Internalization, Clearing and Settlement, and Stock Market Liquidity$^1$

Hans Degryse$^2$, Mark Van Achter$^3$, and Gunther Wuyts$^4$

November 2010

$^1$We would like to thank Cecilia Caglio, Sarah Draus, Jérémie Lefebvre, Christine Parlour, Erik Theissen, Christian Voigt, Michel van der Wel, participants at the 2010 EFA meeting (Frankfurt), the 2010 European Summer Symposium in Financial Markets (Gerzensee), the 2010 Erasmus Liquidity Conference (Rotterdam), the 2010 Conference on the Industrial Organization of Securities Markets (Frankfurt), the 2010 AMF workshop (Paris), the 2010 AFM workshop (Amsterdam) and the 2010 KUL/UCL workshop (Brussels), as well as seminar participants at ESSEC Paris and at the Universities of Louvain, Mannheim, Stavanger and Tilburg for helpful comments and suggestions. Hans Degryse holds the TILEC-AFM chair on Financial Market Regulation. Gunther Wuyts gratefully acknowledges financial assistance from FWO-Flanders under contract G.0567.10.

$^2$Corresponding author: CentER - Tilburg University, P.O. Box 90153, NL-5000 LE Tilburg, The Netherlands. E-mail: h.degyse@uvt.nl.

$^3$Rotterdam School of Management, Erasmus University, Department of Finance, Burgermeester Oudlaan 50, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: mvanachter@rsm.nl.

$^4$University of Leuven, Faculty of Business and Economics, Department of Accounting, Finance and Insurance, Naamsestraat 69, 3000 Leuven, Belgium. E-mail: gunther.wuyts@econ.kuleuven.be.
Abstract

We study the link between stock market liquidity and post-trade costs (i.e. costs of clearing and settlement). Transactions feature low post-trade costs when clearing and settlement is internalized (e.g. buyer and seller originate from the same broker) and high post-trade costs when clearing and settlement is non-internalized. Traders are affiliated to either a large or a small broker, implying different probabilities of settlement internalization. We investigate two different fee structures imposed by the clearing and settlement agent (CSD). The first is a uniform fee on all trades (internalized and non-internalized) such that the CSD breaks even. The second features a CSD breaking even by charging the internalized and non-internalized trades their respective marginal cost. Our findings indicate that with a uniform fee, traders set quotes that are attractive for all counterparties (i.e. from both brokers). In turn, with a marginal-cost based fee, traders face the following trade-off. On the one hand, targeting all available counterparties (i.e. from both brokers) maximizes their probability of trading. However, to do so they need to set liquid quotes to attract traders with a high post-trade cost. On the other hand, targeting own-broker counterparties only allows them to set less liquid quotes but reduces the probability of trading. The optimal order submission strategies hinge on the magnitude of the marginal cost. Finally, we find that traders’ equilibrium strategies not necessarily correspond to the social optimum. Liquidity is not always a good indicator of welfare.

JEL Codes: G10, G15

Keywords: internalization, brokers, clearing and settlement, liquidity
1 Introduction

The organization of a financial market is an important determinant of its liquidity. Market microstructure, the process by which investors’ latent demands are ultimately translated into prices and volumes, has mainly focused on price formation and price discovery, and on the market design of financial systems. Next to the implicit transaction costs related to trading, explicit transaction costs such as commissions and post-trading infrastructure costs are of considerable importance. Data from Elkins/McSherry, for example, show that explicit transaction costs constitute about three quarters of the total transaction costs (see e.g. Domowitz and Steil (2002)). Further, according to the European Commission, costs of the post-trading infrastructure represent 10 to 20% of total post-trading transaction costs. While it is well-known that post-trade transaction costs are considerable, the market microstructure literature has not yet studied its impact on the liquidity of financial markets. This paper makes a first step to fill this void by analyzing the impact of differences in pricing of clearing and settlement services on stock market liquidity. These price differences stem from different degrees of internalization of order flow by the post-trade infrastructure. In particular, we study how the potential of internalizing trades affects participants’ willingness to supply and consume liquidity. Our paper thus studies how the pricing of back office activities influences the front office, i.e. the stock market liquidity.

Our research is motivated by the recent inclusion of internalization systems at several exchanges and the associated pricing schedules for trading services. Internalization occurs when buyer and seller originate from the same investment firm. This may happen when (i) the investment firm trades on its own account with his client (“client-to-house transaction”), (ii) two different counterparties trade through the same investment firm (“client-to-client transaction”), or (iii) transactions are carried out within the same investment firm (“house-to-house transaction”). In our setting, internalization reduces the fees payable to the post-trading infrastructure, i.e. the clearing and settlement fees. In the US, the DTCC (Depositary Trust and Clearing Corporation) which clears and settles trades of all exchanges observed that an increasing number of investment firms pre-netted their trades such that the order flow observed by the DTCC was not representative for the entire market. One of the recommendations the DTCC made was to adapt the clearing and settlement fees in order to reduce the economic incentive for using pre-netting (see e.g. DTCC (2003)). In Europe, with the implementation of MiFID, the Markets in Financial Instruments Directive, several trading systems have introduced features allowing to internalize clearing and settlement. First, regulated markets have created possibilities for internalization. The London Stock Exchange for example started its SETS internalizer in April 2007. SETS internalizer prevents on-book self-executions from passing through to clearing and settlement, thus avoiding post-trade infrastructure
fees. As a result, all order book executions where both sides of the trade originate from the same investment firm do not pass through to clearing and settlement. The tariff charged is 0.1 bp, which is 87.5% lower than the headline rate.\(^1\) Similarly, Euronext has created an algorithm that induces buy and sell orders originating from the same investment firm to avoid the clearing and settlement fees.\(^2\) Second, systematic internalizers allow to avoid clearing and settlement fees when the trades originate from the same investment firm. A recent report by Oxera (2009) argues that brokers internalize about 10% of their trades and they expect this to increase over time. Our paper addresses how internalization of clearing and settlement may affect stock market liquidity.

Our main insights can be summarized as follows. First, we find that explicit transaction costs such as clearing and settlement fees affect stock market liquidity. In general, higher clearing and settlement fees appear to increase stock market liquidity. The reasoning is that higher clearing and settlement fees induce more aggressive limit order pricing to induce incoming counterparties to trade. This is in line with empirical evidence of Berkowitz, Logue and Noser (1988) who find that larger explicit costs decrease implicit transaction costs. Second, internalization reduces clearing and settlement fees. Investment firms with larger market shares are therefore able to create some benefits as they allow to reduce clearing and settlement fees. However, our results show that when more trades can be internalized stock market liquidity decreases. The intuition behind this result is that an increase in internalization opportunities corresponds to a drop in explicit transaction costs and therefore reduces the aggressiveness of limit order prices. Third, when the clearing and settlement agent imposes marginal cost-based pricing (and thus charges the marginal cost for non-internalized trades and a zero cost for internalized trades) different equilibria result. With low costs of non-internalized trades observed liquidity is high as all traders announce prices attractive for counterparties originating from any investment firm. With intermediate costs of non-internalized trades traders originating from a large broker alter their strategy and quote less liquid quotes only attractive to counterparties of their own investment firm. In contrast, the quotes submitted by traders from a smaller broker remain quite liquid as they aim to attract counterparties from all brokers to maximize their execution probability. When the costs of non-internalized trades traders become substantially high stock market liquidity is harmed even more. Now both traders originating from the large and small broker target own-broker counterparties only (i.e. address those traders not having to bear the high clearing and settlement fee) with traders from the small broker quoting more illiquid prices than traders from the large broker. Finally, we perform a welfare analysis comparing the different settings. We find that for low post-trading costs, the

\(^1\)See page 8 on http://www.londonstockexchange.com/traders-and-brokers/rules-regulations/mifid/pre-trade.pdf

equilibria where all traders target counterparties from all brokers (and not only their own broker) produce a higher welfare, compared to equilibria where some (or all) traders aim to attract only “internal” counterparties (i.e. from their own broker). In contrast, for high post-trading costs, only internalized trades produce welfare, a pricing strategy fully reflecting the CSD’s marginal cost achieves this outcome. Relatedly, a social planner may face a trade-off between liquidity and welfare: a more liquid market may entail lower welfare. Therefore, liquidity measures like the bid-ask spread do not necessarily provide a good proxy for welfare.

To our knowledge no papers exist linking the organization of the post-trading infrastructure to stock market liquidity. Taking a wider perspective, our paper is related to different sets of literature. First, it relates to the literature on order submission strategies in limit order markets such as Foucault (1995, 1999), Parlour (1998), Handa, Schwartz and Tiwari (2003), Foucault, Kadan and Kandel (2005), Goettler, Parlour and Rajan (2005), Roşu (2009) and Van Achter (2009). These papers model how traders choose between market orders and limit orders in different dynamic settings. We extend them by including the impact of heterogeneity in post-trading fees on the optimal quote setting behavior of traders belonging to different brokers. Our paper also relates to the literature on make/take fees as modeled in Foucault, Kadan and Kandel (2009) and Colliard and Foucault (2010). In many markets, providers of liquidity receive a “make fee”, whereas consumers of liquidity pay a “take fee”. Foucault, Kadan and Kandel (2009) show this may induce liquidity cycles to arise, while Colliard and Foucault (2010) analyze how inter-market competition affects these make/take fees and ultimately trader behavior and liquidity. Our paper contributes to this literature by highlighting that outstanding quotes by one broker in the limit order book may induce asymmetries for traders affiliated to different brokers. When the transaction is internalized and implies no clearing and settlement fee, the post-trade cost is low and it is as if the payable take fee is small. In contrast, when a trader of another broker is the counterparty, post-trade costs are high and it is as if the payable take fee is large.

Second, our work contributes to the literature on clearing and settlement. The theoretical papers mostly deal with the optimal pricing strategies when central securities depositories (CSDs) interact, in order to explain the high markups for cross-border transfers of securities or the effects of different degrees of access to the CSDs (see e.g. Rochet (2005), Tapking and Yang (2006), Holthausen and Tapking (2007), Tapking (2007), and Koeppel, Monnet and Temzelides (2009)). We model how a cost-based post-trade infrastructure may affect stock market liquidity in two different ways. First, internalization of order flow reduces fees payable to the CSD and therefore changes the traders’ aggressiveness in the stock market. Second, the way a cost-based pricing structure is implemented by the CSD may lead to different stock market equilibria. In particular,
a pricing strategy fully reflecting the CSD’s marginal cost may lead to an equilibrium where traders opt to only address counterparties from the same broker. This reduces the total number of transactions and decreases market liquidity. Further, the empirical papers on the post-trading infrastructure mainly investigate whether there are economies of scale and scope in the clearing and settlement industry (see e.g. Van Cayseele and Wuyts (2008)). Our paper shows that transactions may exhibit different degrees of difficulty (i.e. cheaper internalized clearing and settlement versus more expensive cross-broker clearing and settlement), hinging on the particular stock market equilibrium that is played.

Third, some papers connect different phases of the trading process. Foucault and Parlour (2004) model how competition between stock exchanges links listing fees and transaction costs on those exchanges. They find that competing exchanges relax competition by choosing different trading technologies and listing fees. Berkowitz, Logue and Noser (1988) link explicit transaction costs to implicit transaction costs and find that paying higher commissions yields lower execution costs (be it non-commensurate). Our paper also links two phases of the trading cycle, i.e. stock market liquidity and post-trade infrastructure.

The remainder of this paper is structured as follows. Section 2 introduces the setup of our model. Sections 3 and 4 present different pricing schemes implemented by the clearing and settlement agent, and the corresponding equilibria. Within Section 5, these equilibria are further analyzed and compared, and a welfare analysis is provided. Finally, Section 6 concludes.

2 Setup

We develop an infinite horizon model to analyze a continuous limit order market listing a single security. Before trading starts, the clearing and settlement agent decides upon the prices of clearing and settlement. Traders take these post-trade clearing and settlement prices as given during the subsequent trading day. Each period in time \( t = 0, 1, \ldots, +\infty \), a single trader arrives who is willing to trade one share of the asset. Traders are risk neutral and expected utility maximizers. Further, traders exhibit an exogenously determined trading orientation which makes them either a buyer or a seller. We assume that the proportion of buyers and sellers in the trader population is equal.\(^3\) Buyers have a private valuation for the asset equal to \( V_h \), whereas sellers have a private valuation \( V_l \). We assume both valuations are non-negative and \( V_h - V_l > 0 \), which implies there are always gains

\(^3\)Our model is easily adjusted for the case where the proportion of buyers and sellers is different from 0.5; however it becomes slightly more complex since buyers and sellers no longer choose symmetric strategies. We prefer equal probabilities as this allows us more easily to identify the impact of different pricing schemes implemented by the clearing and settlement agent.
from trade between both parties. These differences in valuation are an outcome of taxes, liquidity shocks, or other portfolio considerations such as differences in endowment, or in opinions on the expected value of the asset. Each trader is linked to one of two possible brokers which means their individual orders are always sent to the market through this particular broker. More specifically, a fraction \( \gamma \) of the total trader population is linked to broker 1, and a complementary fraction \( 1 - \gamma \) is linked to broker 2. Throughout this paper, we mainly focus on brokers of divergent sizes. Thus, we assume broker 1 is a “large” broker serving a relatively larger fraction of the trader base, whereas broker 2 is a “small” one (i.e. \( \gamma > \frac{1}{2} \)). Broker affiliations are indexed by subscript \( j \in \{\text{large}, \text{small}\} \). Hence, for a trader arriving in a random period \( t \), with probability \( \gamma/2 \) it is or a buyer or a seller from the large broker and with probability \( (1 - \gamma)/2 \) it is or a buyer or a seller from the small broker. We assume broker affiliations are observable to traders.

The post-trading infrastructure, which from now on we denote as CSD (i.e. Central Securities Depository), handles clearing and settlement immediately after each transaction, and is considered to be risk neutral. The CSD has a cost \( c \) per leg of the trade for non-internalized trades, i.e. trades involving different brokers, and a lower cost for internalized trades, i.e. trades involving the same broker, which we normalize to zero. In implementing its pricing scheme, the CSD always aims to break even on average, but does not necessarily charge its true costs on each individual transaction. Overall, depending on the sophistication of the set pricing scheme, a CSD can charge different fees based on the type of transaction that is cleared and settled and thus differentiate between internalized and non-internalized trades. To properly account for this distinction, we consider two different pricing schemes implemented by the CSD. More specifically, micro-foundations are provided for various clearing and settlement fees \( c^i_j \), with superscript \( i \in \{I, NI\} \) indicating different cases regarding the pricing structure of the CSD for internalized (\( I \)) and non-internalized (\( NI \)) trades, and subscript \( j \in \{\text{large}, \text{small}\} \) referring to broker affiliation. The following table provides a summary of the two different pricing schemes:

<table>
<thead>
<tr>
<th>Pricing Scheme CSD</th>
<th>Uniform</th>
<th>Trade-Specific</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c^I_{\text{large}} = c^{NI}<em>{\text{large}} = c^I</em>{\text{small}} = c^{NI}_{\text{small}} )</td>
<td></td>
<td>( c^I_{\text{large}} = c^I_{\text{small}} &lt; c^{NI}<em>{\text{large}} = c^{NI}</em>{\text{small}} )</td>
</tr>
</tbody>
</table>

The “Uniform” pricing scheme means that the CSD charges the same fee to internalized and non-internalized trades. This fee is set optimally such that the CSD breaks even on average. The optimal fee and its impact on quotes will be analyzed in Section 3. Next, “Trade-Specific” pricing, entails that an internalized trade will be charged a different fee, compared to a non-internalized trade. In Section 4, we analyze this pricing scheme in detail.

\(^4\)See Duffie et al. (2005) for further economic interpretations.
An arriving trader bases her order submission strategy on her observation of the standing limit order book (LOB). She has two options at her disposal to trade. On the one hand, she could post a quote by submitting a limit order (LO) which does not offer certainty of execution. Posted LOs stay in the market only for one period and are thus take-or-leave offers for the next trader (see Foucault (1999) for a similar approach). On the other hand, she could submit a market order (MO) which guarantees immediate execution but at the cost of a less favorable execution price. Liquidity-demanding MOs execute against standing liquidity-supplying LOs, so they can only be submitted if a counterparty LO is already present in the LOB. Clearly, the LO’s execution probability is endogenous in the model as it depends on other traders’ order placement strategies. We will further discuss this issue below in this section. Orders are for one unit of the asset, and once submitted cannot be modified or cancelled. New in our model and a key contribution to the existing literature (such as Foucault (1999), Handa, Schwartz and Tiwari (2003), Van Achter (2009), and Colliard and Foucault (2010)) is that traders also account for the pricing scheme implemented by the CSD (and the implied clearing and settlement fee) in choosing their optimal strategy. More specifically, it is argued that conditional upon execution, the utility of trading the asset at price \( P \) for a buyer at broker \( j \) for a transaction of type \( i \) equals \( U(V_h, P) = V_h - P - c^i_j \), while a seller’s utility of trading at broker \( j \) with a transaction of type \( i \) is \( U(V_l, P) = P - V_l - c^i_j \). Hence, as non-trading gains are normalized to zero, \( V_h - c^i_j \) and \( V_l + c^i_j \) reflect the reservation price under the appropriate pricing structure that buyers are willing to pay and that sellers are willing to receive for one share of the asset, respectively. Traders naturally aim to maximize the expected payoff of their trade:

\[
\begin{align*}
V_h - A - c^i_j & \quad \text{for a buyer submitting a MO hitting a standing quote } A; \\
\Gamma(B) \cdot (V_h - B - c^i_j) & \quad \text{for a buyer submitting a LO at quote } B; \\
B - V_l - c^i_j & \quad \text{for a seller submitting a MO hitting a standing quote } B; \\
\Psi(A) \cdot (A - V_l - c^i_j) & \quad \text{for a seller submitting a LO at quote } A.
\end{align*}
\]

accounting for the appropriate clearing and settlement fee \( c^i_j \), and with \( \Gamma(B) \) the execution probability of a buy LO at quote \( B \) (the bid price), and \( \Psi(A) \) the execution probability of a sell LO at quote \( A \) (the ask price), as determined by the respective buyer or seller. In setting the optimal bid or ask quotes when submitting a LO, a trader in general has two possibilities. She could determine quotes that only attract counterparties from her own broker (we label this strategy “own”) or she can opt for a quote that is attractive to all possible counterparties, i.e. traders from her own and from the other broker (we label this strategy “all”). Do note that “attract” in this context means the targeted incoming trader is at least willing to hit the standing LO by submitting a MO. Thus, any trader submitting a LO needs to account for the MO.
strategy of the subsequently arriving trader. Given traders are linked to either a large or a small broker, four possible combinations of strategies can be distinguished:

1. traders of both brokers aim to address counterparties of all brokers: \(\{all, all\}\);
2. traders of the large broker only aim to address counterparties of their own broker, traders of the small broker aim to address counterparties of all brokers: \(\{own, all\}\);
3. traders of the large broker aim to address counterparties of all brokers, traders of the small broker only aim to address counterparties of their own broker: \(\{all, own\}\);
4. traders of both brokers only aim to address counterparties of their own broker: \(\{own, own\}\).

Note that the first element within the mentioned \(\{,\}\) always refers to the strategy of traders of the large broker, and the second element to the strategy of traders of the small broker. As will become clear below, these four possible combinations of strategies result in four possible sub-equilibria of our game. Indeed, for each combination, traders at different brokers may post different bid and ask quotes, and the CSD may charge a different fee. We will show below, however, that not every potential sub-equilibrium materializes under every pricing scheme, because some combination(s) will dominate others.

All parameters of the model, including \(V_h\), \(V_l\), \(\gamma\), and \(c^i_j\) are known to the investors. Moreover, they are constant over time, hence the market is assumed to be in steady state. This allows to solve for a stationary equilibrium within each pricing scheme as in Foucault (1999), Van Achter (2009), or Colliard and Foucault (2010). More specifically, a stationary market equilibrium is defined as a set of mutual order submission strategies (specifying an optimal order type, quote and corresponding execution probability to each possible state of the LOB) such that each trader’s strategy is optimal given the strategies of all other traders. Divergences in pricing rules imply different types of equilibria arise. Both the magnitude of the fees for clearing and settlement as well as the type of equilibrium influence stock market liquidity. In Sections 3 and 4 we provide a thorough analysis of each of the derived stationary equilibria.

\footnote{As such, the LO execution probabilities are endogenous, implying traders are in a game situation. In general, traders’ optimal order submission strategies depend on their LO’s probability of execution, which in turn is determined by their order submission strategies. To properly account for these endogenous linkages between the MO and the LO placement strategies, they will be determined simultaneously.}

\footnote{Do note that by assuming the proportion of buyers and sellers in the trader population to be equal, within a broker we have that buyers and sellers have symmetric strategies. Thus, there is no need to further differentiate the strategies in this respect.}
3 CSD Pricing Scheme 1: Uniform Pricing

Under the uniform pricing scheme, which is denoted by superscript $U$, all transactions are handled by the CSD which charges a uniform fee for both brokers to all orders upon execution. Furthermore, at the level of the CSD, we assume that internalized transactions entail a normalized zero marginal cost. In contrast, transactions stemming from traders from different brokers still imply a cost $c$ for the CSD. Hence, within this particular pricing scheme, the CSD is argued to charge a uniform fee to both brokers such that it breaks even on average over all transactions. Thus, it compensates the losses it makes on the difficult (i.e. non-internalized) order flow stemming from different brokers with gains from the easy order flow stemming from trades that occur within the same broker (i.e. internalized). In fact, by charging a uniform break even fee per transaction, the CSD does neither differentiate between different types of transactions, nor between transactions stemming from different brokers. Denote this break even fee by $c^U$, this pricing scheme then implies that:

$$c^U_{\text{large}} = c^U_{\text{small}} = c^U_{\text{I}} = c^U_{\text{NI}}$$

Under this pricing scheme, it is clear that traders from both brokers will always address all traders. This means that the $\{\text{all, all}\}$ combination of strategies dominates the three other combinations. The reason is that as all traders face a uniform fee, it is impossible to set a quote only attractive to traders of one particular broker. Therefore, when analyzing the equilibrium we only consider the $\{\text{all, all}\}$ combination of strategies.

3.1 Equilibrium

We now turn to the determination of the equilibrium quotes and the optimal fee. We solve the model backward. First, for a given fixed fee $c^U$ we derive traders’ order placement strategies in equilibrium. Second, we solve for the optimal fee. In determining its fee, the CSD correctly anticipates how it affects traders’ order submission behavior.

How do traders set their quotes, taking $c^U$ as given? Given that the $\{\text{all, all}\}$ sub-equilibrium will always prevail and that costs and gains are identical for traders of both brokers, we must have that bid and ask quotes, set by traders of the large and small broker are identical. We denote this as follows:

$$A^U_{\text{large}} = A^U_{\text{small}} = A^U_{\text{I}} = A^U_{\text{NI}}$$

$$B^U_{\text{large}} = B^U_{\text{small}} = B^U_{\text{I}} = B^U_{\text{NI}}$$

---

7 Do note that if playing the own-strategy would be possible, this would still be a sub-optimal strategy as it only reduces execution probabilities without inducing any quote advantage.
where \( A_{\text{large}}^{U,\{\text{all,all}\}} \) refers to the ask price \( (A) \) set by a trader from the large broker \((\text{subscript large})\) with uniform pricing by the CSD \((\text{superscript } U)\) and under the \( \{\text{all, all}\} \) sub-equilibrium \((\text{second superscript})\). The other prices have a similar notation.

Suppose now a buyer arrives in the market. She will set the bid price of her LO such that the next incoming seller is indifferent between hitting the LO (by submitting a sell MO) or submitting a sell LO herself. This implies the expected payoff for the incoming seller of submitting a MO or a LO must be the same. The following equation shows this indifference condition:

\[
B^{U,\{\text{all,all}\}} - V_i - c^U = \frac{1}{2} \left[ A^{U,\{\text{all,all}\}} - V_i - c^U \right]
\]

The left hand side of this equation presents the gain from a sell MO, given the bid price set by the buyer in the previous period. The right hand side is the expected gain of a sell LO, which is the execution probability of this order \( (\text{i.e. } 1/2 \text{ or the probability that the next arriving trader is a buyer who will hit the standing sell LO since the seller optimally also sets her ask price to make the next arriving buyer indifferent}) \) multiplied by the payoff upon execution of her order corrected for the appropriate clearing and settlement fee. Thus, the idea here is that \( B^{U,\{\text{all,all}\}} \) is chosen at the lowest level at which the subsequently arriving seller is just willing to submit a MO, while both accounting for the clearing and settlement fee \( c^U \). In other words, \( B^{U,\{\text{all,all}\}} \) equals the seller’s cutoff price and renders this seller indifferent between hitting the standing LO at \( B^{U,\{\text{all,all}\}} \) and submitting her own LO at \( A^{U,\{\text{all,all}\}} \). Submitting a LO at all other quotes is easily proven to be sub-optimal for this buyer.

Similarly an arriving seller sets her LO quote in order to make a subsequently arriving buyer indifferent between submitting a buy MO at \( A^{U,\{\text{all,all}\}} \) or a buy LO at \( B^{U,\{\text{all,all}\}} \):

\[
V_h - A^{U,\{\text{all,all}\}} - c^U = \frac{1}{2} \left[ V_h - B^{U,\{\text{all,all}\}} - c^U \right]
\]

Solving the system of indifference equations yields the quotes for a given fixed fee \( c^U \). Proposition 1 presents the optimal fee and resulting equilibrium quotes for the uniform pricing scheme. Under the \( \{\text{all, all}\} \) combination of strategies which is played within this pricing scheme, the fee \( c^{*,U} \) at which the CSD breaks even over all transactions could be shown to equal \( 2\gamma \left( 1 - \gamma \right) c \). For the explicit calculation we refer to the proof of Proposition 3. Intuitively, this expression could be seen to capture the costly non-internalized match between counterparties from the large broker \((\gamma)\) and the small broker \((1 - \gamma)\). By charging \( c^{*,U} \) on both legs of every transaction \((\text{internalized and non-internalized})\), the CSD on average indeed breaks even: it gains on transactions for which it does not face marginal costs and loses on transactions where active clearing and settlement takes place. While transactions received from the largest broker more often induce no costs, as
they are more often internalized, the CSD still charges a uniform price to both brokers.

**Proposition 1** When the CSD applies a uniform pricing scheme, i.e. it charges the same fee to both brokers and to internalized and non-internalized trades, the optimal fee announced by the CSD is:

\[ c^{*,U} = 2\gamma (1 - \gamma) c \]

Traders always play the \{all, all\} sub-equilibrium. The optimal ask and bid quotes of the trader are:

\[
A_{\text{large}}^{*,U,\{all,all\}} \equiv A_{\text{small}}^{*,U,\{all,all\}} \equiv A^{*,U,\{all,all\}} = \frac{2V_h + V_l - 2\gamma (1 - \gamma) c}{3}
\]

\[
B_{\text{large}}^{*,U,\{all,all\}} \equiv B_{\text{small}}^{*,U,\{all,all\}} \equiv B^{*,U,\{all,all\}} = \frac{V_h + 2V_l + 2\gamma (1 - \gamma) c}{3}
\]

**Proof.** See Appendix A. ■

We observe that the ask decreases in \( c \), while the bid increases in \( c \). Thus, larger post-trading costs appear to induce more liquid quote-setting behavior and thus improve stock market liquidity. The reasoning behind this remarkable result is that traders submit more aggressive LOs in order to induce the counterparty to submit a MO (which incurs the clearing and settlement fee with certainty). That is, it is as if the counterparty now has a lower willingness to trade resulting from the clearing and settlement fee. Moreover, when both brokers exactly have the same market share (i.e. \( \gamma = 0.5 \)), the quotes are most liquid. Indeed, if this condition is fulfilled, the fee charged by the CSD per trade is largest leading to a more aggressive pricing strategy in equilibrium. Further, as could be expected, when one broker attracts the entire market (\( \gamma = 0 \) or \( \gamma = 1 \)), clearing and settlement fees do not play a role anymore as all trades are then internalized. This would imply we are in a model without clearing and settlement fees, comparable to Foucault (1999).

### 4 CSD Pricing Scheme 2: Trade-Specific Pricing

Under the trade-specific pricing scheme, denoted by superscript \( TS \), we assume the CSD breaks even by pricing according to the marginal costs that are associated with individual transactions. That is, clearing and settlement fees are set to zero for trades with both traders stemming from the same broker, and amount to \( c \) for trades with both traders originating from different brokers. As argued before, note that the zero cost attributed to internalized trades merely represents a normalization. More generally, as long as internalized trades imply lower marginal costs than non-internalized trades, all results
mentioned below hold. In terms of the notation introduced in Section 2, this implies:

\[
\begin{align*}
    c_{\text{large}}^I &= c_{\text{small}}^I = 0 \\
    c_{\text{large}}^{NI} &= c_{\text{small}}^{NI} = c
\end{align*}
\]

A novel implication of this differential pricing structure is then that the quoting behavior of traders linked to the large broker may differ substantially from the strategies of traders affiliated to the small broker. Consider the following example to illustrate this point. Assume a buyer linked to the large broker arrives in the market. On the one hand, she could submit a LO. Her quote choice allows her to choose which counterparties she wants to address: (i) by posting a lower bid (i.e. \(B_{\text{large}}^{TS,\{own,all\}}\) or \(B_{\text{large}}^{TS,\{own,own\}}\), depending on the strategy played by the small broker trader) she only attracts counterparties from the same broker implying a higher payoff with a lower execution probability, whereas (ii) by posting a higher bid (i.e. \(B_{\text{large}}^{TS,\{all,all\}}\) or \(B_{\text{large}}^{TS,\{all,own\}}\), depending on the strategy played by the small broker trader) she also attracts counterparties from the other broker implying a lower payoff with a higher execution probability. Do note e.g. \(B_{\text{large}}^{TS,\{own,all\}}\) is the lowest bid quote at which an incoming seller from the same (i.e. large) broker is willing to submit a MO, while accounting for the according zero clearing and settlement fee and her own LO strategy quoting \(A_{\text{large}}^{TS,\{own,all\}}\), and given that traders from the small broker play an all-strategy. In turn, \(B_{\text{large}}^{TS,\{all,all\}}\) is the lowest bid quote at which an incoming seller from the other (i.e. small) broker is willing to submit a MO, while accounting for the according higher clearing and settlement fee \(c\) and her own LO strategy quoting \(A_{\text{small}}^{TS,\{all,all\}}\), and given that traders from the small broker play an all-strategy. Submitting a LO at any other quote is easily proven to be sub-optimal for this buyer.\(^8\) On the other hand, given the availability of a standing sell LO which is attractive to her, she could also submit a MO. A buyer affiliated to the small broker faces a similar trade-off. Further, as the proportion of buyers and sellers in the trader population is equal, the actions of sellers linked to the large and small broker are completely symmetric, and could be derived in a similar way. As we will see below, the choice between these quotes hinges on market parameters and on the trader’s preferences in the trade-off between quote level, execution probability and clearing and settlement fee. In principle, all four possible combinations of strategies that can be played by traders from both brokers are feasible. In the proof of the equilibrium we will show, however, that the \(\{all,own\}\) combination is never optimal. Therefore, we already exclude it in the discussion below. For the three remaining combinations (i.e. \(\{all,all\}\) \(\{own,all\}\) and \(\{own,own\}\)), we will now determine the according equilibrium quotes set by traders at both brokers. The pricing scheme of the CSD (i.e. zero fee for internalized trades, \(c\) for non-internalized trades), is again taken as given by the traders. Note that \(c\) represents the (exogenous) marginal

\(^8\)That is, higher bid quotes do not increase the execution probability yielding lower expected payoffs.
cost for the CSD and thus in contrast to the previous pricing scheme we now do not need to compute it.

4.1 Equilibrium

Starting with the \{all, all\} combination of strategies, traders at the large broker set their quote to keep the marginal trader indifferent as they want to address all traders. Thus, they account for the transaction fee \(c\). So for buyers and sellers from the large broker, we respectively have:

\[
B_{\text{large}}^{TS,\{all,all\}} - V_i - c = \frac{1}{2} \left[ A_{\text{small}}^{TS,\{all,all\}} - V_i - \gamma c \right]
\]

\[
V_h - A_{\text{large}}^{TS,\{all,all\}} - c = \frac{1}{2} \left[ V_h - B_{\text{small}}^{TS,\{all,all\}} - \gamma c \right]
\]

Thus, within the first indifference condition for instance, the incoming seller from the small broker is kept indifferent between hitting the standing quote \(B_{\text{large}}^{TS,\{all,all\}}\) (by submitting a MO sell) accounting for the appropriate clearing and settlement fee, and submitting her own sell LO (of which the execution probability, the quote and the clearing and settlement fee correctly correspond to the \{all, all\} strategy this seller is playing herself). Similarly, for traders from the small broker, who keep an incoming counterparty trader from the large broker indifferent (thus accounting for the transaction fee \(c\)), we have for buyers and sellers:

\[
B_{\text{small}}^{TS,\{all,all\}} - V_i - c = \frac{1}{2} \left[ A_{\text{large}}^{TS,\{all,all\}} - V_i - (1 - \gamma) c \right]
\]

\[
V_h - A_{\text{small}}^{TS,\{all,all\}} - c = \frac{1}{2} \left[ V_h - B_{\text{large}}^{TS,\{all,all\}} - (1 - \gamma) c \right]
\]

Next, consider the \{own, all\} combination of strategies. Traders at the large broker set their quote only to keep counterparties of their \textit{own} broker indifferent (which implies the transaction fee \(c\) does not need to be accounted for). A buyer (seller) at the large broker keeps the incoming seller (buyer) from her \textit{own} broker indifferent, such that:

\[
B_{\text{large}}^{TS,\{own,all\}} - V_i = \gamma \frac{1}{2} \left[ A_{\text{large}}^{TS,\{own,all\}} - V_i \right]
\]

\[
V_h - A_{\text{large}}^{TS,\{own,all\}} = \gamma \frac{1}{2} \left[ V_h - B_{\text{large}}^{TS,\{own,all\}} \right]
\]

Thus, within the indifference condition stated first for instance, an incoming seller from the large broker is kept indifferent between hitting the standing quote \(B_{\text{large}}^{TS,\{own,all\}}\) (by submitting a MO sell) accounting for the appropriate zero clearing and settlement fee, and submitting her own sell LO (of which the execution probability, the quote and the zero clearing and settlement fee correctly correspond to the \{own, all\} strategy this
sider is playing herself). In contrast, traders from the small broker still aim to keep the marginal trader indifferent. A buyer (seller) from the small broker will then keep an incoming seller (buyer) from the large broker indifferent, leading to:

\[
B^{TS,\{own,all\}}_{small} - V_i - c = \gamma \frac{1}{2} \left[ A^{TS,\{own,all\}}_{large} - V_i \right]
\]

\[
V_h - A^{TS,\{own,all\}}_{small} - c = \gamma \frac{1}{2} \left[ V_h - B^{TS,\{own,all\}}_{large} \right]
\]

The reasoning here is similar to that for the small broker traders under the \{all, all\} combination of strategies, but now the expected LO payoffs of the targeted large broker traders correctly reflect the execution probability, the quote and the zero clearing and settlement fee corresponding to the \{own, all\} strategy these traders are playing themselves.

Finally, within the \{own, own\} combination of strategies, all traders only keep potential counterparties of their own broker indifferent. Hence, all trades are internalized and thus incur a zero clearing and settlement fee. The indifference equations for buyer and seller from the large broker then become:

\[
B^{TS,\{own,own\}}_{large} - V_i = \gamma \frac{1}{2} \left[ A^{TS,\{own,own\}}_{large} - V_i \right]
\]

\[
V_h - A^{TS,\{own,own\}}_{large} = \gamma \frac{1}{2} \left[ V_h - B^{TS,\{own,own\}}_{large} \right]
\]

Thus, within the first indifference condition for instance, the incoming seller from the large broker is kept indifferent between hitting the standing quote \(B^{TS,\{own,own\}}_{large}\) (by submitting a MO sell) accounting for the appropriate zero clearing and settlement fee, and submitting her own sell LO (of which the execution probability, the quote and the zero clearing and settlement fee correctly correspond to the \{own, own\} strategy this seller is playing herself). At these quotes, only traders from the large broker are indifferent. For traders originating from the small broker trading at these quotes is too costly given their higher transaction fee \(c\). Therefore, the execution probabilities are only related to the own broker (i.e. \(\gamma\)).

Similarly, the equations for buyer and seller from the small broker are:

\[
B^{TS,\{own,own\}}_{small} - V_i = (1 - \gamma) \frac{1}{2} \left[ A^{TS,\{own,own\}}_{small} - V_i \right]
\]

\[
V_h - A^{TS,\{own,own\}}_{small} = (1 - \gamma) \frac{1}{2} \left[ V_h - B^{TS,\{own,own\}}_{small} \right]
\]

At these quotes, only traders from the small broker are indifferent. For traders stemming from the large broker trading at these quotes is too costly given their higher transaction fee \(c\). Therefore, the execution probabilities are only related to the own broker (i.e. \(1 - \gamma\)).
Solving the above systems of indifference conditions renders the equilibrium quotes and thus the three distinct sub-equilibria. Comparing expected profits for each of the sub-equilibria, we are also able to determine when each of the sub-equilibria is valid. All these elements are shown in the equilibrium presented in Proposition 2.

**Proposition 2** With a CSD applying trade-specific (marginal cost-based) pricing, traders at both brokers play the following LO strategies hinging on the value of the post-trading cost $c$:

- For low values of $c$, i.e. $c \leq \frac{2(V_h - V_l)}{3(2+\gamma)}$, traders from both brokers target counterparties of all brokers, thus the $\{\text{all, all}\}$ sub-equilibrium is played. The equilibrium quotes are:
  
  \[
  \begin{align*}
  A^*_{TS,\text{large}}^{\{\text{all, all}\}} &= \frac{2V_h + V_l}{3} - (1 - \gamma)c \\
  B^*_{TS,\text{large}}^{\{\text{all, all}\}} &= \frac{V_h + 2V_l}{3} + (1 - \gamma)c \\
  A^*_{TS,\text{small}}^{\{\text{all, all}\}} &= \frac{2V_h + V_l}{3} - \gamma c \\
  B^*_{TS,\text{small}}^{\{\text{all, all}\}} &= \frac{V_h + 2V_l}{3} + \gamma c
  \end{align*}
  \]

- For intermediate values of $c$, i.e. $\frac{2(V_h - V_l)}{3(2+\gamma)} < c \leq \frac{2(V_h - V_l)(1+\gamma)}{(1+\gamma)(2+\gamma)(3-\gamma)}$, traders from the large broker only target counterparties of their own broker whereas traders from the small broker target counterparties of all brokers, thus the $\{\text{own, all}\}$ sub-equilibrium is played. The equilibrium quotes are:

  \[
  \begin{align*}
  A^*_{TS,\text{large}}^{\{\text{own, all}\}} &= \frac{2V_h + \gamma V_l}{2 + \gamma} \\
  B^*_{TS,\text{large}}^{\{\text{own, all}\}} &= \gamma V_h + 2V_l \\
  A^*_{TS,\text{small}}^{\{\text{own, all}\}} &= \frac{2V_h + \gamma V_l}{2 + \gamma} - c = A^*_{TS,\text{large}}^{\{\text{own, all}\}} - c \\
  B^*_{TS,\text{small}}^{\{\text{own, all}\}} &= \frac{\gamma V_h + 2V_l}{2 + \gamma} + c = B^*_{TS,\text{large}}^{\{\text{own, all}\}} + c
  \end{align*}
  \]

- For high values of $c$, i.e. $c > \frac{2(V_h - V_l)(1+\gamma)}{(1+\gamma)(2+\gamma)(3-\gamma)}$, traders from both brokers only target own counterparties, thus the $\{\text{own, own}\}$ sub-equilibrium is played. The equilibrium
quotes are:

\[
\begin{align*}
A_{\text{large}}^{*,TS,\{\text{own, own}\}} &= \frac{2V_h + \gamma V_l}{2 + \gamma} \\
B_{\text{large}}^{*,TS,\{\text{own, own}\}} &= \frac{\gamma V_h + 2V_l}{2 + \gamma} \\
A_{\text{small}}^{*,TS,\{\text{own, own}\}} &= \frac{2V_h + (1 - \gamma) V_l}{3 - \gamma} \\
B_{\text{small}}^{*,TS,\{\text{own, own}\}} &= \frac{(1 - \gamma) V_h + 2V_l}{3 - \gamma}
\end{align*}
\]

Proof. See Appendix A. ■

For low post-trading costs, traders at both brokers target counterparties at all brokers by quoting relatively liquid prices. Still, an interesting divergence arises, traders from the small broker have to quote more liquid prices (as compared to traders from the large broker) to attain this goal as they need to convince traders from the large broker (who face the opportunity to submit a LO featuring lower expected clearing and settlement fees) to accept their LO. Do note that given this quote setting behavior, in case a counterparty from the same broker hits a standing quote, both traders involved in the trade receive a “bonus” as they both do not have to pay $c$. An increase in the large broker’s market share $\gamma$ evidently induces traders from the large broker to quote relatively less liquid prices, whereas traders from the small broker are obliged to quote relatively more liquid prices to remain attractive to the traders from the large broker. Next, for an intermediate range of post-trading costs, traders at the large broker alter their strategy and submit relatively illiquid quotes only targeting traders of their own broker. In contrast, traders from the small broker still prefer to target counterparties at both brokers and thus quote a very liquid quote fully compensating the clearing and settlement fee $c$ a potentially arriving counterparty from the large broker would face. They do so because the gain from increased matching probabilities still outweighs the concessions in terms of aggressive pricing. Evidently, this entails that in case a counterparty from the small broker would hit this standing quote, both traders involved in the trade receive a “bonus” as they both do not have to pay $c$. Finally, for sufficiently large post-trading costs (inducing larger cost savings from internalization), both traders from the large and the small broker only address own-broker counterparties by quoting relatively illiquid prices, with the quotes from the small broker being more illiquid as they face a lower execution probability. All quoted prices are now independent of the clearing and settlement fees as these strategies aim at targeting own-broker counterparties only.
5 Liquidity and Welfare: Comparison of CSD Pricing Schemes

In this section, we compare in the first two subsections the implications of the different pricing schemes by the CSD on market liquidity. We do so by investigating the aggressiveness of ask quotes (the analysis of the bid side is entirely symmetric) in the first subsection, and by focussing on patterns in trading volume in the second subsection. Finally, in a third subsection, we discuss the impact of CSD pricing on welfare. To highlight our main points, we illustrate the results of our model using the following parameter values: \( V_h = 20, V_l = 0 \) and \( \gamma = 0.8 \). The marginal cost \( c \) varies in the interval \([0, 20]\). Important to stress is that all results are general, and do not depend on these specific parameter values.

5.1 Quote Aggressiveness

Propositions 1 and 2 showed the equilibrium quotes under the two CSD pricing schemes. We now compare these schemes with respect to liquidity by investigating the aggressiveness of ask quotes: lower, more aggressive ask quotes correspond to a more liquid market. Do note that the delineated ask quotes reflect observed liquidity, hence not the cum-fee liquidity which hinges on the specific match between traders. In Figure 1, we depict the ask prices as a function of \( c \) for our two CSD pricing schemes. The green line represents ask prices for the uniform pricing scheme (computed from Proposition 1), and the black lines the trade-specific pricing scheme (computed from Proposition 2). For the trade-specific pricing scheme, the different parts correspond with the different sub-equilibria as shown in Proposition 1. Panel A draws the ask prices for traders from the large (full lines) and small broker (dotted lines). In Panel B, we present the “average” ask price by taking a weighted average of the quotes of large and small broker’s traders, using the market share of the respective broker (i.e. \( \gamma \) and \( 1 - \gamma \)) as weights.

Within Panel A we observe, as already argued above in the discussion of the propositions featuring the equilibria for both pricing schemes, that traders from large and small brokers, although in principle identical, may quote different prices because of differences in clearing and settlement fees.

**Corollary 1** Bid and ask quotes of traders hinge upon their broker affiliation, due to clearing and settlement fees.

Panel B of Figure 1 indicates that the CSD pricing scheme as well as the level of \( c \) influence the average observed liquidity. For low levels of \( c \), the average ask price under the trade-specific pricing scheme is most liquid. In contrast, for intermediate and high...
levels of $c$ the market is most liquid under uniform pricing. This finding has policy implications for a regulator who wants to maximize observed liquidity. Technological progress, lowering $c$, may induce a regulator to implement trade-specific pricing and not uniform pricing. Therefore:

**Corollary 2** Regulators can improve market liquidity by imposing a pricing scheme on CSD. The optimal pricing scheme depends on the structural cost $c$.

In Subsection 5.3, we investigate the optimal CSD pricing scheme when a regulator wants to maximize welfare (and not liquidity).

---

Please insert Figure 1 around here.

---

### 5.2 Trading Volume

In the previous subsection, we focused on liquidity as measured by quote aggressiveness. Now, we turn to trading volume, another measure for market liquidity often used in the literature and by practitioners. In doing so, we follow a similar approach as Colliard and Foucault (2010) by focusing on trading rates in a given period. In each period, we observe one out of the following six possible states: (1) a trader from the small broker who submits a limit order; (2) a trader from the small broker who submits a market order that is internalized (i.e. a trader from the small broker that hits a standing limit order submitted by another trader of the small broker); (3) a trader from the small broker who submits a market order that is not internalized (i.e. a trader from the small broker that hits a limit order submitted by a trader from the large broker); (4) a trader from the large broker who submits a limit order; (5) a trader from the large broker who submits a market order that is internalized (i.e. a trader from the large broker that hits a standing limit order submitted by another trader of the large broker); (6) a trader from the large broker who submits a market order that is not internalized (i.e. a trader from the large broker that hits a limit order submitted by a trader from the small broker). We do not need to make a distinction between buyers and sellers because both sides of the market are symmetric in our model. For each pricing scheme of the CSD $k \in \{U, TS\}$, denote the stationary probabilities of each of these six possible states under a given sub-equilibrium $s^k \in S^k$ as $\varphi^{k,s^k} = \{\varphi_1^{k,s^k}, \varphi_2^{k,s^k}, \varphi_3^{k,s^k}, \varphi_4^{k,s^k}, \varphi_5^{k,s^k}, \varphi_6^{k,s^k}\}$. $S^k$ denotes the set of all possible sub-equilibria under pricing scheme $k$; hence for the uniform pricing scheme $S^U = \{all, all\}$, while under trade-specific pricing $S^{TS} = \{\{all, all\}, \{own, all\}, \{own, own\}\}$. In Appendix B, we derive the exact expressions for the various $\varphi^{k,s^k}$. Based on these, we can now easily develop measures for trading volume (the trading rate) and the degree of internalization (the internalization rate) for each pricing scheme and sub-equilibrium.
The trading rate $TR$ in a period under sub-equilibrium $s^k$ of pricing scheme $k$ is the likelihood of a market order initiating a trade in a given period. Clearly, this occurs in states 2, 3, 5 and 6 mentioned above, thus:

$$TR^{k,s^k} = \varphi^{k,s^k}_2 + \varphi^{k,s^k}_3 + \varphi^{k,s^k}_5 + \varphi^{k,s^k}_6$$

In turn, the internalization rate is the likelihood of a market order initiating an internalized trade (possibilities 3 and 6) divided by the trading rate:

$$IR^{k,s^k} = \frac{\varphi^{k,s^k}_3 + \varphi^{k,s^k}_6}{\varphi^{k,s^k}_2 + \varphi^{k,s^k}_3 + \varphi^{k,s^k}_5 + \varphi^{k,s^k}_6}$$

and can be seen as the percentage of trades that is internalized. Similarly, the non-internalization rate is defined as the complement:

$$1 - IR^{k,s^k} = \frac{\varphi^{k,s^k}_3 + \varphi^{k,s^k}_6}{\varphi^{k,s^k}_2 + \varphi^{k,s^k}_3 + \varphi^{k,s^k}_5 + \varphi^{k,s^k}_6}$$

Proposition 3 presents the trading rates and internalization rates for the different pricing schemes.

**Proposition 3** Trading rates and internalization rates for the different CSD pricing schemes.

- **Under uniform pricing:**
  - the trading rate is
    $$TR^{U\{}all,all\} = \frac{1}{3}$$
  
  - the internalization rate is
    $$IR^{U\{}all,all\} = 1 - 2\gamma (1 - \gamma)$$

- **Under trade-specific pricing:**
  - the trading rates for the different sub-equilibria are
    $$TR^{TS\{}all,all\} = \frac{1}{3}$$
    $$TR^{TS\{}own,all\} = \frac{2 - \gamma (1 - \gamma)}{6 + \gamma (1 - \gamma)}$$
    $$TR^{TS\{}own,own\} = \frac{2 - 3\gamma (1 - \gamma)}{6 + \gamma (1 - \gamma)}$$
the internalization rates for the different sub-equilibria are

\[
\begin{align*}
IR^{TS,(all,all)} &= 1 - 2\gamma (1 - \gamma) \\
IR^{TS,(own,all)} &= \frac{2 - \gamma (3 - 2\gamma - \gamma^2)}{2 - \gamma (1 - \gamma)} \\
IR^{TS,(own,own)} &= 1
\end{align*}
\]

**Proof.** See Appendix A. ■

Since \( \gamma > 0.5 \), we obtain that \( TR^{U,(all,all)} = TR^{TS,(all,all)} > TR^{TS,(own,all)} > TR^{TS,(own,own)} \), implying that trading volume is highest when the CSD applies uniform pricing, or when \( c \) is small such that the \( \{all, all\} \) sub-equilibrium applies under trade-specific pricing.

For the internalization rates, we have that \( IR^{U,(all,all)} = IR^{TS,(all,all)} < IR^{TS,(own,all)} < IR^{TS,(own,own)} \). The latter is obviously one since in the \( \{own, own\} \) sub-equilibrium all trades are internalized. Internalized trades are thus least frequent in the \( \{all, all\} \) sub-equilibrium. Further, do note that trading volume is not directly related to market liquidity as measured by quote aggressiveness. This can be seen most easily from the fact that the trading rate is the same for \( U, \{all, all\} \) and \( TS, \{all, all\} \), while the aggressiveness of ask quotes derived in the previous subsection is different. In the next subsection, we will use these trading and internalization rates to derive welfare implications of the different pricing rules.

### 5.3 Overall Welfare

In this section, we characterize ex ante welfare for the two CSD pricing schemes. Our ex ante welfare measure builds on rational trader behavior and is therefore identical to the “mean” realized ex post welfare. We focus on overall welfare \( (OW) \), i.e. the sum of all agents’ expected utilities from trading (see Glosten (1998), Goettler, Parlour and Rajan (2005), Hollifield, Miller, Sandås and Slive (2006), and Degryse, Van Achter and Wuyts (2009) for a similar approach in quantifying welfare). As the CSD always breaks even, in our model, \( OW \) coincides with trader welfare.

Welfare is realized when a trade occurs. An internalized trade generates \( V_h - V_l \) whereas a non-internalized trade produces \( V_h - V_l - 2c \). The prices at which trades occur are not relevant for welfare as they merely represent a redistribution between buyer and seller. In contrast, when non-internalized trades occur in equilibrium, an increase in \( c \) reduces the surplus to be split between buyer and seller involved in the transaction. Expected overall welfare per period within pricing scheme \( k \) and sub-equilibrium \( s^k \) simply follows from multiplying the trading rate \( TR^{k,s^k} \) with the welfare realized in the occurrence of a trade (appropriately weighing internalized and non-internalized trades), or
Proposition 4 summarizes our main results regarding expected overall welfare per period for both CSD pricing schemes.

**Proposition 4** Expected overall welfare per period depends on the CSD pricing scheme.

- **Under uniform pricing,** it equals:
  \[
  OW^{U\{all,all\}} = \frac{1}{3} [V_h - V_l - 4\gamma (1 - \gamma) c]
  \]

- **Under trade-specific pricing,** overall welfare hinges on the sub-equilibria:
  - For low values of \(c\), i.e. \(c \leq \frac{2(V_h - V_l)}{3(2+\gamma)}\) or the \(\{all,all\}\) sub-equilibrium, it equals:
    \[
    OW^{TS\{all,all\}} = \frac{1}{3} [V_h - V_l - 4\gamma (1 - \gamma) c]
    \]
  - For intermediate values of \(c\), i.e. \(\frac{2(V_h - V_l)}{3(2+\gamma)} < c \leq \frac{2(V_h - V_l)(1+\gamma^2)}{(1+\gamma)(2+\gamma)(3-\gamma)}\), or the \(\{own,all\}\) sub-equilibrium:
    \[
    OW^{TS\{own,all\}} = \frac{2 - \gamma (1 - \gamma)}{6 + \gamma (1 - \gamma)} \left[ V_h - V_l - 2c \left( 1 - \frac{2 - \gamma (3 - 2\gamma - \gamma^2)}{2 - \gamma (1 - \gamma)} \right) \right]
    \]
  - For high values of \(c\), i.e. \(c > \frac{2(V_h - V_l)(1+\gamma^2)}{(1+\gamma)(2+\gamma)(3-\gamma)}\), or the \(\{own,own\}\) sub-equilibrium:
    \[
    OW^{TS\{own,own\}} = \frac{2 - 3\gamma (1 - \gamma)}{6 + \gamma (1 - \gamma)} [V_h - V_l]
    \]

**Proof.** See Appendix A. □

We illustrate Proposition 4 graphically in Figure 2 using the same parameter values as before. The green line represents welfare for the uniform pricing scheme, and the black lines for the trade-specific pricing scheme. For the trade-specific pricing scheme, the different parts correspond with the distinct sub-equilibria as shown in Proposition 4.

Please insert Figure 2 around here.

Next, we determine which pricing scheme a social planner prefers depending upon the magnitude of \(c\). The social planner faces the following trade-off. On the one hand, it wants to maximize the trading rate as this increases trading gains. On the other
hand, it prefers internalized trades above non-internalized trades as the former do not generate post-trade costs. Therefore it also cares about the internalization rate. The following corollary presents welfare rankings for the entire range of $c$.

**Corollary 3** Overall welfare ranking for the entire range of $c$.\(^9\)

- For low values of $c$, i.e. $c \leq \frac{V_h - V_l}{2+\gamma}$, we find that $OW^U,\{all,all\} = OW^{TS},\{all,all\} \geq OW^{TS},\{own,all\}$. Only the uniform pricing scheme achieves the \{all, all\} sub-equilibrium for the entire range of $c \leq \frac{V_h - V_l}{2+\gamma}$, the trade-specific pricing scheme achieves the \{all, all\} sub-equilibrium for $c \leq \frac{2(V_h - V_l)}{3(2+\gamma)}$.

- For intermediate values of $c$, i.e. $\frac{V_h - V_l}{2+\gamma} < c \leq \frac{5(V_h - V_l)}{2(6+\gamma - \gamma^2)}$, we find that $OW^U,\{all,all\} = OW^{TS},\{all,all\} \geq OW^{TS},\{own,own\} \geq OW^{TS},\{own,all\}$. Only the uniform pricing scheme achieves the \{all, all\} sub-equilibrium for $\frac{V_h - V_l}{2+\gamma} < c \leq \frac{5(V_h - V_l)}{2(6+\gamma - \gamma^2)}$.

- For high values of $c$, i.e. $c > \frac{2(V_h - V_l)}{(2+\gamma)(3-2\gamma)}$, we find that $OW^{TS},\{own,own\} \geq OW^{TS},\{own,all\} = OW^U,\{all,all\}$. Only the trade-specific pricing scheme achieves the \{own, own\} sub-equilibrium for $c > \frac{2(V_h - V_l)}{(2+\gamma)(3-2\gamma)}$.

Hence when the cost of a non-internalized trade is low or intermediate (i.e., when $c \leq \frac{5(V_h - V_l)}{2(6+\gamma - \gamma^2)}$), the social planner will choose to maximize the trading rate through the \{all, all\} sub-equilibrium. It could do so by imposing uniform pricing. Imposing trade-specific pricing only yields the socially optimal \{all, all\} sub-equilibrium until $c \leq \frac{2(V_h - V_l)}{3(2+\gamma)}$. However, when the cost of a non-internalized transaction becomes too large (i.e. when $c > \frac{2(V_h - V_l)}{(2+\gamma)(3-2\gamma)}$) the social planner wants to prevent expensive non-internalized trades from occurring as these are welfare-reducing and prefers the \{own, own\} sub-equilibrium, thus aiming to maximize the internalization rate. Only the trade-specific pricing scheme succeeds in producing this outcome. Further, do note that with extremely high values of $c$ (or extremely low gains from trade), trade-specific pricing allows to create a market for internalized trades only, whereas markets would collapse under uniform pricing since this yields zero welfare.

Our welfare results are important because they highlight a trade-off for the social planner. Recall from Corollary 2 that the maximum liquidity for high values of $c$ is achieved under the uniform pricing scheme. However, uniform pricing produces lowest welfare in this range of $c$. As a consequence, a social planner potentially has to choose between liquidity and social welfare when setting its regulation for a pricing scheme to be implemented by the CSD: a pricing scheme implying higher market liquidity in

\(^9\)Do note that $\frac{V_h - V_l}{2+\gamma} < \frac{5(V_h - V_l)}{2(6+\gamma - \gamma^2)} < \frac{2(V_h - V_l)}{(2+\gamma)(3-2\gamma)}$ always holds when $\gamma > 0.5$ as assumed in our model.
fact reduces social welfare. Moreover, for very low \( c \), both pricing schemes yield the same welfare, although observed liquidity differs under each scheme. This leads to the following result.

**Corollary 4** The bid-ask spread is not always an appropriate measure for welfare.

### 6 Concluding Remarks

Explicit transaction costs such as the fees related to clearing and settlement are still of considerable importance in today’s financial markets. Both in the US and Europe, policies have been implemented in order to reduce clearing and settlement fees. In this paper, we model how internalization of clearing and settlement affects stock market liquidity. Our main insights can be summarized as follows. First, we find that explicit transaction costs such as clearing and settlement fees impact stock market liquidity. In general, higher clearing and settlement fees tend to increase liquidity. The reasoning is that higher clearing and settlement fees induce more aggressive limit order pricing to convince counterparties to trade. Second, internalization reduces the clearing and settlement fees. Our results show that when more trades can be internalized stock market liquidity decreases. The intuition behind this result is that it represents a drop in explicit transaction costs and therefore reduces the aggressiveness of limit order prices. Third, when the clearing and settlement agent applies marginal cost-based pricing, different equilibria result depending upon the magnitude of the post-trading costs. A low level of marginal costs of non-internalized trades is beneficial for stock market liquidity as all traders announce prices attractive for counterparties originating from any investment firm. In turn, when the costs of non-internalized trades are intermediate traders linked to a large broker quote less liquid quotes only attractive to counterparties of their own investment firm. In contrast, the quotes submitted by traders from a smaller broker remain quite liquid as they aim to attract counterparties from all brokers to maximize their execution probability. In case the marginal costs of non-internalized trades traders are substantially high stock market liquidity is harmed even more. Now both traders linked to the large and small investment firm target own-broker counterparties only. Traders from the small broker quote more illiquid prices than traders from the large broker. Finally, our welfare analysis reveals that with low post-trading costs overall welfare is lower when some (or all) traders only target counterparties from their own broker, compared to the cases where all traders aim to attract all potential counterparties, i.e. traders from all brokers. In contrast, for high post-trading costs only internalized trades produce welfare, marginal cost-based pricing achieves this outcome. Relatedly, a social planner may face a trade-off between liquidity and welfare: a more liquid market may entail lower welfare. Therefore, liquidity measures do not necessarily constitute good
proxies for welfare.
Appendix A: Proofs

Proof of Proposition 1.

The equilibrium bid and ask quotes follow immediately from solving the system of indifference conditions delineated in the main text.

Next, we derive the pricing strategy for which the CSD breaks even when it charges a uniform per-transaction fee, while accounting for the fact that internalized order flow does not imply costs. Within Proposition 3 we compute the internalization rate under uniform pricing as $IR_{\{all,all\}}^U = 1 - 2\gamma (1 - \gamma)$. As $IR_{\{all,all\}}^U$ represents the percentage of trades out of total order flow that is internalized in a given period, its complement stands for the percentage of non-internalized trades, i.e. $1 - IR_{\{all,all\}}^U = 2\gamma (1 - \gamma)$. Clearly, only the fraction $1 - IR_{\{all,all\}}^U$ induces positive marginal costs for the CSD. As the CSD is active on both sides of the market in each transaction, it should charge a fee to both legs of the trade. More specifically, a CSD charging $c^{*,U} = (1 - IR_{\{all,all\}}^U) c = 2\gamma (1 - \gamma) c$
on both legs of every transaction (internalized and non-internalized) on average breaks even: it gains on transactions for which it does not face marginal costs and loses on transactions where active clearing and settlement takes place.

Q.e.d. ■

Proof of Proposition 2.

Solving the systems of indifference equations delineated in the main text, taking clearing and settlement fees as given, results immediately in the quotes for the sub-equilibria. We thus only need to prove existence.

Thus, we now investigate under which conditions the different possible combinations of strategies correspond to a sub-equilibrium. First, the expected limit order payoffs are computed for the different combinations of strategies. Next, we will demonstrate under which conditions the different sub-equilibria will hold. Three distinct possibilities for a sub-equilibrium arise, which one is played depends on the level of the cost of clearing and settlement. As in the main text, we assume the proportion of buyers and sellers in the trader population to be equal. This will imply we only have to analyze the expected payoffs of one market side as quotes and expected payoffs of the other market side are completely symmetric. We first compute the limit order payoffs under the four possible combinations of strategies:

• $\{all, all\}$:
The expected payoff of a buyer linked to the large broker submitting $B_{TS,\{all,all\}}$ under this combination of strategies is:

$$\pi_{TS,\{all,all\}}^{TS,large} = \frac{1}{2} \left[ V_h - \left( \frac{V_h + 2V_l}{3} + (1 - \gamma) c \right) - (1 - \gamma) c \right]$$

Similarly, the expected payoff of a buyer affiliated to the small broker submitting $B_{TS,\{all,all\}}$ under this combination of strategies is:

$$\pi_{TS,\{all,all\}}^{TS,small} = \frac{1}{2} \left[ V_h - \left( \frac{V_h + 2V_l}{3} + \gamma c \right) - \gamma c \right]$$

- $\{own, all\}$:

The expected payoff of a buyer linked to the large broker submitting $B_{TS,\{own,all\}}$ under this combination of strategies is:

$$\pi_{TS,\{own,all\}}^{TS,large} = \gamma \frac{1}{2} \left[ V_h - \left( \frac{\gamma V_h + 2V_l}{2 + \gamma} \right) \right]$$

Similarly, the expected payoff of a buyer affiliated to the small broker submitting $B_{TS,\{own,all\}}$ under this combination of strategies is:

$$\pi_{TS,\{own,all\}}^{TS,small} = \frac{1}{2} \left[ V_h - \left( \frac{\gamma V_h + 2V_l}{2 + \gamma} + c \right) - \gamma c \right]$$

- $\{all, own\}$:

The expected payoff of a buyer linked to the large broker submitting $B_{TS,\{all,own\}}$ under this combination of strategies is:

$$\pi_{TS,\{all,own\}}^{TS,large} = \frac{1}{2} \left[ V_h - \left( \frac{(1 - \gamma) V_h + 2V_l}{3 - \gamma} + (1 - \gamma) c \right) \right]$$

Similarly, the expected payoff of a buyer affiliated to the small broker submitting $B_{TS,\{all,own\}}$ under this combination of strategies is:

$$\pi_{TS,\{all,own\}}^{TS,small} = (1 - \gamma) \frac{1}{2} \left[ V_h - \left( \frac{(1 - \gamma) V_h + 2V_l}{3 - \gamma} \right) \right]$$

- $\{own, own\}$:
The expected payoff of a buyer linked to the large broker submitting $B^{TS,own,own}_{large}$ under this combination of strategies is:

$$\pi_{large}^{TS,own,own} = \frac{1}{2} \left[ V_h - \left( \frac{\gamma V_h + 2 V_l}{2 + \gamma} \right) \right]$$

Similarly, the expected payoff of a buyer affiliated to the small broker submitting $B^{TS,own,own}_{small}$ under this combination of strategies is:

$$\pi_{small}^{TS,own,own} = (1 - \gamma) \frac{1}{2} \left[ V_h - \left( \frac{(1 - \gamma) V_h + 2 V_l}{3 - \gamma} \right) \right]$$

We now derive under which conditions the different sub-equilibria apply:

1. Sub-equilibrium $\{all, all\}$ applies when two conditions are jointly satisfied. First, traders at the large broker should have no incentives to deviate to the own-strategy when traders at the small broker play the all-strategy, i.e. this applies when:

$$\pi_{large}^{TS,all,all} \geq \pi_{large}^{TS,own,all}, \text{ or } c \leq \frac{2 (V_h - V_l)}{3 (2 + \gamma)}$$

Secondly, traders at the small broker should have no incentives to deviate to the own-strategy when traders at the large broker play the all-strategy:

$$\pi_{small}^{TS,all,all} \geq \pi_{small}^{TS,all,own}, \text{ or } c \leq \frac{2 (V_h - V_l)}{3 (3 - \gamma)}$$

Given $\gamma > 0.5$, we have that $c \leq \frac{2(V_h-V_l)}{3(2+\gamma)}$ is binding. If this condition is satisfied, this sub-equilibrium holds.

2. Sub-equilibrium $\{own, all\}$ applies when two conditions are jointly satisfied. First, traders at the large broker should have no incentives to deviate to the all-strategy when traders at the small broker play the own-strategy, i.e. this applies when:

$$\pi_{large}^{TS,own,all} > \pi_{large}^{TS,all,all}, \text{ or } c > \frac{2 (V_h - V_l)}{3 (2 + \gamma)}$$

Secondly, traders at the small broker should have no incentives to deviate to the own-strategy when traders at the large broker play the own-strategy:

$$\pi_{small}^{TS,own,all} \geq \pi_{small}^{TS,own,own}, \text{ or } c \leq \frac{2 (V_h - V_l) (1 + \gamma^2)}{(1 + \gamma) (2 + \gamma) (3 - \gamma)}$$

---

10 An underlying assumption in this derivation is that if traders are indifferent between the payoffs of the all and the own-strategy (which is the case at the cutoff values of $c$), the all-strategy is preferred.
Thus, when \( \frac{2(V_h - V_l)}{3(2 + \gamma)} < c \leq \frac{2(V_h - V_l)(1 + \gamma^2)}{(1 + \gamma)(2 + \gamma)(3 - \gamma)} \) the strategies are deviation-proof, and thus this sub-equilibrium holds.

3. Sub-equilibrium \( \{\text{own, all}\} \) applies (using similar reasoning) when:

\[
\pi_{\text{large}}^{TS,\{\text{all,own}\}} \geq \pi_{\text{large}}^{TS,\{\text{own,own}\}}, \text{ or } c \leq \frac{2 \left( V_h - V_l \right) \left( \gamma^2 - 2\gamma + 2 \right)}{(2 + \gamma) (2 - \gamma) (3 - \gamma)}
\]

and

\[
\pi_{\text{small}}^{TS,\{\text{all,own}\}} > \pi_{\text{small}}^{TS,\{\text{all,all}\}}, \text{ or } c > \frac{2 \left( V_h - V_l \right)}{3 (3 - \gamma)}
\]

For \( \gamma > 0.5 \), both conditions could never be jointly met, hence this combination of strategies will never realize and forms no sub-equilibrium.

4. Sub-equilibrium \( \{\text{own, own}\} \) applies (using similar reasoning) when:

\[
\pi_{\text{large}}^{TS,\{\text{own,own}\}} > \pi_{\text{large}}^{TS,\{\text{own,all}\}}, \text{ or } c > \frac{2 \left( V_h - V_l \right) \left( \gamma^2 - 2\gamma + 2 \right)}{(2 + \gamma) (2 - \gamma) (3 - \gamma)}
\]

and

\[
\pi_{\text{small}}^{TS,\{\text{own,own}\}} > \pi_{\text{small}}^{TS,\{\text{own,all}\}}, \text{ or } c > \frac{2 \left( V_h - V_l \right) (1 + \gamma^2)}{(1 + \gamma) (2 + \gamma) (3 - \gamma)}
\]

Further comparison shows that \( c > \frac{2(V_h - V_l)(1 + \gamma^2)}{(1 + \gamma)(2 + \gamma)(3 - \gamma)} \) is the most stringent condition, thus if it is satisfied this sub-equilibrium holds.

Q.e.d.

**Proof of Proposition 3.**

The proof is immediate by filling in the stationary probability distribution results of Appendix B in the definition of trading rate and internalization rate.

Q.e.d.

**Proof of Proposition 4.**

The proof is immediate by filling in the computed trading rates and internalization rates (see Proposition 3) in the overall welfare definition.

Q.e.d.
Appendix B: Infinite Markov chain in this model

At any given discrete point in time $t$, the market can be in six possible states: (1) a trader from the small broker arrives and submits a limit order; (2) a trader from the small broker arrives and submits an internalized market order; (3) a trader from the small broker arrives and submits a non-internalized market order; (4) a trader from the large broker arrives and submits a limit order; (5) a trader from the large broker arrives and submits an internalized market order; (6) a trader from the large broker arrives and submits a non-internalized market order. These six states form a finite state space. For each possible sub-equilibrium $s^k$ corresponding to pricing scheme $k$, a Markov chain (with the property that the next state depends only on the current state) could be constructed with transition matrix $\hat{M}^{k,s^k}$, which is a $6 \times 6$ matrix capturing all transitions from one state to another (see Colliard and Foucault (2010) for a similar approach). These matrices reflect the transition probabilities corresponding to the equilibrium decisions under the considered sub-equilibrium, and could be written as follows:

$$\hat{M}^{U\{all,all\}} = \hat{M}^{TS\{all,all\}} = \begin{bmatrix} \frac{1-\gamma}{2} & \frac{1-\gamma}{2} & 0 & \frac{\gamma}{2} & 0 & \frac{\gamma}{2} \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ \frac{1-\gamma}{2} & 0 & \frac{1-\gamma}{2} & \frac{\gamma}{2} & \frac{\gamma}{2} & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \end{bmatrix}$$

$$\hat{M}^{TS\{own,all\}} = \begin{bmatrix} \frac{1-\gamma}{2} & \frac{1-\gamma}{2} & 0 & \frac{\gamma}{2} & 0 & \frac{\gamma}{2} \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ \frac{1-\gamma}{2} & 0 & \frac{1-\gamma}{2} & \frac{\gamma}{2} & \frac{\gamma}{2} & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \end{bmatrix}$$

$$\hat{M}^{TS\{all,own\}} = \begin{bmatrix} \frac{1-\gamma}{2} & \frac{1-\gamma}{2} & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ \frac{1-\gamma}{2} & 0 & \frac{1-\gamma}{2} & \frac{\gamma}{2} & \frac{\gamma}{2} & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \\ 1-\gamma & 0 & 0 & \gamma & 0 & 0 \end{bmatrix}$$

11 We do not need to make a distinction between buyers and sellers because both sides of the market are symmetric in our model.
\[
\begin{bmatrix}
\frac{1-\gamma}{2} & \frac{1-\gamma}{2} & 0 & \gamma & 0 & 0 \\
1-\gamma & 0 & 0 & \gamma & 0 & 0 \\
1-\gamma & 0 & 0 & \gamma & 0 & 0 \\
1-\gamma & 0 & 0 & \gamma & 0 & 0 \\
1-\gamma & 0 & 0 & \gamma & 0 & 0 \\
1-\gamma & 0 & 0 & \gamma & 0 & 0 \\
\end{bmatrix}
\]

From each of these right stochastic transition matrices, in which each row sums to one and all elements are non-negative, it is possible to derive the stationary probability distribution over all states. More specifically, the stationary distribution \( \varphi^{k,s_k} \) is a row vector satisfying \( \varphi^{k,s_k} = \varphi^{k,s_k} \overline{M}^{k,s_k} \), i.e. \( \varphi^{k,s_k} \) is a normalized left eigenvector of \( \overline{M}^{k,s_k} \) associated with the eigenvalue 1. Do note that as this Markov chain is irreducible and aperiodic, the stationary distribution \( \varphi^{k,s_k} \) is unique. Let \( \varphi_m^{k,s_k} \) with \( m = 1, ..., 6 \) be the stationary probability of occurrence of state \( m \) under the considered sub-equilibrium. Then the stationary probability distribution could be denoted as \( \varphi^{k,s_k} = (\varphi_1^{k,s_k}, \varphi_2^{k,s_k}, \varphi_3^{k,s_k}, \varphi_4^{k,s_k}, \varphi_5^{k,s_k}, \varphi_6^{k,s_k}) \). This distribution \( \varphi^{k,s_k} \) could be derived for each of the sub-equilibria as:

\[
\begin{align*}
\varphi^{U,\{all,all\}} &= \varphi^{TS,\{all,all\}} = \left( \frac{2(1-\gamma)}{3}, \frac{(\gamma-1)^2}{3}, \frac{\gamma(1-\gamma)}{3}, \frac{2\gamma}{3}, \frac{\gamma^2}{3}, \frac{\gamma(1-\gamma)}{3} \right); \\
\varphi^{TS,\{own,all\}} &= \left( \frac{2(\gamma-1)}{\gamma-3}, \frac{\gamma^2-2\gamma+1}{3-\gamma}, \frac{4\gamma}{(\gamma+2)(3-\gamma)}, \frac{2\gamma^2}{(\gamma+2)(3-\gamma)}, \frac{\gamma(1-\gamma)}{\gamma-3} \right); \\
\varphi^{TS,\{all,own\}} &= \left( \frac{4(\gamma-1)}{(\gamma+2)(\gamma-3)}, \frac{2(\gamma^2-2\gamma+1)}{(\gamma+2)(3-\gamma)}, \frac{(\gamma+1-\gamma)}{\gamma+2}, \frac{2\gamma}{\gamma+2}, \frac{\gamma^2}{\gamma+2}, \frac{0}{\gamma+2} \right); \\
\varphi^{TS,\{own,own\}} &= \left( \frac{2(\gamma-1)}{\gamma-3}, \frac{\gamma^2-2\gamma+1}{3-\gamma}, \frac{0}{\gamma+2}, \frac{2\gamma}{\gamma+2}, \frac{\gamma^2}{\gamma+2}, \frac{0}{\gamma+2} \right).
\end{align*}
\]

and could also be seen as the proportion of time spent in each state within the considered sub-equilibrium.
References


Figure 1: Ask Quotes Under CSD Pricing Schemes

Note: This figure illustrates the results of our model using the following parameter values: $V_t = 20$, $V_l = 0$ and $\gamma = 0.8$. The marginal cost $c$, shown on the x-axis, varies in the interval $[0, 20]$. Panel A presents ask quotes of traders of the large broker (full lines) and small broker (dotted lines) under Uniform (Green) and Trade Specific (Black) CSD Pricing Schemes. Panel B shows the weighted average ask quotes across traders of the large and small brokers, with the weights being the market shares of the respective brokers ($\gamma$ and $1-\gamma$). The ask quotes are computed in Proposition 1 (Uniform Pricing) and Proposition 2 (Trade Specific Pricing).
Figure 2: Welfare

Note: This figure illustrates the results of our model using the following parameter values: $V_b = 20$, $V_l = 0$ and $\gamma = 0.8$. The marginal cost $c$, shown on the $x$-axis, varies in the interval $[0, 20]$. The figure presents overall welfare under Uniform (Green) and Trade Specific (Black) CSD Pricing Schemes as computed in Proposition 3.