Efficiency, Insurance, and Redistribution Effects of Government Policies

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Introduction

Motivation

Government policies affect social welfare through many channels:

- They distort production and allocation of goods
- They redistribute resources across agents

Goal. Given two feasible allocations x^* (status quo) and x^{**} (reform), decompose

$$\mathcal{W}(x^{**}) - \mathcal{W}(x^*) = \underbrace{R(x^*, x^{**})}_{\text{Redistribution}} + \underbrace{E(x^*, x^{**})}_{\text{Efficiency}}$$

and further split E into insurance, production efficiency, allocative efficiency, ...

Want Operator

$$\Delta \mathcal{W} = R + E$$

The decomposition satisfy consistency requirements rooted in Pareto efficiency and other desirable properties

- **Pareto-consistency**: $E = 0$ along the frontier; $E < 0$ when moving inside
- **Income-based redistribution**: R $\longleftrightarrow$ changes in distribution of real consumption/disposable incomes
- **Numeraire invariance**, **direction invariance**

Setup

Environment

Agents and goods. I consumers, K final goods. Consumer i has utility $u_i : \mathbb{R}_+^K \to \mathbb{R}$, strictly increasing, strictly concave, twice differentiable.

Allocations. $x \in \mathbb{R}_+^{I \times K}$. $x_{i,k}$ = consumption of good k by i. Aggregate: $X = \sum_i x_i$.

Production. Convex, compact set $\mathcal{Y} \subset \mathbb{R}^K$ with free disposal.

Allocation x is feasible if $X \in \mathcal{Y}$.

Represent $\mathcal{Y}$ using $F(X, A) \leq 0$, where A are economy-wide endowments.

Scope. This framework covers:

- Deterministic and stochastic economies (goods $\equiv$ state-contingent claims)
- Multiple sectors, intermediate goods, production networks
- Distortions: incomplete markets, monopoly power, taxes, borrowing constraints

Social Welfare and the Decomposition Problem

Social welfare function: $\mathcal{W}(x) = \sum_i \bar{\alpha}_i \, u_i(x_i)$, with weights $\bar{\alpha}_i \geq 0$ chosen by the researcher.

More general $\mathcal{W}$. Linearity is for exposition only. Everything goes through for any Pareto-monotone aggregator $\mathcal{W} = \mathcal{G}(\{u_i\}_i)$ increasing in each argument — $\mathcal{W}$ can be reported in any units (e.g. % consumption).

The problem. Given status quo x^* and reform x^{**}, decompose:

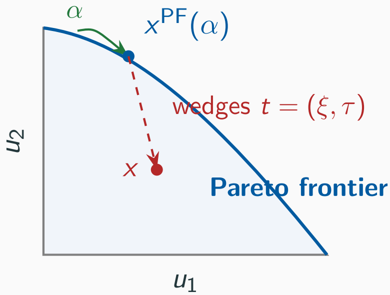
$$\mathcal{W}(x^{**}) - \mathcal{W}(x^*) \;=\; \underbrace{R}_{\text{Redistribution}} + \underbrace{E}_{\text{Efficiency}}$$

Requirements: "Efficiency" anchored to *Pareto efficiency*; "Redistribution" reflects changes in *real disposable income*.

Coordinate System

Main Idea: Coordinate system

Represent every feasible allocation x with coordinates (α, t):



- α: Generalized Pareto–Negishi (PN) weights $\longrightarrow$ captures **inequality**
- $t = (\xi, \tau)$: production and allocation wedges $\longrightarrow$ captures **distortions**

The coordinate system allows "decomposing" changes in welfare into contributions coming from α vs. changes in t.

PN Weights as Coordinates on the Pareto Frontier

An efficient allocation x solves $\max_{x,X} \sum_i \alpha_i u_i(x_i)$ s.t. $\sum_i x_i \leq X$, $X \in \mathcal{Y}$, for PN weights α.

Key insight. The map $\alpha \longmapsto x^{\text{PF}}(\alpha)$ is a coordinate system for the Pareto frontier:

- Each α indexes a unique point on the frontier
- PN weights α capture implicit inequality

Changes in allocation along the frontier $\longleftrightarrow$ changes in α $\longleftrightarrow$ pure redistribution.

Some Objects: Prices, Income, and Indirect Utility

Supporting prices. The Pareto problem generates Lagrange multipliers $\mathrm{P}(\alpha) \in \mathbb{R}_+^K$ on feasibility. These act as implicit prices: $p \propto \mathrm{P}(\alpha)$.

Income at allocation x. Individual income $y_i := \sum_k p_k x_{i,k}$; aggregate income $Y := \sum_i y_i = \sum_k p_k X_k$.

Maximum income. $Y^{\max}(p) := \max_{X \in \mathcal{Y}} \sum_k p_k X_k$ — aggregate income at efficient production.

Indirect utility. $V_i(p, y_i) := \max_{\tilde{x}_i} \; u_i(\tilde{x}_i)$ s.t. $\sum_k p_k \tilde{x}_{i,k} \le y_i$.

Extending the Coordinates beyond the Frontier

Idea. Associate with each interior point x

- a point on the frontier $x^{\mathsf{PF}}(\alpha)$
- a vector of wedges t that capture efficiency distortions relative to $x^{\mathsf{PF}}(\alpha)$

Distortions t are defined so that a reform that moves inside the frontier has a negative efficiency contribution.

Extending the Coordinates beyond the Frontier

Lemma. For any feasible $x \in \mathcal{Y}$, there exist (α, ξ, τ) such that, with $p \propto \mathrm{P}(\alpha)$, (Y, y, x) satisfies:

(i′) **Production wedge**: $Y = (1-\xi)\, Y^{\max}(p), \quad \xi \in [0,1]$

(ii) **Income distribution**: y solves $\max \sum_i \alpha_i V_i(p, \tilde{y}_i)$ s.t. $\sum_i \tilde{y}_i \leq Y$

(iii′) **Allocation wedges**: $1 + \tau_{i,k} = \dfrac{p_1}{p_k} \dfrac{u_{i,k}(x_i)}{u_{i,1}(x_i)}$ for $k > 1$

(α, ξ, τ) separate the inequality content of x (in α) from the distortions relative to the frontier (in ξ, τ).

Welfare in Coordinates

Write $t = (\xi, \tau)$. Social welfare expressed in coordinates:

$$W(\alpha, t) \;:=\; \sum_i \bar{\alpha}_i \, u_i\big(\mathcal{X}_i(\alpha, t)\big)$$

Lemma (Distortions reduce welfare). For all feasible x with coordinates (α, t):

$$\boxed{W(\alpha, 0) \;\geq\; W(\alpha, t)}$$

$(\alpha, 0)$ is the Pareto frontier point associated with α — welfare is maximized at $t = 0$.

This is what makes t a valid measure of efficiency loss: holding α fixed, any nonzero t strictly hurts welfare.

Welfare Decomposition

Marginal Decomposition

Fix x^* and consider $x^{**} = x^* + \varepsilon \hat{x}$. Write $\alpha^* = \alpha(x^*)$, $t^* = t(x^*)$.

$$\hat{\mathcal{W}} = W_\alpha \hat{\alpha} + W_t \hat{t}$$

Redistribution | **Efficiency**

$$\hat{\mathcal{W}} = \boxed{\hat{R} := W_\alpha \hat{\alpha}} + \boxed{\hat{E} := W_t \hat{t}}$$

- $\hat{R}/\hat{\mathcal{W}}$: fraction of marginal welfare change from redistribution
- $\hat{E}/\hat{\mathcal{W}}$: fraction from efficiency
- They sum to one; each can be negative or exceed one

Global Decomposition: Shapley Values

For non-marginal reforms, W is generally non-separable in (α, t).

Shapley values provide a symmetric attribution:

$$R(x^*, x^{**}) = \frac{1}{2}\left[W(\alpha^{**}, t^*) - W(\alpha^*, t^*)\right] + \frac{1}{2}\left[W(\alpha^{**}, t^{**}) - W(\alpha^*, t^{**})\right]$$

$$E(x^*, x^{**}) = \frac{1}{2}\left[W(\alpha^*, t^{**}) - W(\alpha^*, t^*)\right] + \frac{1}{2}\left[W(\alpha^{**}, t^{**}) - W(\alpha^{**}, t^*)\right]$$

$$\boxed{W(x^{**}) - W(x^*) = R(x^*, x^{**}) + E(x^*, x^{**})}$$

- Averages marginal contribution of α (resp. t) across both orderings
- Converges to the marginal decomposition for small reforms

Properties of the Decomposition

(i) **Weak Pareto-consistency**: x^*, x^{**} both on PF $\Rightarrow$ $E = 0$

(ii) **Strong Pareto-consistency**: x^* on PF, x^{**} not $\Rightarrow$ $E < 0$

(iii) **Numeraire invariance**: Ratios $R/(W^{**} - W^*)$ and $E/(W^{**} - W^*)$ independent of price normalization and good labeling

(iv) **Direction invariance**: Same shares whether reform goes $x^* \to x^{**}$ or $x^{**} \to x^*$

(v) **Income-based redistribution**: If $\sum_k p_k^* x_{i,k}^* = \sum_k p_k^* x_{i,k}^{**}$ for all i, then $R = 0$

Properties (i)–(ii) formalize the Pareto-based notion of efficiency. Property (v) links redistribution to changes in *real disposable income*.

Extensions

Finer Decompositions of Efficiency

Production vs. allocative. $t = (\xi, \tau)$ splits E into Shapley contributions of ξ (production) and τ (allocative): $W(x^{**}) - W(x^{*}) = R + E^{\mathrm{pr}} + E^{\mathrm{al}}$.

Production: wasteful G, misallocated intermediates. Allocative: labor wedges, consumption taxes, incomplete markets.

Isolate insurance. In stochastic economies ($K = N \times S$), full-insurance benchmark x_i^{ins} re-uses each agent's per-good expenditure across states. Factor:

$$1 + \tau_{i,n}(s) \;=\; \underbrace{(1 + \tau^{g}_{i,n})}_{\text{goods (across } n)} \quad \underbrace{(1 + \tau^{\mathrm{ins}}_{i,n}(s))}_{\text{insurance (across } s)}$$

Shapley on $(\alpha, \xi, \tau^{g}, \tau^{\mathrm{ins}})$ gives $\hat{W} = \hat{R} + \hat{E}^{\mathrm{pr}} + \hat{E}^{g} + \hat{E}^{\mathrm{ins}}$.

All Prop.-1 properties extend; $\hat{E}^{\mathrm{ins}}$ further splits into aggregate vs. idiosyncratic.

From Coordinates to Policies

Why Map to Policies?

So far: Decomposed welfare using coordinates (α, t).

Now: Any feasible allocation can be viewed as a CE with fictitious instruments:

- Government purchases G, lump-sum transfers T
- Consumer-specific taxes ι (mapped to allocation wedges τ)
- Producer taxes ς

Benefits:

- Reinterprets efficiency in terms of *deadweight losses* and *fiscal externalities*
- Connects to public finance (Diamond–Mirrlees, Harberger)
- Clarifies income effects, substitution effects, marginal cost of public funds

Competitive Equilibrium with Policies

Government policy: (G, T, ι, ς). CRS technology $F(X - A) \leq 0$.

A competitive equilibrium given government policy consists of prices r, allocation x, and net output Z such that

- Consumer i: $\max u_i(\tilde{x}_i)$ s.t. $\sum_k (1 + \iota_{i,k}) r_k (\tilde{x}_{i,k} - a_{i,k}) \leq T_i$
- Firm: $\max \sum_k (1 - \varsigma_k) r_k Z_k$ s.t. $F(Z) \leq 0$
- Government budget

$$\sum_i \underbrace{\left[T_i - \sum_k \iota_{i,k} r_k \left(x_{i,k} - a_{i,k}\right)\right]}_{:= T_i^{net}} = \underbrace{\sum_k \varsigma_k r_k Z_k - \sum_k r_k G_k}_{:= S} .$$

A feasible allocation x can be supported by a competitive equilibrium for some government policy (G, T, ι, ς).

Decomposition in Policy Space

Simple case: reform stays on the PPF ($\hat{S} = 0$, since $\xi = S / \sum_k p_k A_k$) and supporting prices are constant ($\mathrm{P}(\alpha)$ independent of α). WLOG $\iota = \tau$, $r = p$.

Definitions.

- $\eta_i = \sum_k p_k \tau_{i,k} \frac{\partial x_{i,k}}{\partial m}$ — wgt. income elasticity
- $\vartheta^c_{i,k} = \sum_l \tau_{i,l}\, p_l\, x_{i,l}\, \zeta^c_{i,lk}$ — wgt. compensated elasticity

$$\hat{W} = \underbrace{\sum_i \bar{\alpha}_i \frac{u_{i,1}}{1-\eta_i} \hat{T}_i^{\mathrm{net}}}_{\hat{R}} + \underbrace{\sum_i \bar{\alpha}_i \frac{u_{i,1}}{1-\eta_i} \sum_k \vartheta^c_{i,k} \ln\widehat{(1+\tau_{i,k})}}_{\hat{E}^{\mathrm{al}}}$$

Extensions: $\hat{S} \neq 0$ (production efficiency) $\mathrm{P}(\alpha)$ varies with α

Decomposition in Policy Space: Interpretation

$$\hat{W} = \underbrace{\sum_i \bar{\alpha}_i \frac{u_{i,1}}{1-\eta_i} \hat{T}_i^{\mathrm{net}}}_{\hat{R}} + \underbrace{\sum_i \bar{\alpha}_i \frac{u_{i,1}}{1-\eta_i} \sum_k \vartheta^c_{i,k} \ln\widehat{(1+\tau_{i,k})}}_{\hat{E}^{\mathrm{al}}}$$

Interpretation:

- $\hat{R}$: changes in net transfers $\hat{T}_i^{\mathrm{net}}$, weighted by social marginal utility $\bar{\alpha}_i u_{i,1}$
- $\hat{E}^{\mathrm{al}}$: marginal deadweight loss (MDWL) from consumer tax changes; agent i's contribution is a compensated-elasticity-weighted change in tax wedges
- Factor $1/(1-\eta_i)$ in both terms = fiscal-externality multiplier from income-effect feedback

Comparison with Alternative Decompositions

Two Prominent Alternatives

Benabou–Floden (BF). Efficiency ω_E: solution to $u\big((1+\omega_E)\bar{c}^*, \bar{\ell}^*\big) = u\big(\bar{c}^{**}, \bar{\ell}^{**}\big)$

- Based on per-capita aggregates; no social weights
- Measures how much numeraire good compensates for aggregate changes

Dávila–Schaab (DS). Efficiency $d\bar{c} = \frac{1}{I}\sum_i dc_i$, where $dc_i = \hat{c}_i + \frac{u_\ell}{u_c}\hat{\ell}_i$

- Money-metric valuation of individual utility changes, summed with equal weights
- Marginal decomposition accumulated via line integral

Common feature: Both measure efficiency contributions *without* social welfare weights.

$\Rightarrow$ Both are subject to the aggregation critique from Section 6.

Example: Capitalist–Worker Economy

Setup. Two types of households, linear technology.

- **Capitalist** (type 1): zero labor productivity, endowment $a_1 > 0$
- **Worker** (type 2): unit productivity, possibly in debt ($a_2 < 0$)

Laissez-faire allocation x^*. Consider taxing the capitalist by T:

	Policy I	**Policy II**
Transfer to worker	Lump-sum	Proportional labor subsidy
Pareto frontier?	x_T^I on PF $\forall T$	x_T^{II} off PF for $T \neq 0$
Dominance	—	x_T^I Pareto-dominates x_T^{II}

Both deliver the same redistribution; Policy I is non-distortionary.

What Should We Expect?

Policy I: lump-sum transfers.

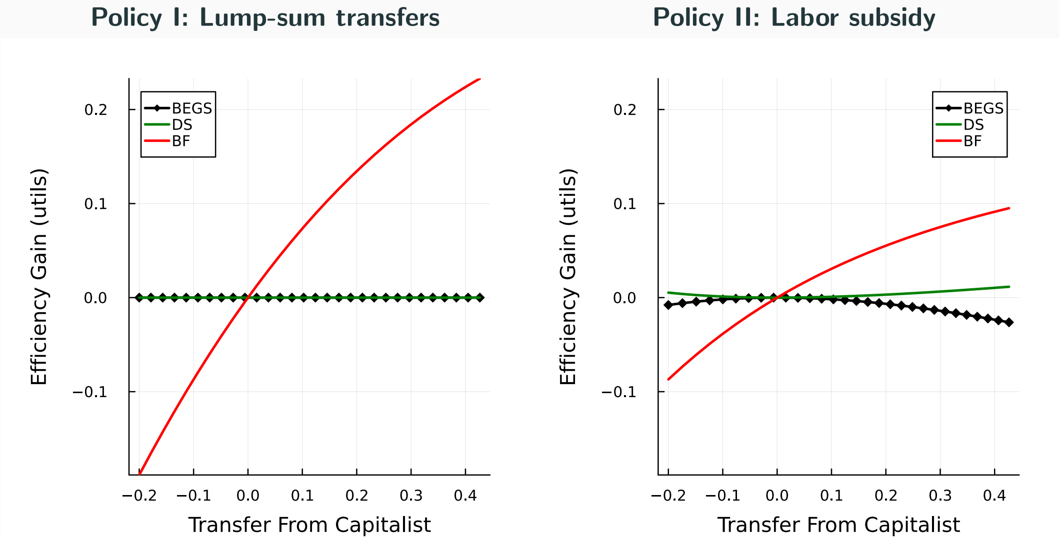
- Pure redistribution: moves along the Pareto frontier
- $\Rightarrow E = 0$ and $R = \hat{W}$ (100% redistribution) for any parameters and any T

Policy II: labor subsidy.

- Distortionary: moves the allocation inside the frontier for $T \neq 0$
- Deadweight loss is convex in the implied subsidy/tax rate
- $\Rightarrow E(T) < 0$ for $T \neq 0$, $E(0) = 0$, hump-shaped (inverse-U) as T varies

Test: does each decomposition (BEGS, BF, DS) deliver these patterns?

Capitalist–Worker Example: Efficiency contribution



x-axis: transfer T from capitalists; y-axis: efficiency gain $E(x^*, x_T^I)$ resp. $E(x^*, x_T^{II})$ (utils). BEGS in black,

Summary across Decompositions

Policy I (lump-sum) — predicted $E = 0$:

- **BEGS:** $E = 0$ always
- **BF:** $E \neq 0$ when worker has debt
- **DS:** $E = 0$ at interior

Policy II (labor subsidy) — predicted $E < 0$, hump-shaped, $E(0) = 0$:

- **BEGS:** negative, inverse-U
- **BF:** switches sign at $T = 0$
- **DS:** U-shaped — finds efficiency gains from distortions; ranks II $\succ$ I

Only BEGS recovers both predictions and ranks the non-distortionary policy as more efficient.

Quantitative Applications

Environment

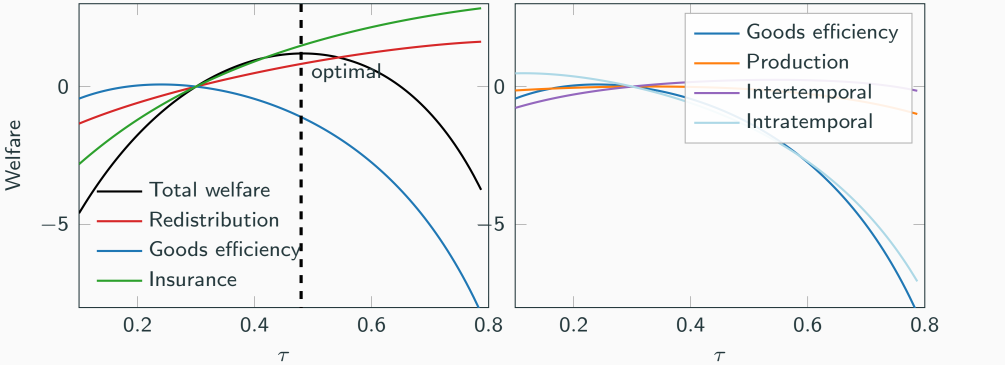
Standard incomplete-markets economy:

- Continuum of households with idiosyncratic productivity $\varepsilon_{i,t} = \varepsilon^P_{i,t}\varepsilon^T_{i,t}$
- Persistent component: $\log \varepsilon^P_{i,t} = \rho^P \log \varepsilon^P_{i,t-1} + \eta^P_{i,t}$
- Preferences: $\mathbb{E}_0 \sum_t \beta^t \left[\frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \Psi \frac{l_{i,t}^{1+\gamma}}{1+\gamma}\right]$
- Budget: $c_{i,t} + a_{i,t+1} = (1-\tau_t)(r_t a_{i,t} + w_t \varepsilon_{i,t} l_{i,t}) + a_{i,t} + T_t, \quad a_{i,t+1} \geq \underline{a}$
- Cobb–Douglas production: $Y_t = AK_t^\theta L_t^{1-\theta}$

Calibration: $\sigma = 1$, $\gamma = 2$, $\beta = 0.96$, $\theta = 0.36$, $\delta = 0.10$.
Baseline: $\tau^* = 30\%$, $B/Y = 100\%$, $G/Y = 15\%$. Earnings process from Krueger et al. (2016).

Application 1: Income-Tax Reform

Reform: $\tau^* = 30\% \to \tau^{**}$, financed by lump-sum transfers; utilitarian optimum $\tau^{**} = 48\%$.



Efficiency Subcomponents

Why goods efficiency is inverse-U shaped:

- **Intratemporal distortion** (labor wedge): Higher $\tau \Rightarrow$ larger labor-leisure distortion $\Rightarrow$ monotonically worsening
- **Intertemporal smoothing**: Incomplete markets cause excessive precautionary saving $\Rightarrow$ overaccumulation of capital, depressed interest rates. Higher τ with transfers *mitigates* this $\Rightarrow$ improving
- **Production efficiency**: Sub-optimal capital-labor ratio—small and negative throughout

Components of Welfare Change

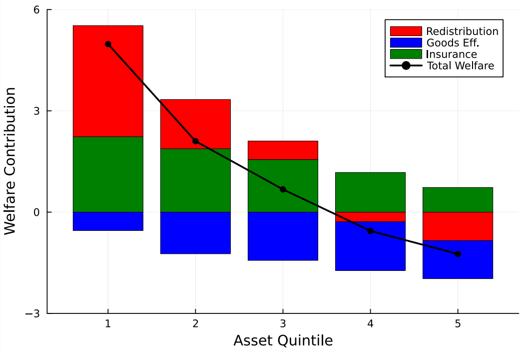
Decomposition at the optimum (reform: $\tau = 30\% \to 48\%$):

Component	**Share of ΔW**	**Sign**
Redistribution	$+68\%$	$+$
Insurance	$+124\%$	$+$
Goods-related efficiency	-92%	$-$
– *intratemporal (labor)*	*dominant negative*	$-$
– *intertemporal (smoothing)*	*modest positive*	$+$
– *production*	*small negative*	$-$

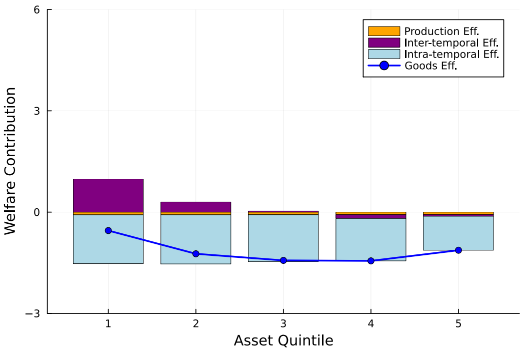
Bottom line: At the optimal tax rate, efficiency losses are dominated by the intratemporal labor distortion. Redistribution-neutral optimum: $\tau = 43\%$ (vs. 48% with redistribution).

Cross-Sectional Patterns

By initial wealth quintile (reform: $\tau = 30\% \to 48\%$):



Total, redistribution, insurance, goods efficiency



Goods efficiency: production, intertemporal, intratemporal

Redistribution drives the gradient; insurance positive throughout; low-wealth benefit from intertemporal smoothing, high-wealth bear the labor wedge.

Conclusion

Summary

1. Coordinate system. Map any feasible allocation to (α, t): PN weights capture inequality; wedges (ξ, τ) capture distortions.

2. Welfare decomposition. Attribute welfare changes to redistribution $(\Delta\alpha)$ and efficiency (Δt), with further splits into production, allocative, and insurance components.

3. Desirable properties. Weak & strong Pareto-consistency, numeraire invariance, direction invariance, income-based redistribution.

4. Alternative decompositions (BF, DS) violate these properties—they can find efficiency gains from adding distortions, and are sensitive to the choice of numeraire.

5. Application. Income-tax reform: redistribution and insurance drive welfare gains; the labor wedge dominates efficiency losses.

Thank you!

Visualizing the α Fixed Point: Edgeworth Box

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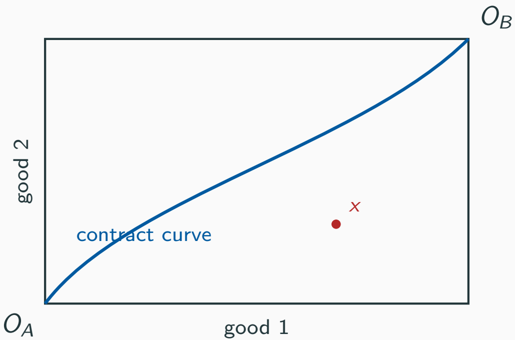

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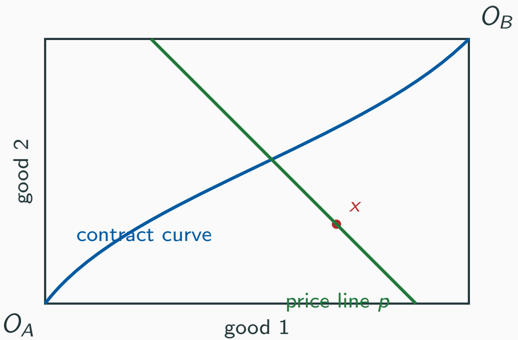

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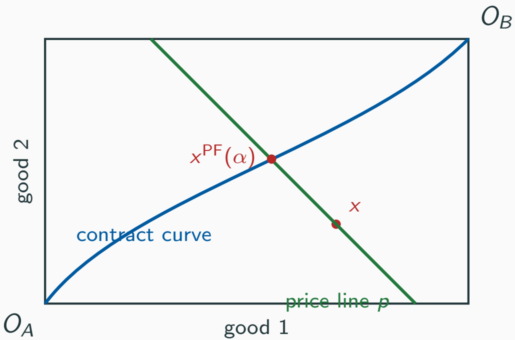

α fixed point. Given interior x, the price line through x meets the contract curve at $x^{\text{PF}}(\alpha)$ with

$$y_i \;=\; p \cdot x_i \;=\; p \cdot x_i^{\text{PF}}(\alpha) \quad \forall\, i,$$

i.e., x and $x^{\text{PF}}(\alpha)$ share the same real wealth distribution — moving along p is *not* redistributive. Here $p \propto P(\alpha)$ supports $x^{\text{PF}}(\alpha)$ with $\mathrm{MRS}_A = \mathrm{MRS}_B = p_1/p_2$.

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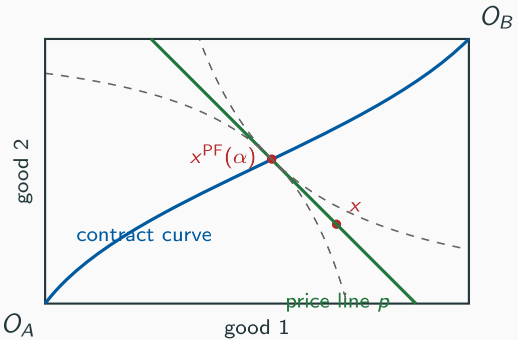

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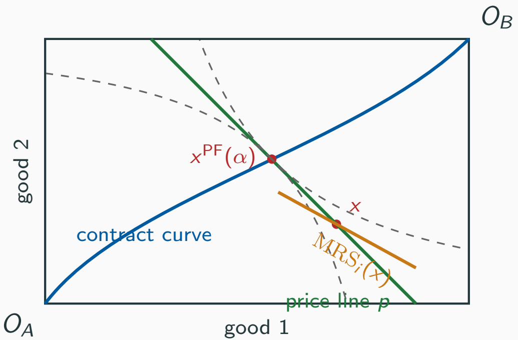

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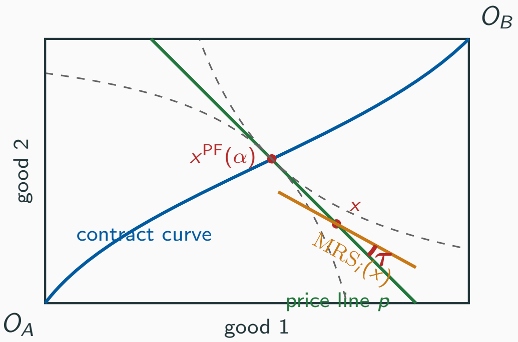

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τ wedge. Gap between $\text{MRS}_i(x)$ and the price ratio:

$$1 + \tau_{i,2} = \frac{p_1}{p_2}\,\frac{u_{i,2}(x_i)}{u_{i,1}(x_i)}.$$

α: inequality — τ: distortion.

Appendix: When the Coordinate System Is Not Unique

1. Forward map $x \mapsto \alpha$. PN weights solve $\alpha_i \propto V_{i,y}\big(\mathrm{P}(\alpha), \sum_k \mathrm{P}_k(\alpha)\, x_{i,k}\big)^{-1}$. Existence by Brouwer; *uniqueness* when $\mathrm{P}(\alpha)$ depends only weakly on α (Gorman, CRRA/CARA, linear tech). In general the fixed point can be non-unique.

Selection. Status quo: α^* minimizing $\mathcal{W}(\alpha^*, 0) - \mathcal{W}(\alpha^*, t^*)$ (ties: closeness to $\bar{\alpha}$). Reform: α^{**} minimizing $\|\alpha^{**} - \alpha^*\|^2$.

2. Inverse map $(\alpha, t) \mapsto x$. A *separate* result: if

$$\sum_k p_k \frac{\partial x_{i,k}}{\partial m}(q, m) \; > \; 0 \quad \forall\, p, q, m,$$

then each (α, t) has a unique x. Sufficient: all goods normal.

Appendix: Policy-Space Decomposition with $\hat{S} \neq 0$

Reform moves off the PPF: $\xi = S/\sum_k p_k A_k$ changes, where $S = \sum_k \varsigma_k r_k Z_k - \sum_k r_k G_k$. A new term $\hat{E}^{\mathrm{pr}}$ appears; $\hat{R}$ picks up a fiscal-externality offset.

Inequality-preserving weights: $\omega_i = \dfrac{V_{i,y}/V_{i,yy}}{\sum_{i'} V_{i',y}/V_{i',yy}}$.

$$\hat{W} = \underbrace{\sum_i \bar{\alpha}_i \tfrac{u_{i,1}}{1-\eta_i}\left(\hat{T}_i^{\mathrm{net}} - \omega_i \hat{S}\right)}_{\hat{R}} + \underbrace{\sum_i \bar{\alpha}_i \tfrac{u_{i,1}}{1-\eta_i}\,\omega_i \hat{S}}_{\hat{E}^{\mathrm{pr}}} + \underbrace{\sum_i \bar{\alpha}_i \tfrac{u_{i,1}}{1-\eta_i} \sum_k \vartheta^c_{i,k} \ln\widehat{(1+\tau_{i,k})}}_{\hat{E}^{\mathrm{al}}}$$

Reading. $\hat{E}^{\mathrm{pr}}$ = value of wasted resources, split inequality-preservingly across agents. $\hat{R}$ now measures transfers *beyond* the inequality-preserving share of $\hat{S}$.

Back

Appendix: Policy-Space Decomposition When Prices Vary with α

When $\mathrm{P}(\alpha)$ moves with α, separate real from nominal income changes: price movements (driven by $\hat{\alpha}$) load onto *redistribution*; real effects stay in efficiency.

Real quantities. With $\mathcal{S} = S / \sum_k p_k A_k$, $\hat{S}^r = \widehat{\mathcal{S}} \sum_k p_k^* A_k$ and $\hat{T}_i^{\mathrm{net},r} = \widehat{T_i^{\mathrm{net}}} - \sum_k \hat{p}_k x_{i,k}^{\mathrm{net},*} = \sum_k p_k^* \hat{x}_{i,k}$.

Decomposition (same $\eta_i, \vartheta_{i,k}^c, \omega_i$):

$$\hat{R} = \sum_i \bar{\alpha}_i \tfrac{u_{i,1}}{1-\eta_i} \Big(\hat{T}_i^{\mathrm{net},r} - \omega_i \hat{S}^r + \sum_k \vartheta_{i,k}^c \widehat{\ln p_k} \Big),$$

$$\hat{E}^{\mathrm{pr}} = \sum_i \bar{\alpha}_i \tfrac{u_{i,1}}{1-\eta_i} \omega_i \hat{S}^r, \quad \hat{E}^{\mathrm{al}} = \sum_i \bar{\alpha}_i \tfrac{u_{i,1}}{1-\eta_i} \sum_k \vartheta_{i,k}^c \ln(\widehat{1+\tau_{i,k}}).$$

Two changes vs. constant-price case: $\hat{S}$, $\hat{T}^{\mathrm{net}}$ become real, and $\hat{R}$ gains $\sum_k \vartheta_{i,k}^c \widehat{\ln p_k}$.

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Insurance with Aggregate Risk

Setup. Single good, CRRA. Aggregate shocks: $\sum_i x_{i,1}(s) = A_1(s)$. Prices: $p_1(s) \propto \Pr(s) A_1(s)^{-\sigma}$, normalized $\sum_s p_1(s) = 1$.

Risk-neutral expectation: $\tilde{\mathbb{E}}[Z] := \sum_s p_1(s) Z(s)$. Income: $y_i = \tilde{\mathbb{E}}[x_{i,1}]$.

Decomposition:

$$\hat{W} = \underbrace{\sum_i \bar{\alpha}_i \, \mathbb{E}\left[v'(x^*_{i,1})\, x^*_{i,1} \ln \widehat{\left(\frac{\tilde{\mathbb{E}} x_{i,1}}{\tilde{\mathbb{E}} X_1} \right)} \right]}_{\hat{R}} + \underbrace{\sum_i \bar{\alpha}_i \, \mathbb{E}\left[v'(x^*_{i,1})\, x^*_{i,1} \ln \widehat{\left(\frac{x_{i,1}}{\tilde{\mathbb{E}} x_{i,1}} \right)} \right]}_{\hat{E}^{\text{ins}}}$$

Second-order (using $\tilde{\mathbb{E}}[Z] \approx \mathbb{E}[Z] - \sigma \operatorname{cov}(Z, \ln X_1)$):

$\hat{R}$: $\approx \sum_i \bar{\alpha}_i (\mathbb{E} x^*_{i,1})^{1-\sigma} \left[\widehat{\ln \mathbb{E} x_{i,1}} - \sigma \operatorname{cov}(\widehat{\ln x_{i,1}}, \ln X_1) \right]$ — covariance = hedging value

$\hat{E}^{\text{ins}}$: $\approx -\frac{\sigma}{2} \sum_i \bar{\alpha}_i (\mathbb{E} x^*_{i,1})^{1-\sigma} \operatorname{var}(\widehat{\ln x_{i,1} - \ln X_1})$ — insurance relative to aggregate

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Appendix: BEGS–Debreu

Following Debreu (1951, 1959).

Scaled production set (fraction $1-\rho$ of resources wasted):

$$\mathcal{Y}(\rho) \;=\; \{\rho X : X \in \mathcal{Y}\}, \qquad \rho \in [0,1]$$

Utility possibility set at efficiency level ρ:

$$\mathcal{U}(\rho) \;=\; \left\{\vec{u} : \exists\, x \text{ with } u_i(x_i) = \vec{u}_i,\ \textstyle\sum_i x_i \in \mathcal{Y}(\rho)\right\}$$

Under standard assumptions:

- $\mathcal{U}(\rho)$ is convex, compact, continuous in the Hausdorff metric
- Nested: $\rho_1 < \rho_2 \Rightarrow \mathcal{U}(\rho_1) \subset \mathcal{U}(\rho_2)$
- $\partial\mathcal{U}(\rho)$ is the Pareto frontier for technology $\mathcal{Y}(\rho)$

Appendix: BEGS–Debreu – Coordinate System (ω, ρ)

Forward map $x \mapsto (\omega(x), \rho(x))$:

- **Efficiency:** $\rho(x) = \inf\{\rho : \vec{u}(x) \in \mathcal{U}(\rho)\}$ (radial distance to first-best frontier)
- **Redistribution:** $\omega(x)$ = Pareto–Negishi weights rationalizing $\vec{u}(x)$ on $\partial\mathcal{U}(\rho(x))$, with $\sum_i \omega_i = 1$

Inverse map $(\omega, \rho) \mapsto \tilde{u}$:

$$\tilde{u}(\omega, \rho) \; = \; \arg\max_x \sum_i \omega_i u_i(x_i) \quad \text{s.t.} \quad \sum_i x_i \in \mathcal{Y}(\rho)$$

Social welfare in new coordinates:

$$\mathcal{W}(\omega, \rho) \; = \; \sum_i \bar{\alpha}_i \, \tilde{u}_i(\omega, \rho)$$

Appendix: The BEGS–Debreu Decomposition

Status quo $x^* \to (\omega^*, \rho^*)$, reform $x^{**} \to (\omega^{**}, \rho^{**})$. Apply two-player Shapley value on the grid of coordinates:

Redistribution component:

$$\mathrm{R}^{BD} = \tfrac{1}{2}\left[\mathcal{W}(\omega^{**}, \rho^*) - \mathcal{W}(\omega^*, \rho^*)\right] + \tfrac{1}{2}\left[\mathcal{W}(\omega^{**}, \rho^{**}) - \mathcal{W}(\omega^*, \rho^{**})\right]$$

Efficiency component:

$$\mathrm{E}^{BD} = \tfrac{1}{2}\left[\mathcal{W}(\omega^*, \rho^{**}) - \mathcal{W}(\omega^*, \rho^*)\right] + \tfrac{1}{2}\left[\mathcal{W}(\omega^{**}, \rho^{**}) - \mathcal{W}(\omega^{**}, \rho^*)\right]$$

Exact decomposition (scale-affine preferences):

$$\boxed{W(x^{**}) - W(x^*) \;=\; \mathrm{R}^{BD}(x^*, x^{**}) + \mathrm{E}^{BD}(x^*, x^{**})}$$

Averages the marginal contribution of ω (resp. ρ) across the two orderings $\omega \to \rho$ and $\rho \to \omega$.

Appendix: Properties of the BEGS–Debreu Decomposition

(i) Weak Pareto-consistency. If x^*, x^{**} are both on $\partial\mathcal{U}(1)$, then $\mathrm{E}^{BD} = 0$.

(ii) Strong Pareto-consistency. If x^* is on the frontier but x^{**} is not, then $\mathrm{E}^{BD}(x^*, x^{**}) \leq 0$ (strict when all $\bar{\alpha}_i > 0$). Symmetric statement when roles reversed.

(iii) Direction invariance (symmetry).

$$\frac{\mathrm{R}^{BD}(x^*, x^{**})}{W(x^{**}) - W(x^*)} = \frac{\mathrm{R}^{BD}(x^{**}, x^*)}{W(x^*) - W(x^{**})}$$

and analogously for E^{BD}.

(iv) Numeraire invariance. Trivial: coordinates are defined on utilities.

Property (v) fails: Income-based redistribution. Because (ω, ρ) live in *utility* space, reforms with zero real-income transfer can shift ω and generate $\mathrm{R}^{BD} \neq 0$.

Appendix: Example 1 – Insurance (Setup)

Setup. Two agents, log utility. Agent 1 deterministic: $c_1 = K$. Agent 2 state-contingent $c_2(s)$ with $\mathbb{E}c_2 = 1$. Total $C = K + 1$.

Certainty equivalent: $\ln c_2^{ce} = \sum_s \Pr(s) \ln c_2(s)$.

Debreu coordinates:

$$\rho = \frac{K + c_2^{ce}}{K + 1}, \quad \omega_1 = \frac{K}{K + c_2^{ce}}, \quad \omega_2 = \frac{c_2^{ce}}{K + c_2^{ce}}$$

with $\tilde{u}_i(\omega, \rho) = \ln \omega_i + \ln \rho + \ln C$.

Appendix: Example 1 – Insurance (Decomposition)

Risk-reducing reform: $\hat{c}_2^{ce} > 0$, $\hat{u}_1 = 0$, $\hat{u}_2 = \hat{c}_2^{ce}/c_2^{ce}$:

$$\hat{W} = \bar{\alpha}_2 \frac{\hat{c}_2^{ce}}{c_2^{ce}} = \underbrace{\frac{\hat{c}_2^{ce}}{K + c_2^{ce}}}_{\hat{\mathrm{E}}^{BD}} + \underbrace{\bar{\alpha}_1 \widehat{\ln \omega_1} + \bar{\alpha}_2 \widehat{\ln \omega_2}}_{\hat{\mathrm{R}}^{BD}}$$

Pathologies of BEGS–Debreu shares:

- $K \to \infty$: $\hat{\mathrm{E}}^{BD} \to 0 \Rightarrow$ redistribution absorbs 100% of $\hat{W}$
- $\bar{\alpha}_2 \to 0$: $\hat{W} \to 0$ with $\hat{\mathrm{E}}^{BD} > 0 \Rightarrow$ efficiency share $= +\infty\%$, redistribution $= -\infty\%$

No physical resources cross agents $\Rightarrow$ our decomposition finds $\mathrm{R} = 0$.

Appendix: Example 2 – Labor Tax (Setup)

Setup. Log-log preferences $u_i = \ln c_i + \ln \ell_i$. Agent 1 has K effective-leisure units (unaffected). Agent 2 has one unit; labor taxed at τ with lump-sum rebate. Constraint $C + L \leq K + 1$.

Equilibrium: $c_1 = \ell_1 = K/2$; $c_2 = \frac{1-\tau}{2-\tau}$, $\ell_2 = \frac{1}{2-\tau}$.

Map to Debreu frontier: the distorted allocation yields the same utilities as the frontier point with $x_1 = K$, $x_2 = 2\sqrt{1-\tau}/(2-\tau)$:

$$\rho = \frac{K + x_2}{K + 1}, \quad \omega_1 = \frac{K}{K + x_2}, \quad \omega_2 = \frac{x_2}{K + x_2}$$

Appendix: Example 2 – Labor Tax (Decomposition)

Tax reduction ($\hat{\tau} < 0$ from $\tau > 0$): $\hat{x}_2 > 0$, $\hat{u}_1 = 0$, $\hat{u}_2 = 2\hat{x}_2/x_2$:

$$\hat{W} = 2\bar{\alpha}_2 \frac{\hat{x}_2}{x_2} = \underbrace{\frac{2\hat{x}_2}{K + x_2}}_{\hat{\mathrm{E}}^{BD}} + \underbrace{2\bar{\alpha}_1 \widehat{\ln \omega_1} + 2\bar{\alpha}_2 \widehat{\ln \omega_2}}_{\hat{\mathrm{R}}^{BD}}$$

Same pathology: $K \to \infty$ gives 100% redistribution; $\bar{\alpha}_2 \to 0$ gives $\pm\infty$% shares.

This is Example 1 in the main text – a *pure* distortion. BEGS–Debreu attributes it (partly) to redistribution. ×

Appendix: Pareto Monotonicity (Property vi)

An alternative property for reforms inside the frontier:

Property (vi) (*Pareto Monotonicity*). If x^{**} weakly Pareto dominates x^*, then $E(x^*, x^{**}) \geq 0$. Symmetric statement when roles reversed.

Appeal. Reforms that make everyone weakly better off "should" be classified as efficiency gains, not redistribution.

Question. Can a decomposition satisfy both income-based redistribution (v) *and* Pareto monotonicity (vi)?

Answer: No, given Pareto-consistency (i). (See next slide.)

Appendix: An Impossibility Result

Proposition. Any differentiable decomposition satisfying (i) cannot satisfy both (v) and (vi).

Intuition. A Pareto improvement inside the frontier may combine: removing distortions for some agents, adding distortions for others, and transferring resources so all gain.

- (v) pins down the resource-neutral piece: any welfare change is efficiency.
- (i) pins down the piece between Pareto-efficient endpoints: efficiency is zero.

Together, (i) and (v) can force $E < 0$ on a Pareto improvement, violating (vi).

BEGS satisfies (i)–(v) and forgoes (vi). The Debreu-coefficient alternative satisfies (i)–(iv) and (vi) but not (v) — and gives $-\infty\%$ to 100% "redistribution" in Example 1.