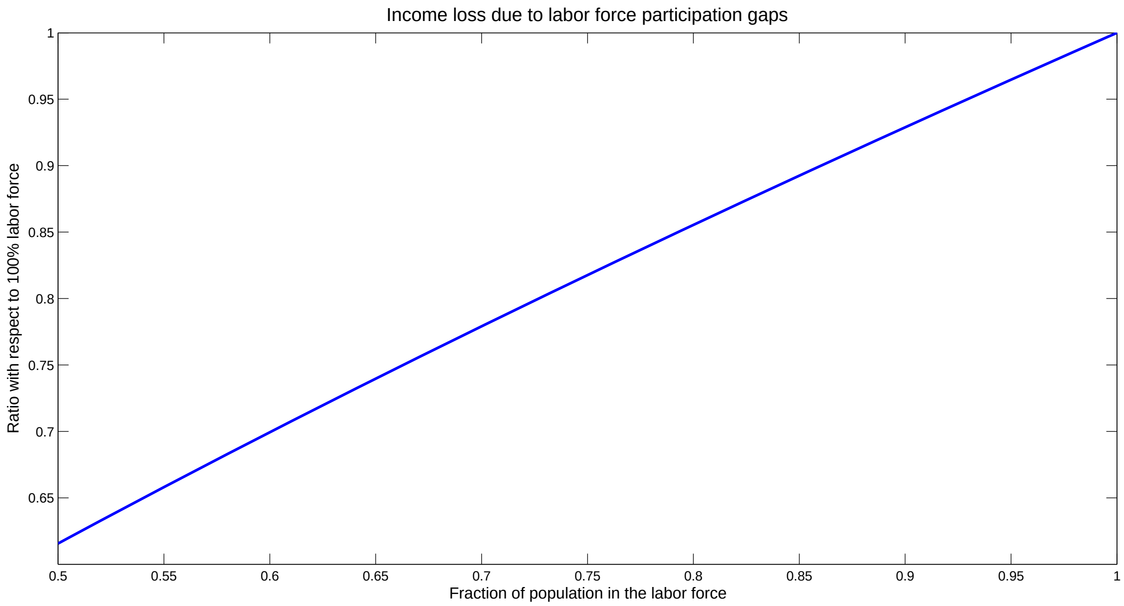


Figure 7: Effects of gender gaps in labour force participation

Span-of-control parameter η. To our knowledge, Bohacek and Rodriguez-Mendizabal (2011) and Bhattacharya et al. (2011) are the only available studies providing estimates for the span-of-control parameter. Bohacek and Rodriguez-Mendizabal (2011) use a value of $\eta = 0.912$ for the entrepreneurial control parameter since this is the value estimated by Burnside (1996) using output data. In our setting, this leads to $\rho = 10.36$ in order to satisfy Zipf's law, and to $\bar{z} = 1.94 * B$, in order to match the fraction of employers in the data. In this case, we get that the loss in output per worker due to gender gaps in entrepreneurship is only around 4%. On the other hand, there is no change in the income per capita loss due to gender inequality in labour force participation because the loss associated to the gap in labour force participation is independent of the span of control associated with managers. Bhattacharya et al. (2011), on the other hand, use $\eta = 0.76$, which imply $\rho = 3.17$ and $\bar{z} = 14 * B$ in our setting. This parametrization leads to a loss in output per worker - and income per capita - due to entrepreneurial gender gaps of almost 15%. So our benchmark parameter values can be interpreted as a middle point with respect to these two alternative parametrizations.

Capital share parameter, α. Setting α equal to 0.375 in order to make $\alpha\eta$ equal to 30% implicitly assumes that the entrepreneurs' income is part of the labour income in the national accounts. Another possibility is to consider it part of the capital income, which implies making

Table 2: Robustness analysis

	Baseline simulation	$\eta$from Bohacek, Rodriguez (2011)	$\eta$from Battacharya et al. (2011)	Capital share $\alpha\eta + (1-\eta) = 0.3$
ρ	4	10.4	3.17	4
η	0.8	0.912	0.76	0.8
$\bar{z}/B$	7.8	1.94	14	7.2
Productivity loss due to entrepreneurs' gender gaps	0.12	0.045	0.145	0.11
Income loss due to labor force gender gaps	0.38	0.38	0.38	0.46

$\alpha\eta + (1-\eta) = 0.3$. Using $\eta = 0.8$, $\rho = 4$ implies $\alpha = 0.125$. Assuming $\bar{z} = 7.8 * B$as in the baseline simulation, we get a larger loss in income per capita due to gender gaps in labour force participation, and a slightly smaller loss in output per worker due to gender gaps in entrepreneurs, 11% instead of 12%.

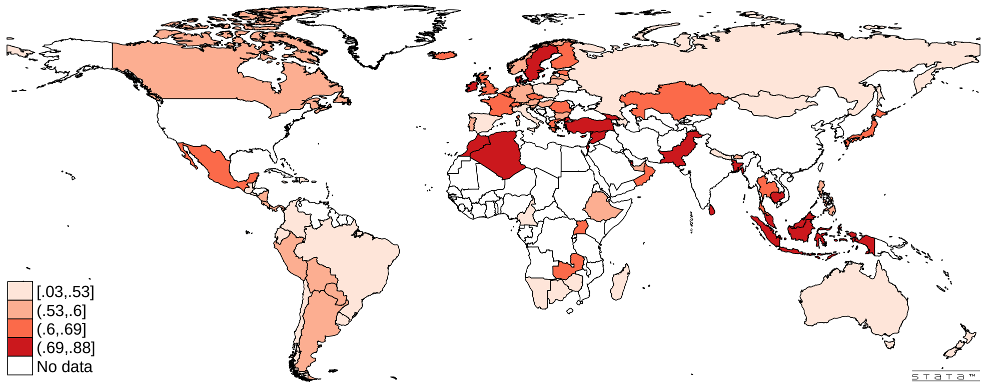
4 Cross-country results

In this section, we use data on employers as a proxy for managers and labour force participation by gender to estimate the effects of labour market gender gaps on the income per capita of 92 countries for the latest available year.[12] The data on employers is from the International Labour Organization (Table 3 - Status in Employment, by sex) for the latest available year, which includes both developed and developing countries, while the data on labour force participation is from the World Bank's World Development Indicators. Specifically, the entrepreneurs' gender gap is defined as

$$MGG_i \equiv 1\text{-}\frac{\text{female employers over female employment in country } i}{\text{male employers over male employment in country } i},$$

[12] An alternative would be to use data on the variable self-employed, which is defined as the sum of employers and own-account workers. The problem with this variable is that a manager in our model is a person with a relatively high talent which then decides to hire several workers, so the model is not meant to capture the decision to work as a self-employed. If we use data on self-employed to measure entrepreneurship we would obtain lower gender gaps and much smaller income losses, especially for developing countries.

Figure 8: Entrepreneurship gender gap world map



and the labour force force gender gap is defined as

$$LFGG_i \equiv 1\text{-}\frac{\text{female labor force participation in country } i}{\text{male labor force participation in country } i}.$$

It is important to note that we compute the gender gap in entrepreneurship by taking the difference in the fraction of entrepreneurs among male and females and normalizing it using the difference in labour force participation between males and females.[13] To make the reading easier, we will refer to these variables as the percentage of women *excluded* from entrepreneurship and from the labour force, respectively, but we do not know whether this *exclusion* is voluntary or not. In other words, if men and women were identical in all dimensions, these two variables could be interpreted as the percentage of women who are excluded from entrepreneurship and the labour force respectively, but there may be reasons other than discrimination that explain these gaps.

Figures (8) and (9) show the value of the entrepreneurship and labour force participation gender gaps in all countries included in the sample. We can see that the countries with the highest gender gaps are located in the Middle East, North Africa, South Asia, and to a lesser extent, in Europe and Latin America. The total income loss caused by these gender gaps in each country of the sample is shown in figure (10).

Tables (3) and (4) show the results for different groups of countries.[14] The variable in

[13]Note that, because of the way we calculate the gender gaps in entrepreneurship, we could then have negative gender gaps (i.e. larger fraction of working women in employer positions), in which case we just proceed as if there was no gap at all; in our sample, this actually only happens in two countries, Fiji and Zaire.

[14]Appendix D shows the results of our simulations for every country included in the sample. Notable

Figure 9: Labour force participation gender gap world map

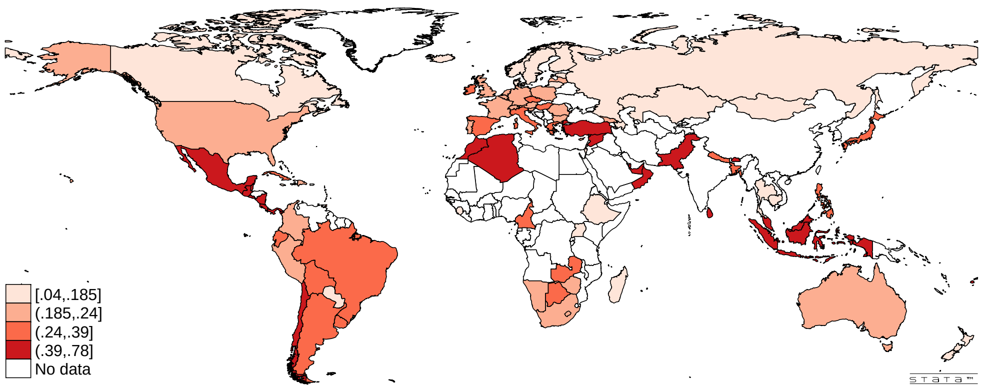


Figure 10: World map of total income loss due to gender gap

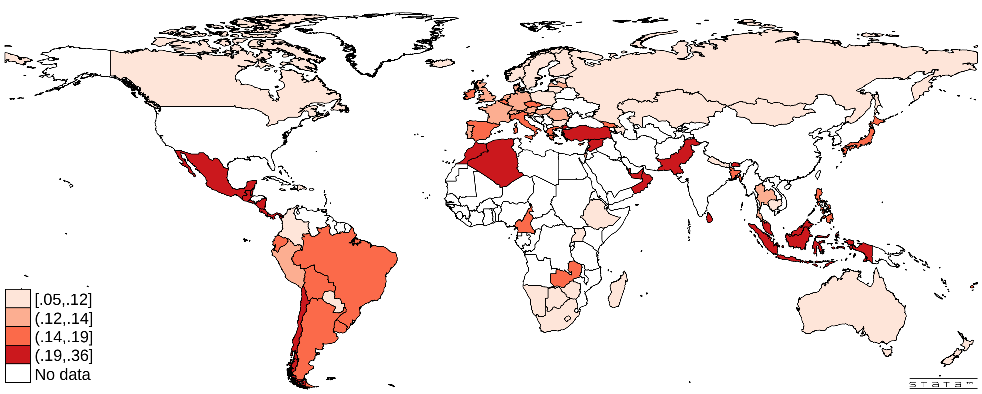


Table 3: Income loss due to gender gaps, by income groups

	Number countries	Entrepreneurs' gender gap	Labour force part. gender gap	Income loss (total)	Income loss (entrepr.)	Income loss (lfp)
Low Income	11	0.542	0.259	0.145	0.051	0.094
Lower-Middle	25	0.559	0.340	0.175	0.051	0.124
Upper-Middle	23	0.549	0.337	0.172	0.050	0.122
High Income	30	0.632	0.249	0.150	0.061	0.089

column 3, *Entrepreneurs' gender gap,* is defined as the gap between males and females in the fraction of entrepreneurs in the working population, while the variable *Labour force participation gender gap* is defined as the gap in labour force participation between men and women. The variable *Income loss (total)* gives us the percentage loss in income per capita due to gender gaps in the labour market in the form of entrepreneurs and labour force participation. The variable *Income loss (entrepr.)* gives the percentage loss in income per capita due to the gender gap in entrepreneurship, while the variable *Income loss (lfp)* gives the percentage loss in income per capita due to the gender gap in labour force participation, so it is the difference between *Income loss (total)* and *Income loss (entrepr.)*.

Table (3) displays the average of these five variables for four groups of countries, according to their income level. In low income countries, for example, more than 50% of women are excluded from entrepreneurship which, according to our calculations, creates an average income loss of 5.1%; the percentage of women that are excluded from the labour force, on the other hand, is 26% which generates an output loss of 9.4%. The sum of these two output losses gives the total income loss due to gender gaps in the labour market, which is equal to 14.5% for low income countries. Perhaps somewhat surprisingly, these losses are non-monotonic across income groups, being indeed higher for lower and upper-middle income countries than for low or high income ones.

In table (4) we split our sample of countries in geographic regions (*East Asia and Pacific*, *Europe and Central Asia*, *Latin America and the Caribbean*, *Middle East and North Africa*, *South Asia*, and *Sub-Saharan Africa*). We find some remarkable differences in gender gaps, which lead to some significant differences in the implied income losses. The region with the largest income loss due to gender gaps is *Middle East and North Africa* where, according to our estimates, the entrepreneurs' gap is 76.6% whereas the labour participation gap is

omissions are the United states, China and India for which the ILO does not provide data on the number of employers. In a companion paper, Cuberes and Teignier (2012) calculate the productivity costs of gender gaps in the labour market in India, using a unique dataset.

Table 4: Income loss due to gender gaps, by regional groups

	Number countries	Entrepreneurs' gender gap	Labour force part. gender gap	Income loss (total)	Income loss (entrepr.)	Income loss (lfp)
East Asia & Pacific	12	0.568	0.276	0.153	0.054	0.099
Europe & Central Asia	33	0.606	0.231	0.141	0.058	0.083
Latin America & Caribbean	20	0.531	0.333	0.168	0.048	0.120
Middle East & North Africa	8	0.766	0.533	0.265	0.068	0.197
South Asia	5	0.586	0.475	0.225	0.051	0.174
Sub-Saharan Africa	10	0.444	0.220	0.120	0.042	0.079

53.3%. These differences between men and women generate an income loss of 6.9% and 19.7% respectively, so a total income loss of 26.5. *South Asia* has the second largest income losses due to gender gaps (22.5%), mostly due to its large gender gap in labour force participation (47.5%), while Europe and Central Asia and Sub-Saharan Africa present the lowest income losses, 14.1% and 12%, respectively.

5 Conclusion

This paper quantifies the effects of gender gaps in the labour market on aggregate productivity and income per capita. Our numerical results show that the gender gap in entrepreneurs has significant effects on resource allocation and aggregate productivity, while the gap from formal employment does not affect productivity but has large effects on income per capita. Specifically, if no women works as an entrepreneur, output per worker would drop by around 12%, while if the labour force participation of women was zero, income per capita would decrease by almost 40%.

When we do the country-by-country analysis, we find that gender gaps do not differ much across income groups, but there are very important differences across geographical regions. To sum up our results, gender inequality in low income countries creates an average loss of almost 14% in GDP per capita, which can be decomposed in losses due to gaps in entrepreneurs (5%) and losses due labour force participation gaps (9%). The region with the largest income loss due to gender inequality is Middle East and North Africa, with a total income loss of 27%, 7% coming from entrepreneurs' gaps and 20% from labour force participation. South Asia experiences the second largest income loss due to gender inequality (23%).

Obviously, more work needs to be done to interpret these differences across income and geographical levels. It is clear that our measure of entrepreneurs and total labour gaps may reflect differences in the labour market (due to both demand and supply factors) as well as cultural factors. The goal of this exercise is to provide some quantitative estimates of the magnitude of these gaps and of their impact on the aggregate economy, not to identify discrimination against women per se.

In terms of future research, we are considering to extend this framework in two different directions. First, introducing a household production sector leading to a division of labour between husbands and wives, as in Becker (1981), which could explain some of the observed differences in labour participation and access to entrepreneurship between males and females without interpreting it as gender discrimination. Second, we plan to make a dynamic version of the span-of-control model presented in this paper (see, for instance, Caselli and Gennaioli 2012), which would make possible to quantify the effects of gender gaps on capital accumulation and economic growth.

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A Derivation of functions $H(z,w)$ and $G(z,w)$

A.1 Three equilibrium conditions in unknowns (z,w,r)

If we substitute equations (5) and (6) into equations (7), (8) and (9), we get the three equilibrium equations which determine the three unknowns (z,w,r). First, when we replace (5) and (6) into (7),

$$\begin{aligned} & z\left(\left[z\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right]^{\frac{\alpha}{1-\eta}}\left[z\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right]^{\frac{1-\alpha}{1-\eta}}\right)^{\eta} \\ & -w\left[z\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right]^{\frac{1}{1-\eta}}-r\left[z\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right]^{\frac{1}{1-\eta}} \quad = \quad w \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} & \left[z\left(\frac{r^{\alpha}w^{1-\alpha}}{\eta\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{-\eta}\right]^{\frac{1}{1-\eta}} \quad -\left[z\alpha\eta\left(\left(\tfrac{\alpha}{1-\alpha}\right)^{1-\alpha}r^{\alpha}w^{1-\alpha}\right)^{-\eta}\right]^{\frac{1}{1-\eta}} \\ & \qquad -\left[z(1-\alpha)\eta\left(\left(\tfrac{1-\alpha}{\alpha}\right)^{\alpha}r^{\alpha}w^{1-\alpha}\right)^{-\eta}\right]^{\frac{1}{1-\eta}} \quad = w \end{aligned}$$

$\Leftrightarrow$

$$z^{\frac{1}{1-\eta}}\left(r^{\alpha}w^{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}\Phi = w \tag{22}$$

where Φ is defined as

$$\Phi \equiv \left(\eta\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)^{\frac{\eta}{1-\eta}}-\left(\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\right)^{\frac{1}{1-\eta}}-\left(\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\right)^{\frac{1}{1-\eta}}.$$

Second, when we substitute (6) into (8),

$$\int_{z}^{\bar{z}}\left[x\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right]^{\frac{1}{1-\eta}}d\Gamma(x)=\frac{K}{N}$$

$\Leftrightarrow$

$$\left(\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right)^{\frac{1}{1-\eta}}\Omega\int_{z}^{\bar{z}}x^{\frac{1}{1-\eta}}d\Gamma(x)=\frac{K}{N} \tag{23}$$

where Ω is defined as

$$\Omega \equiv \left(\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\right)^{\frac{1}{1-\eta}}.$$

Third, when we substitute (5) into (9),

$$\int_{z}^{\bar{z}}\left[x\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right]^{\frac{1}{1-\eta}}d\Gamma(x)=\Gamma(z)$$

$\Leftrightarrow$

$$\left(\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right)^{\frac{1}{1-\eta}}\Psi\int_{z}^{\bar{z}}x^{\frac{1}{1-\eta}}d\Gamma(x)=\Gamma(z) \tag{24}$$

where $\Psi$is defined as

$$\Psi \equiv \left(\eta\left(1-\alpha\right)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\right)^{\frac{1}{1-\eta}}.$$

A.2 Functions $H\left(z,w\right)$ and $G\left(z,w\right)$

Using then equations (23) and (24) we can write the equilibrium interest rate r as a function of the other two unknowns:

$$r=\frac{\alpha}{1-\alpha}\frac{\Gamma\left(z\right)}{k}w.$$

If we then replace it in equations (22) and (24), the three equilibrium conditions in equations (22), (23) and (24) can be summarized in the two equations $G(z,w)=0$ and $H(z,w)=0$. First, when we substitute $r=\frac{\alpha}{1-\alpha}\frac{\Gamma(z)}{k}w$ into (24),

$$\left[x\eta(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}w^{\alpha\eta-1}\left(\frac{1-\alpha}{\alpha}\frac{k}{w\Gamma\left(z\right)}\right)^{\alpha\eta}\right]^{\frac{1}{1-\eta}}\int_{z}^{\bar{z}}x^{\frac{1}{1-\eta}}d\Gamma(x)=\Gamma(z)$$

$\Leftrightarrow$

$$H(z,w)\equiv\Gamma\left(z\right)^{\frac{1-\eta(1-\alpha)}{1-\eta}}-\left[\eta\left(1-\alpha\right)\frac{k^{\alpha\eta}}{w}\right]^{\frac{1}{1-\eta}}\int_{z}^{\bar{z}}x^{\frac{1}{1-\eta}}d\Gamma(x)=0,$$

Second, when we substitute into $r = \frac{\alpha}{1-\alpha}\frac{\Gamma(z)}{k}w$ into (22),

$$z^{\frac{1}{1-\eta}}\left(\left(\frac{\alpha}{1-\alpha}\frac{\Gamma(z)}{k}w\right)^{\alpha}w^{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}\Phi = w$$

$\Leftrightarrow$

$$G(z,w) \equiv w^{\frac{1}{1-\eta}} - \Phi z^{\frac{1}{1-\eta}}\left(\frac{\alpha}{1-\alpha}\frac{\Gamma(z)}{k}\right)^{\frac{-\alpha\eta}{1-\eta}} = 0.$$

A.3 Slopes of $H(z,w) = 0$ and $G(z,w) = 0$

From equation $H(z,w) = 0$, we can obtain the following two partial derivatives:

$$\frac{\partial H(z,w)}{\partial w} = \frac{w^{\frac{\eta-2}{1-\eta}}}{1-\eta}\left[\eta(1-\alpha)k^{\alpha\eta}\right]^{\frac{1}{1-\eta}}\int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}}d\Gamma(x) > 0$$

$$\frac{\partial H(z,w)}{\partial z} = \left(\frac{1-\eta(1-\alpha)}{1-\eta}\Gamma(z)^{\frac{\eta\alpha}{1-\eta}} + \left[\eta(1-\alpha)\frac{k^{\alpha\eta}}{w}\right]^{\frac{1}{1-\eta}}z^{\frac{1}{1-\eta}}\right)\Gamma'(z) > 0$$

which imply that $\frac{dw}{dz}\mid_{H(z,w)=0} = -\frac{\frac{\partial H(z,w)}{\partial z}}{\frac{\partial H(z,w)}{\partial w}} < 0.$

From equation $G(z,w) = 0$, we get the following two partial derivatives:

$$\begin{aligned}\frac{\partial G(z,w)}{\partial w} &= \frac{1}{1-\eta}w^{\frac{\eta}{1-\eta}} > 0\\ \frac{\partial G(z,w)}{\partial z} &= \frac{\Phi}{1-\eta}z^{\frac{1}{1-\eta}}\Gamma(z)^{\frac{-\alpha\eta}{1-\eta}}\left(\frac{\alpha}{1-\alpha}\frac{1}{k}\right)^{\frac{-\alpha\eta}{1-\eta}}\left(-\frac{1}{z} + \alpha\eta\frac{\Gamma'(z)}{\Gamma(z)}\right)\end{aligned}$$

Thus, if $\alpha\eta\frac{z\Gamma'(z)}{\Gamma(z)} < 1$, then $\frac{\partial G(z,w)}{\partial z} < 0$ and $\frac{dw}{dz}\mid_{G(z,w)=0} = -\frac{\frac{\partial G(z,w)}{\partial z}}{\frac{\partial G(z,w)}{\partial w}} > 0$. On the other hand, if $\alpha\eta\frac{z\Gamma'(z)}{\Gamma(z)} > 1$, then $\frac{\partial G(z,w)}{\partial z} > 0$ and $\frac{dw}{dz}\mid_{G(z,w)=0} = -\frac{\frac{\partial G(z,w)}{\partial z}}{\frac{\partial G(z,w)}{\partial w}} < 0.$[15]

[15]For the talent distribution assumed in Section 4, that is the Pareto function, $\frac{z\Gamma'(z)}{\Gamma(z)} = \frac{\rho B^{\rho}z^{-\rho}}{1-B^{\rho}z^{-\rho}} = \frac{\rho}{\left(\frac{z}{B}\right)^{\rho}-1}$, so the condition $\alpha\eta\frac{z\Gamma'(z)}{\Gamma(z)} < 1$ is more likely to be satisfied for large values of z.

B Derivations of functions $\widetilde{H}(z,w)$ and $\widetilde{G}(z,w)$

B.1 Functions $\widetilde{H}(z,w)$ and $\widetilde{G}(z,w)$

If we proceed as before and substitute equations (5) and (6) into equations (7), (15) and (16), we get three equilibrium equations which determine the three unknowns (z,w,r). These equations can then be summarized in two equations in the two unknowns (z,w):

$$\widetilde{H}(z,w) \equiv 1-\theta+\tilde{\Gamma}(z) - \left(\frac{\eta(1-\alpha)}{w}\right)^{\frac{1}{1-\eta}} \left(\frac{1-\theta+\tilde{\Gamma}(z)}{k}\right)^{\frac{-\alpha\eta}{1-\eta}} \int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\tilde{\Gamma}(x) = 0$$

$$\widetilde{G}(z,w) \equiv w^{\frac{1}{1-\eta}} - \Phi z^{\frac{1}{1-\eta}} \left(\frac{\alpha}{1-\alpha}\frac{1-\theta+\tilde{\Gamma}(z)}{k}\right)^{\frac{-\alpha\eta}{1-\eta}} = 0$$

B.2 Slopes of $\widetilde{H}(z,w)=0$ and $\widetilde{G}(z,w)=0$

Taking the partial derivatives of $\widetilde{H}(z,w)$ with respect to z and w, we can easily see that $\frac{\partial \widetilde{H}(z,w)}{\partial w} > 0$ and $\frac{\partial \widetilde{H}(z,w)}{\partial z} > 0$. It is also straightforward to see that $\frac{\partial \widetilde{G}(z,w)}{\partial w} > 0$, while the sign of $\frac{\partial \widetilde{G}(z,w)}{\partial z}$ depends on the sign of $1-\alpha\eta\frac{z\tilde{\Gamma}'(z)}{1/\theta-1+\tilde{\Gamma}(z)}$:

$$\begin{aligned}\frac{\partial \widetilde{G}(z,w)}{\partial z} &= \frac{1}{1-\eta}\Phi z^{\frac{\eta}{1-\eta}}\left(\frac{\alpha}{1-\alpha}\frac{1}{k}\right)^{\frac{-\alpha\eta}{1-\eta}}\left(1-\theta+\tilde{\Gamma}(z)\right)^{\frac{-\alpha\eta}{1-\eta}} \\ &\quad -\frac{\alpha\eta}{1-\eta}\Phi z^{\frac{1}{1-\eta}}\left(\frac{\alpha}{1-\alpha}\frac{1}{k}\right)^{\frac{-\alpha\eta}{1-\eta}}\left(1-\theta+\tilde{\Gamma}(z)\right)^{\frac{-\alpha\eta}{1-\eta}-1}\tilde{\Gamma}'(z)\end{aligned}$$

$\Rightarrow$

$$sign\left(\frac{\partial \widetilde{G}(z,w)}{\partial z}\right) = sign\left(1-\alpha\eta\frac{z\tilde{\Gamma}'(z)}{1-\theta+\tilde{\Gamma}(z)}\right)$$

Therefore, $\frac{dw}{dz}\mid_{\widetilde{H}(z,w)=0} = -\frac{\frac{\partial \widetilde{H}(z,w)}{\partial z}}{\frac{\partial \widetilde{H}(z,w)}{\partial w}} < 0$, which implies that $\widetilde{H}(z,w)$ is downward sloping in the (z,w)-diagram. And, if $\alpha\eta\frac{z\Gamma'(z)}{1/\theta-1+\Gamma(z)} < 1$, $\frac{dw}{dz}\mid_{\widetilde{G}(z,w)=0} = -\frac{\frac{\partial \widetilde{G}(z,w)}{\partial z}}{\frac{\partial \widetilde{G}(z,w)}{\partial w}} > 0$, so $\widetilde{G}(z,w)$ is upward sloping.

B.3 Effects of changes in θ

To analyse the effect of a change in θ on the equilibrium talent threshold z, we can use $\widetilde{H}(z,w)$ and $\widetilde{G}(z,w)$ to obtain a new equilibrium condition, $F(\cdot)$ in terms of the unknown z:

$$\widetilde{F}(z) \equiv -\theta\left(\eta\left(1-\alpha\right)\right)^{\frac{1}{1-\eta}} \int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\Gamma(x) + z^{\frac{1}{1-\eta}}\left(1-\theta+\theta\Gamma\left(z\right)\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{-\alpha\eta}{1-\eta}} \Phi = 0$$

We can now easily see the sign of the partial derivatives of $\widetilde{F}(z,w)$:

$$\frac{\partial \widetilde{F'}(z)}{\partial z} = \theta\left(\eta\left(1-\alpha\right)\right)^{\frac{1}{1-\eta}} z^{\frac{1}{1-\eta}}\Gamma\left(z\right) + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{-\alpha\eta}{1-\eta}} \Phi\left[\frac{1}{1-\eta} z^{\frac{\eta}{1-\eta}}\left(1-\theta+\theta\Gamma\left(z\right)\right) + \theta\Gamma'\left(z\right) z^{\frac{1}{1-\eta}}\right] > 0$$

$$\frac{\partial \widetilde{F}(z)}{\partial \theta} = -\left(\eta\left(1-\alpha\right)\right)^{\frac{1}{1-\eta}} \int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\Gamma(x) + z^{\frac{1}{1-\eta}}\left(\Gamma\left(z\right)-1\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{-\alpha\eta}{1-\eta}} \Phi < 0.$$

Hence, there is a positive relation between z and θ:

$$\frac{dz}{d\theta}\,|_{\widetilde{F}(z)=0} = -\frac{\frac{\partial \widetilde{F}(z)}{\partial \theta}}{\frac{\partial \widetilde{F}(z)}{\partial z}} > 0.$$

C Results for Pareto talent distribution

As explained in Section 3, we assume that the entrepreneurial talent of agents is distributed according to the bounded Pareto distribution

$$\Gamma\left(x\right) = \frac{1-B^{\rho}x^{-\rho}}{1-B^{\rho}\bar{z}^{-\rho}}, \text{ for } 0 \leq x \leq \bar{z}.$$

where $\overline{z}$ denotes the upper bound of the distribution. Using this functional form, we get that

$$\int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} d\Gamma(x) = \int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}} \frac{\rho B^{\rho} x^{-\rho-1}}{1-B^{\rho}\bar{z}^{-\rho}} dx = \int_{z}^{\bar{z}} \rho B^{\rho} \frac{x^{\frac{1}{1-\eta}-\rho-1}}{1-B^{\rho}\bar{z}^{-\rho}} dx = \frac{\rho B^{\rho}}{\frac{1}{1-\eta}-\rho}\left[\frac{\bar{z}^{\frac{1}{1-\eta}-\rho} - z^{\frac{1}{1-\eta}-\rho}}{1-B^{\rho}\bar{z}^{-\rho}}\right].$$

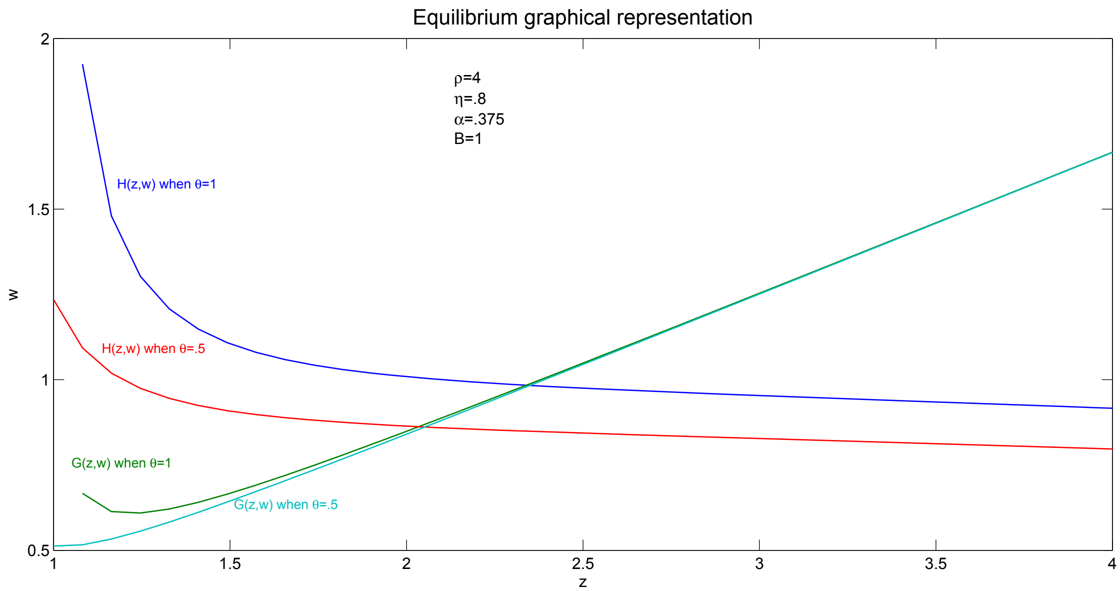

Figure 11: Effects of gender gaps in labour force participation

As a result, the market clearing conditions in equations (23) and (24) become:

$$\left[\eta\alpha\left(\frac{1-\alpha}{\alpha}\right)^{\eta(1-\alpha)}\frac{r^{\eta(1-\alpha)-1}}{w^{\eta(1-\alpha)}}\right]^{\frac{1}{1-\eta}}\frac{\rho B^{\rho}}{\frac{1}{1-\eta}-\rho}\left[\frac{\bar{z}^{\frac{1}{1-\eta}-\rho}-z^{\frac{1}{1-\eta}-\rho}}{1-B^{\rho}\bar{z}^{-\rho}}\right]=\frac{K}{N} \tag{25}$$

$$\left[\eta\left(1-\alpha\right)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha\eta}\frac{w^{\alpha\eta-1}}{r^{\alpha\eta}}\right]^{\frac{1}{1-\eta}}\frac{\rho B^{\rho}}{\frac{1}{1-\eta}-\rho}\left[\bar{z}^{\frac{1}{1-\eta}-\rho}-z^{\frac{1}{1-\eta}-\rho}\right]=1-B^{\rho}z^{-\rho} \tag{26}$$

And output per capita is equal to

$$\begin{aligned}\frac{Y}{N} &= \int_{z}^{\bar{z}} x\left(k(x)^{\alpha}n(x)^{1-\alpha}\right)^{\eta}d\Gamma\left(x\right)=\left[\eta\frac{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{r^{\alpha}w^{1-\alpha}}\right]^{\frac{\eta}{1-\eta}}\int_{z}^{\bar{z}} x^{\frac{1}{1-\eta}}d\Gamma(x)\\ &= \left[\eta\frac{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{r^{\alpha}w^{1-\alpha}}\right]^{\frac{\eta}{1-\eta}}\frac{\rho B^{\rho}}{\frac{1}{1-\eta}-\rho}\left[\frac{\bar{z}^{\frac{1}{1-\eta}-\rho}-z^{\frac{1}{1-\eta}-\rho}}{1-B^{\rho}\bar{z}^{-\rho}}\right].\end{aligned}$$

Under the Pareto talent distribution and the parametrization described in Section 3, the function $\widetilde{H}\left(z,w\right)$ is downward sloping and the function $\widetilde{G}\left(z,w\right)$ is upward sloping for the relevant range of z. Figure (11) plots them for θ equal to 1 and equal 0.5.

D Country-by-country results[16]

	Year	Income group	Region group	Entrepr. gender gap	LFP gender gap	Income loss	Loss due to entrepr. GG	Loss due to LFP GG
Algeria	2004	2	MENA	0.7519	0.5603	0.2705	0.0650	0.2055
Argentina	2006	3	LAC	0.5576	0.3474	0.1752	0.0502	0.1250
Australia	2007	4	EAP	0.3626	0.2028	0.1050	0.0329	0.0721
Austria	2007	4	EUCA	0.5455	0.2309	0.1336	0.0513	0.0823
Azerbaijan	2007	2	EUCA	0.3220	0.1549	0.0843	0.0294	0.0549
Bangladesh	2005	1	SA	0.6966	0.3407	0.1879	0.0654	0.1225
Barbados	2004	3	LAC	0.5976	0.1557	0.1139	0.0587	0.0552
Belgium	2007	4	EUCA	0.6393	0.2383	0.1465	0.0616	0.0850
Belize	2005	3	LAC	0.4673	0.4457	0.2011	0.0393	0.1618
Bhutan	2005	1	SA	0.5790	0.5383	0.2451	0.0481	0.1971
Bolivia	2002	2	LAC	0.5974	0.2488	0.1454	0.0566	0.0888
Botswan	2003	3	SSA	0.2888	0.2594	0.1178	0.0251	0.0927
Brazil	2006	2	LAC	0.5213	0.2878	0.1507	0.0476	0.1031
Bulgari	2007	2	EUCA	0.5821	0.1875	0.1228	0.0562	0.0666
Cambodia	2001	1	EAP	0.7057	0.0847	0.1033	0.0735	0.0298
Cameroo	2001	2	SSA	0.4661	0.3143	0.1543	0.0415	0.1128
Canada	2007	4	NAM	0.5867	0.1438	0.1086	0.0577	0.0509
Chile	2007	3	LAC	0.5036	0.4583	0.2091	0.0425	0.1666
Colombia	2007	2	LAC	0.5296	0.1949	0.1196	0.0503	0.0693
Costa Rica	2007	3	LAC	0.5383	0.4544	0.2111	0.0460	0.1651
Cyprus	2007	4	MENA	0.8346	0.2329	0.1685	0.0855	0.0830
Czech Rep.	2007	3	EUCA	0.6154	0.2500	0.1478	0.0586	0.0892
Denmark	2007	4	EUCA	0.7350	0.1465	0.1273	0.0754	0.0518

[16] Variable 3: World Bank income group (1: Low Income, 2: Lower-Middle, 3: Upper-Middle, 4: High Income); variable 4: World Bank region (EAP: East Asia and Pacific, EUCA: Europe and Central Asia, LAC: Latin America and the Caribbean, MENA: Middle East and North Africa, NAM: North America and Mexico, SA: South Africa, SSA: Sub-Saharan Africa); variable 5: gender gap in entrepreneurs (fraction of women excluded from entrepreneurship relative to men; source: own calculations from ILO data); variable 6: gender gap in labour force participation (fraction of women excluded from the labour force relative to men; source: own calculations from WDIdata); variable 7: total income loss due to gender gaps (source: own results); variable 8: income loss due to entrepreneurs' gender gap (source: own results); variable 9: income loss due to labour force participation gender gap (source: own results).

	Year	Income group	Region group	Entrepr. gender gap	LFP gender gap	Income loss	Loss due to entrepr. GG	Loss due to LFP GG
Dominican Rep	2007	2	LAC	0.4625	0.2260	0.1231	0.0426	0.0805
Ecuador	2006	2	LAC	0.4730	0.3418	0.1646	0.0417	0.1229
El Salvaldor	2006	2	LAC	0.5161	0.4177	0.1958	0.0445	0.1513
Estonia	2007	3	EUCA	0.6320	0.1662	0.1214	0.0624	0.0589
Ethiopi	2006	1	SSA	0.5938	0.1330	0.1058	0.0588	0.0470
Fiji Is.	2005	2	EAP	0	0.5025	0.1834	0	0.1834
Finland	2007	4	EUCA	0.6224	0.1138	0.1027	0.0626	0.0402
France	2007	4	EUCA	0.6505	0.1968	0.1338	0.0639	0.0699
Georgia	2005	2	EUCA	0.7825	0.2397	0.1641	0.0786	0.0855
Germany	2007	4	EUCA	0.5717	0.2227	0.1337	0.0543	0.0793
Greece	2007	4	EUCA	0.6337	0.3338	0.1786	0.0586	0.1200
Guatema	2002	2	LAC	0.3387	0.5890	0.2424	0.0258	0.2167
Hong Kong, China	2007	4	EAP	0.7355	0.2371	0.1574	0.0729	0.0846
Hungary	2007	3	EUCA	0.5515	0.2593	0.1440	0.0514	0.0926
Iceland	2007	4	EUCA	0.6392	0.1100	0.1035	0.0646	0.0388
Indonesia	2007	2	EAP	0.7109	0.4233	0.2180	0.0647	0.1533
Ireland	2007	4	EUCA	0.6961	0.2611	0.1608	0.0675	0.0933
Israel	2007	4	MENA	0.7755	0.1617	0.1374	0.0801	0.0573
Italy	2007	4	EUCA	0.5294	0.3623	0.1775	0.0470	0.1306
Jamaica	2006	2	LAC	0.4884	0.0672	0.0718	0.0481	0.0236
Japan	2007	4	EAP	0.6853	0.3333	0.1841	0.0643	0.1198
Kazakhstan	2004	2	EUCA	0.6198	0.1480	0.1138	0.0615	0.0524
Latvia	2007	3	EUCA	0.5474	0.2232	0.1312	0.0517	0.0795
Lithuania	2007	3	EUCA	0.5887	0.1639	0.1156	0.0575	0.0581
Luxembourg	2007	4	EUCA	0.5612	0.2563	0.1440	0.0525	0.0915
Madagascar	2005	1	SSA	0.3042	0.0562	0.0485	0.0287	0.0197
Malaysia	2007	3	EAP	0.7344	0.4413	0.2269	0.0668	0.1601
Malta	2007	4	EUCA	0.6880	0.5206	0.2497	0.0594	0.1903
Mauritius	2007	3	SSA	0.6859	0.4494	0.2244	0.0612	0.1632
Mexico	2007	3	LAC	0.6554	0.4850	0.2337	0.0570	0.1767
Mongolia	2003	1	EAP	0.4745	0.0508	0.0647	0.0469	0.0179

	Year	Income group	Region group	Entrepr. gender gap	LFP gender gap	Income loss	Loss due to entrepr. GG	Loss due to LFP GG
Morocco	2007	2	MENA	0.7957	0.6912	0.3221	0.0652	0.2568
Namibia	2004	2	SSA	0.3659	0.1900	0.1009	0.0334	0.0675
Nepal	2001	1	SA	0.0340	0.3091	0.1136	0.0027	0.1109
Netherlands	2007	4	EUCA	0.6309	0.2028	0.1336	0.0614	0.0721
New Zealand	2007	4	EAP	0.5131	0.1797	0.1126	0.0488	0.0638
Nicaragua	2006	2	LAC	0.5587	0.5782	0.2577	0.0453	0.2125
Norway	2007	4	EUCA	0.5690	0.1211	0.0990	0.0562	0.0428
Oman	2000	3	MENA	0.6881	0.7050	0.3165	0.0542	0.2623
Pakista	2007	1	SA	0.8368	0.7553	0.3496	0.0671	0.2825
Panama	2007	3	LAC	0.5675	0.4013	0.1951	0.0500	0.1451
Paraguay	2007	2	LAC	0.6040	0.1583	0.1155	0.0594	0.0561
Peru	2007	2	LAC	0.5757	0.2220	0.1338	0.0548	0.0790
Philippines	2007	2	EAP	0.5985	0.3775	0.1900	0.0538	0.1362
Poland	2007	3	EUCA	0.4646	0.2295	0.1246	0.0428	0.0818
Portugal	2007	4	EUCA	0.5782	0.2000	0.1266	0.0555	0.0711
Qatar	2004	4	MENA	0.8796	0.5582	0.2840	0.0793	0.2047
Romania	2007	3	EUCA	0.6073	0.2283	0.1395	0.0582	0.0814
Russian Federation	2007	3	EUCA	0.4086	0.1725	0.0991	0.0379	0.0612
Singapore	2007	4	EAP	0.6312	0.2947	0.1649	0.0593	0.1056
Slovenia	2007	4	EUCA	0.6028	0.1938	0.1273	0.0584	0.0689
South Africa	2007	3	SSA	0	0.2167	0.0771	0	0.0771
Spain	2007	4	EUCA	0.5345	0.3088	0.1593	0.0486	0.1108
Sri Lanka	2007	2	SA	0.7847	0.4293	0.2285	0.0728	0.1556
Sweden	2007	4	EUCA	0.7378	0.1217	0.1195	0.0765	0.0430
Switzerland	2007	4	EUCA	0.6175	0.2027	0.1320	0.0599	0.0721
Syrian Arab Rep	2001	2	MENA	0.8598	0.7783	0.3604	0.0686	0.2918
Thailand	2007	2	EAP	0.6593	0.1787	0.1288	0.0653	0.0634
Trinidad and Tobago	2005	3	LAC	0.5584	0.2842	0.1533	0.0516	0.1017
Turkey	2007	3	EUCA	0.8603	0.6514	0.3146	0.0735	0.2411
Uganda	2003	1	SSA	0.6309	0.1055	0.1010	0.0638	0.0372
United Arab Emirates	2005	4	MENA	0.5443	0.5796	0.2569	0.0439	0.2130

	Year	Income group	Region group	Entrepr. gender gap	LFP gender gap	Income loss	Loss due to entrepr. GG	Loss due to LFP GG
United Kingdom	2007	4	EUCA	0.6250	0.2029	0.1329	0.0608	0.0721
Uruguay	2007	3	LAC	0.5125	0.2973	0.1531	0.0465	0.1066
Zambia	2000	1	SSA	0.6663	0.2728	0.1613	0.0638	0.0976
Zimbabwe	2002	1	SSA	0.4417	0.2038	0.1133	0.0408	0.0725