Leverage and Value Creation in Holding-Subsidiary Structures

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Abstract

This paper determines optimal capital structure and value of Holding-Subsidiary structures (HS), when there is a trade-off between bankruptcy costs and taxation. HS have higher firm value than their stand alone counterparts, as the holding provides a guarantee to its subsidiary's lenders which lowers expected bankruptcy costs, at a given debt level. HS may also reach higher debt capacity, which further increases their value by reducing the tax burden below that of stand alone firms. Optimal debt in the holding and in the subsidiary are respectively lower and higher than in their stand-alone counterparts. Such debt diversity preserves the holding ability to rescue its subsidiary, hence value creation by HS, even with perfect cash flow correlation.

Keywords: holding, subsidiary, groups, guarantees, debt, taxes, bankruptcy costs, limited liability, capital structure

JEL Classifications: G32, G34, L22

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1 Introduction

Companies are often organized as holding-subsidiary structures (HS). These consist of activities which are connected by both ownership links and a common financial management despite being separate entities. Holding corporations routinely guarantee the debt obligations of their subsidiaries (Bodie and Merton, 1992) otherwise they would not be responsible for them due to their limited liability. Such intercorporate guarantees, that can be either informal or contractual[1], are the focus of our paper, which studies how HS create value relative to stand alone organizations thanks to intercorporate support.

For this purpose, we model an entrepreneur who considers organizing her two activities in order to maximize the proceeds from the issue of debt. Debt allows to deduct interest from taxes but increases bankruptcy probability, as in the standard trade-off theory of capital structure. Specific to our model is consideration for the fact that the holding company supports its insolvent subsidiary if the entrepreneur chooses the HS structure. However, there is selective default of the subsidiary when the holding has insufficient resources for rescue. This assumption is consistent with the observation that business groups tend to terminate support to struggling subsidiaries once group profitability turns negative (see e.g. Dewaelheyns and Van Hulle (2006)). It also appears that the first affiliated firm in a group becomes bankrupt when it experiences severe negative shocks to its profitability, shocks large in comparison with the total equity value of the other firms in the group (see e.g. Gopalan et al (2007))[2]. In our model stand alone organizations provide instead no guarantee to each other, as in Merton (1974) and Leland (2007).

In this setting, we show that the total value of an HS arrangement exceeds that of stand-alone companies: the guarantee - which permits to save on bankruptcy costs - has non negative value, because it is *conditional* on the survival of the holding company which enjoys limited liability when needed. This is a key insight of our analysis.

We also show that the entrepreneur further increases HS value through her leverage choice. The guarantee reduces the subsidiary insolvency probability, at the optimal debt of two comparable stand alone firms,

[1] Khanna and Palepu (2000) and Gopalan Nanda and Seru (2007) observe that Indian group firms assist each other in times of financial distress. See also Chang and Hong (2000) for Korean chaebols. Deloof and Vershueren (2006) analyze contractual guarantees in a sample of Belgian group affiliates, while Samson (2001) discusses how guarantees affect credit ratings.

[2] Herring et al. (2009), who describe the range of informal and formal guarantees in financial groups, also discuss recent instances when banks walked away from insolvent subsidiaries.

without affecting the bankruptcy probability of the holding. This implies that the subsidiary can increase its own debt financing relative to the stand alone case so that the higher interest payment reduces its tax burden. We present conditions ensuring that total debt capacity in HS exceeds that of stand alone organizations, even though the optimal holding debt turns out to be lower than that of its stand alone counterpart. It follows that value creation in HS structures results from an initial reduction in bankruptcy costs - due to the guarantee - which boosts HS debt and the associated tax shield. Thus our model fits into the literature highlighting the "bright side" of internal capital markets. In our setting they allow to optimize the tax- bankruptcy cost trade-off, while prior research focuses on their role in circumventing imperfections arising from asymmetric information (Stein, 1997; Gertner, Scharfstein and Stein, 1994).

Several empirical studies focus on one type of HS organization - the traditional business group[3] - finding that they rely on debt rather than equity financing (Bae, Kang and Kim, 2002; Chang, 2003; Dewaelheyns et al., 2007; de Jong et al., 2009). Group affiliates also appear to have higher leverage than stand alone firms (Masulis et al., 2008), especially when there are intragroup guarantees (Deloof and Vershueren, 2006). Another stylized fact concerning groups is that their shares often trade at lower values than in comparable stand-alone firms (Bennedsen and Nielsen, 2006; Claessens et al., 2002). This evidence is puzzling because shareholders seem to give up value that can be created by spinning off the subsidiary. In our model equity values are lower than in stand alone arrangements. Yet there is no puzzle as the entrepreneur, who originally owns the activities, gains with respect to the stand alone case by cashing in a higher market value of debt. Two effects generate the lower average value of HS equity in our model. On the one hand, the guarantee implies that the holding shareholders expect to transfer cash to subsidiary lenders. This reduces the value of equity in the holding below that of a stand alone with the same level of debt, leaving unaffected the value of subsidiary equity. On the other hand, both the holding and the subsidiary modify their leverage in order to optimize the tax-bankruptcy trade-off, leading to higher debt than in the stand alone case. Since equity is a call option, increasing debt reduces its value. Thus our model contributes to the literature on business groups by relating their

[3]This is common in both emerging markets (Khanna and Yafeh, 2007) and continental European countries (Barca and Becht, 2001). HS structures are present in innovative industries in the US and the UK (Allen, 1998; Sahlman, 1990; Mathews and Robinson, 2008), in the private equity industry (Jensen, 2007; Kaplan, 1989) as well as in the banking industry (Dell'Ariccia and Marquez, 2008).

existence, their higher debt capacity and their lower equity value to the observed provision of intercorporate guarantees.

Numerical simulations, for activities with equal cash flow distributions, allow to appreciate the effect of the reliability of the guarantee on leverage and value creation. We first consider a guarantee which is expost verifiable and enforceable in court, i.e. a situation when the lender assigns probability one to the guarantee being honoured by the holding company. We then decrease such probability so as to capture the case of informal guarantees[4], such as comfort letters, which assure subsidiaries' lenders that the holding would assist them in distress but are legally unenforceable. In the first case HS debt is almost twice the total debt of the original stand alone activities, so that tax savings and value are far higher. Moreover, the holding company turns out to be unlevered - thus avoiding bankruptcy costs altogether - for most parameter values. Debt is shifted onto the subsidiary which - being overburdened by the service of debt - is almost never able to distribute dividends irrespective of intercorporate ownership, resembling a LBO target firm. Under the informal guarantee, instead, the subsidiary becomes able to pay a dividend, thanks to a lower debt service which makes it more similar to a traditional group affiliate. In turn the holding levers up for most parametric configurations, the more so the higher are its dividend receipts. This reminds of business groups. Total HS debt is smaller than previously, hence HS tax burden and value get closer to the stand alone case.

Further numerical experiments clarify how differences across activities affect the value of the intercorporate guarantee, also indicating which company ought to provide support. When activities differ in risk, the holding company should be the safer one, consistent with several empirical studies. This is because shielding income from taxes through debt has higher value in the activity with riskier cash flow, due to the asymmetric nature of taxation. The holding company ought to be the one with higher exogenous bankruptcy costs, because it has lower leverage and incurs less often in insolvency. Interestingly, the value differential between HS and stand-alone organization increases in both the volatility and bankruptcy costs differentials. Thus, a further by-product of our analysis is the characterization of holding and subsidiary firms.

Previous theories of groups focus on fund provision by minority shareholders rather than by lenders. Groups emerge when the entrepreneur prefers to fund activities indirectly, through another company, rather

[4]Herring and Carmassi (2009) cite cases of financial institutions providing additional funds to troubled SIV, despite the absence of legal obligations, so as to protect their reputation.

than directly. This is the case when the present value of the activity, net of diversion, is negative: the equity discount thus reflects the risk of expropriation of minority shareholders (Almeida and Wolfenzon, 2006). This explanation for the existence of groups may be appropriate when affiliated firms are listed on public exchanges with loose enforcement of securities regulation. However, it remains unclear why groups thrive in strict-enforcement countries, such as Scandinavian ones, and why unlisted groups are quite common[5]. By shifting the focus from minority shareholders onto lenders our model can account for these situations. So as to emphasize the different rationale for groups, we will allow for just one type of shareholder - the entrepreneur - and we will usually refer to the case of equal cash flows from the two activities. In such a situation, Almeida and Wolfenzon (2006) imply that groups do not exist.

Our model extends Leland's (2007) analysis of gains from mergers to the case of holding-subsidiary structures. In so doing, we borrow his assumptions concerning operational cash flows, tax rates and bankruptcy costs with no *ad hoc* modification. Leland (2007) shows that diversification gains from a merger disappear when cash flow correlation across two symmetric activities is perfect. Our model strikingly implies that gains from HS obtain even in this case: it is still possible for H to rescue S because debt from the holding is optimally lower than the subsidiary one. Leland also identifies cases when merging two stand alone firms reduces value, when activities differ. In those situations, it is still possible to create value through a HS structure by suitably choosing the company that provides support.

While our model posits exogenous operating cash flows, several papers study how agency problems in internal capital markets affect product market competition and investment choice. Most focus however on aspects, such as cash-flow pooling, that are typical of both conglomerate mergers and groups without making any explicit distinction between the two organizations. See, among others, Rajan Servaes Zingales (2000), Inderst and Mueller (2003) and Faure Grimaud and Inderst (2005). Cestone and Fumagalli (2005) and Bianco and Nicodano (2006) analyze the specificities of group internal capital market by assuming limited liability of the holding, as we do, but in an asymmetric information setting. In the first paper the benefit of HS stem from improved managerial effort, rather than a better tax-bankruptcy trade-off as in our paper. The second paper characterizes optimal capital structure when raising debt from subsidiaries may involve risk shifting, but posits a fixed debt capacity which we endogenize.

[5]For instance, only 20.1% of pyramidal structures contain a listed vehicle in France. See DeJong et al (2009).

Another related literature studies tax avoidance and corporate finance (see the survey in Graham (2003)), that often exploits arbitrage in unequal tax rates. For instance, multinational groups raise more debt from subsidiaries in high-tax countries (Huizinga et al. (2008)). In our model, groups minimize the tax burden through debt even when there is no tax rate differential between the holding and its subsidiary. Thus, we point out a powerful tax avoidance tool which, to our knowledge, has not yet been analyzed.

The paper is organized as follows. Section 2 analyzes the organizational modes for two activities, in the case of an enforceable guarantee and either infinitesimal or positive intercorporate ownership. We indicate how the value of debt and equity (Propositions 1, 3) and of optimal debt (Propositions 5 and 6) change due to the conditional guarantee. We also compare with mergers. Section 3 presents numerical simulations for the case of equally distributed cash flows, while Section 4 examines activities differing in mean cash flow, volatility and bankruptcy costs. Section 5 studies the interplay of informal guarantees and positive intercorporate ownership. Section 6 concludes.

2 The model

We consider a no arbitrage environment with two dates $t = \{0, T\}$, in which every payoff is evaluated according to its expected discounted value, under the risk neutral measure.

An entrepreneur owns two production units, and each activity i generates a random operating (net) cash flow value X_i at time T. X_i is a continuous random variable, endowed with the first two moments ($X \in L^2$), that may take both negative and positive values: having denoted as F_i its distribution function, this means $0 < F_i(0) < 1$.

The riskfree interest rate over the time period T is $r_T > 0$, and ϕ denotes the corresponding discount factor, $\phi = (1 + r_T)^{-1} < 1$.

The owner can "walk away" from negative cash flows thanks to limited liability. Thus the (pre-tax) value of each activity with limited liability is

$$H_{0i} = \phi \mathbb{E} X_i^+ \tag{1}$$

where $X_i^+ = \max(X_i, 0)$, and the pre-tax value of limited liability is $L_{0i} = H_{0i} - \phi \mathbb{E} X_i \geq 0$.

Now consider a tax rate on future cash flows equal to $0 < \tau_i < 1$. The aftertax value of the unlevered firm, which corresponds to its equity value, is

$$V_{0i} = (1 - \tau_i) H_{0i}.$$

The present value of taxes it pays, named tax burden in the sequel, is

$$T_{0i}(0) = \tau_i H_{0i} \tag{2}$$

At time $t = 0$ the entrepreneur can lever the firm by issuing zero-coupon debt so as to maximize the value of his claims to the cash flows. Let the debt principal value be $P_i \geq 0$, and assume it is due, with absolute priority, at $t = T$. The value at $t = 0$ of such debt, $D_{0i}(P_i)$, is cashed-in by the entrepreneur at issuance. We assume that there is an incentive to issue debt, as interest is a deductible expense. The promised interest payment is equal to:

$$P_i - D_{0i}(P_i) \tag{3}$$

Taxable income is the operating one net of interest payment, $X_i - (P_i - D_{0i}(P_i))$. The zero-tax level of cash flow, X_i^Z, is therefore equal to:

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{4}$$

We assume that no tax refunds are paid by the tax authority to the owners of the activity if $X_i < X_i^Z$.

Operating cash flows, net of tax payments, are

$$X_i^n = X_i^+ - \tau_i(X_i - X_i^Z)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{5}$$

The tax burden of the levered firm is equal to:

$$T_{0i}(P_i) = \tau_i \phi \mathbb{E}(X_i - X_i^Z)^+ \tag{6}$$

Clearly, some value gains obtain when (6) is lower than (2). However, issuing debt has costs as well. Similarly to Merton (1974), default occurs when net operating cash flow is smaller than the face value of the debt, namely $X_i^n < P_i$. Such default triggering condition can be restated, in terms of the pre-tax cash flows, as $X_i < X_i^d$, where the default threshold X_i^d is defined as:

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1-\tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1-\tau_i} \tag{7}$$

In the event of default, we assume that bondholders will receive a fraction $0 < 1 - \alpha_i < 1$ of operating cash flow, X_i, when this is positive; the remaining fraction is instead lost upon liquidation. There is then a trade-off between the dissipative default costs, $\alpha_i X_i$, and the tax savings possibly generated by debt.[6]

The entrepreneur chooses the face value of debt, P_i, in the two activities, given this tax-bankruptcy cost trade-off, so as to maximize the time-zero combined value of the two units. Let the levered value of equity and debt - computed as expected present values of the corresponding cash flows - be denoted as E_{0i} and D_{0i}. Such cash flows - which we denote as E_i and D_i - vary with the organization- specific guarantees, which we discuss below. As a consequence, also E_{0i}, D_{0i} and firm values do.

2.1 The stand alone case

Stand alone (SA) firms - being separately incorporated and independently managed - are typically not liable for each others' debt. We therefore follow Leland (2007) in modelling stand-alone activities as never providing support to each other. Thus, the entrepreneur maximizes the levered firm value, $\nu_{0i}(P_i)$, of his two stand alone activities, $i = 1, 2$, with respect to non-negative face values of debt:

$$\sum_{i=1}^{2} \nu_{0i}(P_i) = \sum_{i=1}^{2} [E_{0i}(P_i) + D_{0i}(P_i)] \tag{8}$$

We now determine the payoff to financiers at time T which enter the value of equity and debt, i.e. the two elements on the right-hand side of equation (8). The cash flow to shareholders at T is operating cash flow less taxes and the repayment of principal, when the difference is positive[7]:

$$E_i(P_i) = (X_i^n - P_i)^+ \tag{9}$$

Indeed, limited liability ensures that shareholders bear no responsibility when it is negative.

The cash flows to lenders at time T are equal to:

$$D_i(P_i) = \begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases} \tag{10}$$

[6] We are borrowing from Leland (2007) the assumption that cash flows are exogenous and that the firm receives no tax refunds when they are negative. In the real world, companies may carry forward some losses, in order to reduce the asymmetric nature of taxation - which however remains substantial. Similarly, we follow a large credit risk literature that models bankruptcy costs as proportional to cash-flows.

[7] Notice that E_{0i} is a call option with underlying X_i^n and exercise price P_i. It depends on debt principal both directly and indirectly, through the tax shield X_i^Z that enters the underlying.

Lenders receive reimbursement equal to P_i when the firm is solvent, i.e. $X_i \geq X_i^d$. When the firm is insolvent, they receive cash flows net of bankruptcy costs, $(1-\alpha_i)X_i$, if gross cash flows are positive but cash flows net of interests are less than or equal to zero ($X_i \leq X_i^Z$). When they are positive, that is $X_i^Z < X_i < X_i^d$, the government has priority for tax payments, and debtholders also bear a tax liability, $\tau_i(X_i - X_i^Z)$.

Appendix A studies the SA optimization problem given such payoffs.[8]

2.2 Holding-Subsidiary Structures

The two stand alone companies described in the previous section do not guarantee each others' debt. In this section we model the case when one activity - the holding company ($i = H$) - supports its insolvent subsidiary ($i = S$) through a cash transfer.[9] Such cash transfer is however conditional on the survival of the holding, which uses its limited liability otherwise. Corporate limited liability is a common characteristic across major jurisdictions [10]. Blumberg (1989) reports that the theories of the separate legal personalities of corporations and the limited liability of shareholders were already applied to groups in the US during the twentieth century. Courts expanded the concept of limited liability to protect each layer in the HS from the liability of the junior company. [11]

We assess how the presence of such conditional guarantee affects the value of the two activities, initially assuming negligible ownership links - and thus no dividends - between the two companies. Before proceeding we determine the minimum amount of the transfer that allows to rescue an insolvent subsidiary, as well as the subset of states when the transfer occurs. For the sake of notational simplicity, we assume throughout that tax rates and default costs do not differ between the holding and its subsidiary ($\alpha_S = \alpha_H = \alpha, \tau_S = \tau_H = \tau$).

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[8] Due to default costs and tax savings, debt is a portfolio of plain vanilla puts and the present value of the principal. Moreover (10) is an implicit equation, since X_i^Z and X_i^d are themselves function of D_{0i} through (4) and (7). Numerical methods are thus necessary to find a solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}. Appendix A determines some general properties of such fixed point.

[9] While an external guarantor may substitute for intercorporate guarantees, Bodie and Merton (1992) observe that H may use management methods that would not be feasible for an external guarantor. Unlike an external party, H can have access to the subsidiary's proprietary information, reducing problems of asymmetric information.

[10] See Hadden (1996) on Britain, France, Germany and the US, and Hill (1996) on Australia.

[11] The exception to this rule is the "piercing of the corporate veil " that requires to prove both the lack of separate existence of the subsidiary and a holding company's conduct "akin to fraud".

A necessary condition for the transfer is that the subsidiary is unable to meet its debt obligations, $X_S < X_S^d$. Limited liability implies that (a) there is rescue only if the operating cash flows of the subsidiary are non-negative, else the holding would bear an operating loss that it can avoid:

$$0 < X_S < X_S^d \tag{11}$$

(b) the transfer is conditional on the holding company ability to meet both its own and its subsidiary debt obligations. This is the case only if its surplus is greater than the subsidiary deficit:

$$\begin{cases} X_H > X_H^d \\ X_H^n - P_H > P_S - X_S^n \end{cases} \tag{12}$$

Such conditions can be written as $X_H > h(X_S)$, where

$$h(X_S) = \begin{cases} X_H^d + \frac{P_S}{1-\tau} - \frac{X_S}{1-\tau} & X_S < X_S^Z \\ X_H^d + X_S^d - X_S & X_S > X_S^Z \end{cases} \tag{13}$$

It follows that the conditional transfer from the holding to the subsidiary, associated with the guarantee, is equal to:

$$(P_S - X_S^n)\mathbf{1}_{\left\{0<X_S<X_S^d, X_H>h(X_S)\right\}} \tag{14}$$

where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function and $\left\{0 < X_S < X_S^d, X_H > h(X_S)\right\}$ is the support or rescue region.

The entrepreneur - who initially owns both activities - maximizes levered firm value, $\nu_{0i}(P_H, P_S)$, of his holding and subsidiary ($i = H, S$) with respect to the face values of debt:

$$\nu_{0,HS}(P_H, P_S) = \sum_{i=H}^{S} \nu_{0i}(P_H, P_S) = \sum_{i=H}^{S} \left[E_{0i}(P_H, P_S) + D_{0i}(P_H, P_S)\right] \tag{15}$$

taking into consideration the conditional transfer from the holding to the subsidiary.[12] Such optimal debt will in general differ from the stand alone one. We postpone its characterization until section 2.2.3, and first analyze the value of HS and SA arrangements when the face value of debt is exogenous but comparable. Thus, we assume that it is equal for the holding and its stand alone counterpart (i.e. $P_H = P_1$) as well as for the subsidiary and its stand alone correspondent (i.e. $P_S = P_2$). Also the corresponding cash flows are assumed to be equal in distribution. For the time being, we are also assuming no intercorporate dividends.

[12] Due to such transfer, we expect debt and equity of both firms to depend on both face values of debt: this explains why we use the notation $E_{0i}(P_H,P_S), D_{0i}(P_H,P_S)$ in (15).

2.2.1 The value of equity

The payoff to shareholders of the holding company at time T is equal to cash flows after the service of debt and taxes, reduced by the conditional transfer from the holding to its subsidiary. It can be written as:

$$E_H(P_H, P_S) = E_1(P_H) - (P_S - X_S^n)\mathbf{1}_{\left\{0<X_S<X_S^d, X_H>h(X_S)\right\}} \tag{16}$$

where $E_1(P_H)$ is the cash flow to shareholders of its benchmark, i.e. comparable, stand alone. The last term highlights that the payoff to the holding shareholders never exceeds the one of the benchmark stand-alone, because of state-contingent support. It follows that also the equity value of a holding company, $E_{0H}(P_H, P_S)$, cannot exceed the one of a stand alone with the same nominal debt. Equity holders of the subsidiary are unaffected, as the transfer occurs for the sake of servicing debt. As a consequence, the subsidiary equity value is unchanged: $E_S(P_H, P_S) = E_2(P_S)$. An immediate consequence of the fact that $E_H < E_1, E_S = E_2$, is the following:

Proposition 1 *Consider a holding company, its subsidiary and two stand alone firms with the same face value of debt of H and S ($P_1 = P_H, P_2 = P_S$) and the same operating profits ($X_1 = X_H, X_2 = X_S$). Then the average equity price of stand-alone firms exceeds that of HS affiliated counterparts, if the transfer occurs with positive probability.*

This is our first rationale for the observation that equity values are often lower in business groups than in stand alone structures.

Remark 2 *In Proposition 6 below we are going to show that at the optimum the support region is non-empty. The positive probability of the transfer, requested in proposition 1 and below, obtains if the distribution of cash flows has positive density on the positive orthant. This happens, for instance, when cash flows are normally distributed, as in Leland (2007) and in our numerical examples below.*

2.2.2 The value of debt and the total value of HS

The value of subsidiary debt, $D_{0S}(P_H, P_S)$, is the present expected value of the following final payoffs:

$$\begin{aligned} D_S(P_H, P_S) = & \begin{bmatrix} X_S(1-\alpha)+ \\ +\tau(X_S - X_S^Z)\mathbf{1}_{\left\{X_S>X_S^Z\right\}} \end{bmatrix} \mathbf{1}_{\left\{0<X_S<X_S^d, X_H<h(X_S)\right\}}+ \\ & +P_S\left[\mathbf{1}_{\left\{0<X_S<X_S^d,\ X_H>h(X_S)\right\}} + \mathbf{1}_{\left\{X_S>X_S^d\right\}}\right] \end{aligned} \tag{17}$$

The first square bracket refers to the case when the subsidiary defaults and the holding does not support it because its own cash flow is insufficient ($X_H < h(X_S)$). In this situation, lenders have to pay taxes only if cash flows exceed the tax shield ($X_S > X_S^Z$). The first term in the second square bracket takes into account that the subsidiary is able to reimburse its debt thanks to the holding transfer.

It is easy to prove that (17) is equal to:

$$D_S(P_H, P_S) = D_2(P_S) + \Big[P_S - X_S(1-\alpha) - \\ -\tau(X_S - X_S^Z)\mathbf{1}_{\{X_S > X_S^Z\}}\Big]\mathbf{1}_{\{0<X_S<X_S^d,\ X_H>h(X_S)\}} \tag{18}$$

Subsidiary lenders obtain, on top of what accrues to lenders of a comparable stand-alone, the nominal value of debt thanks to the guarantee (first term in square bracket) while losing the cash flow net of bankruptcy costs and taxes (second and third term). Figure 1 shows that the payoffs to subsidiary lenders exceed the stand alone ones due to the transfer, when this occurs.

The payoff to lenders of the holding does not change with respect to the stand alone case, as the transfer to the subsidiary occurs only after the service of the holding debt. Thus debt of the holding is unaffected: $D_H(P_H, P_S) = D_1(P_H)$. It follows that the average value of debt is higher in a HS arrangement than in stand alone companies, given exogenous face levels of debt. We are going to show below that the increase in debt value prevails over the corresponding decrease in equity, leading to higher HS value.

Before stating this result formally, we allow for the presence of non-negligible intercorporate ownership. Let the ownership share be ω, $0 \leq \omega \leq 1$, and let the subsidiary (firm 2) pay to its holding company (firm 1) a dividend equal to $d_j^+ = \omega(X_j^n - P_j)^+, j = s, 2$. Then the payoffs to lenders and shareholders of H (firm 1) change as follows:

$$D_i(P_i, P_j, \omega) = D_i + (1-\alpha)\, d_j^+ 1_{\{X_i < X_{i,\omega}^d\}} + P_i 1_{\{X_{i,\omega}^d < X_i < X_i^d\}}$$

$$E_i(P_i, P_j, \omega) = E_i + d_j^+ 1_{\{X_i > X_{i,\omega}^d\}} + \left(d_j^+ + X_i - P_i\right) 1_{\{X_{i,\omega}^d < X_i < X_i^d\}}$$

where $i = 1, H; j = 2, S$, and $X_{i,\omega}^d$ are the new default thresholds.[13]. The payoff to H (firm 1) lenders is equal to the corresponding zero-ω

[13]These are bounded above by the old ones, because dividends may help the service of debt. To see this, consider the holding case. $X_{H,\omega}^d$ is the level of operating cash flows, net of taxes but gross of dividends, that equals P_H:

$$X_{H,\omega}^d - \tau(X_{H,\omega}^d - X_H^Z)^+ + d_S^+ = P_H \tag{19}$$

value plus larger recovery upon default of the holding (second term) and a higher expected value of debt service (third term). The cum-dividend payoff to H (1) shareholders is equal to the zero-ω value plus the dividends, in all states when H (1) survives thanks to its own operating profits (first and second term), plus the dividends net of debt service when dividends prevent H (1) default.

It follows immediately that the holding (and firm 1) value increase with respect to the no-dividend case:

$$\nu_{0,H}(P_H, P_S, \omega) = D_{0H}(P_H, P_S, \omega) + E_{0H}(P_H, P_S, \omega) > \nu_{0,H}(P_H, P_S)$$
$$\nu_{01}(P_1, \omega) = D_{01}(P_1, P_2, \omega) + E_{01}(P_1, P_2, \omega) > \nu_{01}(P_1)$$

because an otherwise insolvent company (H or 1) avoids bankruptcy thanks to dividends from its participated firm (S or 2).[14]

The ex-dividend value of the subsidiary and of firm 2 are lower than in the $\omega = 0$ case, and are respectively equal to:

$$\nu_{0,S}(P_H, P_S, \omega) = D_{0S}(P_H, P_S) + (1-\omega)E_{0S}(P_S) < \nu_{0,S}(P_H, P_S)$$
$$\nu_{02}(P_2, \omega) = D_{02}(P_2) + (1-\omega)E_{02}(P_2) < \nu_{02}(P_2)$$

Nonetheless, given that the effect of dividends is the same in the HS and SA case, the difference in value between the two organizations is unaffected. This result is not surprising, because dividends are paid out from the subsidiary to the holding only if it is not defaulting and only if it is not receiving rescue funds. Thus, they are paid out to the holding in the same states in which a stand alone pays dividends to another stand-alone corporation which, while having a stake in the company, is independently managed.

Next section will study in depth such value difference, which stems from the guarantee and is identified with it.

Solving for $X^d_{H,\omega}$ we obtain the generalized default threshold for H:

$$X^d_{H,\omega} = \begin{cases} X^d_H & 0 < X_S < X^d_S \\ X^d_H - \omega\left(X_S - X^d_S\right) & X^d_S < X_S < X^\circ_S \\ P_H - \omega(1-\tau)\left(X_S - X^d_S\right) & X^\circ_S < X_S \end{cases}$$

where

$$X^\circ_S = X^d_S - \frac{X^Z_H - P_H}{\omega(1-\tau)}$$

It is easy to demonstrate that such default threshold is bounded above by X^d_H. The cash flow combinations $\{X_S, X_H\}$, such that the holding defaults are represented in Figure 1, bottom panel.A similar reasoning applies to firm 1.

[14] Thus, dividends work as a guarantee pledged by the subsidiary's shareholders in favour of its holding's lenders. This is a role for dividends (and intercorporate ownership) that has gone unnoticed so far in the literature.

2.2.3 Value Creation in HS

We can now measure the value differential between HS, $\nu_{0,HS}(P_H, P_S, \omega)$, and two comparable stand alone units, $\nu_{01}(P_1, \omega) + \nu_{02}(P_2, \omega)$. For any level of intercorporate dividends, this is equal to the discounted expected value of the following payoff:

$$D_S - D_2 + E_H - E_1 = \tag{20}$$

$$= [P_S - X_S(1-\alpha) - \tau(X_S - X_S^Z)^+ - (P_S - X_S^n)^+]\mathbf{1}_{\{0<X_S<X_S^d,\ X_H>h(X_S)\}} =$$

$$= \alpha X_S \mathbf{1}_{\{0<X_S<X_S^d,\ X_H>h(X_S)\}}$$

Value creation therefore coincides with the discounted bankruptcy cost that is avoided. We therefore name it as the value of the guarantee G:

$$G(P_H, P_S) \equiv \alpha\phi\mathbb{E}\left[X_S \mathbf{1}_{\{0<X_S<X_S^d,\ X_H>h(X_S)\}}\right] \tag{21}$$

Equation (21) directly implies the following result:

Proposition 3 *Consider a holding-subsidiary structure and two corresponding stand alone firms, with the same operating profits ($X_1 = X_H; X_2 = X_S$) and intercorporate ownership ω. If the conditional transfer has positive probability at $P_H = P_1, P_S = P_2$, then the value of the HS structure exceeds the value of two comparable SA firms.*

Proof. When the support region has positive probability and the subsidiary is levered, the value of the guarantee in (21) is positive. This holds for any fixed face values of debt, including the optimal ones for stand alone firms ($P_H = P_1^*, P_S = P_2^*$). ■

Proposition 3 implies the following:

Corollary 4 *Assume that the conditional transfer has positive probability for any principal choice. An entrepreneur prefers to incorporate her activities as holding and subsidiary rather than as stand alone companies.*

Proof. Denote the optimized value of the guarantee as $G(P_H^*, P_S^*)$ and observe that $G(P_H^*, P_S^*) \geq G(P_1^*, P_2^*) > 0$ - according to the previous proposition. Then the optimal HS value, corresponding to the optimal choice of debt for the holding and subsidiary, is higher than the sum of two stand alone values. ■

The above results indicate that HS gain value relative to SA organizations by providing a conditional guarantee to subsidiary lenders that saves on bankruptcy costs, even before any optimal HS capital structure decision. Observe also that HS value exceeds that of comparable stand alone firms even if, by Proposition 1, its equity value is lower. These two propositions thus reconcile the paradoxical findings that groups are common (see e.g. La Porta et al., 1999) despite lower equity values.

Appendix B establishes the following properties of the guarantee: it is non-increasing in the holding debt, it can be both increasing or decreasing in the subsidiary one. Indeed, for any joint cash flow distribution and any capital structure, reducing debt in the holding enlarges - or at least does not reduce - the rescue area. The effect associated with changes in subsidiary debt is less obvious. Raising P_S^* on the one hand contributes to the value of the guarantee by increasing the value of S cash flows that are saved when H succeeds in rescuing it. On the other hand raising P_S^* reduces the value of the guarantee by making it less likely that H cash flows will be sufficient to service S debt. When debt in the subsidiary diverges, the second effect dominates and the marginal value of the guarantee is negative.

Using these properties, we are now ready to assess the optimal choice of debt in HS structures, by addressing the tax-bankruptcy trade-off.

2.3 Debt capacity: Holding-Subsidiary versus Stand Alone firms

In this section we first show that optimal capital structure entails a partial shift of debt from the holding onto its subsidiary and that the overall principal - i.e. the debt capacity - in HS is higher than in SA firms with the same cash-flows distributions. Then we present conditions ensuring that the shift is total, while still preserving a higher debt capacity. Throughout, we focus on the simpler case of infinitesimal ownership.

The intuition is straightforward. At positive debt levels, P_1^*, P_2^*, the marginal tax savings equal the marginal default costs in the two stand-alone firms. In the corresponding HS, marginal tax savings and the marginal default cost in H are the same as in the stand alone firms, while the marginal default cost in S is lower than that in the corresponding SA thanks to the guarantee. Debt in the subsidiary must be larger than in the corresponding stand alone in order to re-establish equality with tax savings - thus $P_S > P_2^*$. Furthermore, since the guarantee is non-increasing in the holding debt, it pays to reduce the latter, transferring one unit of debt from H to S, implying $P_H < P_1^*$. Showing that total debt capacity increases is less straightforward, because - as proved in Appendix B - S debt has an ambiguous effect on the guarantee.

Two preliminary steps are necessary. First, following Leland (2007), we rewrite the levered firm value in a SA as unlevered firm value, V_{0i}, plus tax savings from interest deduction less default costs:

$$\nu_{0i}(P_i) = V_{0i} + TS_i(P_i) - DC_i(P_i) \tag{22}$$

where $TS_i(P_i)$ is the present value of tax savings, equal to the differential tax burden of the unlevered and the levered firm:

$$TS_i(P_i) = T_i(0) - T_i(P_i) = \tau_i \phi \left[\mathbb{E}X_i^+ - \mathbb{E}(X_i - X_i^Z)^+ \right] \tag{23}$$

and $DC_i(P_i)$ is the present value of the default costs incurred in because of leverage:

$$DC_i(P_i) = \alpha_i \phi \mathbb{E} \left[X_i \; \mathbf{1}_{\left\{0<X_i<X_i^d\right\}} \right] \tag{24}$$

Tax savings are increasing in the face value of debt, since the latter enlarges interest deductions and the associated tax shield. Default costs too increase in the face value of debt, because the set of default states gets larger.[15] The optimal SA debt results from trading off these effects, as known from seminal results by Kim (1978), among others. Appendix A gives necessary and sufficient conditions for the solution of the SA problem:

$$\min_{P_i \geq 0} \left[T_i(P_i) + DC_i(P_i) \right] \tag{25}$$

Moreover, it shows that the optimal SA leverage is positive if an optimum exists. [16]

Second, it can be shown that maximizing HS value is equivalent to minimizing the sum of tax burdens and default costs for the stand alone companies net of the guarantee:

$$\min_{P_H \geq 0, P_S \geq 0} \left[T_1(P_H) + T_2(P_S) + DC_1(P_H) + DC_2(P_S) - G(P_H, P_S) \right] = \tag{26}$$

We assume a convex objective function - consistent with the SA case of Leland - so as to ensure the existence of an optimum. However, we still do not introduce assumptions on the joint distribution function of cash flows[17].

We now prove that a shift of debt from H to S at (P_1^*, P_2^*) is profitable and give a locally sufficient condition for larger debt capacity in HS.

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[15] Tax savings are short a call option on X_i with strike X_i. The call is decreasing in debt, since the strike is increasing in it. Default costs are a barrier call option on X_i with zero strike and barriers equal to zero and X_i^d. The call is increasing in debt, since the upper barrier is increasing in it.

[16] A sufficient condition for existence, that we maintain below, is that the sum of the tax burden and default costs is convex in debt.

[17] We simply assume that a technical, but innocuous condition holds: the function $xf(x, y)$, where $f(x, y)$ is the joint cash flow density, satisfies the dominated convergence condition when x diverges and $y > 0$.This allows us to exchange limits and integration in the proofs of Appendix B.

Proposition 5 *Let P_1^*, P_2^* be the optimal debt levels for two stand alone companies. Then, locally, (i) the holding debt can be decreased with respect to P_1^* - and the subsidiary increased wrt P_2^* - so as to increase the overall group value; (ii) provided that the value of the guarantee is decreasing in P_S at P_2^*, with $\partial G\left(P_1^*, P_2^*\right)/\partial P_S > \partial G\left(P_1^*, P_2^*\right)/\partial P_H$, then the overall debt capacity is greater than $P_1^* + P_2^*$.*

Proof. See Appendix B. ■

Consistent with this proposition, Masulis et al. (2008) find that group affiliates are more levered than stand alone firms and that holding companies are less levered than comparable stand alone.

We then give necessary and sufficient conditions for the subsidiary to bear all of HS debt and for it to exceed total debt in SA firms. Denote as X_S^{Z**}, X_S^{d**} the tax shield and default threshold of a subsidiary with debt $P_1^* + P_2^*$.

Proposition 6 *If tax burdens and default costs net of the guarantee are convex in debts (i) There cannot be a local minimum for the HS problem in which the subsidiary is unlevered (ii) the holding is optimally unlevered ($P_H^* = 0$) (iii) the subsidiary principal - and, a fortiori, the HS one - is higher than in two stand alone companies ($P_S^* = P_S^* + P_H^* > P_1^* + P_2^*$) if and only if the ratio of default costs to the tax rate is bounded above by a constant Q.*

Proof. See Appendix B, which also provides the expression for Q. ■

Remark 7 *When proposition 6 holds, the support region is non-empty. Proposition 3 and its corollary apply under the conditions of remark 2.*

A condition which implies the one in part *(iii)* of proposition 6 is the following:

$$\alpha X_S^{d**} \begin{pmatrix} \frac{dX_S^{d**}}{dP_S} \Pr\left(X_s = X_S^{d**}, X_H < 0\right) + \\ + \frac{\partial h}{\partial P_S} \Pr(0 < X_s < X_S^{d**}, X_H = h(X_S)) \end{pmatrix}$$

$$< \tau \Pr(X_S > X_S^{Z**}) \frac{dX_2^{Z**}}{dP_S}$$

The left hand side is an upper bound to the effect of changing S debt on default costs, as it measures its impact over the boundary of the rescue region in Figure 1. It considers both the shift of the support region, due to the change in the boundary h and proportional to the probability of being along it, $0 < X_S < X_S^{d**}, X_H = h(X_S)$, and the shift of the default boundary, $X_S = X_S^{d**}, X_H < 0$, proportional to its own probability[18]. The right-hand side measures the marginal effect on the tax burden, which is proportional to the probability of paying taxes, $\Pr(X_S > X_S^{Z**})$. Thus debt in the subsidiary (and overall debt, since the holding is optimally unlevered) - is higher than in the two stand alone firms if - at $P_S = P_1^* + P_2^*$ - the marginal increase in default costs is lower than the marginal savings in taxes.

We will see that debt capacity increases in the base case of Leland - when firms cash flows are equal in distribution and Gaussian. Consistently, we will also find that the optimal debt in H is zero. Only when we abandon the base case and H is much larger than its subsidiary we find positive - albeit small - leverage for H.

Before turning to numerical results, we complete our analysis of competing organizational forms with ongoing focus on intercorporate guarantees.

2.4 HS versus the Conglomerate Merger

An alternative to the HS structure is a simple merger (M), that incorporates activities as one firm with cash flow $X_m = X_1 + X_2$ making them jointly liable vis-à-vis lenders. The resulting conglomerate my be able to increase debt capacity P_m above the one of the two stand alone firms, when cash flow pooling between the two activities allows for diversification gains. This brings enhanced tax advantages because of interest deductions, as predicted by Lewellen (1971). It also allows to use the losses from one unit to offset taxable income from the other unit, thus reducing the negative impact of tax asymmetries (Majd and Myers, 1987). However, one unprofitable division may absorb the cash flows of a profitable one - the "Sarig effect". Leland (2007) shows that the Sarig

[18] The default boundary effect is made by two parts. On the one side, increasing P_S increases marginal default costs, according to

$$\alpha X_S^{d**} \frac{dX_S^{d**}}{dP_S} f_2\left(X_S^{d**}\right),$$

On the other, it lowers the guarantee, according to

$$-\alpha \frac{dX_S^{d**}}{dP_S} \int_0^{+\infty} X_S^{d**} f\left(X_S^{d**}, y\right) dy.$$

The net effect is measured by

$$\alpha X_S^{d**} \frac{dX_S^{d**}}{dP_S} \Pr\left(X_S = X_S^{d**}, X_H < 0\right).$$

effect tends to dominate when cash flow volatility differs, or when the correlation between activities' cash flow is so high that diversification opportunities are limited.

The value comparison of HS and conglomerate mergers does not deliver straightforward results in the absence of parametric or distributional restrictions, even when we assume comparable debt levels in the two organizations, i.e. $P_H + P_S = P_m$. The reason is that limited liability favours HS over mergers, while the tax burden is higher in HS because loss offsetting is imperfect. We are however able to show that HS create value relative to a merger if either (i) activities cash flows are equal and perfectly correlated, or (ii) they are Gaussian with $\rho^Q < \rho \leq 1$ and (common) volatility $\sigma > \sigma_L$, where

$$\begin{cases} \rho^Q < 1 \\ \sigma_L = \arg\min \nu^*(P_m) \end{cases}$$

or (iii) they are Gaussian, with $\rho^R < \rho \leq 1$ and distinct volatilities: $\sigma_H \neq \sigma_S$. [19]

Case (i) highlights an important feature of HS, namely that it allows for debt diversity thanks to the activities' separate incorporation. If cash flows and debt in the two activities were instead the same, the holding would not be able to rescue its subsidiary ever. This is precisely the situation in a merger, because activities are incorporated as one firm and cannot have different debt levels. As Leland (2007) shows, the value of a merger coincides with the value of two stand alone firms when diversification opportunities vanish. In HS, it becomes possible for H to rescue S as one unit of debt gets transferred from the holding onto its subsidiary: thus default costs fall, the optimal debt increases and the tax burden drops. Debt diversity preserves the value of the guarantee when diversification opportunities vanish.

In the other two cases, the Sarig effect is large enough to make a merger less desirable than two stand alone firms (Leland (2007)). The value of HS remains larger than in stand alone firms (due to Proposition 3) because cash flow pooling - i.e. the guarantee - is conditional due to separate incorporation. A merger instead forces each activity to provide an unconditional guarantee to the other.

3 Numerical analysis

The base-case parameters for the numerical analysis in Table 1 are equal to Leland (2007), which is consistent with a firm issuing BBB-rated

[19] We do not include the proof for reasons of space. It is available upon request.

unsecured debt. The horizon T is 5 years, the per annum interest rate is 5%, which gives a compound rate of 27.6% over 5 years. Operating cash flow for each activity, which is normally distributed, has expected value $Mu = 127.6$, and expected present value $X_0 = 100$. Operating cash flow at the end of 5 years has standard deviation (Std) of 49.2, consistent with an annualized standard deviation of cash flows equal to 22.0 ($= 49.2/\sqrt{5}$) if annual cash flows are independently distributed in time. Henceforth we express volatility σ as an annual percent of initial activity value X_0, e.g. $\sigma = 22\%$. The correlation coefficient between the units' cash flows is set equal to 0.2. The tax rate, $\tau = 20\%$, and the default cost parameter, $\alpha = 23\%$, generate optimal leverage and recovery rates consistent with the BBB choice. This gives $V_0 = 80.05$ and $L_0 = 0.057$.

Table 1 shows the optimal capital structure and value. The first column reports results for a stand alone. The second, third and fourth columns refer to holding, subsidiary and an "average" affiliated company ($0.5(H+S)$) respectively, while the last column to half of a conglomerate so as to allow for comparison[20]. Results are insensitive to intercorporate ownership ω.

3.1 HS versus Stand Alone

The optimal face value of debt, which is equal to 57.1 for a stand alone company, reaches 110 in the average HS affiliate, consistent with our analytical results establishing higher debt capacity for HS. Accordingly, expected tax savings are smaller in the former (2.32) than in the latter case (7.31) and, as a result, the average HS affiliate value (83.29) exceeds that of a SA (81.47). The beneficiary is the initial owner of the two activities, who sells them for more. This simulation reveals that the increase in debt allowed for by the guarantee generates higher expected default costs in HS than in SA (4.07 and 0.89).

Intercorporate guarantees reduce the average market value of equity below that of stand alone counterparts. Proposition 1 emphasizes the transfer from H to S, while in this exercise we appreciate the interplay between the transfer and the level of debt. Specifically, in Table 1 we see that the value of equity in the stand alone is larger than in the subsidiary (39.23 instead of 0.07), because of its much lower level of debt (57.1 versus 220). On the contrary, the value of equity in the holding is larger than the stand alone one (49.46 versus 39.23), even if part of its cash flow is being transferred to the subsidiary lenders, because its debt

[20]Results differ slightly from Leland's as we used integration bounds of $\pm 5\sigma$, instead of $\pm 3\sigma$.

is lower (0 versus 57.1).[21] This fact could mistakenly be interpreted, by an outside observer such an econometrician, as the consequence of an inefficiency in HS corporate choices - which is instead absent.

It is worth noting the similarity between this type of HS and leveraged buy-outs where our assumption of no agency costs applies reasonably well.[22] The tax burden of debt drops from 17.70% of operating cash-flow in a stand alone to 12.70% on average for HS and 5.39% for the subsidiary alone. In firms taken private in the first wave of US LBOs, the tax burden dropped from 20% to 1% in the first two years and to 4.8% in the third year (Kaplan, 1989). In Table 1, the default threshold for a subsidiary is 249, way above the mean operating income. In a sample of distressed highly leverage transactions, all sample firms had operating margins in excess of the industry median (Andrade and Kaplan, 1998). Last but not least, the model implies leverage in excess of 95% in subsidiaries. This is also observed in the first wave of LBOs, .

3.2 Capital structure and value with changing correlation

Leland (2007) shows that gains from a merger disappear together with diversification: as the correlation coefficient between activities' cash flows tends to 1, the default costs of the merger converge to those of stand alone firms, so does its debt level and overall value. The HS structure also exploits diversification. One may have expected that, as correlation among cash flows increases, the transfers from the H to S will become less likely and the optimal face value of debt will converge to the stand alone level.

The first part of this reasoning is correct: as the correlation coefficient ρ increases from -0.8 to 0.8, the (unreported) probability of a transfer from H to S halves. The second part of the argument is however incorrect: debt in HS continues to be much larger than in SA, and very diverse between H and S. Figure 2 displays such result in the bottom right panel.[23]

The optimal face value of debt in HS actually increases in ρ. Con-

[21] We will see in later sections that the opposite may happen, i.e. $E_H^* < E_1^* < E_S^*$. However, it will still be the case that the average value of SA equity exceeds that of HS, as stated in the Proposition.

[22] Private equity partners often need to raise new funds in the market because of the limited temporal commitments of financiers, and this is possible only if their reputation is good. Moreover, subsidiary managers receive bonuses only when they repay their debt obligations. See Jensen (2007).

[23] Contrary to the face value, the market value of S debt falls as correlation increases (see bottom left panel of Figure 2): lenders required spread grows in ρ in anticipation of reduced support by H.

sider in fact that expected default costs $\mathbb{E}\left[\alpha X_S \mathbf{1}_{\ 0<X_S<X_S^d}\right]$ are increasing in X_S^d, not only because the probability of default increases but also because conditional default costs, αX_S, are larger. The larger is ρ, the likelier it is that H cash flow suffices to rescue S, i.e. $X_H > h(X_S)$, when conditional default costs are also large. In other words, the guarantee has highest value. This is why the total value differential between HS and stand alone firms, which is always positive (top left panel of Figure 2), achieves a maximum for $\rho = 1$.

Consistent with implications of our Proposition 1, HS average equity value is lower than in the case of stand alone firms for all correlation coefficients (upper right panel of Figure 2), because transfers from H stockholders to S lenders are always positive.

4 Optimal HS when activities differ

Numerical results so far refer to two symmetric activities, that differ only because H is assumed to support S. The analysis below refers to cases when activities differ in either cash flow volatility (σ), or in size (Mu) or in proportional bankruptcy costs (α).

This investigation deserves attention for two reasons. First, Rajan et al. (2000) point to potential inefficiencies stemming from diversity - in size and investment opportunities - across conglomerate activities. While they focus on inefficiencies arising from capital budgeting, Leland (2007) highlights that cash flow diversity has a cost in conglomerates because of the larger foregone value of limited liability: conglomerate may turn out to have lower value than stand alone organizations only when activities are asymmetric. We now assess whether HS are more or less valuable when activities are diverse.

Second, we observe that strategic alliances (Robinson, 2008), venture capital funds (Sahlman, 1990) and innovative firms (Allen, 1998) often adopt a HS structure, with riskier ventures incorporated as subsidiaries. The same is true in traditional business groups (Bianco and Nicodano, 2006; Masulis et al., 2008). This suggests that HS value is not invariant to the relative features of H and S. Below, we endogenously derive the characteristics of holding and subsidiaries that maximize HS value.

The following proposition summarizes our main numerical findings:

Conclusion 8 *Assume different cash-flow distributions for BBB calibrated companies. Then (i) the optimal HS structure has higher value than competing organizations, both M and SA, and value gains increase with risk and bankruptcy costs asymmetries between activities; (ii) in the optimal HS structure, the exogenous default cost parameter and the* *size of the holding are at least as large as those of its subsidiary. The subsidiary, in turn, has at least as risky a cash-flow as the holding.*

These results hold for the case, displayed in Tables 2 to 4, when correlation between cash-flows is equal to 0.2, as well as for the unreported range {-0.8, +0.8}.

Table 2 displays numerical results when the two activities have proportional bankruptcy costs respectively equal to 23%, as in the base case, and 75%. With larger bankruptcy costs, the optimal value of a stand alone firm drops from 81.47 (see the second column) to 80.83 (first column) as its face value of debt reduces from 57.1 to 33. In the case of HS, by contrast, it turns out that the activity with larger bankruptcy costs should be the holding company, because - under the optimal capital structure - H never pays them. Both the optimal capital structure and group value do not change as default costs in the holding increase from 23% to 75%. It follows that the value difference between HS and SA increase from 3.64 in Table 1 to 4.29 in Table 2 with asymmetries in default costs across activities.

Table 3 concerns the case of different risk, with one unit having an annualized cash flow volatility of 44% as opposed to 22% of the other. We know that the tax shield has higher value with a riskier cash flow, because the firm pays taxes when earnings after interests are positive, but does not get a comparable tax refund in the opposite situation. It is therefore unsurprising to find higher optimal debt, and associated tax shield, in all organizations. A comparison between the first and the second column reveals that the higher volatility unit, when incorporated as a stand alone, has higher face value of debt (83 instead of 57), and is accordingly charged a much higher spread (6.2% as opposed to 1.23%) by lenders. This implies increased tax savings (4.66 versus 2.32), and a higher value of the riskier stand alone (84.84 versus 81.47) - even though its equity value drops from 39.23 to 36.1. The total value of these two stand alone is equal to 166.31. We also find that subsidiaries are optimally riskier than their holding companies, consistent with empirical evidence. A riskier holding would not be able to rescue its subsidiary as often as a safer one, and would suffer more from tax asymmetries as it uses less the tax shield of debt. Similarly to the SA case, subsidiary debt increases to 221 (up from 220 in the base volatility case). Total HS value is now equal to 170.91, exceeding its value in the base case (166.58), when the subsidiary is less risky. Importantly, value gains relative to SA increase from 3.24 to 4.6.

The last case we examine (Table 4) is the one of differing size, as the expected cash flow of one activity is five times the other. In all the organizations, value falls relative to the symmetric case (SA from 162.46

to 162.44; HS from 166.58 to 166.13). The smaller activity should be the one receiving conditional support. The smaller subsidiary - see the fourth column - raises less debt than the one of the base case (125 as opposed to 220). The larger holding company can now raise debt as well (64) so as to reduce its own tax burden, without compromising the provision of support to its subsidiary. A HS with a larger subsidiary would be suboptimal, as the HS would raise less debt with a corresponding reduction in both the tax shield and value (163.25) - which remains higher than in the competing organizations.

All the previous qualitative results under asymmetry hold true when correlation varies: we conclude that the ability of HS to create value for the entrepreneur by optimally trading off taxes and bankruptcy costs is a robust result.

5 Informal guarantees

So far we studied a guarantee which is enforceable in court. Often guarantees are not legally binding, though: comfort letters, for instance, are legally unenforceable promises of rescue sent by the parent to subsidiary's lenders. The latter nonetheless expect support from the holding company with some positive probability, as the holding trades-off the benefits from improved future credit conditions with the costs of reduced financial integrity in deciding whether to honor comfort letters (Boot et al. (1993)).[24] In this section, we assume that lenders attribute an exogenous probability $\pi < 1$ to ex post rescue.

Table 5 reports numerical results when the probability, π, is equal to 0.5 instead of 1, which is the value implicit in Tables 1 to 4. Intercorporate dividends are equal to zero ($\omega = 0$).

The observations concerning the comparison between stand alone firms and HS hold true (and carry over to other positive values of the probability π). The subsidiary is still able to raise a larger amount of debt (69) relative to its stand alone counterpart (57.2). Total debt capacity also increases in HS (124 versus 114.4), implying higher HS value (163.24 versus 162.46).

However, total debt capacity is drastically smaller than in the contractual case (124 instead of 220) and its distribution between the holding and its subsidiary is more balanced (55 as opposed to zero; 69 as opposed to 220), due to the uncertainty regarding the actual rescue. Correspondingly, total value falls from 166.58 to 163.24. The holding raises more debt than in the contractual case: lenders would charge too

[24] Consistent with this view, Gopalan Nanda and Seru (2007) observe a worsening of credit conditions for all group affiliates when one firm goes bankrupt.

high a spread if the entrepreneur shifted all of HS debt onto the subsidiary[25]. Finally, it is still the case that average equity prices in group affiliated firms (35.80) fall short of stand-alone equity valuation (39.23).

We now allow for intercorporate dividends ($0 < \omega \leq 1$) and keep $\pi = 0.5$. In relative terms, dividends do not affect our main conclusions. Unreported numerical results confirm that HS structures still have a greater value than two corresponding SA companies, for all cash flow correlations if $\omega < 1$, and for moderate correlation ($|\rho| < 0.8$) when $\omega = 1$. The H is still less indebted than a SA receiving the same amount of dividends.

In absolute terms, while dividends do not affect HS value when the guarantee is legally binding, they do affect it when the guarantee is informal. For $\pi = 0.5$, Table 6 reveals that the larger is ω, the larger is total group value. For $\rho = 0.2$, total value grows from 163.23 (for $\omega = 0$) to 163.71 (for $\omega = 1$). This value gain stems from a reduction in the probability that the holding defaults, at any given level of its debt, thanks to the dividend transfer from its subsidiary[26]. Consistent with this conjecture, we observe an increase in the optimal face value of debt in the holding - which restores equality between the marginal tax benefit and the marginal bankruptcy cost. P_H now ranges from 55 at $\omega = 0$ to 83 at $\omega = 1$.

Total group debt increases at a slower pace (from 124 to 135), as the level of debt in the subsidiary falls (from 69 to 52). This reduction, in turn, stems from the higher holding leverage which reduces both its net cash flow after interest and its ability to rescue its subsidiary.

The behavior of tax savings, default costs and equity prices for the subsidiary is non-linear. However, tax savings in S are lower at $\omega = 1$ than at $\omega = 0$ because of lower optimal debt which translates into a reduced tax shield. This holds true for all values of ρ. The opposite occurs for tax savings in the holding.

Perhaps counterintuitively, default costs in the holding are higher at $\omega = 1$ than at $\omega = 0$ if $\rho > -0.8$. This is because the holding has higher debt now; at the same time, H and S have similar debt levels, implying that dividends are of little help in avoiding H insolvency unless cash flows are highly negatively correlated. On the contrary, default costs in the holding are lower at $\omega = 1$ than at $\omega = 0$ for $\rho = -0.8$. In other words, the capital structure in HS becomes more similar to the conglomerate one when $\omega > 0$ and $\pi < 1$. Gains from debt diversity, that show up

[25] This may explain why both holding and subsidiaries raise debt in Belgian and Italian groups (Dewaelheyns and Van Hulle, 2007; Bianco and Nicodano, 2006).

[26] De Jong et al. (2009) support the hypothesis that dividends help servicing the holding debt.

in Tables 1 through 4, are reduced and the typical reasoning relating to diversification dominates again.

Importantly, unreported results show that conglomerate value (163.15) exceeds HS value in the base case when $\omega = 0$, for $\pi = 0.1$.The optimal HS debt is now lower than the merger debt as lenders - anticipating uncertain support by H - charge higher spreads to both the holding and its subsidiary relative to the merger. The tax burden is almost equal but expected default costs are higher in HS because the unconditional guarantee in mergers always works while the conditional one in HS is less reliable.

We can summarize these numerical findings as follows.

Conclusion 9 *Consider a BBB-calibrated stand alone company. (a) Let* $\pi = 1$. *Then HS value achieves its maximum at* $\rho = 1$, *for any admissible* ω *(b) Now let* $0 < \pi < 1$.*Then HS achieves its maximum value, which is decreasing in* ρ, *for* $\omega = 1$ *(c)The entrepreneur always prefers HS over SA (d) There exists a* $\pi'(\omega = 0, \rho)$ *such that the entrepreneur prefers M to HS for all* $\pi < \pi'$, *when activities cash flows are equal in distribution.*

6 Concluding comments

This paper contributes to our understanding of firm scope, by clarifying the role of intercorporate guarantees in affecting capital structure and value creation. Guarantees determine the overall debt capacity of the organization, its tax burden and expected default costs. These, in turn, affect the value of lenders' and shareholders' claims to cash-flows, which add up to the total value of the organization to the entrepreneur.

Holding- subsidiary structures embed a conditional guarantee which increases debt capacity relative to stand alone counterparts, thus improving the trade-off between taxes and default costs. The drivers of value creation relative to competing organizations are the possibility of leveraging in a different manner the holding and the subsidiary, together with the possibility of letting the second selectively default when rescue is not viable.

Our framework allows to understand further specificities of HS. Their value increases when diversification opportunities between two activities vanish, a feature that clearly distinguishes them from conglomerate mergers. HS value is also enhanced by asymmetric cash flows deriving from the activities, provided that the holding company has lower cash flow volatility and/or higher bankruptcy costs than its subsidiary. On the contrary, such asymmetries may reduce the value of a conglomerate merger below that of stand alone organizations.

In the recent crisis, some institutions were interconnected as HS and had large leverage. Our simulations do indicate that HS appear to incur into much higher bankruptcy costs than competing organization, despite the provision of a guarantee to lenders. The comparative welfare properties of HS thus appear to deserve further attention, as HS might be socially wasteful despite being value maximizing. Moreover, they derive some value gains from tax avoidance.

Importantly, our model is just a first step towards a better understanding of intercorporate guarantees in holding- subsidiary structures, as it relies on a simple static setting with two activities, no agency problems and exogenous contracts. Developments relying on more general settings are postponed to further work.

7 Appendix A -

7.1 The stand alone optimization problem

This Appendix studies the maximization of the SA value with respect to non-negative debt levels, $P_i \geq 0$, with $i = 1, 2$, through its equivalent problem, namely the minimization of the tax burden plus default costs (25). We first establish some properties of the market value of debt for stand alone companies.

Lemma 10 *Debt is increasing less than proportionally in the face value of debt:*

$$0 \leq dD_{0i}(P_i)/dP_i < 1 \qquad \text{with} \qquad \lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} > 0$$

Proof. Observe first that, as default costs and taxes approach zero, we have:

$$0 \leq dD_{0i}(P_i)/dP_i = (1 - F_i(P_i))\phi \leq \phi < 1$$

In particular, when the face value of debt tends to zero, we have:

$$\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} = (1 - F_i(0))\,\phi > 0$$

since the probability that X_i is positive is positive ($F_i(0) < 1$). For positive default cost and tax rates, when closed form expressions for $D_{0i}(P_i)$ do not obtain, we use the fact that risky debt D_{0i} can be written as the difference between the corresponding riskless debt, $P_i\phi$, and lenders' discounted expected loss.

The first part of the proof proceeds by contradiction. Thus assume instead $dD_{0i}(P_i)/dP_i \geq 1$. Observe that the derivative of riskless debt wrt the face value of debt (ϕ) is smaller than one. In order for the risky debt to have a derivative not smaller than one, the discounted expected loss should have a derivative smaller than zero, i.e. it should decrease in the face value of debt. This contradicts the minimal requirement that both default probability and expected default costs increase in the face value of debt.

As for the other property, let $\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} \leq 0$, when default costs and taxes are finite. This implies that the discounted expected loss has a derivative, when $P_i \to 0+$, positive and not smaller than ϕ. This in turn implies that lenders' expected loss has a derivative greater than one with respect to debt , which is absurd. ■

This Lemma implies that both the tax shield and the default threshold are increasing in the face value of debt:

$$0 < \frac{dX_i^Z}{dP_i} = 1 - \frac{dD_{0i}(P_i)}{dP_i} \leq 1, \tag{27}$$

$$\frac{dX_i^d}{dP_i} = 1 + \frac{\tau_i}{1-\tau_i}\frac{dD_{0i}(P_i)}{dP_i} \geq 1 \tag{28}$$

Now we show that the stand alone is optimally unlevered (levered), without (with) taxes. Indeed, the Kuhn-Tucker (KT) conditions for problem (25) are

$$\begin{cases} \frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i} \geq 0 \\ P_i^* \geq 0 \\ \left[\frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i}\right] P_i^* = 0 \end{cases} \tag{29}$$

Conditions 29 are necessary and sufficient if $T_i + DC_i$ is convex in $P_i \geq 0$, as assumed in the text.

The derivative of tax burdens and default costs, appearing on the lhs of 29, is equal to:

$$\frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} = -\tau_i(1 - F_i(X_i^Z))\frac{dX_i^Z}{dP_i}\phi + \alpha X_i^d f_i(X_i^d)\frac{dX_i^d}{dP_i}\phi \tag{30}$$

where f_i is the density of X_i.

If $\tau_i = 0$, a minimum exists, with $P_i^* = X_i^{d*} = X_i^{Z*} = 0$. When there is taxation ($\tau_i > 0$), then a minimum at $P_i = 0$ cannot exist, since the first condition in 29 is violated. The optimum is interior, and (30) is set to zero.

8 Appendix B -

8.1 Proofs of propositions 5 and 6

Letting $f(x,y)$ be the joint density of the cash flows (X_S, X_H), we can write the guarantee as

$$G(P_H, P_S) = \alpha\phi \int_0^{X_S^d} \int_{h(x)}^{+\infty} x f(x,y) dx dy =$$

$$= \alpha\phi \left[\int_0^{X_S^Z} x \int_{X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}}^{+\infty} f(x,y) dy dx + \int_{X_S^Z}^{X_S^d} x \int_{X_H^d + X_S^d - x}^{+\infty} f(x,y) dy dx \right]$$

We first prove the following lemma which characterizes G.

Lemma 11 *The guarantee a) is non increasing in P_H and has a null derivative if and only if $P_S = 0$; b) has a null derivative with respect to P_S at $P_S = 0$; c) is decreasing in P_S when the latter diverges.*

Proof. Part (a) requires the guarantee to be non increasing in P_H, that is:

$$\frac{\partial G}{\partial P_H} = -\alpha\phi \times \frac{dX_1^d}{dP_1} \times$$

$$\times \begin{bmatrix} \int_0^{X_S^Z} x f\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \\ + \int_{X_S^Z}^{X_S^d} x f(x, X_H^d + X_S^d - x) dx \end{bmatrix} \leq 0 \tag{31}$$

Equality in (31) holds if and only if the third term is zero, that is $X_S^d = X_S^Z = 0$, which in turn happens if and only if $P_S = 0$.

As concerns part (b), we compute:

$$\frac{\partial G}{\partial P_S} = \alpha\phi \times \left\{ -\frac{1}{1-\tau} \int_0^{X_S^Z} x f\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right. \tag{32}$$

$$\left. -\frac{dX_2^d}{dP_2} \int_{X_S^Z}^{X_S^d} x f(x, X_H^d + X_S^d - x) dx + \frac{dX_2^d}{dP_2} \int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy \right\} =$$

$$= \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ -\int_0^{X_S^Z} x f\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right.$$

$$\left. + \left(1 - \tau + \tau \frac{dD_{02}(P_2)}{dP_2}\right) \begin{bmatrix} -\int_{X_S^Z}^{X_S^d} x f(x, X_H^d + X_S^d - x) dx + \\ + \int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy \end{bmatrix} \right\}$$

When $P_S = 0$, then $X_S^d = X_S^Z = 0$, all the integrals vanish and the previous derivative is null.

As concerns part (c), namely

$$\lim_{P_S \to +\infty} \frac{\partial G(P_H, P_S)}{\partial P_S} < 0,$$

consider that, when $P_S \to +\infty$, definition (7) implies that

$$\lim_{P_S \to +\infty} X_S^d = +\infty$$

For fixed y, the convergence condition $\lim_{x \to +\infty} x f(x, y) = 0$ - which follows from the fact that f is a density - implies that, for any sequence x_n which goes to $+\infty$, then $x_n f(x_n, y)$ converges to zero. We suppose that the function $f_n(y)$ satisfies the dominated convergence property. This allows us to exchange integration and limit:

$$\lim_{n \to +\infty} \int_{X_H^d}^{+\infty} x_n f(x_n, y) dy = \int_{X_H^d}^{+\infty} \lim_{n \to +\infty} x_n f(x_n, y) dy = 0$$

and, as a consequence,

$$\lim_{X_S^d \to +\infty} \int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy = 0$$

Together with (27) this entails

$$\lim_{P_S \to +\infty} \frac{\partial G}{\partial P_S} =$$

$$= \lim_{P_S \to +\infty} \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ - \int_0^{X_S^Z} x f\left(x, X_H^d + \frac{P_S}{1-\tau} - \frac{x}{1-\tau}\right) dx - \right.$$

$$\left. - \left(1 - \tau + \tau \frac{dD_{02}(P_2)}{dP_2}\right) \int_{X_S^Z}^{X_S^d} x f(x, X_H^d + X_S^d - x) dx \right\} < 0$$

and proves part (c). ■

We then consider proposition 5.

Proof. i) In order to increase value, we need to decrease the following function:

$$T_{HS}(P_H, P_S) + DC_{HS}(P_H, P_S) =$$

$$= T_1(P_H) + T_2(P_S) + DC_1(P_H) + DC_2(P_S) - G(P_H, P_S)$$

Since the derivative of both $T_1 + DC_1$ and $T_2 + DC_2$ wrt their own arguments is null at the optimum leverage of the SA, P_1^*, P_2^*, then the impact of a local variation depends on the sign of the derivatives of the guarantee. We know from lemma 11 that decreasing the holding debt increases the guarantee (at any positive leverage of the subsidiary, including the optimal stand alone one), and therefore reduces $T_{HS} + DC_{HS}$, as needed. Given this, one can reduce the holding debt and increase the subsidiary one so that $T_{HS} + DC_{HS}$ decreases, as follows. We have:

$$d\left(T_{HS}\left(P_1^*, P_2^*\right) + DC_{HS}\left(P_1^*, P_2^*\right)\right) = -\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_H} dP_H - \frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S} dP_S \tag{33}$$

and the differential is negative if and only if

$$+\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_H} dP_H + \frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S} dP_S > 0$$

Consider expression (32) for $\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S}$, neglect $\alpha\phi$ and recognize that such derivative has a negative and a positive part. Define them as follows:

$$G_S^n := -\frac{1}{1-\tau}\int_0^{X_S^Z} x f\left(x, h(x)\right) dx +$$
$$-\frac{dX_2^d}{dP_2}\int_{X_S^Z}^{X_S^d} x f(x, h(x)) dx$$
$$G_S^p := \frac{dX_2^d}{dP_2}\int_{X_H^d}^{+\infty} X_S^d f(X_S^d, y) dy$$

Then the differential (33) is negative **if** there exists a couple $dP_H < 0, dP_S > 0$ such that

$$\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_H} dP_H + G_S^n dP_S = 0$$

For such couple (in particular, for $dP_S > 0$), indeed, $G_2^p dP_S > 0$. It suffices to take

$$dP_H < 0, dP_S = -\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_H}\frac{1}{G_S^n} dP_H$$

since the last differential is positive.

ii) provided that $\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S} < 0$, in order to demonstrate the assert we need to show that there exists a variation in P_S, starting from P_2^*, of the type $dP_S = -dP_H + \varepsilon, \varepsilon > 0, dP_H < 0$, such that the differential (33) is negative. This is value increasing and the corresponding principals represent a locally preferred debt capacity. This in turn happens, for $dP_H < 0$ and $\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S} < 0$, if and only if

$$\varepsilon < -\frac{\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_H} - \frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S}}{\frac{\partial G\left(P_1^*, P_2^*\right)}{\partial P_S}} dP_H$$

The right hand side of the last expression is positive, as required, as soon as $\partial G\left(P_1^*, P_2^*\right)/\partial P_H - \partial G\left(P_1^*, P_2^*\right)/\partial P_S < 0$. ■

We are now ready for the proof of proposition 6.

Proof. Part i): let us examine the Kuhn Tucker (KT) conditions for a minimum of $T_{HS} + DC_{HS}$ with respect to non-negative subsidiary debt. Recall that such conditions are necessary and sufficient, under the convexity assumption of the proposition.

$$\begin{cases} \frac{\partial T_{HS}(P_H^*, P_S^*)}{\partial P_S} + \frac{\partial DC_{HS}(P_H^*, P_S^*)}{\partial P_S} = \frac{dT_2(P_S^*)}{dP_S} + \frac{dDC_2(P_S^*)}{dP_S} - \frac{\partial G(P_H^*, P_S^*)}{\partial P_S} \geq 0 \\ P_S^* \geq 0 \\ \left[\frac{dT_2(P_S^*)}{dP_S} + \frac{dDC_2(P_S^*)}{dP_S} - \frac{\partial G(P_H^*, P_S^*)}{\partial P_S}\right] P_S^* = 0 \end{cases} \tag{34}$$

The derivative of tax burdens and default costs paid by the subsidiary with respect to its own debt, which appears in the first and third condition above, is equal to:

$$\frac{dT_2(P_S)}{dP_S} + \frac{dDC_2(P_S)}{dP_S} = \tag{35}$$

$$= -\tau(1 - F_2(X_S^Z))\frac{dX_2^Z}{dP_2}\phi + \alpha X_S^d f_2(X_S^d)\frac{dX_2^d}{dP_2}\phi$$

where F_2, f_2 are respectively the distribution and density functions of $X_2 = X_S$.

Let us examine whether the KT conditions are satisfied at $P_S = 0$. As a consequence of part (b) of the previous lemma, if $P_S^* = 0$ we have

$$\frac{dT_2(0)}{dP_S} + \frac{dDC_2(0)}{dP_S} - \frac{\partial G(P_H^*, 0)}{\partial P_S} = \tag{36}$$

$$= \frac{dT_2(0)}{dP_S} + \frac{dDC_2(0)}{dP_S} = -\tau(1 - F_2(0))\left[1 - \frac{dD_{02}}{dP_2} \mid_{P_2=0}\right]\phi$$

where the last term follows from (35), since $X_S^d = X_S^Z = 0$ when $P_S = 0$. Such derivative is negative, since $F_2(0) < 1$ and

$$\lim_{P_2 \to 0+} \frac{dD_{02}}{dP_2} < 1 \tag{37}$$

by lemma 10 in the text. The KT conditions are then violated when the subsidiary is unlevered. This concludes the proof of part i).

Let us now prove the other parts of the proposition. The KT conditions for a minimum of $T_{HS} + DC_{HS}$, with respect to non-negative debt for both the holding and the subsidiary, under the constraint

$$P_H^* + P_S^* \geq P_1^* + P_2^* := K \tag{38}$$

are equal to:

$$\begin{cases} \frac{\partial T_{HS}(P_H^*,P_S^*)}{\partial P_H} + \frac{\partial DC_{HS}(P_H^*,P_S^*)}{\partial P_H} = & \\ = \frac{dT_1(P_H^*)}{dP_H} + \frac{dDC_1(P_H^*)}{dP_H} - \frac{\partial G(P_H^*,P_S^*)}{\partial P_H} = \mu_1 + \mu_3 & (i) \\ P_H^* \geq 0 & (ii) \\ \mu_1 P_H^* = 0 & (iii) \\ \frac{\partial T_{HS}(P_H^*,P_S^*)}{\partial P_S} + \frac{\partial DC_{HS}(P_H^*,P_S^*)}{\partial P_S} = & \\ = \frac{dT_2(P_S^*)}{dP_S} + \frac{dDC_2(P_S^*)}{dP_S} - \frac{\partial G(P_H^*,P_S^*)}{\partial P_S} = \mu_2 + \mu_3 & (iv) \\ P_S^* \geq 0 & (v) \\ \mu_2 P_S^* = 0 & (vi) \\ P_H^* + P_S^* \geq K & (vii) \\ \mu_3(P_H^* + P_S^* - K) = 0 & (viii) \\ \mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0 & (ix) \end{cases} \tag{39}$$

In order to demonstrate part (ii), we temporarily ignore constraints (vii) and $(viii)$, and set $\mu_3 = 0$ in $(i), (iv)$ and (ix). We want to demonstrate that there exists a point $(0, P_S^*)$ which solves them. All the conditions but *(iv)* are easy to discuss.

Consider all the conditions except *(iv)* first. Having $\mu_3 = 0$, condition (i) becomes

$$\frac{dT_1(P_H^*)}{dP_H} + \frac{dDC_1(P_H^*)}{dP_H} - \frac{\partial G(P_H^*, P_S^*)}{\partial P_H} = \mu_1 \tag{40}$$

The left-hand side is positive at $P_H = P_1^*$, since the first two derivatives are null and $-\partial G/\partial P_H > 0$ (by part (a) of lemma (11) and part (i) of this proposition, which rules out $P_S^* = 0$). The first two terms on the left hand side are negative, if $P_H^* < P_1^*$, given convexity of $T_i + DC_i$ for a stand-alone. We also know that the third term $(-\partial G/\partial P_H)$ is still positive if $P_H^* < P_1^*$. When $P_H^* \to 0$, the left-hand side of (40) cannot be negative, since this would contradict the convexity assumption on the objective function. Thus $P_H^* = 0$ and conditions (i, ii, iii) are satisfied by letting μ_1 equal to the (non-negative) difference between $\frac{dT_1(P_H^*)}{dP_H} + \frac{dDC_1(P_H^*)}{dP_H}$ and $\frac{\partial G(P_H^*,P_S^*)}{\partial P_H}$.

If later we choose $P_S^* > 0$, also conditions (v, vi) are satisfied, provided that we select $\mu_2 = 0$. Given that we chose $\mu_1 \geq 0, \mu_2 = \mu_3 = 0$, condition (ix) holds. Let us turn to condition (iv), which has to provide us with a choice $P_S^* > 0$. In view of the other conditions, (iv) becomes

$$\frac{dT_2(P_S^*)}{dP_S} + \frac{dDC_2(P_S^*)}{dP_S} - \frac{\partial G(P_H^*, P_S^*)}{\partial P_S} = 0 \tag{41}$$

Consider its left-hand side as a function of P_S, denoting it with $\zeta(P_S)$ We know from the limit behavior of the guarantee (part (b) of lemma (11)) and from convexity of the stand-alone taxes and default costs $(T_2 + DC_2)$ that ζ has a negative limit when the subsidiary debt tends to zero, and a positive limit (even non finite) when P_S diverges. It follows that there exists a positive debt level which satisfies condition (41). This proves part ii) of the proposition, since all the KT conditions are satisfied.

Let us turn to part iii). We want to demonstrate that there exists a point $(0, P_S^*)$, with $P_S^* > K$, which solves conditions (i) to (ix). As above, we start by considering all the conditions but *(iv),* which requires some caution.

We are interested in a solution for which the constraint *(vii)* is not binding, implying $\mu_3 = 0$ in condition *(viii)*. As above, we choose $P_H^* = 0$ and let μ_1 be equal to the (non-negative) difference between $\frac{dT_1(P_H^*)}{dP_H} + \frac{dDC_1(P_H^*)}{dP_H}$ and $\frac{\partial G(P_H^*, P_S^*)}{\partial P_H}$. Thus P_H^* and $\mu_1 \geq 0$ satisfy conditions (i, ii, iii). If later we also choose $P_S^* > K$, conditions (v, vi, vii) are satisfied as well, provided that we select $\mu_2 = 0$. Given that we chose $\mu_1 \geq 0, \mu_2 = \mu_3 = 0$, both conditions $(viii, ix)$ hold.

Let us turn to condition (iv), which has to provide us with a choice $P_S^* > K$. In view of the other conditions, (iv) becomes again (41). Consider again $\zeta(P_S)$. We are going to show that, under the conditions posited sub (iii), $\zeta(P_1^* + P_2^*) < 0$, which implies that $P_S^* > P_1^* + P_2^*$. We have:

$$\zeta(P_1^* + P_2^*) = \phi \left\{ -\tau(1 - F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dP_S} + \alpha X_S^{d**} f_2(X_S^{d**})\frac{dX_S^{d**}}{dP_S} + \right.$$

$$+\frac{\alpha}{(1-\tau)} \int_0^{X_S^{Z**}} x f\left(x, \frac{P_1^* + P_2^*}{1-\tau} - \frac{x}{1-\tau}\right) dx +$$

$$\left. -\alpha \frac{dX_S^{d**}}{dP_S} \times \left[-\int_{X_S^{Z**}}^{X_S^{d**}} x f(x, X_S^{d**} - x) dx + \int_0^{+\infty} X_S^{d**} f(X_S^{d**}, y) dy \right] \right\}$$

where X_S^{d**} and X_S^{Z**} are the default and tax shield thresholds corresponding to $P_S = P_1^* + P_2^*$, and dX_S^{d**}/dP_S is evaluated at $P_S = P_1^* + P_2^*$.

Omitting ϕ, we can write the condition $\zeta < 0$ in a more compact way as

$$-\tau(1-F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dP_S}+\alpha\frac{dX_S^{d**}}{dP_S}X_S^{d**}\int_{-\infty}^{0}f(X_S^{d**},y)dy+$$

$$+\frac{\alpha}{(1-\tau)}\int_0^{X_S^{Z**}}xf\left(x,h(x)\right)dx+\alpha\frac{dX_S^{d**}}{dP_S}\int_{X_S^{Z**}}^{X_S^{d**}}xf(x,h(x))dx<0$$

or, recognizing that both $1/\left(1-\tau\right)$ and $\frac{dX_S^{d**}}{dP_S}$ are $\partial h/\partial P_S$,

$$-\tau(1-F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dP_S}+\alpha\frac{dX_S^{d**}}{dP_S}X_S^{d**}\int_{-\infty}^{0}f(X_S^{d**},y)dy+$$

$$+\alpha\frac{\partial h}{\partial P_S}\int_0^{X_S^{d**}}xf(x,h(x))dx<0$$

The last formulation can be written as

$$\alpha/\tau<\frac{\left(1-F_2(X_S^{Z**})\right)\frac{dX_S^{Z**}}{dP_S}}{X_S^{d**}\frac{dX_S^{d**}}{dP_S}\int_{-\infty}^{0}f(X_S^{d**},y)dy+\frac{\partial h}{\partial P_S}\int_0^{X_S^{d**}}xf(x,h(x))dx}$$

$$=\frac{\Pr(X_S>X_S^{Z**})\frac{dX_2^{Z**}}{dP_S}}{X_S^{d**}\frac{dX_S^{d**}}{dP_S}\Pr\left(X_S=X_S^{d**},X_H<0\right)+\frac{\partial h}{\partial P_S}\int\limits_0^{X_S^{d**}}xf(x,h(x)dx}\equiv Q$$

which is the condition in the proposition statement. This proves part iii). ■

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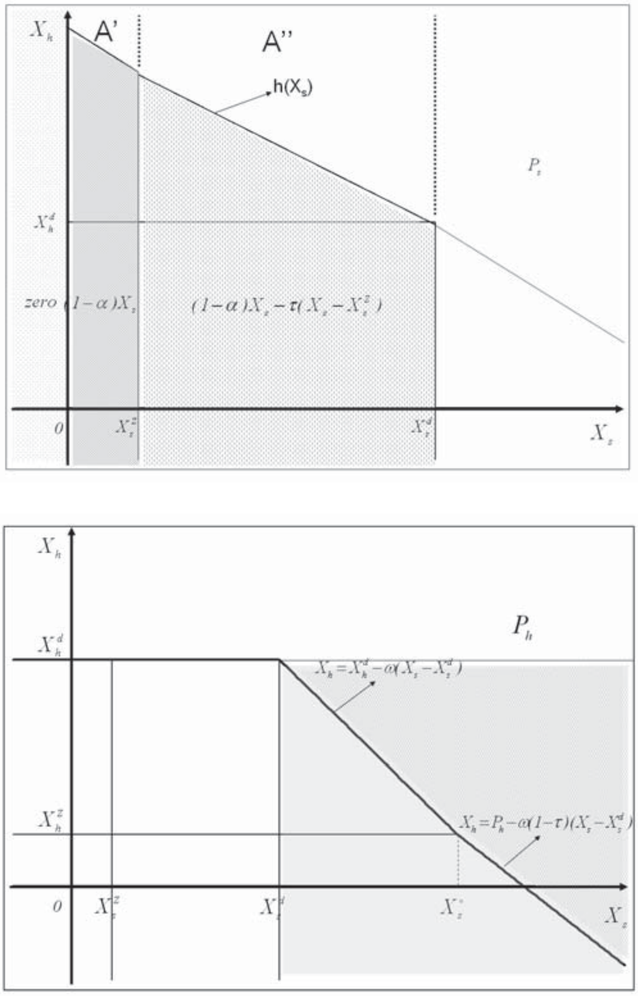

FIGURE 1. This figure represents the payoffs to lenders as a function of both S (horizontal axis) and the H cash flows (vertical axis). The upper panel focusses on S lenders when intercorporate ownership is zero. When X_H is lower than the holding default threshold X_H^d the holding is unable to help its subsidiary. The area of the transfer is $A' \cup A''$ and is bounded by the linear function $h(X_S)$. In A', the subsidiary saves on both default costs and taxes thanks to the transfer, while in A'' it must pay taxes. Lenders get $(1-\alpha)X_S$ in the dark grey zone, and $(1-\alpha)X_S - \tau(X_S - X_S^Z)$ in the pale grey zone. The bottom panel depicts the payoff to H lenders, when there is positive intercorporate ownership. In the shaded area H would default ($X_H < X_H^d$), but dividends - proportional to its ownership share, ω - allow its survival in the shaded area with dots.

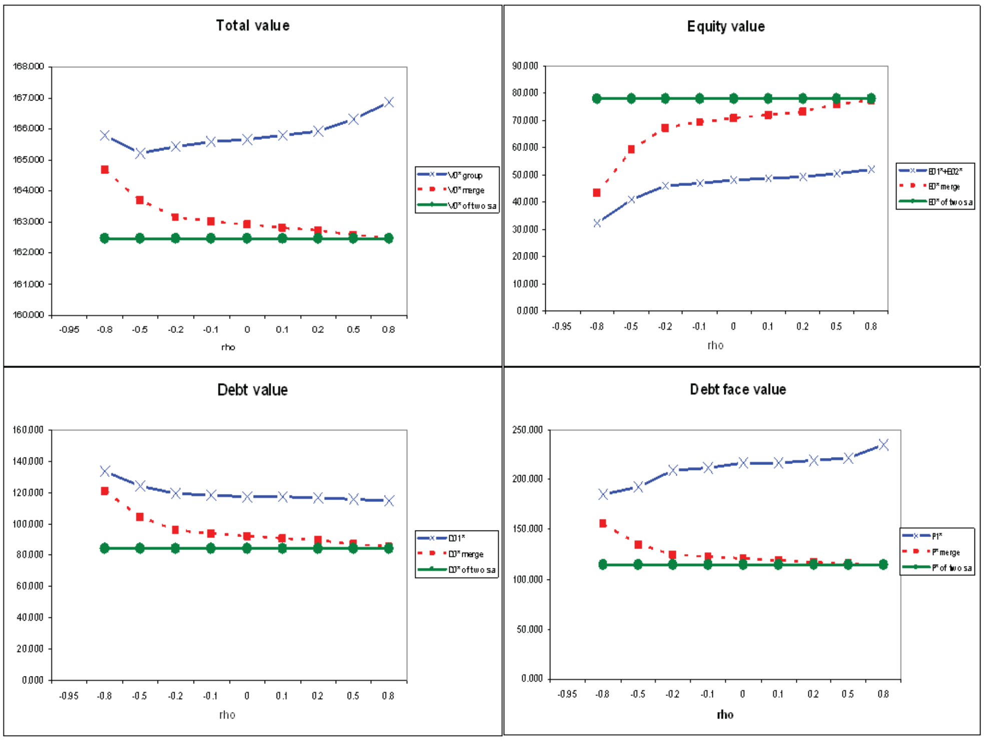

FIGURE 2 .The upper left panel displays the value of an HS (stars), a conglomerate (big and small dots) and two stand alone firms (dotted) as the correlation coefficient between the activities cash glows varies between -0.8 and +0.8. Similarly, the upper right panel displays the value of equity, the lower left panel the market value of debt and the last one the face value of debt.

Table 1: Optimal Capital Structure and Value

	Symbols	Values				
		Stand Alone	Holding	Subsidiary	1/2 HS	1/2 Conglom
Face Value of Debt	P^*	57.10	0	220	110	58.5
Default Threshold	X^{d*}	67.65	0	249.27	-	69.64
No Tax Profit Level	X^{Z*}	14.89	0	102.93	-	13.94
Value of Debt	D_0^*	42.21	0	117.06	58.53	44.56
Leverage Ratio (%)	D_0^*/ν_0^*	51.81	0	99.9	70.26	54.62
Annual Yield Spread of Debt (%)	y	1.23	//	8.45	-	0.6
Value of Equity	E_0^*	39.26	49.46	0.07	24.76	37.01
Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.47	49.46	117.13	83. 29	81.57
Tax Burden	T_0^*	17.70	20.01	5.39	12. 70	17.77
Tax Savings of Leverage	TS_0^*	2.32	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.89	0	8.13	4.07	0.61

Table 2: Asymmetric default costs: capital structure and value

Variables	Symbols	Values					
		S. Alone	S. Alone	Holding	Subsidiary	1/2 HS	1/2 Conglom
Default Costs (%)	α	75	23	75	23		23
Face Value of Debt	P^*	33	57.10	0	220	110	58.5
Default Threshold	X^{d*}	39.247	67.65	0	249.27	-	69.64
No Tax Profit Level	X^{Z*}	8.01	14.89	0	102.93	-	13.94
Value of Debt	D_0^*	24.99	42.21	0	117.06	58.53	44.56
Value of Equity	E_0^*	55.84	39.26	49.46	0.07	24.76	37.01
Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80.83	81.47	49.46	117.13	83.29	81.57
Leverage Ratio (%)	D_0^*/ν_0^*	30.92	51.81	0	99.9	70.26	54.62
Annual Yield Spread of Debt (%)	y	0.7	1.23	//	8.45	-	0.6
Tax Burden	T_0^*	18.76	17.70	20.01	5.39	12.70	17.77
Tax Savings of Leverage	TS_0^*	1.25	2.32	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.46	0.89	0	8.13	4.07	0.61

Table 3: Asymmetric volatilities: capital structure and value, $\sigma_s = 44\%, \sigma_h = 22\%$

Variables	Symbols	Values					
		S.A.($\sigma = 44\%$)	S. A.($\sigma = 22\%$)	Holding	Subsidiary	1/2 HS	1/2 Conglom
Optimal Face Value of Debt	P^*	83	57.10	0	221	110.5	59
Default Threshold	X^{d*}	95.19	67.65	0	248.169	-	34.75
No Tax Profit Level	X^{Z*}	34.25	14.89	0	102.32	-	8.50
Value of Optimal Debt	D_0^*	48.75	42.21	0	107.16	53.57	41. 98
Value of Optimal Equity	E_0^*	36.10	39.26	60.44	3.31	31.88	39. 64
Optimal Levered Firm Value	$\nu_0^*= D_0^*+E_0^*$	84.84	81.47	60.44	110.47	85.45	81. 62
Optimal Leverage Ratio (%)	D_0^*/ν_0^*	57.46	51.81	0	97	62.69	51.4
Tax Burden	T_0^*	16.05	17.70	20.01	7.29	13.65	17.45
Tax Savings of Leverage	TS_0^*	4.66	2.32	0	13.41	6.71	2.60
Expected Default Costs	DC_0^*	2.64	0.89	0	5.39	2.70	1.18
Annual Yield Spread of Debt (%)	y	6.2	1.23	//	10.9	-	2%

Table 4: Asymmetric size: capital structure and value $V_{h0} = 167, V_{s0} = 33$

Variables	Symbols	Values					
		S. Alone(1/3)	S. Alone(5/3)	Holding	Subsidiary	1/2 HS	1/2 Conglom
Optimal Face Value of Debt	P^*	19	95	64	125	94.5	57.5
Default Threshold	X^{d*}	22. 50	112. 54	76.20	142.69	-	67. 81
No Tax Profit Level	X^{Z*}	4.98	24.85	15.21	59.26	-	13. 765
Value of Debt	D_0^*	14.05	70.24	48.79	70.74	59.77	43.24
Value of Equity	E_0^*	13.11	65.54	46.60	0	23. 30	38. 255
Levered Firm Value	$\nu_0^*= D_0^*+E_0^*$	27.16	135.78	95.39	70.74	83. 07	81.49
Leverage Ratio (%)	D_0^*/ν_0^*	51.73	51.73	51.14	100	71.95	53.06
Annual Yield Spread of Debt (%)	y	1.23	1.23	0.6	6.5	-	1.1
Tax Burden	T_0^*	5.90	29.50	30.99	0.36	15.32	17. 78
Tax Savings of Leverage	TS_0^*	0.77	3.85	2.37	6.31	4.34	2.23
Expected Default Costs	DC_0^*	0.30	1.48	0.39	2.27	1.33	0.75

Note: HS principal is calculated as the sum of holding and subsidiary principals.

Table 5: Optimal Capital Structure and Value, different levels of π, $\omega = 0$

	Symbols	Values							
	π			0.5			1		
		Stand Alone	Holding	Subsid	1/2 HS	Holding	Subsid	1/2 HS	1/2 Congl
Default Costs (%)	α	23	23	23	23	23	23	23	23
Face Value of Debt	P^*	57.10	55	69	62	0	220	110	58.5
Default Threshold	X^{d*}	67.65	65.21	81.69	-	0	249.27	-	69.64
No Tax Profit Level	X^{Z*}	14.89	14.16	18.23	-	0	102.93	-	13.94
Value of Debt	D_0^*	42.21	40.84	50.77	45.84	0	117.06	58.53	44.56
Leverage Ratio (%)	D_0^*/ν_0^*	51.81	50.58	61.56	56.13	0	99.9	70.26	54.62
Annual Yield Spread of Debt (%)	y	1.23	1.13	1.33	-	//	8.45	-	0.6
Value of Optimal Equity	E_0^*	39.26	39.91	31.70	35.80	49.46	0.07	24.76	37.01
Levered Firm Value	$\nu_0^*= D_0^*+E_0^*$	81.47	80.75	82.48	81.62	49.46	117.13	83. 29	81.57
Tax Burden	T_0^*	17.70	17.81	17.18	17.49	20.01	5.39	12. 70	17.77
Tax Savings of Leverage	TS_0^*	2.32	2.20	2.83	2.51	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.89	0.77	1.11	0.94	0	8.13	4.07	0.61

Note: both the stand alone and the conglomerate cases are invariant to π. Holding and subsidiary figures coincide with stand alone figures for $\pi = 0$.HS figures obtain by summing up the holding and the subsidiary figures.

Table 6: Capital structure and value with intercorporate dividends, $\pi = 0.5$

Variables	Symbols	ρ								
		-0.8			-0.2			0		
Omega	ω	0	0.5	1	0	0.5	1	0	0.5	1
Face Value of S. Debt	P^*_{0S}	74	59	48	72	62	52	71	63	53
Face Value of H Debt	P^*_{0H}	56	82	119	55	71	88	58	70	86
Value of S Debt	D^*_{0S}	54.90	44.69	36.69	53.12	46.40	39.37	52.23	46.89	39.96
Value of H Debt	D^*_{0H}	41.49	63.39	92.37	40.84	53.81	67.13	42.79	52.62	64.93
Levered HS Value	ν^*_{0HS}	163.34	164.13	165.03	163.29	163.64	163.97	163.27	163.55	163.82
HS Leverage Ratio (%)	D^*_{0HS}/ν^*_{0HS}	59.00	65.85	78.21	57.54	61.23	64.95	58.20	60.84	64.02
S. Leverage. Ratio (%)	D^*_{0S}/ν^*_{0S}	65.71	54.14	44.80	64.00	56.31	48.06	63.14	56.99	48.83
H Equity Value	E^*_H	38.31	37.86	35.96	39.45	45.44	57.47	37.76	46.35	58.92
S Equity Value .	E^*_S	28.65	37.12	45.22	29.87	35.99	42.53	30.49	35.85	41.88
HS Equity Value	E^*_{HS}	66.96	74.98	81.18	68.32	81.43	100.00	68.25	82.20	90.80
Default Threshold	X^d_g	154.10	168.02	199.26	150.49	158.05	166.62	152.75	157.88	165.22
No Tax Profit Level	X^Z_{HS}	33.62	32.92	37.94	33.04	33.04	33.50	33.98	34.82	34.11
Tax Burden	T^*_0	34.80	34.91	34.14	34.89	34.92	34.82	34.85	34.82	34.72
H Tax Savings of Lever	TS^*_{0H}	2.26	2.89	4.13	2.20	2.67	3.24	2.36	2.70	3.27
S. Tax Savings of Lever	TS^*_{0S}	2.97	2.23	1.76	2.93	2.43	1.97	2.92	2.50	2.03
H Exp. Default Costs	DC^*_{0H}	0.82	0.51	0.68	0.77	0.78	0.86	0.93	0.88	1.04
S. Exp. Default Costs	DC^*_{0S}	1.13	0.51	0.27	1.16	0.73	0.42	1.18	0.82	0.49
H Yield Spread (%)	y_H	1.18	0.28	0.20	1.13	0.70	0.56	1.27	0.87	0.78
S Yield Spread (%)	y_S	1.15	0.71	0.52	1.27	1.97	0.72	1.33	1.08	0.81

Table 6: Capital structure and value with intercorporate dividends, $\pi = 0.5$

Variables	Symbols	ρ					
		0.2			0.8		
Omega	ω	0	0.5	1	0	0.5	1
Face Value of S. Debt	P^*_{0S}	69	63	52	68	58	49
Face Value of H Debt	P^*_{0H}	55	68	83	51	66	89
Value of S. Debt	D^*_{0S}	50.77	46.73	39.17	38.15	42.98	36.81
Value of H Debt	D^*_{0H}	40.84	50.85	62.26	49.53	48.45	64.02
Levered HS Value	ν^*_{0HS}	163.23	163.48	163.71	163.10	163.22	163.55
HS Leverage. Ratio (%)	D^*_{0g}/ν^*_{0HS}	56.13	59.69	61.96	53.75	56.01	61.65
S Leverage. Ratio (%)	D^*_{0S}/ν^*_{0S}	61.56	56.89	47.93	46.58	52.65	45.20
H Equity Value	E^*_H	39.91	48.19	62.30	43.04	52.46	62.72
S Equity Value	E^*_S	31.70	35.41	42.56	32.38	38.65	44.62
HS Equity Value	E^*_{HS}	71.61	83.60	104.86	75.42	91.11	107.34
Default Threshold	X^d_{HS}	146.90	155.40	160.36	140.92	146.86	163.21
No Tax Profit Level	X^Z_{HS}	32.38	33.42	33.57	31.32	32.57	37.17
Tax Burden	T^*_0	34.99	34.83	34.81	35.15	34.96	34.25
H Tax Savings of Lever	TS^*_{0H}	2.20	2.66	3.22	1.99	2.73	3.87
S. Tax Savings of Lever	TS^*_{0S}	2.83	2.53	2.00	2.86	2.34	1.90
H Exp. Default Costs	DC^*_{0H}	0.77	0.91	1.09	0.60	1.10	1.86
S. Exp. Default Costs	DC^*_{0S}	1.11	0.87	0.49	1.25	0.83	0.51
H Yield Spread (%)	y_H	1.13	0.98	0.92	negative	1.38	1.81
S Yield Spread (%)	y_S	1.33	1.16	0.83	7.25	1.18	0.89

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