Learning about Bond Prices

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Learning mechanism

- Risk-neutral bond pricing

$$Q_t^n = \mathrm{E}_t^{\mathscr{P}}\left(\frac{P_t}{P_{t+1}} Q_{t+1}^{n-1}\right); \qquad Y_t^n \approx -\frac{1}{n}\log Q_t^n$$

- Perceived term premium under Internal Rationality

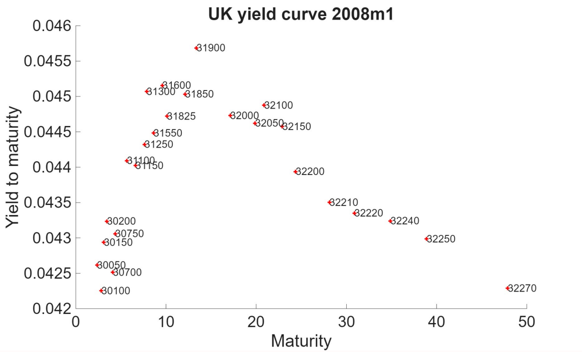
$$Y_t^n - Y_t^1 = (n-1)s_t + u_t^n; \quad s_t = s_{t-1} + w_t; \quad \mathrm{E}_t^{\mathscr{P}} s_t = \mathrm{E}_t^{\mathscr{P}} s_{t-1} + g(s_{t-1} - \mathrm{E}_t^{\mathscr{P}} s_{t-1})$$

- Term premium realised in market

$$Y_t^n - Y_t^1 = \left[n\left(\frac{n-1}{n}\right)^2 - 1\right]\mathrm{E}_t^{\mathscr{P}} s_t$$

Estimating yield curves

- Zero coupon yields estimated from gilt prices

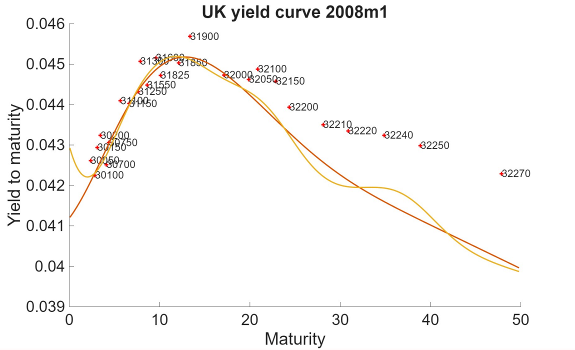


- Not obvious that intercept+slope model captures this well

What they do in finance

- 3 factor models with intercept+slope+curvature terms
- Impose stiffness on long end of yield curve
- Bank of England use Waggoner (1997)
- Gürkaynak, Sack and Wright use Nelson-Siegel (1987) and Svensson (1994)

Effect of stiffness



- *Negative comment:* Model of perceived term premium too restrictive
- *Positive comment:* Misperception that drives result really hard to falsify

A good model of the yield curve?

- Learning generates volatility in yield curve with nice properties
- Yield curve on average upward-sloping - what about first moment?
- Unconditional expectation $\mathrm{E}s_t = 0$ so $\mathrm{E}(Y_t^n - Y_t^1) = 0$
- No unconditional term premium
- Can learning rationalise both first and second moments of yield curve?

Ellison-Tischbirek (2001)

- Decomposition of unconditional term premium

$$\mathrm{E}(Y_t^2 - Y_t^1) = \frac{1}{2}[-\mathrm{Cov}(m_{t+1}, m_{t+1}) + \mathrm{Cov}(\mathrm{E}_t m_{t+1}, \mathrm{E}_{t+1} m_{t+2})]$$

- Depends on two covariances
  1. between successive stochastic discount factors
  2. between successive expectations of stochastic discount factors
- Learning injects persistence into expectations but makes them less volatile
- We generate unconditional term premia by strategic interactions

Ellison-Tischbirek (2001)

- Conditional term premium

$$Y_t^2 - Y_t^1 = -\frac{1}{2}\mathrm{Cov}_t(m_{t+1}, \mathrm{E}_{t+1} m_{t+2})$$

- Need time-varying covariance to generate volatility - difficult!
- Full information, partial information, noisy information, news shocks, strategic interactions à la Angeletos, Collard and Dellas (2008) do not work
- We need time-varying strategic interactions to generate movement in yield curve