Information Sales and Insider Trading with Long-lived Information *

Giovanni Cespa †

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Abstract

Fundamental information resembles in many respects a durable good. Hence, the effects of its incorporation into stock prices depend on who is the agent controlling its flow. Similarly to a durable goods monopolist, a monopolistic analyst selling information intertemporally competes against herself. This forces her to partially relinquish control over the information flow to traders. Conversely, an insider solves the intertemporal competition problem through vertical integration, thus exerting a tighter control over the flow of information. Comparing market patterns I show that a dynamic market where information is provided by an analyst is *thicker* and *more informative* than one where an insider trades.

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†CSEF, Università di Salerno and CEPR. E-mail: `gcespa@unisa.it`.

1 Introduction

Organized stock markets facilitate the exchange of assets among traders hence allowing a firm's fundamental information to be impounded into prices. There are mainly two ways by which this occurs: either traders acquire information from a specialized provider (e.g., an *analyst*), or they obtain it thanks to a particular relationship they have with the firm (i.e. they are *insiders*). Far from being irrelevant, the way through which information is gathered to the market dramatically affects the characteristics of stock prices. This paper shows that the dynamic properties of a market closely depend on *who* is the agent exerting *control* over the flow of information.

Fundamental information resembles in many respects a durable good. Indeed, a trader holding a signal on a firm's pay-off can use it during several trading rounds. Also, as most durable goods, the value of such a signal depreciates as a result of its use, due to price information transmission. However, differently from a durable good, information *cannot* be rented. Therefore, the ability of its provider (be it an analyst or an insider) to overcome the traditional self-competition problem (see Bulow 1982, 1986, Coase 1972, and Waldman 1993) directly impacts the properties of the underlying asset market.

Consider an analyst selling information. As the durable goods monopolist – who in order to extract consumer surplus may artificially shorten the life of the product she sells – the analyst, after distributing a signal of a given quality is tempted to increase the quality of the signals she sells in the periods to come. In particular, in a two-period market, I show that once the first signal has been sold to competitive traders, the analyst distributes a new signal which, in order to be palatable to potential buyers, must render partially "obsolete" the signal sold in the first period. The seller thus *impoverishes* the quality of the first period information she sells (so to reduce the level of its durability and weaken future self-competition), while consistently *enhancing* the one sold in the second period (so to force the first period signal obsolescence). This, in turn, attenuates the severity of the market makers' adverse selection problem along the two periods, implying a pattern of *increasing* market depth.

Consider now the case of an insider. Being the end-user of the information he possesses enables him to *choose* the rate at which the market learns it. In particular, as he directly exploits his informational advantage, he avoids the effect of intertemporal self-competition, fully internalizes the negative effect of aggressive speculation, and trades less intensely.

The analyst thus acts in a way that is much akin to the durable goods monopolist that, being forced to sell rather than rent, handles her intertemporal self-competition problem strategically choosing the quality of the goods she markets; the insider, on the other hand, attenuates competition through vertical integration: the producer and the final user of the information good, in his case, coincide. [1] Comparing market patterns, the insider's tighter control over the information flow makes the market in the second period *thinner* and prices *less* informative than those that obtain in the analyst's market. In a dynamic market, therefore, trading by an insider *worsens* stock price accuracy and *impairs* market depth compared to a market where information is provided by an analyst.

Several papers analyze dynamic trading in markets with asymmetric information and assess the relevance of information flows in determining the behavior of market patterns. Yet, in all of these works the information flow is either *exogenously given*, as if traders were born endowed with their private signals, or *determined* by traders' endogenous decisions to acquire signals of a given constant precision. [2] However, as information is a valuable good, its distribution is likely to depend on the decisions of agents who, given traders' time-varying desire to become informed, optimally set the quality of the signals they release. If this is the case, then the dynamic properties of a market should be analyzed by explicitly modeling such decisions.

In this paper I take a first step at addressing this issue by studying a dynamic asset market with risk-averse, competitive agents, in which *control over the information flow* is exerted by a monopolistic analyst selling *long-lived information*. In every period the analyst optimally chooses the quality of the information she distributes to the agents in the asset market. Within this framework, I characterize the optimal solution to the analyst's intertemporal profit maximization problem and investigate how this affects agents' trading behavior and the dynamic properties of the asset market. This has an independent interest since, to the best of my knowledge, this is the first paper that provides such an analysis within a discrete-time, dynamic rational expectations

[1] Alternatively, it may be useful to think of the insider as of the monopolistic producer that *rents* instead of selling. Indeed, the monopolistic renter by keeping the ownership of the goods she markets, fully internalizes the negative effect of overproduction and thus cuts back on the quantities she releases; similarly, the insider, by holding on to his informational advantage, directly bears the negative effects of an excessively aggressive behavior, and speculates less intensely. Other authors have adopted the durable goods monopolist paradigm to explore traditional finance problems (see e.g. DeMarzo and Urošević 2003).

[2] Examples of the first type include He and Wang (1995), Vives (1995a, 1995b) and Cespa (2002); examples of the second type include Admati and Pfleiderer (1988b), Holden and Subrahmanyam (1996), and Foster and Viswanathan (1996).

equilibrium model. In a 2-period setup, I show that optimality on the side of the analyst calls for an increasing pattern of signal quality. This, in turn, implies an increasing pattern of market depth and a rapid devaluation of the information sold. [3]

The paper contributes to the literature on insider trading that, starting with the pioneering work of Kyle (1985), has devoted attention to gauge the impact of trading by a strategic agent on price efficiency. Leland (1992) shows that insider trading accelerates the resolution of fundamental uncertainty. Fishman and Hagerty (1992), in a model where the insider is not the only agent possessing fundamental information, argue that the presence of a better informed insider may discourage costly research from market professionals and, under some parameter configurations, lead to a less informative stock price. [4] The present work complements this argument by questioning – in the case of long-lived information – whether trading by an insider allows information impounding into asset prices in the most "effective" way.

The paper also has important empirical and policy implications. First, it predicts that insiders should rather base their trading activity on long-lived information. Indeed, as argued above, thanks to his superior ability to control the flow of such information, an insider is likely to face a lower number of (potentially) competing agents, and enjoy the possibility of slowly exploiting his informational advantage. This suggests that insider trading should be based on information that can be repeatedly exploited before it becomes publicly known. [5]

Second, the paper strengthens the case against insider trading, showing that in contrast to what most of the literature on the subject traditionally maintains (see e.g., Carlton and Fischel 1983, Leland 1992, and Manne 1966), in a dynamic context insider trading, far from *accelerating* the resolution of uncertainty, may actually *slow down* information impounding into prices, yielding a thinner market. This adds to the standard arguments calling for strict insider trading regulation. Indeed, the durable

[3]Numerical simulations show that the result carries over to the general $N > 2$-period market.

[4]Other authors have emphasized the effects that insider trading has on the welfare of market participants (see e.g., Bhattacharya and Nicodano 2001 and Medrano and Vives 2004).

[5]The evidence on insider trading patterns provides some support for this prediction. Surveying the empirical literature on insider trading, Huddart, Ke, and Petroni (2003) observe that "... insiders know of price-relevant events months and even years before public disclosure of the event" and that "... abnormal trade by insiders generally is found to concentrate in the two quarters prior to the disclosure." Furthermore, in their study of insider trading patterns in the Milan stock exchange in the years from 1991 to 1999, Bagliano, Favero, and Nicodano (2001) conclude that insider trading episodes started taking place on average 39.3 days before the resolution of the relevant uncertainty. Finally, Cornell and Sirri (1992) in their detailed analysis of the Anheuser-Busch's 1982 tender offer for Campbell Taggart, document how insiders repeatedly traded during a month before information about the merger was made public.

goods monopolist, by *renting* manages to keep up the price of the good he supplies, extracting a higher surplus from consumers. Similarly, an insider by exerting a tighter control over the information flow, manages to keep up market thinness, *extracting* higher rents from liquidity traders. [6] A legislation designed to effectively curb insider trading may thus facilitate the transmission of fundamental information into prices. This, in turn, may eventually enhance the efficiency of the market and reduce the market impact of trades, implying lower trading costs and improving market liquidity.

Finally, this work also contributes to the literature on financial markets information sales. This has mainly focused on the *static* problem faced by a monopolistic information provider selling signals either directly, as in the case of an investment advisor, or indirectly, as in the case of a mutual fund (see Admati and Pfleiderer 1986, 1988a, and 1990). Fishman and Hagerty (1995) show that a strategic agent can use information sales as a commitment device to trade aggressively against a symmetrically informed peer. Allen (1990) shows that the credibility problem faced by an information seller needing to prove his access to superior information may leave room for financial intermediaries to appropriate part of the seller's information value. Simonov (1999) studies the effect of competition among analysts in the Admati and Pfleiderer (1986)'s context, showing that externalities in information transmission may lead to counterintuitive results. [7] Little attention has been devoted to study the *dynamics* of the information sales problem. A notable exception is represented by Naik (1997) who studies the single-shot problem of an analyst selling a flow of information in a continuous time model. However, as in Naik the analyst's decision is made "once-and-for-all," no intertemporal competition problem arises there.

The paper is organized as follows. In the next section I present the static benchmark where I review the results of Admati and Pfleiderer (1986) and prove that in a static setup a market where information is sold by a monopolistic analyst and one where an insider trades generate the *same* patterns of depth and price informativeness. In section 3 I present the 2-period model with long-lived information and in section 4 I study the analyst's optimal sales policy. In section 5 I compare patterns of depth and price informativeness across the two markets and analyze numerically the properties of the general $N > 2$-period model. Finally, in section 6 I discuss the effects of market segmentation and public announcements on the analyst's control of

[6] Incidentally, this argument provides a formalization to Carlton and Fischel (1983)'s intuition that an insider is better able to control the flow of information generated within the firm. Furthermore, it shows that such control comes at the cost of a thinner and less efficient market.

[7] Recently, García and Vanden (2005) analyze competition among mutual funds.

the information flow. A final section contains concluding remarks while most of the proofs are relegated to the appendix.

2 The Static Benchmark

Consider a market where a single risky asset with liquidation value $v \sim N(\bar{v}, \tau_v^{-1})$ and a riskless asset with unitary return are traded. In this market competitive speculators *or* an insider trade along with noise traders against a competitive, risk-neutral market making sector.

In the former case there is a continuum of informed traders in the interval $[0,1]$. Every informed trader i (potentially) receives a signal $s_i = v + \epsilon_i$, where $\epsilon_i \sim N(0, \tau_\epsilon^{-1})$, v and ϵ_i are independent and errors are also independent across agents. Let the informed traders' preferences over final wealth W_i be represented by a CARA utility function $U(W_i) = -\exp\{-W_i/\gamma\}$, where $\gamma > 0$ denotes the coefficient of constant absolute risk tolerance and $W_i = X_i(v - p)$ indicates the profit of buying X_i units of the asset at price p.

In the market with the insider, a risk-neutral, strategic agent holds a perfect signal about the liquidation value v and trades a quantity X_I to maximize his expected final wealth.

In both markets noise traders submit a random demand u (independent of all other random variables in the model), with $u \sim N(0, \tau_u^{-1})$. Finally, assume that in the competitive market, given v, the average signal $\int_0^1 s_i di$ equals v almost surely (i.e. errors cancel out in the aggregate: $\int_0^1 \epsilon_i di = 0$).

2.1 The Equilibrium in the Competitive Market

In this section I present a version of the traditional large-market noisy rational expectations equilibrium market, as studied by Admati (1985), Grossman and Stiglitz (1980), Hellwig (1980), and Vives (1995a).

To find the equilibrium in this market, assume that each informed trader submits a price contingent order $X_i(s_i, p)$ specifying the desired position in the risky asset for any price p and restrict attention to linear equilibria where $X_i(s_i, p) = as_i - bp$. Competitive, risk-neutral market makers observe the aggregate order flow $L(p) = \int_0^1 X_i(s_i, p)di + u = av + u - bp$ and set a semi-strong efficient price. If we let $z_C = av + u$ denote the informational content of the order flow, then the following result applies:

Proposition 1 *In the competitive market there exists a unique linear equilibrium. It is symmetric and given by* $X_i(s_i, p) = a(s_i - p)$ *and* $p = E[v|z_C] = \lambda_C z_C + (1 - \lambda_C a)\bar{v}$, *where* $a = \gamma\tau_\epsilon$, $\lambda_C = a\tau_u/\tau_C$ *and* $\tau_C = (\text{Var}[v|z_C])^{-1} = \tau_v + a^2\tau_u$.

Proof. See Admati (1985) and Vives (1995a). QED

Intuitively, an informed speculator's trading aggressiveness a increases in the precision of his private signal and in the risk tolerance coefficient. Market makers' reaction to the presence of informed speculators $\lambda_C = a\tau_u/\tau$ is captured by the OLS regression coefficient of the unknown payoff value on the order flow. As is common in this literature, λ_C measures the reciprocal of market depth (see e.g., Kyle 1985 and Vives 1995a), and its value determines the extent of noise traders' expected losses: $E[u(v - p)] = -\lambda_C\tau_u^{-1}$. The informativeness of the equilibrium price is measured by the reciprocal of the payoff conditional variance given the order flow: $(\text{Var}[v|z_C])^{-1} = \tau_C$. The higher τ_C, the smaller the uncertainty on the *true* payoff value once the order-flow has been observed.

2.2 The Equilibrium in the Strategic Market

The linear equilibrium of the strategic market is given by the well known result due to Kyle (1985). Assume the insider submits a linear market order $X_I(v) = \alpha + \beta v$ to the market making sector indicating the desired position in the risky asset. [8] Upon observing the aggregate order flow $z_I = x_I + u$, market makers set the semi-strong efficient equilibrium price. Restricting attention to linear equilibria, the following result holds:

Proposition 2 *In the strategic market there exists a unique linear equilibrium given by* $X_I(v) = \beta(v - \bar{v})$ *and* $p = E[v|z_I] = \lambda_I z_I + \bar{v}$, *where* $\beta = \sqrt{\tau_v/\tau_u}$, $\lambda_I = (1/2)\sqrt{\tau_u/\tau_v}$, *and* $\tau_I = (\text{Var}[v|z_I])^{-1} = 2\tau_v$.

Proof. See Kyle (1985). QED

Owing to *camouflage* opportunities, the insider's aggressiveness β is larger (smaller), the more (less) dispersed is the distribution of noise traders' demand. Conversely, market makers' reaction to the presence of the insider (λ_I) is harsher (softer) the more concentrated is the demand of noise traders. A *noisier* market thus spurs a

[8] As shown by Rochet and Vila (1994), assuming that the insider submits a price contingent order does not change the equilibrium result.

more *aggressive* insider's trading; owing to the insider's risk-neutrality, these two countervailing effects exactly cancel out. As a consequence, price informativeness does not depend on τ_u and is given by $\tau_I = 2\tau_v$. [9]

2.3 The Information Market

Suppose now as in Admati and Pfleiderer (1986) that the private signal each trader observes in the competitive asset market is sold by a monopolistic *buy-side* analyst who has a *perfect* knowledge of the asset pay-off realization. [10] Furthermore, assume that *(i)* the analyst *does not* trade on the information she sells, and *(ii)* she *truthfully* provides the information she promises to traders. The last assumption clearly simplifies the analysis. Indeed, recent research has outlined the tendency displayed by sell-side analysts to provide biased information. However, differently from their sell-side counterparts, buy-side analysts *privately* provide investment advice services to their clients (mutual funds and pension funds). Therefore, absent the need to preserve privileged access to companies' information, they are unlikely to feel the pressure towards issuing *public* investment recommendations that please firms' managers. Furthermore, their firms do not perform investment banking or brokerage services. Hence, their research output is likely to be less biased than the one provided by sell-side analysts. [11]

The error affecting each trader's signal can be thought as an interpretation mistake that the trader commits when processing the information he receives (see Admati and Pfleiderer 1986). An analyst providing vague predictions embeds a low precision τ_ϵ in the signal she sells. The lower (higher) is τ_ϵ, the more (less) vague is the analyst's information release, and the more (less) is each trader's information likely to be incorrectly interpreted. Given that the analyst holds all the bargaining power, in order to receive information each trader i pays a price that makes him indifferent

[9] Subrahmanyam (1991) shows that if the insider is risk-averse, this result does not hold.

[10] Admati and Pfleiderer (1986) also consider the case in which the analyst is not perfectly informed. While the static case can be handled under such assumption, the dynamic extension I consider in section 4 quickly becomes intractable.

[11] Sell-side analysts working at investment banks and brokerage firms are likely to face a conflict of interests mainly for three reasons. First, they may tip investors towards buying stock of a current or potential investment banking client. Also, they may provide over optimistic research results to boost brokerage commissions. Finally, as their access to relevant information often depends on contacts with firms' insiders, they may be unwilling to provide negative information on a firm in order not to compromise such contacts. See Cheng, Liu, and Qian (2004) and Groysberg, Healy, Chapman, and Gui (2005).

between observing or not the signal s_i. Denoting by ϕ such a price

$$E[E[U(W_i - \phi)|\{s_i, p\}]] = E[E[U(W_i)|p]].$$

Standard normal calculations show that

$$\phi = \frac{\gamma}{2} \ln \frac{\tau_{iC}}{\tau_C}, \tag{2.1}$$

where $\tau_{iC} = (\mathrm{Var}[v|s_i, p])^{-1} = \tau_C + \tau_\epsilon$. Thus, each trader pays a price which is a monotone transformation of the informational advantage he acquires over market makers by observing the signal. The analyst faces a trade-off: on the one hand she would like to make each trader's informational advantage as large as possible, increasing τ_ϵ and thus τ_{iC}. On the other hand, as each trader's speculative aggressiveness is directly related to his signal's precision, increasing τ_ϵ enhances price efficiency (τ_C), and thus reduces the signal's value. Maximizing (2.1) with respect to τ_ϵ the analyst finds the precision that optimally balances the above offsetting effects:

$$\hat{\tau}_\epsilon = \frac{1}{\gamma}\sqrt{\frac{\tau_v}{\tau_u}}. \tag{2.2}$$

Hence, the analyst sells a signal that is more (less) informative the higher (lower) is the unconditional noise-to-signal ratio and the more risk-averse the traders are – poorer ex-ante information and/or noisier markets allow the analyst to release less vague predictions.

Note that $\hat{\tau}_\epsilon$ minimizes λ_C^{-1}. The intuition is straightforward: the analyst seeks to extract the maximum aggregate surplus from informed traders. Such surplus, in turn, increases in the informational advantage traders have vis-à-vis market makers. When such advantage is maximal, market depth is at its minimum, and traders are also willing to pay the highest price.

Furthermore, according to (2.2), the equilibrium market parameters replicate those obtained in the strategic market of the previous section. Indeed, the aggregate trading aggressiveness $a = \int_0^1 a\, di = \sqrt{\tau_v/\tau_u}$; thus, price informativeness $\tau_C = \tau_v + a^2\tau_u = 2\tau_v = \tau_I$, and the reciprocal of market depth $\lambda_C = (1/2)\sqrt{\tau_u/\tau_v} = \lambda_I$. Summarizing:

Proposition 3 *In the static information market, the analyst sells a signal with precision $\hat{\tau}_\epsilon = (1/\gamma)\sqrt{\tau_v/\tau_u}$; such information quality minimizes market depth replicating the equilibrium properties of an asset market with a single, risk-neutral insider.*

The equivalence between the analyst's and the insider's problems can be best understood by rewriting (2.1) as follows:

$$\phi = \frac{\gamma}{2} \ln \left(1 + \frac{1}{\gamma} \frac{\lambda_C}{\tau_u} \right).$$

The analyst who wishes to maximize her expected profits chooses a signal quality $\hat{\tau}_\epsilon$ such that the stock market is as thin as possible. In this way she maximizes the aggregate rents she extracts from competitive traders which, given the "zero-sum" nature of the market game, are just the flip side of the coin of noise traders' expected losses. However, this is the same result obtained in a market with a risk-neutral insider that *in equilibrium* sees his ex-ante profits (i.e. the expected losses of noise traders) maximized when the impact of his trades (as measured by λ_I) is as large as possible. [12] Therefore, in a static information market, the way in which a perfectly informed agent conveys fundamental information to the market *does not matter*. [13]

3 A Dynamic Asset Market with Long Lived Information

Consider now a 2-period extension of the market analyzed in the previous section. In particular, assume that assets are traded for two periods and that in period 3 the risky asset is liquidated and the value v collected (thus, $p_3 = v$).

In the competitive market, every informed trader i in each period n (potentially) receives a private signal $s_{in} = v + \epsilon_{in}$, where $\epsilon_{in} \sim N(0, \tau_{\epsilon_n}^{-1})$, v and ϵ_{in} are independent, and errors are also independent across agents and periods (therefore private information is "long lived"). Assume that a trader i's preferences over final wealth W_{i3} are represented by a CARA utility function $U(W_{i3}) = -\exp\{-W_{i3}/\gamma\}$, where $W_{i3} = \sum_{n=1}^{3} \pi_{in} = \sum_{n=1}^{3} X_{in}(p_{n+1} - p_n)$ indicates the profit of buying X_{in} units of the asset at price p_n.

[12] This is immediate as in any linear equilibrium noise traders' ex-ante expected losses are given by $E[u(v-p)] = -\lambda_I \tau_u^{-1}$, and, owing to the semi-strong efficiency of the market, when the insider trades with aggressiveness β, $\lambda_I = \beta\tau_u/(\beta^2\tau_u + \tau_v)$. The insider, thus, sees his equilibrium ex-ante profits (i.e. the losses of noise traders) maximized when choosing β such that λ_I is as large as possible.

[13] This provides a different interpretation to Admati and Pfleiderer's (1986) result showing the superiority of "personalized" information allocations over "newsletters." Indeed, it is only by selling diverse signals that the information provider exerts the same control over the information leakage obtained by an insider.

In the strategic market, before the first period, the insider observes v and then chooses X_{In}, in every period n to maximize his expected final wealth.

In both markets noise traders demand follows an independently and identically normally distributed process $\{u_n\}_{n=1}^{2}$ (independent of all other random variables in the model), with $u_n \sim N(0, \tau_u^{-1})$ in every period n. Finally, assume that in the competitive market given v and for every n, the average signal $\int_0^1 s_{in} di$ equals almost surely v (i.e. errors cancel out in the aggregate: $\int_0^1 \epsilon_{in} di = 0$).

3.1 The Equilibrium in the Dynamic Competitive Market

Let us indicate with s_i^n and p^n respectively, the sequence of private signals and prices a trader has observed up to period n. In every period $n = 1, 2$ an informed trader submits a price contingent order $X_{in}(s_i^n, p^{n-1}, \cdot)$ indicating the position desired in the risky asset at every price p_n. Restricting attention to linear equilibria it is possible to show that the strategy of an agent i in period n depends on $\tilde{s}_{in} = (\sum_{t=1}^{n} \tau_{\epsilon t})^{-1}(\sum_{t=1}^{n} \tau_{\epsilon t} s_{it})$ and on the sequence of equilibrium prices: $X_{in}(\tilde{s}_{in}, p^n) = a_n \tilde{s}_{in} - \varphi_n(p^n)$, where $\varphi_n(p^n)$ is a linear function of the sequence p^n. Market makers in every period observe the net aggregate order flow: $L_n(\cdot) = \int_0^1 X_{in} di - \int_0^1 X_{in-1} di + u_n = z_{Cn} + \varphi_n(p^n) - \varphi_{n-1}(p^{n-1})$, where $z_{Cn} = \Delta a_n v + u_n$ indicates the informational content of period n net order flow, and set a semi-strong efficient equilibrium price conditional on past and current information $p_n = E[v | z_C^{n-1}, z_{Cn}]$. [14]

Proposition 4 *In the 2-period competitive market, there exists a unique linear equilibrium. The equilibrium is symmetric and given by* $X_{in}(s_i^n, p^n) = a_n(\tilde{s}_{in} - p_n)$, *and* $p_n = \lambda_{Cn} z_{Cn} + (1 - \lambda_{Cn} \Delta a_n) p_{n-1}$, $n = 1, 2$, *where* $a_n = \gamma(\sum_{t=1}^{n} \tau_{\epsilon t})$, $\tilde{s}_{in} = (\sum_{t=1}^{n} \tau_{\epsilon t})^{-1}(\sum_{t=1}^{n} \tau_{\epsilon t} s_{it})$, $z_{Cn} = \Delta a_n v + u_n$, $\lambda_{Cn} = \Delta a_n \tau_u / \tau_n$, *and* $\tau_{Cn} = (Var[v|p^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_n)^2$.

Proof. See Vives (1995a). QED

In every period n an informed trader speculates according to the sum of the precisions of his private signals weighted by the risk tolerance coefficient; market makers observe the (net) aggregate order flow and set the semi-strong efficient price p_n attributing weight $\lambda_{Cn} = \Delta a_n \tau_u / \tau_{Cn}$ to its informational content $z_{Cn} = \Delta a_n v + u_n$.

[14] It can be shown that in every linear equilibrium, the sequences p^n and z_C^n are observationally equivalent (see Vives, 1995a).

The information impounded in the equilibrium price is thus reflected in the public precision $\tau_{Cn} = (\mathrm{Var}[v|z_C^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_n)^2$.

3.2 The Equilibrium in the Dynamic Strategic Market

Assume that in every period n the insider submits a linear market order $X_{In}(v) = \alpha_n + \beta_n v$ indicating the position desired in the risky asset. Market makers observe the (sequence of) aggregate order flow(s) $z_{In} = x_{In} + u_n$ (z_I^n) and set the semi-strong efficient equilibrium price $p_n = E[v|z_I^{n-1}, z_{In}]$. Restricting attention to linear equilibria the following result holds:

Proposition 5 *In the 2-period strategic market there exists a unique linear equilibrium given by* $X_{In}(v, p_{n-1}) = \beta_n(v - p_{n-1})$ *and* $p_n = \lambda_{In} z_{In} + p_{n-1}$, $n = 1, 2$, *where* $z_{In} = x_{In} + u_n$

$$\beta_1 = \frac{2K-1}{\lambda_{I1}(4K-1)}, \qquad \beta_2 = \frac{1}{2\lambda_{I2}},$$

$$\lambda_{I1} = \frac{1}{4K-1}\sqrt{\frac{2\tau_u K(2K-1)}{\tau_v}}, \qquad \lambda_{I2} = \frac{1}{2}\sqrt{\frac{\tau_u}{\tau_{I1}}},$$

$\tau_{I1} = (Var[v|z_{I1}])^{-1} = (4K-1)\tau_v/2K$, $\tau_{I2} = (Var[v|z_{I1}, z_{I2}])^{-1} = 2\tau_{I1}$ *and*

$$\frac{\lambda_{I2}}{\lambda_{I1}} \equiv K = \frac{1}{6}\left\{1 + 2\sqrt{7}\cos\left(\frac{1}{3}\left(\pi - \arctan\left(3\sqrt{3}\right)\right)\right)\right\} \approx 0.901.$$

Proof. See Huddart, Hughes, and Levine (2001). QED

As more information is impounded in the price, the severity of the adverse selection problem decreases, and market makers set a less steep price schedule: $\lambda_{I2} < \lambda_{I1}$. As a consequence, profit opportunities decline, and the insider turns to a more aggressive trading behavior: $\beta_2 > \beta_1$.

4 A Dynamic Market for Information

In this section I use the results of section 3.1 to determine the optimal policy of the information provider. This is done in two steps: first, I obtain a trader i's value for the sequence of signals $\{s_{i1}, s_{i2}\}$; second, I solve for the analyst's optimal information sales policy.

4.1 The Value of Long Lived Information

As done in section 2, assume now that the signal each trader receives in every period n is sold by a monopolistic analyst who has perfect knowledge of the asset pay-off realization v, and does not trade on such information. Furthermore, assume the analyst truthfully provides the information she promises to each trader. As in every period n she extracts all the surplus, the analyst sets the price ϕ_n for the signal s_{in} equal to value that leaves the trader indifferent between acquiring or not the signal:

Proposition 6 *In the 2-period information market, the maximum price a trader i is willing to pay to buy a signal s_{in} in each period $n = 1, 2$ is given by ϕ_1, ϕ_2, where*

$$\begin{aligned}\phi_1 &= \phi(s_{i1}||p_1) + \phi(s_{i1}||p_1, p_2) \\ &= \frac{\gamma}{2} \ln \frac{\tau_{iC1}}{\tau_{C1}} + \frac{\gamma}{2} \ln \frac{\tau_{C2} + \tau_{\epsilon_1}}{\tau_{C2}},\end{aligned} \tag{4.3}$$

$$\phi_2 = \frac{\gamma}{2} \ln \frac{\tau_{iC2}}{\tau_{C2} + \tau_{\epsilon_1}}, \tag{4.4}$$

where $\tau_{iCn} = (Var[v|s_i^n, p^n])^{-1} = \tau_{Cn} + \sum_{t=1}^{n} \tau_{\epsilon_t}$.

Proof. See the appendix. QED

The first period signal price is the sum of two components capturing the trader's informational advantage vis-à-vis market makers that the signal allows in the first *and* in the second period. The intuition is as follows. In period 1 a trader buys s_{i1} and establishes a position in the risky asset $X_{i1}(s_{i1}, p_1)$. The expected utility of his final wealth then depends on the position $X_{i1}(\cdot)$ (times the return from buying/selling the asset at p_1 and liquidating it at v) *plus* the change in the first period position he will eventually make at time two (times the return from changing the position at p_2 and liquidating such change at v). However, the latter component depends on the change in price which, in turn, depends on the arrival of private information in period two. As the trader cannot anticipate such "new" information in period one, his expected utility from acquiring s_{i1} depends only on the informational advantage the signal gives him in that period: [15]

$$E\left[U\left(X_{i1}(s_{i1}, p_1)(v - p_1) + \Delta X_{i2}\left(s_i^2, p^2\right)(v - p_2)\right)\right] = -\left(\frac{\tau_{C1}}{\tau_{iC1}}\right)^{1/2}.$$

[15] Indeed, absent a price change that informed traders *cannot* anticipate in period one, it would be suboptimal to establish a position X_{i1} and already plan to change it in period two.

The price the trader is willing to pay to *use* s_{i1} in period one is thus the one that makes him indifferent between having and not having the signal:

$$\phi(s_{i1}||p_1) = \frac{\gamma}{2} \ln \frac{\tau_{iC1}}{\tau_{C1}}.$$

The signal s_{i1} has however an *added* value, as it allows the trader to keep an informational advantage in the second period as well when the analyst sells the second signal (without having to buy a second signal). Such added value is given by the price the trader would be ready to pay in order to have s_{i1} and observe $\{p_1, p_2\}$:

$$\phi(s_{i1}||p_1, p_2) = \frac{\gamma}{2} \ln \frac{\tau_{C2} + \tau_{\epsilon_1}}{\tau_{C2}}.$$

In the second period, as a signal has already been sold, the trader compares the precision of the forecast she obtains from buying one additional signal to the one she gets from *not* buying it and using *both* period's prices *and* the first period signal.

Remark 1 The solution proposed in proposition 6 generalizes Admati and Pfleiderer (1986). In particular, if $\tau_{\epsilon_2} = 0$, then $\phi_1 = \phi$ as no new information is released by the analyst in period two, and thus the first period signal has no "added" value.

4.2 The Analyst's Optimal Policy

As argued in section 2.3, in order to make information sales profitable, the analyst "adds" some noise to the information she possesses. Thus, in a dynamic setup, in every period n the analyst chooses the precision τ_{ϵ_n} of the normal random variable ϵ_n from which the error term is drawn.

Using the expressions for the price of information obtained in proposition 6 and starting from the second period, given any τ_{ϵ_1}

$$\tau^*_{\epsilon_2} \in \arg\max_{\tau_{\epsilon_2}} \int_0^1 \phi_2 di,$$

which gives as a unique positive solution

$$\tau^*_{\epsilon_2} = \frac{1}{\gamma}\sqrt{\frac{\tau_{iC1}}{\tau_u}}.$$

Note that $\tau^*_{\epsilon_2}$ has the same functional form as $\hat{\tau}_\epsilon$. However, $\tau^*_{\epsilon_2} > \hat{\tau}_\epsilon$. Indeed, given *any* τ_{ϵ_1}, the analyst's second period profit maximization problem is similar to the one she faces in the static market. However, as the precision of the information traders hold before buying the second period signal (i.e. τ_{iC1}) is strictly higher than the one they hold prior to acquiring information in a static market (i.e. τ_v), the signal quality the analyst chooses in the former case must be strictly higher than the one she sets in the latter.

In the first period the analyst then chooses τ_{ϵ_1} to solve

$$\max_{\tau_{\epsilon_1}} \int_0^1 \frac{\gamma}{2}\left(\ln\frac{\tau_{iC1}}{\tau_{C1}} + \ln\frac{\tau_{C2}(\tau^*_{\epsilon_2})+\tau_{\epsilon_1}}{\tau_{C2}(\tau^*_{\epsilon_2})} + \ln\frac{\tau_{iC2}(\tau^*_{\epsilon_2})}{\tau_{C2}(\tau^*_{\epsilon_2})+\tau_{\epsilon_1}}\right)di \tag{4.5}$$
$$= \max_{\tau_{\epsilon_1}} \int_0^1 \frac{\gamma}{2}\left(\ln\frac{\tau_{iC1}}{\tau_{C1}} + \ln\frac{2\tau_{iC1}+\tau^*_{\epsilon_2}}{\tau_{C1}+\tau_{iC1}}\right)di.$$

The next proposition characterizes the solution to (4.5), comparing it with the static benchmark.

Proposition 7 *In the 2-period information market, there exists a unique sequence of optimal signal precisions* $\{\tau^*_{\epsilon_1}, \tau^*_{\epsilon_2}\}$ *that solves the analyst's profit maximization problem, where*

1. $\tau^*_{\epsilon_1}$ *is the unique positive solution to* (4.5), $\tau^*_{\epsilon_2} = (1/\gamma)\sqrt{\tau^*_{iC1}/\tau_u}$, *where* $\tau^*_{iC1} = \tau_{iC1}(\tau^*_{\epsilon_1})$;

2. $\tau^*_{\epsilon_1} < \hat{\tau}_\epsilon < \tau^*_{\epsilon_2}$.

Proof. See the appendix. QED

In a dynamic market an analyst is faced with two problems: first, and *similarly* to the one-shot information sales case, she needs to take into account the negative effect that the price externality induced by the sale of information has on *both* period profits. [16] Second, and *differently* from the one-shot case, she faces an intertemporal self-competition problem. As a durable goods monopolist (Bulow 1982, 1986, and Coase 1972) once the first signal has been sold to informed traders, in order to make a new signal palatable to potential buyers, she must render partially obsolete the first period signal. The analyst thus scales down the quality of the first period information, and increases the quality of the information sold in the second period.

[16] In this case the problem is actually worsened by the compound negative effects that the first period signal sale has on first and second period profits.

To describe this in more detail, when the analyst chooses the second period signal quality she solves

$$\max_{\tau_{\epsilon_2}} \int_0^1 \frac{\gamma}{2} \ln\left(\frac{\tau_{iC2}}{\tau_{C2}+\tau_{\epsilon_1}}\right) di \Leftrightarrow \max_{\tau_{\epsilon_2}} \int_0^1 \frac{\gamma}{2}\left(\ln\frac{\tau_{iC2}}{\tau_{C2}} - \ln\frac{\tau_{C2}+\tau_{\epsilon_1}}{\tau_{C2}}\right) di,$$

for any given first period signal quality τ_{ϵ_1}. Thus, the price traders are willing to pay in order to get s_{i2} captures the informational advantage they have in the second period vis-à-vis market makers *net* of the informational advantage they would have holding s_{i1} and observing both period equilibrium prices $\{p_1, p_2\}$. [17] To maximize her profit, the analyst has thus an incentive to market a signal that in a way "kills-off" the second-hand market for the first period signal. [18] She does so by selling a signal whose precision $\tau^*_{\epsilon_2}$ is *strictly* higher than the precision of the first period signal.

Going back to period one, the analyst now faces the following problem:

$$\max_{\tau_{\epsilon_1}} \int_0^1 \frac{\gamma}{2}\left(\ln\frac{\tau_{iC1}}{\tau_{C1}} + \ln\frac{2\tau_{iC1}+\tau^*_{\epsilon_2}}{\tau_{C1}+\tau_{iC1}}\right) di$$
$$\Leftrightarrow \max_{\tau_{\epsilon_1}} \int_0^1 \frac{\gamma}{2}\left(\ln\left(1+\frac{1}{\gamma}\frac{\lambda_{C1}}{\tau_u}\right) + \ln\left(1+\frac{1}{\gamma}\frac{\tau_{\epsilon_1}}{\tau_{\epsilon_2}}\frac{\lambda_{C2}}{\tau_u}+\frac{1}{\gamma}\frac{\lambda_{C2}}{\tau_u}\right)\right) di.$$

As in the static case, she is interested in choosing a signal that makes the first period market as thin as possible. However, she must now take into account two additional contrasting effects. Increasing the first period signal precision allows traders to grab a *higher share* of second period noise traders' losses and this, in turn, increases the price they are willing to pay to get s_{i1}. On the other hand, a higher first period signal precision inevitably increases second period market depth, thus *reducing the size* of the second period rents the analyst can extract from traders. As the second effect is stronger than the first, the analyst chooses $\tau^*_{\epsilon_1} < \hat{\tau}_\epsilon$. [19]

Therefore, the analyst sells a pair of signals that *impoverishes* first period information quality while consistently *enhancing* second period private information. As long lived information is a durable good that *cannot* be rented, the analyst needs to

[17] We can interpret the term $(\gamma/2)\ln(\tau_{iC2}/\tau_{C2})$ as the *gross* informational advantage traders have in the second period vis-à-vis market makers.

[18] The expression "second-hand" market here is used by way of analogy with the durable goods monopolist literature. Actually, traders do not resell their signals. However, we can always interpret the fact that traders are able to use in period two the signal they acquired in period one, as a second-hand market in which each trader resells to himself the signal previously acquired.

[19] An alternative intuition for this result is the following one. When setting $\tau^*_{\epsilon_1}$ the analyst tries to extract as much surplus as possible from traders but at the same time she also tries to limit the competition she expects to face in the second period owing to the information traders bought in period one. As a result, she scales down the quality of the first period signal.

force the obsolescence of her first period signal. She does so combining a low first period signal quality (hence, reducing the product durability as in Bulow 1986) and introducing high second period signal quality (hence, marketing a new product that makes the old one obsolete as in Waldman 1993). [20]

Denote by $\phi_1(\tau^*_{\epsilon_1}), \phi_2(\tau^*_{\epsilon_1})$, respectively the optimal price of the first and second period signal and with $\phi(\hat{\tau}_\epsilon)$ the optimal price in the static market. The next proposition derives the implications of the optimal solution for the price of information and the depth of the market.

Proposition 8 *The information allocation chosen by the analyst prescribes that*

1. $\phi_1(\tau^*_{\epsilon_1}) > \phi(\hat{\tau}_\epsilon) > \phi_2(\tau^*_{\epsilon_1})$;
2. $\lambda_C(\hat{\tau}_\epsilon) > \lambda_{C1}(\tau^*_{\epsilon_1}) > \lambda_{C2}(\tau^*_{\epsilon_1})$.

Therefore, while the price of private information decreases across trading periods, depth increases.

Proof. See the appendix. QED

As the analyst kills-off the second-hand market for the first period signal, traders' net informational advantage vis-à-vis market makers decreases and the price they are willing to pay to buy s_{i2} ends up being lower than the one they pay to get s_{i1}. The flip side of the coin is that the adverse selection problem faced by market makers becomes less severe and market depth increases.

Remark 2 Increasing patterns of market depth have been documented at the inter-daily level by the empirical finance literature (see Foster and Viswanathan 1993). Theoretical explanations of this phenomenon have always been related to the *strategic* trading of insiders facing some form of *competitive pressure*, that speeds-up the market makers' learning process. Foster and Viswanathan (1990) show that a single insider is forced to spend his informational advantage at a faster pace than he would otherwise do, owing to the presence of impending public information. Holden and Subrahmanyam (1992) consider a market where the competition among symmetrically informed insiders forces more aggressive trading and a faster unfolding of

[20]The signal durability here refers to the need that traders have to acquire additional information over time. To be sure, a fully revealing signal is infinitely durable (as it kills traders' need to receive further information in the future), while an infinitely noisy signal is infinitely perishable (as it does not affect traders' demand for additional information).

the underlying uncertainty. According to this paper, in contrast, increasing levels of depth may be *entirely compatible* with an asset market where *no trader* has market power, and forthcoming public information poses *no threat* to informed traders' speculative abilities. In such a market, instead, the information flow is controlled by a *monopolistically informed* agent who, owing to the nature of the information she sells, intertemporally competes against herself. [21]

5 Insider Trading and Information Sales

We are now ready to contrast the dynamic properties of the competitive market where information is sold with those of the market with a strategic trader. An immediate consequence of proposition 5 is the following:

Proposition 9 *In the 2-period asset market:*

1. $\beta_2 < \gamma\tau^*_{\epsilon_2}$;
2. $\lambda_{I2} > \lambda_{C2}$;
3. $\tau_{I2} < \tau_{C2}$.

Proof. See the appendix. QED

Therefore, as opposed to the static market result, in a dynamic market an insider induces different patterns for second period depth and price informativeness. In particular, as he directly uses his informational advantage, he avoids the effect of intertemporal self-competition, fully internalizes the negative effect of aggressive speculation, and trades less intensely. This, in turn, makes the second period market *thinner* and its price *less* informative. [22]

[21] Therefore, as in the literature on vertical control (Tirole, 1988) – where consumers may face a competitive industry controlled by a monopolistic supplier of the intermediate good influencing the price of the final good – here we can think of liquidity traders as facing a sector of competitive traders whose behavior is controlled by a monopolistic supplier of information exerting a (partial) control over market depth.

[22] A simple intuition for this result – although only partially correct since trading aggressiveness differ across the equilibria in the two markets – is the following one. Owing to intertemporal competition, the informativeness of the second period price induced by the analyst is given by $\tau_{C2} = 2\tau_{C1}(\tau^*_{\epsilon_1}) + \tau^*_{\epsilon_1}$ while, according to proposition 5, an insider trades in a way that second period public precision is "only" twice as high as in the first period.

The insider's second period problem is akin to the problem he faces in the static market. The equilibrium solution prescribes that he trades in a way to minimize second period market depth. The information monopolist, instead, chooses the second period information quality to minimize second period depth but, as argued above, *also* to minimize the second period value competitive traders attach to their first period signal. To see this, rewrite (4.4) as follows

$$\phi_2 = \frac{\gamma}{2} \ln \left(1 + \frac{\tau_{C2}}{\tau_{C2} + \tau_{\epsilon_1}} \frac{1}{\gamma} \frac{\lambda_{C2}}{\tau_u} \right).$$

Therefore, τ_{ϵ_2} must make noise traders' second period expected losses as large as possible *while* slashing the information advantage traders have in the second period thanks to the signal they bought in period 1. As $(\tau_{C2}/(\tau_{C2}+\tau_{\epsilon_1}))$ is strictly decreasing in τ_{ϵ_1}, this forces the analyst to sell a signal whose precision is strictly higher than the one minimizing $(1/\lambda_{C2})$.

According to proposition 9 and differently from proposition 3, in a dynamic market the *way* through which a monopolistically informed agent conveys information about the fundamentals to the market *does matter*. In particular, whether such information is exploited directly or sold to competitive traders changes the patterns of depth and price efficiency. In contrast to the view according to which insider trading improves the accuracy of stock prices (see e.g., Carlton and Fischel 1983, and Manne 1966), the above result shows instead that a single insider can exploit his monopolistic position in such a way as to *choose the rate* at which the market learns the fundamental, in this way *impairing* second period liquidity *and* price efficiency.

Conversely, a monopolistic analyst, owing to intertemporal competition, loses control over the information flow and speeds up the market learning process. In the spirit of the durable goods monopolist interpretation, the insider thus acts in a way that is much akin to the monopolistic producer that *rents* instead of selling. Indeed, the monopolistic renter fully internalizes the negative effect of overproduction by keeping the ownership of the goods he markets and thus cuts back on the quantities he releases. The insider, on the other hand, by holding on to his informational advantage, directly bears the negative effects of an excessively aggressive behavior, and speculates less intensely.

Remark 3 As noted in proposition 7 in the first period the analyst reduces the quality of the information she sells. It is easy to show that this makes first period depth and price informativeness in the competitive market lower than in the strategic market. As I will argue in the next section, this result only affects the first period: when $N > 2$ numerical simulations show that starting from the second round of trade, the competitive market is always deeper than the strategic market; furthermore, price informativeness in the competitive market is always higher than in the strategic market for all $n = 1, 2, \ldots N$.

5.1 The General N-Period Information Market

The intuition gained in the previous section shows that in a dynamic market an insider is able to retain strong control over the information leakage produced by his trades. Conversely, an analyst facing intertemporal competition, is forced to give up most of such control to information buyers. If that is the case, as the number of trading rounds increases this lack of control should be exacerbated.

In this section, I compare the multiperiod versions of the 2-period market of section 3.2. As is well known, both the results in propositions 4, and 5 can be generalized to an arbitrary number of periods $N > 2$ (see, respectively Vives 1995a, and Kyle 1985). Building on these extensions, consider now the general, $N \geq 2$-period case and suppose that in every period n the analyst sells a signal of a different (conditional) precision τ_{ϵ_n}, charging a price ϕ_n. The next proposition gives an explicit expression for ϕ_n, generalizing proposition 6.

Proposition 10 *In the $N \geq 2$-period information market, the maximum price ϕ_n an agent i is willing to pay to buy a signal s_{in} in each period n is given by*

$$\phi_n = \frac{\gamma}{2}\left(\ln \frac{\tau_{iCn}}{\tau_{Cn} + \sum_{t=1}^{n-1} \tau_{\epsilon_t}} + \sum_{\substack{n+1 \leq t \leq N \\ n+1 < N}} \ln \frac{\tau_{Ct} + \sum_{k=1}^{n} \tau_{\epsilon_k}}{\tau_{Ct} + \sum_{k=1}^{n-1} \tau_{\epsilon_k}}\right), \tag{5.6}$$

where $\tau_{Cn} = (Var[v|p^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_n)^2$, and $\tau_{iCn} = (Var[v|s_i^n, p^n])^{-1} = \tau_{Cn} + \sum_{t=1}^{n} \tau_{\epsilon_t}$.

Proof. See the appendix. QED

According to (5.6), ϕ_n can be decomposed as follows:

$$\phi_n = \frac{\gamma}{2}\left(\ln\frac{\tau_{iCn}}{\tau_{Cn}} - \ln\frac{\tau_{Cn}+\sum_{t=1}^{n-1}\tau_{\epsilon_t}}{\tau_{Cn}}\right) + \\ \frac{\gamma}{2}\left(\sum_{\substack{n+1\le t\le N\\ n+1<N}}\left(\ln\frac{\tau_{Ct}+\sum_{k=1}^{n}\tau_{\epsilon_k}}{\tau_{Ct}} - \ln\frac{\tau_{Ct}+\sum_{k=1}^{n-1}\tau_{\epsilon_k}}{\tau_{Ct}}\right)\right).$$

Thus, in the N-period market, in every period n a signal is useful *both* because of the increase in informational advantage it allows a trader to hold in the same period n (the first term in the above expression) *and* because of the increase in the informational advantage it determines in every future period $k = n+1, n+2, \ldots, N$ (the second term).

Given any trading length N, the last period optimal precision is thus given by $\tau^*_{\epsilon_N} = (1/\gamma)\sqrt{\tau_{iCN-1}/\tau_u}$. Recursive substitution of $\tau^*_{\epsilon_N}$ into every period n's profit function, shows that the analyst solves a sequence of maximization problems such that at every time $n = 1, 2, \ldots, N-1$ she chooses

$$\tau^*_{\epsilon_n} \in \arg\max_{\tau_{\epsilon_n}}\left(\sum_{t=n}^{N-1}\phi_t + \phi^*_N\right) \\ \equiv \frac{\gamma}{2}\left(\sum_{k=n}^{N-1}\ln\frac{\tau_{iCk}}{\tau_{Ck}+\sum_{j=1}^{n-1}\tau_{\epsilon_j}} + \ln\frac{2\tau_{iCN-1}+\tau^*_{\epsilon_N}}{\tau_{CN-1}+\sum_{j=1}^{n-1}\tau_{\epsilon_j}+\tau_{iCN-1}}\right),$$

given the sequence $\{\tau^*_{\epsilon_t}\}_{t=n+1}^{N-1}$.

Using the above expression for the value of information I run numerical simulations for the case $N = 4$. The aim is to verify that the results obtained in proposition 9 still hold when the number of trading rounds increases. Letting $\tau_v, \tau_u, \gamma \in \{.2, .4, .6, .8, 1, 4, 6\}$, in all of the simulations the analyst induces a more aggressive traders' behavior than that displayed by the insider. Hence, the effect of intertemporal competition leads the analyst to lose control over the information flow, whereas the insider, lacking competitive pressure, can trade less aggressively. As a result from the second trading round onwards, the competitive market is more liquid than the strategic market (see figure 1).

[Figure 1 about here.]

As to price informativeness, the numerical simulations show that the competitive market leads to a more rapid resolution of the fundamentals' uncertainty than the strategic market starting from the *first* trading round. The intuition is straightforward: as the number of trading rounds increases, traders are willing to pay a higher price for the first period signal. This, in turn, shifts upwards the information quality supplied by the analyst, thus increasing competitive traders' aggressiveness (see figure 2).

[Figure 2 about here.]

6 Extensions

In order to increase her grip over the information flow, the analyst may want to consider two different strategies. She may try and *segment* the first period information market, so to reduce the fraction of traders that already possess a signal in the second period. Also, she may want to publicly release some information at the beginning of period two in order to reduce the informational advantage that traders have acquired in period one. Both strategies attempt to reduce the competitive pressure the analyst faces in the second period. However, as shown in this section, none of them can increase the analyst's profit.

6.1 Market Segmentation

Consider an extension of the 2-period market analyzed in section 3 in which every informed trader i in each period n (potentially) receives a private signal $s_{in} = v + \epsilon_{in}$, where $\epsilon_{in} \sim N(0, \tau_{\epsilon_{in}}^{-1})$. All the remaining assumptions are kept as in section 3. Under these conditions, the following result holds: [23]

Proposition 11 *In the 2-period competitive market, there exists a unique linear equilibrium. The equilibrium is given by* $X_{in}(s_i^n, p^n) = a_{in}(\tilde{s}_{in} - p_n)$, *and* $p_n = \lambda_{Cn} z_{Cn} + (1 - \lambda_{Cn} \Delta a_n) p_{n-1}$, $n = 1, 2$, *where* $a_{in} = \gamma(\sum_{t=1}^{n} \tau_{\epsilon_{it}})$, $\tilde{s}_{in} = (\sum_{t=1}^{n} \tau_{\epsilon_{it}})^{-1} \times (\sum_{t=1}^{n} \tau_{\epsilon_{it}} s_{it})$, $z_{Cn} = \Delta a_n v + u_n$, $\Delta a_n = \int_0^1 a_{in} - a_{in-1} di$, $\lambda_{Cn} = \Delta a_n \tau_u / \tau_{Cn}$, *and* $\tau_{Cn} = (Var[v|p^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_n)^2$.

Therefore, the heterogeneity of signals' precisions is reflected into traders' speculative aggressiveness. In the above market the analyst may decide to provide each

[23] Proposition 11 extends the dynamic equilibrium result in Vives (1995a) to the case in which traders hold signals of different precisions. Its proof is available from the author upon request.

trader with a signal of a different precision. The following proposition shows that this is never optimal: [24]

Proposition 12 *In the 2-period information market with heterogeneous signal precision, in every period $n = 1, 2$ the analyst sells to all traders a signal of the same precision.*

Proof. See the appendix. QED

The proof is based on two arguments. First, notice that in every period $n = 1, 2$ price informativeness τ_{Cn} only depends on informed agents' *average* signal precision. Thus, τ_{Cn} is invariant with respect to a distribution of signals' precisions that leaves its average unchanged. Next, in the first period the analyst's objective function is concave in the informational advantage each trader holds over market makers in every period n (τ_{iCn}/τ_{Cn}). Thus, owing to Jensen's inequality, given two information allocations yielding the *same* average total precision, in every period n the analyst obtains a higher profit when she sells to *all* traders a signal with the *same* precision (thus providing all traders with the *same* private precision) than when she sells signals with *diverse* precisions. It then follows that in every optimal information allocation, τ_{iC1} is the same across all traders, and $\tau^*_{\epsilon_{i2}}(\tau_{iC1}) = \tau^*_{\epsilon_2}$ for every trader $i \in [0, 1]$.

A direct implication of the above argument, is that the analyst never finds it profitable to segment the market – i.e. to sell information of precision $\tau^*_{\epsilon_1} > 0$ ($\tau^*_{\epsilon_1} = 0$) to a fraction $0 < \mu < 1$ ($1 - \mu$) of traders in the first period. Indeed, such information allocation is dominated by one in which all traders in the first period receive a signal of precision $\mu\tau^*_{\epsilon_1}$. Intuitively, market segmentation yields two contrasting effects. On the one hand, by reducing the fraction of traders that receive information in the first period, the analyst faces a reduced pressure to sell a better signal in the second period, as part of the population that buys information in the second period holds no previous signal. This, in turn, slows down information devaluation, increasing the analyst's profit. On the other hand, since equilibrium prices reflect fundamental information, the *value* that each trader assigns to a signal in the second period – after having observed the price sequence – is lower. This, in turn, limits the price that the analyst can extract from those traders that did not receive a signal in the first period. As the second effect is always stronger than the first, market segmentation never pays.

[24] This result thus strengthens Admati and Pfleiderer's (1986) conclusion that in a single period information market vertical differentiation is never profitable.

6.2 Public Disclosure

In a large market with differential information, disclosing to *each* trader i the signal *each* trader j has received ($j \neq i$) is practically unfeasible. A possible way out is for the analyst to reveal the aggregate signal she sold to traders in the first period (namely $\bar{s}_1 = \int_0^1 s_{i1} di$). Notice, however, that given the analyst's perfect knowledge of the fundamental v, such a strategy leads to complete information revelation, preventing the sale of a new signal in period 2. [25]

Based on these considerations, I address the issue of information disclosure in the following way: suppose that at the beginning of period 2 the analyst discloses *one* of the signals she sold in period 1, say $s_{j1} = v + \epsilon_{j1}$ (i.e. the analyst chooses at random which signal to communicate to the market). In a large market each trader assigns zero probability to the event that his signal will be made public. Therefore, in order to determine the price of information in this setup we can focus on the equilibrium in which each trader $i \in [0, 1]$ anticipates observing a (public) signal s_{j1}, $j \neq i$ at the beginning of period 2.

Proposition 13 *In the 2-period competitive market with disclosure, there exists a unique linear equilibrium. The equilibrium is symmetric and given by* $X_{i1}(s_1, p_1) = a_1(s_{i1} - p_1)$, $X_{i2}(s_i^2, p^2; s_{j1}) = a_2(\tilde{s}_{i2} - p_2)$, $p_1 = \lambda_{C1} z_{C1} + (1 - \lambda_{C1} a_1)\bar{v}$, $p_2 = \alpha E[v|z_C^2] + (1 - \alpha)s_{j1}$, *where* $a_n = \gamma(\sum_{t=1}^n \tau_{\epsilon_t})$, $E[v|z_C^2] = \lambda_{C2} z_{C2} + (1 - \lambda_{C2}\Delta a_2)p_1$, $\tilde{s}_{in} = (\sum_{t=1}^n \tau_{\epsilon_t})^{-1}(\sum_{t=1}^n \tau_{\epsilon_t} s_{it})$, $z_{Cn} = \Delta a_n v + u_n$, $\lambda_{Cn} = \Delta a_n \tau_u / \tau_{Cn}$, $\tau_{Cn} \equiv (Var[v|p^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^n (\Delta a_n)^2$, $\alpha = \tau_{C2}/\hat{\tau}_{C2}$, *and* $\hat{\tau}_{C2} \equiv (Var[v|z^2; s_{j1}])^{-1} = \tau_{C2} + \tau_{\epsilon_1}$.

Proof. See the appendix. QED

Information disclosure does not change the nature of the strategies that traders adopt in the no-disclosure equilibrium. On the other hand, it improves the market maker's estimation. While in the no-disclosure model second period public precision is given by $\text{Var}[v|z^2]^{-1} \equiv \tau_{C2} = \tau_v + \tau_u \sum_{t=1}^2 (\Delta a_t)^2$, in the model with disclosure $\text{Var}[v|z^2; s_{j1}]^{-1} \equiv \hat{\tau}_{C2} = \tau_{C2} + \tau_{\epsilon_1}$: the precision incorporated in the public signal

[25] Assuming a richer information structure does not help. For, suppose the analyst knew $v + w$ with $w \sim N(0, \tau_w^{-1})$ and independent from all the other random variables in the model. Then, first period signals would take the form $s_{i1} = v + w + \epsilon_{i1}$. The analyst could therefore disclose the average signal at interim (i.e. $\bar{s}_1 = \int_0^1 s_{i1} di = v + w$) without making the equilibrium fully revealing. Such a strategy would, however, again prevent the sale of any further signal, since $s_{i2} = v + w + \epsilon_{i2}$ would be a *noisier* signal than the one the analyst disclosed. As a consequence, no trader would be ready to buy it.

increases the quality of the public forecast. This, in turn, affects the price each trader is willing to pay in order to buy *both* signals:

$$\hat{\phi}_1 = \frac{\gamma}{2}\ln\frac{\tau_{iC1}}{\tau_{C1}} + \frac{\gamma}{2}\ln\frac{\hat{\tau}_{C2}+\tau_{\epsilon_1}}{\hat{\tau}_{C2}},$$
$$\hat{\phi}_2 = \frac{\gamma}{2}\ln\frac{\hat{\tau}_{iC2}}{\hat{\tau}_{C2}+\tau_{\epsilon_1}},$$

where $\hat{\tau}_{iC2} = \hat{\tau}_{C2} + \tau_{\epsilon_1} + \tau_{\epsilon_2}$. A straightforward calculation shows then that $\hat{\phi}_n < \phi_n$, $n = 1, 2$. Therefore,

Proposition 14 *The analyst never finds it profitable to publicly disclose information in the second period.*

The intuition is as follows: second period information disclosure has two effects. First, it reduces the added value that the first period signal has in the second period, in this way making more desirable the acquisition of further information in the second period: [26]

$$\hat{\phi}(s_{i1}||p_1, p_2; s_{j1}) = \frac{\gamma}{2}\ln\frac{\hat{\tau}_{C2}+\tau_{\epsilon_1}}{\hat{\tau}_{C2}} < \phi(s_{i1}||p_1, p_2) = \frac{\gamma}{2}\ln\frac{\tau_{C2}+\tau_{\epsilon_1}}{\tau_{C2}}.$$

However, at the same time it also reduces the uncertainty over the asset value v, and thus the *gross* informational advantage that traders acquire when they buy a new signal. [27] This, in turn, reduces traders' value for new information:

$$\frac{\gamma}{2}\ln\frac{\hat{\tau}_{iC2}}{\hat{\tau}_{C2}} < \frac{\gamma}{2}\ln\frac{\tau_{iC2}}{\tau_{C2}}.$$

The latter effect is always stronger than the former. Hence, with information disclosure the maximum price the analyst can extract for s_{i2} is lower. [28]

Remark 4 Propositions 8, 12, and 14 show that while the analyst's and the durable goods monopolist's problem share various common features, they also display a number of differences. First, note that as opposed to the durable goods producer, the

[26] Notice that this effect *reduces* the price a trader is willing to pay to buy the first signal.

[27] See footnote 17.

[28] The result in proposition 14 is robust to a different information structure. Assuming that traders receive the same signal in every period (with Admati and Pfleiderer's 1986 terminology, considering the dynamic "newsletters" model) leads exactly to the same conclusion. In this model the case against information disclosure is even stronger, for the anticipation of a useless first period signal in the second period makes traders unwilling to pay any extra amount in order to buy it. Computations for this case are available upon request.

analyst *does not* produce the fundamental information on which the signals she sells are based. In other words, she only transforms a raw-material whose production is located at the upstream level. As a consequence, the strategy of accelerating the first period signal decay also impacts on her ability to sell further signals in the future. This, in turn, implies that a policy of increasing such a rate of decay through public disclosure is never profitable. [29]

Also, differently from a durable goods monopolist, the analyst finds it optimal to serve the whole market in both periods. Indeed, segmenting the first period information market relaxes second period competition but also reduces the profits the analyst reaps from first period traders. According to proposition 12 the latter effect is always stronger than the former.

7 Discussion and Concluding Remarks

In this paper I have argued that as fundamental information resembles in many respects a durable good, the effects of its incorporation into stock prices depend on who is the agent controlling its flow. A monopolistic analyst selling information in a *dynamic* market tackles an intertemporal self-competition problem that leads her to partially release the control over the information flow to traders. Conversely, an insider acts "as if" he would rent the information he possesses to the market, thus securing a tighter control over the information flow. As a result, for a given piece of information, a market where information is provided by an analyst is deeper and more efficient than one where information is transmitted by an insider.

A number of issues are left for future research. Among these, competition between different analysts deserves special consideration. Indeed, in a *static* market, competition among analysts may lower the pressure to provide signals of a better quality (Simonov 1999). To be sure, when signals are correlated, traders may place a higher value in holding the signal bundle. This, in turn, relaxes competition, allowing the analysts to reduce the precision they embed in their signals. As a consequence, traders base their strategies on information of a lower quality, potentially negatively affecting the properties of the underlying stock market. In a dynamic market, on the

[29] Keeping the analogy with the durable-goods monopolist literature, publicly disclosing a signal is akin to the strategy of an artist who, to convince buyers that future production will be limited, makes a litograph and destroys the plates (see Bulow 1982). Notice, however, that by doing so the artist *does not* affect the value of the durable good. Conversely, as argued above, information disclosure reduces the value of the "good" the analyst can sell in the future.

other hand, the intertemporal competition effect I uncover will still be there, accelerating the resolution of the underlying uncertainty. Therefore, the overall impact of competition on market quality will depend on the interplay between the *competition-stifling* effect due to signal complementarity, and the *competition-enhancing* effect due to the long-lived nature of information.

A related issue refers to the properties of a market where either competing analysts or multiple insiders provide information. In the latter case the existing literature has shown that the effect of competition on market quality depends on the correlation structure of the insiders' information and on the possibility of coordination. [30] This suggests that the comparison between the properties of a market where competing analysts provide information and one with multiple insiders should heavily depend on the posited information structure.

Also, in the paper I have assumed that the decision to trade on or sell privileged information is exogenous. However, the paper's main result raises the issue of why information sales occur at all in financial markets. In other words, one may wonder why the analyst does not find a way to internalize the negative effect of excessive speculation so to exploit more efficiently her information. For example, she could choose either to directly act as an insider, or (for instance if faced with a capital constraint) to *indirectly* sell her information by setting up a mutual fund. In addressing this issue, however, one may want to consider as well the benefits of direct information sales brought up by the literature. Indeed, Fishman and Hagerty (1995) argue that faced with informed competitors, an agent may use information sales as a commitment device to trade aggressively in the stock market. This strategy, in turn, secures the analyst a lager share of the reduced total market profits. [31] Also, Admati and Pfleiderer (1990) show that direct sales of information allow better surplus extraction vis-à-vis the set-up of a mutual fund, and may thus be preferred as a means to distribute information. [32] A formal analysis of the conditions under which the cost

[30] Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993a) show that increasing the number of strategic, informed traders accelerates price discovery in a Kyle (1985) market. However, competition can be dampened both when insiders hold different, correlated signals (Foster and Viswanathan 1996) and if the coordination properties of public disclosure are exploited (Huddart, Hughes, and Levine 2005).

[31] According to my model, dynamic sales should strengthen this competitive effect, potentially providing a further reason for information sales to occur. I am grateful to an anonymous referee for suggesting this interpretation of my analysis.

[32] Kane and Marks (1990) also compare direct sales of information to the establishment of a mutual fund, proving that the existence of a borrowing constraint makes the analyst always prefer the former way to deliver information to the latter. In their framework, however, information sales do not affect

of direct information sales brought up by my model is offset *either* by their strategic benefit, *or* by the enhanced surplus-extraction ability they allow, is beyond the scope of this paper and is left for future research.

Finally, the paper focuses on the single asset case. As traders typically hold portfolios of assets, a natural application of the present work is to the analysis of the multi-security case. [33] I leave this and other extensions for further investigation.

the value of the analyst's signal.

[33] See Admati (1985), Caballé and Krishnan (1994), and Cespa (2004) for static models of stock markets where traders exchange vectors of assets.

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Appendix

Proof of proposition 6.

Start from the second period. Owing to the assumption of a CARA utility function and the normality of the random variables, a trader's expected utility from using the signal she bought in period 1 (together with first and second period equilibrium prices) is given by $E[U(X_{i2}(s_{i1}, p_1, p_2)(v-p_2))|\{s_{i1}, p_1, p_2\}] = -\exp\{-a_1^2(s_{i1}-p_2)^2/(2\gamma^2(\tau_{C2}+\tau_{\epsilon_1}))\}$. On the other hand if the trader chooses to acquire the second period signal as well, her expected utility is given by $E[U(X_{i2}(s_{i1}, s_{i2}, p_1, p_2)(v-p_2))|\{s_{i1}, s_{i2}, p_1, p_2\}] = -\exp\{-a_2^2(\tilde{s}_{i2}-p_2)^2/(2\gamma^2\tau_{iC2})\}$. Using a standard result from normal theory (see e.g., Danthine and Moresi 1992), prior to deciding whether or not to buy s_{i2}, the expected utility the trader earns in the first case is given by $E[U(X_{i2}(s_{i1}, p_1, p_2)(v-p_2))] = E[E[U(X_{i2}(s_{i1}, p_1, p_2)(v-p_2))|\{s_{i1}, p_1, p_2\}]] = -(\tau_{C2}/(\tau_{C2}+\tau_{\epsilon_1}))^{1/2}$, whereas in the second case

$$
\begin{aligned}
E[U(X_{i2}(s_{i1}, s_{i2}, p_1, p_2)(v-p_2))] &= E\left[E\left[U\left(X_{i2}\left(s_i^2, p^2\right)(v-p_2)\right) \mid \left\{s_i^2, p^2\right\}\right]\right] \\
&= -\left(\frac{\tau_{C2}}{\tau_{iC2}}\right)^{1/2}.
\end{aligned}
$$

Therefore, denoting with $\phi_2(s_{i2}||s_{i1}, p_1, p_2)$ the maximum price the trader is willing to pay in order to acquire s_{i2} once she has already acquired the first signal, the trader's certainty equivalent for the second period signal is given by the solution of $\exp\{\phi_2(s_{i2}||s_{i1}, p_1, p_2)/\gamma\}(\tau_{C2}/\tau_{iC2})^{1/2} = (\tau_{C2}/(\tau_{C2}+\tau_{\epsilon_1}))^{1/2}$, or

$$
\phi_2 = \phi(s_{i2}||s_{i1}, p_1, p_2) = \frac{\gamma}{2}\ln\frac{\tau_{iC2}}{\tau_{C2}+\tau_{\epsilon_1}}.
$$

In the first period a trader that buys s_{i1}, uses it in both period 1 and 2, and plans to buy s_{i2} earns an expected utility given by

$$
\begin{aligned}
&E\left[U\left(X_{i1}(s_{i1}, p_1)(p_2-p_1) + X_{i2}\left(s_i^2, p^2\right)(v-p_2)\right)\right] = \\
&\qquad = E\left[E\left[U\left(X_{i1}(p_2-p_1) + \frac{a_2^2}{2\gamma\tau_{iC2}}(\tilde{s}_{i2}-p_2)^2\right) |\{s_{i1}, p_1\}\right]\right] \\
&\qquad = E\left[U\left(\frac{a_1^2}{2\gamma\tau_{iC1}}(s_{i1}-p_1)^2\right)\right] \\
&\qquad = -\left(\frac{\tau_{C1}}{\tau_{iC1}}\right)^{1/2},
\end{aligned}
$$

whereas a trader that plans to buy no signal makes zero expected profits (as the information she ends up holding coincides with the one of the market makers that, under the competitive assumption earn zero profits). Therefore, the maximum price a trader is willing to pay for using the first period signal in period one is given by

$$\phi(s_{i1}||p_1) = \frac{\gamma}{2}\ln\frac{\tau_{iC1}}{\tau_{C1}}.$$

However, the trader can also use the same signal in period two, insofar as it allows him to have an informational advantage vis-à-vis market makers *independently* from buying the second signal. The expected utility the trader expects to earn from observing $\{s_{i1}, p_1, p_2\}$ is given by $E[U(X_{i2}(s_{i1}, p_1, p_2)(v - p_2))] = -(\tau_{C2}/(\tau_{C2} + \tau_{\epsilon_1}))^{1/2}$ which compared with the expected utility he earns only observing equilibrium prices gives

$$\phi(s_{i1}||p_1, p_2) = \frac{\gamma}{2}\ln\frac{\tau_{C2} + \tau_{\epsilon_1}}{\tau_{C2}}.$$

QED

Proof of proposition 7.

Given traders' willingness to pay, the analyst is faced with the problem of choosing the optimal sequence of signals' precisions $\{\tau^*_{\epsilon_1}, \tau^*_{\epsilon_2}\}$. Starting from the second period he solves

$$\max_{\tau_{\epsilon_2}} \int_0^1 \phi(s_{i2}||s_{i1}, p_1, p_2)di.$$

The first order condition for the second period signal precision is given by

$$\frac{\gamma(\tau_{\epsilon_1} + \gamma^2\tau^2_{\epsilon_1}\tau_u + \tau_v - \gamma^2\tau_{\epsilon_2}\tau_u)}{2\tau_{iC1}\tau_{iC2}} = 0, \tag{7.7}$$

and its unique positive solution gives $\tau^*_{\epsilon_2} = (1/\gamma)\sqrt{\tau_{iC1}/\tau_u}$. To see that this solution is a maximum, let $F_1(\tau_{\epsilon_2}) = \tau_{C2} + \tau_{\epsilon_1}$. Then (7.7) can be rewritten as follows: $\psi(\tau_{\epsilon_2}) = (F_1(\tau_{\epsilon_2})(\tau_{\epsilon_2} + F_1(\tau_{\epsilon_2})))^{-1}\gamma(F_1(\tau_{\epsilon_2}) - 2\gamma\tau^2_{\epsilon_2}\tau_u)$. Differentiating the previous expression with respect to τ_{ϵ_2} gives

$$\begin{aligned}\frac{\partial\psi(\cdot)}{\partial\tau_{\epsilon_2}} \propto\ & (F_1'(\tau_{\epsilon_2}) - 4\gamma^2\tau_{\epsilon_2}\tau_u)F_1(\tau_{\epsilon_2})(\tau_{\epsilon_2} + F_1(\tau_{\epsilon_2}))\\ & -(F_1(\tau_{\epsilon_2}) - 2\gamma^2\tau^2_{\epsilon_2}\tau_u)(F_1'(\tau_{\epsilon_2})(\tau_{\epsilon_2} + F_1(\tau_{\epsilon_2})) + F_1(\tau_{\epsilon_2})(1 + F_1'(\tau_{\epsilon_2}))),\end{aligned}$$

and evaluating it at optimum $(\partial\psi(\cdot)/\partial\tau_{\epsilon_2})|_{\tau_{\epsilon_2}=\tau^*_{\epsilon_2}} \propto (F_1'(\tau^*_{\epsilon_2}) - 4\gamma^2\tau^*_{\epsilon_2}\tau_u)F_1(\tau^*_{\epsilon_2})(\tau^*_{\epsilon_2} + F_1(\tau^*_{\epsilon_2}))$. As one can check, the sign of the above expression is always negative, and the proposed solution is indeed a maximum.

Consider now the first period. Using $\tau^*_{\epsilon_2}$ the analyst's objective function becomes

$$\int_0^1 \phi_1 + \phi_2 di = \int_0^1 \frac{\gamma}{2}\left(\ln\frac{\tau_{iC1}}{\tau_{C1}} + \ln\frac{2\tau_{iC1}+\tau^*_{\epsilon_2}}{\tau_{C1}+\tau_{iC1}}\right) di.$$

Let

$$\begin{aligned}F(\tau_{\epsilon_1}) &= \frac{\partial(\phi_1+\phi_2)}{\partial\tau_{\epsilon_1}}\\ &= \frac{\gamma}{2}\left(\frac{\tau_v-\gamma^2\tau^2_{\epsilon_1}\tau_u}{\tau_{C1}\tau_{iC1}} - \frac{2\gamma^2\tau^2_{\epsilon_1}\tau_u(3+2\gamma(\gamma\tau_{\epsilon_1}\tau_u+\sqrt{\tau_u\tau_{iC1}}))+\tau_{\epsilon_1}(1+4\gamma^2\tau_u\tau_v)-4\gamma\tau_v\sqrt{\tau_u\tau_{iC1}}}{2\tau_u\tau^*_{\epsilon_2}(\tau_{C1}+\tau_{iC1})(2\tau_{iC1}+\tau^*_{\epsilon_2})}\right)\end{aligned}$$

Then, as one can check, $F(0)=(\tau_v+2\gamma\sqrt{\tau_u\tau_v^3})^{-1}(1+3\gamma\sqrt{\tau_u\tau_v})>0$, and $F(\hat{\tau}_\epsilon)<0$. Hence, as $F(\tau_{\epsilon_1})$ is continuous in τ_{ϵ_1}, there exists a $\tau^*_{\epsilon_1}\in(0,\hat{\tau}_{\epsilon_1})$ such that $F(\tau^*_{\epsilon_1})=0$ and $F'(\tau^*_{\epsilon_1})<0$. To see that such a point is unique indicate with $F_1(\tau_{\epsilon_1})=(\gamma/2)(\partial\ln(\tau_{iC1}/\tau_{C1})/\partial\tau_{\epsilon_1})$ and with $F_2(\tau_{\epsilon_1})=(\gamma/2)(\partial\ln((\tau_{C1}+\tau_{iC1})^{-1}(2\tau_{iC1}+\tau^*_{\epsilon_2})/\partial\tau_{\epsilon_1})$. Hence $F(\tau_{\epsilon_1})=F_1(\tau_{\epsilon_1})+F_2(\tau_{\epsilon_1})$. Now, both $(\gamma/2)\ln(\tau_{iC1}/\tau_{C1})$ and $(\gamma/2)\ln(\tau_{C1}+\tau_{iC1})^{-1}(2\tau_{iC1}+\tau^*_{\epsilon_2})$ are unimodal in τ_{ϵ_1}, in particular $F(\tau_{\epsilon_1})>0\Leftrightarrow\tau_{\epsilon_1}<(1/\gamma)\sqrt{\tau_v/\tau_u}$, while $F_2(\tau_{\epsilon_1})>0\Leftrightarrow\tau_{\epsilon_1}<\tilde{\tau}_{\epsilon_1}<(1/\gamma)\sqrt{\tau_v/\tau_u}$. Thus, as $\tau^*_{\epsilon_1}\in(0,(1/\gamma)\sqrt{\tau_v/\tau_u})$, then for any $\eta>0$, there is a $\tilde{\tilde{\tau}}_{\epsilon_1}\in(\tau^*_{\epsilon_1},\tau^*_{\epsilon_1}+\eta)$ such that $F_i(\tau^*_{\epsilon_1})>F_i(\tilde{\tilde{\tau}}_{\epsilon_1})$ for $i=1,2$. Hence $0=F_1(\tau^*_{\epsilon_1})+F_2(\tau^*_{\epsilon_1})>F_1(\tilde{\tilde{\tau}}_{\epsilon_1})+F_2(\tilde{\tilde{\tau}}_{\epsilon_1})$ and the latter inequality implies that $\tau^*_{\epsilon_1}$ is unique.

The second part of the proposition is immediate as $(\gamma\tau^*_{\epsilon_1})^2\tau_u<\tau^*_{iC1}$.

QED

Proof of proposition 8.

For the first part, notice that $\phi_1-\phi_2\geq 0\Leftrightarrow G(\tau_{\epsilon_1})\equiv 4\tau^3_{iC1}-\tau_{C1}(\tau_{C1}+\tau_{iC1})(2\tau_{iC1}+\tau^*_{\epsilon_2})\geq 0$. Evaluating $G(0)=-(2\tau_v^2/\gamma)\sqrt{\tau_v/\tau_u}<0$, while $G((1/\gamma)\sqrt{\tau_v/(3\tau_u)})>0$. Hence as $G(\cdot)$ is continuous in τ_{ϵ_1}, there is a $\tilde{\tau}_{\epsilon_1}\in(0,(1/\gamma)\sqrt{\tau_v/(3\tau_u)})$ such that $G(\tilde{\tau}_{\epsilon_1})=0$ and $G'(\tilde{\tau}_{\epsilon_1})>0$. Furthermore as one can check $G(\tau_{\epsilon_1})=\tau^*_{\epsilon_2}(\tau_{iC1}+\tau_{C1})(2\gamma\tau_{\epsilon_1}\sqrt{\tau_u\tau_{iC1}}-\tau_{C1})+2\gamma\tau^2_{iC1}\tau_{\epsilon_1}$ and as all of the terms of the previous expression are increasing in τ_{ϵ_1}, the point $\tilde{\tau}_{\epsilon_1}$ is unique. Now, evaluating $F((1/\gamma)\sqrt{\tau_v/(3\tau_u)})>0$, hence it must be that $\tilde{\tau}_{\epsilon_1}<(1/\gamma)\sqrt{\tau_v/(3\tau_u)}<\tau^*_{\epsilon_1}$ and as for any $\tau_{\epsilon_1}>\tilde{\tau}_{\epsilon_1}$, $G(\tau_{\epsilon_1})>0$, the result follows.

To see that $\phi_1(\tau^*_{\epsilon_1})>\phi(\hat{\tau}_\epsilon)$, notice that

$$\phi_1=\frac{\gamma}{2}\left(\ln\frac{\tau_{iC1}}{\tau_{C1}}+\ln\frac{2\tau_{iC1}}{\tau_{C1}+\tau_{iC1}}\right),$$

and its unique maximum coincides with the one of the static information market, i.e. $\hat{\tau}_\epsilon = (1/\gamma)\sqrt{\tau_v/\tau_u}$. Now, $(1/\gamma)\sqrt{\tau_v/3\tau_u} < \tau^*_{\epsilon_1} < \hat{\tau}_\epsilon$, hence to prove that $\phi_1(\tau^*_{\epsilon_1}) > \phi(\hat{\tau}_\epsilon)$ it is sufficient to show that $\phi(\hat{\tau}_\epsilon) < \phi_1((1/\gamma)\sqrt{\tau_v/3\tau_u})$. Evaluating, $\phi(\hat{\tau}_\epsilon) < \phi_1((1/\gamma)\sqrt{\tau_v/3\tau_u})$ if and only if

$$\frac{2\gamma\tau_v(3\sqrt{3}-4)+\sqrt{\tau_v/\tau_u}(3-\sqrt{3})}{2\gamma\tau_v(\sqrt{3}+8\gamma\sqrt{\tau_u\tau_v})} > 0,$$

a condition which is always satisfied. Next, to see that $\phi_2(\tau^*_{\epsilon_1}) < \phi(\hat{\tau}_\epsilon)$, notice that

$$\phi_2(\tau^*_{\epsilon_1}) = \frac{\gamma}{2}\ln\left(1+\frac{1}{2\gamma\sqrt{\tau_u\tau_{iC1}(\tau^*_{\epsilon_1})}}\right),$$

and a direct comparison with $\phi(\hat{\tau}_\epsilon)$ gives the desired result.

For the second part, notice that $\lambda_{C1}(\tau^*_{\epsilon_1}) > \lambda_{C2}(\tau^*_{\epsilon_1})$ if and only if $a_1\tau_{C2} > \Delta a_2\tau_{C1} \Leftrightarrow a_1^2\tau_u(\tau_{C1}+\tau_{iC1})^2 > \tau_{C1}^2\tau_{iC1}$. Define $H(\tau_{\epsilon_1}) = a_1^2\tau_u(\tau_{C1}+\tau_{iC1})^2 - \tau_{C1}^2\tau_{iC1}$, and notice that $H(0) = -\tau_v^3$, and that $\lim_{\tau_{\epsilon_1}\to\infty} H(\tau_{\epsilon_1}) = \infty$. Hence, there is a $\hat{\tau}_{\epsilon_1}$ such that $H(\hat{\tau}_{\epsilon_1}) = 0$. Furthermore, $H(\hat{\tau}_{\epsilon_1}) = 0 \Rightarrow H'(\hat{\tau}_{\epsilon_1}) > 0$, and as $H'(\tau_{\epsilon_1}) = \gamma a_1\tau_u(18a_1^4\tau_u^2 + 2\tau_v^2 + 4\tau_{\epsilon_1}^2 + 15a_1^2\tau_u\tau_{\epsilon_1} + 20a_1^2\tau_u\tau_v + 6\tau_{\epsilon_1}\tau_v) - \tau_v^2$, $\hat{\tau}_{\epsilon_1}$ is unique. Consider then the point $\hat{\hat{\tau}}_{\epsilon_1} = (1/\gamma)\sqrt{\tau_v/3\tau_u}$ and notice that $F(\hat{\hat{\tau}}_{\epsilon_1}) > 0$ which implies that $\tau^*_{\epsilon_1} > \hat{\hat{\tau}}_{\epsilon_1}$. Evaluating $H(\hat{\hat{\tau}}_{\epsilon_1}) = \tau_v^2/(9\gamma^2\tau_u)$, which implies that $\hat{\tau}_{\epsilon_1} < \hat{\hat{\tau}}_{\epsilon_1} < \tau^*_{\epsilon_1}$ or, equivalently, that $\lambda_{C1}(\tau^*_{\epsilon_1}) > \lambda_{C2}(\tau^*_{\epsilon_1})$.

To see that $\lambda_C(\hat{\tau}_\epsilon) > \lambda_{C1}(\tau^*_{\epsilon_1})$, notice that $\hat{\tau}_\epsilon > \tau^*_{\epsilon_1}$ and as for $\tau_\epsilon \leq \hat{\tau}_\epsilon$, $\lambda_{C1}(\cdot)$ increases in τ_ϵ, the result follows.

QED

Proof of proposition 9.

Given the expressions for the equilibrium parameters, start from the second part of the claim. To see that $\lambda_{I2} > \lambda_{C2}(\tau^*_{\epsilon_1})$, notice that given $\tau^*_{\epsilon_2}$, $\lambda_{C2} = (\tau_{C1} + \tau_{iC1})^{-1}(\tau_u\tau_{iC1})^{1/2}$, hence $(\partial\lambda_{C2}/\partial\tau_{\epsilon_1}) < 0$ and $\lambda_{C2}(\tau^*_{\epsilon_1}) < \lambda_{C2}((1/\gamma)(\tau_v/3\tau_u))$. Thus, as one can check, $\lambda_{C2}((1/\gamma)(\tau_v/3\tau_u)) < \lambda_{I2}$. Next, $\beta_2 = (1/2\lambda_{I2}) < (1/2\lambda_{C2})$, while $\gamma\tau^*_{\epsilon_2} > (1/2\lambda_{C2})$. Therefore, $\gamma\tau^*_{\epsilon_2} > \beta_2$. Finally, as $\lambda_{I2} > \lambda_{C2}(\tau^*_{\epsilon_1})$, and $\lambda_{I2} = \beta_2\tau_u\tau_{I2}^{-1}$, we have that $\beta_2\tau_u\tau_{I2}^{-1} > \Delta a_2\tau_u\tau_{C2}^{-1}(\tau^*_{\epsilon_1})$. However, as $\beta_2 < \Delta a_2$, then it must be that $\tau_{I2}^{-1} > \tau_{C2}^{-1}(\tau^*_{\epsilon_1})$ or that $\tau_{I2} < \tau_{C2}(\tau^*_{\epsilon_1})$.

QED

Proof of proposition 10.

Without loss of generality, the proof is given for the case $N = 3$. Starting from $n = 3$, an information buyer that has already observed $\{s_{i1}, s_{i2}\}$, has to decide whether to acquire s_{i3}. If he does so, then according to proposition 4, $X_{i3}(\tilde{s}_{i3}, p_3) = a_3(\tilde{s}_{i3} - p_3)$, with $a_3 = \gamma \sum_{t=1}^{3} \tau_{\epsilon_t}$, $E[U(X_{i3}(v - p_3))|\tilde{s}_{i3}, p^3] = -\exp\{-(a_3^2/2\gamma^2 \tau_{iC3})(\tilde{s}_{i3} - p_3)^2\}$, and

$$E\left[E\left[U(X_{i3}(v - p_3))|\left\{\tilde{s}_{i3}, p^3\right\}\right]\right] = -\left(\frac{\tau_{C3}}{\tau_{iC3}}\right)^{1/2}.$$

On the other hand, if the trader does not buy s_{i3}, then it is easy to see that $X_{i3}(\tilde{s}_{i2}, p_3) = a_2(\tilde{s}_{i2} - p_3)$,

$$E\left[U(X_{i3}(v - p_3))|\left\{\tilde{s}_{i2}, p^3\right\}\right] = -\exp\left\{-\left(\frac{a_2^2}{2\gamma^2(\tau_{C3} + \sum_{t=1}^{2} \tau_{\epsilon_t})}\right)(\tilde{s}_{i2} - p_3)^2\right\}, \tag{7.8}$$

and

$$E\left[E\left[U(X_{i3}(v - p_3))|\left\{\tilde{s}_{i2}, p^3\right\}\right]\right] = -\left(\frac{\tau_{C3}}{\tau_{C3} + \sum_{t=1}^{2} \tau_{\epsilon_t}}\right)^{1/2}. \tag{7.9}$$

Therefore, indicating with $\phi_3(s_{i3}||s_i^2, p^3)$ the maximum price the trader is willing to pay in order to acquire s_{i3} once he has already acquired the first and second period signals, his certainty equivalent for the third period signal is given by the solution to $\exp\{\phi_2(s_{i3}||s_i^2, p^3)/\gamma\}(\tau_{C3}/\tau_{iC3})^{1/2} = (\tau_{C3}/(\tau_{C3} + \sum_{t=1}^{2} \tau_{\epsilon_t}))^{1/2}$, or

$$\phi_3 = \phi\left(s_{i3}||s_i^2, p^3\right) = \frac{\gamma}{2} \ln \frac{\tau_{iC3}}{\tau_{C3} + \sum_{t=1}^{2} \tau_{\epsilon_t}}.$$

Stepping back to period 2, the price a trader is willing to pay to acquire s_{i2} is the sum of the price he would pay to exploit the informational advantage in *(i)* period two and *(ii)* in period three. Starting from *(ii)*, as shown above if the trader possesses s_{i2}, then his expected utility from trading in period 3 is given by (7.9). On the other hand if the trader only has s_{i1}, then it is easy to see that $X_{i3}(s_{i1}, p^3) = a_1(s_{i1} - p_3)$ and computing the ex-ante expected utility in this case,

$$E\left[E\left[U(X_{i3}(v - p_3))|\left\{s_{i1}, p^3\right\}\right]\right] = -\left(\frac{\tau_{C3}}{\tau_{C3} + \tau_{\epsilon_1}}\right)^{1/2}.$$

Therefore, the value of s_{i2} in period 3 is given by

$$\phi\left(s_{i2}||s_{i1}, p^3\right) = \frac{\gamma}{2} \ln \frac{\tau_{C3} + \sum_{t=1}^{2} \tau_{\epsilon_t}}{\tau_{C3} + \tau_{\epsilon_1}}. \tag{7.10}$$

To address point *(i)*, we first need to find the trader's second period strategy if he observes $\{s_{i1}, s_{i2}\}$ and if he only observes s_{i1}. Start from $X_{i2}(\tilde{s}_{i2}, p^2)$, that by dynamic optimality is the maximizer of

$$
\begin{aligned}
E[U(&X_{i2}(p_3 - p_2) + X_{i3}(v - p_3))|\{\tilde{s}_{i2}, p^2\}] \qquad (7.11)\\
&= E\left[-\exp\left\{-\frac{1}{\gamma}\left(X_{i2}(p_3 - p_2) + \frac{a_2^2(\tilde{s}_{i2} - p_3)^2}{2\gamma(\tau_{C3} + \sum_{t=1}^{2}\tau_{\epsilon t})}\right)\right\} \mid \{\tilde{s}_{i2}, p^2\}\right].
\end{aligned}
$$

Letting $F = (2\gamma^2(\tau_{C3} + \sum_{t=1}^{2}\tau_{\epsilon t}))^{-1}a_2^2$, the argument in the above exponential can be rewritten as follows:

$$
\begin{aligned}
F(p_3 - \mu)^2 &+ ((X_{i2}/\gamma) + 2F(\mu - \tilde{s}_{i2}))(p_3 - \mu)\\
&+ ((X_{i2}/\gamma) + F(2\tilde{s}_{i2} - \mu))\mu + F\tilde{s}_{i2} - (X_{i2}/\gamma)p_2,
\end{aligned}
$$

where $p_3 - \mu$ is normally distributed (conditionally on $\{\tilde{s}_{i2}, p^2\}$) with mean zero and variance Σ (i.e. $\mu = E[p_3|\tilde{s}_{i2}, p^2]$), where

$$
\mu = \frac{\Delta\tau_{C3}(\sum_{t=1}^{2}\tau_{\epsilon_t})\tilde{s}_{i2} + \tau_{C2}(\tau_{C3} + \sum_{t=1}^{2}\tau_{\epsilon_t})p_2}{\tau_{C3}\tau_{iC2}}, \qquad \Sigma = \frac{\Delta\tau_{C3}(\tau_{C3} + \sum_{t=1}^{2}\tau_{\epsilon_t})}{\tau_{iC2}\tau_{C3}^2}.
$$

Using a standard property of normal random variables, it can be shown that (7.11) is equal to $(\Sigma^{-1} + 2F)^{-1/2}\Sigma^{-1/2}$ times

$$
\begin{aligned}
-\exp\Big\{-\big(&(\mu^2F + ((X_{i2}/2) - 2F\tilde{s}_{i2})\mu + F\tilde{s}_{i2}^2 - (X_{i2}/\gamma)p_2) \qquad (7.12)\\
&-(1/2)((X_{i2}/\gamma) - 2F(\tilde{s}_{i2} - \mu))^2\left(\Sigma^{-1} + 2F\right)^{-1}\big)\Big\}
\end{aligned}
$$

The first order condition to maximize (7.12) with respect to X_{i2} yields

$$
X_{i2} = \gamma\left((\mu - p_2)\left(\Sigma^{-1} + 2F\right) + 2F(\tilde{s}_{i2} - \mu)\right), \qquad (7.13)
$$

and using the above expressions for μ and Σ one finds that

$$
X_{i2}(\tilde{s}_{i2}, p_2) = a_2(\tilde{s}_{i2} - p_2). \qquad (7.14)
$$

Substituting (7.13) in (7.12), rearranging and using (7.14)

$$
\begin{aligned}
E[U(&X_{i2}(p_3 - p_2) + X_{i3}(v - p_3))|\{\tilde{s}_{i2}, p^2\}]\\
&= -\left((\Sigma^{-1} + 2F)^{-1/2}\Sigma^{-1/2}\right)\exp\big\{-\big((1/2)(\mu - p_2)^2(\Sigma^{-1} + 2F)\\
&\qquad + 2F(\tilde{s}_{i2} - \mu)(\mu - p_2) + F(\tilde{s}_{i2} - \mu)^2\big)\big\}\\
&= -\left((\Sigma^{-1} + 2F)^{-1/2}\Sigma^{-1/2}\right)\exp\left\{-\frac{a_2^2}{2\gamma^2\tau_{iC2}}(\tilde{s}_{i2} - p_2)^2\right\}.
\end{aligned}
$$

Finally, computing the ex-ante expected utility yields

$$E\left[E\left[U(X_{i2}(p_3-p_2)+X_{i3}(v-p_3))|\left\{\tilde{s}_{i2},p^2\right\}\right]\right]=-\left(\frac{\tau_{C2}}{\tau_{iC2}}\right)^{1/2}.$$

Analogously one can find that $X_{i2}(s_{i1},p_2)=a_1(s_{i1}-p_2)$ and that

$$E\left[E\left[U(X_{i2}(p_3-p_2)+X_{i3}(v-p_3))|\left\{s_{i1},p^2\right\}\right]\right]=-\left(\frac{\tau_{C2}}{\tau_{C2}+\tau_{\epsilon_1}}\right)^{1/2}.$$

Therefore, the value of s_{i2} in period 2 is given by

$$\phi(s_{i2}||s_{i1},p^2)=\frac{\gamma}{2}\ln\frac{\tau_{iC2}}{\tau_{C2}+\tau_{\epsilon_1}}. \tag{7.15}$$

The price of the second period signal is then obtained summing (7.10) and (7.15):

$$\phi_2=\frac{\gamma}{2}\left(\ln\frac{\tau_{C3}+\sum_{t=1}^{2}\tau_{\epsilon_t}}{\tau_{C3}+\tau_{\epsilon_1}}+\ln\frac{\tau_{iC2}}{\tau_{C2}+\tau_{\epsilon_1}}\right).$$

Along the same lines of what done for ϕ_2 one finds that

$$\phi_1=\frac{\gamma}{2}\left(\ln\frac{\tau_{iC1}}{\tau_{C1}}+\ln\frac{\tau_{C2}+\tau_{\epsilon_1}}{\tau_{C2}}+\ln\frac{\tau_{C3}+\tau_{\epsilon_1}}{\tau_{C3}}\right).$$

QED

Proof of proposition 12.

Starting from the second period, the analyst solves

$$\max_{\tau_{\epsilon_{i2}}}\ln\frac{\tau_{iC2}}{\tau_{C2}+\tau_{\epsilon_{i1}}},$$

for every trader in the market, where $\tau_{C2}=\tau_v+(\int_0^1 a_{i1})^2\tau_u+(\int_0^1(a_{i2}-a_{i1})di)^2\tau_u$ and $\tau_{iC2}=\tau_{C2}+\sum_{t=1}^{2}\tau_{\epsilon_{it}}$. Solving the maximization problem yields $\tau^*_{\epsilon_{i2}}=(1/\gamma)\sqrt{\tau_{iC1}/\tau_u}$. Therefore, the second period optimal precision depends on the distribution of the first period signal precision across traders. In particular, if τ_{iC1} is the same for every $i\in[0,1]$, then $\tau^*_{\epsilon_{i2}}=\tau^*_{\epsilon_2}$ for every trader $i\in[0,1]$.

Consider now the analyst's first period objective function:

$$\int_0^1\ln\frac{\tau_{iC1}}{\tau_{C1}}+\ln\frac{\tau_{iC2}}{\tau_{C2}}di.$$

Notice that for $\tau_{\epsilon_{i2}}=\tau^*_{\epsilon_{i2}}$, the above is a function of $\tau_{\epsilon_{i1}}$. Also, given that $\tau_{C1}=\tau_v+(\int_0^1 a_{i1}di)^2\tau_u$ both the first and second period public precisions only depend on informed agents' average signal precision; hence, they are invariant to a distribution of signals' precisions that leaves its average unchanged. Let $\bar{\tau}_{iCn} = \int_0^1 \tau_{iCn} di$ for some given distribution of first period signals precisions. Then, for such information allocation owing to Jensen's inequality, the following holds:

$$\int_0^1 \ln \frac{\tau_{iCn}}{\tau_{Cn}}\ di \leq \ln \int_0^1 \frac{\tau_{iCn}}{\tau_{Cn}}\ di = \ln \frac{\bar{\tau}_{iCn}}{\tau_{Cn}},$$

for $n = 1, 2$. In words: given two information allocations yielding the *same* average total precision, the analyst obtains a higher profit when she sells to all traders a signal with the *same* precision (thus providing all traders with the *same* private precision) than when she sells signals with *diverse* precisions. It then follows that in every optimal information allocation, τ_{iC1} is the same across all traders and $\tau^*_{\epsilon_{i2}} = \tau^*_{\epsilon_2}$ for every trader $i \in [0, 1]$.

QED

Proof of proposition 13.

Let $W_{i2} = X_{i1}(p_2 - p_1) + X_{i2}(v - p_2)$ denote the final wealth of an agent i. The agent chooses X_{i1}, X_{i2} to maximize $E[U(W_{i2})] = -E[\exp\{-\gamma^{-1}W_{i2}\}]$.

Using backward induction, at time 2 trader i chooses X_{i2} to maximize

$$-\exp\{-\gamma^{-1}X_{i1}(p_2 - p_1)\}E[\exp\{\gamma^{-1}X_{i2}(v - p_2)\}|\tilde{s}_{i2}, p_2; s_{j1}],$$

given X_{i1}. Normality of the random variables and negative exponential utility yield $X_{i2} = a_2(\tilde{s}_{i2} - p_2)$, where $a_2 = \gamma(\sum_{t=1}^{2} \tau_{\epsilon_t})$. Substituting the optimal period 2 strategy in the second period objective function and simplifying

$$E[\exp\{-\gamma^{-1}X_{i2}(v - p_2)\}|\tilde{s}_{i2}, p_2; s_{j1}] = \exp\left\{-\frac{a_2^2}{2\gamma^2\hat{\tau}_{iC2}}(\tilde{s}_{i2} - p_2)^2\right\},$$

where $\hat{\tau}_{iC2} \equiv (\mathrm{Var}[v|\tilde{s}_{i2}, z_C^2; s_{j1}])^{-1} = \hat{\tau}_2 + \tau_{\epsilon_1} + \tau_{\epsilon_2}$, and $\hat{\tau}_2 \equiv (\mathrm{Var}[v|z^2; s_{j1}])^{-1} = \tau_v + \tau_u \sum_{t=1}^{2}(\Delta a_t)^2 + \tau_{\epsilon_1}$. In the first period, the agent chooses X_{i1} to maximize

$$\begin{aligned}-E[E[\exp\{-\gamma^{-1}X_{i1}(p_2 - p_1)\} \exp\{-\gamma^{-1}X_{i2}(v - p_2)\}|\tilde{s}_{i2}, p_2; s_{j1}]|s_{i1}, p_1]\\ = -E\left[\exp\left\{-\gamma^{-1}\left(X_{i1}(p_2 - p_1) + \frac{a_2^2}{2\gamma^2\hat{\tau}_{iC2}}(\tilde{s}_{i2} - p_2)^2\right)\right\}|s_{i1}, p_1\right].\end{aligned}$$

The expression in the curly braces of the latter formula is a quadratic form of the bivariate vector $\psi = (\tilde{s}_{i2} - p_2 - \mu_1, p_2 - \mu_2)'$ which is normally distributed conditional on $\{s_{i1}, p_1\}$ with zero mean and variance-covariance matrix Σ:

$$\left(X_{i1}(p_2 - p_1) + \frac{a_2^2}{2\gamma^2\hat{\tau}_{iC2}}(\tilde{s}_{i2} - p_2)^2\right) = c + b'\psi + \psi' A\psi,$$

where

$$\Sigma = \begin{pmatrix} \frac{\hat{\tau}_{i2}(\hat{\tau}_2\tau_{\epsilon_2}\tau_{i1}+\Delta\hat{\tau}_2\tau_{\epsilon_1}^2-\tau_1\tau_{\epsilon_1}\tau_{\epsilon_2})}{(\sum_{t=1}^2\tau_{\epsilon_t})^2\hat{\tau}_2\tau_{i1}} & -\frac{\tau_{\epsilon_1}\hat{\tau}_{i2}\Delta\hat{\tau}_2}{\tau_{i1}\hat{\tau}_2^2(\sum_{t=1}^2\tau_{\epsilon_t})} \\ -\frac{\tau_{\epsilon_1}\hat{\tau}_{i2}\Delta\hat{\tau}_2}{\tau_{i1}\hat{\tau}_2^2(\sum_{t=1}^2\tau_{\epsilon_t})} & \frac{\Delta\hat{\tau}_2(\hat{\tau}_2+\tau_{\epsilon_1})}{\tau_{i1}\hat{\tau}_2^2} \end{pmatrix},$$

$c = (\mu_2 - p_1)X_{i1} + (a_2\mu_1)^2/(2\gamma\hat{\tau}_{iC2})$, $b = (a_2^2\mu_1/(\gamma\hat{\tau}_{iC2}), X_{i1})'$, and A is a 2×2 matrix with $a_{11} = a_2^2/(2\gamma\hat{\tau}_{iC2})$ and the rest zeroes. Using a standard result from normal theory (see e.g., Danthine and Moresi 1992), it follows that

$$-E\left[\exp\left\{-\gamma^{-1}\left(X_{i1}(p_2-p_1)+\frac{a_2^2}{2\gamma^2\hat{\tau}_{iC2}}(\tilde{s}_{i2}-p_2)^2\right)\right\}|s_{i1},p_1\right]$$
$$= -|\Sigma|^{-1/2}\left|\Sigma^{-1}+2\gamma^{-1}A\right|^{-1/2}\times\exp\left\{-\gamma^{-1}\left(c-\frac{1}{2\gamma}b'\left(\Sigma^{-1}+2\gamma^{-1}A\right)^{-1}b\right)\right\}.$$

Maximizing the above function with respect to X_{i1} and indicating with h_{ij} the elements of $H \equiv (\Sigma^{-1} + 2\gamma^{-1}A)^{-1}$ yields

$$X_{i1} = \gamma\left(\frac{\mu_2-p_1}{h_{22}} - \frac{h_{12}a_2^2\mu_1}{h_{22}\hat{\tau}_{i2}}\right). \tag{7.16}$$

Standard normal calculations yield

$$\mu_1 = \left(\frac{\tau_{C1}\hat{\tau}_{iC2}\tau_{\epsilon_1}}{\hat{\tau}_{C2}\tau_{iC1}\left(\sum_{t=1}^2\tau_{\epsilon_t}\right)}\right)(s_{i1}-p_1),$$
$$\mu_2 - p_1 = \left(\frac{(\Delta\hat{\tau}_{C2})\tau_{\epsilon_1}}{\hat{\tau}_{C2}\tau_{iC1}}\right)(s_{i1}-p_1),$$
$$h_{22} = \left(\frac{\left(\sum_{t=1}^2\tau_{\epsilon_t}\right)^2}{\tau_{\epsilon_2}}\right)\left|\Sigma^{-1}+2\gamma^{-1}A\right|^{-1},$$
$$h_{12} = -\left(\frac{\tau_{\epsilon_1}\sum_{t=1}^2\tau_{\epsilon_t}}{\tau_{\epsilon_2}}\right)\left|\Sigma^{-1}+2\gamma^{-1}A\right|^{-1},$$
$$\left|\Sigma^{-1}+2\gamma^{-1}A\right| = \frac{(\hat{\tau}_{C2}\tau_{iC1}-\tau_{C1}\tau_{\epsilon_1})\left(\sum_{t=1}^2\tau_{\epsilon_t}\right)^2}{(\Delta\hat{\tau}_{C2})\tau_{\epsilon_2}},$$

where $(\Delta\hat{\tau}_{C2}) \equiv \hat{\tau}_{C2} - \tau_{C1} = (\Delta a_2)^2\tau_u + \tau_{\epsilon_1}$. Using these expressions in (7.16) and simplifying yields $X_{i1} = a_1(s_{i1} - p_1)$, where $a_1 = \gamma\tau_{\epsilon_1}$.

As to equilibrium prices, in the first period market makers observe the aggregate order flow, extract its informational content $z_{C1} = a_1 v + u_1$, and set $p_1 = E[v|z_1]$. In the second period, besides the aggregate order flow, the public signal s_{j1} becomes available. Thus, market makers set the equilibrium price equal to $E[v|z_C^2; s_{j1}] = \alpha E[v|z_C^2] + (1-\alpha)s_{j1}$, where $\alpha = \tau_{C2}/\hat{\tau}_{C2}$.

QED

Figure 1: Comparing depth with a single, risk-neutral insider (continuous line) and with a monopolistic information seller (dotted line), when $\tau_v = \tau_u = \gamma = 1$ and $N = 4$.

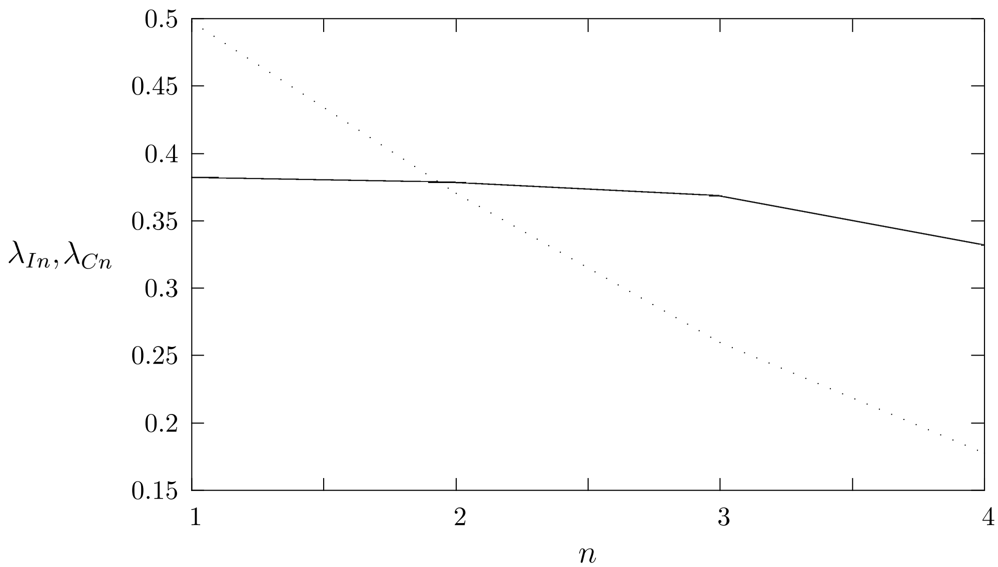

Figure 2: Comparing price informativeness with a single, risk-neutral insider (continuous line) and with a monopolistic information seller (dotted line), when $\tau_v = \tau_u = \gamma = 1$ and $N = 4$.