Social Value of Coordination and Information*

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[Very incomplete and preliminary]

Abstract

This paper examines the social value of coordination and information. Whereas the equilibrium level of coordination depends merely on the degree of strategic complementarity, the socially optimal level of coordination depends also on a second-order external effect. The equilibrium level of coordination is suboptimally low if and only if this second-order effect is weak enough. Turning to the welfare effects of information, we identify higher "transparency" with an increase in the relative precision of public information. At the optimal allocation transparency is socially valuable if and only if coordination is valuable. At the equilibrium allocation, on the other hand, the welfare effects of transparency depend also on the co-variation between the equilibrium level of activity and the first-order externality: welfare increase with transparency if and only if this co-variation is high enough.

We next discuss the implications of the results for various applications, such as economies with production externalities or price complementarities.

We finally examine the properties of optimal fiscal/stabilization policies.

*For useful comments we thank ... .

1 Introduction

[to be completed]

2 The Model

Payoffs. There is a mass-one continuum of agents, distributed over $[0,1]$, each choosing an action $k \in \mathbb{R}$. Let Ψ denote the c.d.f. of the distribution of k in the population. Individual utility is given by

$$u = u(k, K, \tilde{\theta}),$$

where $K = \int k d\Psi(k)$ is the average action, $\tilde{\theta} = (\tilde{\theta}_1, ..., \tilde{\theta}_n) \in \tilde{\Theta} \subseteq \mathbb{R}^n$ are exogenous parameters, and $u : \mathbb{R}^{n+2} \to \mathbb{R}$ is a quadratic function.[1] We interpret k as investment (or effort, or any other economic activity) and $\tilde{\theta}$ as the exogenous productivity (the underlying economic fundamentals for the relevant environment).

Assuming an utilitarian welfare aggregator,[2] social welfare is given by

$$w = w(\tilde{\theta}; \Psi) = \int u(k, K, \tilde{\theta}) d\Psi(k).$$

In principle, welfare depends on the entire distribution Ψ. However, since u is quadratic in k and depends on Ψ only though K, the impact of Ψ on welfare can be summarized by its two first moments, the cross-sectional mean $K = \int k d\Psi(k)$ and the cross-sectional variance $var = \int (k-K)^2 d\Psi(k)$. To see this, note that the second-order Taylor expansion of u around $k = K$ is exact:

$$u(k, K, \tilde{\theta}) = u(K, K, \tilde{\theta}) + u_k(K, K, \tilde{\theta}) \cdot (k - K) + \frac{u_{kk}}{2} \cdot (k - K)^2.$$

Aggregating across agents then gives

$$w = W(K, \tilde{\theta}) + \frac{u_{kk}}{2} \cdot var$$

where $W(K, \tilde{\theta}) \equiv u(K, K, \tilde{\theta})$. This term represents welfare when all agents choose the same action, whereas the additional term $\frac{u_{kk}}{2} \cdot var$ captures the welfare loss associated with cross-sectional heterogeneity in k.

Externalities, complementarities, and concavity. Strategic complementarities emerge whenever $u_{kK} > 0$/ An externality, on the other hand, is present whenever $u_K \neq 0$; we impose no restriction on u_K. We however restrict $u_{kk} < 0$ and $W_{KK} < 0$, which ensure concavity at the individual and the aggregate level; without concavity, either the individual or the social objective would be unbounded.We finally put an upper bound on the degree of complementarity by letting

[1] That is, $u(k, K, \tilde{\theta}) = vMv'$ where M is a $(n+3) \times (n+3)$ real matrix and $v = (1, k, K, \tilde{\theta})$.

[2] Welfare then coincides with expected individual utility "behind the veil of ignorance."

$\alpha \equiv -u_{kK}/u_{kk}$ be strictly smaller than one; this bound is necessary and sufficient for the existence of a unique stable equilibrium under complete information.[3]

Note that $W_{KK} = u_{kk} + 2u_{kK} + u_{KK}$. In the absence of complementarity and externality aggregate welfare inherits the same concavity as individual utility: $W_{KK} = u_{kk}$ when $u_{kK} = u_K = 0$.[4] Introducing a complementarity naturally affects the curvature of aggregate welfare by the addition of the term $2u_{kK}$ in W_{KK}. Allowing then for an external second-order effect, $u_{KK} \neq 0$, affects the curvature of aggregate welfare beyond and above the complementarity, without affecting individual incentives. The complementarity u_{kK} and the external effect u_{KK} will turn out to play a critical role in the social value of coordination and information. We find it useful to let $\eta \equiv u_{KK}/u_{kk}$; the restriction $W_{KK} < 0$ can then be restated as $2\alpha - \eta < 1$, which puts an additional upper bound on the degree of complementarity.

Information. The fundamentals $\tilde{\theta} \in \mathbb{R}^n$ are not known at the time investment decisions are made. They are drawn for a joint Normal distribution with mean $\mu_{\tilde{\theta}}$ and covariance matrix $\Sigma_{\tilde{\theta}}$. For any given $\tilde{\theta} \in \mathbb{R}^n$, let $\theta \in \mathbb{R}$ be the unique solution to $u_k(\theta, \theta, \tilde{\theta}) = 0$. θ is a linear function of $\tilde{\theta}$ and has a nice interpretation: it is the equilibrium level of investment under complete information.[5]

Instead of assuming that agents receive private and public signals about $\tilde{\theta}$, we find it more convenient to assume that they receive signals directly about θ – indeed, as shown in Lemma 1 below, what matters for individual best responses and therefore for equilibrium is merely beliefs about θ. In particular, each agent observes a private signal $x = \theta + \sigma_x \xi_i$ and a public signal $y = \theta + \sigma_y \varepsilon$, where ξ and ε are standard Normal noises. The noises are independent of each other, as well as of θ; ξ is also independent across agents.

We also find it convenient to introduce the random variable $z = \lambda y + (1 - \lambda)\mu_\theta$, where $\lambda = \sigma_y^{-2}/(\sigma_y^{-2} + \sigma_\theta^{-2})$, μ_θ is the prior mean of θ, and σ_θ^2 the corresponding variance. z is a sufficient statistic for the public information (the prior and the public signal). We accordingly let $\sigma_z = (\sigma_y^{-2} + \sigma_\theta^{-2})^{-1/2}$ measure the noise in public information.

Let $\delta \equiv \sigma_z^{-2}/(\sigma_x^{-2} + \sigma_z^{-2})$ and $\sigma \equiv (\sigma_x^{-2} + \sigma_z^{-2})^{-1/2} = (\sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2})^{-1/2}$. The posterior belief of an agent about θ is normal with mean $\mathbb{E}[\theta|x, y] = (1 - \delta)x + \delta z$ and variance $Var[\theta|x, y] = \sigma^2$. Literally interpreted, the dependence of $\mathbb{E}[\theta|x, y]$ on x is the result of the observation of private signals about θ. More generally, however, x introduces idiosyncratic variation in market expectations about the fundamentals and may thus be read also as heterogeneity in the filtering and

[3] While we focus on the case of strategic complements ($\alpha > 0$), the analsis directly extends to substitutes ($\alpha < 0$), provided $\alpha \in (-1, 1)$.

[4] Note that $u_K = 0$ implies $u_{KK} = 0$, but not vice-versa.

[5] To see this, note that, if $\tilde{\theta}$ were known, the best response for an individual would be given by the solution to the FOC $u_k(k, K, \tilde{\theta}) = 0$. The unique (and symmetric) equilibrium under complete information is the unique fixed point of this best response function, or equivalently $k = K = \theta$ where θ solves $u_k(\theta, \theta, \tilde{\theta}) = 0$; equivalently, $\theta = -(u_{kk} + u_{kK})^{-1}[u_k(0, 0, 0)+ < u_{k\tilde{\theta}}, \tilde{\theta} >]$, where $u_{k\tilde{\theta}} = (u_{k\tilde{\theta}_1}, ..., u_{k\tilde{\theta}_n})$ is the gradient of $u_k(k, K, \tilde{\theta})$ with respect to $\tilde{\theta}$ and $< \cdot, \cdot >$ denotes the inner product of two vectors.

interpretation of commonly available information. In this sense, δ measures the level of conformity in market expectations, whereas σ the quality of available information.

The information structure is parametrized by $(\mu_{\tilde{\theta}}, \Sigma_{\tilde{\theta}}; \sigma_x, \sigma_y)$ or equivalently by $(\mu_{\tilde{\theta}}, \Sigma_{\tilde{\theta}}; \delta, \sigma)$. We will see that the welfare losses associated with incomplete information depend on the signal structure (δ, σ) but not on the prior structure $(\mu_{\tilde{\theta}}, \Sigma_{\tilde{\theta}})$. In the following, we focus on comparative statics with respect to (δ, σ) and interpret an increase in δ as an increase in the publicity or transparency of available information and a reduction in σ as an increase in the overall quality of information. We later translate the results in terms of comparative statics with respect to (σ_x, σ_y).[6]

3 Equilibrium

Each agent chooses k so as to maximize his expected utility: $k = \arg\max_{\tilde{k}} \mathbb{E}_{x,y}[u(\tilde{k}, K, \tilde{\theta})]$, where $\mathbb{E}_{x,y}[\cdot]$ is a short-cut for $\mathbb{E}[\cdot|x, y]$. The solution to this optimization problem gives the best response for the individual.

Lemma 1 *The best response is*

$$k = (1-\alpha)\mathbb{E}_{x,y}[\theta] + \alpha\mathbb{E}_{x,y}[K], \tag{1}$$

where $\alpha \equiv -u_{kK}/u_{kk}$ parametrizes the (equilibrium) degree of coordination.

Proof. The FOC, $\mathbb{E}_{x,y}[u_k(k, K, \tilde{\theta})] = 0$, is both necessary and sufficient. Since u is quadratic, for any (k, K, θ), $u_k(k, K, \tilde{\theta}) = u_k(0, 0, \tilde{\theta}) + u_{kk}k + u_{kK}K$. The FOC $\mathbb{E}_{x,y}[u_k(k, K, \tilde{\theta})] = 0$ can thus be expressed as

$$\mathbb{E}_{x,y}[u_k(0, 0, \tilde{\theta})] + u_{kk}k + u_{kK}\mathbb{E}_{x,y}[K] = 0.$$

Similarly, since $u_k(\theta, \theta, \tilde{\theta}) = 0$ for all $(\theta, \tilde{\theta})$, it is also true that $\mathbb{E}_{x,y}[u_k(\theta, \theta, \tilde{\theta})] = 0$, or equivalently

$$\mathbb{E}_{x,y}[u_k(0, 0, \tilde{\theta})] + u_{kk}\mathbb{E}_{x,y}[\theta] + u_{kK}\mathbb{E}_{x,y}[\theta] = 0.$$

Combining the two equations gives (1). ■

The best response thus has a very simple interpretation: the optimal k is a convex combination of the expectation of the complete-information equilibrium level of investment and the expectation of the actual aggregate level of investment. The weight on the latter, α, is given by the degree of complementarity.

[6] What we have in mind is that the transparency of public announcements, policy measures, and news in the media may affect either the noise in publicly available information (a reduction in σ_y for given σ_x(, or the extent to which such information is interpreted differently across market participants (an increase in δ for given σ). In most examples of interest, the results are not sensitive on which interpretation we take.

Given the normality of posterior beliefs about θ, equilibrium investment decisions are linear in x and z, so that $k = \beta x + \gamma z$, where β and γ are constants determined in equilibrium. Then, $K = \beta\theta + \gamma z$ and thus

$$k = (1 - \alpha + \alpha\beta)\left[(1-\delta)x + \delta z\right] + \alpha\gamma z.$$

It follows that $\beta = (1-\alpha)(1-\delta)/[1-\alpha(1-\delta)]$ and $\gamma = \delta/[1-\alpha(1-\delta)]$. Clearly, this is the unique symmetric linear (rational expectations) equilibrium. Furthermore, since best responses are linear in $\mathbb{E}_{x,y}[\theta]$ and $\mathbb{E}_{x,y}[K]$, there do not exist equilibria other than this one.[7]

Proposition 1 *The equilibrium exists, is unique and is given by $k = \beta x + \gamma z$, where*

$$\beta = 1 - \delta - \rho, \qquad \gamma = \delta + \rho, \qquad \text{and} \qquad \rho = \frac{\alpha\delta(1-\delta)}{1-\alpha(1-\delta)}. \tag{2}$$

Note that equilibrium strategies depend only on α and δ, the degree of complementarity and the relative precision of public information. When $\alpha = 0$, $\beta = 1-\delta$ and $\gamma = \delta$. The term ρ thus measures the excess sensitivity of equilibrium allocations to public information as compared to the case where there are no complementarities. Moreover, ρ is increasing in α. Stronger complementarities thus lead to a higher sensitivity of investment to public information. This is a direct implication of the fact that, in equilibrium, the public signal is a relatively better predictor of aggregate behavior than the private signal.

If information were complete, all agents would choose $k = K = \theta$. Incomplete information distorts equilibrium behavior in two ways. First, aggregate noise generates (non-fundamental) volatility in the aggregate level of investment around the complete-information counterpart. Second, idiosyncratic noise generates heterogeneity in level of investment in cross-section. As we show in the next section, both distortions contribute to welfare losses; from an ex ante perspective, the first can be measured by $Var(K-\theta)$, the second by $Var(k-K)$.

Proposition 2 (**i**) *Non-fundamental volatility increases with δ (for given σ, σ_θ) if and only if $\alpha < 1/(1+\delta)$, and decreases with σ_y (for given σ_x, σ_θ) if and only if $\sigma_y^{-2} < (1-\alpha)\sigma_x^{-2} - \sigma_\theta^{-2}$.*

(**ii**) *Heterogeneity decreases with δ (for given σ, σ_θ) and increases with σ_y (for given σ_x, σ_θ).*

Proof. See Appendix. ■

Hence, a higher transparency in public information (higher δ or lower σ_y) necessarily leads to less heterogeneity. However, when transparency is low, this comes at the cost of more volatility. More precise private information, on the other hand, has ambiguous effects on both volatility and heterogeneity.

[7] See Morris and Shin (2002); although their model is a special case of ours and the welfare implications are different, the structure of the best responses is identical.

4 Welfare analysis

In this section we examine aggregate welfare at the equilibrium allocation.

Proposition 3 *Equilibrium welfare satisfies*

$$\mathbb{E}w = \mathbb{E}W(\theta,\tilde{\theta}) + Cov\left[W_K(\theta,\tilde{\theta}),(K-\theta)\right] + \frac{W_{KK}}{2}\cdot Var(K-\theta) + \frac{u_{kk}}{2}\cdot Var(k-K). \quad (3)$$

Proof. Recall that welfare is given by $w = w(\tilde{\theta};\Psi) = W(K,\tilde{\theta}) + \frac{u_{kk}}{2}\cdot var$, where $W(K,\tilde{\theta}) \equiv u(K,K,\tilde{\theta})$, $K = \int_{-\infty}^{\infty} kd\Psi(k)$, $var = \int_{-\infty}^{\infty}(k-K)^2 d\Psi(k)$, and Ψ is the distribution of k in the population. A quadratic expansion of $W(K,\tilde{\theta})$ around $K=\theta$, which is exact since u and thus W is quadratic, gives

$$W(K,\tilde{\theta}) = W(\theta,\tilde{\theta}) + W_K(\theta,\tilde{\theta})\cdot(K-\theta) + \frac{W_{KK}}{2}\cdot(K-\theta)^2.$$

It follows that

$$\mathbb{E}w = \mathbb{E}W(\theta,\tilde{\theta}) + \mathbb{E}\left[W_K(\theta,\tilde{\theta})\cdot(K-\theta)\right] + \frac{W_{KK}}{2}\cdot\mathbb{E}(K-\theta)^2 + \frac{u_{kk}}{2}\cdot var.$$

In equilibrium, $k = \beta x + \gamma z$, implying that $K = \beta\theta + \gamma z$ and $var = Var(k-K|\theta,z) = (\beta\sigma_x)^2$, so that $\mathbb{E}[K-\theta] = 0$ and $\mathbb{E}var = Var(k-K)$, which gives the result. ■

The above has a simple interpretation. The first term, $\mathbb{E}W(\theta,\tilde{\theta})$, is expected welfare under complete information; the other three terms measure the welfare losses generated by incomplete information. In particular, the last two terms are the second-order losses associated with aggregate and idiosyncratic volatility, whereas the covariance term captures a first-order effect.

That idiosyncratic and aggregate volatility generate losses is not surprising given the concavity of preferences; what is interesting is that their relative weight may differ. If there were no complementarity and no externality ($u_{kK} = u_K = 0$), social welfare would simply inherit the curvature of individual utility, so that $W_{KK} = u_{kk}$; substituting idiosyncratic for aggregate volatility at a one-to-one rate would then leave ex ante welfare unaffected. Other things equal, the introduction of a positive complementarity ($u_{kK} > 0$) necessarily reduces the weight on aggregate volatility, whereas a negative complementarity $u_{kK} < 0$ increases it. The complementarity of course influences also equilibrium behavior and therefore affects not only the relative weight that welfare attributes to volatility and heterogeneity, but also the magnitudes of $Var(K-\theta)$ and $Var(k-K)$. A second-order externality ($u_{KK} \neq 0$), on the other hand, can change the relative weight on aggregate volatility without affecting equilibrium behavior.

The covariance term is perhaps less familiar but equally intuitive. Recall that the complete-information equilibrium satisfies $u_k(\theta,\theta,\tilde{\theta}) = 0$ and that $W_K(K,\tilde{\theta}) = u_k(K,K,\tilde{\theta}) + u_K(K,K,\tilde{\theta})$. When there is no externality, $u_k(\theta,\theta,\tilde{\theta}) = 0$ implies $W_K(\theta,\tilde{\theta}) = 0$: the complete-information equilibrium coincides with the first-best allocation. In this case, the covariance term would be zero; this is merely an implication of the fact that small deviations around a maximum have zero first-order effects. In the presence of an externality, instead, $W_K(\theta, \tilde{\theta}) = u_K(\theta, \theta, \tilde{\theta}) \neq 0$ and hence the covariance of $W_K(\theta, \tilde{\theta})$ with $K - \theta$ need not be zero. Welfare then increases (decreases) when $K - \theta$ and $u_K(\theta, \theta, \tilde{\theta})$ are positively (negatively) correlated, that is, if when the "error" in aggregate investment due to incomplete information tends to move in the same (opposite) direction with the "social" return to investment, thus partly offsetting (exacerbating) the first-order loss associated with the externality.

To examine how the combination of these effects affects welfare, let $\mathcal{L} \equiv [-2u_{kk}^{-1}][\mathbb{E}W(\theta, \tilde{\theta}) - \mathbb{E}w]$ denote the welfare loss due to incomplete information normalized by the curvature of the utility function. Next, let $\phi \equiv (-u_{kk}^{-1})Cov(u_K(\theta, \theta, \tilde{\theta}), \theta)/Var(\theta)$ denote the coefficient of the regression of $u_K(\theta, \theta, \tilde{\theta})$ on θ, also normalized by the curvature of the utility function. This coefficient measures how much the externality (evaluated at the complete-information equilibrium) co-varies with the complete-information equilibrium level of activity. We can then show the following.

Proposition 4 *In equilibrium, the welfare losses due to incomplete information are given by*

$$\mathcal{L} = -2\phi Cov(\theta, K - \theta) + (1 - 2\alpha + \eta)Var(K - \theta) + Var(k - K) \quad (4)$$

$$= \left\{ \frac{2\phi[1 - \alpha(1 - \delta)] + (1 - 2\alpha + \eta)\delta + (1 - \alpha)^2(1 - \delta)}{[1 - \alpha(1 - \delta)]^2} \right\} \sigma^2 \quad (5)$$

where ϕ parametrizes the co-variance of the first-order external effect with θ, η the second-order external effect, and α the complementarity.

Proof. See Appendix. ■

The next then analyzes the comparative statics of welfare with respect to the transparency of available information.

Proposition 5 *There exist $\underline{\phi}, \bar{\phi} : \mathbb{R}^2 \to \mathbb{R}$, with $\underline{\phi} \leq \bar{\phi}$, such that:*[8]

(i) When $\phi \geq \bar{\phi}(\alpha, \eta)$, $\mathcal{L}$ monotonically decreases with $\delta \in [0, 1]$.

(ii) When $\phi \in (\underline{\phi}(\alpha, \eta), \bar{\phi}(\alpha, \eta))$, $\mathcal{L}$ initially increases with δ, reaches a maximum at some $\delta \in (0, 1)$, and thereafter decreases with δ.

(iii) When $\phi \leq \underline{\phi}(\alpha, \eta)$, $\mathcal{L}$ monotonically increases with $\delta \in [0, 1]$.

(iv) $\bar{\phi}(\alpha, \eta) < [>]0$ if and only if $\eta < [>]\alpha(2 - \alpha)$.

Proof. See Appendix. ■

The following special cases are particularly useful when we turn to applications.

[8] The monotonicities in (i) and (ii) are strict as long as one of the inequalities is strict.

Corollary 1 *When $\phi \geq 0$, $\eta \leq \alpha$ suffices for welfare to increase with transparency for all $\delta \in [0,1]$.*

Corollary 2 *When $\phi = 0$, $\eta \geq 2\alpha$ suffices for welfare to decrease with transparency for δ small enough.*

Note that the sign of ϕ is the same as the sign of the correlation between the externality $u_K(\theta, \theta, \tilde{\theta})$ and the equilibrium level of investment θ. Moreover, we will show that $\eta \leq \alpha$ is necessary and sufficient for the optimal relative weight on public information to be no less than the equilibrium one ($\gamma^*/\beta^* \geq \gamma/\beta$), or equivalently for coordination to be as valuable at the social level as at the private level. We can thus say that the welfare effect of transparency is positive if (i) the correlation between the externality and the equilibrium level of investment is non-negative; and (ii) coordination is valued socially at least as much as privately. The latter in turn is true unless there is a strong enough second-order external effect from volatility.

Finally, note that when ϕ is sufficiently below zero welfare may decrease not only with a higher δ but also with a lower σ. That is, when the equilibrium externality is strongly negatively correlated with the equilibrium level of activity, information per se can be undesirable.

5 The social value of coordination

In this section we characterize the properties of optimal allocations and relate them to the social value of coordination.

For any given $\tilde{\theta}$, let $\hat{\theta}$ denote the complete-information first-best level of activity, that is, the unique solution to $W_K(\hat{\theta}, \tilde{\theta}) = 0$, or equivalently to $u_k(\hat{\theta}, \hat{\theta}, \tilde{\theta}) + u_K(\hat{\theta}, \hat{\theta}, \tilde{\theta}) = 0$. Since u is quadratic, $\hat{\theta}$ is a linear transformation of $\tilde{\theta}$, like θ; but it differs from θ as long as $u_K(\theta, \theta, \tilde{\theta}) \neq 0$.

For simplicity, we first focus on the case that $\tilde{\theta}$ is unidimensional. In this case, there is a linear function $t : \mathbb{R} \to \mathbb{R}$ such that $\hat{\theta} = t(\theta)$ for all realization of $\tilde{\theta}$. By implication, if the prior for θ is Normal with mean μ_θ and variance σ_θ^2, the prior for $\hat{\theta}$ is Normal with mean $\mu_{\hat{\theta}} = t(\mu_\theta)$ and variance $\sigma_{\hat{\theta}}^2 = (t'\sigma_\theta)^2$. A similar transformation applies to the signal structure: it is as if agents observe private signals $\hat{x} = t(x)$ and a public signal $\hat{y} = t(y)$. The posterior beliefs about $\hat{\theta}$ are thus normal with mean $\mathbb{E}_{x,y}\hat{\theta} = \delta\hat{x} + (1-\delta)\hat{z}$ and variance $\hat{\sigma}^2$, where $\hat{x} = t(x)$, $\hat{z} = t(z)$, $\hat{\sigma} = (t'\sigma)^2$, and δ as before.[9]

Consider now strategies that are indexed by $\hat{\alpha} \in [-1, +1]$ and can be expressed in the following form:

$$k = (1 - \hat{\alpha})\mathbb{E}_{x,y}\hat{\theta} + \hat{\alpha}\mathbb{E}_{x,y}K. \tag{6}$$

These strategies are similar to the best responses in (1) in that they prescribe the agents to follow a weighted average of their belief about a complete-information "target" level and their belief about

[9] That is, $\delta = (t'\sigma_x)^{-2}/((t'\sigma_x)^{-2} + (t'\sigma_x)^{-2}) = \sigma_x^{-2}/(\sigma_x^{-2} + \sigma_z^{-2})$.

the other agents' activity. But they differ in two respects: the "target" is the first-best rather than the equilibrium level of activity; and the degree of coordination is $\hat{\alpha}$ rather than α.

We then ask the following question: among the class of strategies described above, which one maximizes welfare?

Definition 1 *The optimal degree of coordination is the coefficient $\hat{\alpha}$ that maximizes welfare among the class of strategies described by (6).*

The answer turns out to be particularly simple.

Proposition 6 *The optimal degree of coordination is $\hat{\alpha} = 2\alpha - \eta$.*

Proof. For any given $\hat{\alpha}$, the solution to (6) gives

$$k = \hat{\beta}\hat{x} + \hat{\gamma}\hat{z} = t(\hat{\beta}x + \hat{\gamma}z)$$

where

$$\hat{\beta} = 1 - \delta - \hat{\rho}, \qquad \hat{\gamma} = \delta + \hat{\rho}, \qquad \text{and} \qquad \hat{\rho} = \frac{\hat{\alpha}\delta(1-\delta)}{1-\hat{\alpha}(1-\delta)}. \tag{7}$$

(This is the analogue of Proposition 1 replacing (θ, x, z) with $(\hat{\theta}, \hat{x}, \hat{z})$ and α with $\hat{\alpha}$.) Volatility and heterogeneity are then given by

$$Var(K - \hat{\theta}) = \frac{\delta}{[1-\alpha(1-\delta)]^2}\sigma^2 \tag{8}$$

$$Var(k - K) = \frac{(1-\alpha)^2(1-\delta)}{[1-\alpha(1-\delta)]^2}\sigma^2 \tag{9}$$

Welfare in turn can be expressed as follows:

$$\begin{aligned} \mathbb{E}w &= \mathbb{E}W(\hat{\theta}, \tilde{\theta}) + \frac{W_{KK}}{2} \cdot Var(K - \hat{\theta}) + \frac{u_{kk}}{2} \cdot Var(k - K) = \\ &= \mathbb{E}W(\hat{\theta}, \tilde{\theta}) - \frac{u_{kk}}{2}\hat{\mathcal{L}} \end{aligned}$$

where $W(\hat{\theta}, \tilde{\theta})$ is the first-best level of welfare under complete information and

$$\hat{\mathcal{L}} = (1 - 2\alpha + \eta)Var(K - \hat{\theta}) + Var(k - K) \tag{10}$$

is the welfare loss due to incomplete information. (Note that now there is no-first order effect because, by definition of $\hat{\theta}$, $W_K(\hat{\theta}, \tilde{\theta}) = 0$.)

Since $W(\hat{\theta}, \tilde{\theta})$ is independent of $\hat{\alpha}$, the optimal $\hat{\alpha}$ is simply the one that minimizes $\hat{\mathcal{L}}$. Substituting (8) and (9) into (10) yields

$$\hat{\mathcal{L}} = \frac{(1 - 2\alpha + \eta)\delta + (1 - \hat{\alpha})^2(1 - \delta)}{(1 - \hat{\alpha}(1 - \delta))^2}\hat{\sigma}^2$$

It follows that

$$\frac{\partial \hat{\mathcal{L}}}{\partial \hat{\alpha}} = (\hat{\alpha} - (2\alpha - \eta)) \frac{2\delta(1-\delta)\hat{\sigma}^2}{(1-\hat{\alpha}(1-\delta))^3},$$

which is negative iff $\hat{\alpha} < 2\alpha - \eta$ and positive iff $\hat{\alpha} > 2\alpha - \eta$. Therefore, welfare is indeed maximized at $\hat{\alpha} = 2\alpha - \eta$.[10] ■

We conclude that in the absence of a second-order externality ($\eta = 0$) it would have been socially optimal for the agents to coordinate twice as much as they actually do ($\hat{\alpha} = 2\alpha$). This is because, as we discussed earlier, the complementarity reduces the curvature W_{KK} of aggregate welfare with respect to aggregate investment and therefore reduces the welfare loss associated with volatility. On the other hand, a second-order externality that increases the curvature of aggregate welfare ($\eta > 0$) reduces the social value of coordination. The socially optimal level of coordination remains higher than the actual one as long as this second-order effect is weak enough, namely $\hat{\alpha} > \alpha$ if and only if $\eta < \alpha$.

The analysis above characterized optimal allocations restricting attention to strategies that can be described by (6). Alternatively we could consider strategies that are linear in $\hat{x}$ and $\hat{z}$ (or equivalently in x and z) :

$$k = \hat{\beta}\hat{x} + \hat{\gamma}\hat{z}$$

for some $(\hat{\beta}, \hat{\gamma})$. We could then ask what are the weights $(\hat{\beta}, \hat{\gamma})$ that maximize welfare. As indicated by the proof of the proposition above, the answer turns out to be essentially the same: the optimal $(\hat{\beta}, \hat{\gamma})$ are given by (7) with $\hat{\alpha} = 2\alpha - \eta$. There is thus a one-to-one mapping between the socially optimal degree of coordination $\hat{\alpha}$ and the socially optimal excess weight on public information. In particular, the equilibrium puts less (more) weight on public information than what the optimal allocations would do if and only if $\eta < (>) 2\alpha$.

Finally, note that welfare losses evaluated at the optimal allocation (at $\hat{\alpha} = 2\alpha - \eta$) reduce to

$$\hat{\mathcal{L}} = \frac{1}{1 + \delta\hat{\alpha}/(1-\hat{\alpha})}\hat{\sigma}^2,$$

which is monotonic in δ.

Proposition 7 *At the optimal allocation, welfare increases with δ if and only if the optimal degree of coordination $\hat{\alpha}$ is positive.*

This result also helps explain the welfare effect of public information at the equilibrium allocation, as indicated earlier.

`[to be completed]`

[10] Any $\hat{\alpha}$ is trivially optimal in the extreme cases that $\delta = 0$ or $\delta = 1$.

6 Applications

[to be completed]

6.1 Investment (production externalities)

The canonical model of production externalities/investment complementarities can be nested as follows:

$$u(k, K, \tilde{\theta}) = A(K, \tilde{\theta})k - c(k),$$

where $A(K, \tilde{\theta}) = (1-a)\tilde{\theta} + aK$ represents the private return to investment, with $a \in (0, 1/2)$ and $\tilde{\theta} \in \mathbb{R}$, and $c(k) = \frac{1}{2}k^2$ the private cost of investment. Accordingly, welfare is given by

$$w = A(K, \tilde{\theta})K - \frac{1}{2}K^2 - \frac{1}{2}var = (1-a)\tilde{\theta}K - \frac{1}{2}(1-2a)K^2 - \frac{1}{2}var.$$

Variants of this specification appear in Arrow (19??), Bryant (1983), Romer (198?), Cooper and John (1988), Acemoglu (1993), Benhabib and Farmer (1994), and others; the important ingredient is that the private return to investment increases with the aggregate level of investment.[11]

In this example, $\eta = 0$ and $\alpha = \phi = a$. That is, there is no second-order external effect, but there is a first-order externality that is positively correlated with the equilibrium level of investment.

Corollary 3 *In the investment example described above, coordination is inefficiently low. Moreover, welfare unambiguously increases with δ and decreases with σ.*

6.2 Price complementarities

[to be completed]

6.3 Cournot competition

[to be completed]

6.4 No externalities

In the examples considered above, the complementarities came together with externalities. Consider now economies where there is a complementarity but the complete-information equilibrium involves no externality, at either the first or the second order: $u_K(\theta, \theta, \tilde{\theta}) = u_{KK} = 0$. In this case, the complete-information equilibrium is optimal and $\phi = \eta = 0$.

Corollary 4 *Suppose there is no externality as described above. Coordination is inefficiently low and welfare unambiguously increases with δ and decreases with σ.*

[11] This was the example we examined in Angeletos and Pavan (2004).

6.5 Undesirable coordination

We next consider environments in which the complete-information equilibrium is optimal and, in addition, the agents' desire to coordinate under incomplete information is not warranted from a social perspective. The first condition is satisfied if and only if $u_K(\theta, \theta, \tilde{\theta}) = 0$ (and hence $\phi = 0$); by the second we mean $u_{KK} = -2u_{kK}$ (equivalently, $\eta = -2\alpha$) so that volatility and heterogeneity are equally weighted in welfare losses.

An example of this short can be constructed by modifying the investment game as follows: let individual utility be given by

$$u(k, K, \tilde{\theta}) = A(K, \tilde{\theta})k - c(k) - B(K, \tilde{\theta}),$$

where A and c are as before and $B(K, \tilde{\theta}) = (\tilde{\theta} - A(K, \tilde{\theta}))K$. Welfare then reduces to

$$w = \int (\tilde{\theta}k - c(k))d\Psi(k)$$

From a social perspective, it is then as if utility were given simply by $u = \tilde{\theta}k - c(k)$, in which case there is of course no value to coordination.[12]

Corollary 5 *Suppose $u_K(\theta, \theta, \tilde{\theta}) = 0$ but $u_{KK} = -2u_{kK}$, so that the optimal degree of coordination is zero. Then welfare increases with δ iff $\delta > (1 - \alpha)/(2 - \alpha)$ and is maximized iff either $\delta = 0$ or $\delta = 1$.*

6.6 Inefficient fluctuations

`[to be completed]`

7 Policy

`[to be completed]`

8 Concluding Remarks

`[to be completed]`

[12] In this case, we have $\eta = 2\alpha$, $q = \alpha/(1 - \alpha)$, $\phi = 0$, and

$$\mathcal{L} = Var(K - \theta) + Var(k - K) = \frac{(1 - \alpha)^2(1 - \delta) + \delta}{[1 - \alpha(1 - \delta)]^2}\sigma^2.$$

Note that now $W_{KK} < 0$ iff $\alpha < 1$. $\mathcal{L}$ is positive and increasing in σ, but non-monotonic in δ : $\mathcal{L}$ attains its minimum $(= \sigma)$ at $\delta \in \{0, 1\}$ and its maximum at $\delta = \frac{1-\alpha}{2-\alpha}$; $\partial\mathcal{L}/\partial\delta > 0$ for $\delta < \frac{1-\alpha}{2-\alpha}$; and $\partial\mathcal{L}/\partial\delta < 0$ for $\delta > \frac{1-\alpha}{2-\alpha}$.

9 Appendix

Proof of Proposition 2. Since $K-\theta=\gamma(z-\theta)=\gamma\left[\lambda(y-\theta)+(1-\lambda)(\mu_\theta-\theta)\right]$, non-fundamental volatility is given by

$$\begin{aligned} Var(K-\theta) &= \gamma^2(\lambda^2\sigma_y^2+(1-\lambda)^2\sigma_\theta^2)=\gamma^2\sigma_z^2 \\ &= \frac{\sigma_y^{-2}+\sigma_\theta^{-2}}{[\sigma_y^{-2}+\sigma_\theta^{-2}+(1-\alpha)\sigma_x^{-2}]^2} \\ &= \frac{\delta}{[1-\alpha(1-\delta)]^2}\sigma^2 \end{aligned}$$

and since $k-K=\beta(x-\theta)$, heterogeneity is given by

$$\begin{aligned} Var(k-K) &= (\beta\sigma_x)^2 \\ &= \frac{(1-\alpha)^2\sigma_x^{-2}}{[\sigma_y^{-2}+\sigma_\theta^{-2}+(1-\alpha)\sigma_x^{-2}]^2} \\ &= \frac{(1-\alpha)^2(1-\delta)}{[1-\alpha(1-\delta)]^2}\sigma^2. \end{aligned}$$

(Here we have used $\sigma_x=\sigma/\sqrt{1-\delta}$, $\sigma_z=\sigma/\sqrt{\delta}$, and the formulas for β and γ). ■

Proof of Proposition 4. In the complete-information equilibrium, $W_K(\theta,\tilde{\theta})=u_K(\theta,\theta,\tilde{\theta})$. Since both θ and $W_K(\theta,\tilde{\theta})$ are linear transformations of $\tilde{\theta}$ and the latter is jointly Normal, regressing $W_K(\theta,\tilde{\theta})$ on θ gives

$$W_K(\theta,\tilde{\theta})=const+\hat{\phi}\theta+error$$

where $\hat{\phi}=Cov(W_K(\theta,\tilde{\theta}),\theta)/Var(\theta)$ and *error* is a linear combination of $\tilde{\theta}$ that is orthogonal to θ. It follows that

$$Cov(W_K(\theta,\tilde{\theta}),K-\theta)=\hat{\phi}Cov(\theta,K-\theta)+Cov(error,K-\theta)$$

Next, note that $K-\theta$ is a linear combination of θ and the public signal error ε. Since the latter, by assumption, is orthogonal to $\tilde{\theta}$, and since *error*, by construction, is a linear combination of $\tilde{\theta}$ that is orthogonal to θ, we have that $Cov(error,K-\theta)=0$. It follows that

$$Cov(W_K(\theta,\tilde{\theta}),K-\theta)=\hat{\phi}Cov(\theta,K-\theta)=(-u_{kk})\phi Cov(\theta,K-\theta)$$

where $\phi\equiv(-u_{kk}^{-1})\hat{\phi}=(-u_{kk}^{-1})Cov(W_K(\theta,\tilde{\theta}),\theta)/Var(\theta)$. In the special case that $\tilde{\theta}\in\mathbb{R}$ (unidimensional fundamentals), $\phi=\alpha-\eta+q(1-\alpha)$, where $q\equiv u_{K\tilde{\theta}}/u_{k\tilde{\theta}}$.

Next, recall that $W_{KK}=u_{kk}+2u_{kK}+u_{KK}=u_{kk}(1-2\alpha+\eta)$. Substituting in (3) and using the definition of $\mathcal{L}$ gives (4). Finally, note that $k-K=\beta(x-\theta)$, and $K-\theta=\gamma(z-$

$\theta) = \gamma\left[\lambda(y-\theta)+(1-\lambda)(\mu_\theta-\theta)\right]$. It follows that $Var(k-K)=\beta^2\sigma_x^2$, $Var(K-\theta)=\gamma^2\sigma_z^2$, and $Cov(\theta,K-\theta)=-\gamma(1-\lambda)\sigma_\theta^2$. Substituting the equilibrium values of β and γ gives

$$Var(k-K) = \frac{(1-\alpha)^2(1-\delta)}{[1-\alpha(1-\delta)]^2}\sigma^2 \tag{11}$$

$$Var(K-\theta) = \frac{\delta}{[1-\alpha(1-\delta)]^2}\sigma^2 \tag{12}$$

$$Cov(\theta,K-\theta) = -\frac{1}{1-\alpha(1-\delta)}\sigma^2 \tag{13}$$

Together with (4) the above gives (5). ■

Proof of Proposition 5. Taking the derivative of $\mathcal{L}$ with respect to δ, we have that

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\delta} &= \frac{-\alpha(2-3\alpha+\alpha^2(1-\delta))+\eta(1-\alpha(1+\delta))-2\phi(1-\alpha(1-\delta))}{(1-\alpha(1-\delta))^3} = \\ &= 2(1-\alpha(1-\delta))^{-3}\cdot[f(\alpha,\eta,\delta)-\phi],\end{aligned}$$

where

$$f(\alpha,\eta,\delta)\equiv\frac{-\alpha(2-3\alpha+\alpha^2(1-\delta))+\eta(1-\alpha(1+\delta))}{2(1-\alpha(1-\delta))}.$$

Parts (i)-(iii) then follow immediately if we let

$$\underline{\phi}(\alpha,\eta)\equiv\min_{\delta\in[0,1]}f(\alpha,\eta,\delta)\quad\text{and}\quad\bar{\phi}(\alpha,\eta)\equiv\max_{\delta\in[0,1]}f(\alpha,\eta,\delta).$$

Next, to understand the properties of the bounds $\underline{\phi}$ and $\bar{\phi}$, note that the sign of the derivative of f with respect to δ is the same as that of $\alpha-\eta$:

$$\frac{\partial f}{\partial\delta}=\frac{\alpha(1-\alpha)(\alpha-\eta)}{(1-\alpha(1-\delta))^2}.$$

When $\eta<\alpha$, f is increasing in δ and

$$\begin{aligned}\underline{\phi}(\alpha,\eta) &= f(\alpha,\eta,0)=\tfrac{1}{2}(-\alpha(2-\alpha)+\eta)<-\tfrac{1}{2}\alpha(1-\alpha)<0\\ \bar{\phi}(\alpha,\eta) &= f(\alpha,\eta,1)=\tfrac{1}{2}(-\alpha(2-3\alpha)+\eta(1-2\alpha))\leq-\min\{\alpha,\tfrac{1}{2}\}(1-\alpha)^2<0.\end{aligned}$$

(The last two inequalities in the formula for $\bar{\phi}$ follow from the following: if $\alpha\leq1/2$, $f(\alpha,\eta,1)$ is non-decreasing in η, in which case $\eta<\alpha$ implies $f(\alpha,\eta,1)<f(\alpha,\alpha,1)=-\alpha(1-\alpha)^2<0$; if instead $\alpha>1/2$, $f(\alpha,\eta,1)$ is decreasing in η, in which case $\eta>-1+2\alpha$ (equivalently, $W_{KK}<0$) implies $f(\alpha,\eta,1)<f(\alpha,-1+2\alpha,1)=-\frac{1}{2}(1-\alpha)^2<0$.). When instead $\eta=\alpha$, f is independent of δ and

$$\underline{\phi}(\alpha,\eta)=\bar{\phi}(\alpha,\eta)=-(1-\alpha)<0.$$

Finally, when $\eta>\alpha$, f is decreasing in δ,

$$\begin{aligned}\underline{\phi}(\alpha,\eta) &= f(\alpha,\eta,1)=\tfrac{1}{2}(-\alpha(2-3\alpha)+\eta(1-2\alpha))\\ \bar{\phi}(\alpha,\eta) &= f(\alpha,\eta,0)=\tfrac{1}{2}(-\alpha(2-\alpha)+\eta).\end{aligned}$$

Hence, $\bar{\phi} > 0$ if and only if $\eta > 0$ and $-\alpha(2-\alpha)+\eta > 0$, or equivalently if and only if $\eta > \alpha(2-\alpha)$, which completes the proof. ■