Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian Model*

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Abstract

This paper estimates and solves a multi-country version of the standard DSGE New Keynesian (NK) model. The country-specific models include a Phillips curve determining inflation, an IS curve determining output, a Taylor Rule determining interest rates, and a real effective exchange rate equation. The IS equation includes a real exchange rate variable and a country-specific foreign output variable to capture direct inter-country linkages. In accord with the theory all variables are measured as deviations from their steady states, which are estimated as long-horizon forecasts from a reduced-form cointegrating global vector autoregression. The resulting rational expectations model is then estimated for 33 countries on data for 1980Q1-2006Q4, by inequality constrained IV, using lagged and contemporaneous foreign variables as instruments, subject to the restrictions implied by the NK theory. The multi-country DSGE NK model is then solved to provide estimates of identified supply, demand and monetary policy shocks. Following the literature, we assume that the within country supply, demand and monetary policy shocks are orthogonal, though shocks of the same type (e.g. supply shocks in different countries) can be correlated. We discuss estimation of impulse response functions and variance decompositions in such large systems, and present estimates allowing for both direct channels of international transmission through regression coefficients and indirect channels through error spillover effects. Bootstrapped error bands are also provided for the cross country responses of a shock to the US monetary policy.

Keywords: Global VAR (GVAR), New Keynesian DSGE models, supply shocks, demand shocks, monetary policy shocks.

JEL Classification : C32, E17, F37, F42.

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1 Introduction

Business cycle fluctuations transmit both domestically and through the international economy and it is important to determine the extent to which macroeconomic fluctuations result from exogenous national or global shocks to demand, supply or monetary policy. In this paper we provide measures of the effects of such shocks using a multi-country New Keynesian model. While there is a literature using multi-country vector autoregressions (VARs) to model the international transmission of shocks, including Canova and Ciccarelli, 2009 and Dees, di Mauro, Pesaran and Smith (2007, DdPS) who use a global VAR, (GVAR), so far it has proved difficult to use such reduced-form multi-country VARs to examine the effects of structural shocks with clear economic interpretation.

To identify and measure the relative importance of different types of structural shocks much recent literature has used New Keynesian dynamic stochastic general equilibrium, DSGE, models.[1] Within this literature, structural shocks are associated with errors in the log-linearised versions of the first order conditions for households and firms' optimisation problems, with the variables measured as deviations from the steady states. The number and naming of the shocks tend to be model specific. For instance Smets and Wouters (2007) consider a model with seven shocks that they label as technology, risk premium, investment, government spending, wage mark-ups, price mark-ups and monetary policy, though they can be grouped into supply, demand and monetary policy shocks. These models are then used to examine the effect of identified shocks.[2]

Most of the literature attempting to measure the effect of structural shocks has assumed a closed-economy setting. Carabenciov *et al.* (2008, p.6) who consider developing multi-country models, state that "Large scale DSGE models show promise in this regard, but we are years away from developing empirically based multi-country versions of these models". The open economy contributions have tended to use either models for two economies of comparable size, such as the euro area and the US (as in de Walque *et al.* , 2005, for example), or small open economy, SOE, models where the rest of the world is treated as exogenous. Lubik and Schorfheide (2007), building on the theoretical contributions of Gali and Monacelli (2005), estimate small-scale structural general equilibrium models, similar to the ones estimated below, for Australia, Canada, New Zealand and UK, but, unlike the approach taken in this paper, they treat each of these economies separately, not allowing for the interactions between them.[3] Given the questions they were concerned

[1] Other approaches to identification of structural shocks have also been considered in the literature. These include the structural VARs proposed by Blanchard and Quah (1989) and identification by sign restrictions on the impulse responses proposed by Uhlig (2005). It is unclear how these approaches can be extended to multi-country models. The structural VAR identification scheme has also been recently criticised by Carlstrom et al. (2009).

[2] As a matter of convenience, we refer to this NK model as structural, since under certain standard theoretical assumptions, its parameters can be related to a set of 'deeper' parameters of technology and tastes, but in this paper we do not take a particular position on this interpretation or on the rational expectations and representative agent assumptions on which these models are based: our goal is to generalise the standard DSGE NK model to a multi-country setting.

[3] The DSGE models considered by Lubik and Schofheide (2007) also lack any backward components and have been criticised by Fukac and Pagan (2010) as being dynamically misspecified.

with, this was not a problem; but when one wishes to measure the effects of structural shocks in a multi-country framework as we do, one needs to allow for the interactions across economies. Compared to a two-country model, an interacting multi-country model raises a range of additional conceptual and technical questions, in particular about the cross-country correlation of shocks and the determination of exchange rates.

This paper answers these conceptual and technical questions within a general framework for the structural analysis of multi-country interactions. The approach is implemented by estimating and solving a relatively large multi-country New Keynesian (MCNK) model, comprising 33 countries on quarterly data over the period 1979Q1-2006Q4. The country-specific models include a Phillips curve representing the aggregate supply equation, an IS curve representing the aggregate demand equation, a Taylor rule for the monetary policy equation, and a reduced-form real effective exchange rate equation. The IS equation includes an exchange rate variable and a country-specific foreign output variable to capture direct inter-country linkages. The US economy is treated differently because the US dollar is used as the numeraire for exchange rates. As a result the US real exchange rate is equal to the inverse of the US price level, and although a Phillips curve determining inflation is estimated for the US, the model is solved in terms of the log US price level, so that country-specific real exchange rates can be determined.

In accord with the theory all variables are measured as deviations from their steady states, which are estimated explicitly as the long-horizon forecasts obtained from a reduced-form cointegrating global vector autoregressive (GVAR) model advanced in Pesaran, Schuermann and Weiner (2004) and further developed in DdPS. The steady states derived under this approach, by taking full account of any stochastic trends and cointegrating relations that might be present in the historical observations, yield deviations that are stationary as required by the theory. The steady states also have the advantage that they are based on long-run economic models that are theoretically consistent with the short-run DSGE model based on the resultant deviations as discussed in Dees, Pesaran, Smith and Smith (2009, DPSS).

The parameters of the structural equations for each country are estimated by the instrumental variable (IV) method subject to inequality restrictions implied by the macroeconomic theory. These include a restriction on the sum of the coefficients of the backward and forward inflation components of the Phillips curve (PC) in order to avoid indeterminate solutions. There has been some concern in the literature as to whether the parameters of such DSGE models are in fact identified, e.g. Canova and Sala (2009). DPSS argue that a multi-country perspective can help identification by using trade-weighted averages of foreign variables as instruments. When there is a large number of countries, these foreign averages can be treated as weakly exogenous for estimation, even though they are endogenous to the system as a whole. This allows consistent estimates of the equation parameters and thus of the structural errors. The interpretation of these as structural shocks requires further restrictions on their correlation, discussed below.

The resulting multi-country rational expectations model is then solved for a unique stable solution. It turns out that theory restrictions together with the restriction on the coefficients of the inflation variables in the PC are sufficient to arrive at a unique stable solution. To our knowledge this is the first time that such a multi-country New Keynesian model under rational expectations has been estimated and solved for a unique stable solution.

The solution is then used to obtain estimates of the supply, demand and monetary policy shocks for all the 33 countries (when applicable). In accordance with the literature, we assume that the within country supply, demand and monetary policy shocks are pair-wise orthogonal, though shocks of the same type (e.g. supply shocks across different countries) can be correlated. We also allow for non-zero correlations between the structural shocks and the reduced form real exchange rate shocks. When the model is solved, the variables in the global economy (all taken to be endogenously determined) can be written as functions of current and past values of the structural shocks, enabling us to calculate structural impulse responses and variance decompositions that allow for the possible correlations of supply, demand and monetary policy shocks across countries. The model allows both for direct channels of international transmission of shocks through contemporaneous effects of foreign variables and indirect channels through error spillover effects.

The rich structure of the multi-country model allows one to address many issues of interest; we shall focus on two different sets of questions. We first examine the impact of a US monetary policy shock on inflation and output deviations in the US and how these effects are then transmitted to the rest of the world, in particular to China, Japan and the euro area economies. This is a natural question given the dominant role of the US in the world economy and the large literature on the effects of US monetary policy shocks. Secondly we examine the effects of global demand and supply shocks (defined as PPP GDP weighted averages of country-specific shocks) on output, inflation and interest rates, distinguishing between direct and indirect channels of transmission of shocks in the global economy. We also investigate the importance of direct channels of transmission by considering a MCNK model without foreign output effects, and examine the importance of using GVAR deviations by estimating an alternative MCNK specification where output deviations are computed using the Hodrick-Prescott filter. The results confirm the importance of allowing for direct channels of international transmission as well as using a cointegrating model for the computation of stationary output, inflation and interest rate deviations. The impulse response results and their bootstrapped bounds are in line with the main predictions of the NK macroeconomic theory and tend to be qualitatively similar across countries. A summary of the main findings is provided in the concluding section.

The rest of this paper is set out as follows: Section 2 describes the structure of the model: the form of the country-specific models, how the countries are linked, and the solution of the multi-country rational expectations model. Section 3 explains the framework for global shock accounting used to calculate the impulse response functions and forecast error variance decompositions, which describe the effects of composite shocks on composite variables. Section 4 considers the issue of estimating the deviations from the steady states. Section 5 presents the parameter estimates and discusses the theory restrictions imposed to ensure that the multi-country NK model has a stable solution and the parameter estimates have the signs predicted by the NK macroeconomic theory.

Section 6 examines the effect of various supply, demand and monetary policy shocks. Section 7 considers various extensions and alternative assumptions. Section 8 concludes.

2 The Multi-Country NK model

2.1 Individual equations of the country-specific models

We first describe the individual equations of the country-specific models, then discuss how they are integrated within a multi-country setting and how the resulting rational expectations model is solved. While the framework used is general, the specific model used for illustration is designed to be as close as possible to the standard three equation closed economy New Keynesian models routinely estimated in the literature.[4] This standard model is augmented to allow for inter-country linkages and estimated for 33 countries subject to a number of *a priori* restrictions from economic theory.

In particular, we consider a multi-country model composed of $N+1$ countries, indexed by i, where $i = 0, 1, 2, ..., N$. The US, $i = 0$, is treated differently, since the dollar is used as the numeraire currency. The variables for each country are measured as deviations from the steady states, the measurement of which is discussed in Section 4. For country $i = 1, 2, ..., N$ the variables included are inflation deviations, $\widetilde{\pi}_{it}$, output deviations, $\widetilde{y}_{it}$, the interest rate deviations, $\widetilde{r}_{it}$ and the real effective exchange rate deviations, $\widetilde{re}_{it}$, except for Saudi Arabia where an interest rate variable is not available. The US model includes only the variables: $\widetilde{\pi}_{0t}$, $\widetilde{y}_{0t}$, and $\widetilde{r}_{0t}$, since (as it is shown below) the US real exchange rate is proportional to its price level. We also use country-specific foreign variables, which are trade weighted averages of the corresponding variables for other countries. For example the foreign output variable of country i is defined by $\widetilde{y}^*_{it} = \Sigma_{j=0}^{N} w_{ij}\widetilde{y}_{jt}$, where w_{ij} is the trade weight of country j in the total trade (exports plus imports) of country i. By construction $\sum_{j=0}^{N} w_{ij} = 1$, $w_{ii} = 0$.

The treatment of exchange rates is central to the construction of a coherent multi-country model and a more detailed discussion of the issues involved is in order. Denote the log nominal exchange rate of country i against the US dollar by e_{it}, and the bilateral log exchange rate of country i with respect to country j by e_{ijt}. It is easily seen that $e_{ijt} = e_{it} - e_{jt}$, and the log real effective exchange rate of country i with respect to its trading partners is then given by

$$re_{it} = \sum_{j=0}^{N} w_{ij}(e_{it} - e_{jt}) + \sum_{j=0}^{N} w_{ij}p_{jt} - p_{it},$$

[4] Ireland (2004), for example, notes that 'The development of the forward-looking microfounded New Keynesian model stands, in the eyes of many observers, as one of the past decade's most exciting and significant achievements in macroeconomics.' As examples of this achievement he cites Clarida *et al.* (1999) and Woodford (2003).

where p_{it} is the log general price level in country i. Therefore (recalling that $\sum_{j=0}^{N} w_{ij} = 1$)

$$
\begin{aligned}
re_{it} &= (e_{it} - p_{it}) - \sum_{j=0}^{N} w_{ij}(e_{jt} - p_{jt}), \\
&= ep_{it} - ep_{it}^{*},
\end{aligned}
\tag{1}
$$

where $ep_{it}^{*} = \sum_{j=0}^{N} w_{ij} ep_{jt}$. Deviations from steady states are defined accordingly as $\widetilde{re}_{it} = \widetilde{ep}_{it} - \widetilde{ep}_{it}^{*}$.

For the US, $e_{0t} = 0$, and $ep_{0t} = -p_{0t}$, which is determined by the US Phillips curve equation. Specification of a separate exchange rate equation for the US will not be needed. Accordingly, in what follows we shall consider equations for the log real effective exchange rates for countries $i = 1, 2, ..., N$, and solve for the $N + 1$ log real exchange rates, $\widetilde{ep}_{it}$, $i = 0, 1, 2, ..., N$, with the log US real exchange rate deviations being given by $-\widetilde{p}_{0t}$. It is important that possible stochastic trends in the log US price level are appropriately taken into account when computing $\widetilde{p}_{0t}$. This can be achieved by first estimating $\widetilde{\pi}_{0t}$ and then cumulating the values of $\widetilde{\pi}_{0t}$ to obtain $\widetilde{p}_{0t}$ up to an arbitrary constant.

The equations in the country-specific models include a standard Phillips curve (PC), derived from the optimising behaviour of monopolistically competitive firms subject to nominal rigidities, which determines inflation deviations $\widetilde{\pi}_{it}$, where $\pi_{it} = p_{it} - p_{i,t-1}$. This takes the form

$$
\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{if} E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right) + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st},\ i = 0, 1, ..., N,
\tag{2}
$$

where $E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right) = E\left(\widetilde{\pi}_{i,t+1} \mid \Im_{i,t-1}\right)$. There are no intercepts included in the equations since deviations from steady state values have mean zero by construction. The error term, $\varepsilon_{i,st}$, is interpreted as a supply shock or a shock to the price-cost margin in country i. The parameters are non-linear functions of underlying structural parameters. For instance, suppose that there is staggered price setting, with a proportion of firms, $(1 - \theta_i)$, resetting prices in any period, and a proportion θ_i keeping prices unchanged. Of those firms able to adjust prices only a fraction $(1 - \omega_i)$ set prices optimally on the basis of expected marginal costs. A fraction ω_i use a rule of thumb based on lagged inflation. Then for a subjective discount factor, λ_i, we have

$$
\begin{aligned}
\beta_{if} &= \lambda_i \theta_i \phi_i^{-1},\ \beta_{ib} = \omega_i \phi_i^{-1}, \\
\beta_{iy} &= (1 - \omega_i)(1 - \theta_i)(1 - \lambda_i \theta_i)\phi_i^{-1},
\end{aligned}
$$

where $\phi_i = \theta_i + \omega_i[1 - \theta_i(1 - \lambda_i)]$. Notice that there is no reason for these parameters to be the same across countries with very different market institutions and property rights (which will influence λ_i), so we allow them to be heterogeneous from the start. If $\omega_i = 0$, all those who adjust prices do so optimally, then $\beta_{fi} = \lambda_i$, and $\beta_{bi} = 0$. Since $\theta_i \geq 0$, $\omega_i \geq 0$, $\lambda_i \geq 0$ the theory implies $\beta_{ib} \geq 0$, $\beta_{if} \geq 0$, and $\beta_{iy} \geq 0$, which we impose in estimation. The restriction $\beta_{ib} + \beta_{if} < 1$ ensures a unique rational expectations solution in the case where $\tilde{y}_{it}$ is exogenously given and there are no feedbacks from lagged values of inflation to the output gap. The corresponding condition in a multi-country model is likely to be more complicated. We use the restriction $\beta_{ib} + \beta_{if} \leq 0.99$, where the equality corresponds to a 4% per annum discount rate which is often imposed, but this condition might not be sufficient for the model to have a unique solution.

The aggregate demand or IS curve is obtained by log-linearising the Euler equation in consumption and substituting the result in the economy's aggregate resource constraint. In the standard closed economy case, this yields an equation for the output gap, $\widetilde{y}_{it}$, which depends on the expected future output gap, $E_{t-1}(\widetilde{y}_{i,t+1})$, and the real interest rate deviations, $\widetilde{r}_{it} - E_{t-1}(\widetilde{\pi}_{i,t+1})$. Lagged output will enter the IS equation if the utility of consumption for country i at time t is $u(C_{it} - h_i C_{i,t-1})$ where h_i is a habit persistence parameter. For an open economy model, the aggregate resource constraint will also contain net exports, which in turn will be a function of the real effective exchange rate, $\widetilde{re}_{it}$, and the foreign output gap, $\widetilde{y}^*_{it}$. The open economy version of the standard IS equation is then

$$\widetilde{y}_{it} = \alpha_{ib}\widetilde{y}_{i,t-1} + \alpha_{if}E_{t-1}(\widetilde{y}_{i,t+1}) + \alpha_{ir}[\widetilde{r}_{it} - E_{t-1}(\widetilde{\pi}_{i,t+1})] + \alpha_{ie}\widetilde{re}_{it} + \alpha_{iy*}\widetilde{y}^*_{it} + \varepsilon_{i,dt},\ i = 0, 1, ..., N.$$

The coefficient of the real interest rate, α_{ir}, is interpreted as the inter-temporal elasticity of consumption, see Clarida *et al.* (1999), while $\alpha_{if} = 1/(1 + h_i)$ and $\alpha_{ib} = h_i/(1 + h_i)$. The error, $\varepsilon_{i,dt}$, is interpreted as a demand shock. A number of authors note that unless technology follows a pure random walk process, $\varepsilon_{i,dt}$ may reflect technology shocks, though by conditioning on the foreign output variable the convolution of demand shocks with technology shocks might be somewhat obviated. As discussed further in Section 5, the unrestricted estimates of this equation in the case of many countries resulted in a positive coefficient on the interest rate variable, and given the importance of the interest rate effects in the standard model we decided to impose the restriction $\alpha_{if} = 0$ for all i. Thus the IS equation used in the model is

$$\widetilde{y}_{it} = \alpha_{ib}\widetilde{y}_{i,t-1} + \alpha_{ir}[\widetilde{r}_{it} - E_{t-1}(\widetilde{\pi}_{i,t+1})] + \alpha_{ie}\widetilde{re}_{it} + \alpha_{iy*}\widetilde{y}^*_{it} + \varepsilon_{i,dt},\ i = 0, 1, ..., N, \tag{3}$$

subject to the restrictions $\alpha_{ir} \leq 0$, $\alpha_{iy*} \geq 0$. The analysis of the more general case where $\alpha_{if} \neq 0$, might require consideration of other factors such as financial as well as real variables. But such an extension is beyond the scope of the present paper and will not be pursued here.

The interest rate deviations in country i, $\widetilde{r}_{it}$, (except for Saudi Arabia where interest rate data are not available) are set according to a standard Taylor rule (TR) of the form:

$$\widetilde{r}_{it} = \gamma_{ib}\widetilde{r}_{i,t-1} + \gamma_{i\pi}\widetilde{\pi}_{it} + \gamma_{iy}\widetilde{y}_{it} + \varepsilon_{i,mt},\ i = 0, 1, ..., N. \tag{4}$$

The error $\varepsilon_{i,mt}$ is interpreted as a monetary policy shock.

The log real effective exchange rate deviations, $\widetilde{re}_{it}$, are modelled as a stationary first order autoregression,[5]

$$\widetilde{re}_{it} = \rho_i\widetilde{re}_{i,t-1} + \varepsilon_{i,et},\ |\rho_i| < 1,\ i = 1, 2, ..., N. \tag{5}$$

[5] Since the model explains the exchange rate and the forward rate (from domestic and foreign interest rates) it implicitly defines the uncovered interest parity risk premium.

As noted earlier we do not need a separate exchange rate equation for the US, since the US log real effective exchange rate is given as an *exact* linear combination of the other N log real effective exchange rates.

Putting equations (2) to (5) together for all 33 countries, the total number of variables in the multi-country model is $k = \sum_{i=0}^{N} k_i = 130$, where k_i is the number of variables in country i. For the US, with no exchange rate equation, $k_0 = 3$, for Saudi Arabia, with no interest rate equation, $k_{SA} = 3$, for the other 31 countries $k_i = 4$. With 130 endogenous variables the system is already quite large, but can be readily extended to include oil prices, and financial variables such as real equity prices and long term interest rates. The reduced form GVAR model developed in DdPS does include such variables, but these are excluded from the current exercise since the primary aim here is to analyse a multi-country version of the standard New Keynesian model that excludes financial variables.

The parameters of the multi-country model can be estimated consistently for each country separately by instrumental variables (IV) subject to the theory restrictions referred to above. As instruments, following the argument in DPSS, we use an intercept, the lagged values of the country-specific endogenous variables $\widetilde{y}_{i,t-1}$, $\widetilde{\pi}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}$, the current values of the foreign variables $\widetilde{y}^*_{it}, \widetilde{\pi}^*_{it}, \widetilde{r}^*_{it}$, and the log oil price deviation, $\widetilde{p}^o_t$.[6] The details of the estimation procedure and the estimation results are discussed further in Section 5.

The estimates of the structural parameters can then be used to estimate the country-specific structural shocks, namely the supply, demand and monetary policy shocks as denoted by $\varepsilon_{i,st}, \varepsilon_{i,dt}$ and $\varepsilon_{i,mt}$, respectively, for $i = 0, 1, ..., N$. As far as the cross correlations of the structural shocks are concerned we follow the literature and assume that these shocks are pair-wise orthogonal within each country, but allow for the shocks of the same type to be correlated across countries. In a multi-country context it does not seem plausible to assume that shocks of the same type are orthogonal across countries. Consider neighbouring economies with similar experiences of supply disruptions, or small economies that are affected by the same supply shocks originating from a dominant economy. As discussed in Chudik and Pesaran (2010), it is possible to deal with such effects explicitly by conditioning the individual country equations on the current and lagged variables of the dominant economy (if any), as well as on the variables of the neighbouring economies. This has been done partly in the specification of the IS equations. But following such a strategy more generally takes us away from the standard New Keynesian model and will not be pursued here. Instead we shall try to deal with such cross-country dependencies through suitably restricted error correlations.

We also allow the exchange rate shocks, $\varepsilon_{i,et}$, defined by (5), to have non-zero correlations with the other shocks both within and across the countries. This yields the main case we consider: a block diagonal error covariance matrix which is bordered by non-zero covariances between $\varepsilon_{i,et}$ and $(\varepsilon_{i,st}, \varepsilon_{i,dt}, \varepsilon_{i,mt})$, though we shall also consider other more restricted versions of the covariance

[6] Given the importance of oil prices for the determination of steady state inflation and possibly real exchange rates we included an oil price variable in the reduced form GVAR model which is used for the estimation of the steady states.

matrix. There is also an estimation issue: since the dimension of the endogenous variables, $k = 130$, is larger than the time series dimension, T, an unrestricted (sample) estimate of the variance covariance matrix of the errors is rank deficient and is not guaranteed to be a positive definite matrix. We discuss this further in Section 5.

2.2 Solution of the multi-country RE model

We now consider linking the country-specific models and solving the resultant multi-country RE model. For all countries $i = 0, 1, ..., N$, let $\widetilde{\mathbf{x}}_{it} = (\widetilde{\pi}_{it}, \widetilde{y}_{it}, \widetilde{r}_{it}, \widetilde{ep}_{it})'$ with the associated global $(k+1) \times 1$ vector $\widetilde{\mathbf{x}}_t = (\widetilde{\mathbf{x}}'_{0t}, \widetilde{\mathbf{x}}'_{1t}, ..., \widetilde{\mathbf{x}}'_{Nt})'$, so that $\widetilde{\mathbf{x}}_{0t}$ includes the redundant US real exchange rate variable. This is because although in the US model $\widetilde{ep}_{0t} = -\widetilde{p}_{0t}$ and $\widetilde{\pi}_{0t}$ are related, $\widetilde{ep}_{0t}$ is still needed for the construction of $\widetilde{ep}^*_{it}$, $i = 0, 1, ...N$ that enter the IS equations.

In terms of $\widetilde{\mathbf{x}}_{it}$ the country-specific models based on equations (2), (3), (4) and (5) can be written as

$$\mathbf{A}_{i0}\widetilde{\mathbf{x}}_{it} = \mathbf{A}_{i1}\widetilde{\mathbf{x}}_{i,t-1} + \mathbf{A}_{i2}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1}) + \mathbf{A}_{i3}\widetilde{\mathbf{x}}^*_{it} + \mathbf{A}_{i4}\widetilde{\mathbf{x}}^*_{i,t-1} + \boldsymbol{\varepsilon}_{it}, \text{ for } i = 0, 1, ..., N, \tag{6}$$

where $\widetilde{\mathbf{x}}^*_{it} = (\widetilde{y}^*_{it}, \widetilde{ep}^*_{it})'$, and as before $\widetilde{y}^*_{it} = \sum_{j=0}^{N} w_{ij}\widetilde{y}_{jt}$, and $\widetilde{ep}^*_{it} = \sum_{j=0}^{N} w_{ij}\widetilde{ep}_{jt}$. The expectations are taken with respect to a common global information set formed as the union intersection of the individual country information sets, $\mathfrak{I}_{i,t-1}$.

For US, $i = 0$

$$\mathbf{A}_{00} = \begin{pmatrix} 1 & -\beta_{0y} & 0 & 0 \\ 0 & 1 & -\alpha_{0r} & -\alpha_{0e} \\ -\gamma_{0\pi} & -\gamma_{0y} & 1 & 0 \end{pmatrix}, \quad \mathbf{A}_{01} = \begin{pmatrix} \beta_{0b} & 0 & 0 & 0 \\ 0 & \alpha_{0b} & 0 & 0 \\ 0 & 0 & \gamma_{0b} & 0 \end{pmatrix},$$

$$\mathbf{A}_{02} = \begin{pmatrix} \beta_{0f} & 0 & 0 & 0 \\ -\alpha_{0r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{A}_{03} = \begin{pmatrix} 0 & 0 \\ \alpha_{0y*} & -\alpha_{0e} \\ 0 & 0 \end{pmatrix},$$

and $\boldsymbol{\varepsilon}_{0t} = (\varepsilon_{0,st}, \varepsilon_{0,dt}, \varepsilon_{0,mt})'$. Note that $\mathbf{A}_{04} = \mathbf{0}$, since there is no exchange rate equation for the US.

For the other countries, $i = 1, 2, ..., N$, (except Saudi Arabia where there is no interest rate equation)

$$\mathbf{A}_{i0} = \begin{pmatrix} 1 & -\beta_{iy} & 0 & 0 \\ 0 & 1 & -\alpha_{ir} & -\alpha_{ie} \\ -\gamma_{i\pi} & -\gamma_{iy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{A}_{i1} = \begin{pmatrix} \beta_{ib} & 0 & 0 & 0 \\ 0 & \alpha_{ib} & 0 & 0 \\ 0 & 0 & \gamma_{ib} & 0 \\ 0 & 0 & 0 & \rho_i \end{pmatrix},$$

$$\mathbf{A}_{i2} = \begin{pmatrix} \beta_{if} & 0 & 0 & 0 \\ -\alpha_{ir} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{A}_{i3} = \begin{pmatrix} 0 & 0 \\ \alpha_{iy*} & -\alpha_{ie} \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{A}_{i4} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\rho_i \end{pmatrix},$$

and $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{i,st}, \varepsilon_{i,dt}, \varepsilon_{i,mt}, \varepsilon_{i,et})'$.

Let $\widetilde{\mathbf{z}}_{it} = (\widetilde{\mathbf{x}}'_{it}, \widetilde{\mathbf{x}}^{*\prime}_{it})'$ then the $N+1$ models specified by (6) can be written compactly as

$$\mathbf{A}_{iz0}\widetilde{\mathbf{z}}_{it} = \mathbf{A}_{iz1}\widetilde{\mathbf{z}}_{i,t-1} + \mathbf{A}_{iz2}E_{t-1}\left(\widetilde{\mathbf{z}}_{i,t+1}\right) + \boldsymbol{\varepsilon}_{it}, \text{ for } i = 0, 1, 2, ..., N, \tag{7}$$

where

$$\begin{aligned}
\mathbf{A}_{0z0} &= \left(\mathbf{A}_{00}, \ -\mathbf{A}_{03}\right), \ \mathbf{A}_{0z1} = \left(\mathbf{A}_{01}, \ \mathbf{0}_{k_0\times(k_0+1+k_0^*)}\right), \ \mathbf{A}_{0z2} = \left(\mathbf{A}_{02}, \ \mathbf{0}_{k_0\times(k_0+1+k_0^*)}\right), \text{ for } i = 0, \\
\mathbf{A}_{iz0} &= \left(\mathbf{A}_{i0}, \ -\mathbf{A}_{i3}\right), \ \mathbf{A}_{iz1} = \left(\mathbf{A}_{i1}, \ \mathbf{A}_{i4}\right), \ \mathbf{A}_{iz2} = \left(\mathbf{A}_{i2}, \ \mathbf{0}_{k_i\times(k_i+k_i^*)}\right), \text{ for } i = 1, 2, ..., N.
\end{aligned}$$

The variables $\widetilde{\mathbf{z}}_{it}$ are linked to the variables in the global model, $\widetilde{\mathbf{x}}_t$, through the identity

$$\widetilde{\mathbf{z}}_{it} = \mathbf{W}_i\widetilde{\mathbf{x}}_t, \tag{8}$$

where the 'link' matrices $\mathbf{W}_i$, $i = 0, 1, ..., N$ are defined in terms of the weights w_{ij}. For $i = 0$, $\mathbf{W}_0$ is $(k_0 + 1 + k_0^*) \times (k + 1)$ and for $i = 1, 2, ..., N$, $\mathbf{W}_i$ is $(k_i + k_i^*) \times (k + 1)$ dimensional.

Substituting (8) in (7) now yields

$$\mathbf{A}_{iz0}\mathbf{W}_i\widetilde{\mathbf{x}}_t = \mathbf{A}_{iz1}\mathbf{W}_i\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_{iz2}\mathbf{W}_iE_{t-1}\left(\widetilde{\mathbf{x}}_{t+1}\right) + \boldsymbol{\varepsilon}_{it}, \ i = 0, 1, ..., N,$$

and then stacking all the $N + 1$ country models we obtain the multi-country RE model for $\widetilde{\mathbf{x}}_t$ as

$$\mathbf{A}_0\widetilde{\mathbf{x}}_t = \mathbf{A}_1\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_2E_{t-1}\left(\widetilde{\mathbf{x}}_{t+1}\right) + \boldsymbol{\varepsilon}_t, \tag{9}$$

where the stacked $k \times (k + 1)$ matrices $\mathbf{A}_j$, $j = 0, 1, 2$ are defined by

$$\mathbf{A}_j = \begin{pmatrix} \mathbf{A}_{0zj}\mathbf{W}_0 \\ \mathbf{A}_{1zj}\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_{Nzj}\mathbf{W}_N \end{pmatrix}, \ j = 0, 1, 2, \text{ and } \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_{0t} \\ \boldsymbol{\varepsilon}_{1t} \\ \vdots \\ \boldsymbol{\varepsilon}_{Nt} \end{pmatrix}.$$

The multi-country RE model given by (9) represents a system of k variables in $k + 1$ RE equations, and as noted above, contains a redundant equation in the US model. To deal with this redundancy we consider the new $k \times 1$ vector $\widetilde{\mathring{\mathbf{x}}}_t = (\widetilde{\mathring{\mathbf{x}}}'_{0t}, \widetilde{\mathring{\mathbf{x}}}'_{1t}, ..., \widetilde{\mathring{\mathbf{x}}}'_{Nt})'$, where $\widetilde{\mathring{\mathbf{x}}}_{0t} = (y_{0t}, r_{0t}, ep_{0t})'$ and $\widetilde{\mathring{\mathbf{x}}}_{it} = \widetilde{\mathbf{x}}_{it}$ for $i = 1, 2, ..., N$. In particular, for the US we can relate the 4×1 vector $\widetilde{\mathbf{x}}_{0t}$ to the 3×1 vector $\widetilde{\mathring{\mathbf{x}}}_{0t}$ by ,

$$\widetilde{\mathbf{x}}_{0t} = \mathbf{S}_{00}\widetilde{\mathring{\mathbf{x}}}_{0t} - \mathbf{S}_{01}\widetilde{\mathring{\mathbf{x}}}_{0,t-1},$$

where

$$\mathbf{S}_{00} = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \ \mathbf{S}_{01} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Similarly, $\widetilde{\mathbf{x}}_t = (\widetilde{\mathbf{x}}'_{0t}, \widetilde{\mathbf{x}}'_{1t}, ..., \widetilde{\mathbf{x}}'_{Nt})'$ can be related to the $k \times 1$ global vector $\widetilde{\mathring{\mathbf{x}}}_t = (\widetilde{\mathring{\mathbf{x}}}'_{0t}, \widetilde{\mathring{\mathbf{x}}}'_{1t}, ..., \widetilde{\mathring{\mathbf{x}}}'_{Nt})'$ by

$$\widetilde{\mathbf{x}}_t = \mathbf{S}_0 \widetilde{\mathring{\mathbf{x}}}_t - \mathbf{S}_1 \widetilde{\mathring{\mathbf{x}}}_{t-1}, \tag{10}$$

where

$$\mathbf{S}_0 = \begin{pmatrix} \mathbf{S}_{00} & \mathbf{0}_{4\times(k-3)} \\ \mathbf{0}_{(k-3)\times 3} & \mathbf{I}_{k-3} \end{pmatrix}, \mathbf{S}_1 = \begin{pmatrix} \mathbf{S}_{01} & \mathbf{0}_{4\times(k-3)} \\ \mathbf{0}_{(k-3)\times 3} & \mathbf{0}_{(k-3)\times(k-3)} \end{pmatrix}.$$

Using (10) in (9) we have

$$\mathbf{A}_0 \left(\mathbf{S}_0 \widetilde{\mathring{\mathbf{x}}}_t - \mathbf{S}_1 \widetilde{\mathring{\mathbf{x}}}_{t-1} \right) = \mathbf{A}_1 \left(\mathbf{S}_0 \widetilde{\mathring{\mathbf{x}}}_{t-1} - \mathbf{S}_1 \widetilde{\mathring{\mathbf{x}}}_{t-2} \right) + \mathbf{A}_2 E_{t-1}(\mathbf{S}_0 \widetilde{\mathring{\mathbf{x}}}_{t+1} - \mathbf{S}_1 \widetilde{\mathring{\mathbf{x}}}_t) + \boldsymbol{\varepsilon}_t,$$

or

$$\mathbf{H}_0 \widetilde{\mathring{\mathbf{x}}}_t = \mathbf{H}_1 \widetilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{H}_2 \widetilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_3 E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_{t+1}) + \mathbf{H}_4 E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_t) + \boldsymbol{\varepsilon}_t, \tag{11}$$

where

$$\mathbf{H}_0 = \mathbf{A}_0 \mathbf{S}_0, \mathbf{H}_1 = \mathbf{A}_1 \mathbf{S}_0 + \mathbf{A}_0 \mathbf{S}_1, \mathbf{H}_2 = -\mathbf{A}_1 \mathbf{S}_1, \mathbf{H}_3 = \mathbf{A}_2 \mathbf{S}_0,\ \mathbf{H}_4 = -\mathbf{A}_2 \mathbf{S}_1.$$

For a determinate solution the $k \times k$ matrix $\mathbf{H}_0$ must be non-singular. Pre-multiplying (11) by $\mathbf{H}_0^{-1}$

$$\widetilde{\mathring{\mathbf{x}}}_t = \mathbf{F}_1 \widetilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{F}_2 \widetilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{F}_3 E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_{t+1}) + \mathbf{F}_4 E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_t) + \mathbf{u}_t, \tag{12}$$

where $\mathbf{F}_j = \mathbf{H}_0^{-1} \mathbf{H}_j$, for $j = 1, 2, 3, 4$, and $\mathbf{u}_t = \mathbf{H}_0^{-1} \boldsymbol{\varepsilon}_t$. Using a companion form representation (12) can be written as

$$\boldsymbol{\chi}_t = \mathbf{A} \boldsymbol{\chi}_{t-1} + \mathbf{B} E_{t-1}(\boldsymbol{\chi}_{t+1}) + \boldsymbol{\eta}_t, \tag{13}$$

where $\boldsymbol{\chi}_t = \left(\widetilde{\mathring{\mathbf{x}}}'_t, \widetilde{\mathring{\mathbf{x}}}'_{t-1} \right)'$, and

$$\mathbf{A} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix},\ \mathbf{B} = \begin{pmatrix} \mathbf{F}_3 & \mathbf{F}_4 \\ \mathbf{0} & \mathbf{0} \end{pmatrix},\ \boldsymbol{\eta}_t = \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \end{pmatrix}.$$

The system of equations in (13) is the canonical rational expectations model and its solution has been considered in the literature. Binder and Pesaran (1995, 1997) review the alternative solution strategies and show that the nature of the solution critically depends on the roots of the quadratic matrix equation

$$\mathbf{B} \boldsymbol{\Phi}^2 - \boldsymbol{\Phi} + \mathbf{A} = \mathbf{0}. \tag{14}$$

There will be a unique globally consistent stationary solution if (14) has a real matrix solution such that all the eigenvalues of $\boldsymbol{\Phi}$ and $(\mathbf{I} - \mathbf{B}\boldsymbol{\Phi})^{-1}\mathbf{B}$ lie strictly inside the unit circle. The solution is then given by

$$\boldsymbol{\chi}_t = \boldsymbol{\Phi} \boldsymbol{\chi}_{t-1} + \boldsymbol{\eta}_t. \tag{15}$$

Partitioning $\boldsymbol{\Phi}$ conformably to $\boldsymbol{\chi}_t$, (15) can be expressed as

$$\begin{pmatrix} \widetilde{\mathring{\mathbf{x}}}_t \\ \widetilde{\mathring{\mathbf{x}}}_{t-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix} \begin{pmatrix} \widetilde{\mathring{\mathbf{x}}}_{t-1} \\ \widetilde{\mathring{\mathbf{x}}}_{t-2} \end{pmatrix} + \begin{pmatrix} \mathbf{I}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k \end{pmatrix} \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \end{pmatrix},$$

so that the solution in terms of $\tilde{\mathring{\mathbf{x}}}_t$, is given by

$$\tilde{\mathring{\mathbf{x}}}_t = \mathbf{\Phi}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{\Phi}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\boldsymbol{\varepsilon}_t, \tag{16}$$

where $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}'_{0t}, \boldsymbol{\varepsilon}'_{1t}, ..., \boldsymbol{\varepsilon}'_{Nt})'$. The structural shocks, $\boldsymbol{\varepsilon}_t$, can be recovered by noting that

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_0(\tilde{\mathring{\mathbf{x}}}_t - \mathbf{\Phi}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1} - \mathbf{\Phi}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2}). \tag{17}$$

The covariance matrix of the structural shocks is given by

$$E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t) = \mathbf{\Sigma}_\varepsilon, \tag{18}$$

which can be obtained from the estimated structural shocks.

It will be convenient to reorder the elements of $\boldsymbol{\varepsilon}_t$ in (17) in terms of the different types of shocks as $\boldsymbol{\varepsilon}_t^0 = (\boldsymbol{\varepsilon}'_{st}, \boldsymbol{\varepsilon}'_{dt}, \boldsymbol{\varepsilon}'_{mt}, \boldsymbol{\varepsilon}'_{et})'$, where $\boldsymbol{\varepsilon}_{st}$ and $\boldsymbol{\varepsilon}_{dt}$ are the $(N+1)\times 1$ vectors of supply and demand shocks, and $\boldsymbol{\varepsilon}_{mt}$ and $\boldsymbol{\varepsilon}_{et}$ are the $N \times 1$ vectors of monetary policy shocks (for all countries except Saudi Arabia) and shocks to the real effective exchange rates (for all countries except the US). We can then write

$$\boldsymbol{\varepsilon}_t^0 = \mathbf{G}\boldsymbol{\varepsilon}_t, \tag{19}$$

where $\mathbf{G}$ is a non-singular $k \times k$ matrix with elements 0 or 1. Also $E(\boldsymbol{\varepsilon}_t^0\boldsymbol{\varepsilon}_t^{0\prime}) = \mathbf{\Sigma}_\varepsilon^0 = \mathbf{G}\mathbf{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{G}'$, which can be obtained from $\mathbf{\Sigma}_\varepsilon$ by suitable permutations of its rows and columns.

As discussed above, we assume that there are zero covariances between the supply, demand and monetary policy shocks, though there can be non-zero covariances between the same type of structural shocks in different countries. We allow the exchange rate shocks, $\boldsymbol{\varepsilon}_{et}$, to have non-zero correlations with the other shocks both within and across the countries. The covariance of demand and supply shocks are given by the $(N+1)\times(N+1)$ dimensional matrices $\mathbf{\Sigma}_{ss}$ and $\mathbf{\Sigma}_{dd}$, and the covariance matrices of the monetary policy shocks and exchange rate shocks are given by the $N \times N$ matrices $\mathbf{\Sigma}_{mm}$ and $\mathbf{\Sigma}_{ee}$. The covariances between the exchange rate shocks and the structural shocks are given by $\mathbf{\Sigma}_{es}$, etc. These assumptions yield a block diagonal error covariance matrix which is bordered by non-zero covariances between the exchange rate shocks and the structural shocks, so that $\mathbf{\Sigma}_\varepsilon^0$ has the form:

$$\mathbf{\Sigma}_\varepsilon^0 = \begin{pmatrix} \mathbf{\Sigma}_{ss} & \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_{se} \\ \mathbf{0} & \mathbf{\Sigma}_{dd} & \mathbf{0} & \mathbf{\Sigma}_{de} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_{mm} & \mathbf{\Sigma}_{me} \\ \mathbf{\Sigma}_{es} & \mathbf{\Sigma}_{ed} & \mathbf{\Sigma}_{em} & \mathbf{\Sigma}_{ee} \end{pmatrix}. \tag{20}$$

3 Impulse responses, variance decompositions and shock accounting in the MCNK model

The analysis of the effects of shocks will be represented, as usual, by impulse response functions, IRFs, and forecast error variance decompositions, FEVDs. The system is solved in terms of the

$k \times 1$ vector $\mathring{\widetilde{\mathbf{x}}}_t$, and since $\mathring{\widetilde{\mathbf{x}}}_{0t} = (\widetilde{y}_{0t}, \widetilde{r}_{0t}, \widetilde{ep}_{0t})' = (\widetilde{y}_{0t}, \widetilde{r}_{0t}, -\widetilde{p}_{0t})'$, it includes the US price level and not $\widetilde{\pi}_{0t}$. To compute the effects of shocks on US inflation we can switch back to the $(k+1) \times 1$ vector $\widetilde{\mathbf{x}}_t$, as defined by (10). The standard approach to IRFs and FEVDs needs to be somewhat modified to deal with the cross-country correlation of shocks and below we discuss their calculation.

3.1 Impulse response functions

Impulse response functions provide counter-factual answers to questions concerning either the effects of a particular shock in a given economy, or the effects of a combined shock involving linear combinations of shocks across two or more economies. The effects of the shock can also be computed either on a particular variable in the global economy or on a combination of variables. Denote a composite shock, defined as a linear combination of the shocks, by $\xi_t = \mathbf{a}'\boldsymbol{\varepsilon}_t^0$, and consider the time profile of its effects on a composite variable $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$. The $k \times 1$ vector $\mathbf{a}$ and the $(k+1) \times 1$ vector $\mathbf{b}$ are either appropriate selection vectors picking out a particular error or variable or a suitable weighted average. The error weights, $\mathbf{a}$, can be chosen to define composite shocks, such as a global supply shock; the variable weights, $\mathbf{b}$, to define composite variables such as the real effective exchange rate or a PPP GDP weighted average of the countries in the euro area. The IRFs estimate the time profile of the response by $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$ to a unit shock (defined as one standard error shock of size $\sigma_\xi = \sqrt{\mathbf{a}'\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}$) to $\xi_t = \mathbf{a}'\boldsymbol{\varepsilon}_t^0$, and the FEVDs estimate the relative importance of different shocks in explaining the variations in output, inflation and interest rates from their steady states in a particular economy over time.

Using (16) and (19), we obtain

$$\mathring{\widetilde{\mathbf{x}}}_t = \boldsymbol{\Phi}_{11}\mathring{\widetilde{\mathbf{x}}}_{t-1} + \boldsymbol{\Phi}_{12}\mathring{\widetilde{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\mathbf{G}^{-1}\boldsymbol{\varepsilon}_t^0, \tag{21}$$

and the time profile of $\mathring{\widetilde{\mathbf{x}}}_{t+n}$ in terms of current and lagged shocks can be written as

$$\mathring{\widetilde{\mathbf{x}}}_{t+n} = \mathbf{D}_{n1}\mathring{\widetilde{\mathbf{x}}}_{t-1} + \mathbf{D}_{n2}\mathring{\widetilde{\mathbf{x}}}_{t-2} + \mathbf{C}_n\boldsymbol{\varepsilon}_t^0 + \mathbf{C}_{n-1}\boldsymbol{\varepsilon}_{t+1}^0 + ... + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+n-1}^0 + \mathbf{C}_0\boldsymbol{\varepsilon}_{t+n}^0, \tag{22}$$

where $\mathbf{D}_{n1}$ and $\mathbf{D}_{n2}$ are functions of $\boldsymbol{\Phi}_{11}$ and $\boldsymbol{\Phi}_{12}$, $\mathbf{C}_j = \mathbf{P}_j\mathbf{H}_0^{-1}\mathbf{G}^{-1}$, and $\mathbf{P}_j$ can be derived recursively as

$$\mathbf{P}_j = \boldsymbol{\Phi}_{11}\mathbf{P}_{j-1} + \boldsymbol{\Phi}_{12}\mathbf{P}_{j-2},\ \mathbf{P}_0 = \mathbf{I}_k,\ \mathbf{P}_j = \mathbf{0}, \text{ for } j < 0.$$

Similarly, using (10) and (21), we have

$$\widetilde{\mathbf{x}}_{t+n} = \mathring{\mathbf{D}}_{n1}\mathring{\widetilde{\mathbf{x}}}_{t-1} + \mathring{\mathbf{D}}_{n2}\mathring{\widetilde{\mathbf{x}}}_{t-2} + \mathbf{B}_n\boldsymbol{\varepsilon}_t^0 + \mathbf{B}_{n-1}\boldsymbol{\varepsilon}_{t+1}^0 + ... + \mathbf{B}_1\boldsymbol{\varepsilon}_{t+n-1}^0 + \mathbf{B}_0\boldsymbol{\varepsilon}_{t+n}^0, \tag{23}$$

where

$$\mathring{\mathbf{D}}_{n1} = \mathbf{S}_0\mathbf{D}_{n1} - \mathbf{S}_1\mathbf{D}_{n-1,1},\quad \mathring{\mathbf{D}}_{n2} = \mathbf{S}_0\mathbf{D}_{n2} - \mathbf{S}_1\mathbf{D}_{n-1,2},$$

and

$$\mathbf{B}_0 = \mathbf{S}_0\mathbf{C}_0 \text{ and } \mathbf{B}_\ell = \mathbf{S}_0\mathbf{C}_\ell - \mathbf{S}_1\mathbf{C}_{\ell-1}, \text{ for } \ell = 1, 2, ..., n.$$

Notice that B_ℓ, $\ell = 0, 1, 2, ..., n$ are $(k+1) \times k$, dimensional matrices that transmit the effects of the k shocks in the system to the $(k+1)$ elements of $\widetilde{\mathbf{x}}_{t+n}$ that include both the US price level and the US inflation. Clearly, both representations (22) and (23) can be used to carry out the impulse response analysis. But it is more convenient to use (23) when considering the effects of shocks on US inflation.

The generalized impulse response function for the effect on $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$ of a one standard error shock to $\xi_t = \mathbf{a}'\boldsymbol{\varepsilon}_t^0$ is then

$$\begin{aligned} g_q(n, \sigma_\xi) &= E(q_{t+n} \mid \xi_t = \sigma_\xi = \sqrt{\mathbf{a}'\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}, \Im_{t-1}) - E(\mathbf{b}'\widetilde{\mathbf{x}}_{t+n} \mid \Im_{t-1}) \\ &= \frac{\mathbf{b}'\mathbf{B}_n\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}{\sqrt{\mathbf{a}'\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}}, \ n = 0, 1, 2, \ldots . \end{aligned} \tag{24}$$

While we can identify the IRFs of, say, supply shocks as a group because they are assumed to be orthogonal to demand and monetary policy shocks, we cannot identify the supply shock in any particular country, because they are correlated with the supply shocks in other countries. The issue of how to identify country-specific demand or supply shocks in a multi-country setting is beyond the scope of the present paper. Instead here we focus on the effects of global supply or demand shocks. For instance, a global supply shock uses $\mathbf{a}_s$, which has PPP GDP weights that add to one, corresponding to the supply shocks of each of the $N+1$ countries and zeros elsewhere. For a monetary policy shock, we consider a unit (one standard error) shock to the US interest rate and examine its effects on the US and the rest of the world.[7] We interpret this as the effect of a shock to US monetary policy, which can be justified, for example, in the context of a recursive specification of monetary policy shocks where in the block of interest rate equations the US monetary policy rule is placed first.

3.2 Forecast error variance decomposition

Forecast error variance decomposition (FEVD) techniques can also be used to estimate the relative importance of different types of shocks in explaining the forecast error variance of different variables in the world economy. Such a decomposition can be achieved without having to specify the nature or sources of the cross-country correlations of supply or demand shocks. Additional identifying assumptions will be needed if we also wish to identify the relative importance of country-specific supply shocks, but as noted above such an exercise is beyond the scope of the present paper.

For the FEVD of global shocks we partition $\mathbf{B}_\ell = \begin{pmatrix} \mathbf{B}_{s\ell}, & \mathbf{B}_{d\ell}, & \mathbf{B}_{m\ell}, & \mathbf{B}_{e\ell} \end{pmatrix}$ in (23) conformably with the partitioning of $\boldsymbol{\varepsilon}_t^0 = (\boldsymbol{\varepsilon}_{st}', \boldsymbol{\varepsilon}_{dt}', \boldsymbol{\varepsilon}_{mt}', \boldsymbol{\varepsilon}_{et}')'$, and note that the n step ahead forecast errors can be written as

$$\tilde{\boldsymbol{\upsilon}}_{t+n} = \widetilde{\mathbf{x}}_{t+n} - E\left(\widetilde{\mathbf{x}}_{t+n} \left| \Im_{t-1} \right.\right) = \sum_{j \in shocks} \sum_{\ell=0}^{n} \mathbf{B}_{j,n-\ell}\boldsymbol{\varepsilon}_{j,t+\ell}. \tag{25}$$

[7] Similar issues have been considered by Eichenbaum and Evans (1995) and Kim (2001), but using two-country VARs.

Under the assumption that within country supply, demand and monetary policy shocks are orthogonal we have

$$\begin{aligned}
Var\left(\tilde{\boldsymbol{v}}_{t+n}\left|\Im_{t-1}\right.\right) &= \sum_{\ell=0}^{n}\mathbf{B}_{s,n-\ell}\boldsymbol{\Sigma}_{ss}\mathbf{B}'_{s,n-\ell}+\sum_{\ell=0}^{n}\mathbf{B}_{d,n-\ell}\boldsymbol{\Sigma}_{dd}\mathbf{B}'_{d,n-\ell} \\
&+\sum_{\ell=0}^{n}\mathbf{B}_{m,n-\ell}\boldsymbol{\Sigma}_{mm}\mathbf{B}'_{m,n-\ell}+\sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{ee}\mathbf{B}'_{e,n-\ell} \\
&+\sum_{\ell=0}^{n}\mathbf{B}_{s,n-\ell}\boldsymbol{\Sigma}_{se}\mathbf{B}'_{e,n-\ell}+\sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{es}\mathbf{B}'_{s,n-\ell} \\
&+\sum_{\ell=0}^{n}\mathbf{B}_{d,n-\ell}\boldsymbol{\Sigma}_{de}\mathbf{B}'_{e,n-\ell}+\sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{ed}\mathbf{B}'_{d,n-\ell} \\
&+\sum_{\ell=0}^{n}\mathbf{B}_{m,n-\ell}\boldsymbol{\Sigma}_{me}\mathbf{B}'_{e,n-\ell}+\sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{em}\mathbf{B}'_{m,n-\ell}.
\end{aligned}$$

The first four terms give the contributions to the variance from each of the four shocks; the following six terms arise from the covariances between the exchange rate shocks and the three structural shocks. Using the above FEVD, one can then estimate the importance of supply shocks, demand shocks or monetary policy shocks in the world economy for the explanations of output growth, inflation, interest rates and real effective exchange rates, either for individual variables or any given linear combinations of the variables. These proportions will not add up to unity, due to the non-zero correlations between the real effective exchange rates and the three structural shocks. But as we shall show below, due to the relatively small magnitudes of the covariance terms between the real exchange rates and the structural shocks, the proportion of forecast error variances explained by variances of the four shocks add to a number which is very close to unity.

4 Deviations from steady states

So far we have assumed that the deviations from the steady states are given and are covariance stationary, as required by the NK model. In practice, however, such deviations must be identified and measured consistently. In cases where the variables under consideration are either stationary or trend-stationary, the steady state values are either fixed constants or can be approximated by linear trends, and the deviations in the NK model can be replaced by realised values with constant terms or linear trends added to the equations (as appropriate) to take account of the non-zero deterministic means of the stationary or trend stationary processes. But there exists ample evidence that most macroeconomic variables, including inflation and interest rates, real exchange rate and real output, are likely to contain stochastic trends and could be cointegrated. Common stochastic trends at national and global levels can lead to within country as well as between country cointegration. The presence of such stochastic trends must be appropriately taken into account in the identification and estimation of steady state values (and hence the deviations), otherwise the estimates of the structural parameters and the associated impulse responses can be badly biased even in large samples.

There are a variety of methods that can be used to handle permanent components, some of which have been recently discussed by Fukac and Pagan (2010). Here we follow DPSS and measure the steady states as the long-horizon forecasts from an underlying global vector error correcting model (VECM). We also contrast the results obtained using this approach with the alternative often favoured in the literature where inflation and interest rates and real effective exchange rates are treated as stationary, and the output deviations are computed using the Hodrick-Prescott filter.

The global model is specified in terms of the realised values denoted by $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt})'$, with the deviations given by

$$\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}_t^P,$$

where $\mathbf{x}_t^P$ denotes the permanent component of $\mathbf{x}_t$. $\mathbf{x}_t^P$ is further decomposed into deterministic and stochastic components

$$\mathbf{x}_t^P = \mathbf{x}_{d,t}^P + \mathbf{x}_{s,t}^P, \text{ and } \mathbf{x}_{d,t}^P = \boldsymbol{\mu} + \mathbf{g}t,$$

where $\boldsymbol{\mu}$ and $\mathbf{g}$ are $k \times 1$ vectors of constants and t a deterministic time trend. The steady state (permanent-stochastic component) $\mathbf{x}_{st}^P$, is then defined as the 'long-horizon forecast' (net of the permanent-deterministic component)

$$\mathbf{x}_{s,t}^P = \lim_{h\to\infty} E_t\left(\mathbf{x}_{t+h} - \mathbf{x}_{d,t+h}^P\right) = \lim_{h\to\infty} E_t\left[\mathbf{x}_{t+h} - \boldsymbol{\mu} - \mathbf{g}(t+h)\right].$$

In the case where $\mathbf{x}_t$ is trend stationary then $\mathbf{x}_{s,t}^P = \mathbf{0}$, and we revert back to the familiar case where deviations are formed as residuals from regressions on linear trends. However, in general, $\mathbf{x}_{s,t}^P$ is non-zero and must be estimated from a multivariate time series model of $\mathbf{x}_t$ that allows for stochastic trends and cointegration. Once a suitable multivariate model is specified, it is then relatively easy to show that $\mathbf{x}_{s,t}^P$ corresponds to a multivariate Beveridge-Nelson (1981) decomposition as argued by Garratt *et al.* (2006). The economic model used to provide the long-horizon forecasts is a global VAR (GVAR) which takes account of unit roots and cointegration in the global economy (within as well as across economies). DPSS provide more detail on the GVAR and explain how it can be regarded as the reduced form of a structural model such as the MCNK considered here.

For each country, $i = 0, 1, 2, ..., N$, the global VAR model consists of VARX* models of the form:

$$\mathbf{x}_{it} = \mathbf{h}_{i0} + \mathbf{h}_{i1}t + \mathbf{A}_{i1}\mathbf{x}_{i,t-1} + \mathbf{B}_{i2}\mathbf{x}_{i,t-2} + \mathbf{C}_{i0}\mathbf{x}_{it}^* + \mathbf{C}_{i1}\mathbf{x}_{i,t-1}^* + \mathbf{u}_{it}, \; i = 0, 1, .., N,$$

and the associated VECM, with cointegrating restrictions:

$$\Delta\mathbf{x}_{it} = \mathbf{c}_{i0} - \boldsymbol{\alpha}_i\boldsymbol{\beta}_i'[\mathbf{z}_{i,t-1} - \boldsymbol{\gamma}_i(t-1)] + \mathbf{C}_{i0}\Delta\mathbf{x}_{it}^* + \mathbf{G}_i\Delta\mathbf{z}_{i,t-1} + \mathbf{u}_{it},$$

where $\mathbf{z}_{it} = (\mathbf{x}'_{it}, \mathbf{x}^{*\prime}_{it})'$, $\boldsymbol{\alpha}_i$ is a $k_i \times r_i$ matrix of rank r_i, and $\boldsymbol{\beta}_i$ is a $(k_i + k_i^*) \times r_i$ matrix of rank r_i. This allows for cointegration within $\mathbf{x}_{it}$ and between $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$. Then as with (8) $\mathbf{z}_{it} = \mathbf{W}_i\mathbf{x}_t$, and by the same process as above we can stack the $N+1$ individual country models and solve for the GVAR specification

$$\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{F}_1 \mathbf{x}_{t-1} + \mathbf{F}_2 \mathbf{x}_{t-2} + \mathbf{u}_t. \tag{26}$$

This is a standard VAR specification and can be readily used to derive $\mathbf{x}_t^P$ as the long-horizon forecasts of $\mathbf{x}_t$.

Various GVARs have been widely used for a variety of purposes.[8] The version used to calculate the long-horizon forecasts is estimated over the same sample, 1979Q4-2006Q4, for the same 33 countries, explaining the same variables (output, inflation, short interest rates, and exchange rates), with the addition of the price of oil, which is included as an endogenous variable in the US VARX* model. These 131 endogenous variables are driven by 82 stochastic trends and 49 cointegrating relations. Weak exogeneity of the foreign variables for the individual VARX* equations is rejected only in 8.4% of the cases at the 5% level. Other versions of the GVAR include financial variables, but these have been excluded for comparability with MCNK model which does not include them.[9]

The long-horizon forecasts from this GVAR model provide estimates of the steady states $\mathbf{x}_t^P$, which match the economic concept of a steady state and are derived from a multivariate economic rather than a univariate statistical model, so they will reflect the long-run cointegrating relationships and stochastic trends in the system. The deviations from steady states used as variables in the MCNK model, $\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}_t^P$, are uniquely identified and stationary by construction so avoiding the danger of spurious regression.

The measures of steady state depend on the underlying economic model, which seems a desirable property. However, they may be sensitive to misspecification and it is possible that intercept shifts, broken trends or other forms of structural instability not allowed for in the estimated economic model, will be reflected in the measured deviations from steady state. For instance, Perron and Wada (2009) argue that the difference between the univariate BN decomposition and other methods of measuring trend US GDP are the artifacts created by neglect of the change in slope of the trend function in 1973. Although our estimation period is all post 1973, so this is not an issue, and various tests indicate that the estimated GVARs seem structurally stable, possible structural breaks could be dealt with using the average long-horizon forecasts from models estimated over different samples. The evidence in Pesaran, Schuermann and Smith (2009) indicates that averaging over observation windows improves forecasts. The extension of this procedure from forecasting to the estimation of steady states is an area for further research.

[8] See for instance Dees, di Mauro, Pesaran and Smith (2007), Dees, Holly, Pesaran and Smith (2007), Pesaran, Smith and Smith (2007), or Pesaran, Schuermann and Smith (2009).

[9] Details of the estimated GVAR are provided in a supplement available from the authors upon request.

5 Estimates and solution

The structural equations are estimated for each country separately using the inequality-constrained instrumental variables method.[10] DSGE models are often estimated by Bayesian methods but given the size of the model this would be a demanding task. The parameters are estimated subject to the theory restrictions discussed in Subsection 2.1. Where the constraints are not satisfied, the parameters are set to their boundary values and the choice between any alternative estimates that satisfy the constraints is based on the in-sample prediction errors.[11] As to be expected, there is a considerable degree of heterogeneity in the estimates, with those for Latin American countries often being the outliers.[12]

The estimation sample for all equations starts in $t = 1980Q1$ and ends in 2006Q3 for the Phillips curve and IS equations (due to the presence of future dated variables), and ends in 2006Q4 for the Taylor rule and exchange rate equations. An exception is the Phillips curve for Argentina which is estimated over the sub-sample 1990Q1-2006Q3. The parameters of the structural equations (PC, IS and Taylor rule) are estimated by the IV method using the following as instruments: a vector of ones, $\widetilde{\pi}_{i,t-1}, \widetilde{y}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}, \widetilde{\pi}^*_{it}, \widetilde{y}^*_{it}, \widetilde{r}^*_{it}$, and $\widetilde{p}^o_t$, except for Saudi Arabia where $\widetilde{r}_{i,t-1}$ and $\widetilde{r}^*_{it}$ are excluded as there is no interest series for this country. The European countries belonging to the Economic and Monetary Union (EMU) are here considered separately, but an aggregation of these countries into a single region as in DdPS could also be envisaged. The exchange rate equation is estimated by OLS. Table 1 provides summary statistics for the estimates obtained for all the 33 countries in the global DSGE model, and Table 2 gives detailed estimates for eight major economies.[13] We now comment briefly on the estimates.

5.1 Country-specific parameter estimates

The parameters of the Phillips curve, (2), are estimated subject to the inequality restrictions $\beta_{ib} \geq 0$, $\beta_{if} \geq 0$, $\beta_{ib} + \beta_{if} \leq 0.99$, and $\beta_{iy} \geq 0$. Since under $\beta_{ib} = \beta_{if} = 0$, the third restriction, $\beta_{ib}+\beta_{if} \leq 0.99$, is satisfied, there are 14 possible specifications. All specifications are estimated and from those satisfying the restrictions the one with the lowest in-sample mean squared prediction error is selected. Application of this procedure to Argentina over the full sample resulted in the estimates, $\hat{\beta}_{ib} = \hat{\beta}_{if} = \hat{\beta}_{iy} = 0$, which does not seem plausible and could be due to structural breaks, so the PC for Argentina was estimated over the sub-sample, 1990Q1-2006Q3, which gave the somewhat more plausible estimates of $\beta_{if} = 0.53$, $\hat{\beta}_{ib} = \hat{\beta}_{iy} = 0$. In the case of 7 countries, the

[10] Inference in inequality constrained estimation is non-standard and will not be addressed here. Gouriéroux et al. (1982) consider the problem in the case of least squares estimation.

[11] Pesaran and Smith (1994, p. 708) discuss the relationship between this criterion and the IV minimand.

[12] There is also evidence of misspecification in a number of the estimated equations, but since we wished to consider a tight specification that corresponds to the standard theory we did not add extra lags or more global variables to reduce the extent of the misspecification or to improve the fit of the regressions.

[13] Full details of the country-specific estimates are provided in a supplement which is available from the authors upon request.

IV estimates satisfied all the constraints. Also the coefficient of inflation expectations, β_{if}, turned out to be positive in all cases and is generally much larger than the coefficient of lagged inflation, β_{ib}. The mean value of β_{iy} at 0.11 is very close to the standard prior in the literature, although this average hides a wide range of estimates obtained across countries.

Table 1: Distribution of inequality-constrained IV estimates using GVAR estimates of deviations from steady states

	Mean	# Constrained	UC Mean	Constraint
Phillips curve - Equation (2), N=33				
β_{ib}	0.12	10	0.17	$\beta_{ib}\geq 0$
β_{if}	0.80	0	0.80	$\beta_{if}\geq 0$
β_{iy}	0.11	7	0.14	$\beta_{iy}\geq 0$
$\beta_{ib}+\beta_{if}$	0.93	22	0.80	$\beta_{ib}+\beta_{if}\leq 0.99$
IS curve - Equation (3), N=33				
α_{ib}	0.27	0	0.27	
α_{ir}	-0.20	18	-0.43	$\alpha_{ir}\leq 0$
α_{ie}	0.02	0	0.02	
α_{iy*}	0.79	2	0.84	$\alpha_{iy*}\geq 0$
Taylor Rule - Equation (4), N=32				
γ_{ib}	0.59	0	0.59	
$\gamma_{i\pi}$	0.24	4	0.28	$\gamma_{ir}\geq 0$
γ_{iy}	0.06	11	0.09	$\gamma_{iy}\geq 0$
Exchange rates - Equation (5), N=32				
ρ_i	0.67	0	0.67	$\lvert\rho_i\rvert < 1$

Note: The estimation sample begins in t=1980Q1 and ends in 2006Q3 for the PC and IS equations, and 2006Q4 for the Taylor rule and exchange rate equations. An exception is the Phillips curve in Argentina which is estimated over the sub-sample 1990Q1-2006Q3. N is the number of countries for which the equations were estimated. The column headed "Mean" gives the average over all estimates, constrained and unconstrained. The column headed "# Constrained" gives the number of estimates constrained at the boundary. The column headed "UC Mean" gives the mean of the unconstrained estimates. Individual country results are available in a supplement available upon request. Results for selected countries are provided in Table 2.

Initially the IS equation was estimated with expected future output deviations included. The coefficient of the future output variable, α_{if}, was negative in 3 countries (Germany, New Zealand and Saudi Arabia), insignificantly positive in 16 countries and significantly positive in 14. In only 11 countries was the coefficient of the real interest rate, α_{ir}, negative as would be expected from the theory. There seemed to be an association between a significant coefficient on the future output variable and a positive real interest rate coefficient, since in 10 out of the 14 countries where the coefficient of future output was significant, the coefficient on the real interest rate was positive.

Various restricted versions of the equation were considered, including setting $\alpha_{if} = 1$, $\alpha_{ib} + \alpha_{if} = 1$, and $\alpha_{if} = 0$. The specification that imposed $\alpha_{if} = 0$ gave the maximum number of countries with negative real interest rate effects. Given the importance of having a negative interest rate effect in the IS curve for the monetary transmission mechanism, we opted for the IS specification without the future output variable, and estimated the parameters of (3) subject to the constraints $\alpha_{ir} \leq 0$ and $\alpha_{iy*} \geq 0$, following the same procedure as before. The unrestricted equation was chosen for 14 countries. Including $\widetilde{y}^*_{it}$ tended to produce a more negative and significant estimate of the interest rate effect and, in the case of the US, the estimate of the interest rate coefficient was negative only when $\widetilde{y}^*_{it}$ was included in the IS equation. The estimate of the coefficient of the real exchange rate variable averaged to about zero, but with quite a large range of variations across the different countries.

The Taylor Rule, (4) was estimated subject to the constraints $\gamma_{iy} \geq 0$ and $\gamma_{i\pi} \geq 0$. The unrestricted equation was chosen for 18 countries out of the 32 possible Taylor rule equations. Recall that there is no interest rate equation for Saudi Arabia. In the case of Malaysia a fully constrained specification with $\gamma_{iy} = \gamma_{i\pi} = 0$, resulted, and in 3 other countries we obtained the restricted case with $\gamma_{i\pi} = 0$. In 11 countries, including the US, we ended up with $\gamma_{iy} = 0$.

For the real effective exchange rate equation, (5), the OLS estimates of ρ_i ranged from 0.34 to 0.86, confirming that this is a stable process, as one would expect given that we are using deviations from the steady states.

Table 2 shows the inequality-constrained IV estimates for some of the major economies. There is a very strong output effect in the Chinese Phillips curve, and to a lesser extent in the Japanese and the US Phillips curve (with $\beta_{iy} \geq 0.1$), while this effect is somewhat lower in the other countries. There are strong real interest rate effects in the IS curves for the US and Canada, while the estimates for the European countries are either close to zero or constrained at the boundary. Finally, there are strong foreign output effects in the IS curves for all countries, except for Japan.

Table 2: **Inequality-constrained IV estimates using GVAR estimates of deviations from steady states for eight major economies**

	US	China	Japan	Germany	France	UK	Italy	Canada
Phillips curve - Equation (2)								
β_{ib}	0.22	0.00	0.11	0.10	0.00	0.12	0.38	0.22
β_{if}	0.77	0.86	0.84	0.89	0.99	0.87	0.61	0.77
β_{iy}	0.10	0.14	0.13	0.04	0.08	0.05	0.04	0.02
IS curve - Equation (3)								
α_{ib}	0.21	0.72	0.15	0.03	0.00	0.54	0.23	0.37
α_{ir}	-0.98	-0.47	-0.23	0.00	-0.02	0.00	0.00	-1.41
α_{ie}	-0.01	0.02	0.02	-0.04	-0.01	0.25	-0.02	0.12
α_{iy*}	0.74	0.31	0.15	1.10	0.64	0.95	0.73	0.89
Taylor Rule - Equation (4)								
γ_{ib}	0.79	0.98	0.82	0.62	0.94	0.74	0.82	0.51
$\gamma_{i\pi}$	0.28	0.11	0.21	0.27	0.04	0.20	0.20	0.42
γ_{iy}	0.00	0.00	0.15	0.04	0.03	0.01	0.01	0.00
Exchange rates - Equation (5)								
ρ_i		0.78	0.76	0.54	0.68	0.53	0.73	0.84

Note: The estimation sample begins in t=1980Q1 and ends in 2006Q3 for the PC and IS equations, and 2006Q4 for the Taylor rule and exchange rate equations. Individual country results are available in a Supplement available upon request.

5.2 Solution and covariance matrix of the shocks

Details of the method used to solve (14), $\mathbf{B}\mathbf{\Phi}^2 - \mathbf{\Phi} + \mathbf{A} = \mathbf{0}$, are given in the Appendix. It involves an iterative back-substitution procedure starting with an arbitrary initial choice of $\mathbf{\Phi}$, which was set to an identity matrix. As a check against multiple solutions, we also started the iterations with an initial value of $\mathbf{\Phi}$ that had units along the diagonal and the off diagonal terms were drawn from a uniform distribution over the range -0.5 to +0.5. Both initial values resulted in the same solution.

The multi-country NK model is solved for all time periods in our estimation sample, and allows us to obtain estimates of all the structural shocks in the model. Altogether there are 130 different shocks; 98 structural and 32 reduced form. Denote the shock of type $k = s, d, m, e$ in country $i = 1, 2, ..., 33$ at time $t = 1980Q1 - 2006Q4$ by $\varepsilon_{i,k,t}$. It is now possible to compute pair-wise correlations of any pair of shocks both within and across countries. In Table 3 we provide averages of pair-wise correlations across the four types of shocks. Just to be clear the average pair-wise correlation of supply shocks is computed by averaging over the $(33 \times 32)/2 = 528$ pairs of correlation coefficients from the 33 supply shocks, and similarly the average pair-wise correlation coefficients of supply and demand shocks is computed by averaging over $(33 \times 34)/2 = 561$ pairs of supply-demand shocks.

Table 3: Average pair-wise correlations of shocks using GVAR deviations.

	Supply	Demand	Mon. Pol.	Ex. Rate
Supply	0.495	0.166	0.040	0.048
Demand		0.067	0.063	-0.005
Mon. Pol.			0.139	-0.043
Ex. Rate				0.049

The largest average correlations are among supply shocks, at 0.495; the other correlations are all less than 0.17. By comparison, the average pair-wise correlations of shocks of different types (given as the off-diagonal elements in Table 3) are small, with the largest figure given by the average correlation of demand and supply shocks given by 0.166. The other average correlations across the different types of shocks are small. This is in line with our maintained identifying assumption that supply, demand and monetary policy shocks are orthogonal.

Consider now the problem of consistent estimation of the covariance matrix of shocks defined by (20). One possibility would be to estimate the non-zero blocks $\mathbf{\Sigma}_{kl}$, $k, l = s, d, m, e$ with the sample covariance matrix using the estimates of $\boldsymbol{\varepsilon}_t$, defined by (17) and denoted by $\hat{\boldsymbol{\varepsilon}}_t$. For instance, $\mathbf{\Sigma}_{ss}$ can be estimated by $\sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_{st}\hat{\boldsymbol{\varepsilon}}'_{st}/T$. These estimates of the component matrices can then be inserted in (20) to provide an estimate of $\mathbf{\Sigma}^0_\varepsilon$, say $\hat{\Sigma}^0_\varepsilon$. However, since the dimension of the endogenous variables, $k = 130$, is larger than the time series dimension, $T = 108$, $\hat{\Sigma}^0_\varepsilon$ is not guaranteed to be a positive definite matrix. While the estimates of the individual correlations are consistent, the estimate of the whole matrix is not when $T < N$. This is an important consideration when we come to compute bootstrapped error bands for the impulse response functions. The same issue arises in other contexts including mean-variance portfolio optimisation where the number of assets is large.

A number of solutions have been suggested in the literature. Ledoit and Wolf (2004) consider an estimator which is a convex linear combination of the unrestricted sample covariance matrix and an identity matrix and provide an estimator for the weights. Friedman, Hastie and Tibshirani (2008) apply the lasso penalty to loadings in principal component analysis to achieve a sparse representation. Fan, Fan and Lv (2008) use a factor model to impose sparsity on the covariance matrix. Bickel and Levina (2008) propose thresholding the sample covariance matrix, where the threshold parameter is chosen using cross validation.

The procedure we use, starts from the fact that the diagonal matrix, $diag(\hat{\Sigma}^0_\varepsilon)$, which has $(\hat{\boldsymbol{\sigma}}^2_{1,ss}, ..., \hat{\boldsymbol{\sigma}}^2_{N+1,ss}, \hat{\boldsymbol{\sigma}}^2_{1,dd}, ..., \hat{\boldsymbol{\sigma}}^2_{N,ee})'$ on the diagonal and zeros elsewhere, is certainly positive definite. Thus one can use a convex combination of $\hat{\Sigma}^0_\varepsilon$ and $diag(\hat{\Sigma}^0_\varepsilon)$, which shrinks the sample covariance matrix towards its diagonal, to obtain a positive definite matrix. Such a simple shrinkage estimator of the covariance matrix is given:

$$\hat{\Sigma}^0_\varepsilon(\varrho) = (1 - \varrho) diag\left(\hat{\Sigma}^0_\varepsilon\right) + \varrho\hat{\Sigma}^0_\varepsilon. \tag{27}$$

We experimented with different values of ρ, and found that $\hat{\Sigma}^0_\varepsilon(\varrho)$ is positive definite for all values of $\varrho \leq 0.4$. Accordingly, the initial estimates of the IRFs and FEVDs are based on the shrinkage covariance matrix, $\hat{\Sigma}^0_\varepsilon(0.4)$. We then examine the sensitivity of the IRFs to the choice of covariance matrix. Since calculation of generalised IRFs does not require the covariance matrix to be positive definite we can compare the IRFs from the shrinkage covariance matrix, $\hat{\Sigma}^0_\varepsilon(0.4)$ with the IRFs from $\hat{\Sigma}^0_\varepsilon$, as well as the diagonal covariance matrix, $diag(\hat{\Sigma}^0_\varepsilon)$, and a block diagonal covariance, $Bdiag(\hat{\Sigma}^0_\varepsilon)$ matrix, which sets the covariances between the exchange rate shocks and the structural shocks in $\hat{\Sigma}^0_\varepsilon$, to zero.

6 Analysis of shocks

A large number of possible counter-factual scenarios can be considered differing in the type of shock, the target country, the specification of the error covariance matrix, and the structure of the equations in the global model. We consider the time profiles of the effects of a US monetary policy shock, and global supply and demand shocks on output, inflation and interest rates across the 33 countries. In this section we use the shrinkage covariance matrix estimator defined by (27) with $\varrho = 0.40$. This assumes a bordered covariance matrix, with non-zero covariances between structural shocks of the same type, and with unrestricted covariances between the structural shocks and exchange rate shocks.

Notice that we are measuring the effects of an unexpected one period shock not on the variables, but on their deviations from steady states. To examine the effects of shocks on the variables themselves, we would also need to consider the changes in their steady states. The system is stable and following these shocks the variables converge to their steady state values within 5 to 6 years in the vast majority of cases. Although there are only short lags in the system, no more than one period, and strongly forward looking behaviour in the Phillips curve, there is complicated dynamics and some slow adjustment to shocks. The largest eigenvalue of the system is 0.975**.** Many of the eigenvalues are complex, so adjustments often cycle back to zero. Inflation is a forward-looking variable in this model, so it jumps as expectations adjust to a shock, while interest rates respond strongly to inflation.

6.1 US monetary policy shock

We first consider a contractionary US monetary policy shock, $\mathbf{a}'_m \boldsymbol{\varepsilon}^0_t$, where $\mathbf{a}_m$ has zeros except for the element corresponding to, $\varepsilon_{0,mt}$, which is set to unity. Given that this shock has been widely considered in the literature, it is worth simulating it with the MCNK model for comparison purposes. The US monetary policy shock raises the US interest rate on impact by one standard error (around 22 basis points per quarter), which also simultaneously impacts interest rates in other countries through the contemporaneous dependence of monetary policy shocks as captured by the off diagonal elements of $\hat{\boldsymbol{\Sigma}}_{mm}$.

Figures 1a-1c show the effect of a contractionary US monetary shock on interest rates, inflation and output for 26 countries. The results for the five Latin American countries (Argentina, Brazil, Chile, Mexico, Peru), Indonesia and Turkey are excluded as they tend to be outliers due to the much higher levels of inflation and nominal interest rates experienced in these economies over our estimation sample.[14] Also to focus on the differences across countries, the graphs only show the point estimates. Bootstrapped confidence bounds will be considered below.

The monetary policy shock raises interest rates in the US by one standard error, 22 basis points, and interest rates rise almost everywhere else. The mean change for other countries amounts to an increase of 6 basis points, though this is skewed by Argentina, not shown on the graph, and the median is 2 basis points. Interest rates then move below their steady state values very quickly to offset the shock and by quarter 4 they are lower almost everywhere, by -15 basis points in the US; for the other countries the mean is -16 basis points, the median -10 basis points, with the mean skewed to the left by Chile, not shown on the graph. The effect of the monetary policy shock on interest rates in other countries is of the same order of magnitude as in the US. All the interest rates are close to their steady state values within five years, except for the interest rates in Norway which take longer to settle down.

The US monetary policy shock depresses inflation and output, which is consistent with the standard results, e.g. Kim (2001), and output and inflation return to close to steady state within five years for inflation, and six years for output. By quarter 4, US inflation is -0.18 per cent and US output -0.50 per cent below their steady state values. The reduction in US inflation and output in response to the monetary policy shock has a similar shape to that of Smets and Wouters (2007, Fig. 6). The major difference is that whereas in their model a monetary policy shock causes interest rates to go up then slowly return to zero, in our model the monetary policy shock initially raises interest rates, but this is quickly offset by the effects of the relatively sharp falls in output and inflation. This rapid stabilising response occurs despite the fact that there is quite a lot of inertia in our Taylor rules, which have a coefficient of lagged interest rate that averages 0.59. The effects on inflation and output in other countries are similar to those in the US. On average after four quarters, inflation in countries other than the US is lower by -0.18 per cent (per quarter), the same as the US, and output is lower by -0.64 per cent, rather more than the US. The US variables tend to return to their steady state values relatively quickly compared to other countries. The results show that a US monetary policy shock has a rather large global impact in this model.

[14] The excluded countries show the same qualitative patterns in their impulse response functions.

Figure 1a: Impulse responses of a one standard error US monetary policy shock on interest rates (per cent per quarter)

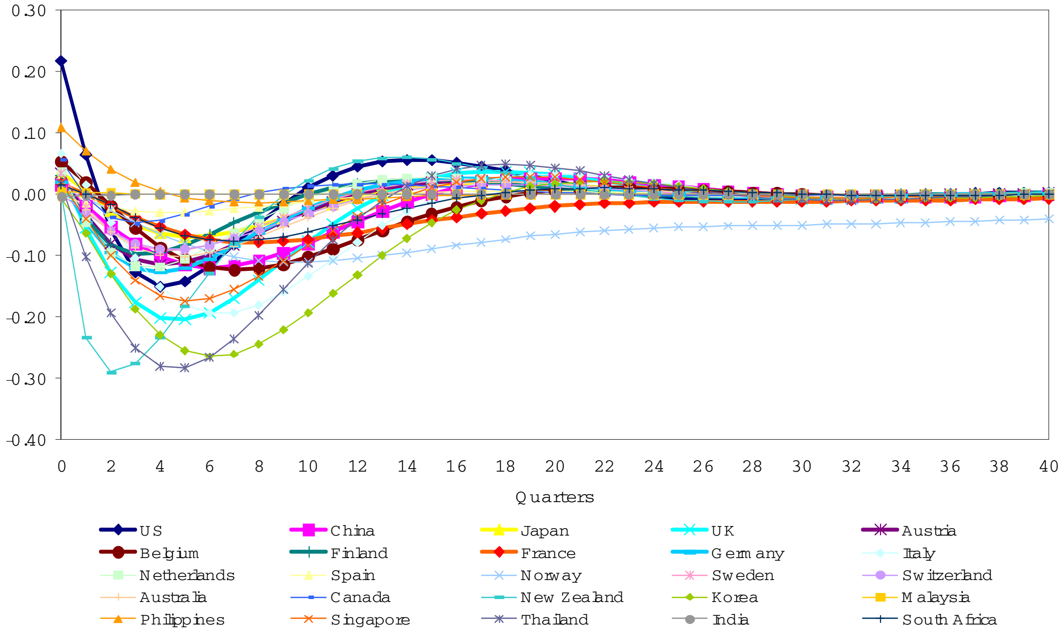


Figure 1b: Impulse responses of a one standard error US monetary policy shock on inflation (per cent per quarter)

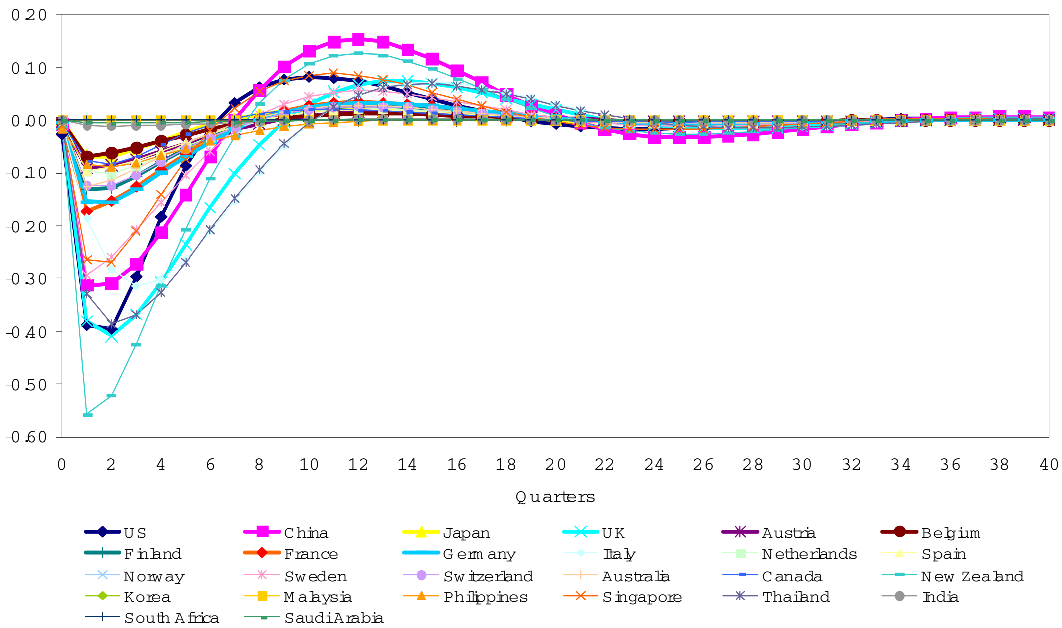
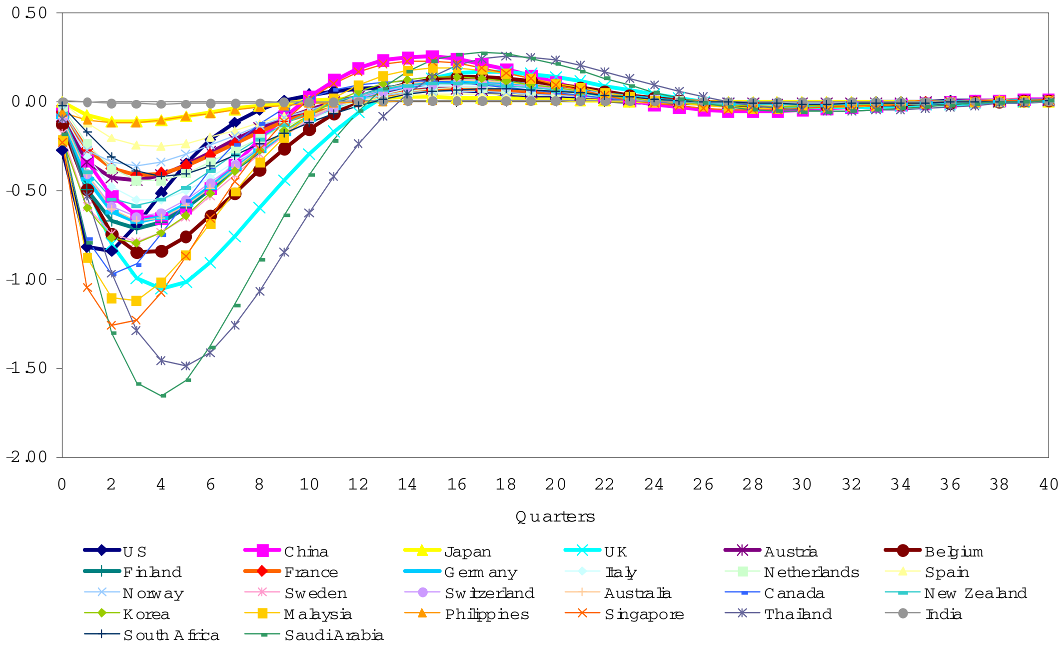


Figure 1c: Impulse responses of a one standard error US monetary policy shock on output (per cent per quarter)



6.2 Global supply and demand shocks

We now consider a global inflationary supply shock, $\mathbf{a}_s'\boldsymbol{\varepsilon}_t^0$, where the non-zero elements of $\mathbf{a}_s$ are PPP GDP weights (that add up to one), associated with the $N+1$ supply shocks, $\varepsilon_{i,st}$, in $\boldsymbol{\varepsilon}_t^0$. Figures 2a-2c show the effects of a unit (one standard error) global supply shock on inflation, output and interest rates across the 26 countries as in Figures 1a-1c. The supply shock causes inflation and interest rates to increase on impact, but then they both fall below their steady state values relatively rapidly, before slowly returning back to the steady states. The global supply shock has quite a large impact. In the US the supply shock increases inflation by 1.4 per cent, which is then reversed to -1.4 per cent after two quarters before returning to its steady state value. The pattern is similar across other countries, though the impact effect on the US is rather higher than the average increase in inflation experienced in other countries, which is 1.0 per cent rather than 1.4 per cent in the US. Similarly, the reduction in US inflation after two quarters is rather larger than the average fall in inflation in other countries.

Figure 2a: Impulse responses of a one standard error global supply shock on inflation (per cent per quarter)

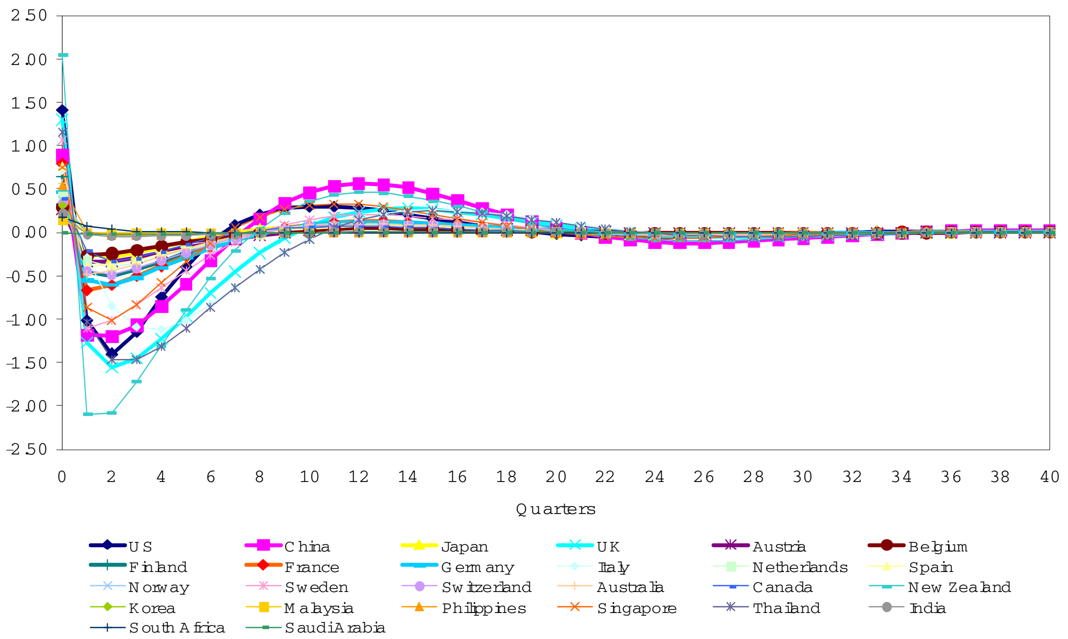


Figure 2b: Impulse responses of a one standard error global supply shock on output (per cent per quarter)

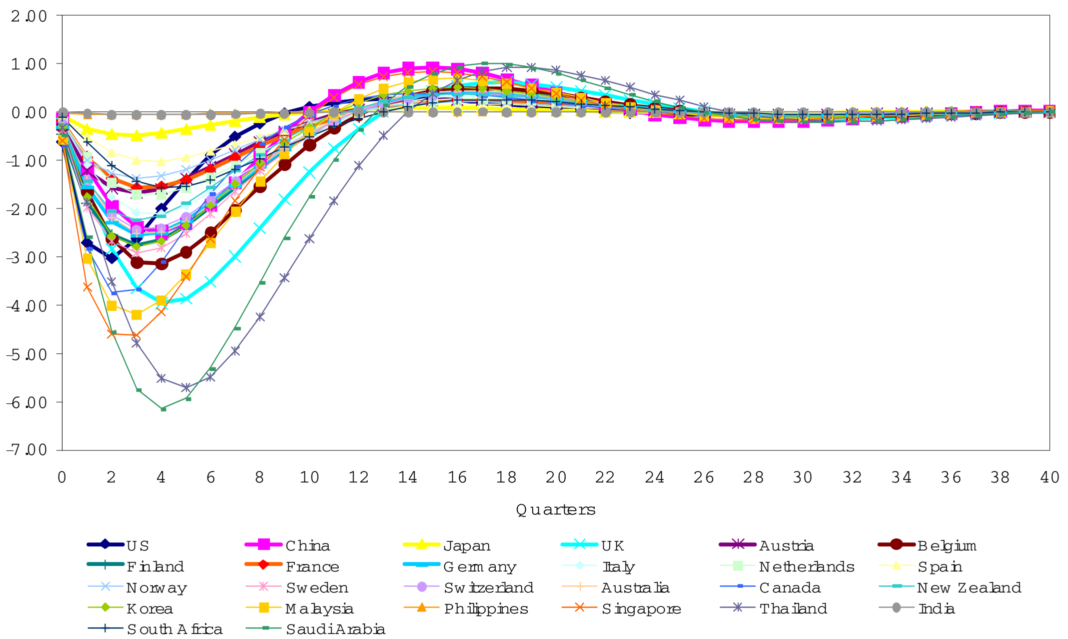
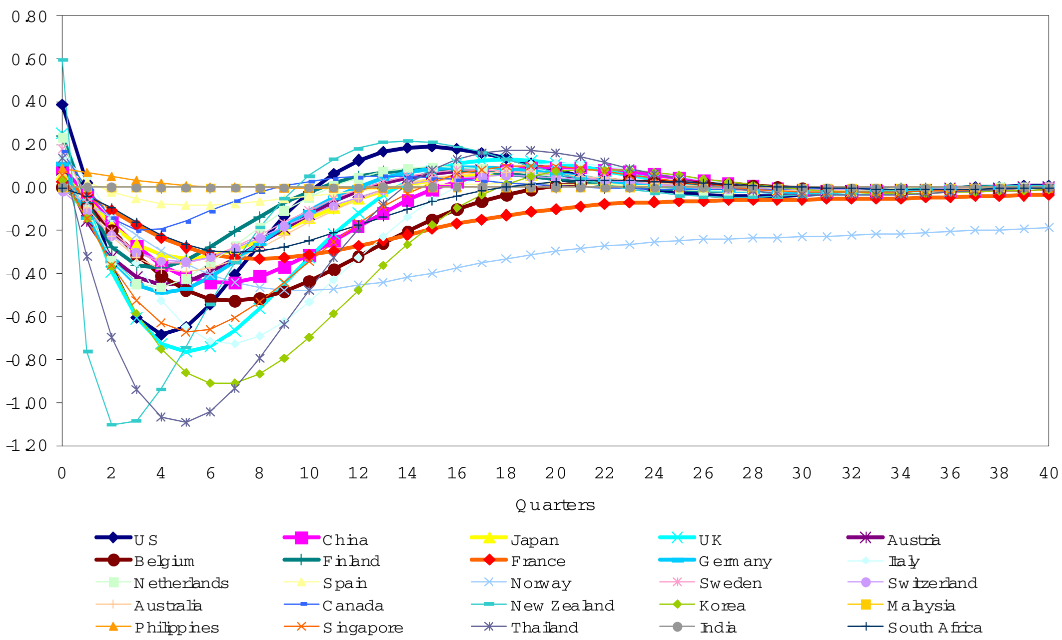


Figure 2c: Impulse responses of a one standard error global supply shock on interest rates (per cent per quarter)



The global supply shock also reduces output across the board with an average effect of -2.4 per cent after 4 quarters. The pattern of dynamic adjustments to the global supply shock is different from the standard closed economy models because cross-variable feedbacks seem to operate at a faster pace: inflation and interest rates rapidly move to offset the effects of the inflationary pressure resulting from the global supply shock.

The effects of a global demand shock (constructed similarly to the global supply shock using PPP GDP weights) on output, inflation and interest rates are summarised in Figures 3a-3c. As expected the demand shock has a positive effect on output, inflation and interest rates. In accord with the theory, in the MCNK model a global demand shock increases output and inflation, while a supply shock reduces output and increases inflation. The global demand shock causes output and interest rates to rise before cycling back to their steady state values. The initial expansionary phase of the shock is relatively long lived and takes around 11 to 15 quarters. The effects of the demand shock across countries are qualitatively similar, but differ markedly in the size of the effects. The effect on US output of 3.7 per cent is in the middle of the cross country distribution of the effects, with the US output returning to its steady state value relatively fast. In some other countries output increases further after impact before returning to steady state. The positive effect on inflation lasts a somewhat shorter period than on output. The shape of the responses by output, inflation and interest rates in the US, to the demand shock are qualitatively similar to those reported by Smets and Wouters (2007, Fig. 2).

Figure 3a: Impulse responses of a one standard error global demand shock on output (per cent per quarter)

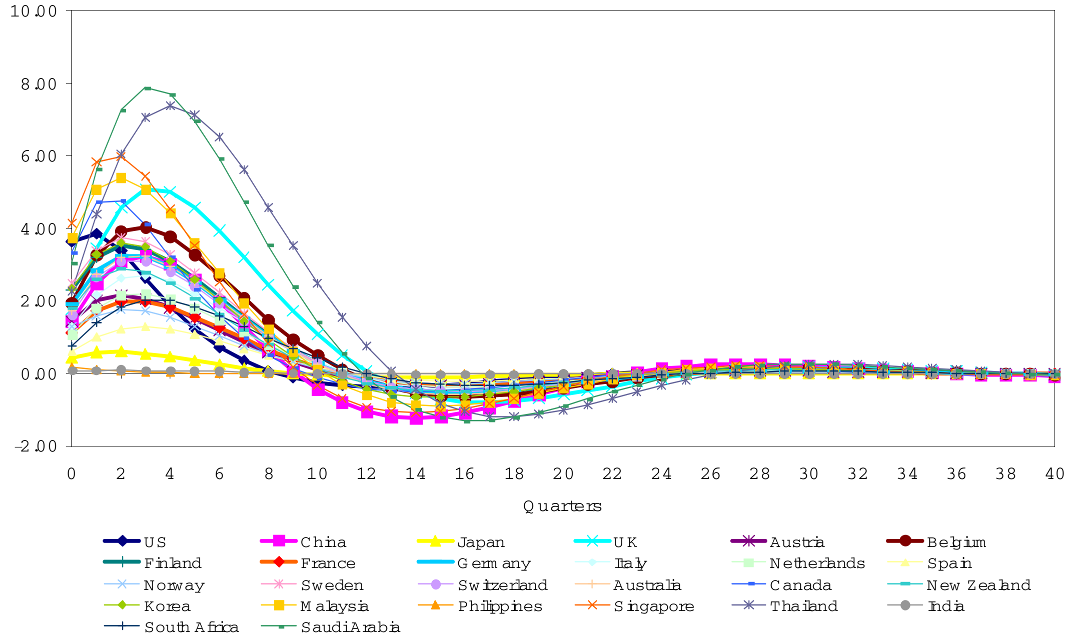


Figure 3b: Impulse responses of a one standard error global demand shock on inflation (per cent per quarter)

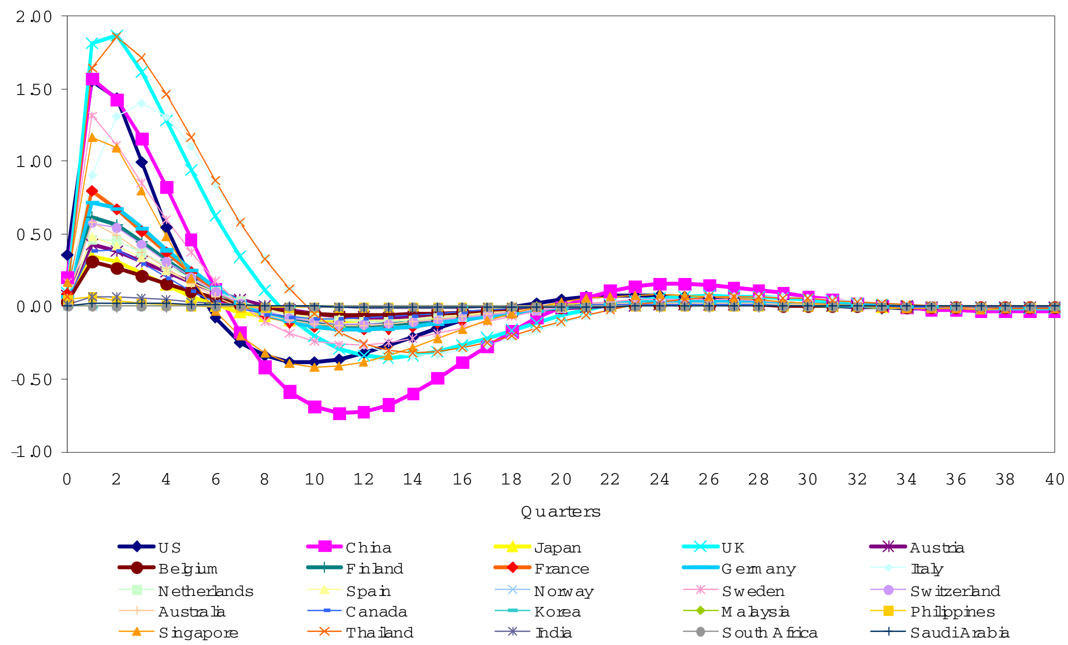
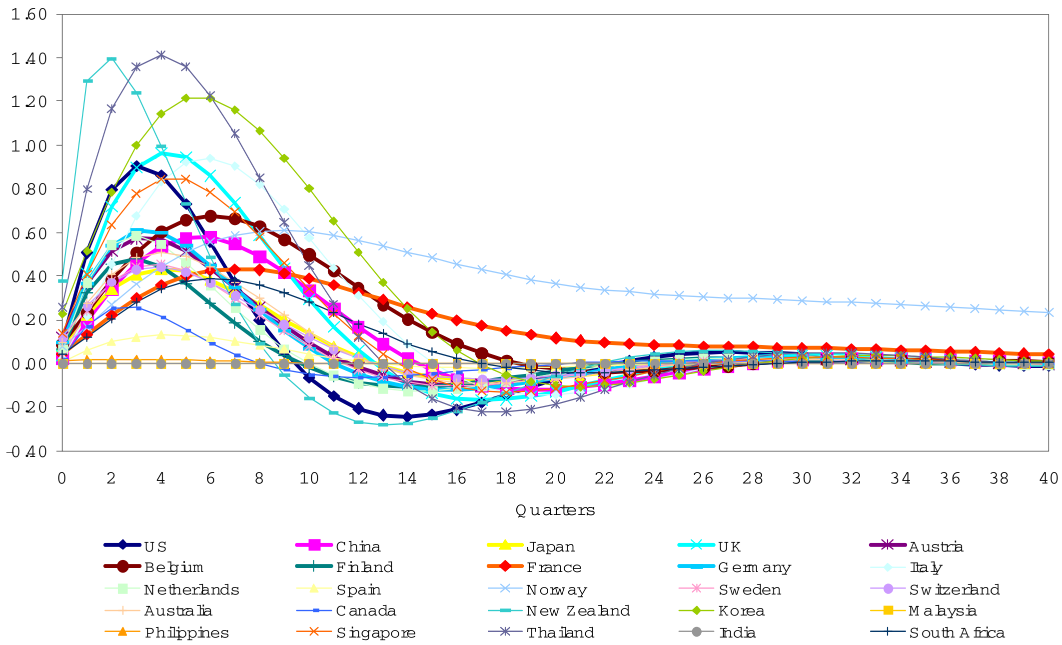


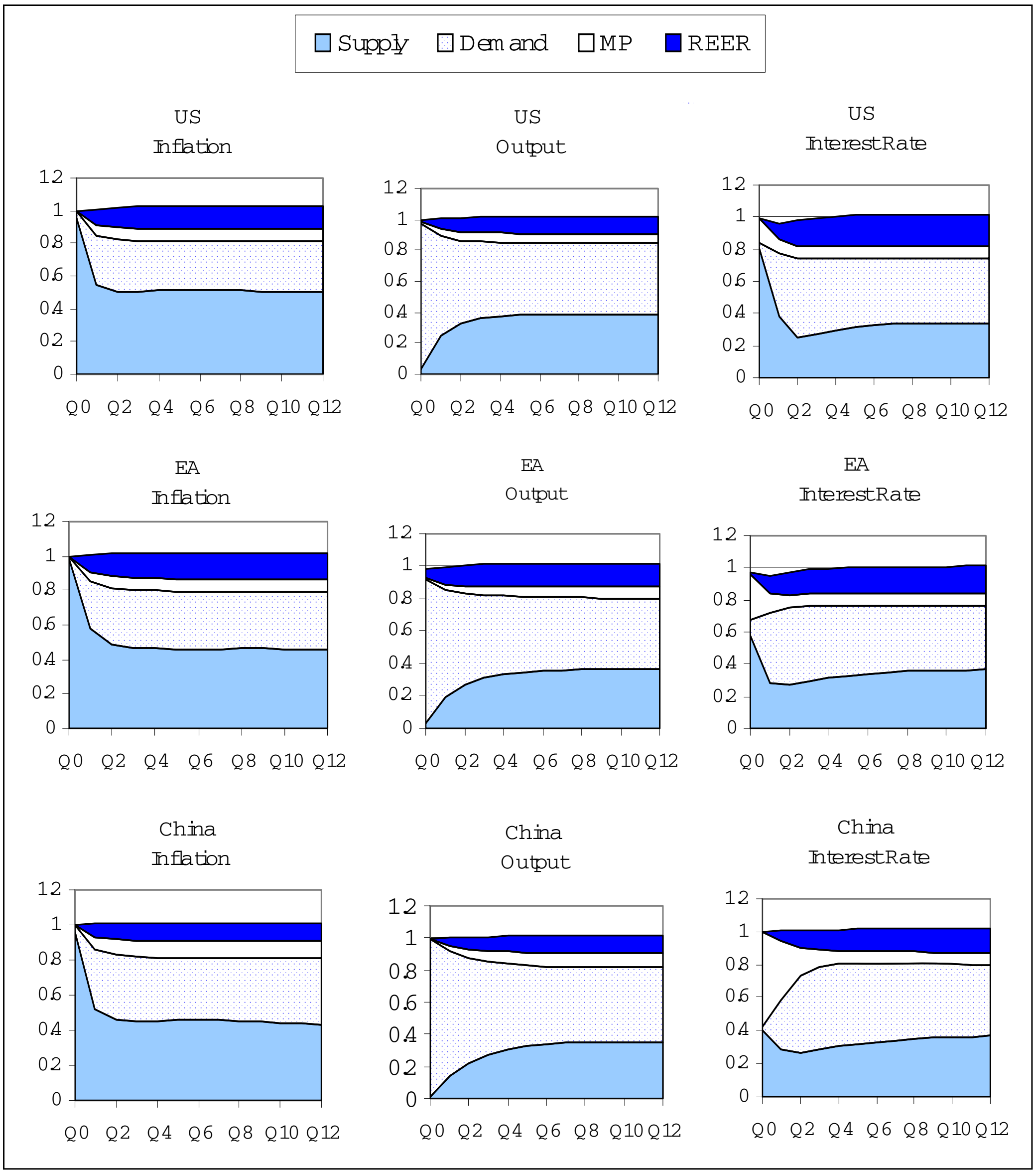
Figure 3c: Impulse responses of a one standard error global demand shock on interest rates (per cent per quarter)



6.3 Forecast error variance decomposition

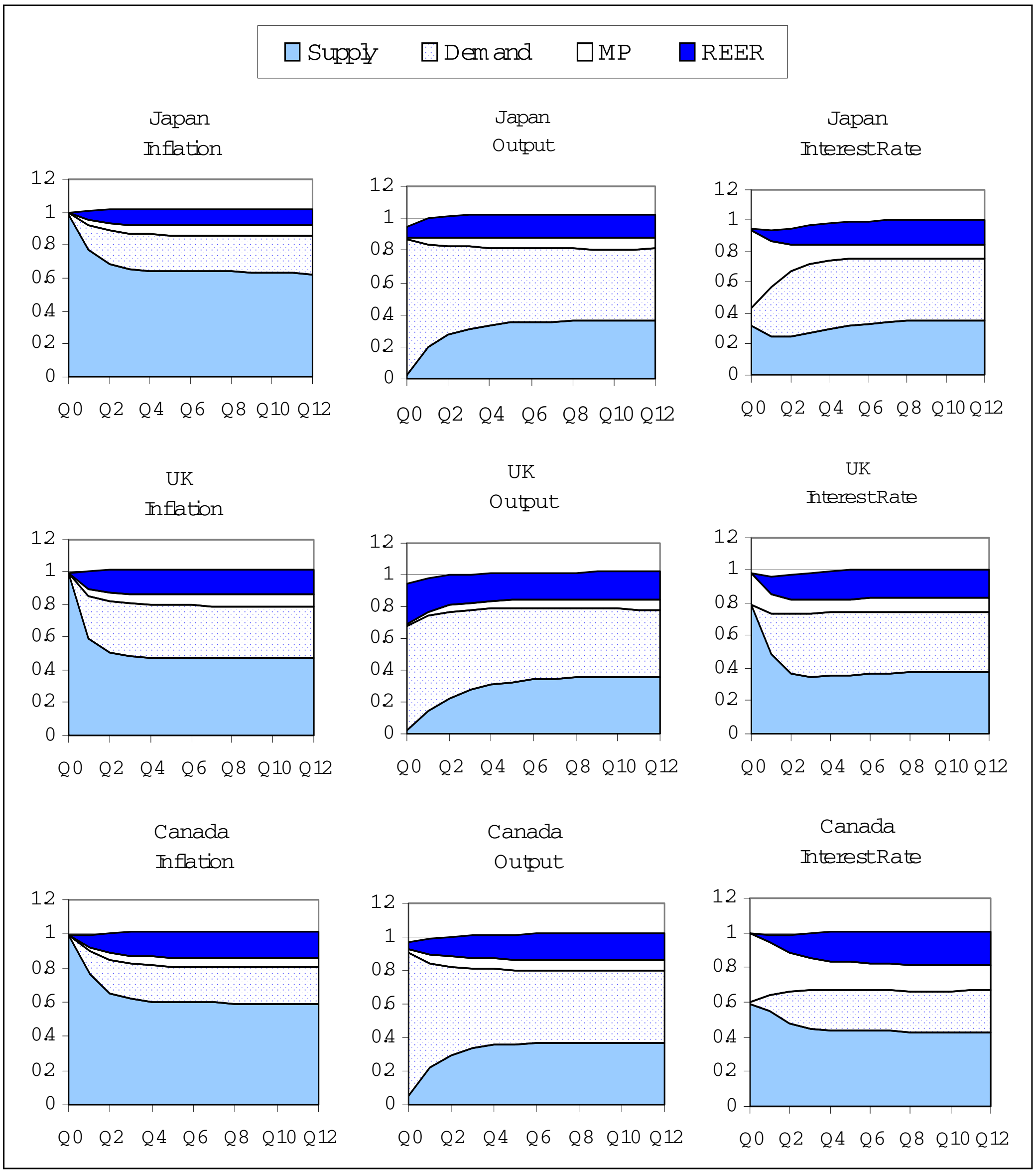
Figures 4a-4b show the FEVDs for selected economies. The euro area estimates are obtained by averaging over the FEVDs of member countries using PPP GDP weights. The FEVDs across the different variables add up close to unity, being a little below on impact and a little above after 12 quarters on average. While there are differences across countries, in all cases supply and demand shocks account for most of the variations in output, inflation and interest rate in the long-run, with monetary policy shocks and exchange rate shocks accounting for relatively little of the variations, around 8-10% each. Monetary policy shocks account for more of the variation in interest rates in Canada than in other countries, though even here it is not a large proportion. On impact supply shocks account for nearly all the variation of inflation, but this drops rapidly and these shocks only account for about half of the variation of inflation in the long-run. Demand shocks account for most of the variations in output on impact, but again this figure drops quite rapidly. Smets and Wouters (2007) also find that monetary policy shocks account for relatively little of the variations in output and inflation in the US.

Figure 4a: Forecast error variance decomposition of the shocks in explaining inflation, output and interest rates for the US, the euro area and China



Note: Q0 refers to the values on impact.

Figure 4b: Forecast error variance decomposition of the shocks in explaining inflation, output and interest rates for Japan, the UK and Canada



Note: Q0 refers to the values on impact.

6.4 Bootstrapped error bands

As noted earlier we also used a bootstrap procedure, set out in detail in the Appendix, to compute 90% error bands for the impulse responses. The results for the effects of US monetary policy shock and global demand and supply shocks on the US and euro area interest rates, output, and inflation are displayed in Figures 5a-5c. As above, the euro area impulse responses are obtained by averaging over the impulse responses of member countries using PPP GDP weights. These figures show the median (which is almost identical to the mean except for India, not shown) and the 5%

and 95% quantiles of the bootstrap distribution. The results indicate that the effects of the shocks are statistically significant in the sense that the 90% bootstrap bands do not always cover zero. Results for other IRFs are similar and available on request.

Figure 5a: Impulse responses of a one standard error US monetary policy shock on US and euro area interest rates, inflation and output (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)

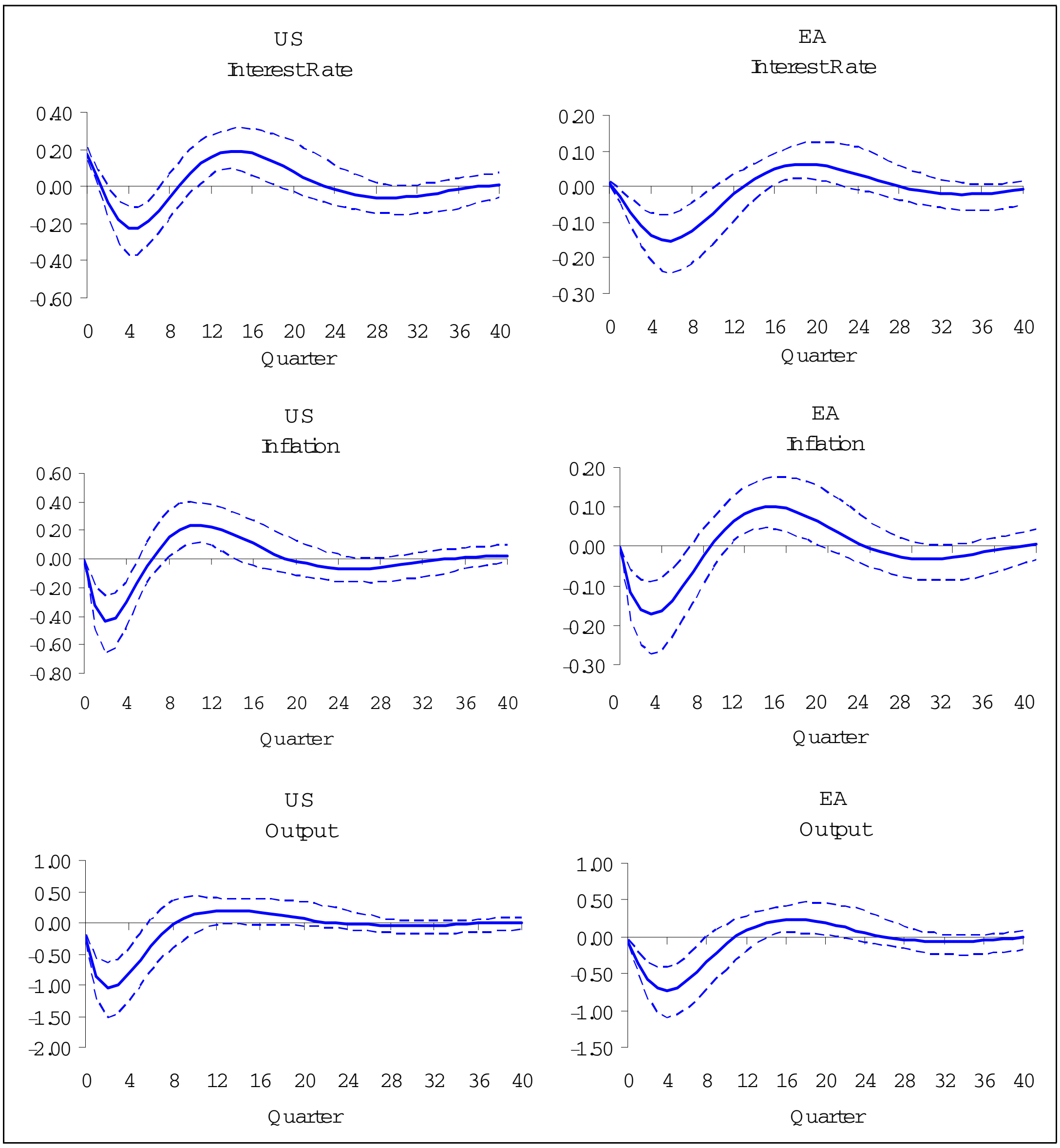


Figure 5b: Impulse responses of a one standard error global supply shock on US and euro area inflation, output and interest rates (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)

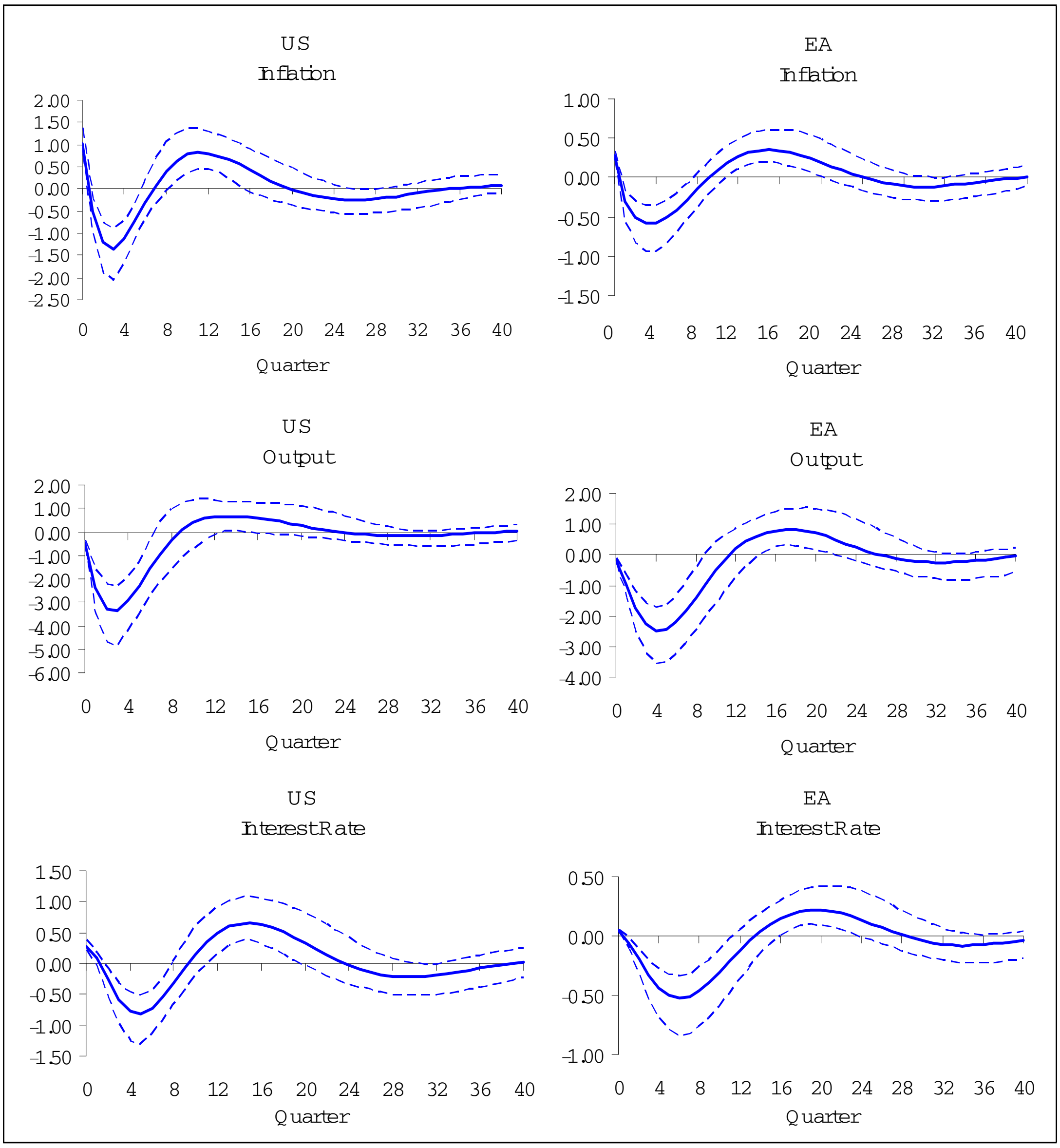
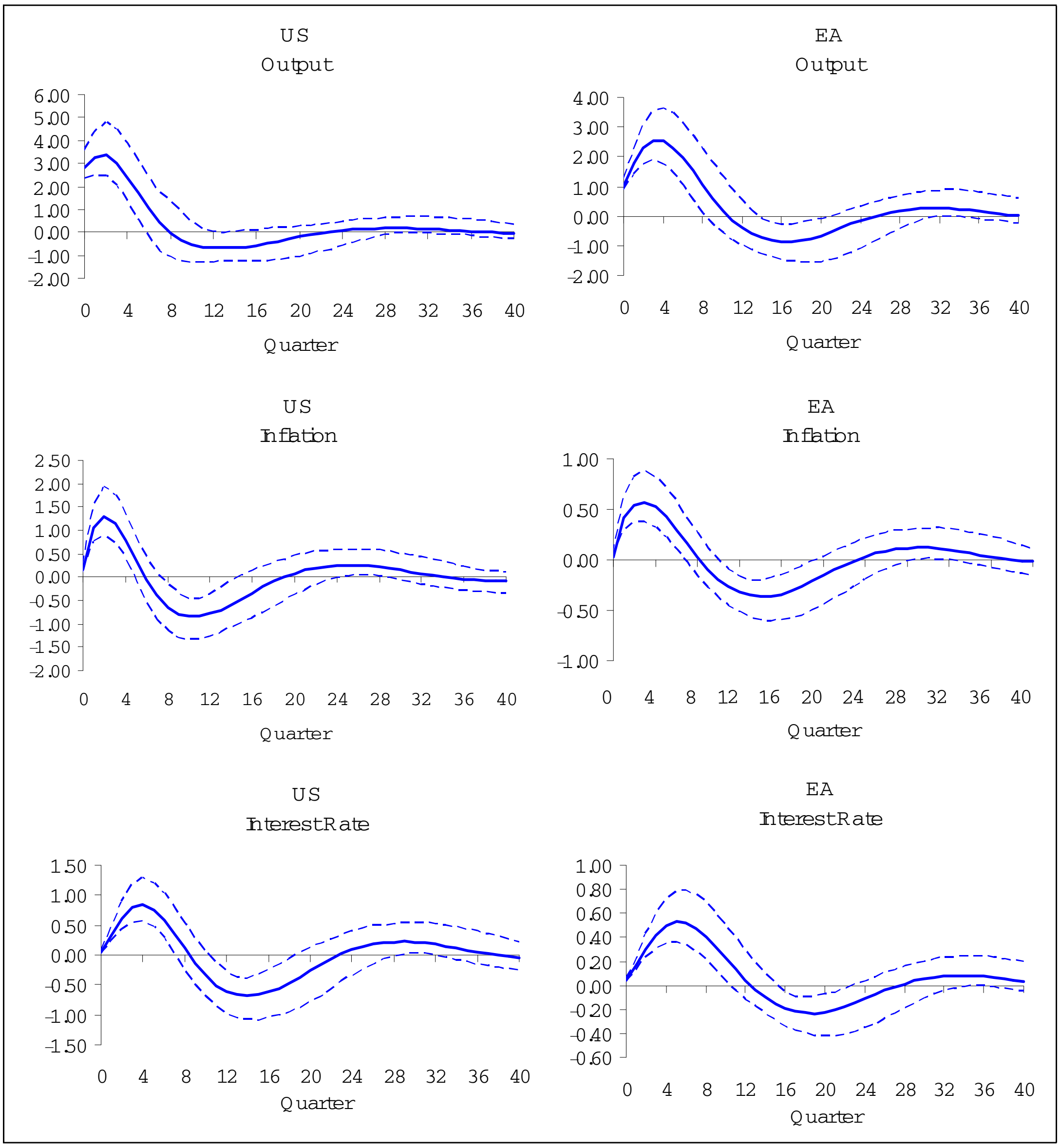


Figure 5c: Impulse responses of a one standard error global demand shock on US and euro area output, inflation and interest rates (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)



7 Alternative specifications

In this paper we have deviated from the empirical NK DGSE modelling literature in two important respects. First, we have estimated the steady states as long horizon expectations using an error correcting GVAR specification, as compared to using a purely statistical de-trending procedure. Second, we have allowed for international linkages across shocks and economies using a full multi-country NK model. In what follows we evaluate the importance of these innovations for our results. We also consider the sensitivity of the impulse responses to alternative specifications of the covariances of the structural shocks.

7.1 Measurement of steady states

As an alternative measure of steady states we considered the familiar Hodrick-Prescott (HP) filter for real output and following the literature assumed that the other variables, namely inflation, interest rate and the real exchange rate are stationary, and thus their steady states can be viewed as constants. We computed the HP filter of log real output using the smoothing parameter of 1600 for all countries. The output deviations based on the HP filter were then computed, which we denote by $\widetilde{y}_{it}^{HP}$, for $i = 0, 1, ..., N$. The country-specific NK models were then estimated by the IV procedure subject to the same theoretical restrictions as above, with an intercept included to allow for the assumed constant steady state values of the other three variables. The instruments used were an intercept, the lagged values of the country specific endogenous variables, $\widetilde{y}_{i,t-1}^{HP}, \pi_{i,t-1}, r_{i,t-1}, re_{i,t-1}$, the current values of the foreign variables $\widetilde{y}_{it}^{HP*}, \pi_{it}^{*}, r_{it}^{*}$,[15] and the first difference of the oil price variable, Δp_{it}^{o}. The results are summarised in Table 4 which has the same format as Table 1.

Table 4: Inequality-constrained IV estimates using HP filtered output deviations with smoothing parameter $\lambda = 1600$ and constant steady states for other variables

	Mean	# Constrained	UC Mean	Constraint
Phillips curve - Equation (2), N=33				
β_{ib}	0.16	6	0.20	$\beta_{ib} \geq 0$
β_{if}	0.81	0	0.81	$\beta_{if} \geq 0$
β_{iy}	0.02	12	0.03	$\beta_{iy} \geq 0$
$\beta_{ib}+\beta_{if}$	0.97	25	0.92	$\beta_{ib}+\beta_{if} \leq 0.99$
IS curve - Equation (3), N=33				
α_{ib}	0.67	0	0.67	
α_{ir}	-0.19	11	-0.28	$\alpha_{ir} \leq 0$
α_{ie}	0.00	0	0.00	
α_{iy*}	0.43	6	0.53	$\alpha_{iy*} \geq 0$
Taylor Rule - Equation (4), N=32				
γ_{ib}	0.80	0	0.80	
$\gamma_{i\pi}$	0.18	1	0.18	$\gamma_{ir} \geq 0$
γ_{iy}	0.07	4	0.08	$\gamma_{iy} \geq 0$
Exchange rates - Equation (5), N=32				
ρ_i	0.95	0	0.95	$\lvert\rho_i\rvert < 1$

See the notes to Table 1.

[15] The foreign output variable based on HP steady state values are computed as $\tilde{y}_{it}^{*HP} = \Sigma_{j=0}^{N} w_{ij} \tilde{y}_{jt}^{HP}$.

The estimates in Table 4 are more backward looking than those obtained using GVAR deviations, with slower adjustments and near unit root autocorrelation coefficients for the real effective exchange rates. The effect of output deviations in the Phillips curve is smaller using the HP filter as compared to using the GVAR measures of the steady states - also documented in DPSS. In the IS curve, in addition to larger estimated coefficients for the lagged variables and thus slower adjustments, domestic output deviations are less responsive to foreign output deviations when using $\tilde{y}_{it}^{HP}$. This significantly reduces an important channel for the international transmission of shocks. The Taylor rule is less responsive to inflation and also adjusts more slowly, the coefficient on the lagged interest rate is 0.8 as compared to 0.6 when using the GVAR deviations.

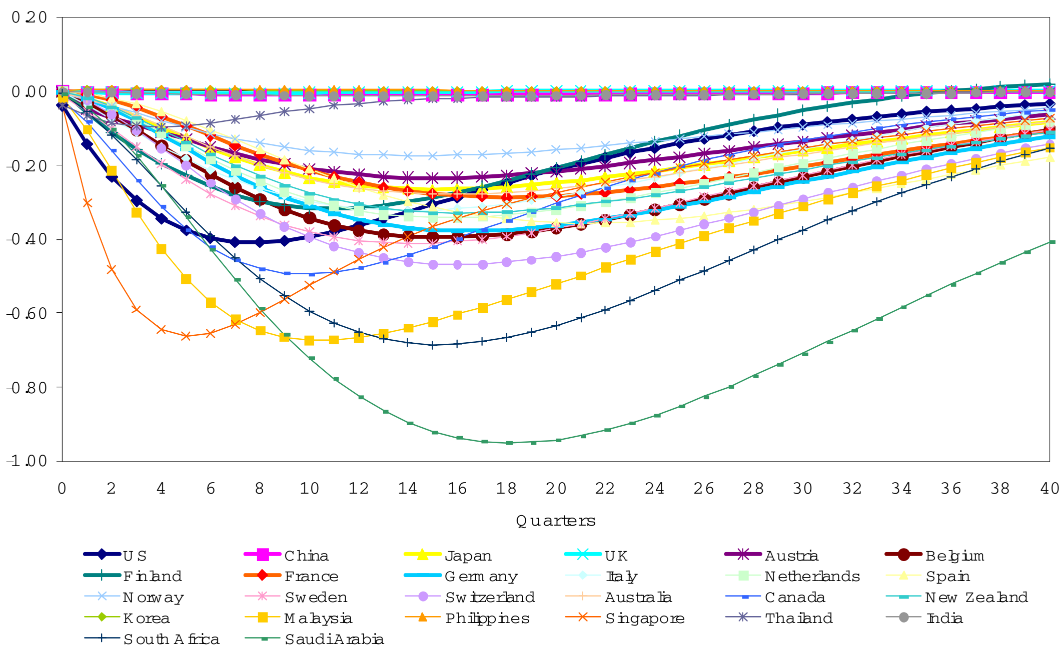
The patterns of average correlations among the estimated shocks, shown in Table 5, is similar to those using GVAR deviations, with the highest average correlation being between supply shocks.

Table 5: Average correlations among the estimated shocks using HP measure of steady state for output and constants for other variables.

	Supply	Demand	Mon. Pol.	Ex. Rate
Supply	0.475	0.173	0.049	0.003
Demand		0.089	0.017	-0.006
Mon. Pol.			0.070	0.001
Ex. Rate				-0.002

Again we used the simple shrinkage estimator of the covariance matrix given by (27) with $\varrho = 0.4$. The effect of the slower adjustment implied by the parameters using HP deviations for output and constant steady states for the other variables can be seen in Figure 6, which shows the response of output in the various countries to a contractionary US monetary policy shock. While output declines as in Figure 1a, the return to equilibrium is much slower using HP deviations than using GVAR deviations, probably because of the slower estimated adjustment. In fact many of the impulse responses fail to converge to their steady state values even after 40 quarters, which could be indicative of the possible non-stationary nature of some of the variables included in the country specific DSGE models. A comparison of the results in Figures 1a and 6 clearly show the importance of the de-trending procedure for the multi-country analysis of shocks and their transmission in the global economy. Similar results are also obtained when the impulse responses of global supply and demand shocks are compared across the two approaches to the identification and estimation of steady states.

Figure 6: Impulse responses of a one standard error US monetary policy shock on output using HP deviations and constant steady states (per cent per quarter)



7.2 Shutting off direct international linkages

To evaluate the importance of allowing for direct international linkages, we estimated the MCNK model (with GVAR deviations) under alternative restrictions on the coefficient of the foreign output variable in the IS curve. Initially, only the coefficient of foreign output in the US was set to zero. This caused the unrestricted estimate of the interest rate coefficient to be positive, and led to a restricted IS curve for the US without an interest rate variable, thus cutting off the main transmission route for the operation of the US monetary policy. We also experimented with dropping the foreign variables from all the IS equations. Not surprisingly, this caused the average pair-wise correlation coefficient across the demand shocks to increase from 0.166, reported in Table 3, to 0.229, thus shifting the burden of the international transmission of shocks to the indirect effects as captured by error covariances. The impulse response functions also became much less sensible as shown in Figures 7a-7c. In response to a US monetary policy shock, interest rates rise almost everywhere, but the response of output and inflation is much more dispersed as compared to the results from the baseline model.

Figure 7a: Impulse responses of a one standard error US monetary policy shock on interest rates in model without foreign output (per cent per quarter)

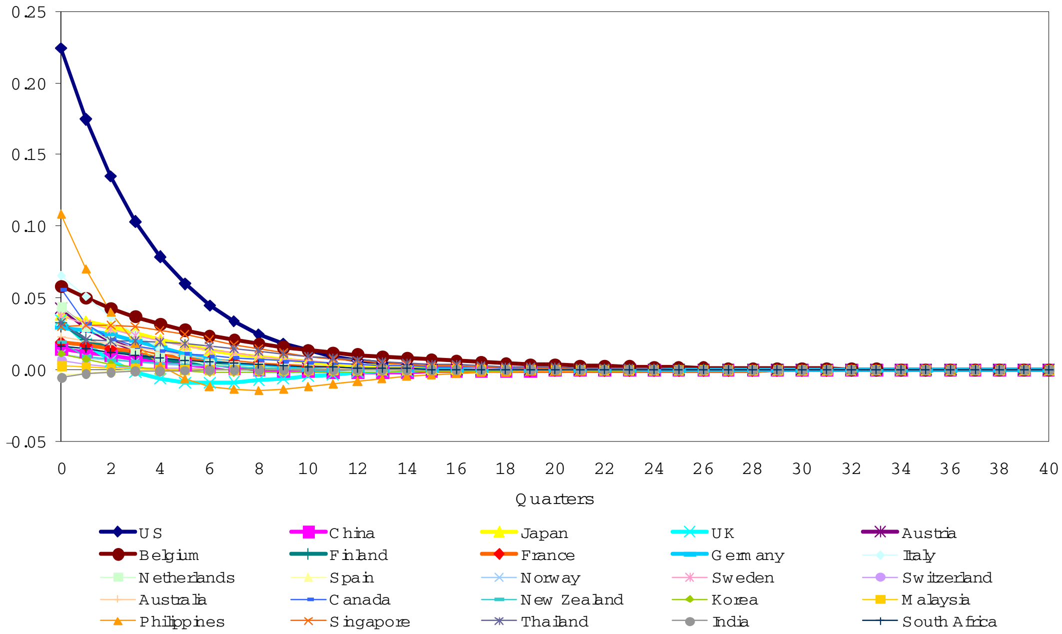


Figure 7b: Impulse responses of a one standard error US monetary policy shock on inflation in model without foreign output (per cent per quarter)

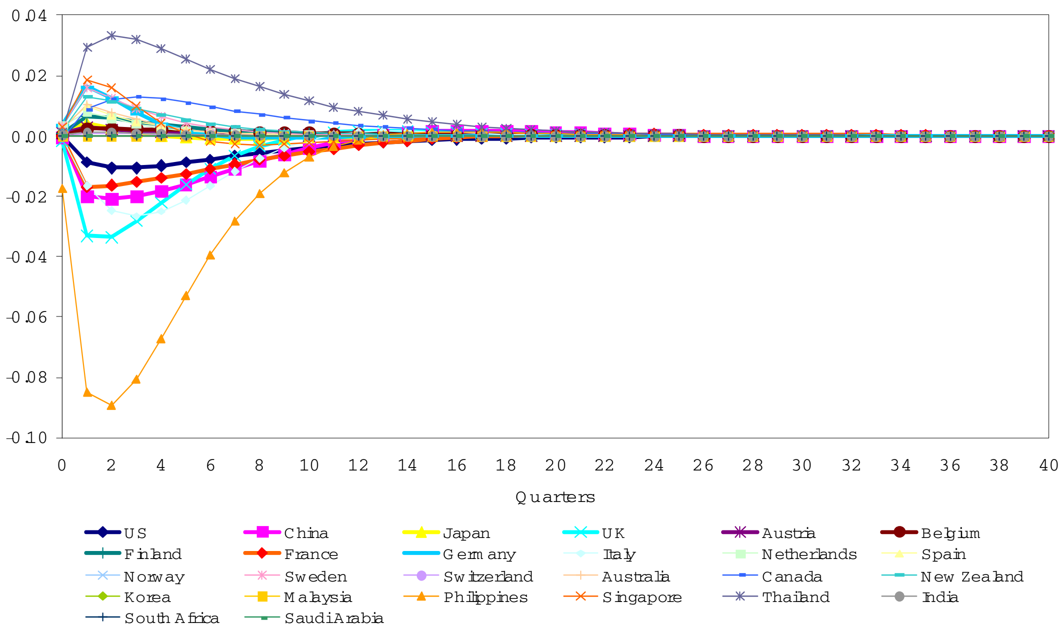
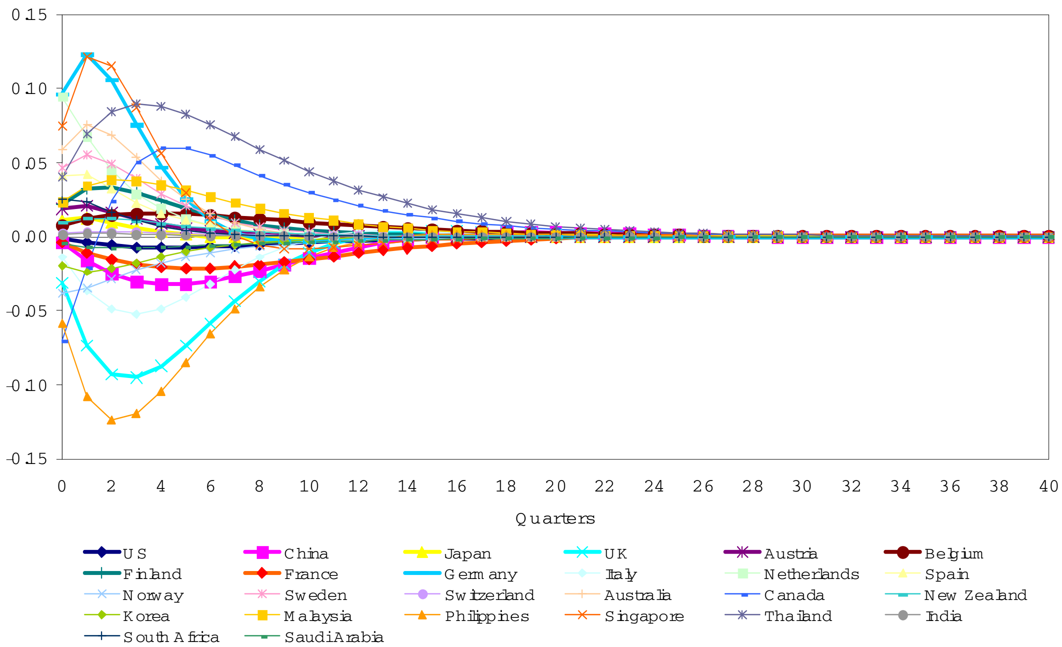


Figure 7c: Impulse responses of a one standard error US monetary policy shock on output in model without foreign output (per cent per quarter)



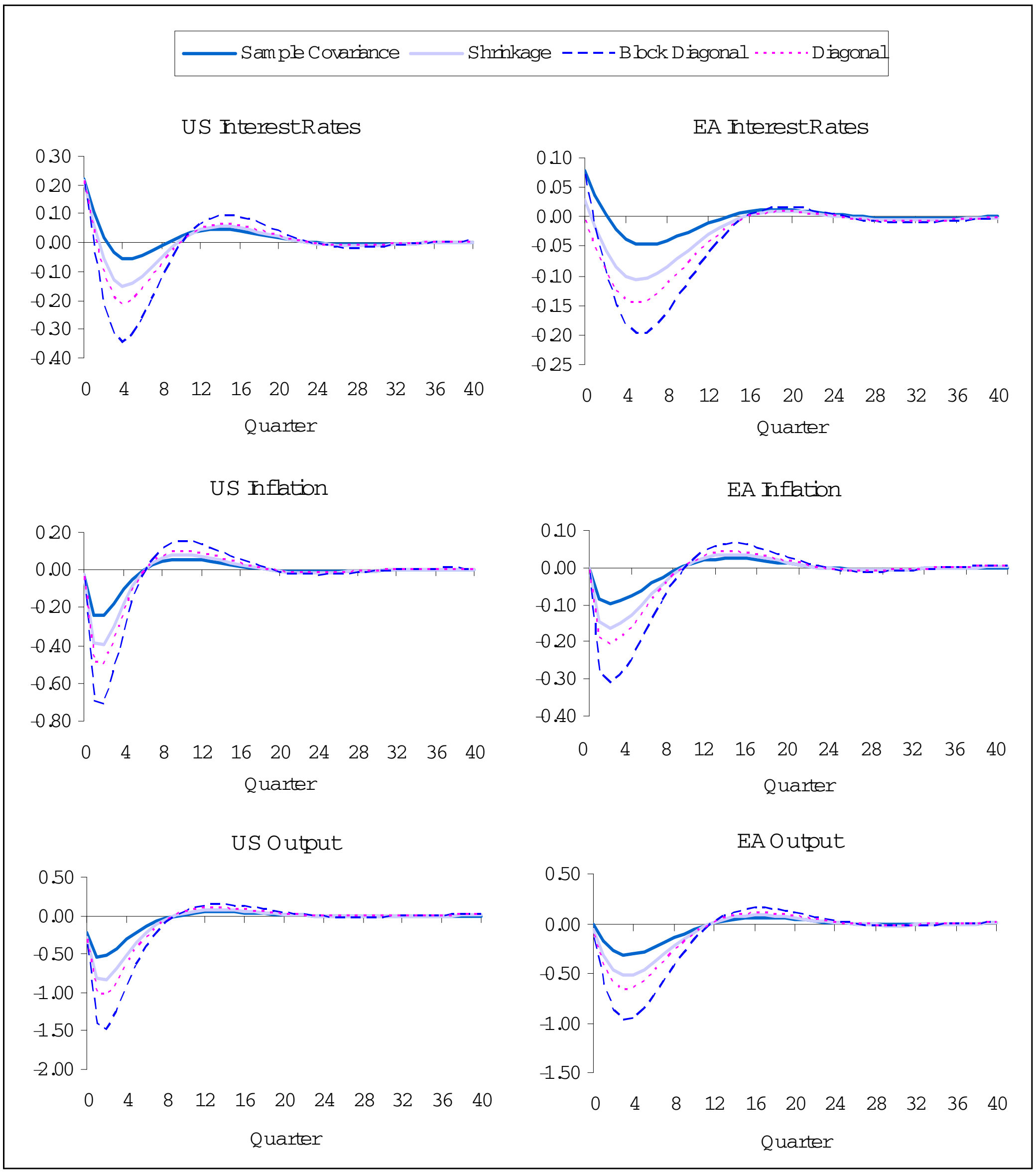
7.3 Choice of error covariance matrices

The impulse responses reported so far are based on the shrinkage covariance matrix, $\hat{\Sigma}_{\varepsilon}^{0}(0.4)$, which uses a weighted average of the sample moment estimate of (20), $\hat{\Sigma}_{\varepsilon}^{0}$, with its diagonal, $diag(\hat{\Sigma}_{\varepsilon}^{0})$. Since the choice of the weight, ϱ, is to some extent arbitrary we thought it is important to investigate the sensitivity of our results to the choice of Σ_{ε}^{0} and how it is estimated. Accordingly, here we consider four alternative estimates of the error covariance matrix: (a) the sample moment estimate, $\hat{\Sigma}_{\varepsilon}^{0}$ (b) the diagonal matrix, $diag(\hat{\Sigma}_{\varepsilon}^{0})$, which cuts off all correlations between shocks, and (c) a block diagonal covariance matrix, $Bdiag(\hat{\Sigma}_{\varepsilon}^{0})$ which imposes zero covariances between exchange rate and other shocks, but allows each type of shock to be correlated within a block, in addition to (d) the shrinkage estimator used above. Setting the covariances of exchange rates with the other shocks to zero, means that the shocks have no effects on exchange rates since there is no direct feedback from the other variables on the real effective exchange rates.

Figure 8 presents IRFs for the effect of a US monetary policy shock on interest rates, inflation and output, using the four estimates of the error covariance matrices. This is the scenario where sensitivity to the choice of the error covariance matrix seems to be the greatest. The different IRFs show the same qualitative pattern, but there seems to be a consistent ranking of the size of the responses, with $\hat{\Sigma}_{\varepsilon}^{0}$ yielding the smallest effects followed by $\hat{\Sigma}_{\varepsilon}^{0}(0.4)$, and $diag(\hat{\Sigma}_{\varepsilon}^{0})$, with $Bdiag(\hat{\Sigma}_{\varepsilon}^{0})$ producing the largest effects. Using the block diagonal covariance matrix, $Bdiag(\hat{\Sigma}_{\varepsilon}^{0})$, where the exchange rate covariances are set to zero amplifies the responses relative both to the sample moment estimate, $\hat{\Sigma}_{\varepsilon}^{0}$, which allows covariances between the exchange rate and other shocks and the fully diagonal covariance matrix, $diag(\hat{\Sigma}_{\varepsilon}^{0})$, where all covariances are set to zero.

The results for the fully diagonal covariance matrix are interesting, because they show the effect of shutting off all international transmissions through the error correlations. With cross error correlations set to zero, there is no indirect instantaneous transmission of the US monetary policy shock to the euro area, so the effect on euro area interest rates on impact is zero. However, direct transmissions through the equations drives euro area interest rates down quite rapidly as the reduction in US output and inflation, reduces euro area output and inflation, prompting an interest rate response. The difference between the block diagonal and the diagonal versions of Σ_{ε}^{0} suggest that the correlations within each of the three blocks of structural shocks amplify the direct responses, while a comparison of the diagonal and block-diagonal choices with $\hat{\Sigma}_{\varepsilon}^{0}$ indicates that allowing for covariances between the exchange rates and the three structural shocks have important moderating influences on the global interactions.

Figure 8: Impulse responses of a one standard error US monetary policy shock on US and euro area interest rates, inflation and output under alternative covariance matrices (per cent per quarter)



8 Conclusion

This paper shows that it is possible to estimate, solve and simulate a forward-looking multi-country New Keynesian model and use it to estimate the effects of identified supply, demand and monetary policy shocks. In constructing such a model it is necessary to be cautious with regard to the assumptions made about exchange rates, particularly the treatment of the numeraire, and about the assumed structure of the shock correlations. For all the economies considered, the qualitative effects of demand and supply shocks are as predicted by the theory. Monetary policy shocks are offset more quickly than is typically obtained in the literature. Global supply and demand shocks are the most important drivers of output, inflation and interest rates in the long run. By contrast monetary or exchange rate shocks have only a short-run role in the evolution of the world economy. Despite the uniformity of the specifications assumed across countries, there are major differences between countries in the size of the effects of the shocks. Changing the degree of international transmission, through the use of alternative covariance matrices and foreign variables in equations changed the estimated size of the effects of the shocks. The results indicate the importance of international connections, including the covariances between monetary policy in different countries: a US monetary policy shock has effects on output and inflation in other countries that are of the same order of magnitude as its effects on the US. Ignoring global inter-connections as country-specific models do, will inevitably cause serious misspecification.

The objective of the current paper is to provide a multi-country version of the standard NK model, which allows the identification of the usual types of shocks and provides a framework within which a range of questions about international transmission of shocks can be answered. There are a number of natural routes for further developments. There may be scope to allow for more global variables in the structural equations, which may reduce the cross-country correlations and allow the identification of country-specific idiosyncratic shocks. There may be advantages in including financial variables like real equity prices and long interest rates. Less structural models, like the GVAR of DdPS indicate the importance of the international transmission of financial shocks. There is currently considerable macro-finance research to extend DSGE models to include explanations of the term premium in interest rates, the equity premium, the role of banks and the role of foreign assets, which is important given the role of the US dollar as an international store of value, not just a unit of account. Another possible development is to introduce international trade variables, such as exports and imports directly rather than indirectly, as is done in this model through including the real effective exchange rate and world output in the IS equation. The model provided in this paper provides a natural framework for such extensions.

Appendix

A.1 Solving and Bootstrapping the MCNK model

A.1.1 Solution

Starting with the canonical representation of the global model given by (13)

$$\boldsymbol{\chi}_t = \mathbf{A}\boldsymbol{\chi}_{t-1} + \mathbf{B}E_{t-1}(\boldsymbol{\chi}_{t+1}) + \boldsymbol{\eta}_t,$$

has the following solution

$$\boldsymbol{\chi}_t = \boldsymbol{\Phi}\boldsymbol{\chi}_{t-1} + \boldsymbol{\eta}_t,$$

where $\boldsymbol{\Phi}$ satisfies the quadratic matrix equation

$$\mathbf{B}\boldsymbol{\Phi}^2 - \boldsymbol{\Phi} + \mathbf{A} = \mathbf{0}. \tag{A.1}$$

Solving the quadratic equation, we therefore obtain the reduced form solution in terms of $\tilde{\mathring{\mathbf{x}}}_t$ and the structural shocks, $\boldsymbol{\varepsilon}_t$, as

$$\tilde{\mathring{\mathbf{x}}}_t = \boldsymbol{\Phi}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1} + \boldsymbol{\Phi}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\Phi}_{11}$ and $\boldsymbol{\Phi}_{12}$ are defined by

$$\boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix}.$$

To solve (A.1) for $\boldsymbol{\Phi}$, we employ a back-substitution procedure which involves iterating on an initial arbitrary choice of $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$, say $\boldsymbol{\Phi}_0$ and $\boldsymbol{\Psi}_0$, and use the recursive relations

$$\boldsymbol{\Phi}_r = (\mathbf{I}_k - \mathbf{B}\boldsymbol{\Phi}_{r-1})^{-1}\mathbf{A}, \tag{A.2}$$

$$\boldsymbol{\Psi}_r = (\mathbf{I}_k - \mathbf{B}\boldsymbol{\Phi}_{r-1})^{-1}\mathbf{B}, \tag{A.3}$$

where $\boldsymbol{\Phi}_r$ and $\boldsymbol{\Psi}_r$ are the values of $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$, respectively, at the r^{th} iteration ($r = 1, 2, ...$) and $\boldsymbol{\Psi}$ is the coefficient matrix in the forward equation

$$\mathbf{z}_t = \boldsymbol{\Psi}E_{t-1}(\mathbf{z}_{t+1}) + \mathbf{v}_t,$$

with

$$\mathbf{z}_t = \boldsymbol{\chi}_t - \boldsymbol{\Phi}\boldsymbol{\chi}_{t-1},$$
$$\mathbf{v}_t = (\mathbf{I}_k - \mathbf{B}\boldsymbol{\Phi})^{-1}\boldsymbol{\eta}_t.$$

See Binder and Pesaran (1995, 1997) for further details. Matlab and Gauss code for this procedure is available at http://ideas.repec.org/c/dge/qmrbcd/73.html. This iterative procedure is continued until one of the following convergence criteria is met

$$\|\boldsymbol{\Phi}_r - \boldsymbol{\Phi}_{r-1}\|_{\max} \leq 10^{-6} \text{ or } \|\boldsymbol{\Psi}_r - \boldsymbol{\Psi}_{r-1}\|_{\max} \leq 10^{-6},$$

where the max norm of a matrix $\mathbf{A} = \{a_{ij}\}$ is defined as $\|\mathbf{A}\|_{\max} = \max_{i,j}\{|a_{ij}|\}$.

It follows from (16) that

$$\mathbf{u}_t = \tilde{\mathring{\mathbf{x}}}_t - \boldsymbol{\Phi}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1} - \boldsymbol{\Phi}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2},$$

and so

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_0\mathbf{u}_t.$$

In numerical calculations all unknown parameters are replaced with the restricted IV estimates described in Section 5.

A.1.2 Computation of bootstrap error bands

We generate B bootstrap samples denoted by $\tilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ from the process

$$\tilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{H}}_0^{-1}\hat{\boldsymbol{\varepsilon}}_t^{(b)},\ t = 1, 2, ..., T, \tag{A.4}$$

by resampling the structural residuals, $\hat{\boldsymbol{\varepsilon}}_t$, and setting $\tilde{\mathring{\mathbf{x}}}_0^{(b)} = \tilde{\mathring{\mathbf{x}}}_0$ and $\tilde{\mathring{\mathbf{x}}}_{-1}^{(b)} = \tilde{\mathring{\mathbf{x}}}_{-1}$, where $\tilde{\mathring{\mathbf{x}}}_0$ and $\tilde{\mathring{\mathbf{x}}}_{-1}$ are the observed initial data vectors that include the US real exchange rate (or equivalently the US price level). Recall that the multi-country rational expectations model is solved in terms of the US price level rather than the US inflation.

We initially orthogonalise the structural shocks, $\hat{\boldsymbol{\varepsilon}}_t$, by using the inverse of the Choleski factor, $\widetilde{\mathbf{P}}$, associated with the Choleski decomposition of the shrinkage covariance matrix, $\hat{\mathbf{\Sigma}}_\varepsilon(0.4)$, defined by (27). This way we obtain the $k \times 1$ orthogonal vector $\hat{\boldsymbol{\upsilon}}_t = \widetilde{\mathbf{P}}^{-1}\hat{\boldsymbol{\varepsilon}}_t$ where its j^{th} element $\hat{\upsilon}_{jt}$, $j = 1, 2, ..., k$, has unit variance. The bootstrap error vector is then obtained as $\boldsymbol{\varepsilon}_t^{(b)} = \widetilde{\mathbf{P}}\hat{\boldsymbol{\upsilon}}_t^{(b)}$, where $\hat{\boldsymbol{\upsilon}}_t^{(b)}$ is the $k \times 1$ vector of re-sampled values from $\{\hat{\upsilon}_{jt}\}_{j=1,2,...,k;t=1,2,...,T}$. Prior to any resampling the structural residuals are recentered to ensure that their bootstrap population mean is zero.

Once a set of $\tilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ are generated, US inflation is computed from the US price level so that $\tilde{\mathbf{x}}_{it}^{(b)}$ is constructed, with the corresponding foreign variables, $\tilde{\mathbf{x}}_{it}^{*(b)}$, computed using the trade weights. For each bootstrap replication the individual country models are then estimated by the inequality constrained IV procedure, subject to the constraints given in Section 5, ensuring that any constraint which binds for the estimates based on historical realisations are also imposed on the bootstrap estimates.

The country specific models in terms of $\tilde{\mathbf{x}}_{it}^{(b)}$ are given by

$$\hat{\mathbf{A}}_{i0}^{(b)}\tilde{\mathbf{x}}_{it}^{(b)} = \hat{\mathbf{A}}_{i1}^{(b)}\tilde{\mathbf{x}}_{i,t-1}^{(b)} + \hat{\mathbf{A}}_{i2}^{(b)}E_{t-1}\left(\tilde{\mathbf{x}}_{i,t+1}^{(b)}\right) + \hat{\mathbf{A}}_{i3}^{(b)}\tilde{\mathbf{x}}_{it}^{*(b)} + \hat{\mathbf{A}}_{i4}^{(b)}\tilde{\mathbf{x}}_{i,t-1}^{*(b)} + \boldsymbol{\varepsilon}_t^{(b)},$$

and are subsequently combined yielding the MCNK model

$$\tilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{F}}_1^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{F}}_2^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{F}}_3^{(b)}E_{t-1}\left(\tilde{\mathring{\mathbf{x}}}_{t+1}^{(b)}\right) + \hat{\mathbf{F}}_4^{(b)}E_{t-1}\left(\tilde{\mathring{\mathbf{x}}}_t^{(b)}\right) + \mathbf{u}_t^{(b)}. \tag{A.5}$$

Solving the quadratic matrix as described earlier, the reduced form solution of (A.5) follows as

$$\tilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \mathbf{u}_t^{(b)},$$

with

$$\hat{\mathbf{u}}_t^{(b)} = \tilde{\mathring{\mathbf{x}}}_t^{(b)} - \hat{\mathbf{\Phi}}_{11}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} - \hat{\mathbf{\Phi}}_{12}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)}$$

and

$$\hat{\boldsymbol{\varepsilon}}_t^{(b)} = \hat{\mathbf{H}}_0^{(b)}\hat{\mathbf{u}}_t^{(b)}.$$

For the first bootstrap replication we begin the iterative back-substitution procedure, using the estimated $\hat{\mathbf{\Phi}}$ from the actual data as an initial value to compute (A.4) and (A.4), so that for $b = 1$, $\mathbf{\Phi}_0^{(1)} = \hat{\mathbf{\Phi}}$. For each subsequent bootstrap replication, b, the initial value is set to the solution of (A.1) obtained under the preceding replication, $b-1$, so that $\mathbf{\Phi}_0^{(b)} = \hat{\mathbf{\Phi}}^{(b-1)}$ and $\mathbf{\Psi}_0^{(b)} = (\mathbf{I}_k - \hat{\mathbf{B}}^{(b)}\hat{\mathbf{\Phi}}^{(b-1)})^{-1}\hat{\mathbf{B}}^{(b)}$. If for a particular bootstrap replication the iterative back-substitution procedure fails to converge after 500 iterations, the initial values for $\mathbf{\Phi}_0^{(b)}$ and $\mathbf{\Psi}_0^{(b)}$ are set to the identity matrix.

For each bootstrap replication $b = 1, 2, ..., B$, having estimated the individual country NK models using the simulated data $\tilde{\mathring{\mathbf{x}}}_t^{(b)}$, the MCNK model is reconstructed as described above and the impulse responses are calculated $g^{(b)}(n)$, for $n = 0, 1, 2, ...$ These statistics are then sorted in ascending order, and the $(1-\alpha)100\%$ confidence interval is calculated by using the $\alpha/2$ and $(1-\alpha/2)$ quantiles, say $q_{\alpha/2}$ and $q_{(1-\alpha/2)}$, respectively of the bootstrap distribution of $g(n)$.

To compute the upper and lower confidence bounds we use 2000 convergent and stationary bootstrap replications. A convergent replication is defined as one where for the corresponding bootstrap sample, the iterative back-substitution procedure described above (see section A.1.1) converges within 500 iterations, whether the initial values for $\mathbf{\Phi}_0^{(b)}$ and $\mathbf{\Psi}_0^{(b)}$ are set to the identity matrix or otherwise. Having achieved convergence, a bootstrap replication is checked to make sure that it yields a stationary solution. If any of the above two conditions is violated, a new bootstrap sample is computed. For our bootstrap results we had to carry out a total of 2311 bootstrap replications, of which 311 where due to non-convergence of the iterative back-substitution procedure. No bootstrap replications were found to be non-stationary.

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