

Macro to Micro and Back Again

Excess Demands + Tâtonnement + Phillips Curves

**Keynote @ 2nd Banca d'Italia Annual Research Conference
by Iván Werning — based on work with Guido Lorenzoni**

MICROECONOMIC THEORY

ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN

MICROECONOMIC THEORY

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Michael Woodford

INTEREST
& PRICES

MICROECONOMIC THEORY

ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN

GE $\leftrightarrow$ Macro

Michael Woodford

INTEREST & PRICES

FOUNDATIONS OF A THEORY
OF MONETARY POLICY

GE ↔ Macro

GE ↔ Macro

■ General Equilibrium Theory...

- ▶ well-developed theory
- ▶ flexible framework
- ▶ applied: Trade, Macro, Public Finance, IO...

$$Z(\bar{p}) = 0$$

equilibrium
(vectors)

GE ↔ Macro

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■ New Keynesian Macroeconomics...

- ▶ Success: micro + nominal rigidities...
- ▶ ... GE with many markets but...
- ▶ → gap with GE theory
- ▶ → gap with basic micro 101 intuition

$$Z(\bar{p}) = 0$$

equilibrium
(vectors)

$$U = u(c) - v(n)$$

$$u'(c_t) = \beta R \mathbb{E}_t u'(c_{t+1})$$

$$p_t^* = \mu_t + \sum_{s \geq 0} \beta^s MC_{t+s}$$

GE ↔ Macro

■ General Equilibrium Theory...

- ▶ well-developed theory
- ▶ flexible framework
- ▶ applied: Trade, Macro, Public Finance, IO...

■ Disequilibrium Literature: initial progress GE + price rigidities... ... but challenges → price dynamics + intractable (2^N regimes)

■ New Keynesian Macroeconomics...

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**Keynes
&
ISLM**

**General
Equilibrium**
(with Money)

Disequilibrium
(Fixed Price)

**New
Keynesian**

1930-40s

1950s & 1960s

1970s

1980s-present

Keynes
Hicks

Patinkin
Clower

Patinkin,
Clower,
Barro-Grossman,
Benassy, Dreze,
Malinvaud,
Green
...

Talk Plan

Part #1. “Tâtonnement and Price Setting
in General Equilibrium”

Part #2.
“Macroeconomics with Excess Demands”

Talk Plan

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$$\{\pi_{n,t}\} \leftrightarrow \{z_{n,t}\}$$

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(excess demand $\neq$ output gap)

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$$\pi_{n,t} = \kappa_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{n,t+1}$$

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more general $\rightarrow$ more intuitive!

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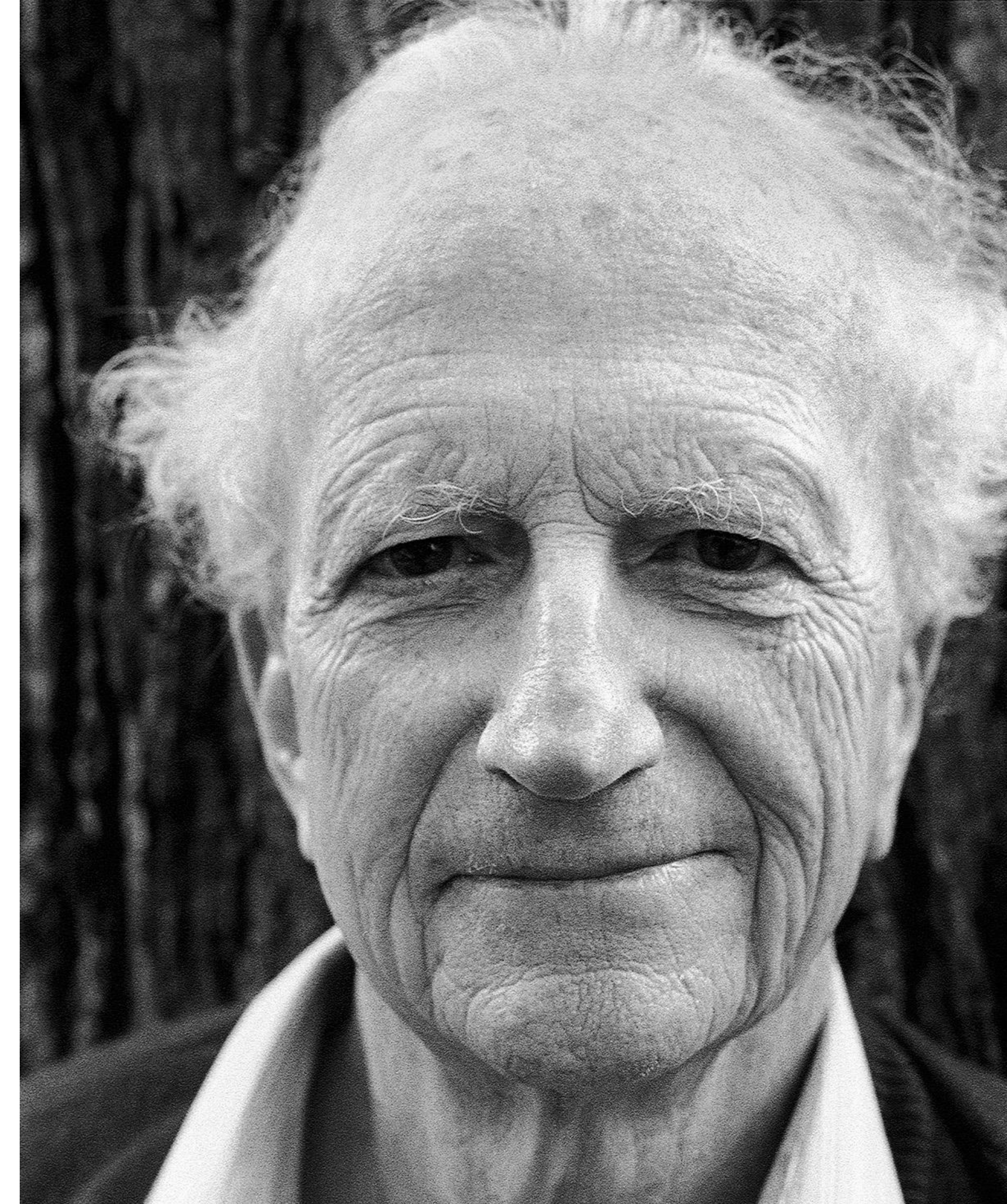
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“Macroeconomics with Excess Demands”



Micro 101...
“Prices Rise with
Excess Demand”

Talk Plan

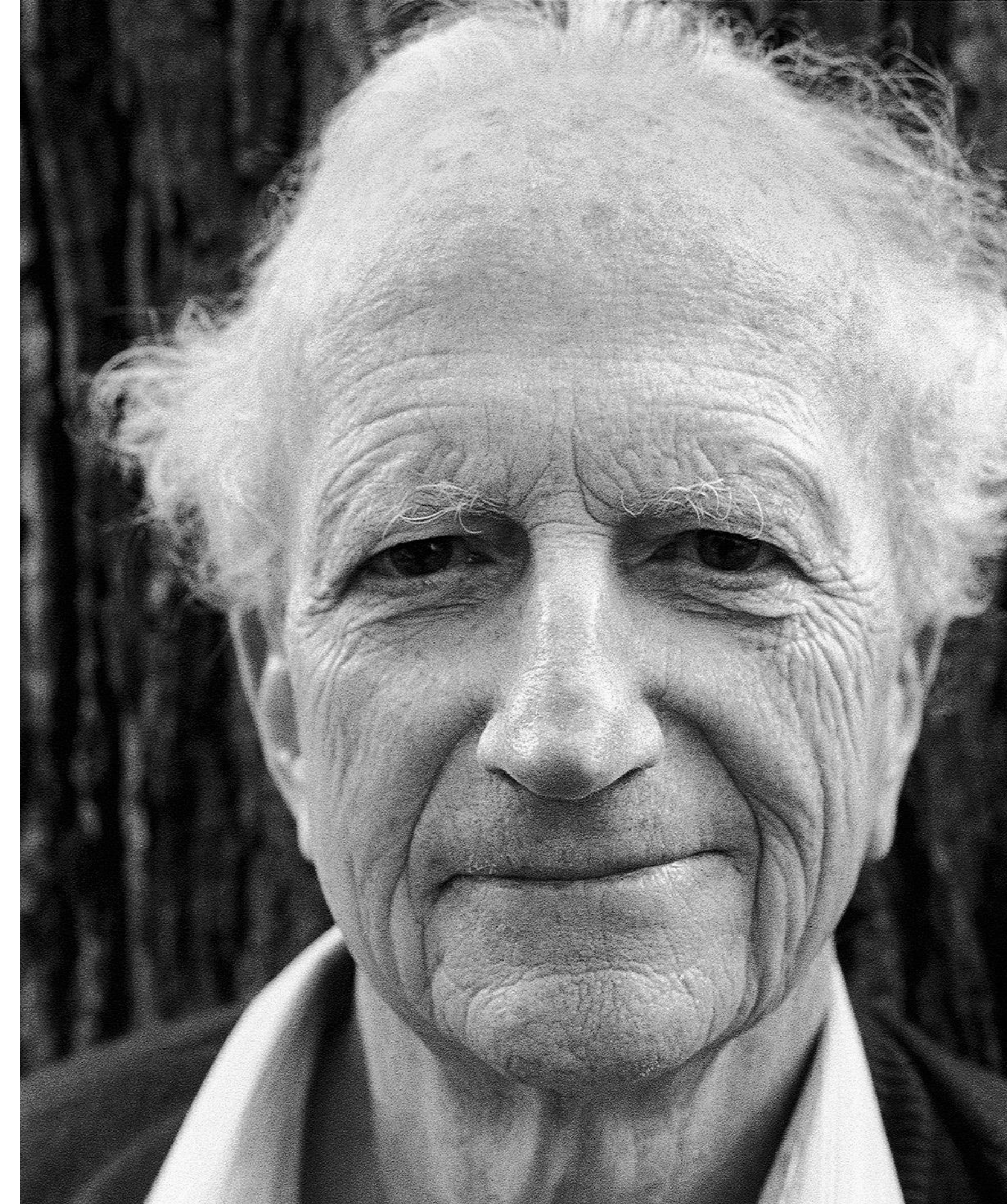
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more general $\rightarrow$ more intuitive!



Heard on the internet...
“Phillips Curves don’t exist
they are dead...”

Micro 101...
“Prices Rise with
Excess Demand”

Talk Plan

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Tâtonnement and Price Setting in General Equilibrium*

IVÁN WERNING
MIT



GUIDO LORENZONI
Chicago Booth

June 2026

Abstract. A foundational question in General Equilibrium theory—when is an equilibrium stable?—has remained largely unresolved for over 150 years despite important contributions. This paper proposes a resolution.

The theory of tâtonnement is the very essence of the theory of price determination.
LÉON WALRAS (letter to Charles Gide)

Talk Plan

Part #1. “Tâtonnement and Price Setting in General Equilibrium”

Doing Macroeconomics with Excess Demands: New Keynesians meet Disequilibrium

GUIDO LORENZONI
CHICAGO BOOTH

IVÁN WERNING
MIT

April 2026

Abstract

This paper proposes a new method that allows for the study of general macroeconomic settings with nominal rigidities in a simple manner and using the main tool of general equilibrium theory: excess demand functions.

and Price Setting Equilibrium*

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Chicago Booth

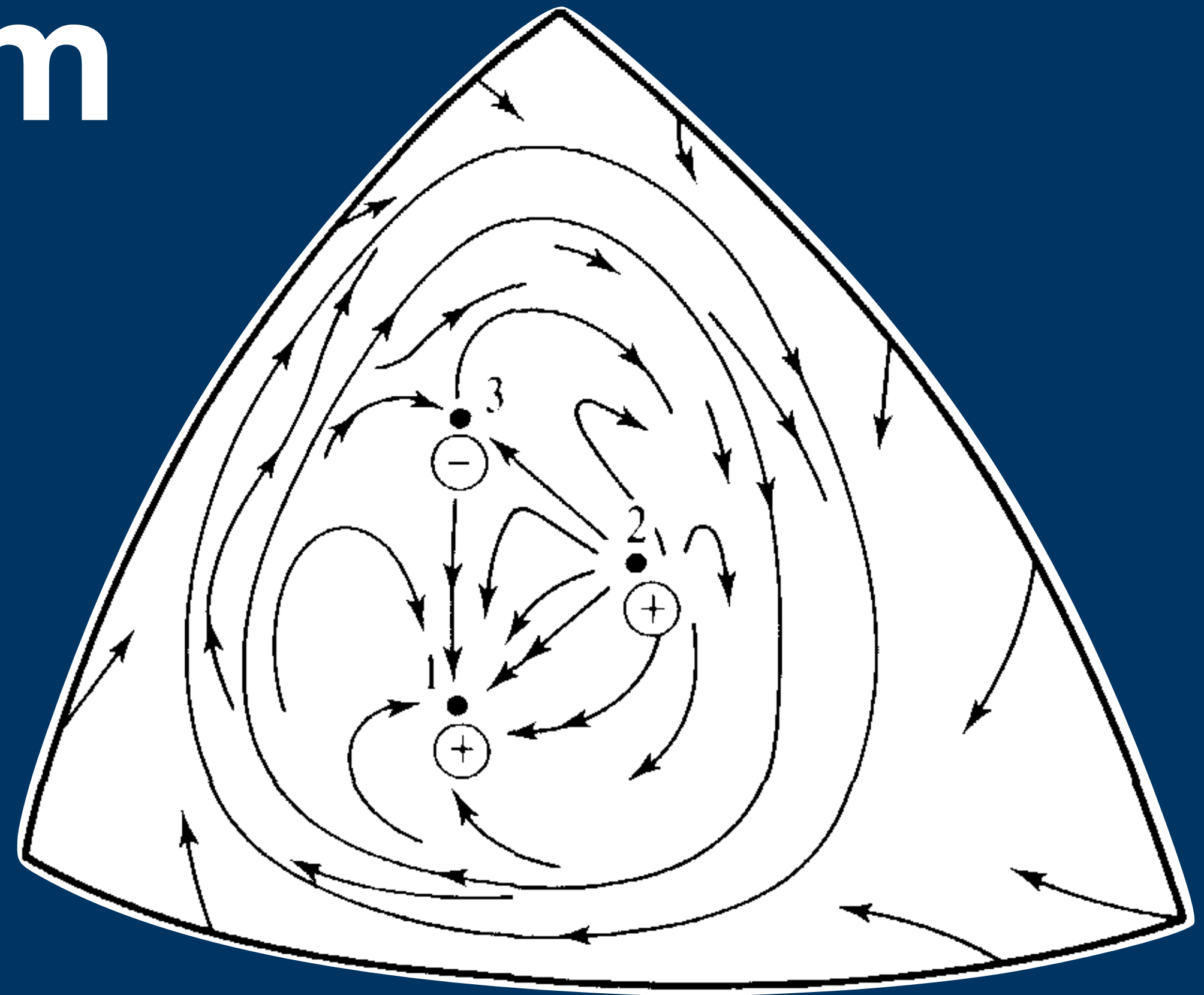
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General Equilibrium theory—when is an equilibrium
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Tâtonnement and Price Setting in General Equilibrium

Guido Lorenzoni and Iván Werning



Tâtonnement History



“Greatest of all economists”
Joseph Schumpeter

Tâtonnement History

■ **Léon Walras 1874-1900** (“Elements of Pure Economics”)...

stability $\leftrightarrow$ existence!... **no auctioneer** in Walras! 🤖

“Our task is very simple: we need only show that the upward and downward movements of prices solve the system of equations of offer and demand by a process of **tâtonnement**.”



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Léon Walras
ELEMENTS OF
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$$\dot{p}_n = d_n Z_n(p) \rightarrow \text{fruitful, but } \mathbf{ad\ hoc}$$

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■ **Our paper:** revisits stability question but...
... using dynamic equilibrium model

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ELEMENTS OF
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Partial Equilibrium

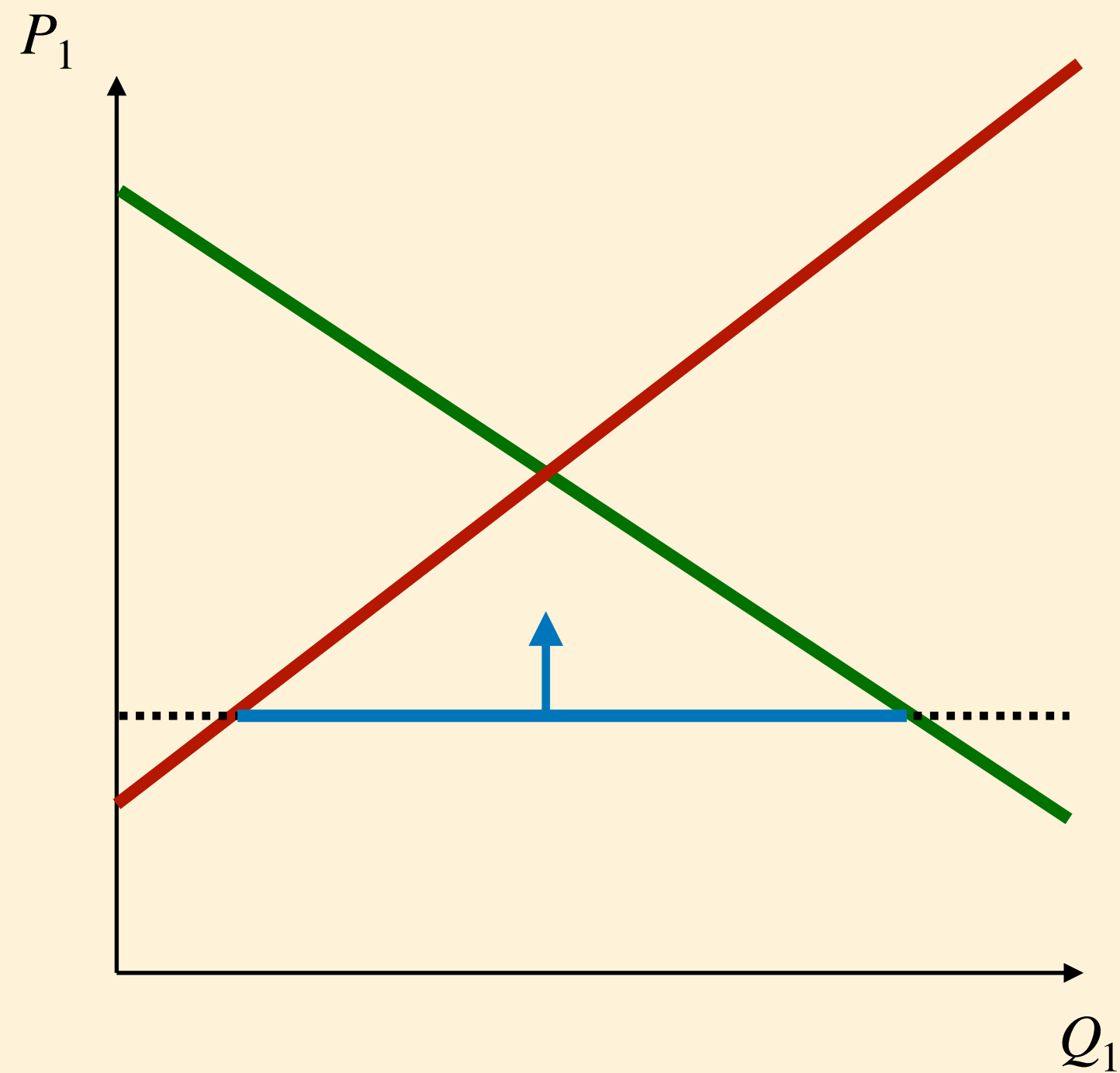


Simple!

Demand vs Supply
slopes



Stable!



Partial Equilibrium

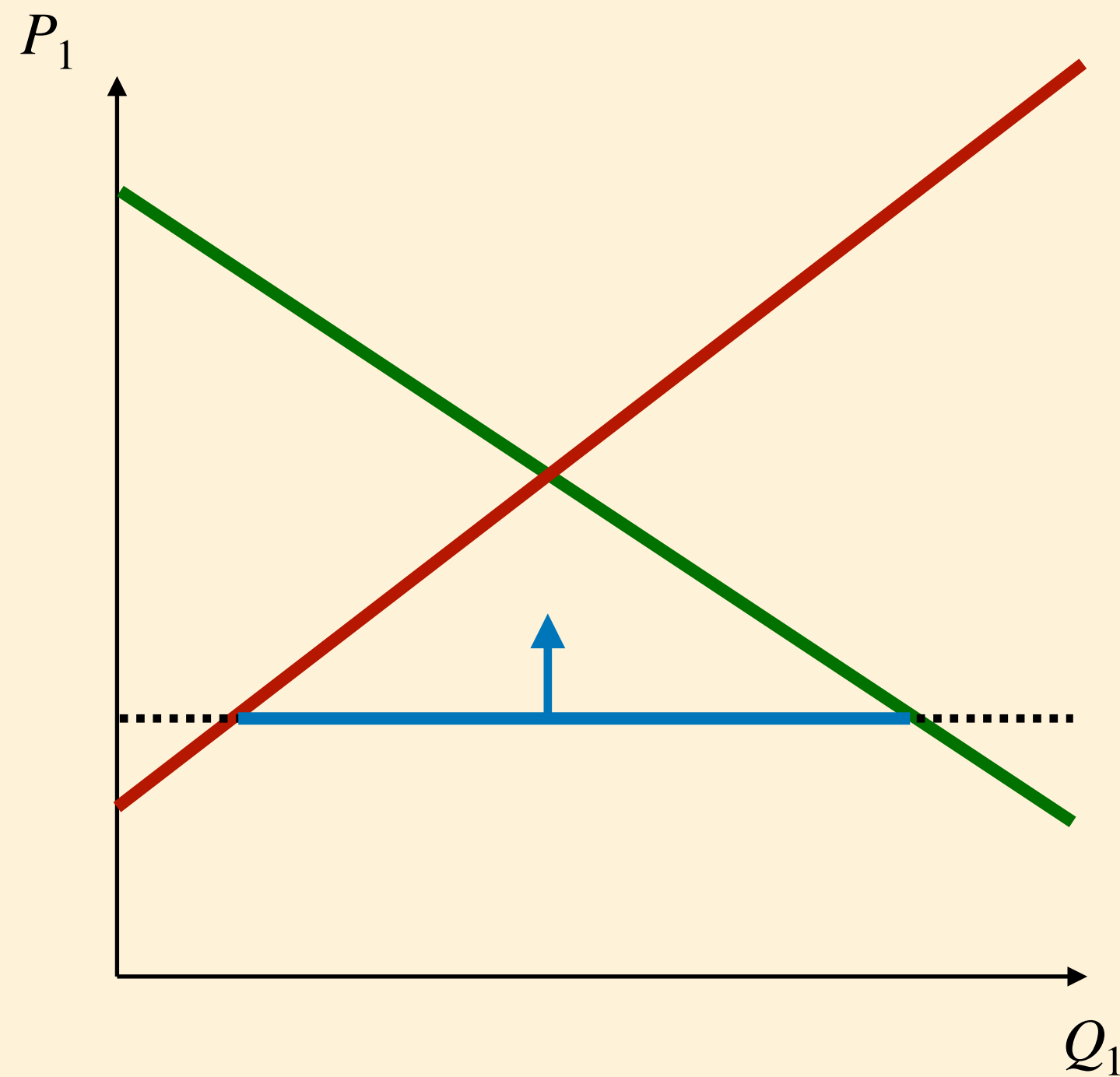


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General Equilibrium?



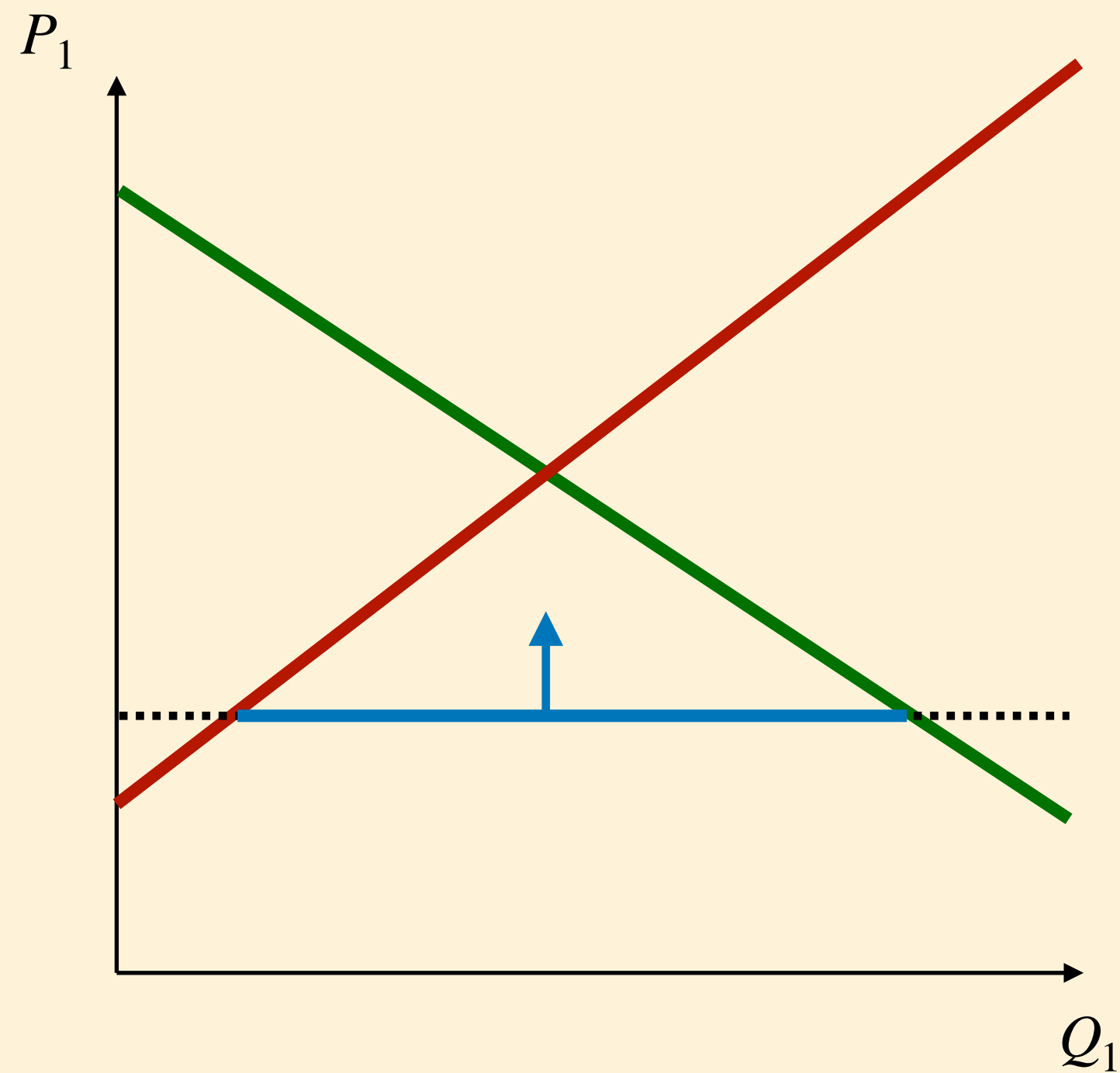
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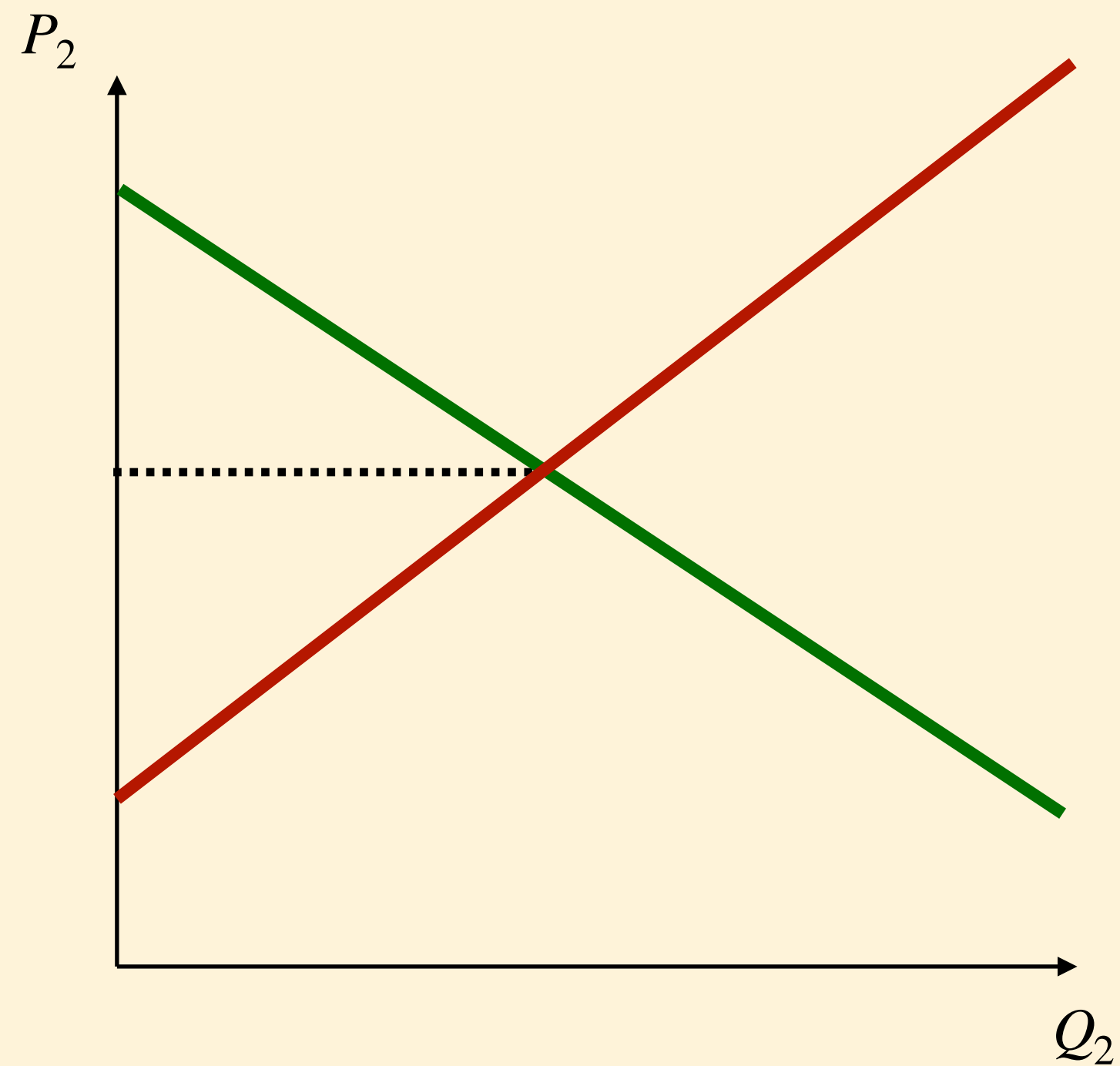
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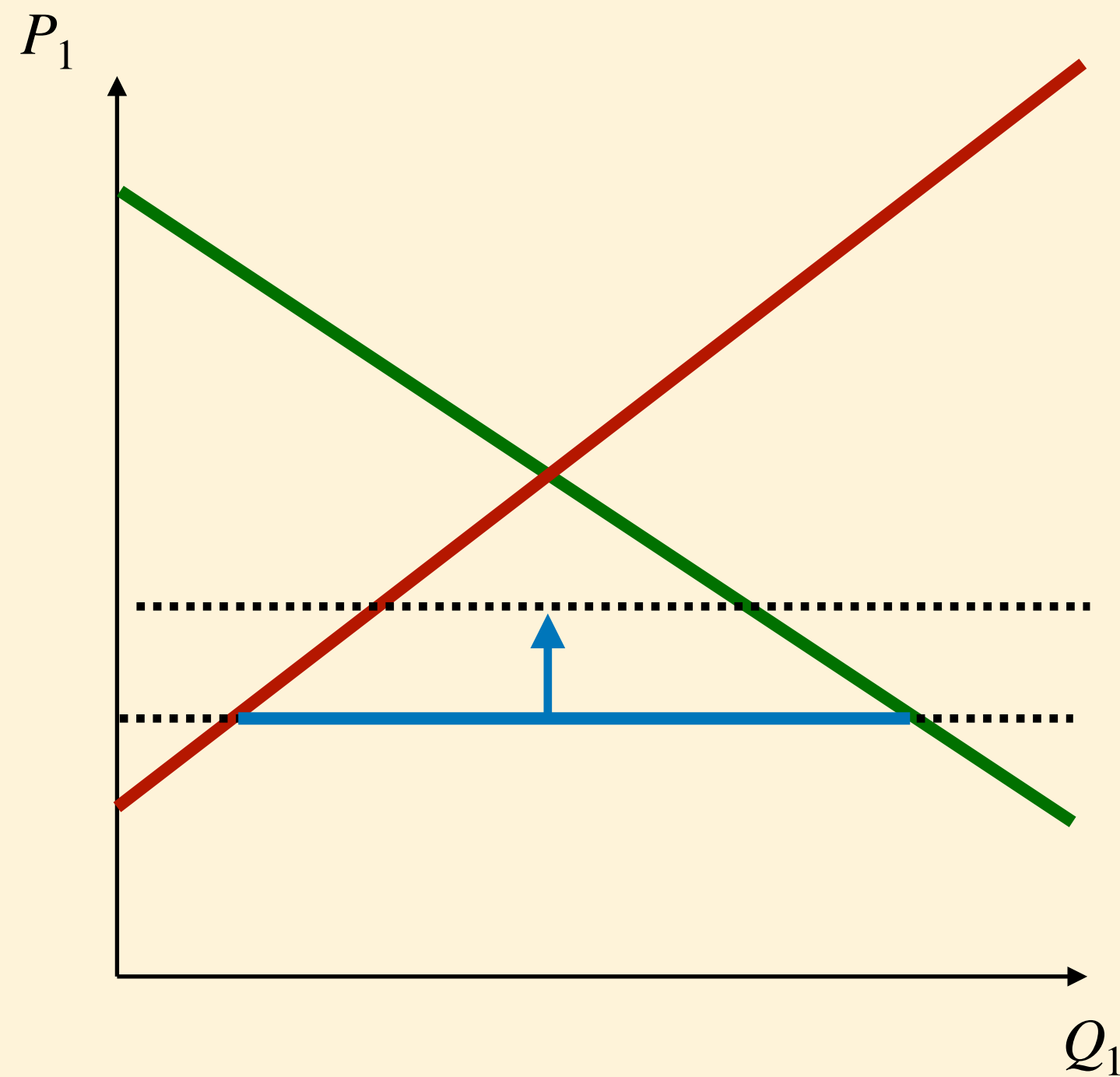
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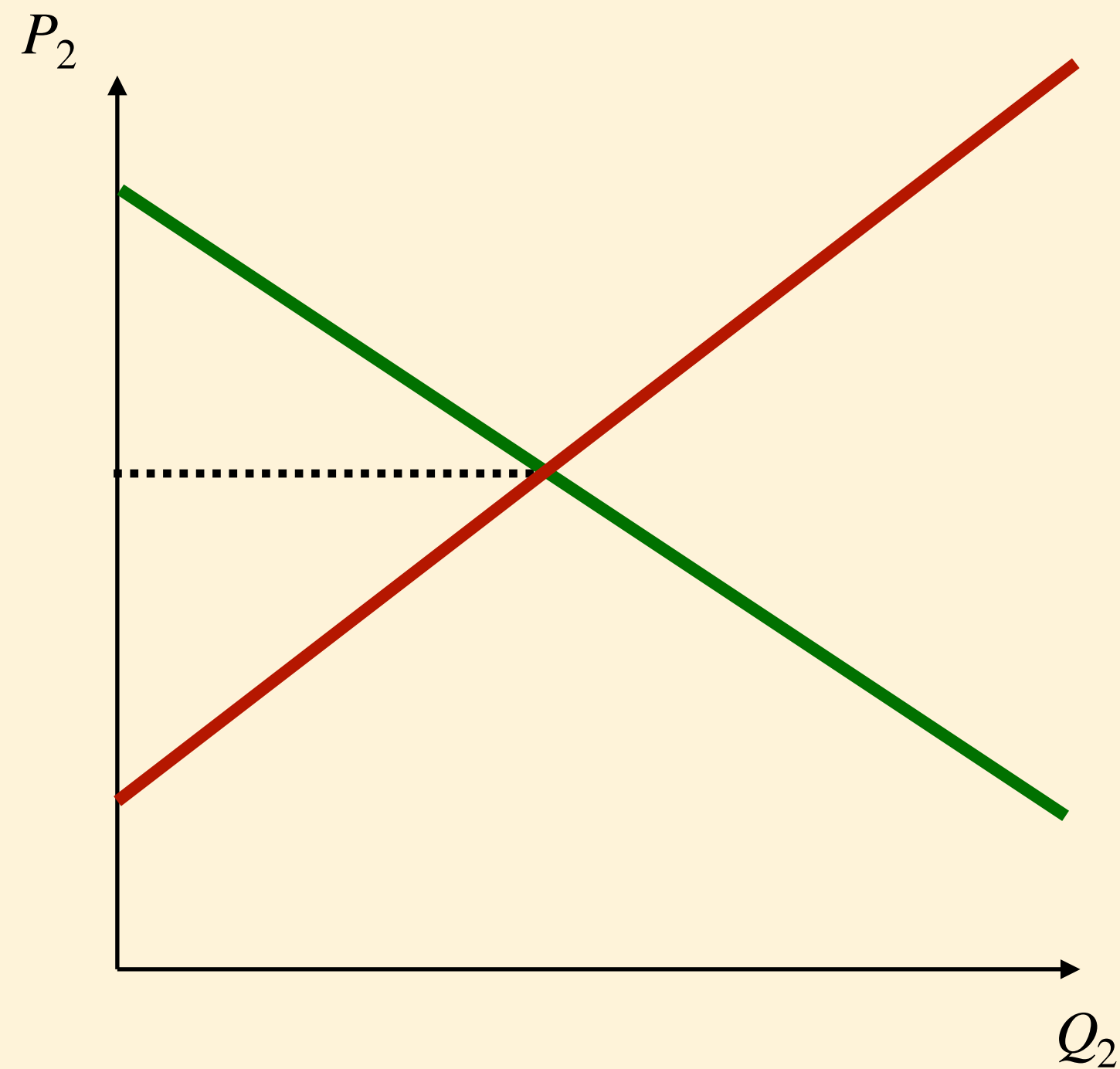
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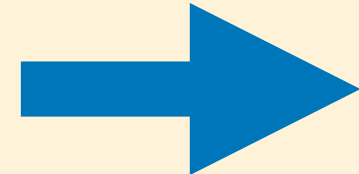
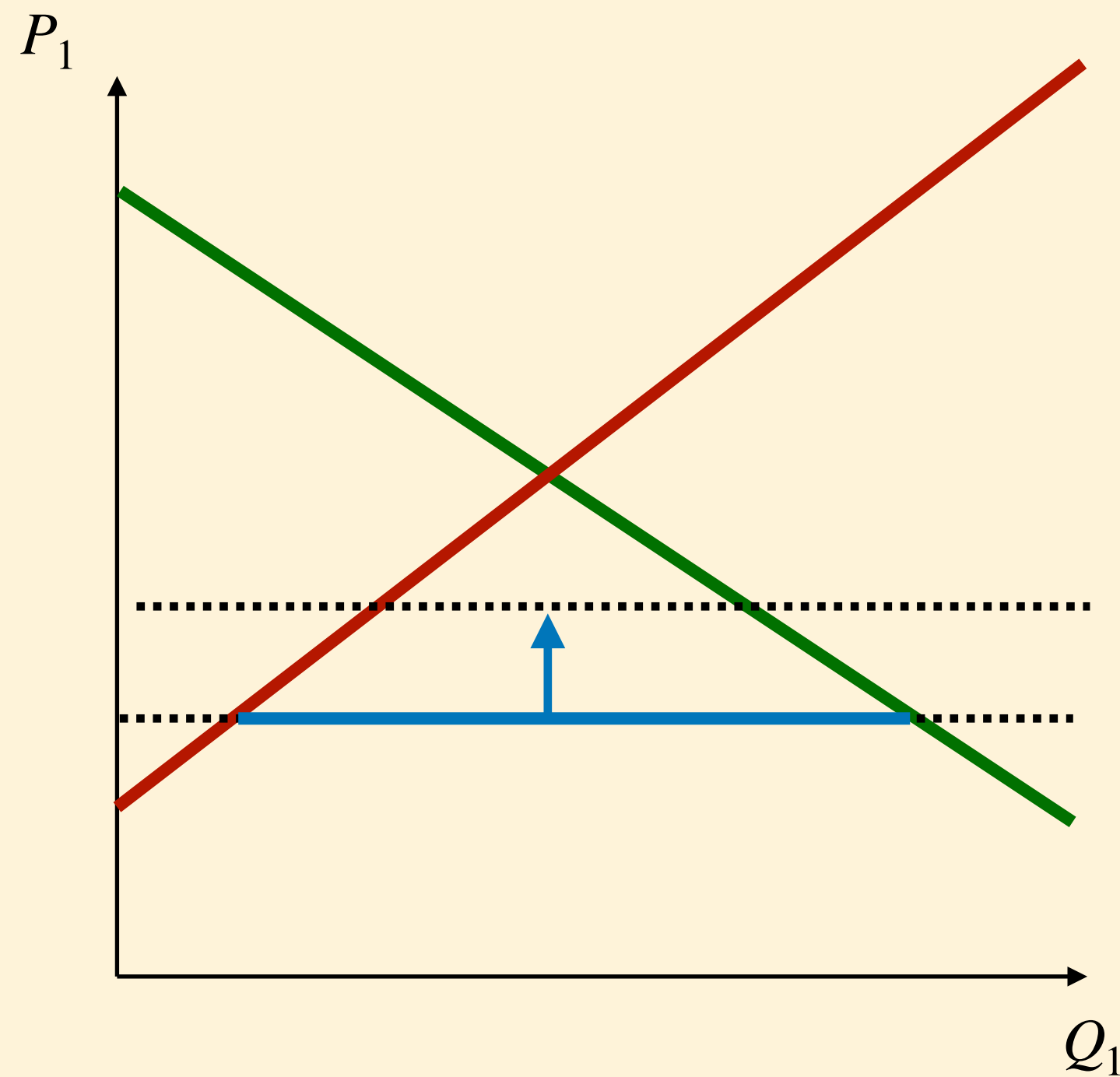
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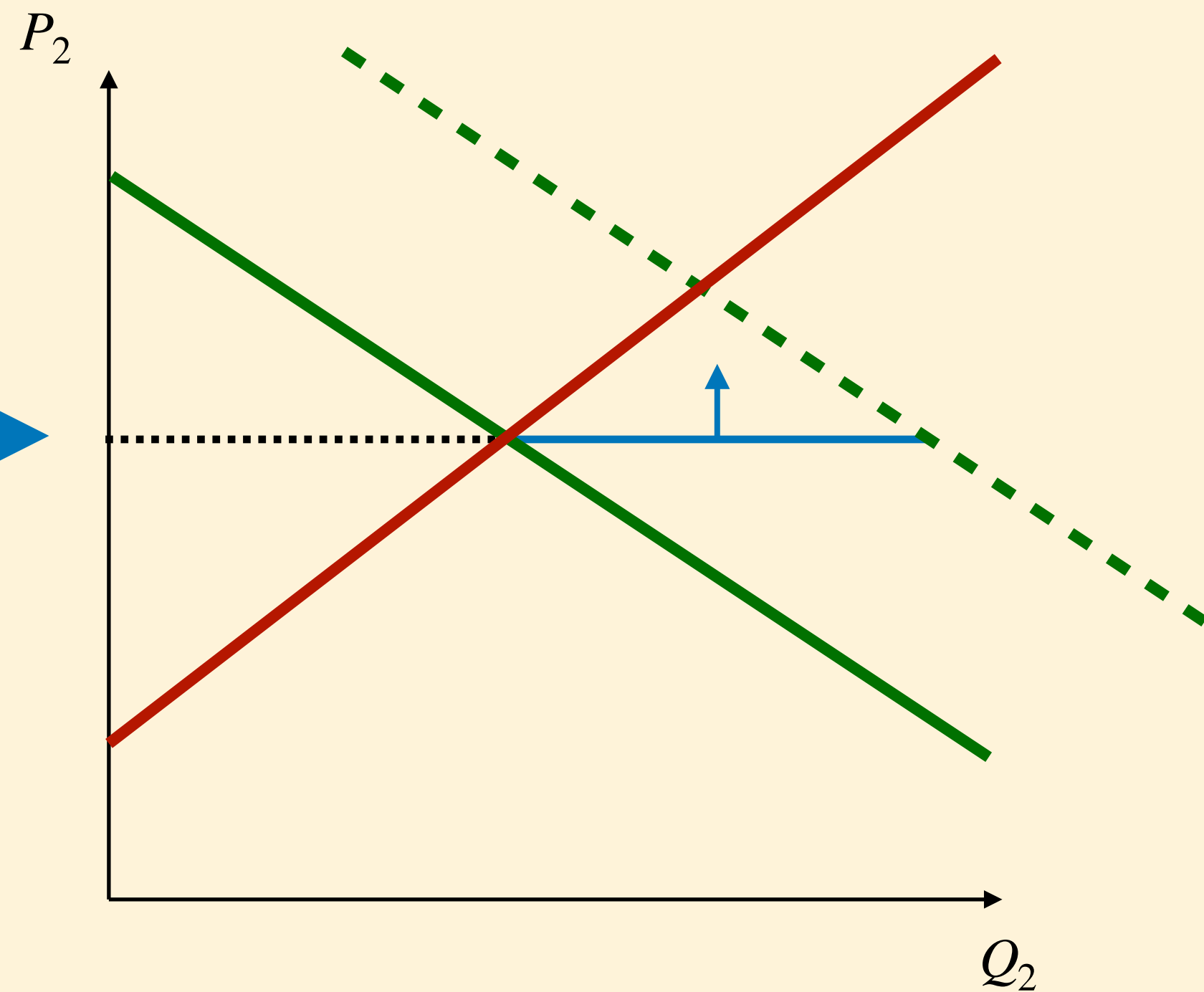
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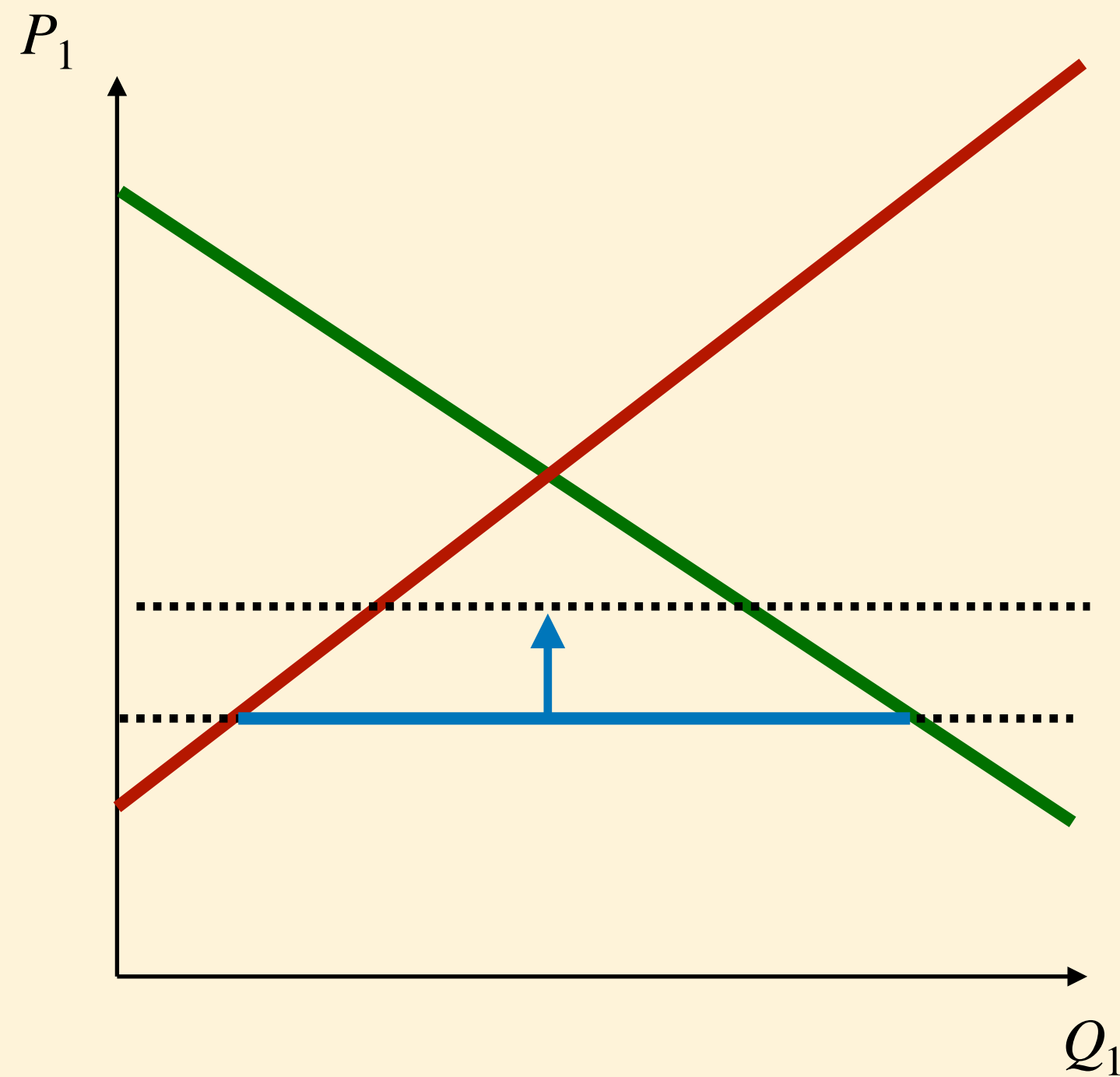
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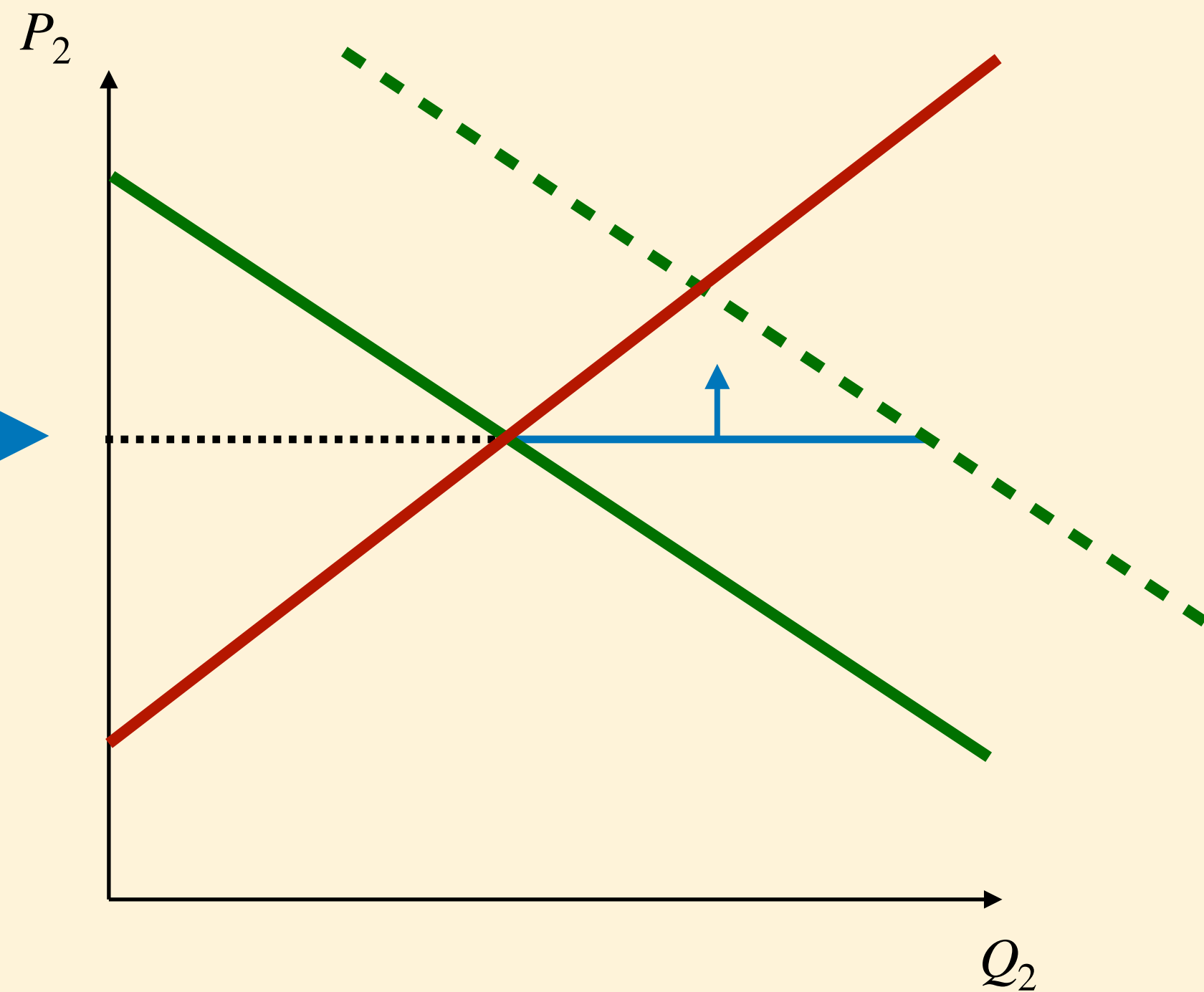
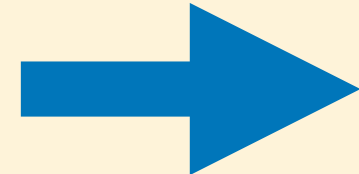
General Equilibrium?



Cross Effects!...
Fix one market...
...break another?



Unstable?



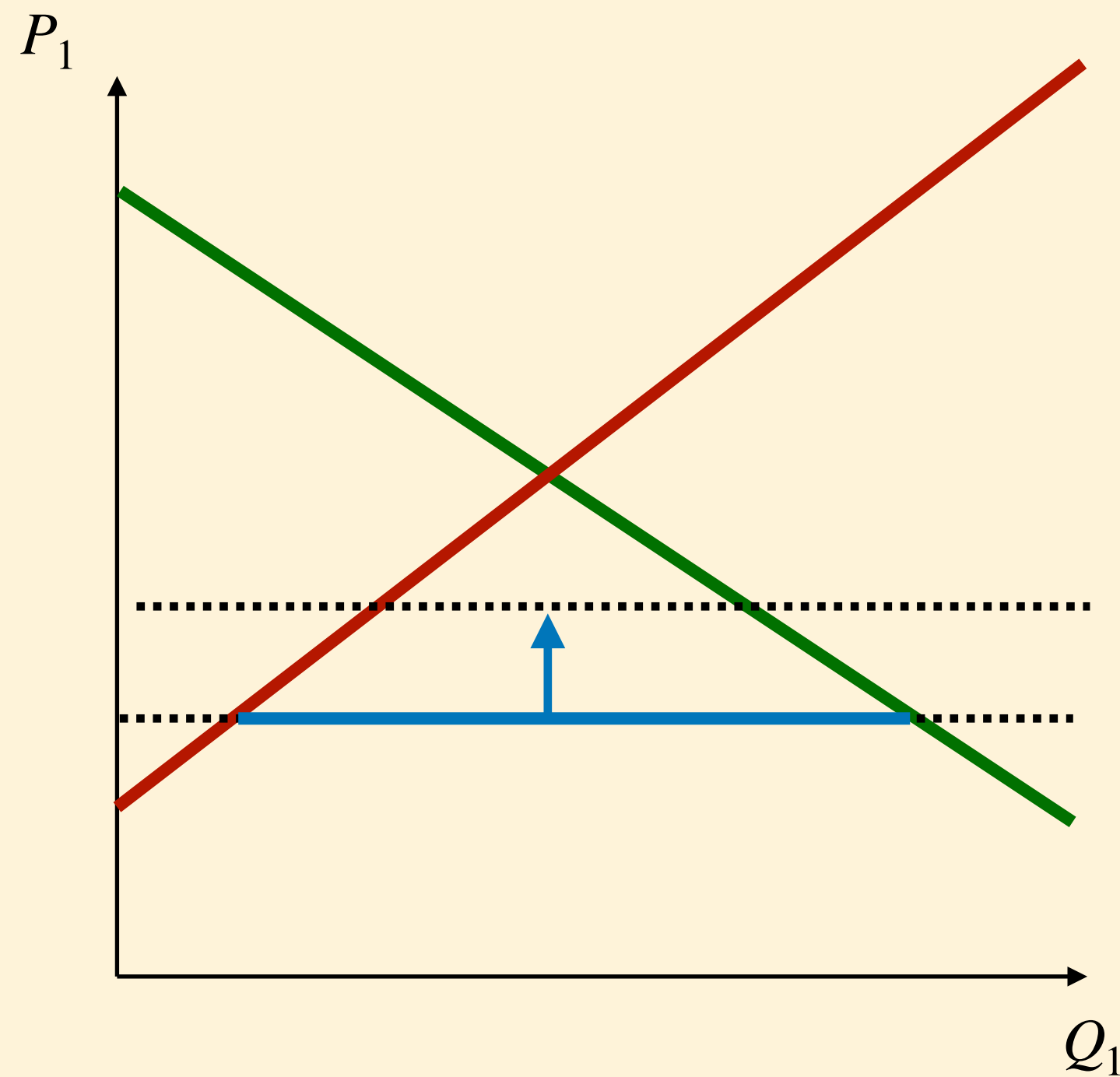
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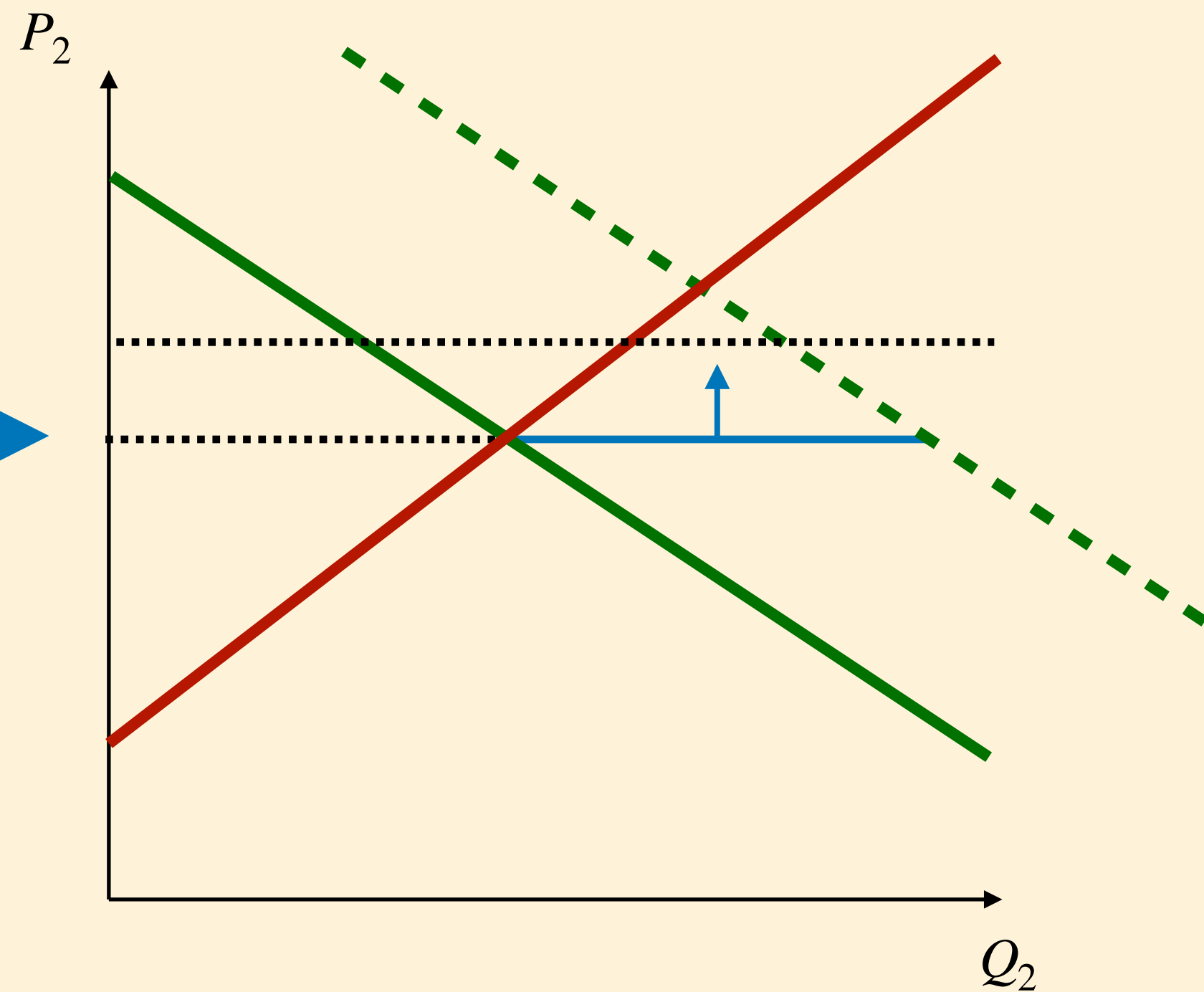
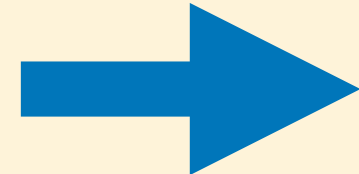
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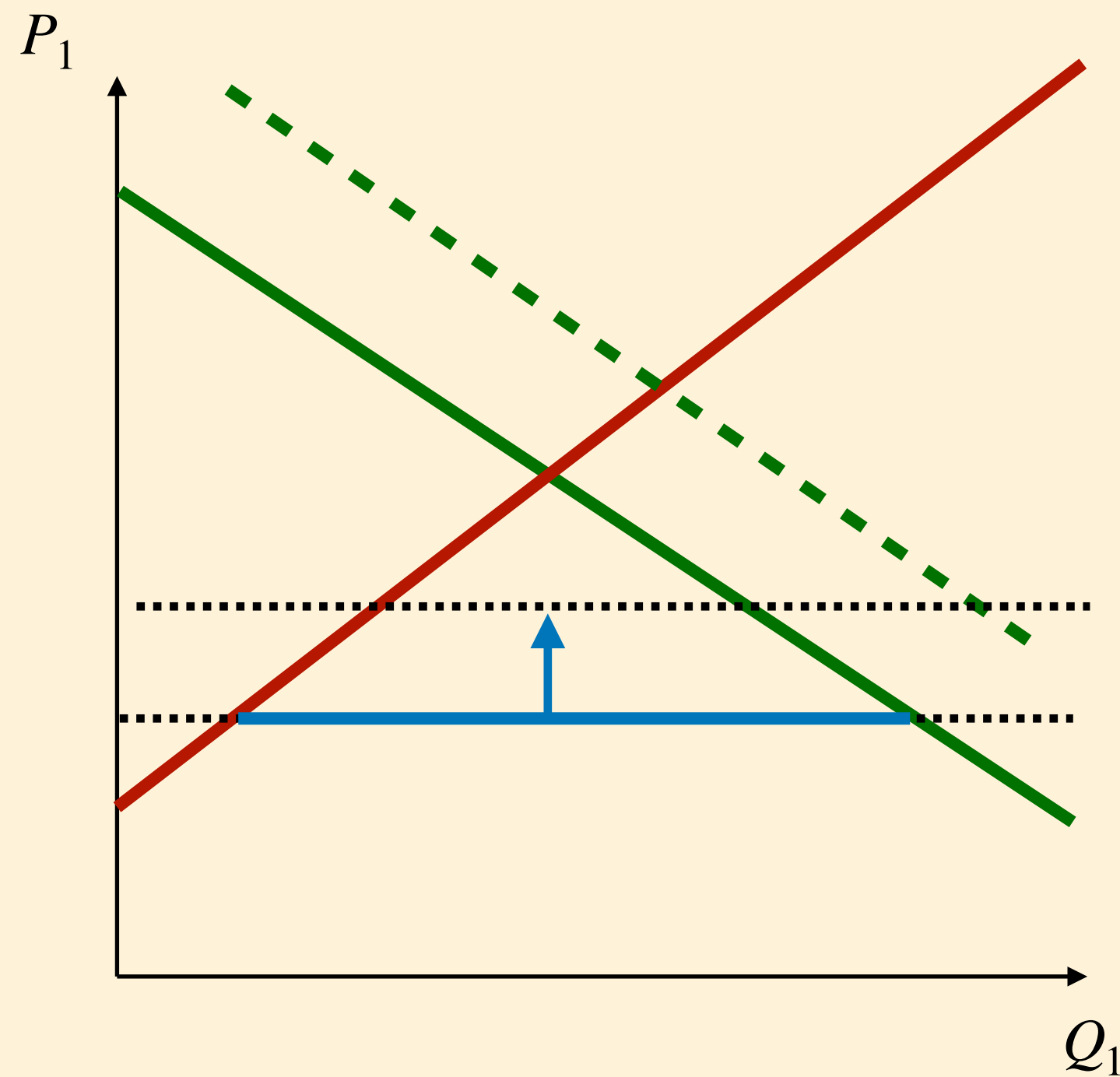
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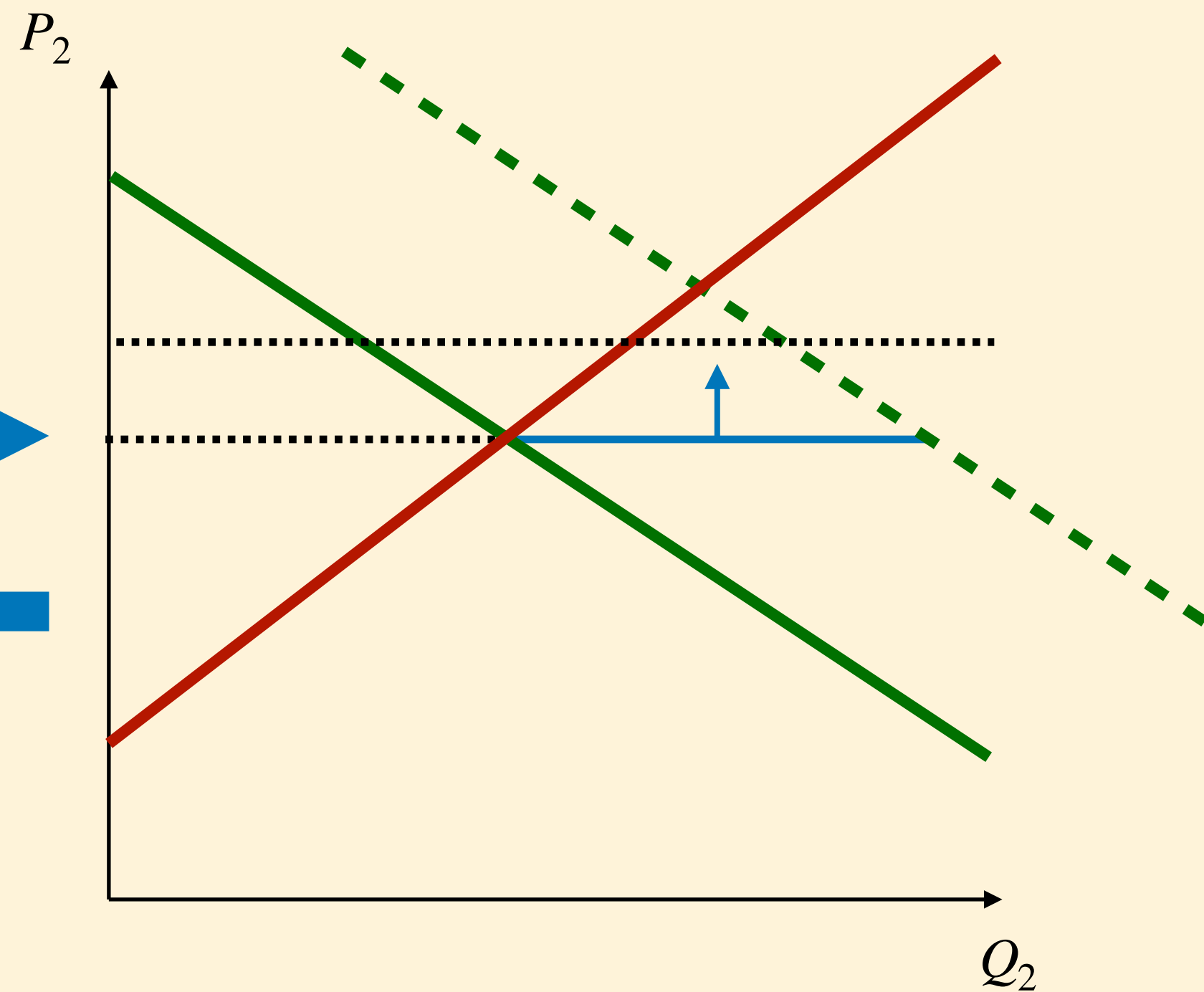
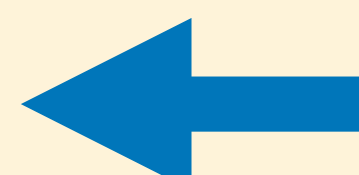
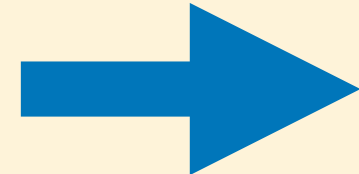
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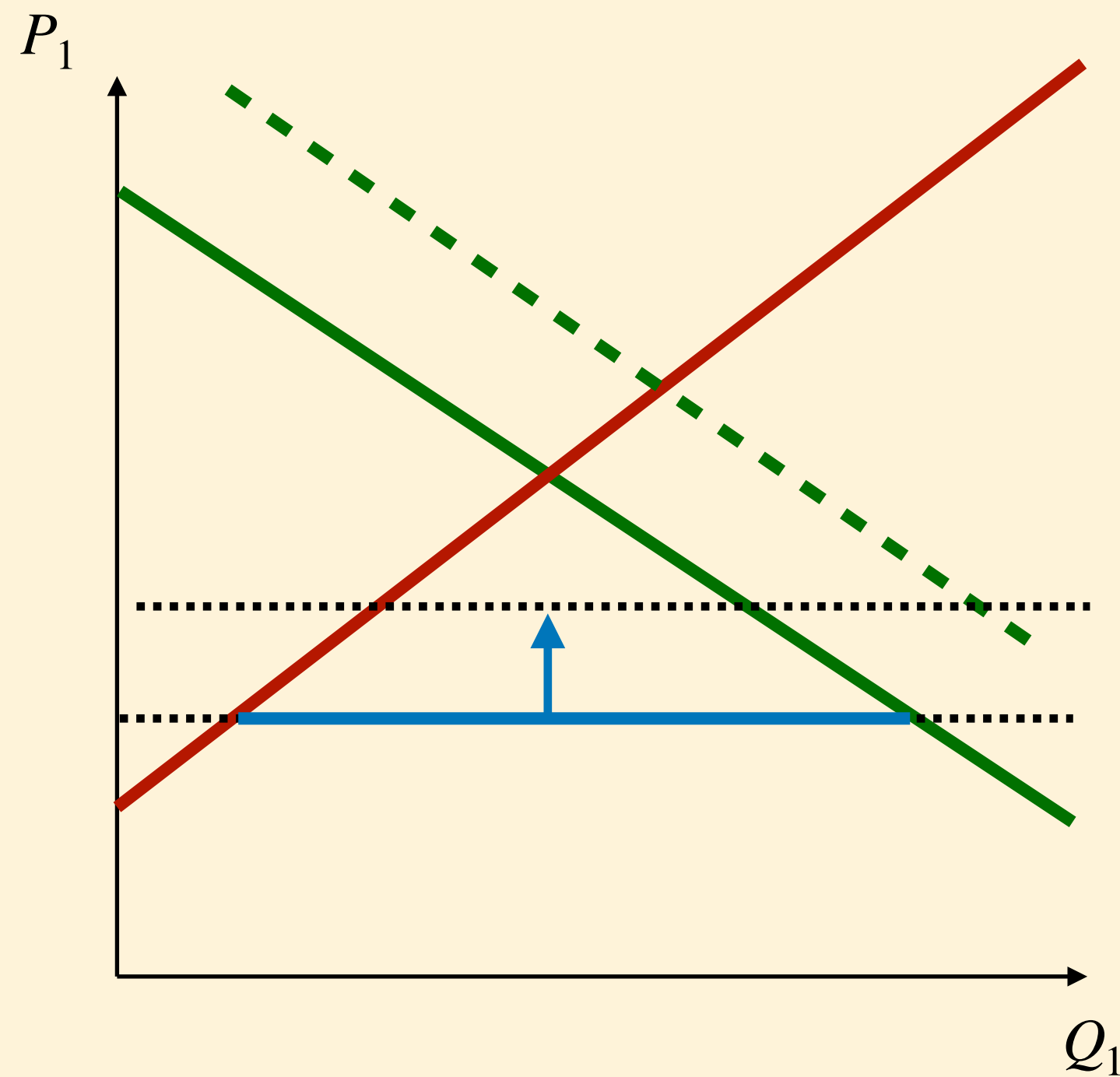
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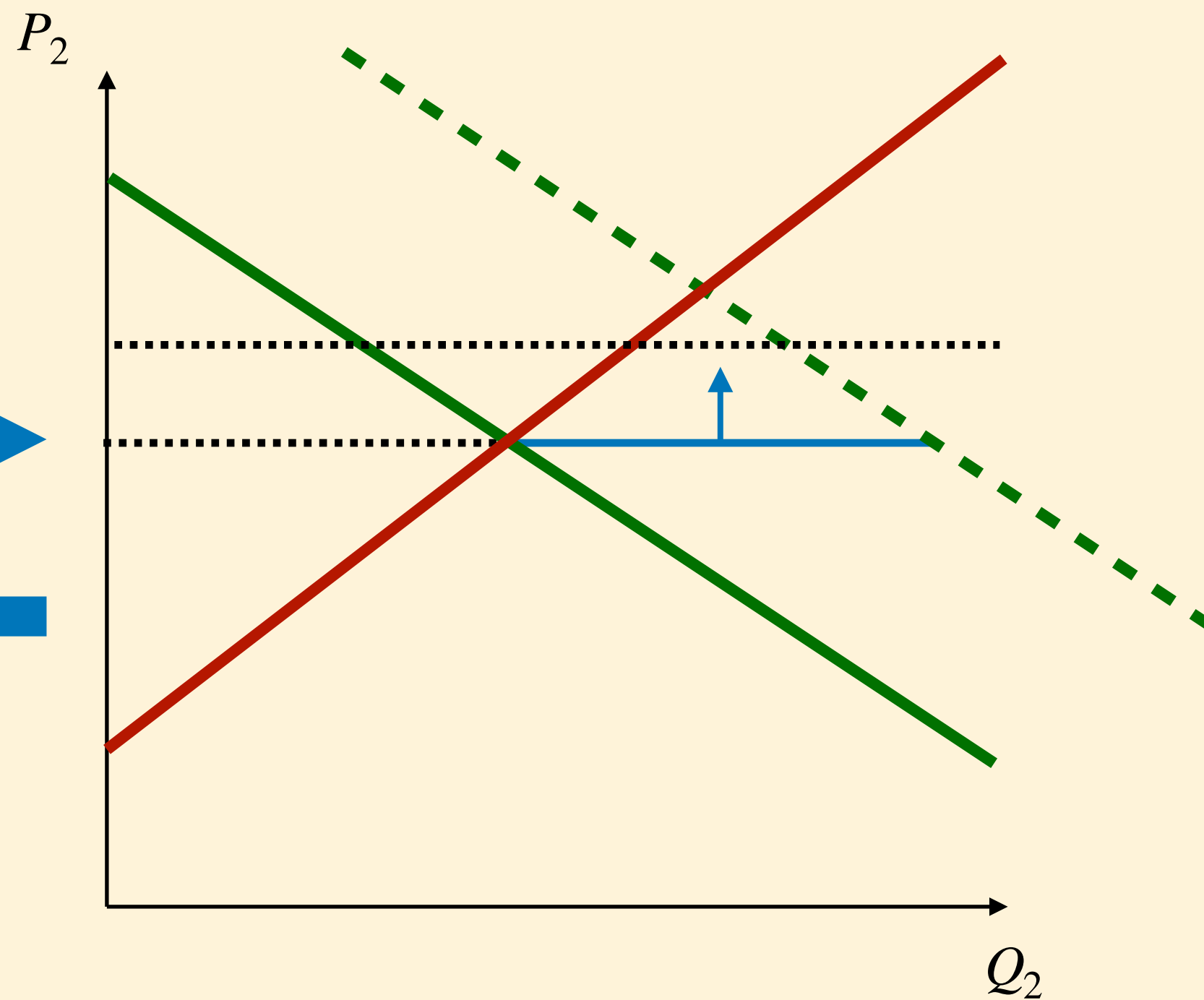
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Unstable?



Hicks Condition

$$\uparrow P_n \rightarrow \downarrow Q_n$$

...if all other prices fixed (PE)
and when *any subset* of other
markets adjusted to
equilibrium.



Parti

Der

General Equilibrium?



Cross Effects!...
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Hicks Condition

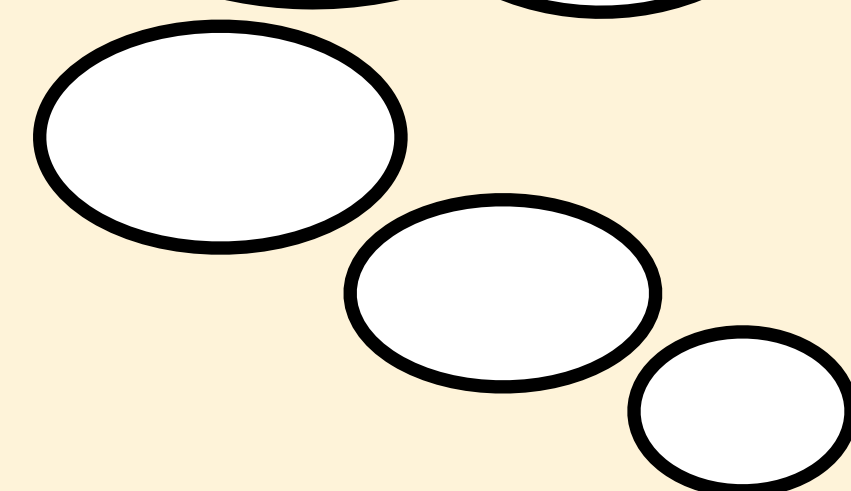
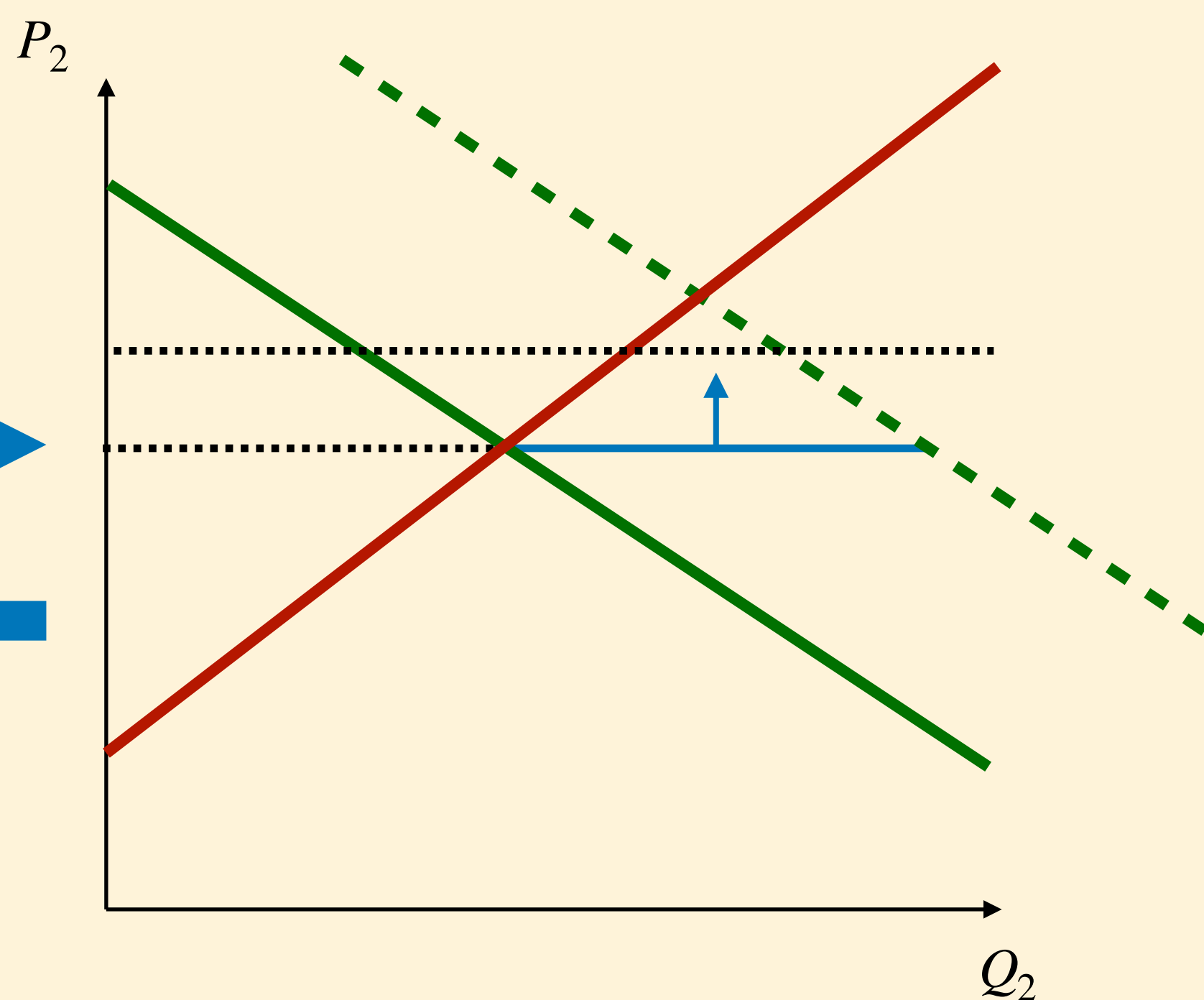
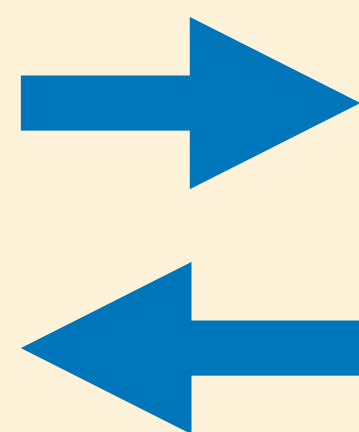
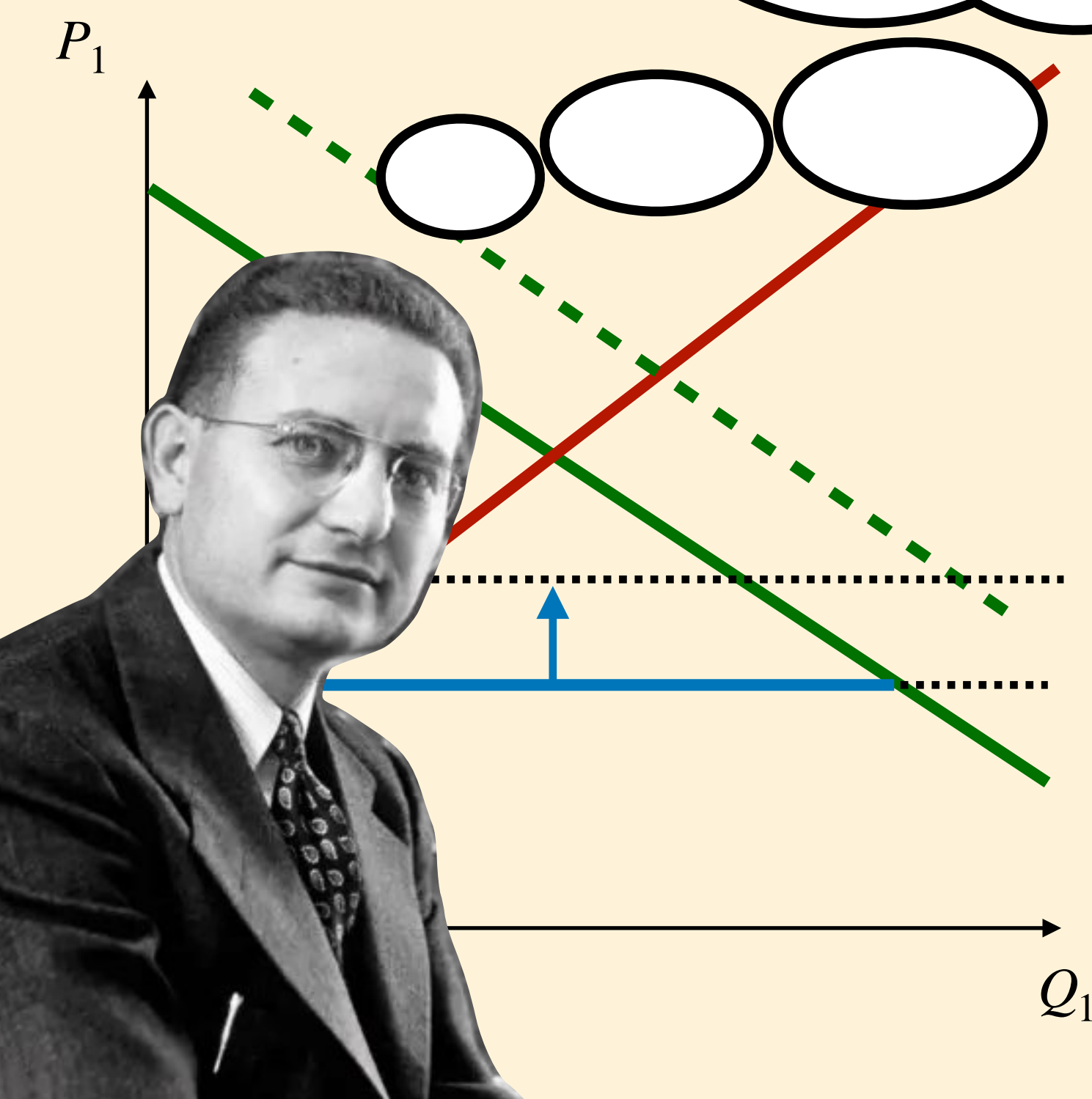
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Hicks is wrong!

Study dynamics...

$$\dot{p}_n = d_n Z_n(p)$$



Tâtonnement History

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■ GE theory...

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■ GE theory...

▶ existence



Tâtonnement History

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▶ existence



▶ uniqueness/multiplicity



Tâtonnement History

■ GE theory...

▶ existence



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▶ stability

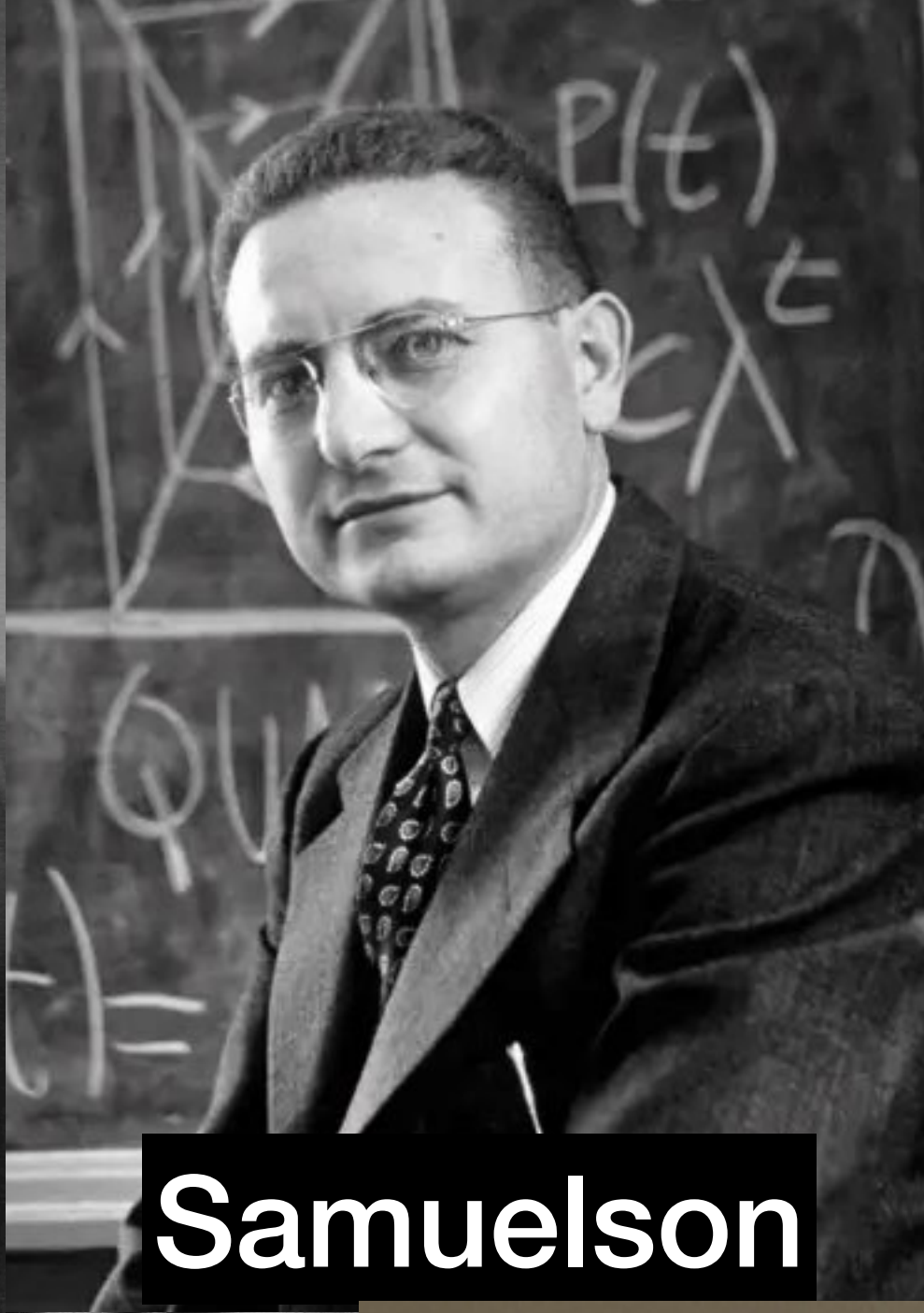


Fail! but not for lack of effort...

$$\dot{p}_n = d_n Z_n(p)$$



Hicks



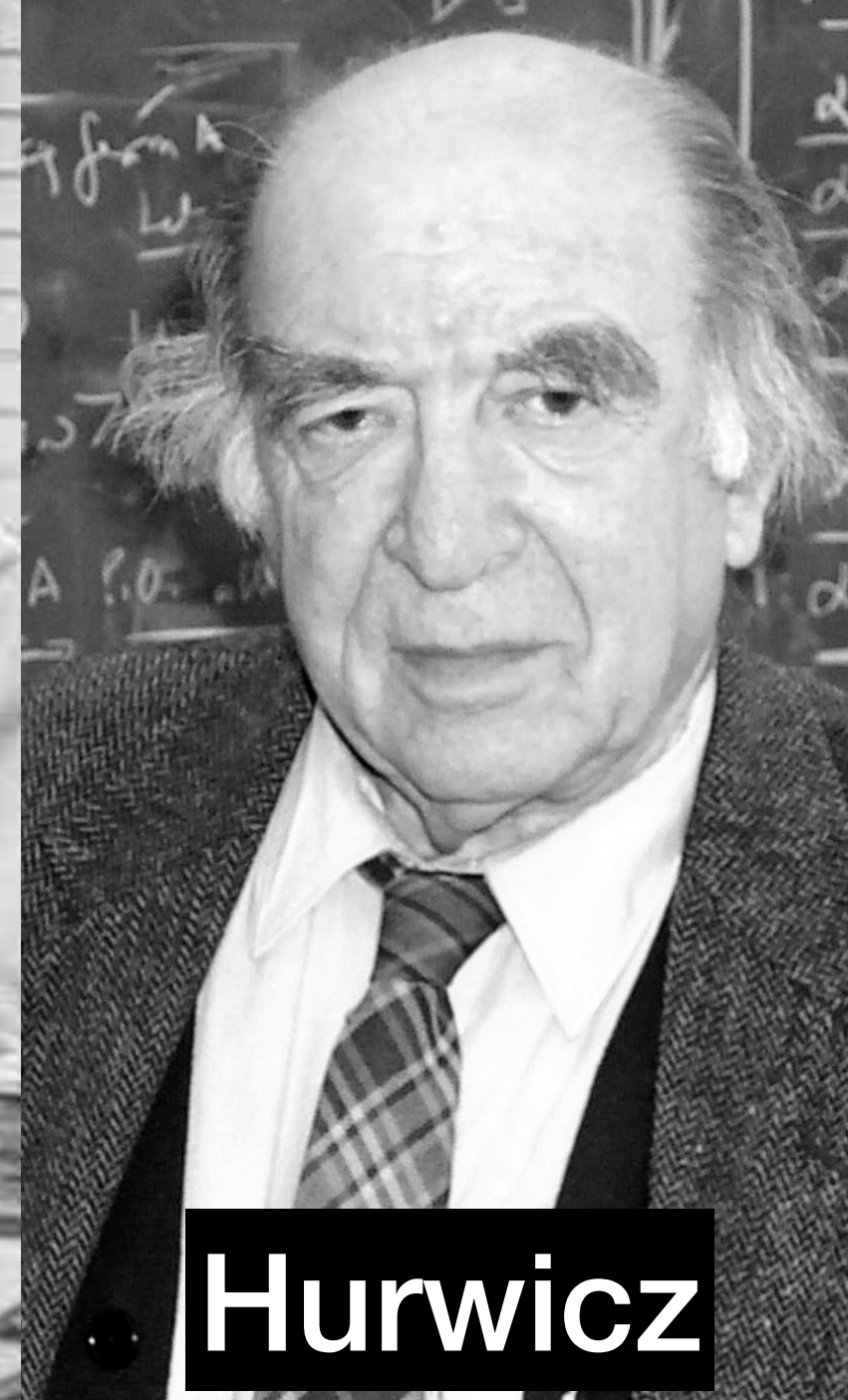
Samuelson



Metzler



Arrow



Hurwicz



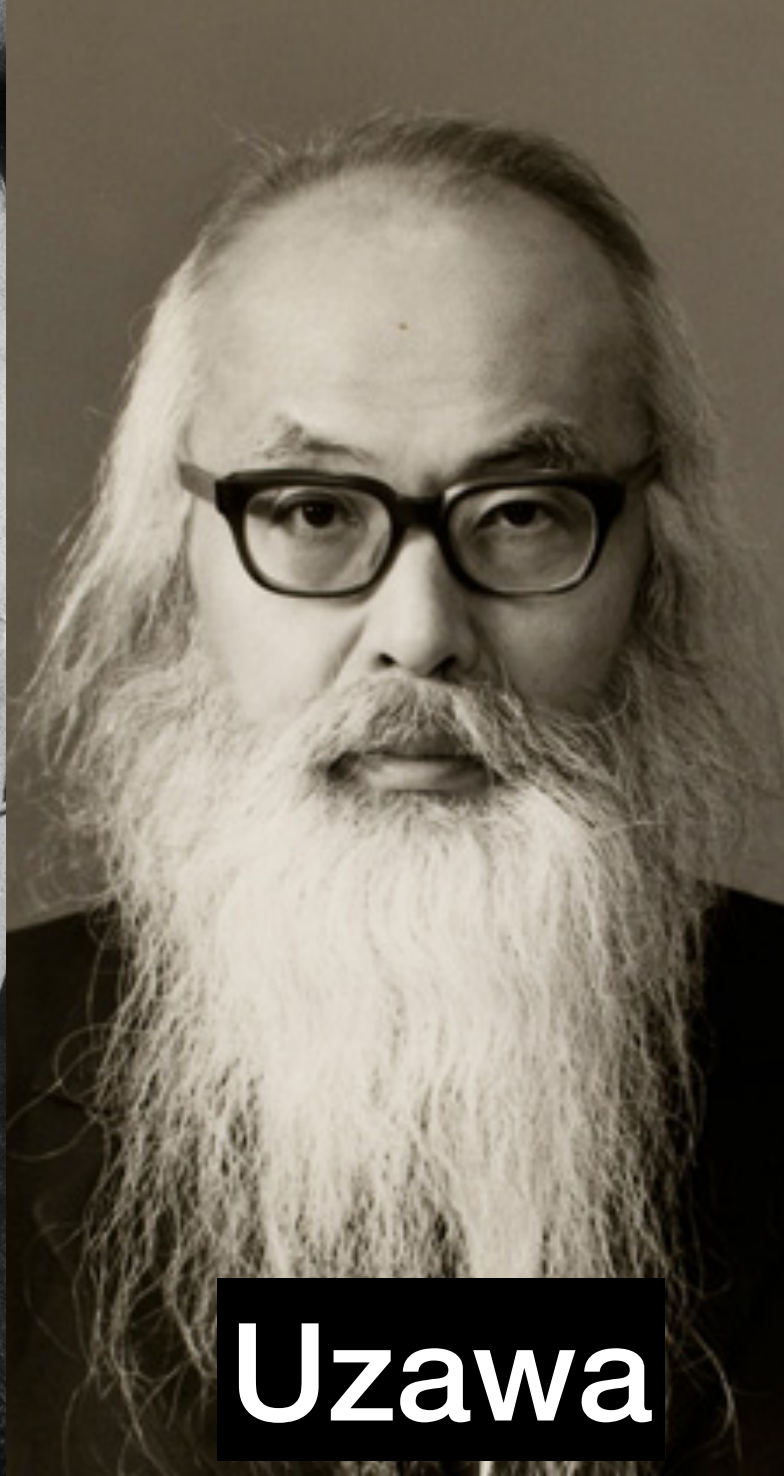
Scarf



Hahn



Fisher



Uzawa



Negishi



Smale



McKenzie



Bewley

Tâtonnement History

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Tâtonnement History

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Fail! but not for lack of effort...

$$\dot{p}_n = d_n Z_n(p)$$

■ Many sophisticated results on stability and instability, math concepts...

■ ... but **open conceptual problems**...

▶ who changes prices? are they reasonable? (alternatives proposed)

▶ consumers and producers optimize quantities given prices...

... but if markets don't clear, they cannot, so why is demand curve right object?

▶ static (not forward looking), rational expectations...? assets and money?

Tâtonnement History

■ GE theory...

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... but if m

▶ static (not

Increasingly Called
into Question...

... Decline 1970s

... Abandoned 1980s

right object?

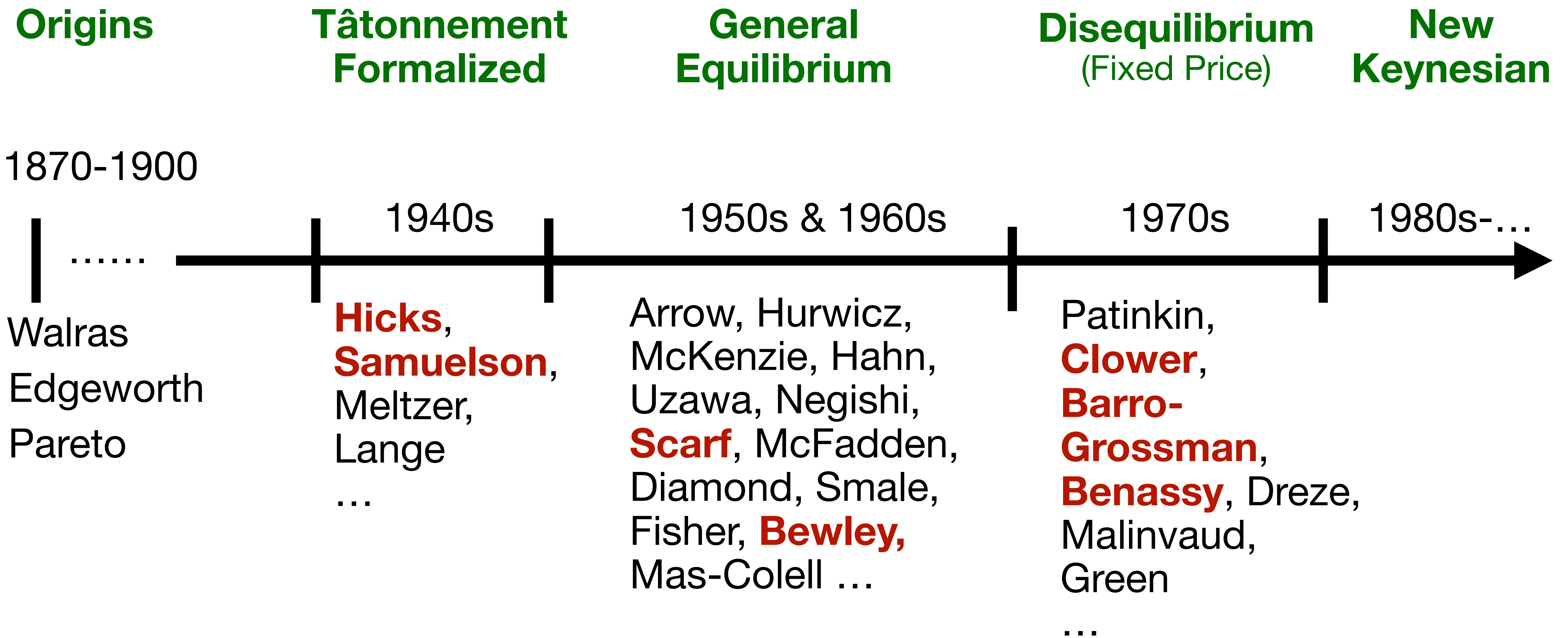
money?

Tâtonnement History

- Literature faded in 70s-80s
- Capstone: Frank Fisher's (1983) **wish list...**
 - ▶ truly dynamic model...
 - ▶ endogenous price adjustments
 - ▶ trade + consumption + production along path...
 - ▶ intertemporal saving/borrowing...
 - ▶ forward-looking agents (not naive myopic)...

■ ... we deliver on this list!





Disequilibrium Literature

Disequilibrium Literature

■ Two motivations...

- ▶ GE → quantities at non-equilibrium prices (e.g. Dreze, Benassy)
- ▶ Macro → Keynesian effects in GE (e.g. Clower, Barro-Grossman)

Disequilibrium Literature

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■ Lesson: Disequilibrium prices → some agents face **quantity constraints**
→ impacts demands for others goods
→ **spillovers!**

Disequilibrium Literature

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No Full Characterization

No Price Dynamics

- Lesson: Disequilibrium prices → some agents face **quantity constraints**
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 - **spillovers!**

Contributions and Results: **Micro** → **Macro**

Contributions and Results: **Macro → Micro**

 **Excess Demand Approach**

Contributions and Results: **Macro** → **Micro**

■ **Excess Demand Approach**

■ **Main Result...**

▶ forward-looking tâtonnement, not myopic...

▶ ... yet: justify “myopic” Samuelson-!!

▶ ... but two differences in $Z_n(P)$...

□ **Frisch** demand (not Marshallian) → “as if” representative agent → **well behaved**

□ **Non-Walrasian** demand → spillovers → **prices affected by other markets**

Contributions and Results: **Macro** → **Micro**

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input-output network connection

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■ Stability...

input-output network connection

▶ unstable possible, but...

▶ Frisch demands → strong force for **stability**

▶ Forward-looking price setting → additional force for **stability!**

Contributions and Results: Macro → Micro

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... Literature Lost Steam 70s

... Just Before...

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... Just Before...



Dixit-Stiglitz
1979

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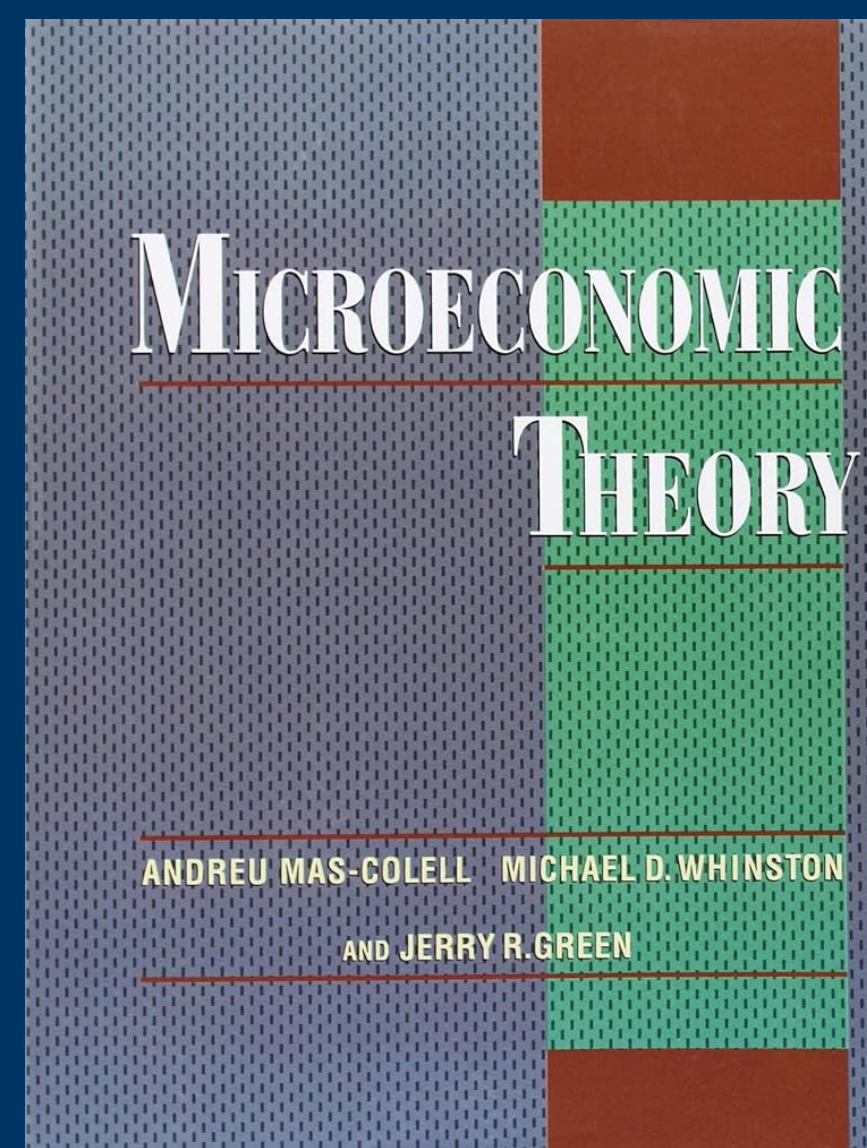
... Just Before...



Dixit-Stiglitz
1979

Guillermo
Calvo
1983





Static: Walrasian Benchmark

Static GE Model

■ GE = **General** Equilibrium

■ General Primitives...

▶ n goods (goods and factors, many labor etc.)

▶ h household types, general preferences

▶ f firms, general technologies (networks, etc.)

$$x = (x_1, \dots, x_N) \geq 0$$

$$y = (y_1, \dots, y_N) \geq 0$$

$$z = (z_1, \dots, z_N) = x - y$$

Math

$$\Pi^f(P) \equiv \max_{x^f, y^f} P \cdot (y^f - x^f)$$
$$(x^f, y^f) \in Y^f$$

$$D_W^f(P), S_W^f(P)$$
$$D_W^h(P), S_W^h(P)$$

$$D_W(P) = \sum_j D_W^j(P)$$
$$S_W(P) = \sum_j S_W^j(P)$$

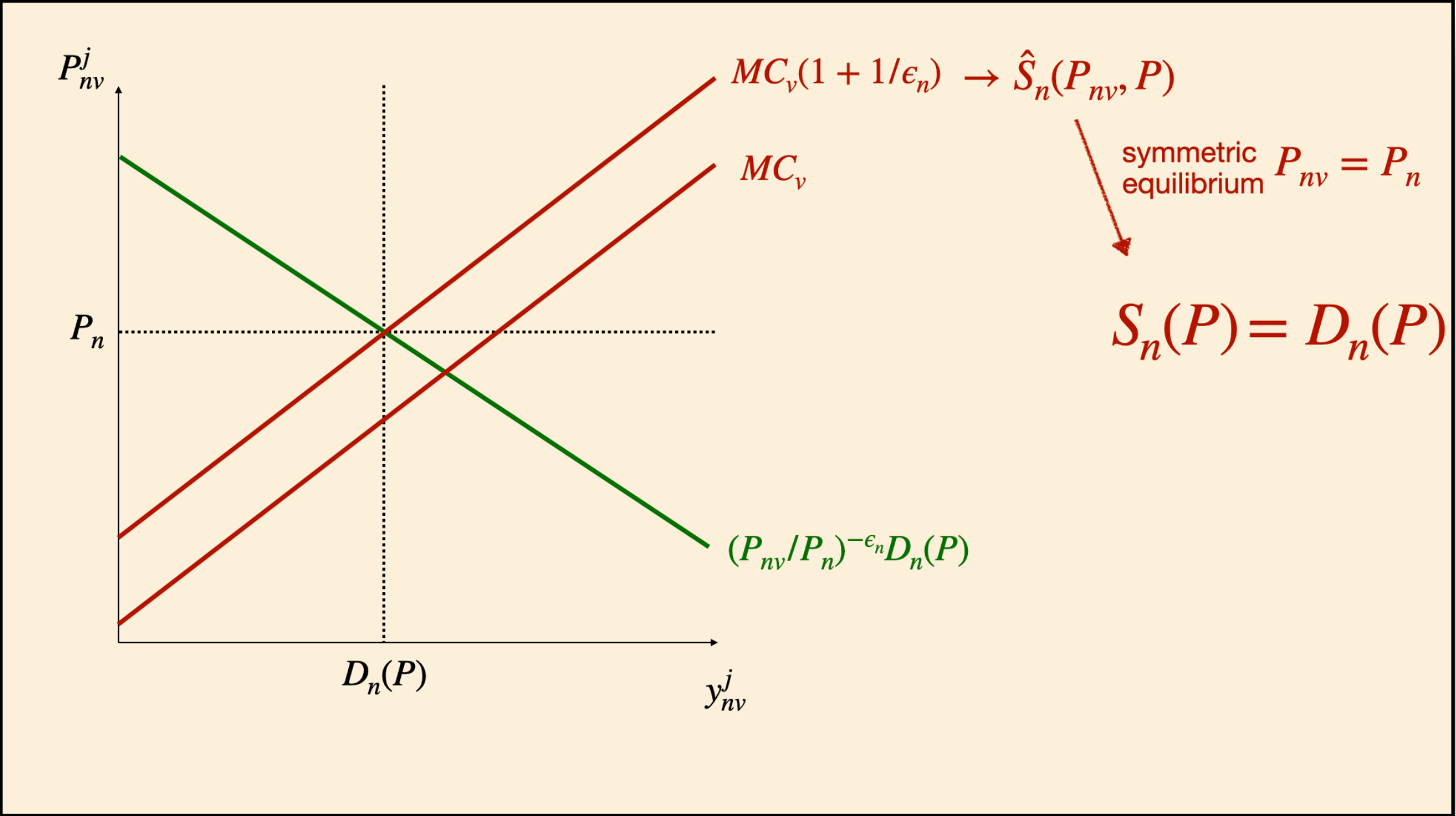
$$D_W(P) = S_W(P)$$

$$\max_{x^h, y^h} U^h(x^h, y^h)$$
$$P \cdot (x^h - y^h) \leq P \cdot a^h + \sum_f \omega^{h,f} \Pi^f(P)$$

$$\dot{P}_n = \alpha_n (D_{Wn}(P) - S_{Wn}(P))$$

Samuelson's ad hoc proposal
to capture Walras' idea...

Figure



$$\Pi^f(P) \equiv \max_{x^f, y^f} P \cdot (y^f - x^f)$$

$$(x^f, y^f) \in Y^f$$



$$D_W^f(P), S_W^f(P)$$



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**Samuelson's ad hoc proposal
to capture Walras' idea...**

$$\Pi^f \equiv \max_{z \in Y^f} -p \cdot z$$

$$\Pi^f \equiv \max - p \cdot z$$



$$z \in Y^f$$

$$Z^{**f}(p)$$

$$\Pi^f \equiv \max - p \cdot z$$



$$z \in Y^f$$

$$Z^{**f}(p)$$

$$U^h(z^h)$$

$$\begin{array}{l} \Pi^f \equiv \max_{z \in Y^f} -p \cdot z \\ \downarrow \\ Z^{**f}(p) \end{array}$$

$$\begin{array}{l} \max_{z^h} U^h(z^h) \\ p \cdot z^h \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f \end{array}$$

$$\Pi^f \equiv \max_{z \in Y^f} -p \cdot z$$



$$z \in Y^f$$

$$Z^{**f}(p)$$

$$Z^{**h}(p)$$



$$\max_{z^h} U^h(z^h)$$

$$p \cdot z^h \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$\Pi^f \equiv \max_{z \in Y^f} -p \cdot z$$

$$\downarrow$$

$$Z^{**f}(p)$$

$$(Z^{**}(p) \equiv \sum_j Z^{**j}(p))$$

$$\uparrow$$

$$\max_{z^h} U^h(z^h)$$

$$p \cdot z^h \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$Z^{**}(p) = 0$$

N equations
 N unknowns

Static: Adding Market Power

Adding Market Power

$$z_{un}(\omega) + z_{vn}(\omega) = 0$$

Adding Market Power

■ Each market $n \rightarrow$ varieties $\omega \in [0,1]$

► **price-taking undifferentiated (u)** vs. **price-setting differentiated (v = varieties)**

$$z_{un}(\omega) + z_{vn}(\omega) = 0$$

Adding Market Power

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► **price-taking undifferentiated (u)** vs. **price-setting differentiated (v = varieties)**

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► **differentiated side $n \rightarrow$ agent type $j \rightarrow$ sets prices $p_{nv}(\omega) \dots$**

Adding Market Power

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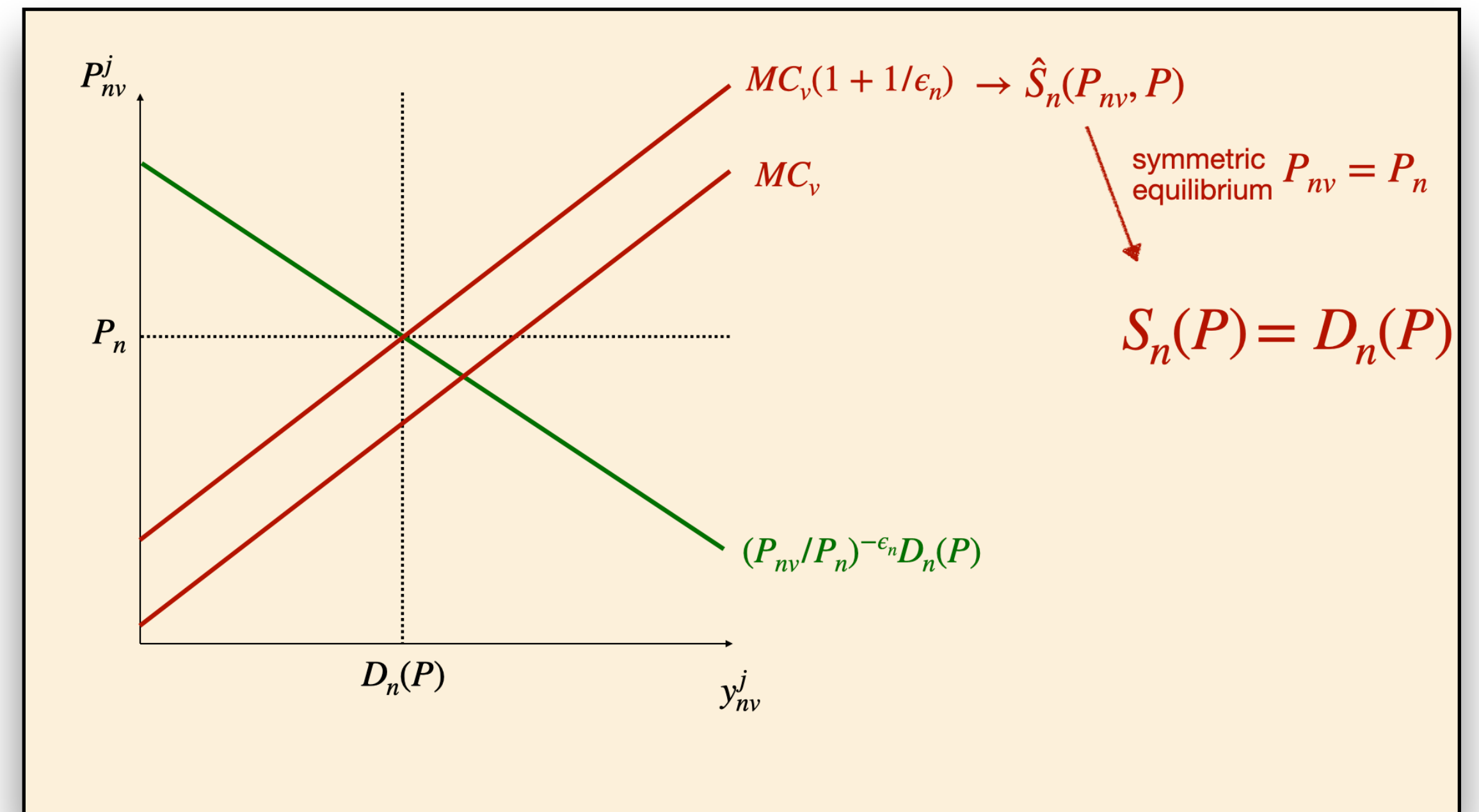
► **undifferentiated side n** : CES aggregator over varieties with elasticity ϵ_n

Math

$$\begin{aligned} \Pi^f(P) &\equiv \max_{x^f, y^f} P \cdot (y^f - x^f) \\ &\quad (x^f, y^f) \in Y^f \\ &\downarrow \\ D_W^f(P), S_W^f(P) &\quad \rightarrow \quad D_W(P) = \sum_j D_W^j(P) \quad \rightarrow \quad D_W(P) = S_W(P) \\ D_W^h(P), S_W^h(P) &\quad \leftarrow \quad S_W(P) = \sum_j S_W^j(P) \\ &\uparrow \\ \max_{x^h, y^h} U^h(x^h, y^h) & \\ P \cdot (x^h - y^h) &\leq P \cdot a^h + \sum_f \omega^{h,f} \Pi^f(P) \end{aligned}$$

$\dot{P}_n = \alpha_n (D_{Wn}(P) - S_{Wn}(P))$
**Samuelson's ad hoc proposal
to capture Walras' idea...**

Figure



$$\Pi^f(P) \equiv \max_{x^f, y^f} P \cdot (y^f - x^f)$$

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$$D_W^f(P), S_W^f(P)$$



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**Samuelson's ad hoc proposal
to capture Walras' idea...**

$$\Pi^f(p) \equiv \max_{z_u, z_{vn}, p_{vn}} \quad - (p \cdot z_u^f + \textcolor{blue}{p_{vn} z_{vn}})$$

$$(z_u^f, \textcolor{blue}{z_{vn}}) \in Y^f$$

$$\textcolor{blue}{z_{vn}} = - (p_{vn}/p_n)^{-\epsilon_n} z_{un}$$

$$\Pi^f(p) \equiv \max_{z_u, z_{vn}, p_{vn}} \quad - (p \cdot z_u^f + \textcolor{blue}{p_{vn}z_{vn}})$$

$$(z_u^f, \textcolor{blue}{z_{vn}}) \in Y^f$$

$$\textcolor{blue}{z_{vn}} = - (p_{vn}/p_n)^{-\epsilon_n} \, z_{un}$$

$$\max_{z_u, z_{vn}, p_{vn}} \quad U^h(z_u^h, \textcolor{blue}{z_{vn}})$$

$$p \cdot z_u^h + \textcolor{blue}{p_{vn}z_{nv}} \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$\textcolor{blue}{z_{vn}} = - (p_{vn}/p_n)^{-\epsilon_n} \, z_{un}$$

$$\begin{aligned}\Pi^f(p) &\equiv \max_{z_u, z_{vn}, p_{vn}} - (p \cdot z_u^f + p_{vn} z_{vn}) \\ (z_u^f, z_{vn}) &\in Y^f \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

Need prices and quantities

p and $z_{un} \dots$

... or do we really? ;)

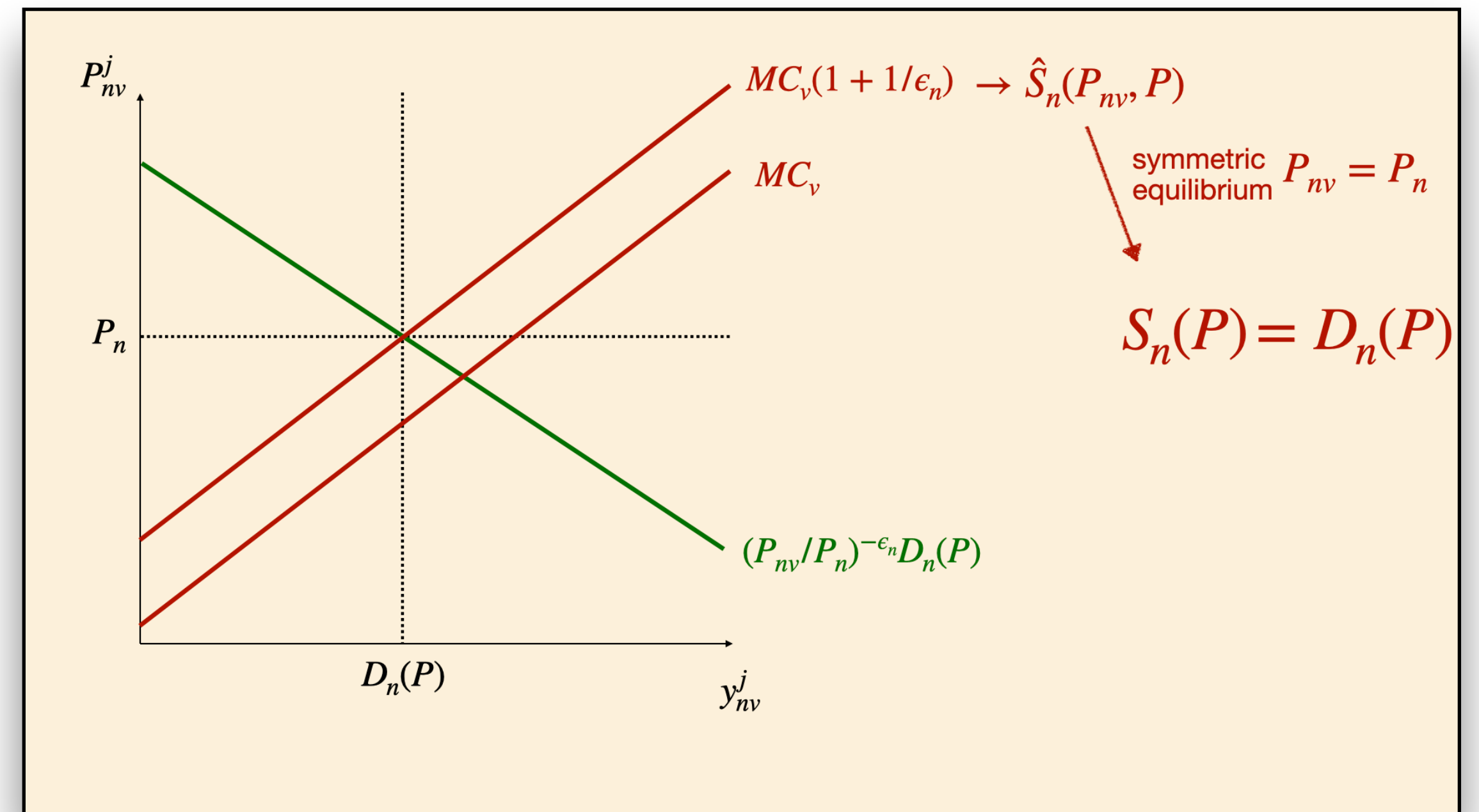
$$\begin{aligned}\max_{z_u, z_{vn}, p_{vn}} \quad & U^h(z_u^h, z_{vn}) \\ p \cdot z_u^h + p_{vn} z_{nv} &\leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

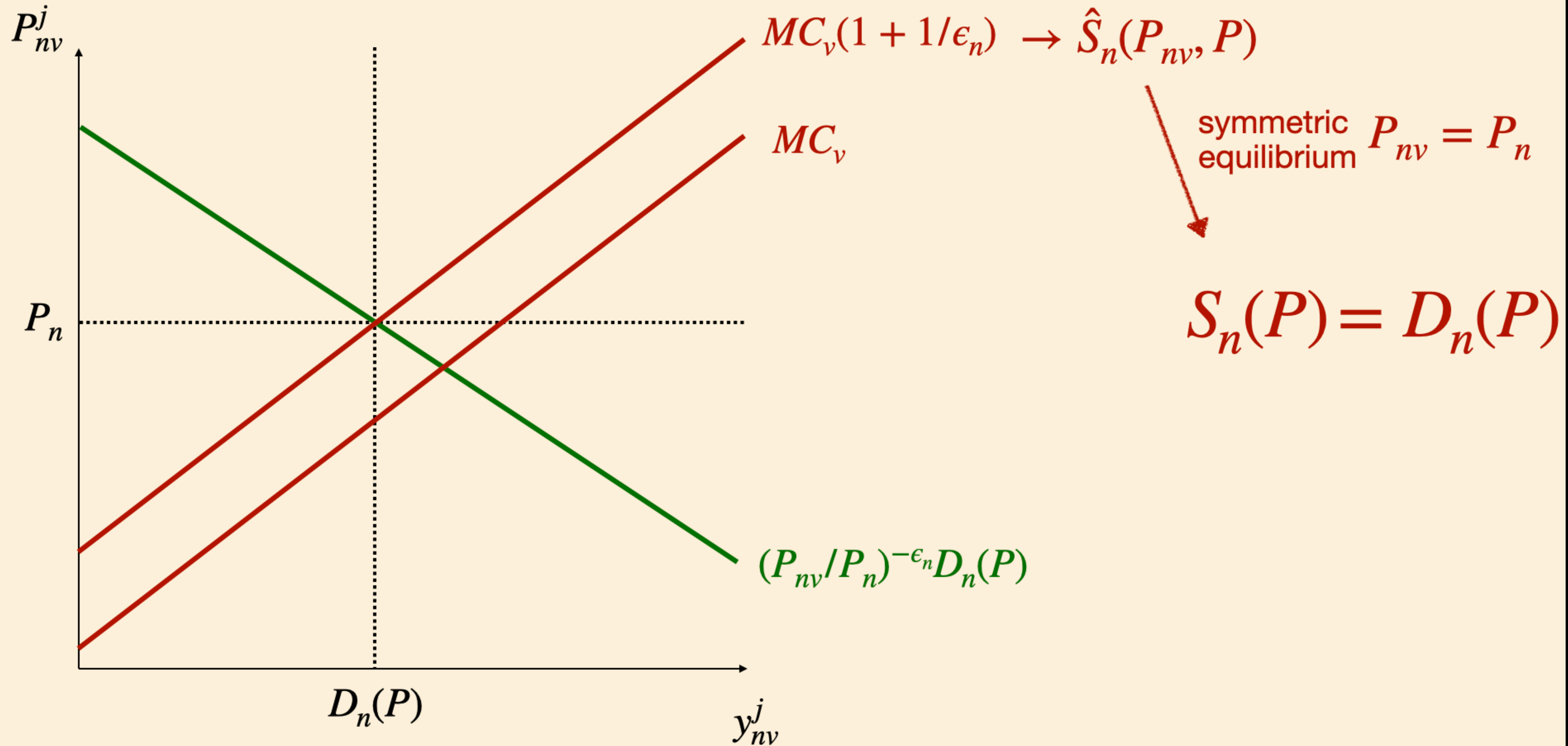
Math

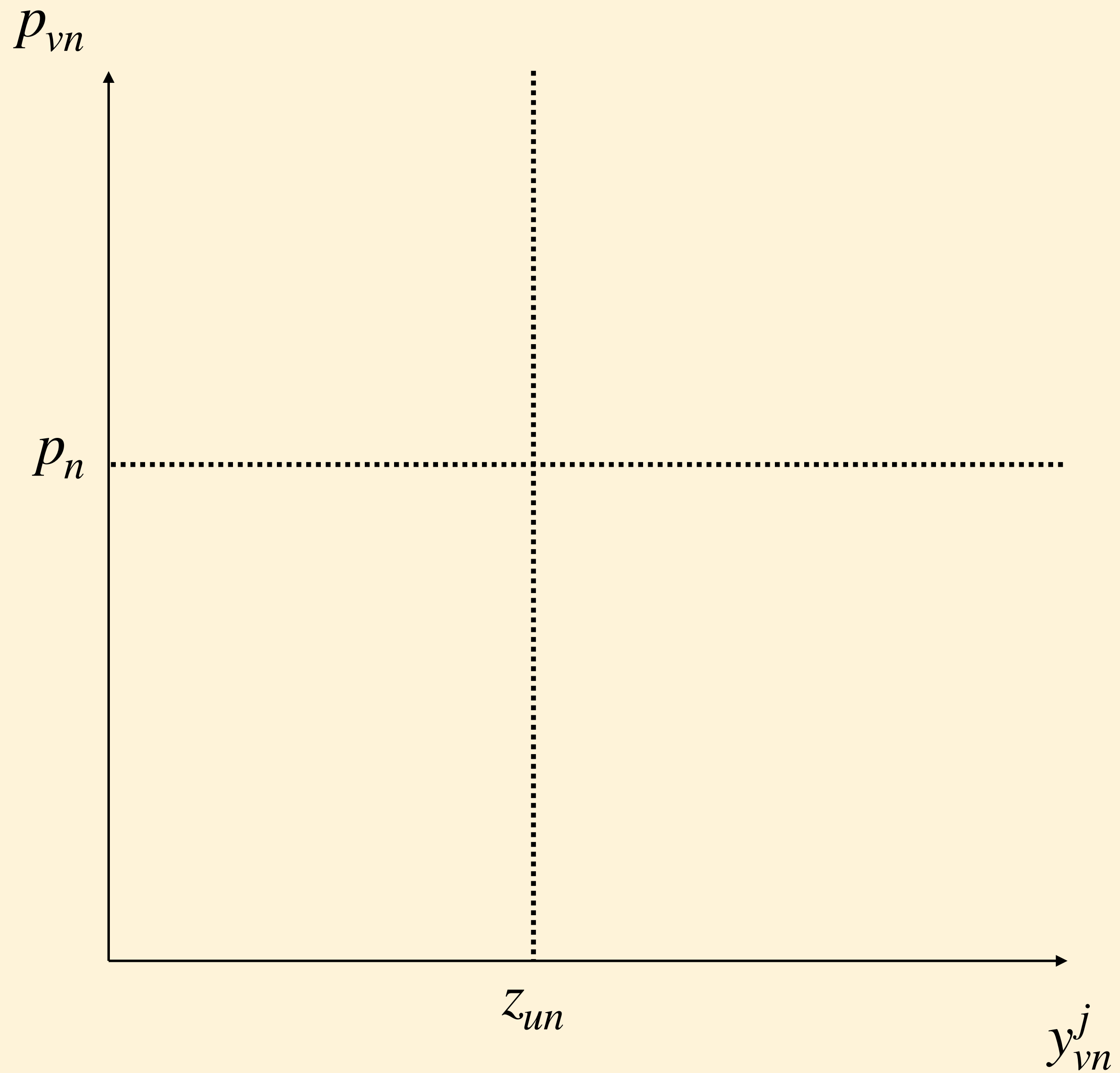
$$\begin{aligned} \Pi^f(P) &\equiv \max_{x^f, y^f} P \cdot (y^f - x^f) \\ &\quad (x^f, y^f) \in Y^f \\ &\downarrow \\ D_W^f(P), S_W^f(P) &\quad \rightarrow \quad D_W(P) = \sum_j D_W^j(P) \quad \rightarrow \quad D_W(P) = S_W(P) \\ D_W^h(P), S_W^h(P) &\quad \leftarrow \quad S_W(P) = \sum_j S_W^j(P) \\ &\uparrow \\ \max_{x^h, y^h} U^h(x^h, y^h) & \\ P \cdot (x^h - y^h) &\leq P \cdot a^h + \sum_f \omega^{h,f} \Pi^f(P) \end{aligned}$$

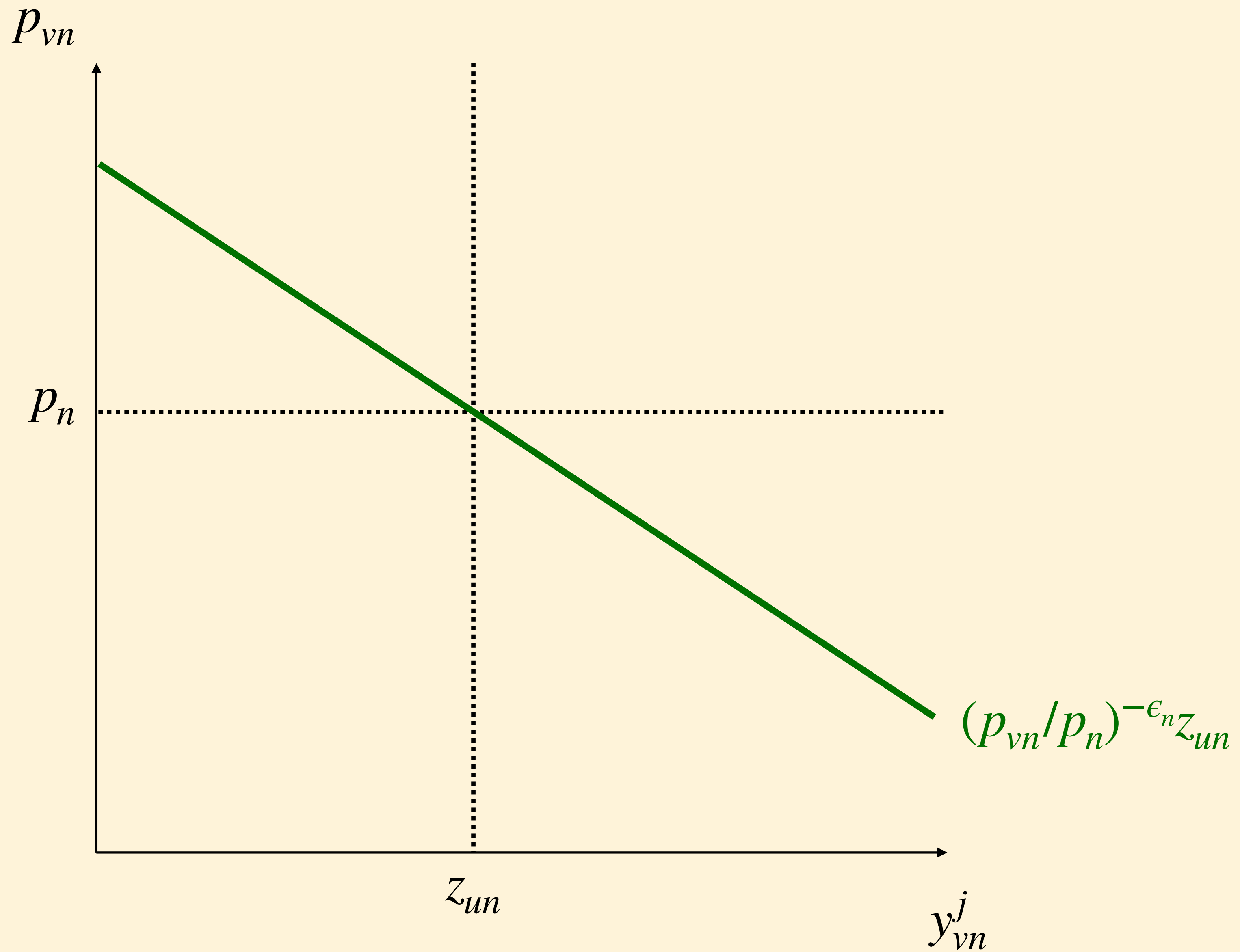
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Samuelson's ad hoc proposal to capture Walras' idea...

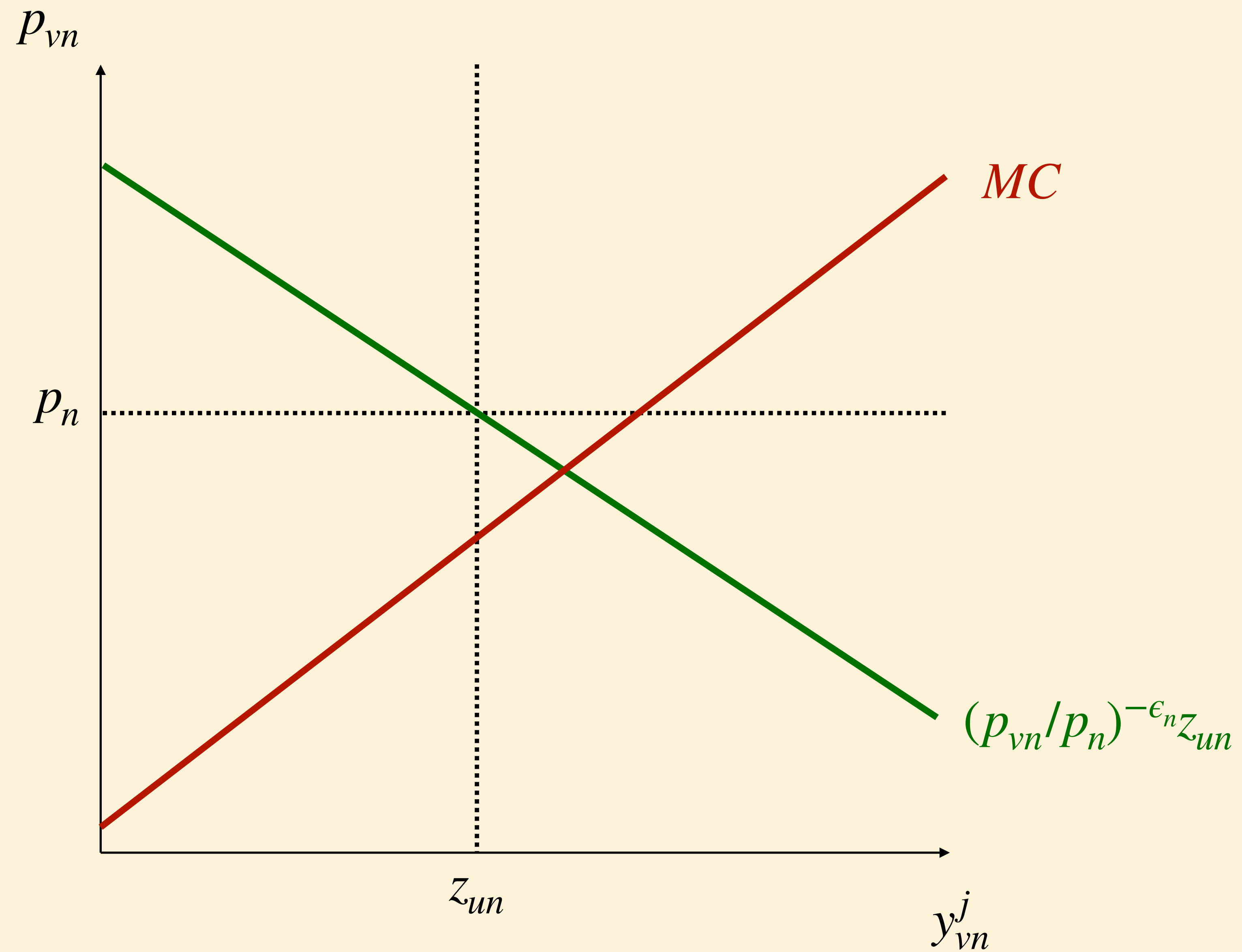
Figure

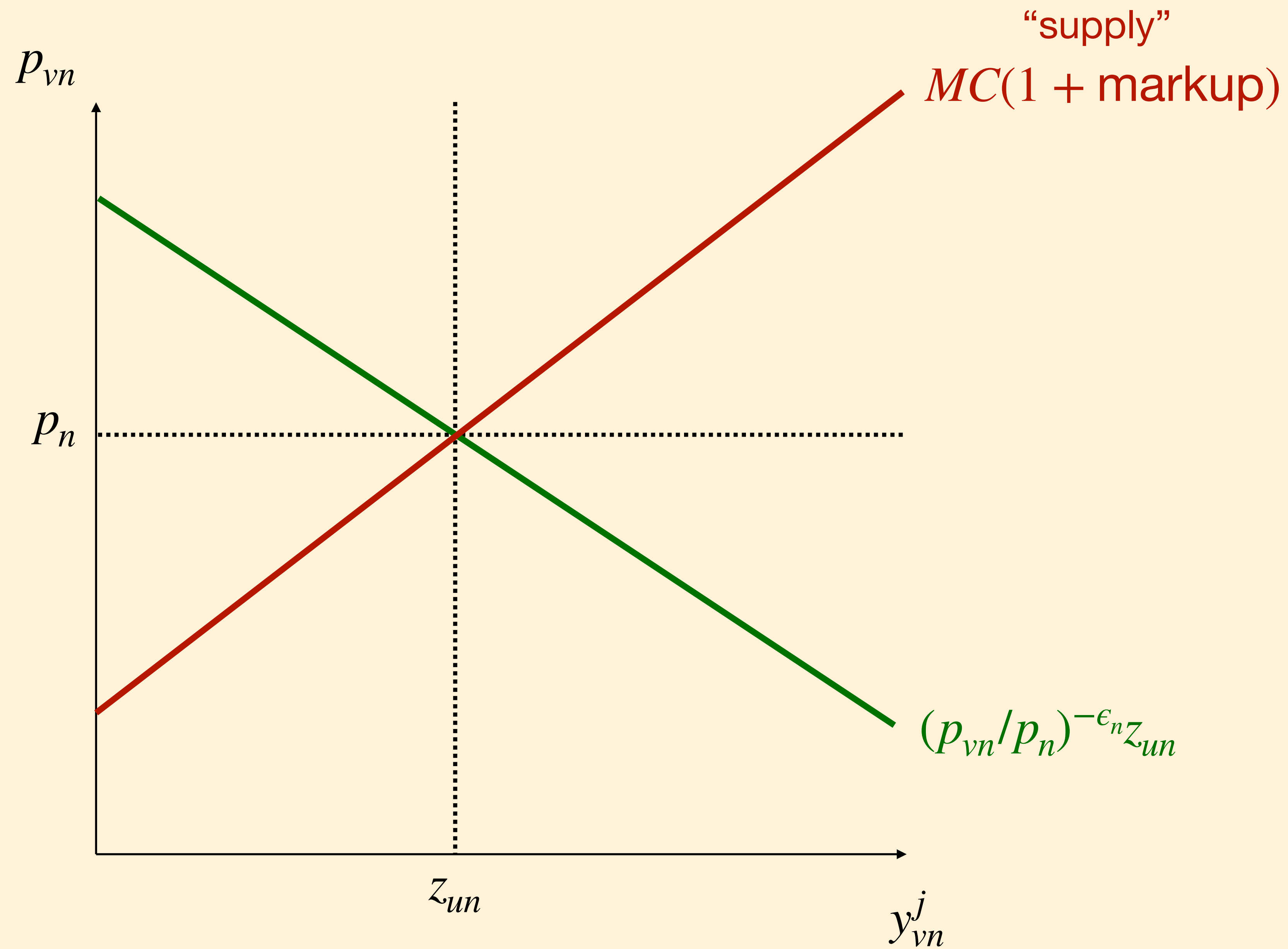


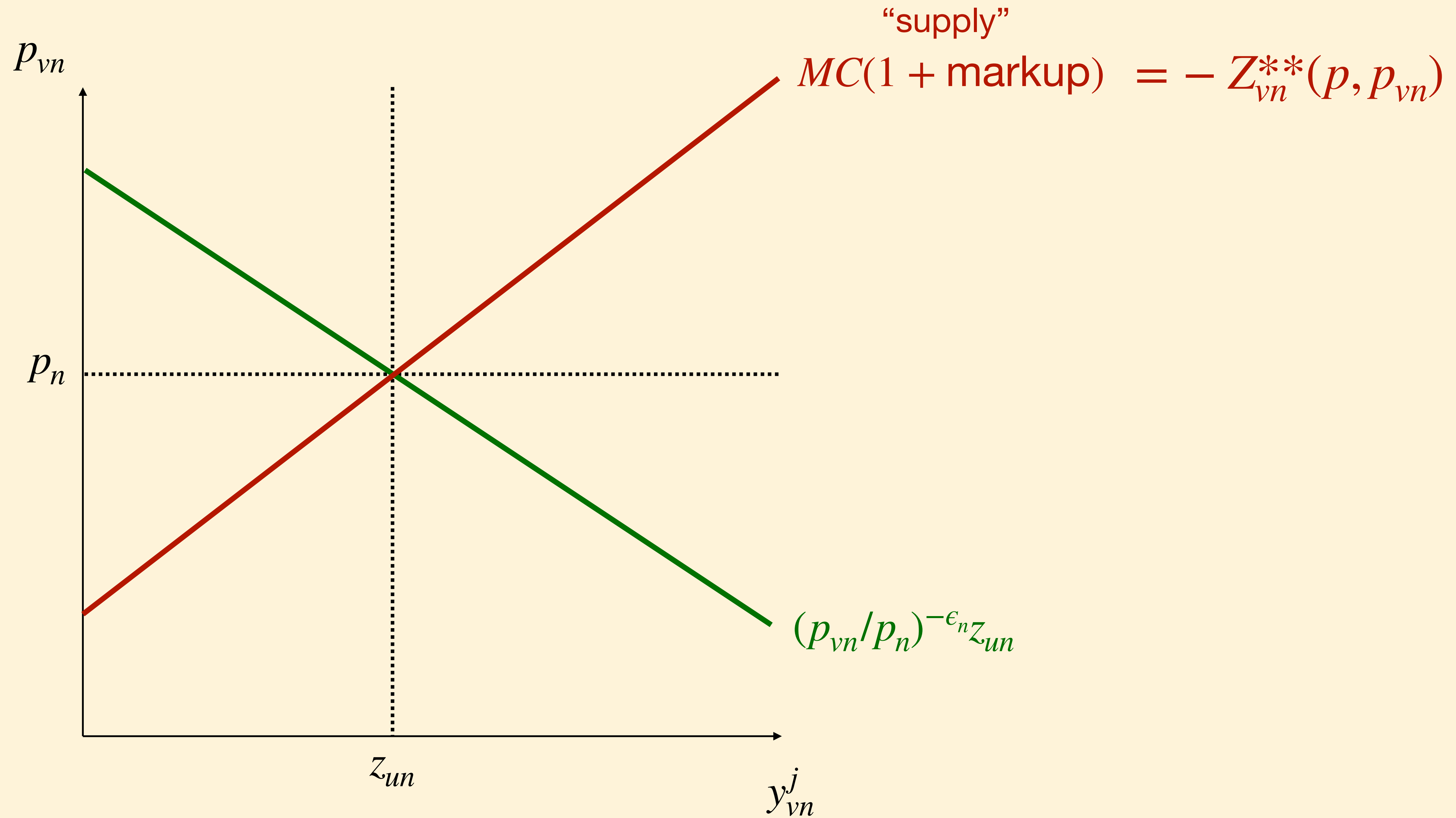


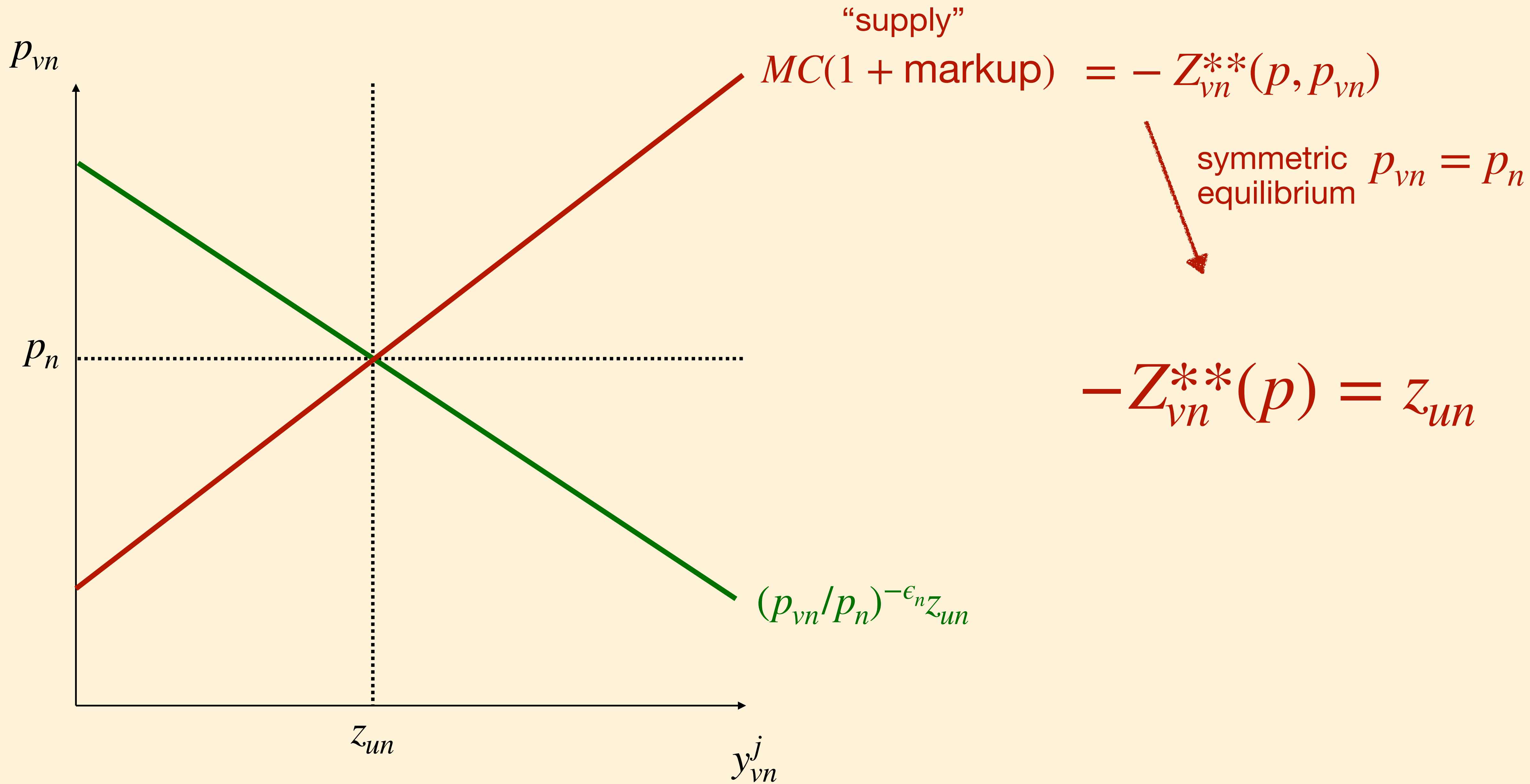












Math

$$\Pi^f(P) \equiv \max_{x^f, y^f} P \cdot (y^f - x^f)$$
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$$D_W^f(P), S_W^f(P)$$
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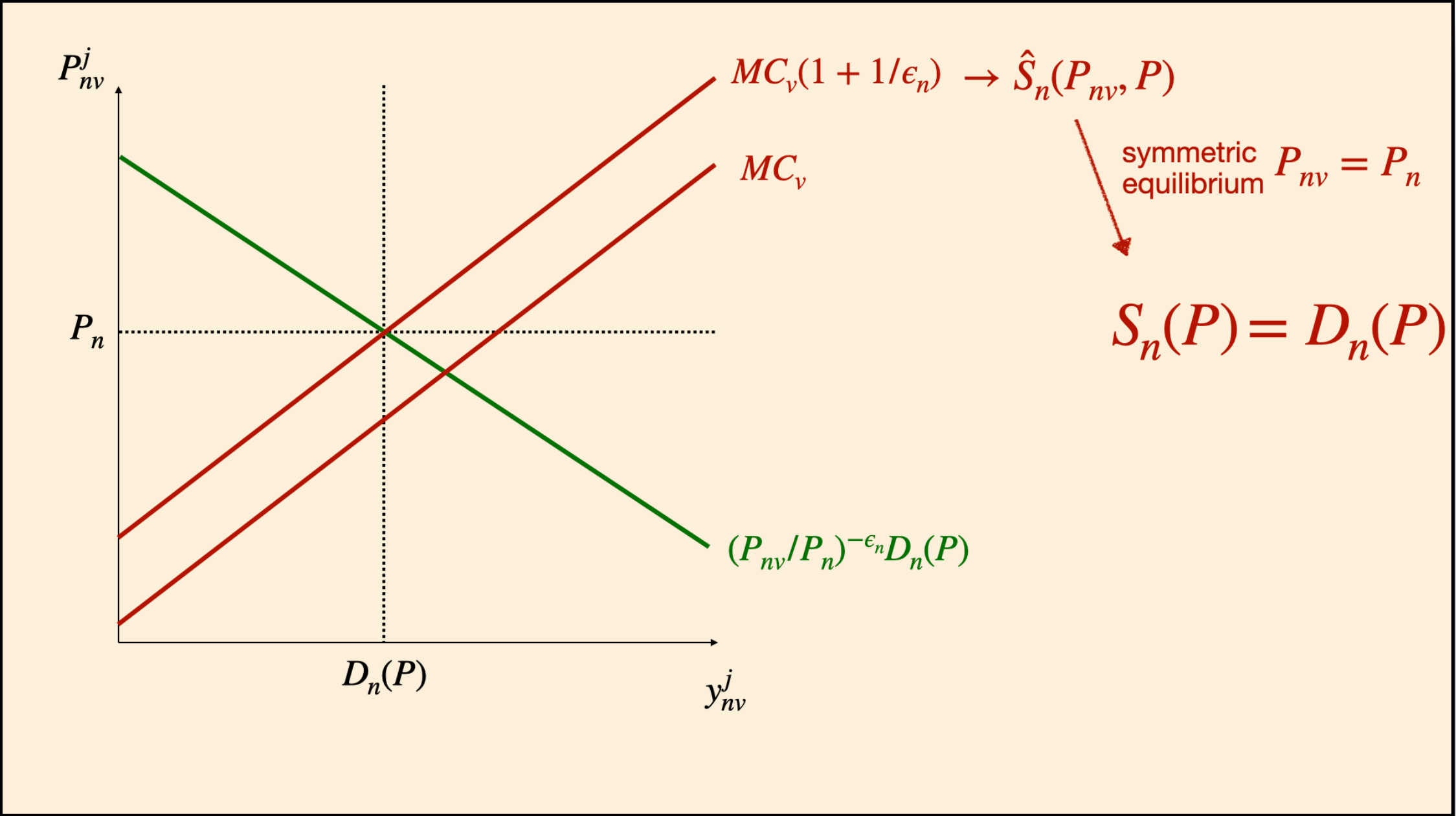
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$$\begin{aligned}\Pi^f(p) &\equiv \max_{(z_u^f, z_{vn}) \in Y^f} - (p \cdot z_u^f + p_{vn} z_{vn}) \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

“as if” competitive...
(+ markup omitted
here for simplicity)

$$\begin{aligned}\max U^h(z_u^h, z_{vn}) \\ p \cdot z_u^h + p_{vn} z_{vn} &\leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f(p) \\ z_{vn} &= - (p_{vn}/p_n)^{-\epsilon_n} z_{un}\end{aligned}$$

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$$Z^{**}(p) \equiv Z_{un}^{**}(p) + Z_{vn}^{**}(p)$$

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“as if” competitive...
(+ markup omitted
here for simplicity)

$$Z^{**}(p) \equiv Z_{un}^{**}(p) + Z_{vn}^{**}(p) \longrightarrow \boxed{Z^{**}(p) = 0}$$

...exactly as if Walrasian!
(new result)

$$\begin{aligned} &\max U^h(z_u^h, z_{vn}^h) \\ &p \cdot z_u^h + p_n z_{vn}^h \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f(p) \end{aligned}$$

Static 2.0: Price Frictions

Non-Walrasian Equilibria

Non-Walrasian Equilibria

■ **Coming...** Price Frictions + Dynamics

■ **First....** Price Frictions

Non-Walrasian Equilibria

- **Coming...** Price Frictions + Dynamics

- **First....** Price Frictions

- Non-Walrasian (temporary) equilibrium...

 - ▶ price pressure market n $\leftrightarrow$ excess demand $\tilde{z}_n = z_{un} + Z_{vn}^{**}(p)$

Non-Walrasian Equilibria

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$$Z(p) = (I - A)^{-1} Z^{**}(p) \quad (A = \text{Spillover Matrix})$$

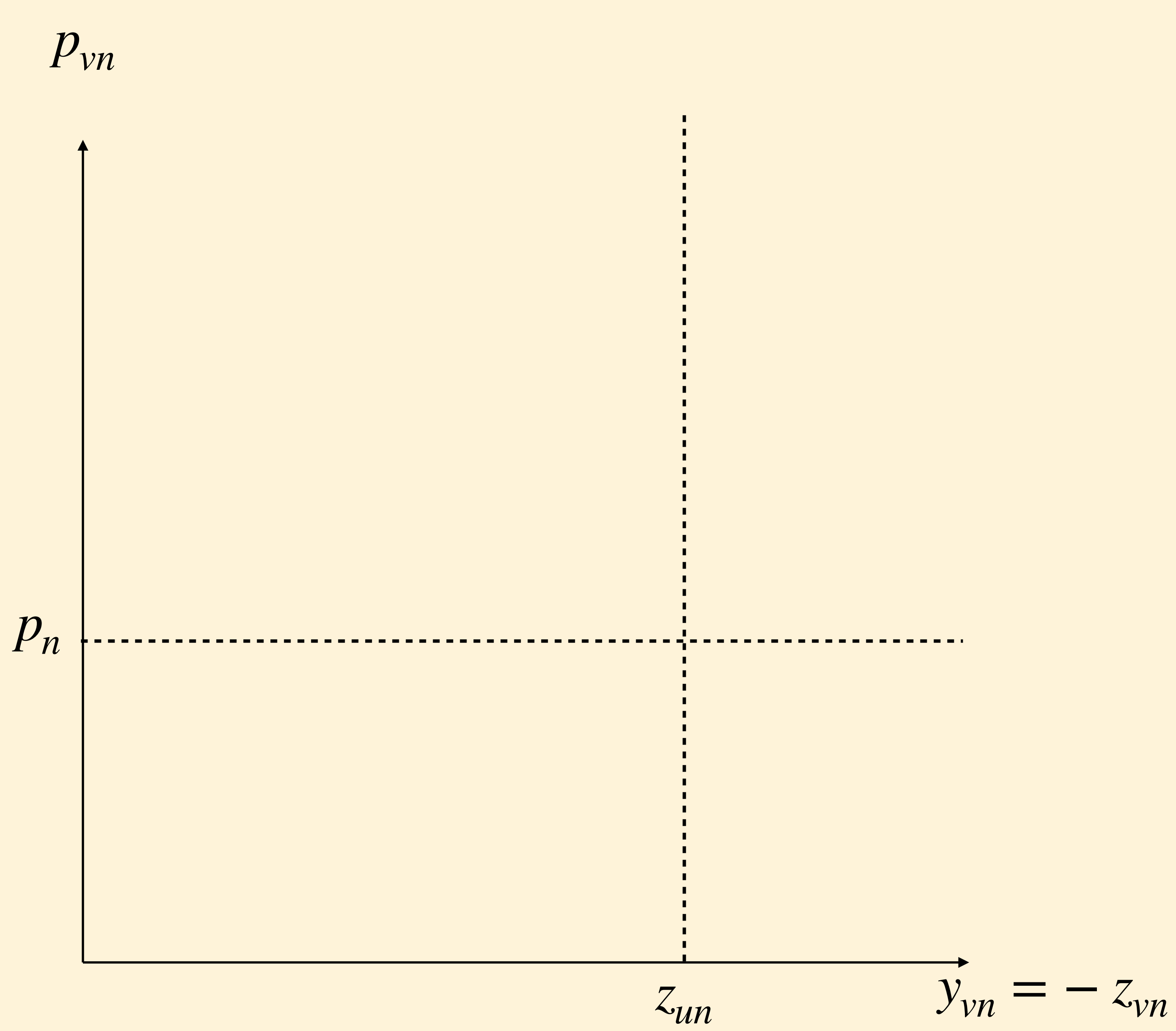
Price Pressure and Excess Demand

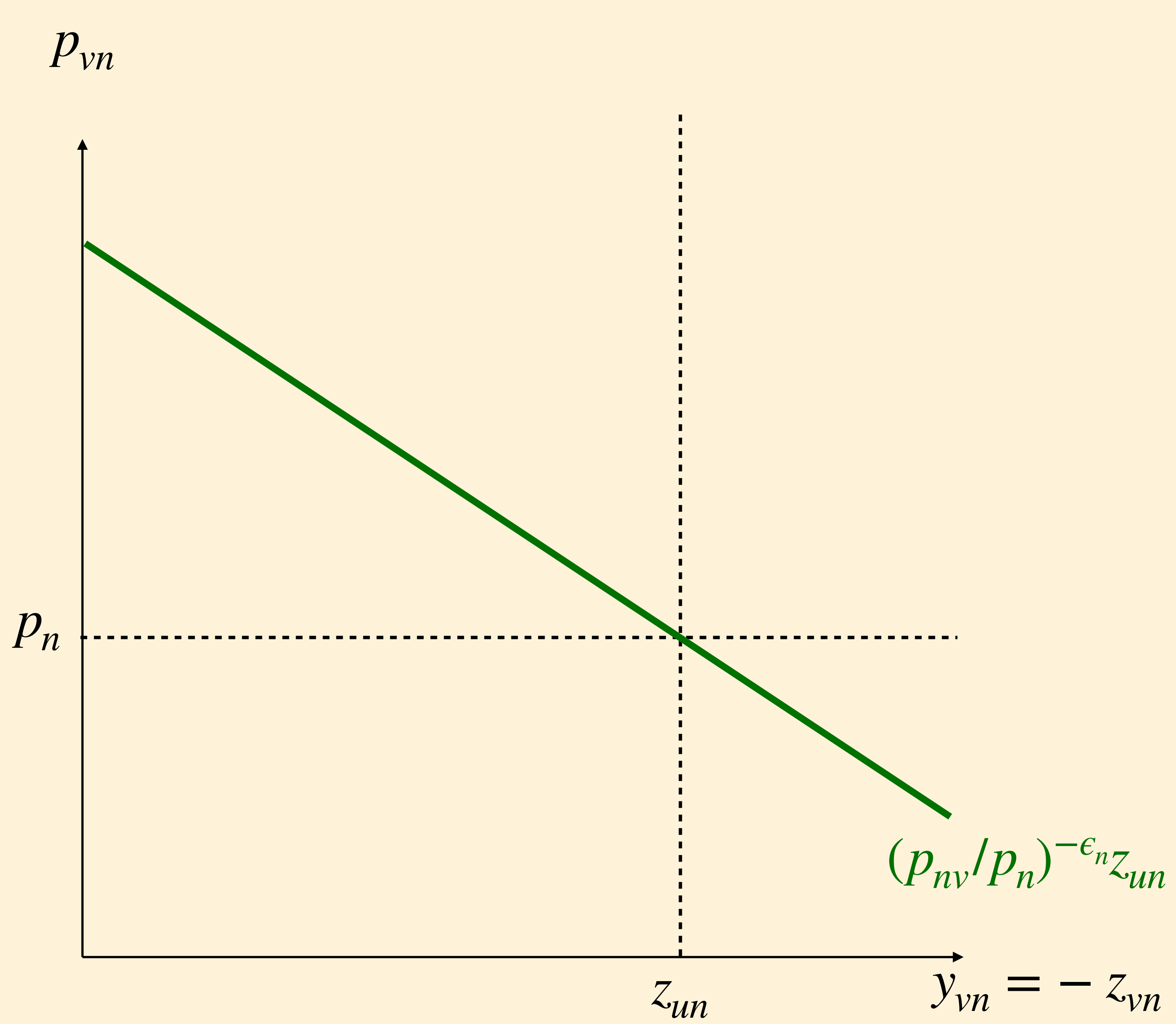
■ Price deviator: $p_{vn} \neq p_n$

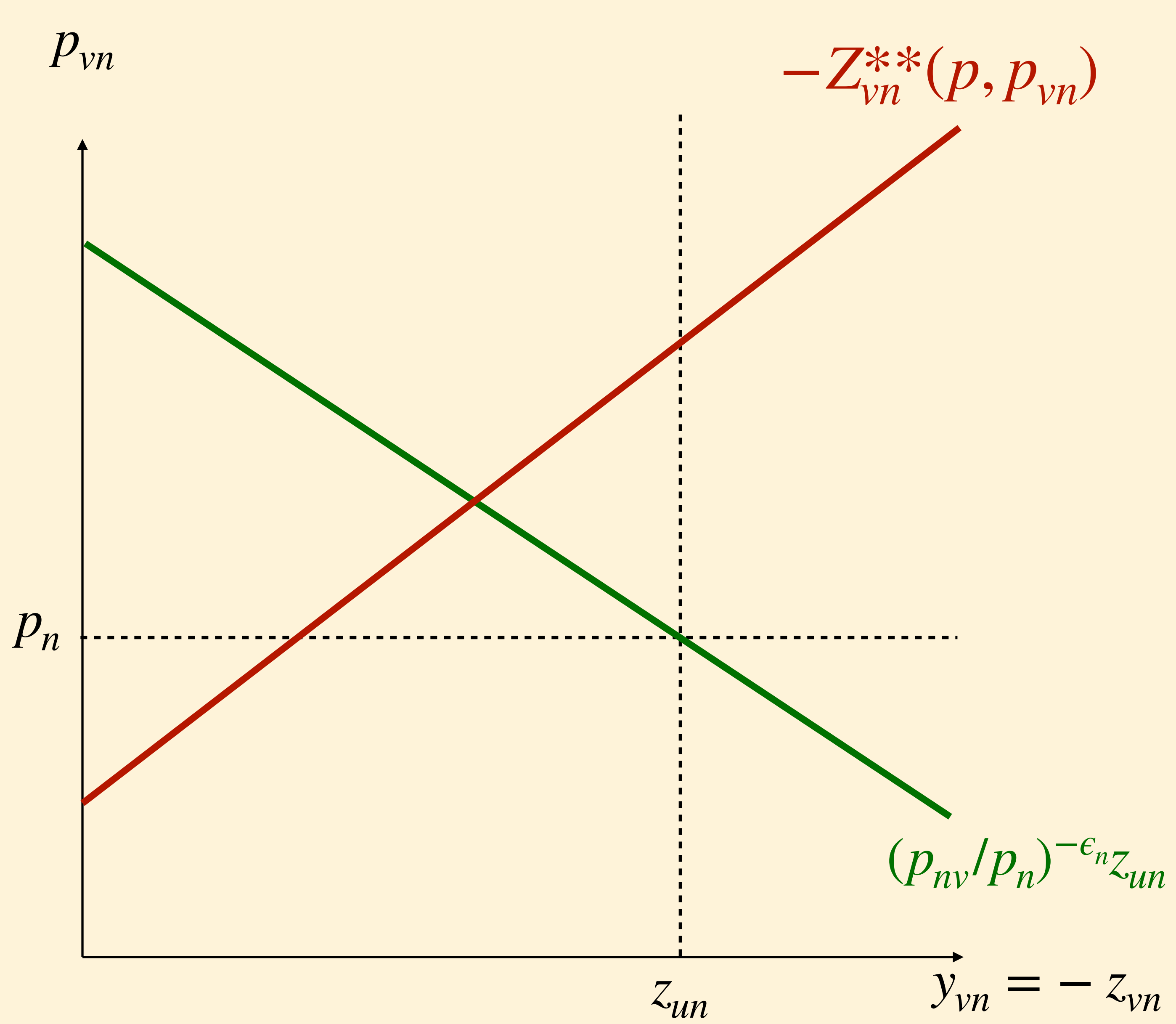
■ Market clearing...

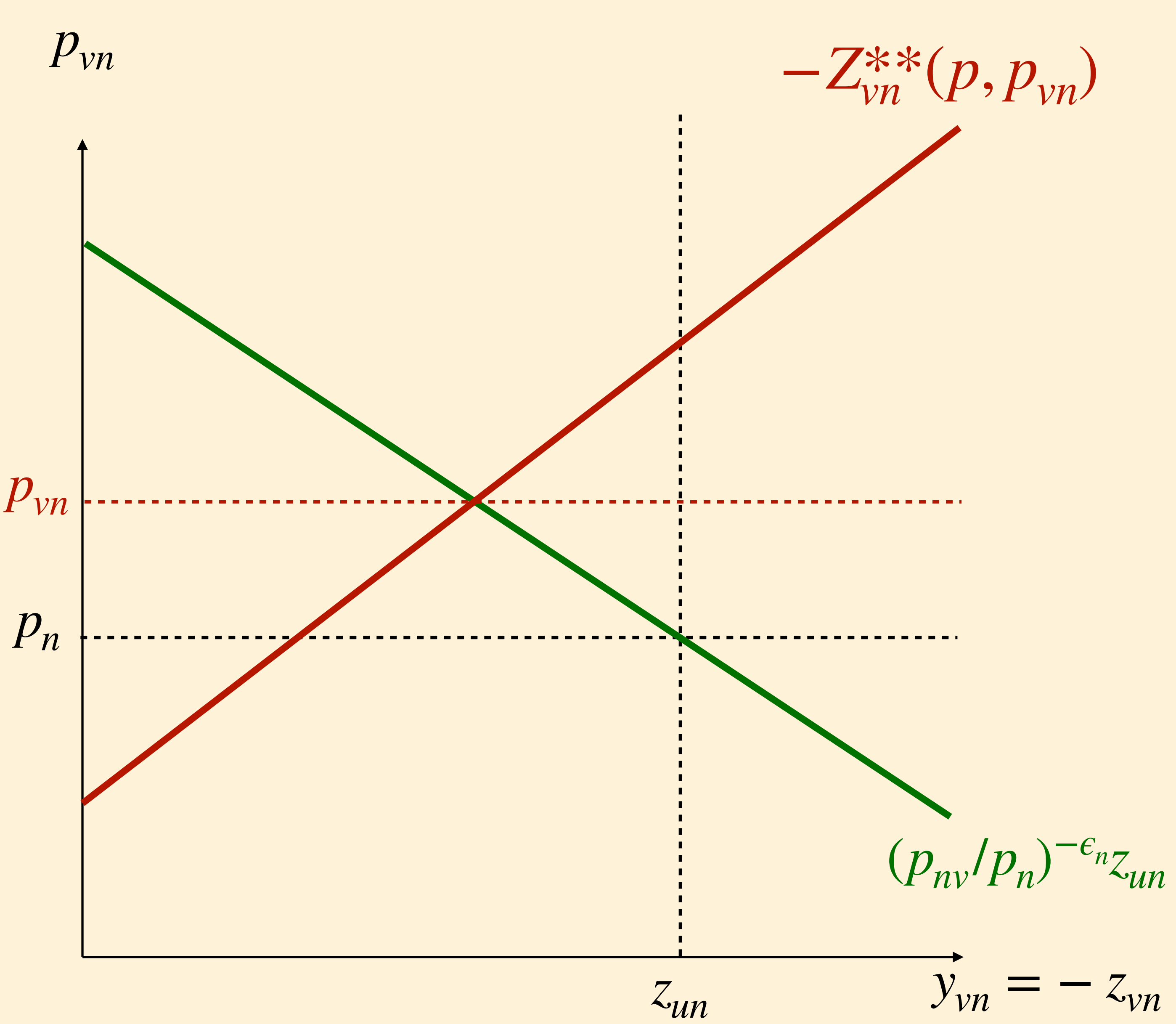
$$(p_{vn}/p_n)^{-\epsilon_n} z_{un} + Z_{vn}^{**}(p, p_{vn}) = 0$$

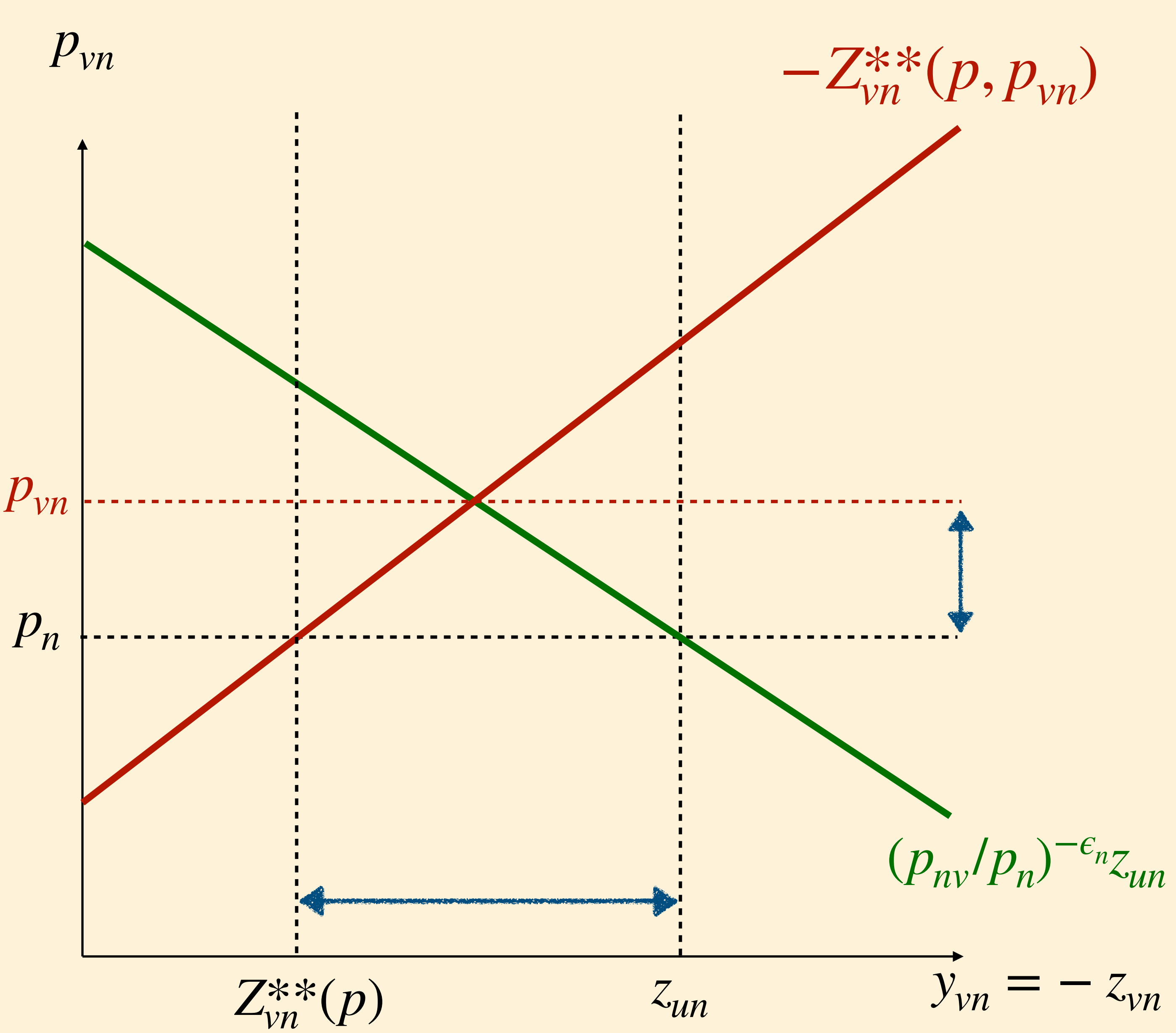
$$Z_v^{**}(p)$$











Price Pressure and Excess Demand

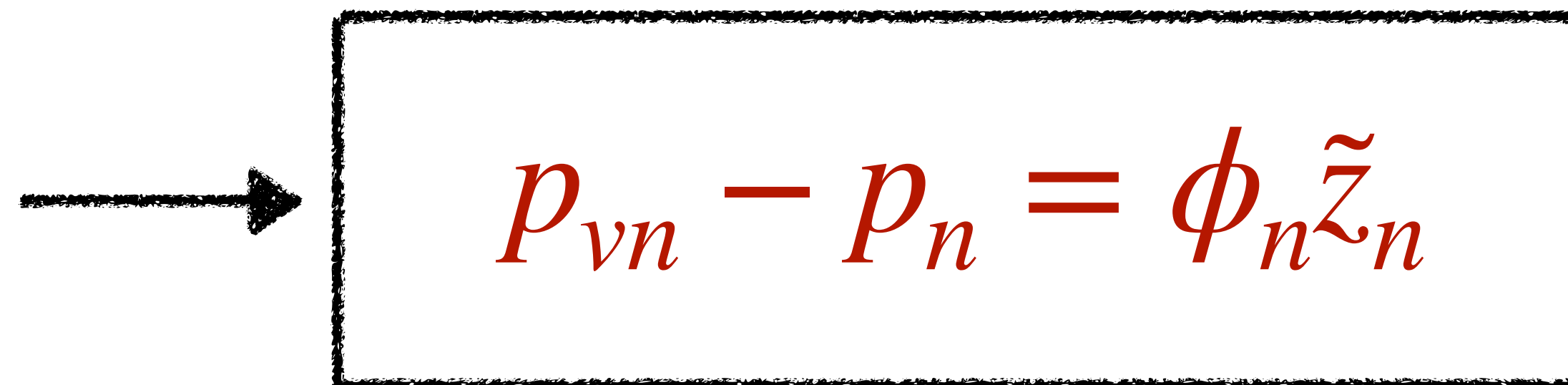
■ Price deviator: $p_{vn} \neq p_n$

■ Market clearing...

$$(p_{vn}/p_n)^{-\epsilon_n} z_{un} + Z_{vn}^{**}(p, p_{vn}) = 0$$

■ Excess (Effective) Demand...

$$\tilde{z}_n \equiv z_{un} + Z_{vn}^{**}(p)$$


$$p_{vn} - p_n = \phi_n \tilde{z}_n$$

(nonlinear in paper)

New Keynesian Literature:
excess demand $\neq$ “gaps”)

Disequilibrium Literature:
did not have this

Full Optimum

$$\rightarrow Z_{vn}^{**}(p)$$

Price Setting $\rightarrow$ “As If” Walrasian Quantity Choice!

Full Optimum

$$\rightarrow Z_{vn}^{**}(p)$$

Price Setting $\rightarrow$ “As If” Walrasian Quantity Choice!

Partial Optimum over $z_u \dots$

$$\rightarrow Z_u^{*j}(p, z_{vn})$$

Conditional Demand

Full Optimum

$$\rightarrow Z_{vn}^{**}(p)$$

Price Setting $\rightarrow$ “As If” Walrasian Quantity Choice!

Partial Optimum over $z_u \dots$

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Conditional Demand

Aggregating...

$$Z_v^{**}(p)$$

$$Z_u^*(p, z_v)$$

Full Optimum

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Price Setting $\rightarrow$ “As If” Walrasian Quantity Choice!

Partial Optimum over $z_u \dots$

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Conditional Demand

Aggregating...

$$Z_v^{**}(p)$$

$$Z_u^*(p, z_v)$$

Up next
solve quantities

Equilibrium Quantities at Fixed Prices

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Equilibrium Quantities at Fixed Prices

■ Fixed price: $p_{vn} = p_n$

$$z_u = Z_u^*(p, z_v)$$

$$z_{un} + z_{vn} = 0$$

$$\rightarrow z_u = Z_u^*(p, -z_u)$$

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Equilibrium Quantities at Fixed Prices

■ Fixed price: $p_{vn} = p_n$

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$$z_{un} + z_{vn} = 0$$

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$$\rightarrow \tilde{z} = Z^{**}(p) + A\tilde{z}$$

↓
spillover
matrix

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

Equilibrium Quantities at Fixed Prices

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

■ Fixed price: $p_{vn} = p_n$

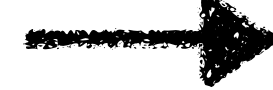
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↓
spillover
matrix



Effective
Demand

Notional
Demand

$$Z(p) = (I - A)^{-1}Z^{**}(p)$$

Equilibrium Quantities at Fixed Prices

$$Z_u^*(p, z_v)$$

$$Z_v^{**}(p)$$

■ Fixed price: $p_{vn} = p_n$

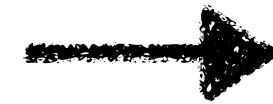
$$z_u = Z_u^*(p, z_v)$$

$$z_{un} + z_{vn} = 0$$

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$$\rightarrow \tilde{z} = Z^{**}(p) + A\tilde{z}$$

spillover
matrix



Effective
Demand

Notional
Demand

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

$$r(A) < 1$$

$$(I - A)^{-1} = I + A + A^2 + \dots$$

Leontief Inverse

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

$$Z(p) = (I - A)^{-1} Z^{**}(p)$$

$$p_{nv} - p_n = \phi_n Z_n(p)$$

(nonlinear in paper)

Dynamic: Adding Price Adjustment

Adding Dynamics

■ Static GE → **Dynamic GE...**

▶ repeated GE spot each t → previous GE model natural reference

▶ intertemporal choices + add price frictions à la Calvo

Adding Dynamics

■ Static GE → **Dynamic GE...**

▶ repeated GE spot each t → previous GE model natural reference

▶ intertemporal choices + add price frictions à la Calvo

▶ NK meets GE...

setup → more eneral (GE tradition)

approach → excess demand (GE tradition)

$$\Pi^f \equiv \max \qquad \qquad \qquad - (p \cdot z_u^f + p_{vn} z_{vn})$$

$$(z_{ut}^f, z_{vnt}) \in Y^f$$

$$z_{vnt} = - \left(p_{vnt}/p_{nt} \right)^{-\epsilon_n} z_{unt}$$

$$\max \qquad \qquad \qquad U^h(z_u^h, z_{vn})$$

$$p \cdot z_u^h + p_{vn} z_{vn} \qquad \leq p \cdot a^h + \sum_f \omega^{h,f} \Pi^f$$

$$z_{vnt} = - \left(p_{vnt}/p_{nt} \right)^{-\epsilon_n} z_{unt}$$

$$\Pi^f \equiv \max - \mathbb{E} \int_0^\infty \textcolor{blue}{Q}_t (p_t \cdot z_{ut}^f + p_{vnt} z_{vnt}) \textcolor{blue}{d}t$$

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s.t. Calvo pricing friction

$$\max \mathbb{E} \int_0^\infty e^{-\rho t} U^h(z_{ut}^h, z_{vnt}) dt$$

$$\mathbb{E} \int_0^\infty Q_t(p_t \cdot z_{ut}^h + p_{vnt} z_{vnt}) dt \leq p_0 \cdot a_0^h + \sum_f \omega^{h,f} \Pi^f$$

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Monetary Policy...

For now...

$$i_t = \rho$$

$$Q_t = e^{-\rho t}$$

$$\max \mathbb{E} \int_0^\infty e^{-\rho t} U^h(z_{ut}^h, z_{vnt}) dt$$

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s.t. Calvo pricing friction

$$(Q_t = e^{-\int_0^t i_s ds})$$

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s.t. Calvo pricing friction

Results: Samuelson Comeback?

$$\Pi^f \equiv \max - \int_0^\infty e^{-\rho t} [p_t \cdot z_{ut} + p_{vnt} y_{vnt}] \, dt$$

$$(z_{ut}, z_{vnt}) \in Y^f$$

$$\max \int_0^\infty e^{-\rho t} U^h(z_{ut}, z_{vnt}) \, dt$$

$$\int_0^\infty e^{-\rho t} [p_t \cdot z_{ut} + p_{nv} z_{nvt} - p_t \cdot a^h] \, dt \leq \sum_f \omega^{h,f} \Pi^f$$

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(Lagrangian)

$$L^j = \mu^j \int_0^\infty e^{-\rho t} \left[\frac{1}{\mu^j} U^j(z_{ut}, z_{vnt}) - p_t \cdot z_{ut} - p_{vnt} z_{vnt} \right] dt$$



μ^j = Lagrange multiplier on agent j budget constraint

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only this matters at a point in time...



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“as if” utility
as in
static analysis...



$$\hat{U}^j = \frac{1}{\mu^j} U^j(z_u, z_{vn}) + I^j$$

“as if”
quasilinear
outside good



“as if” utility
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static analysis...



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$$\begin{aligned} Z(p) \\ Z_u^*(p, z_v) \\ Z_v^{**}(p) \end{aligned}$$

“as if” utility
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static analysis...

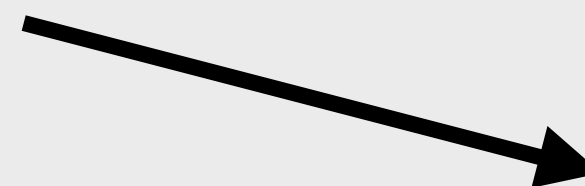


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except now...



Frisch Demands

“As If” Representative Agent
with Quasilinear Utility

Removes income effects
along path

$$\begin{aligned} Z(p) \\ Z_u^*(p, z_v) \\ Z_v^{**}(p) \end{aligned}$$

Some simple math combines...

1. optimal reset price with nominal rigidity
 2. aggregation → price index in market n
- ... to arrive at...


$$\begin{aligned} &Z(p) \\ &Z_u^*(p, z_v) \\ &Z_v^{**}(p) \end{aligned}$$

**2nd-order
Samuelson! ...**

Proposition.

$$\dot{p}_{nt} = d_n Z_n(p_t) + (1/\rho) \ddot{p}_{nt}$$

$$(d_n = \theta_n(1 + \theta_n/\rho)\phi_n)$$



$$Z(p)$$

$$Z_u^*(p, z_v)$$

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Result: Excess Demand Drives Price Adjustment

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Samuelson! ...

...Forward Looking
Samuelson!

$$\rightarrow \dot{p}_{nt} = d_n \int_0^\infty \rho e^{-\rho s} Z_n(p_{t+s}) ds$$

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Proposition. Small θ_n/ρ ...

$$\dot{p}_{nt} = d_n Z_n(p_t)$$

easy for $\rho \rightarrow \infty$

Only θ_n/ρ matters!

but also holds for
 $\theta \rightarrow 0$

... $\rho \rightarrow \infty$
Myopic
Samuelson!

Result: Excess Demand $\neq$ Output Gap Drives Price Adjustment

Proposition.

Now: work “Myopic” limit

Later: forward-looking pricing

$$Z(p)$$
$$(p, z_v)$$
$$*(p)$$

Proposition. Small $\theta_n/\rho \dots$

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Myopic Tâtonnement

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Proposition. Samuelson-like Tâtonnement

$$\dot{p} = D Z(p) \quad (D = \text{diag}(d_n))$$



Myopic Tâtonnement

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■ Same... but **not** the same...

▶ Marshallian → **Frisch**

▶ ~~Walrasian Notional~~ → Non-Walrasian Effective (spillovers)

Myopic Tâtonnement

Proposition. Samuelson-like Tâtonnement

$$\dot{p} = D Z(p) \quad (D = \text{diag}(d_n))$$

$$= D(I - A)^{-1} Z^{**}(p)$$



■ Same... but **not** the same...

▶ Marshallian → **Frisch**

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Results: Stability Analysis

Exchange Economy

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■ Exchange economy... (Yeomen Farmers)

▶ no firms

▶ separable utility $\rightarrow U^j(z_u) + u_v^j(z_{nv})$

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Yeomen farmer $\Rightarrow$ Global

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Foundations for ad hoc
Bewley (1980) analysis: “this
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Foundations for ad hoc
Bewley (1980) analysis: “this
tâtonnement equation cannot really be
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...Now it can!

D

Positive Diagonal
(speeds of adjustment)

 $(I - A)^{-1}$

Leontief Inverse
(spillovers)

$$S_{mn} = \frac{\partial Z_n^{**}}{\partial p_m}$$

Frisch-Notional
Substitution Matrix

$$\dot{p} = D(I - A)^{-1} S p$$

$S \prec 0$ negative definite $\rightarrow$ strong **stability** force

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$$A = 0$$

“Yeomen” Economy



Stable for all D!
i.e. D-Stable

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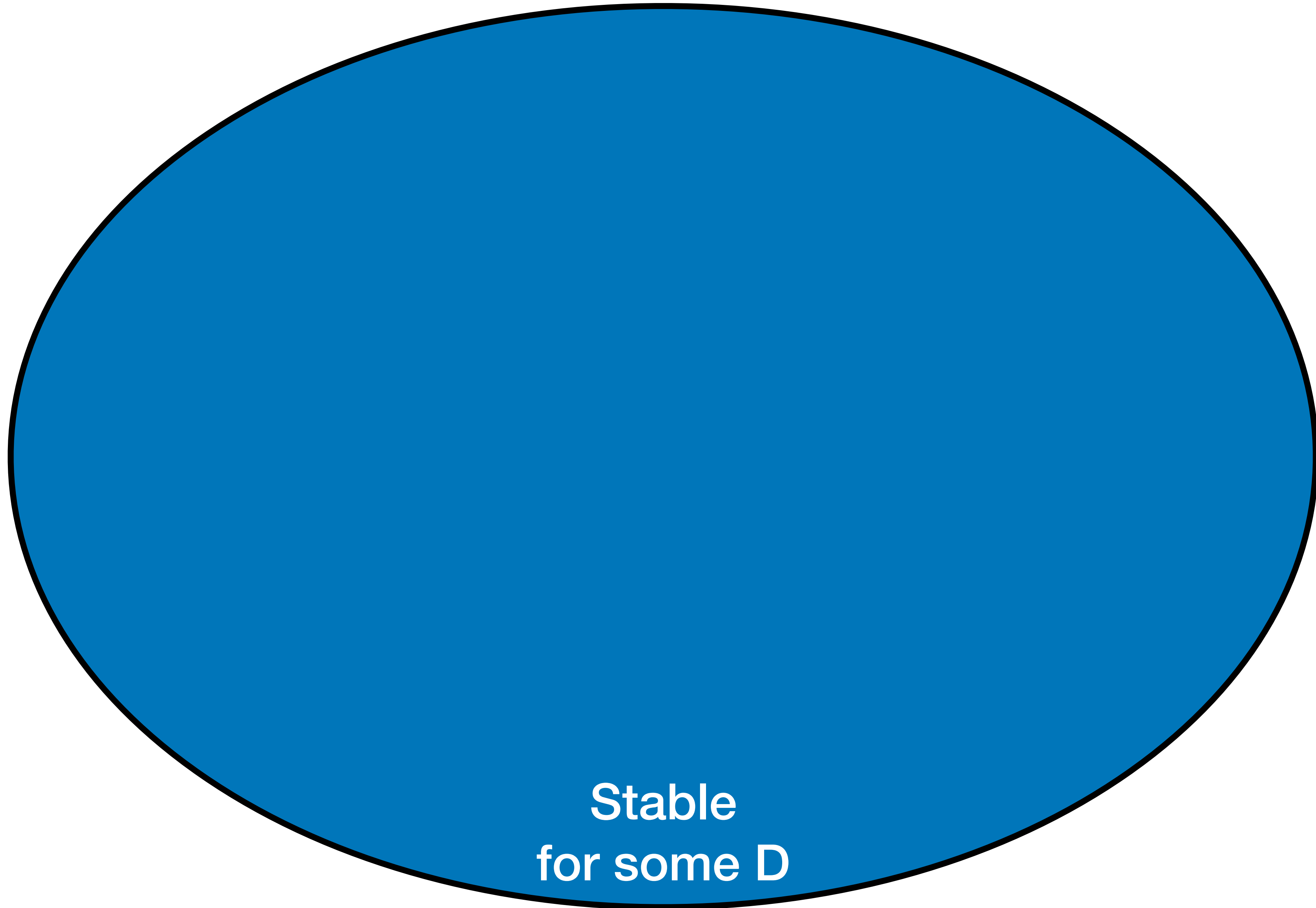
$A = 0$
“Yeomen” Economy



Stable for all D!
i.e. D-Stable

Next: $A \neq 0$ spillovers?

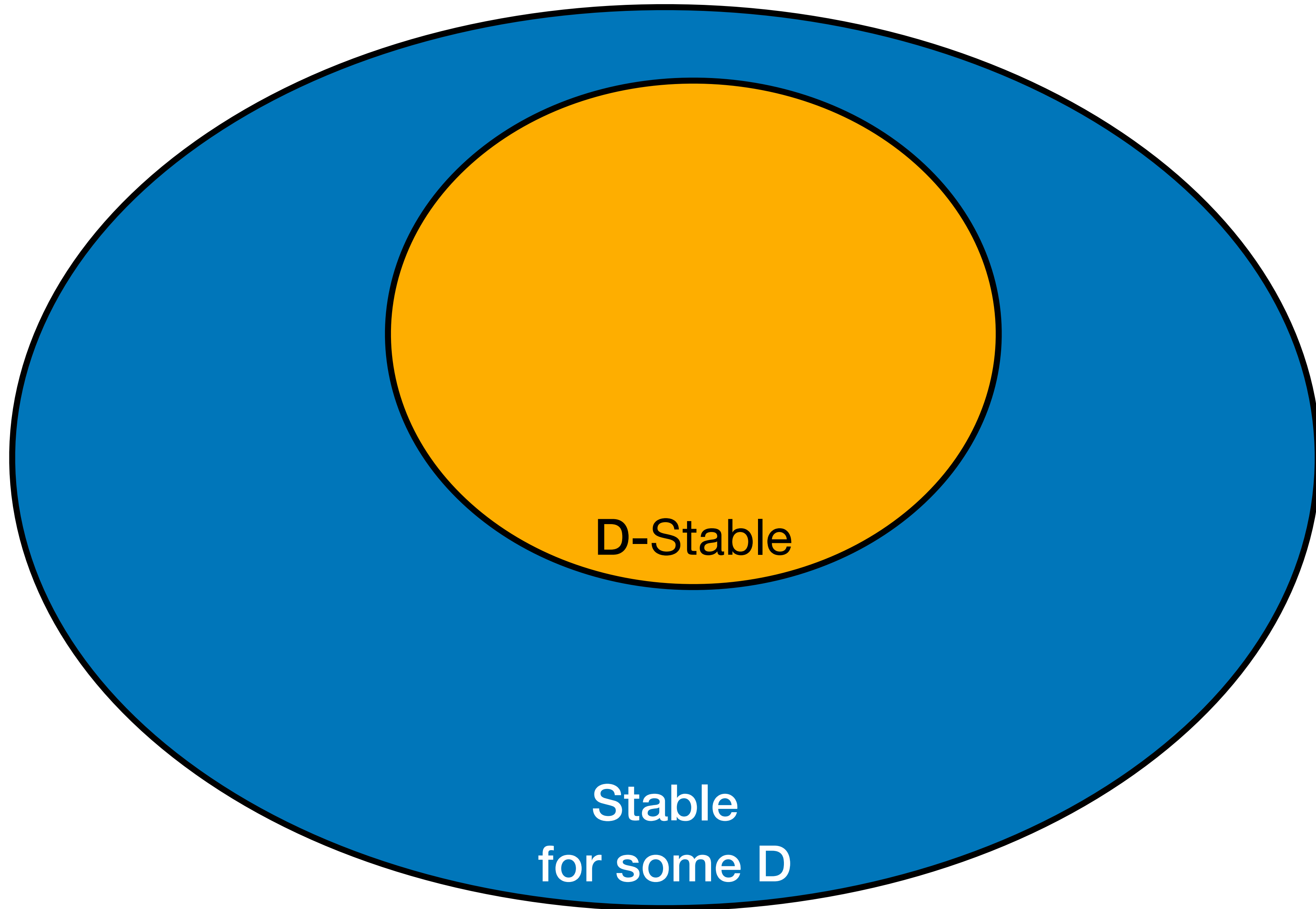
Stability



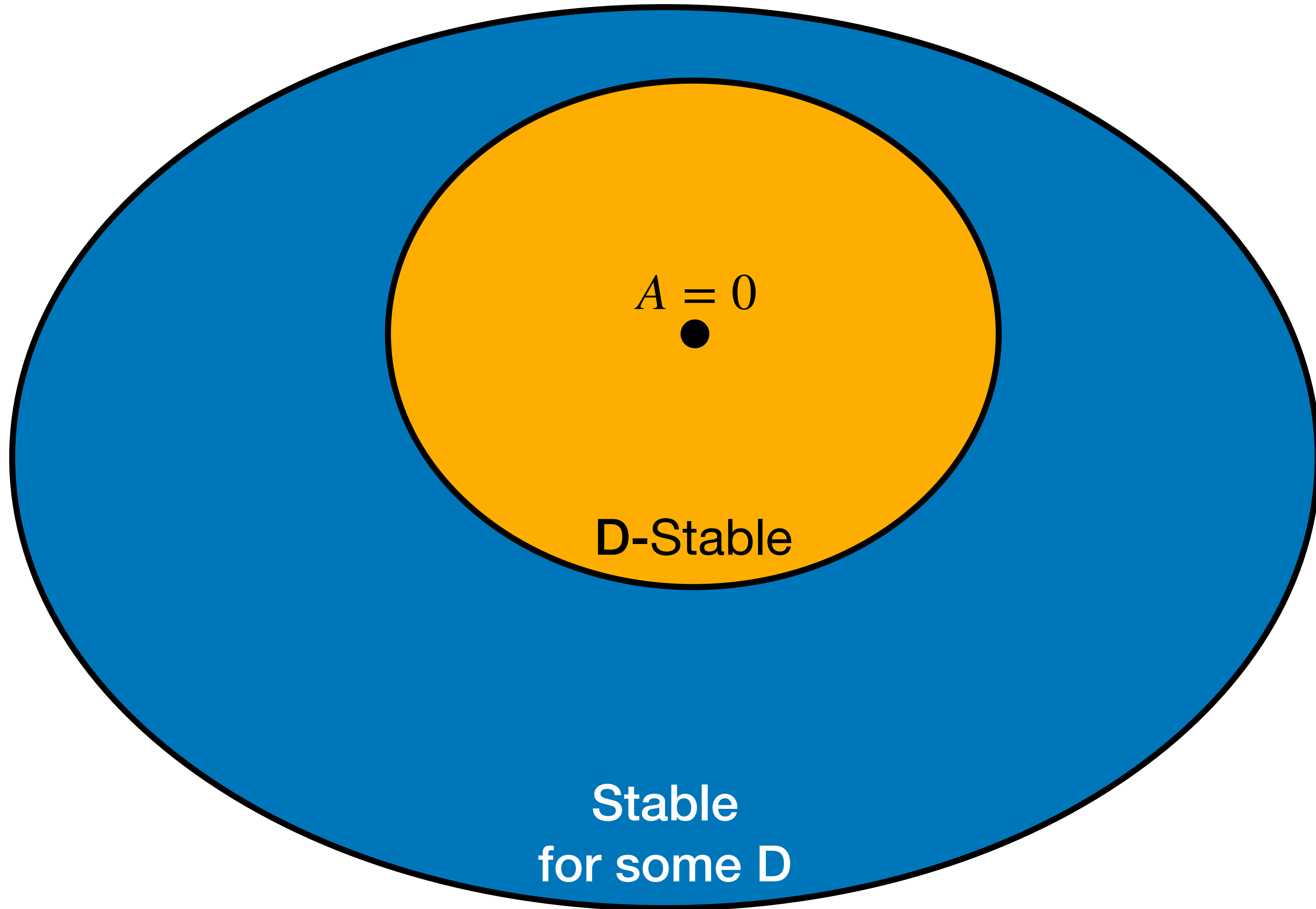
Stable
for some D

Unstable
for all D

Stability

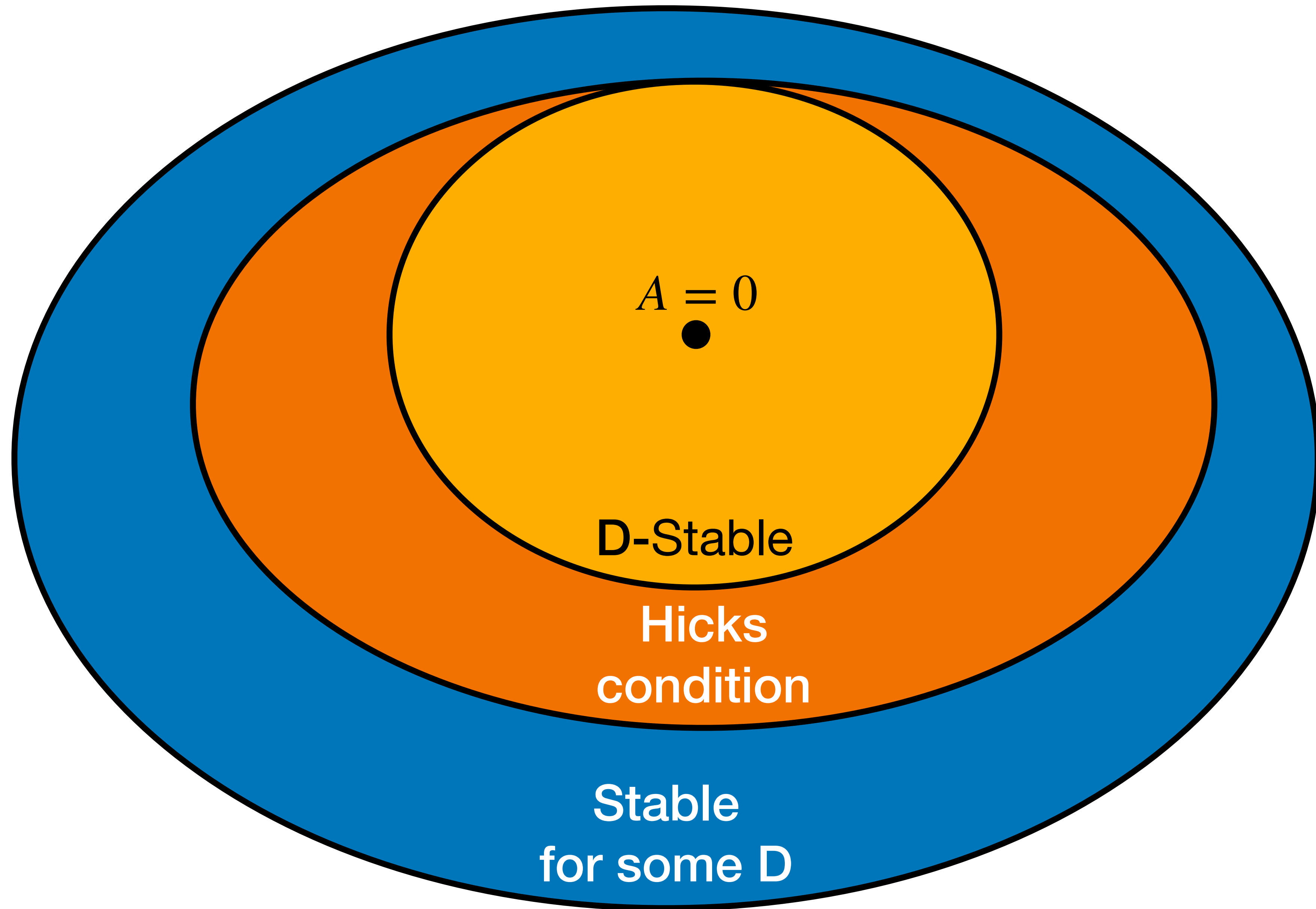


Stability

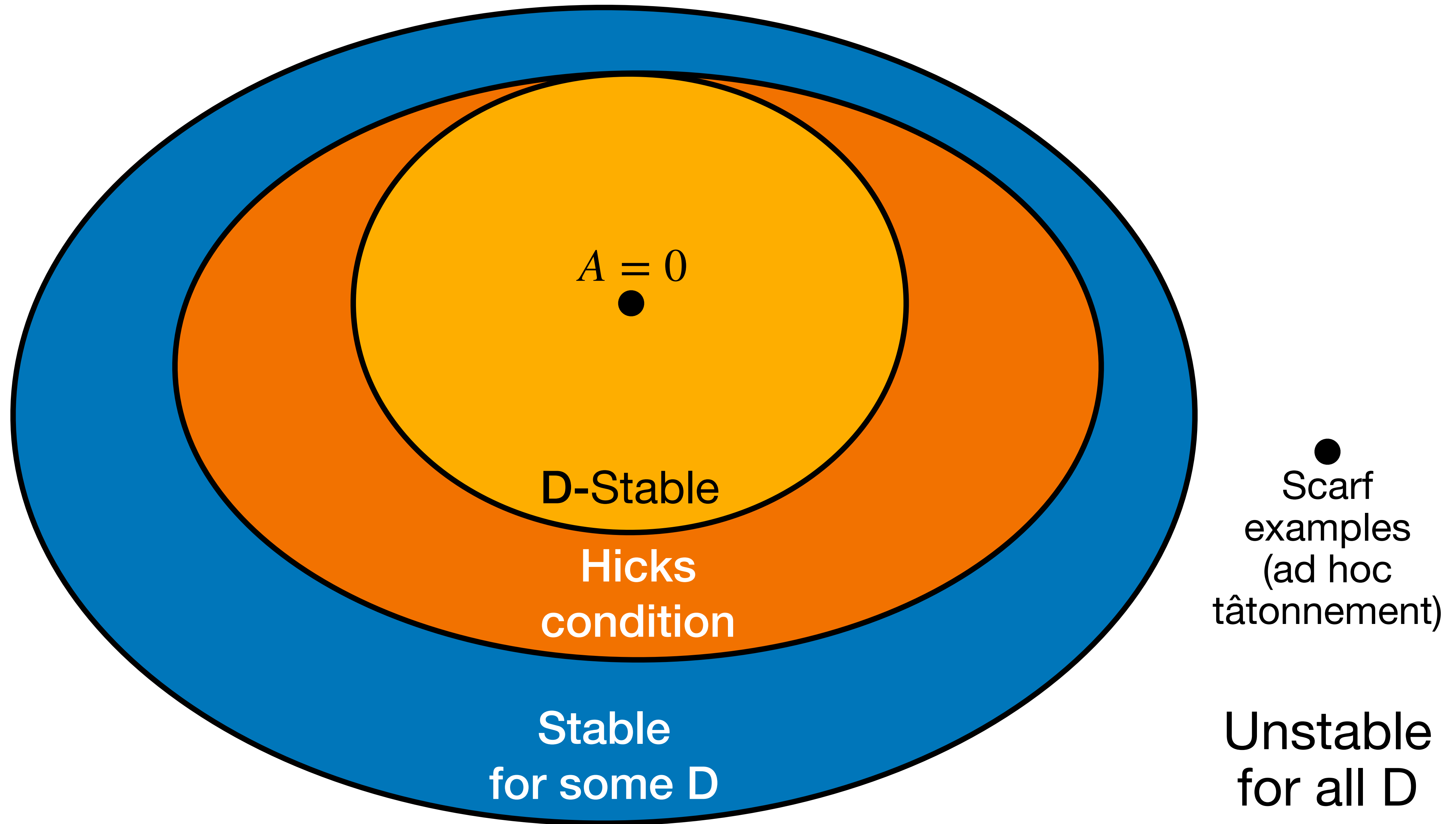


Unstable
for all D

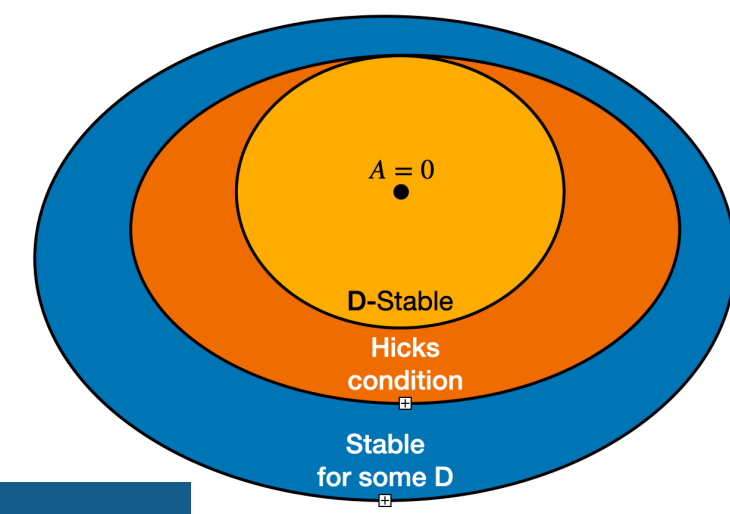
Stability



Stability



1. Potential Stability



Proposition. Under $r(A) < 1$

$A \geq 0$ or A triangular or $A = A^T$
(upstream/downstream)



Stable for a Set of D

Weak
Sufficient
Conditions

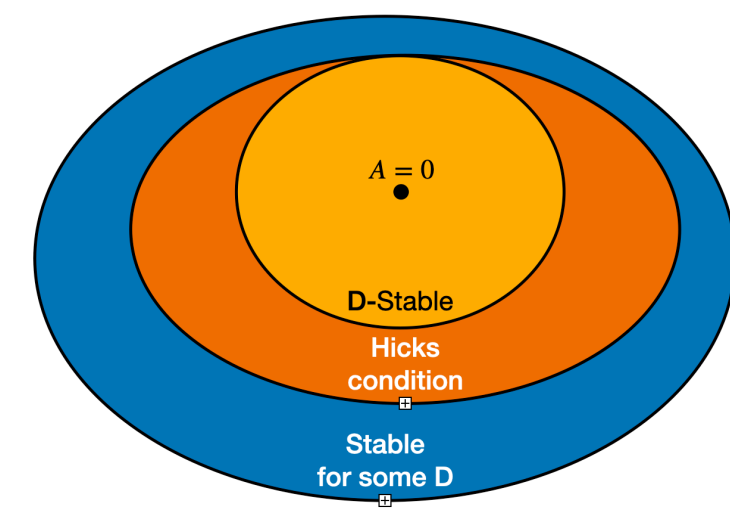
■ Proves $A \neq 0 \rightarrow$ not inherently destabilizing $\rightarrow$ Frisch demand key!

■ Traditional tâtonnement $\rightarrow$ Marshallian demand $\rightarrow$ **Scarf: unstable for all D !**

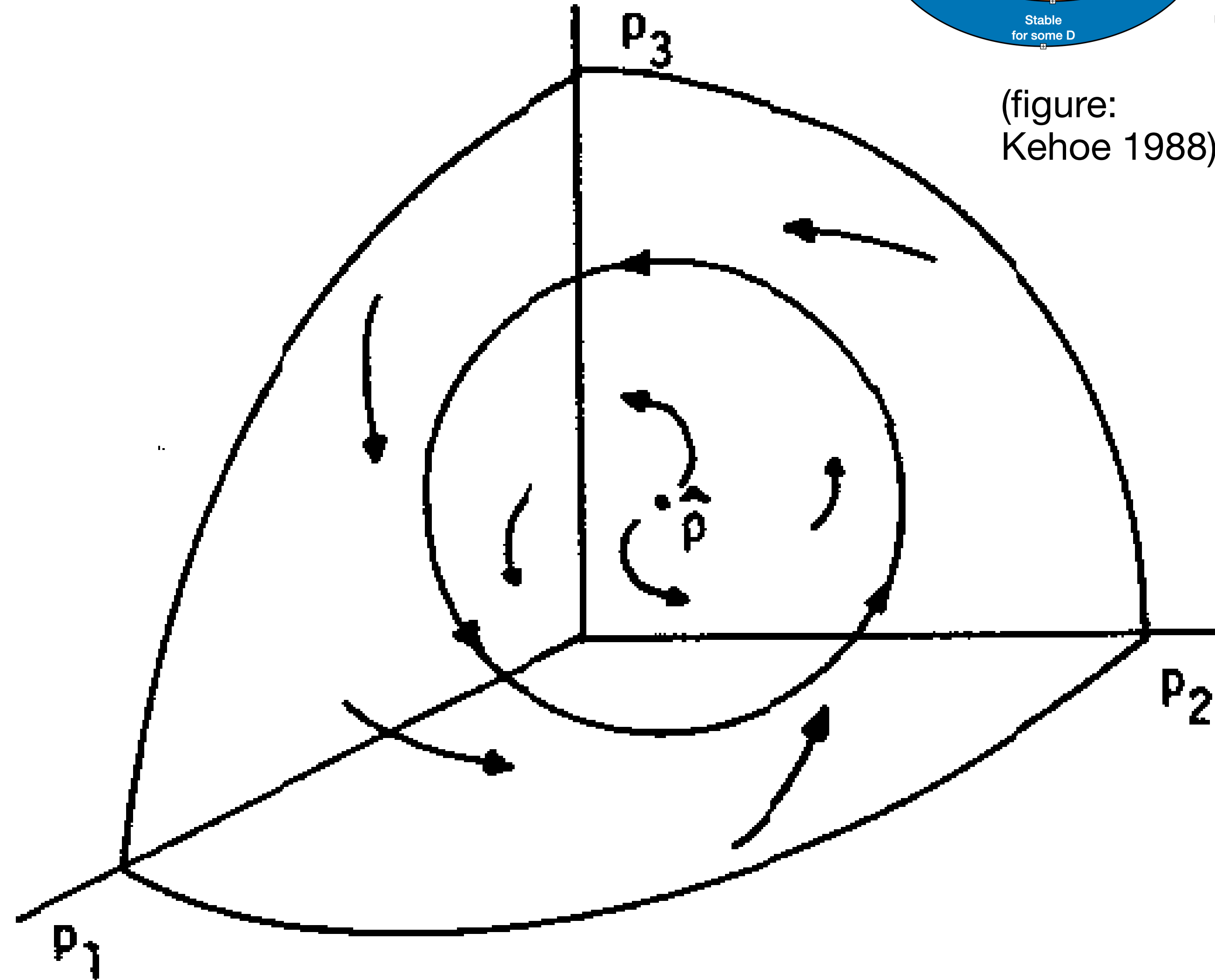
■ Our setting $\rightarrow$ Frisch demand $\rightarrow$ **Scarf: always stable!**

Scarf Instability Example (1960)

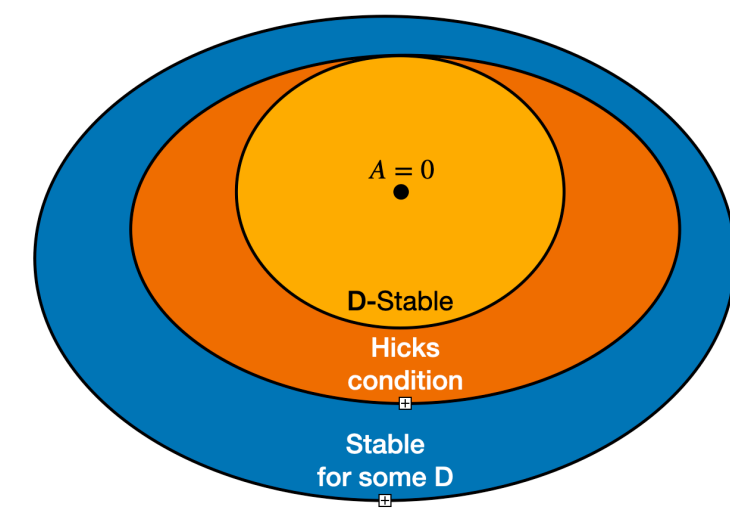
$$\dot{p}_n = Z_n(p)$$



(figure:
Kehoe 1988)

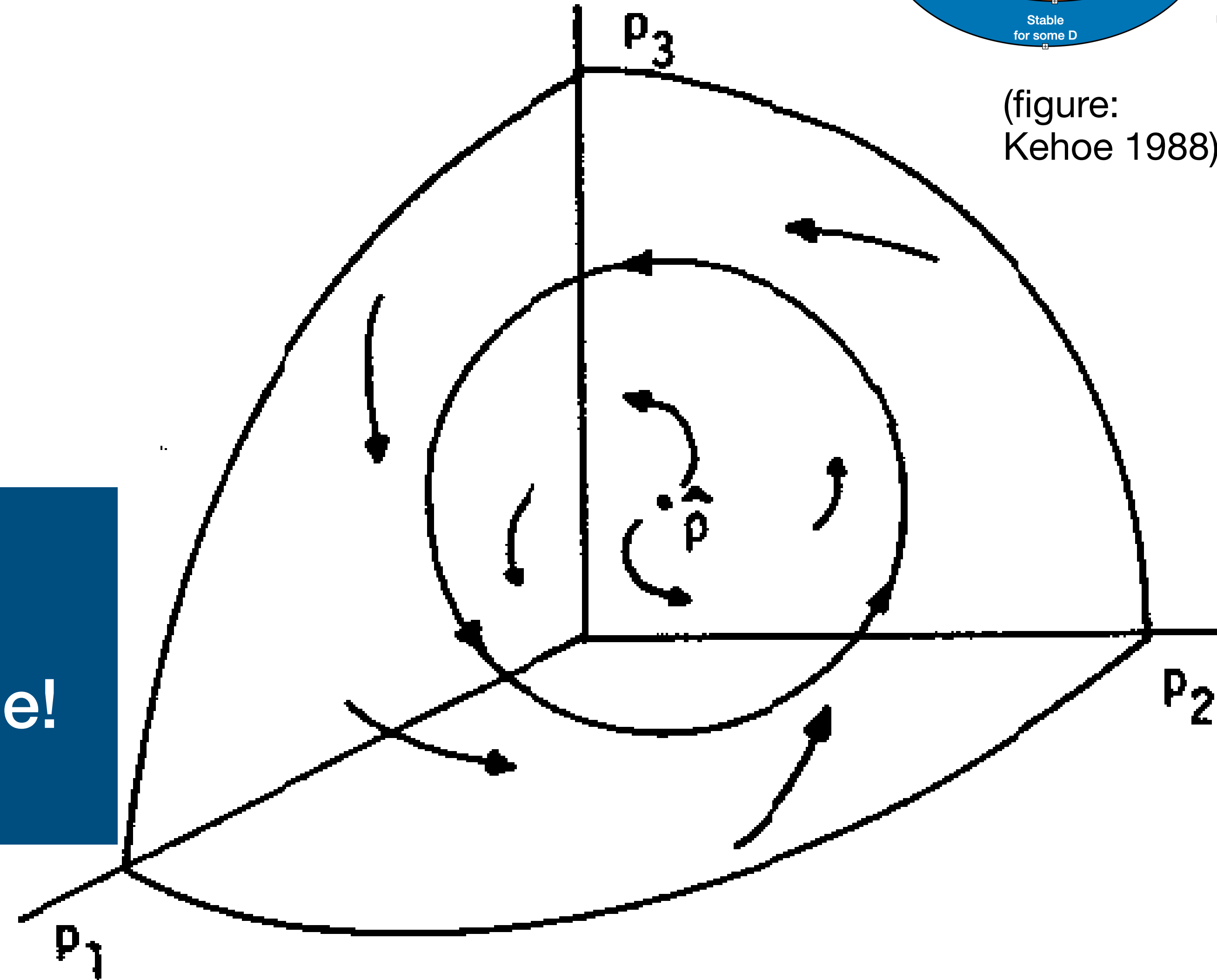


Scarf Instability Example (1960)



(figure:
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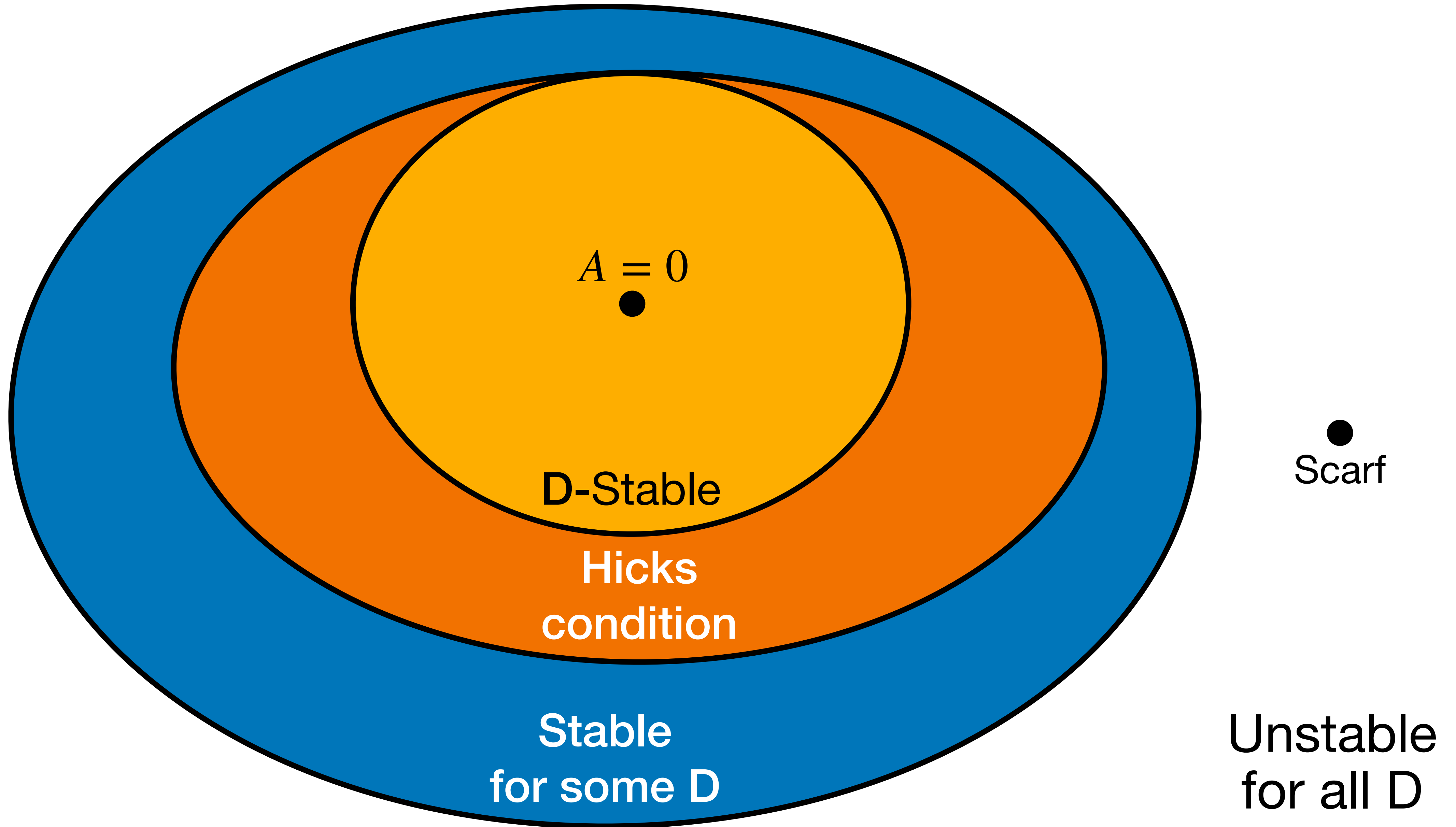
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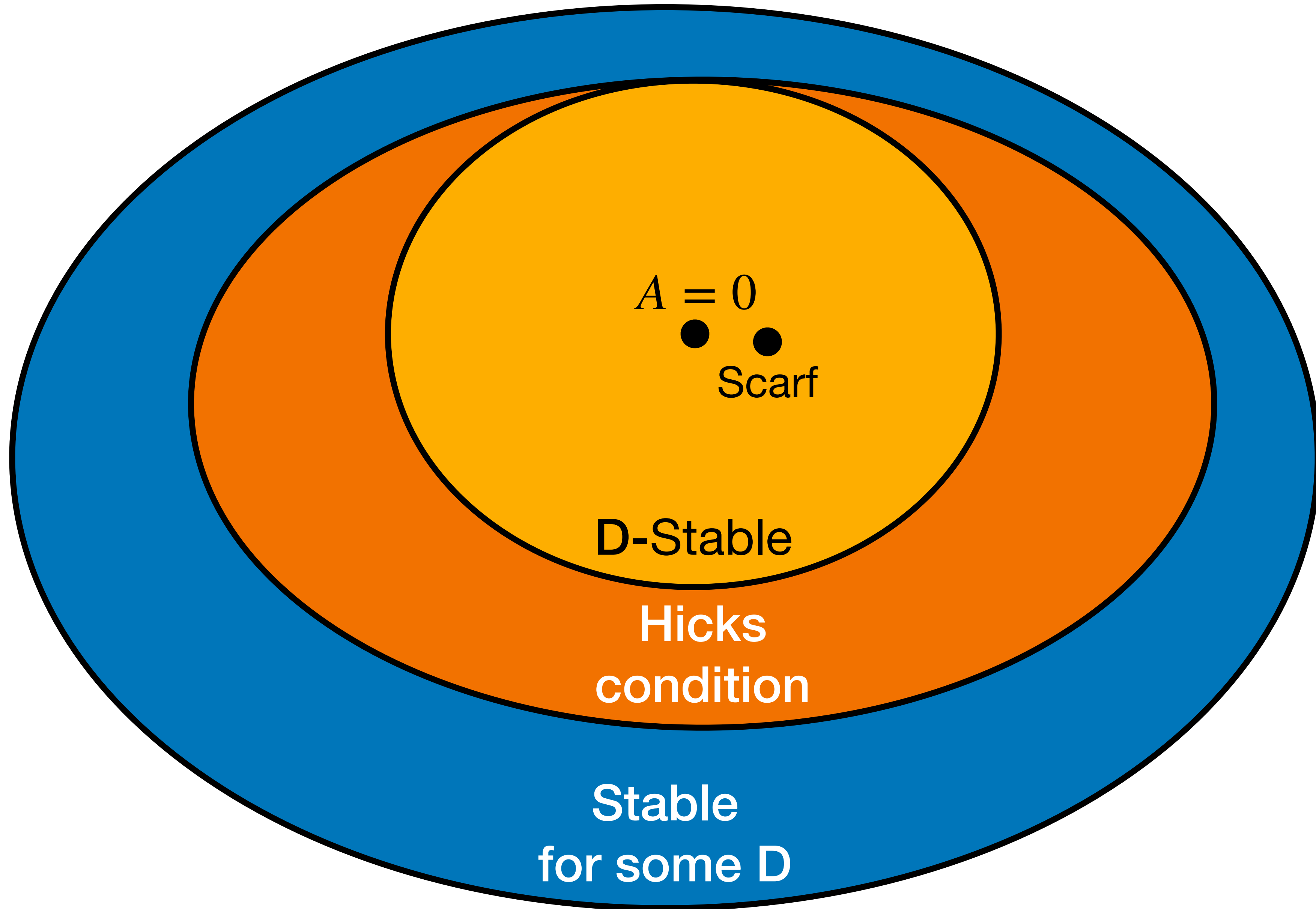
Proposition.

Generalized
Scarf Economies $\Rightarrow$ always stable!

Stability



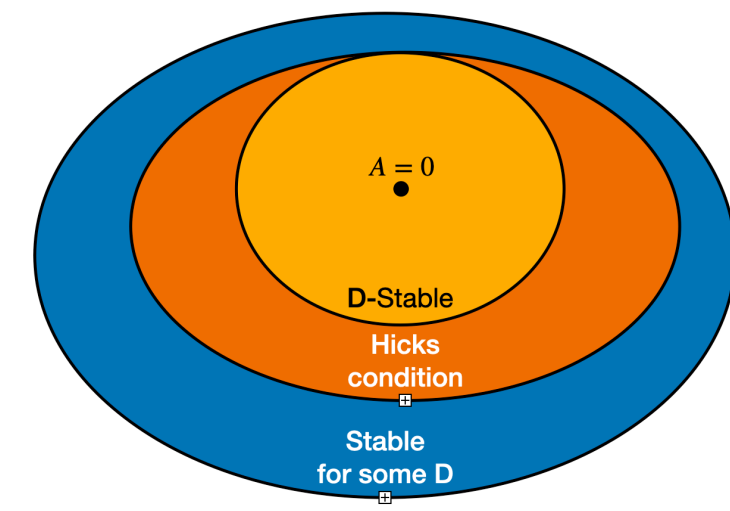
Stability



2. Robust Stability

Consider a change in prices...

$$dp = (dp_1, dp_2, \dots, dp_N)$$

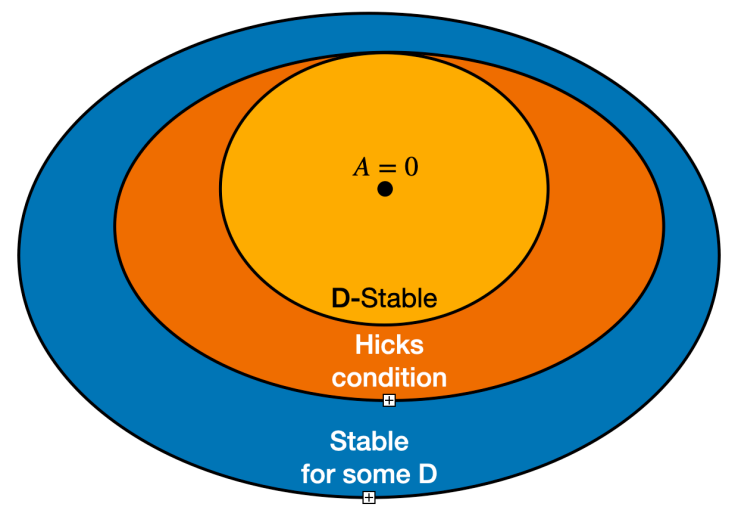


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↓ $\tilde{z} = Z(p)$



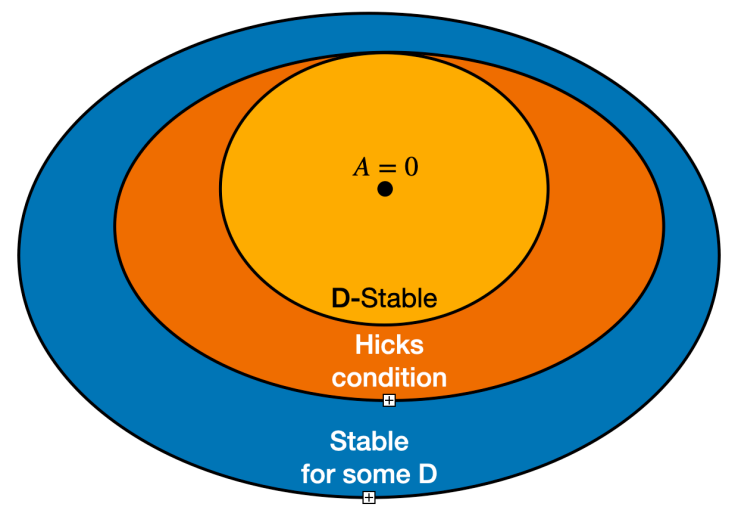
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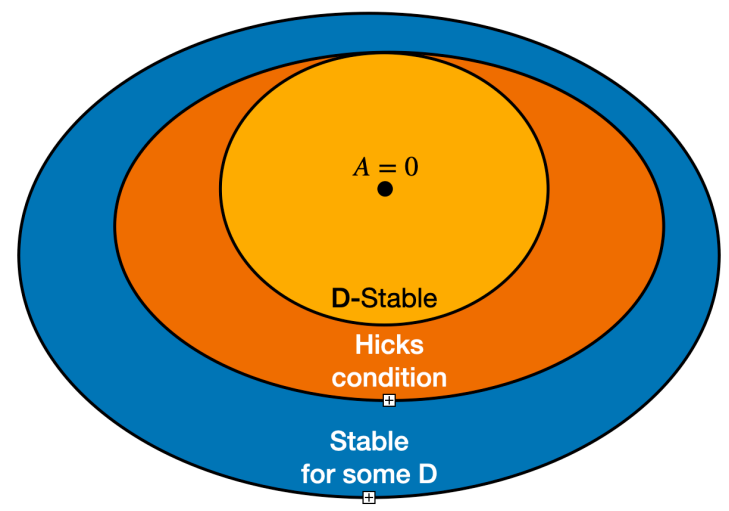
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Two conditions...

$$\sum_n dp_n d\tilde{z}_n < 0$$



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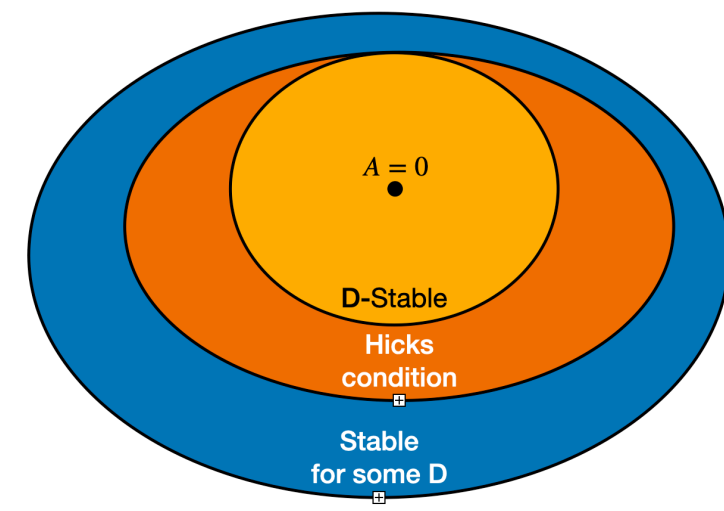
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Average Law of Demand
WARP=Weak Axiom of Revealed Preference



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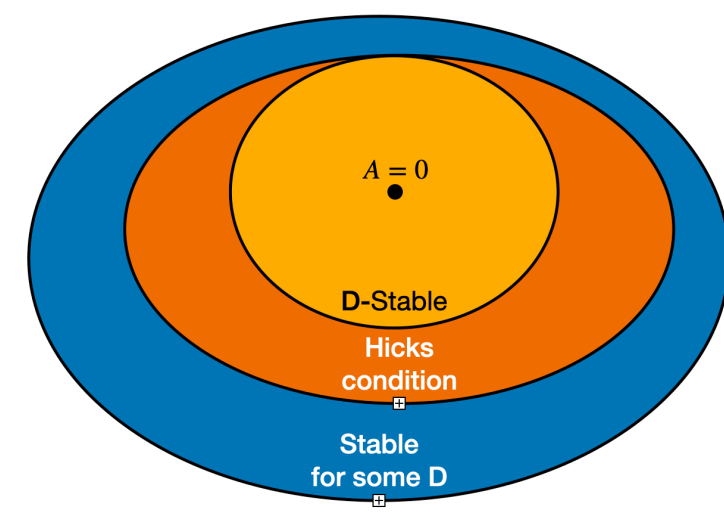
Two conditions...

$$\sum_n dp_n d\tilde{z}_n < 0$$

Average Law of Demand
WARP=Weak Axiom of Revealed Preference



Quasi
Negative Definite
(Arrow-McManus)



2. Robust Stability

Consider a change in prices...

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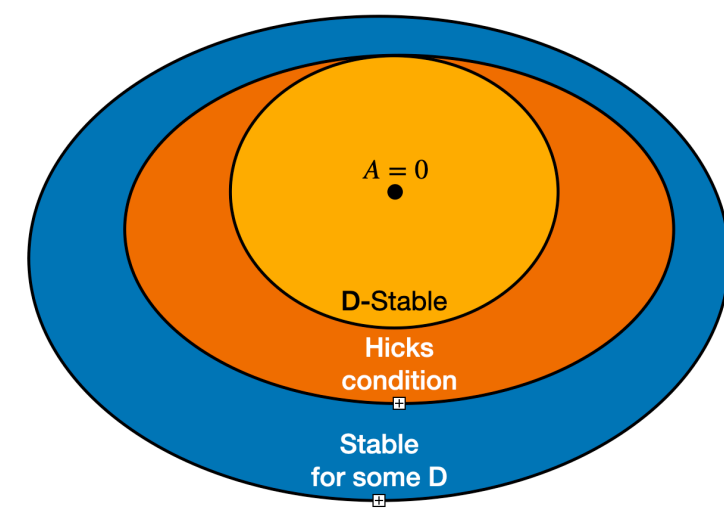
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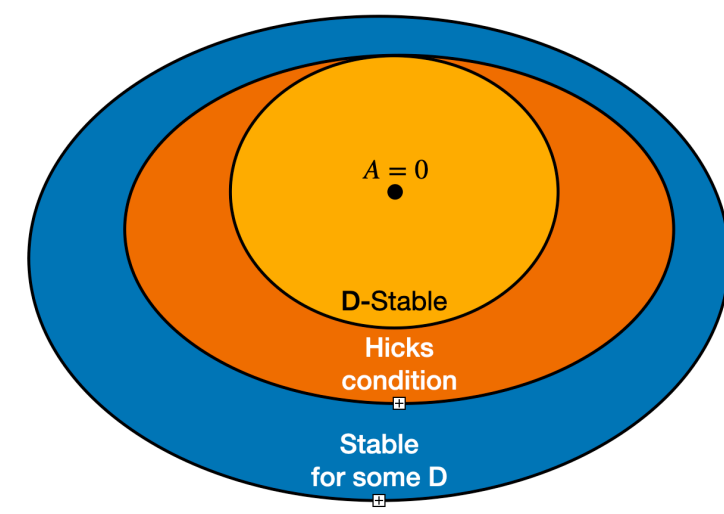
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Weaker Law of Demand
Holds in Some Market!



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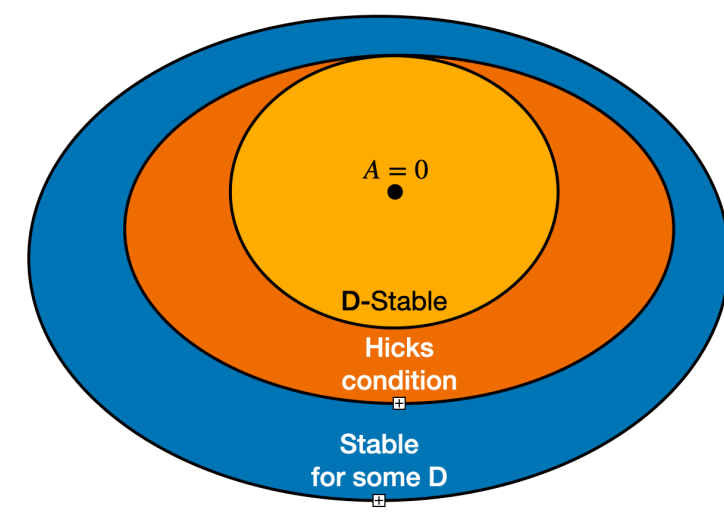
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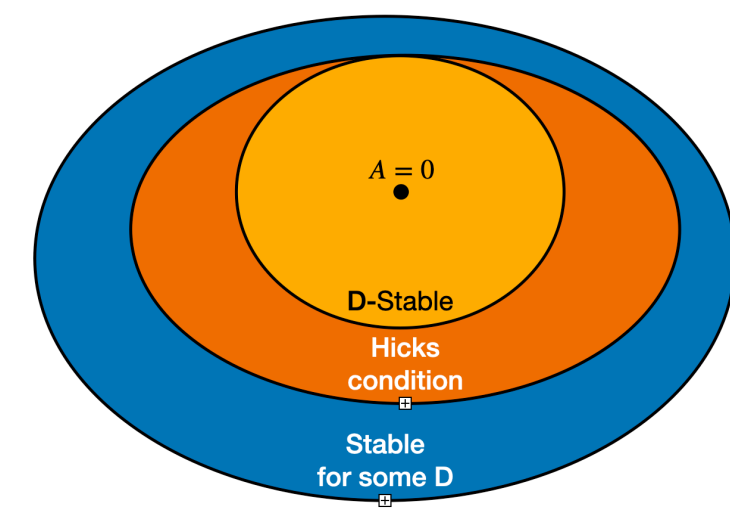
Weaker Law of Demand
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P-matrix...
Hicks Condition!



2. Robust Stability



**Sufficient
Condition**

Proposition.

$$\text{for all } dp \quad \sum_n dp_n d\tilde{z}_n < 0 \quad \Rightarrow \text{D-Stable}$$

Average Law
of Demand

$$S < 0$$

→ holds

$A \neq 0$ small

**Necessary
Condition**

Proposition. (Metzler)

$$\text{D-Stable} \Rightarrow \text{for all } dp \quad \exists n \quad dp_n d\tilde{z}_n < 0$$

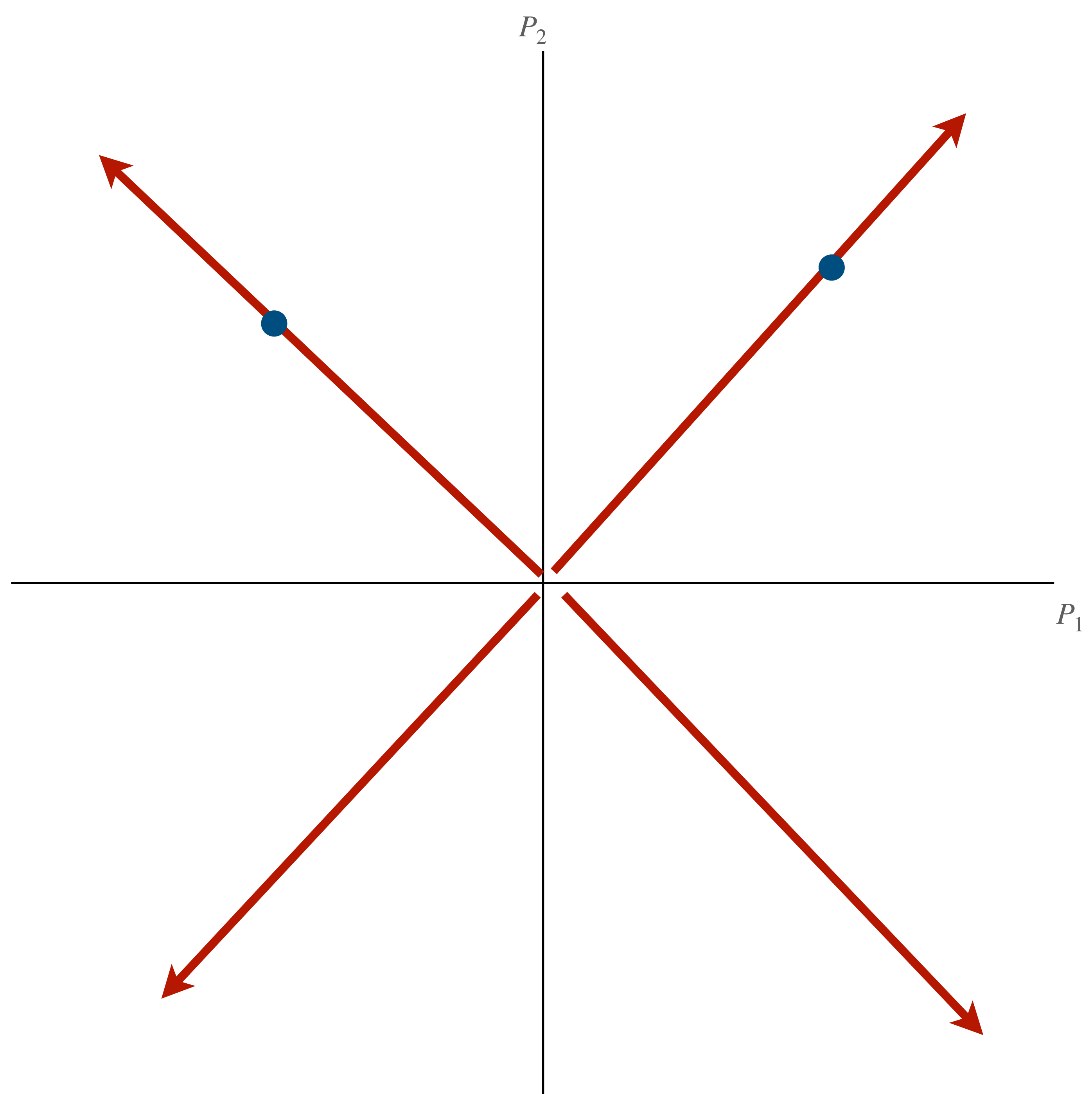
Hicks Condition

Weaker Law
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■ Two ways to **diverge**
(eigenvalues)

● real

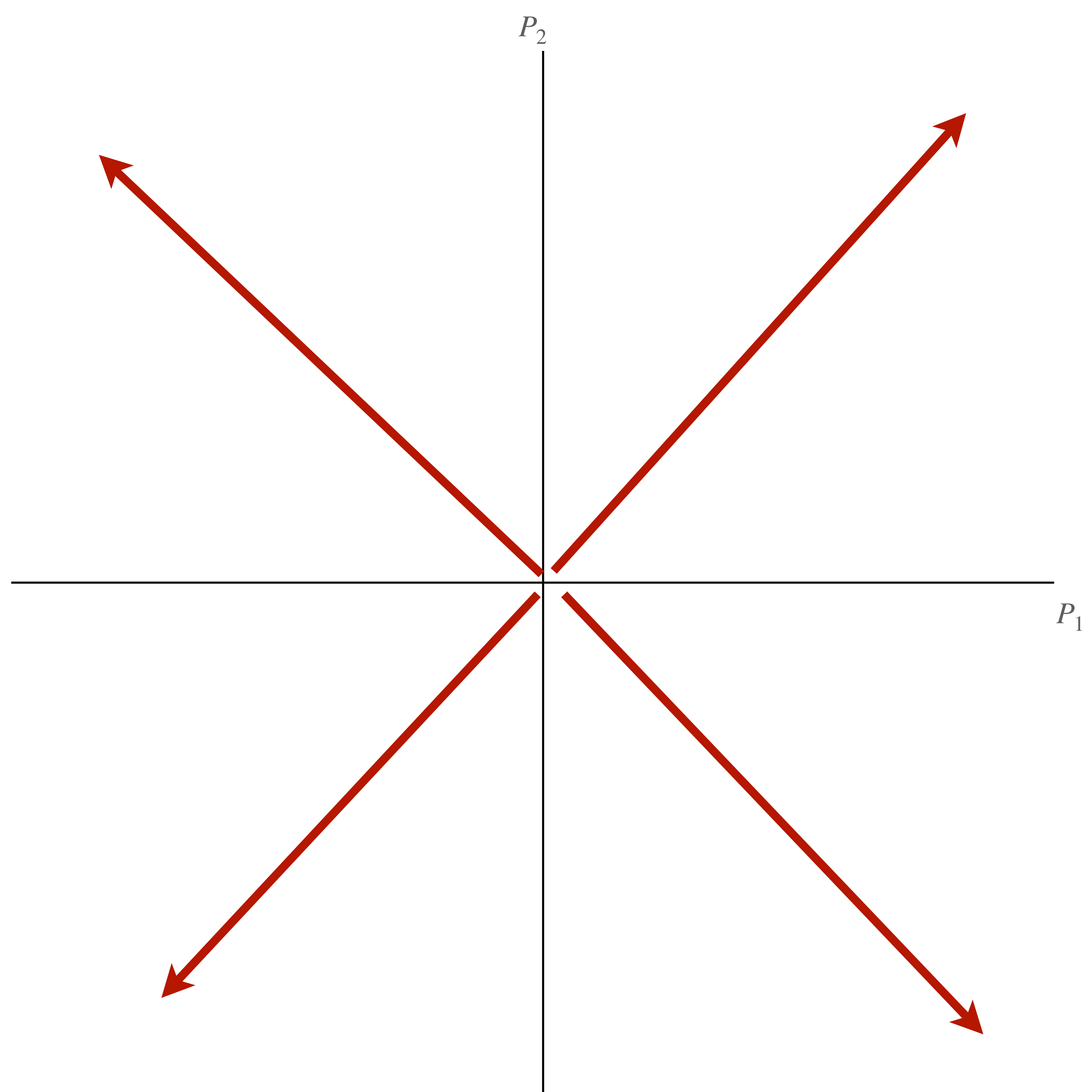
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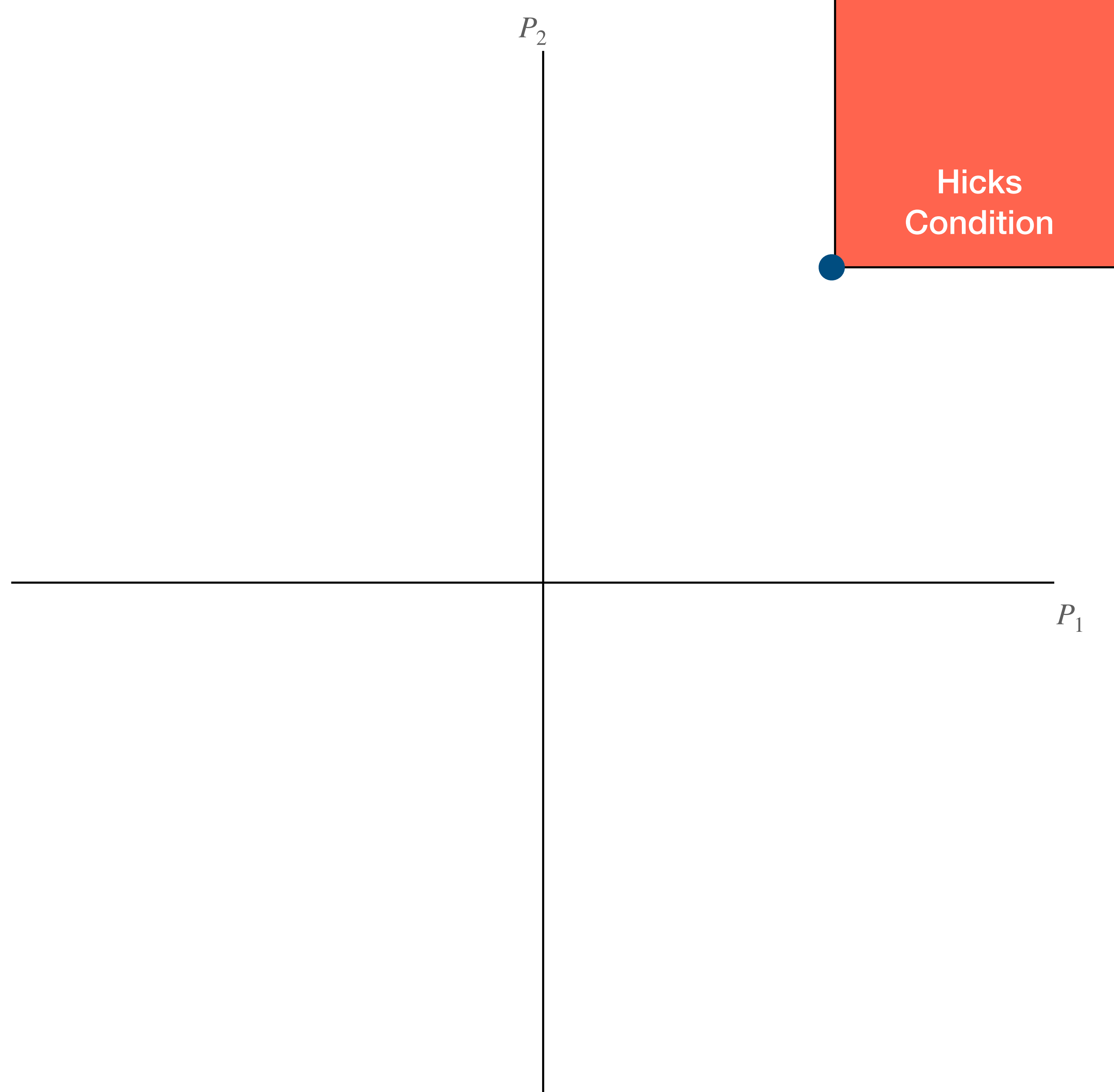
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■ Two ways to **diverge**
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• real → Hicks rules out!

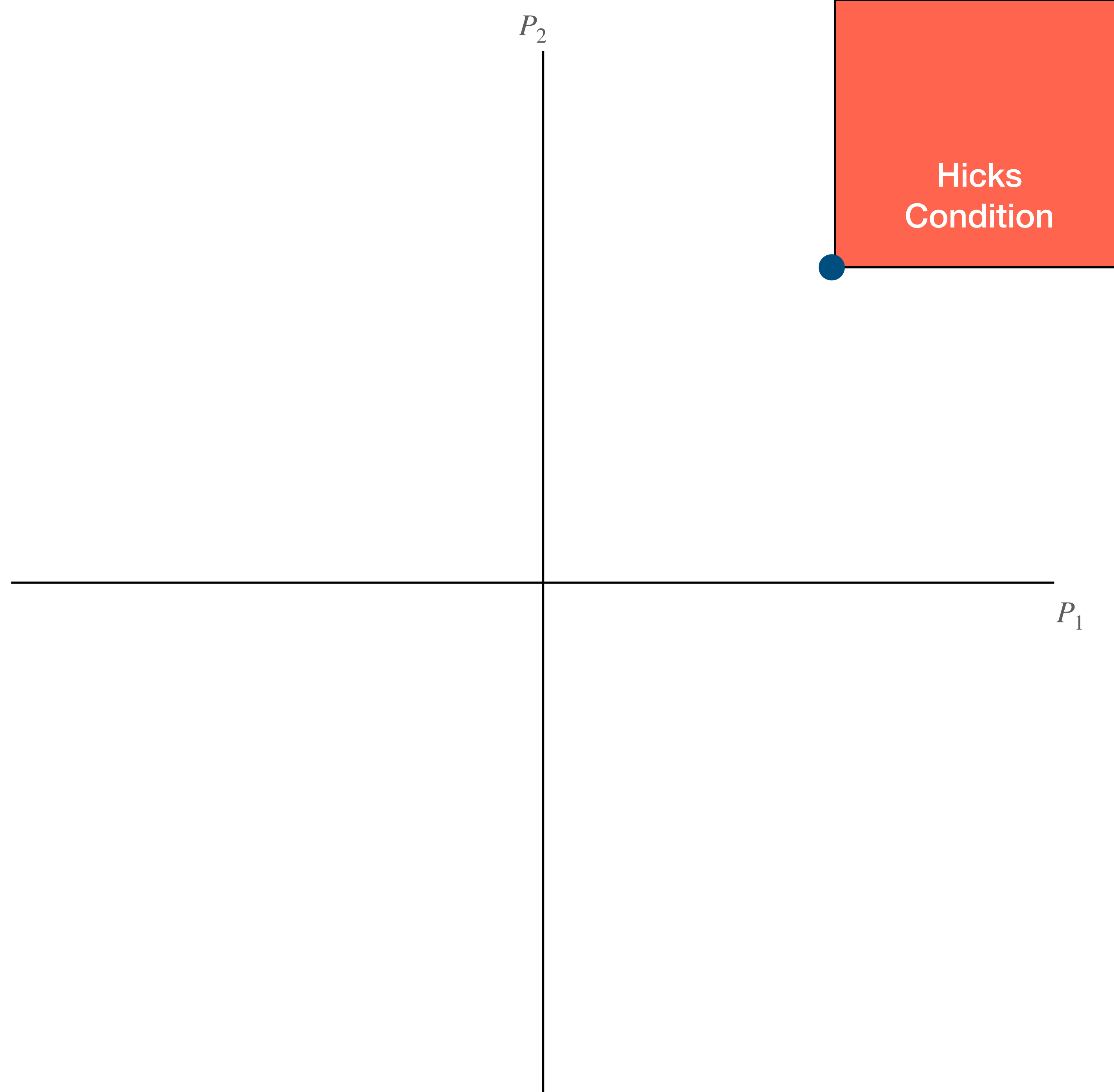
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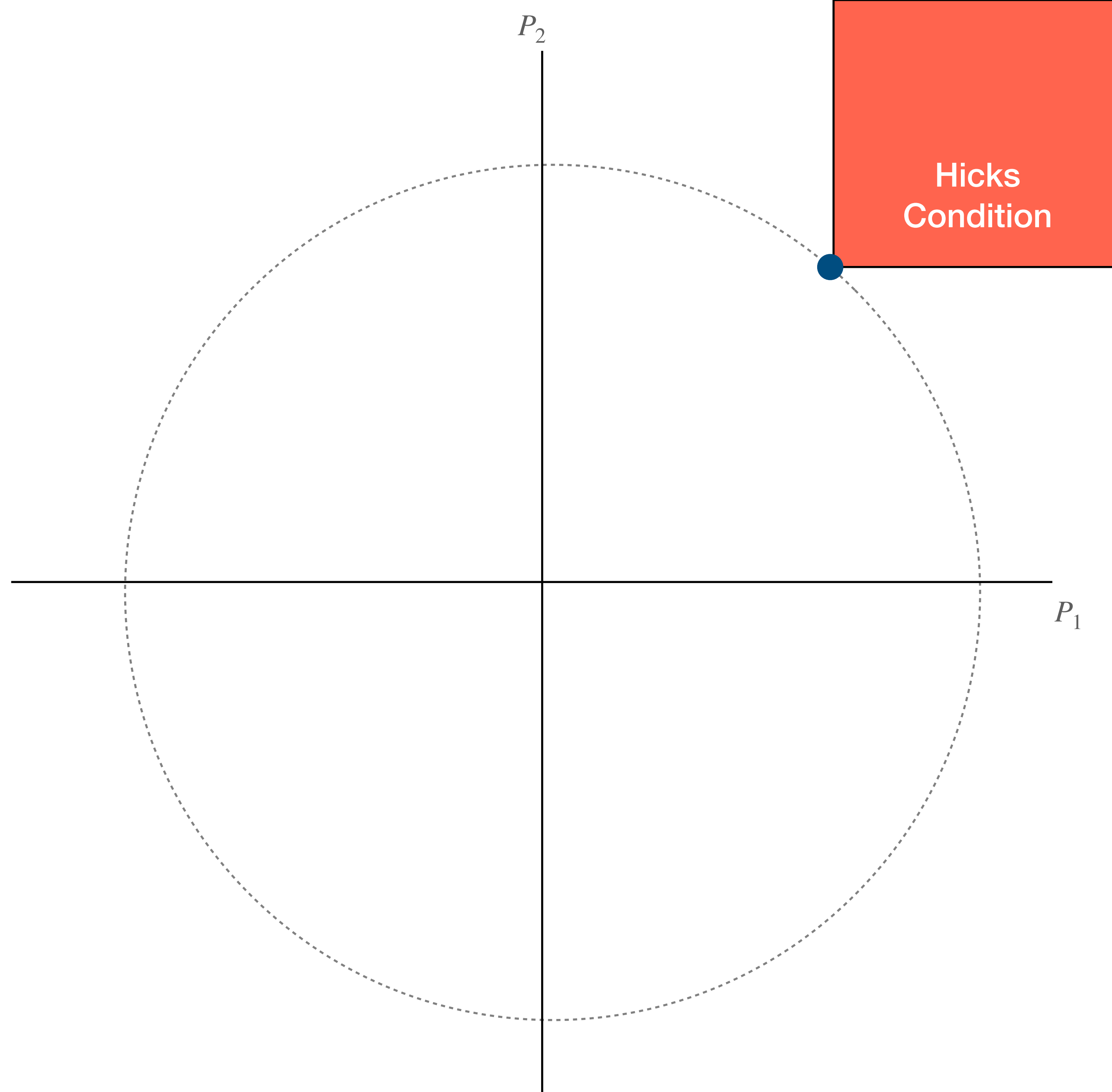
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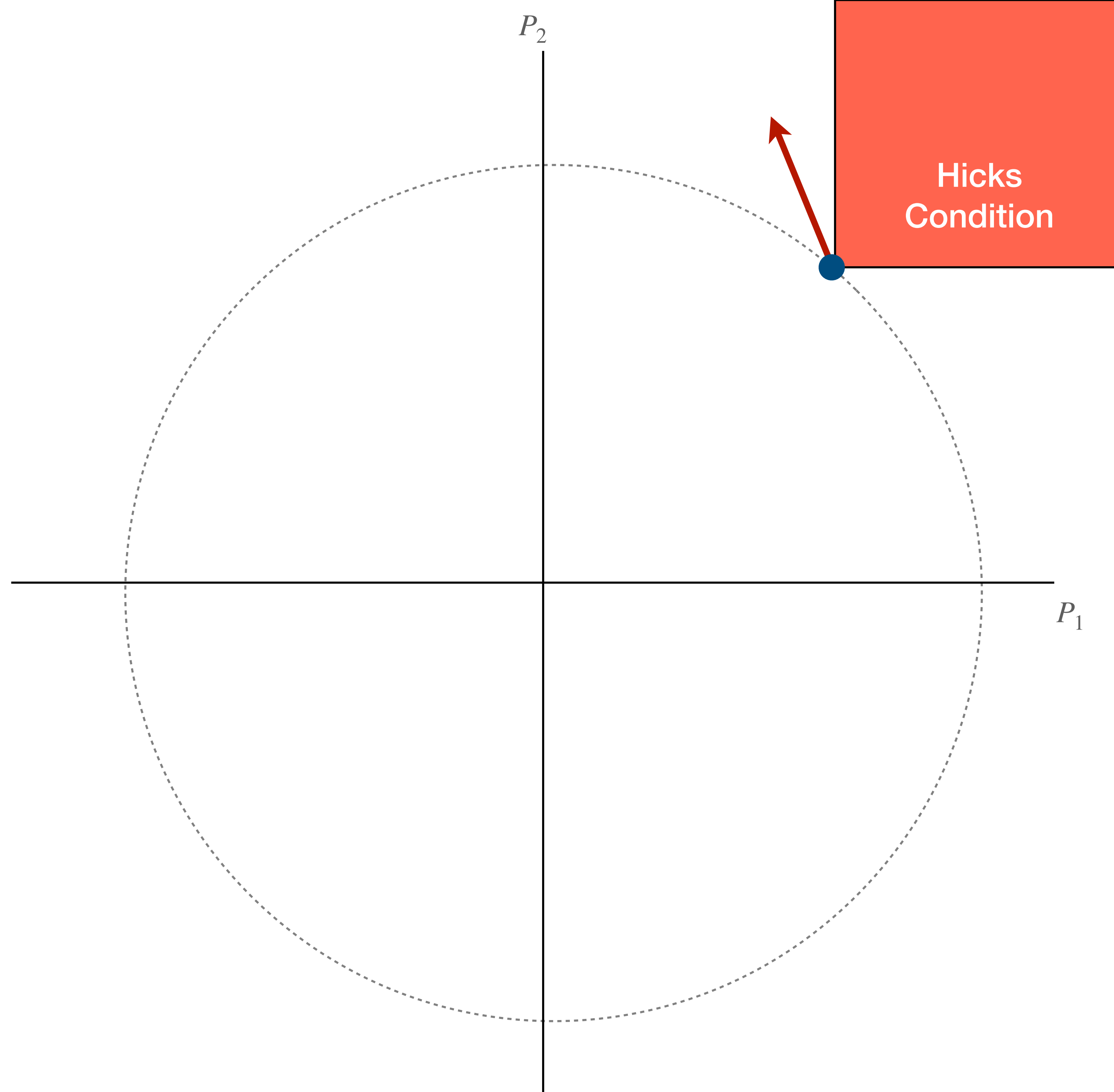
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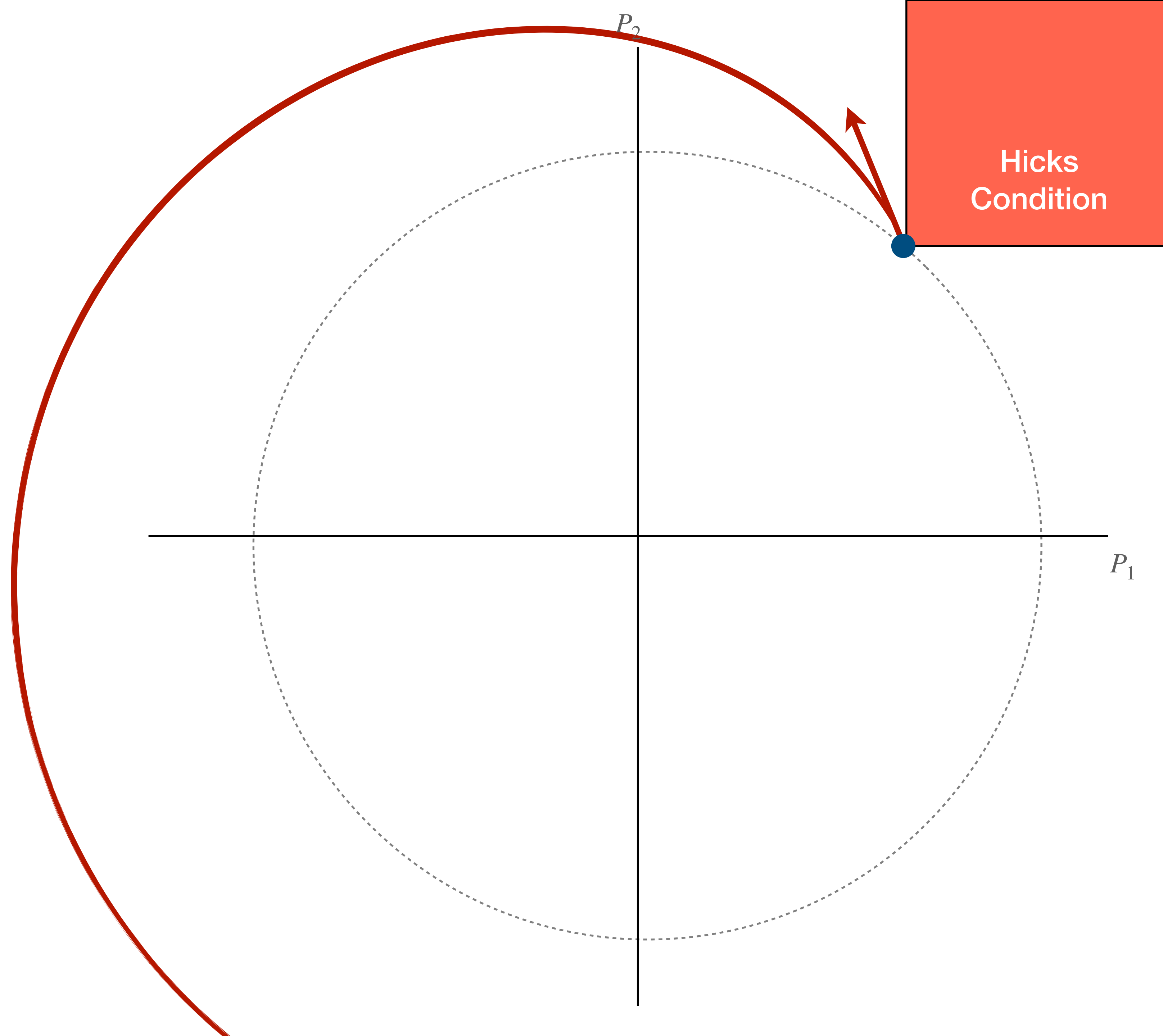
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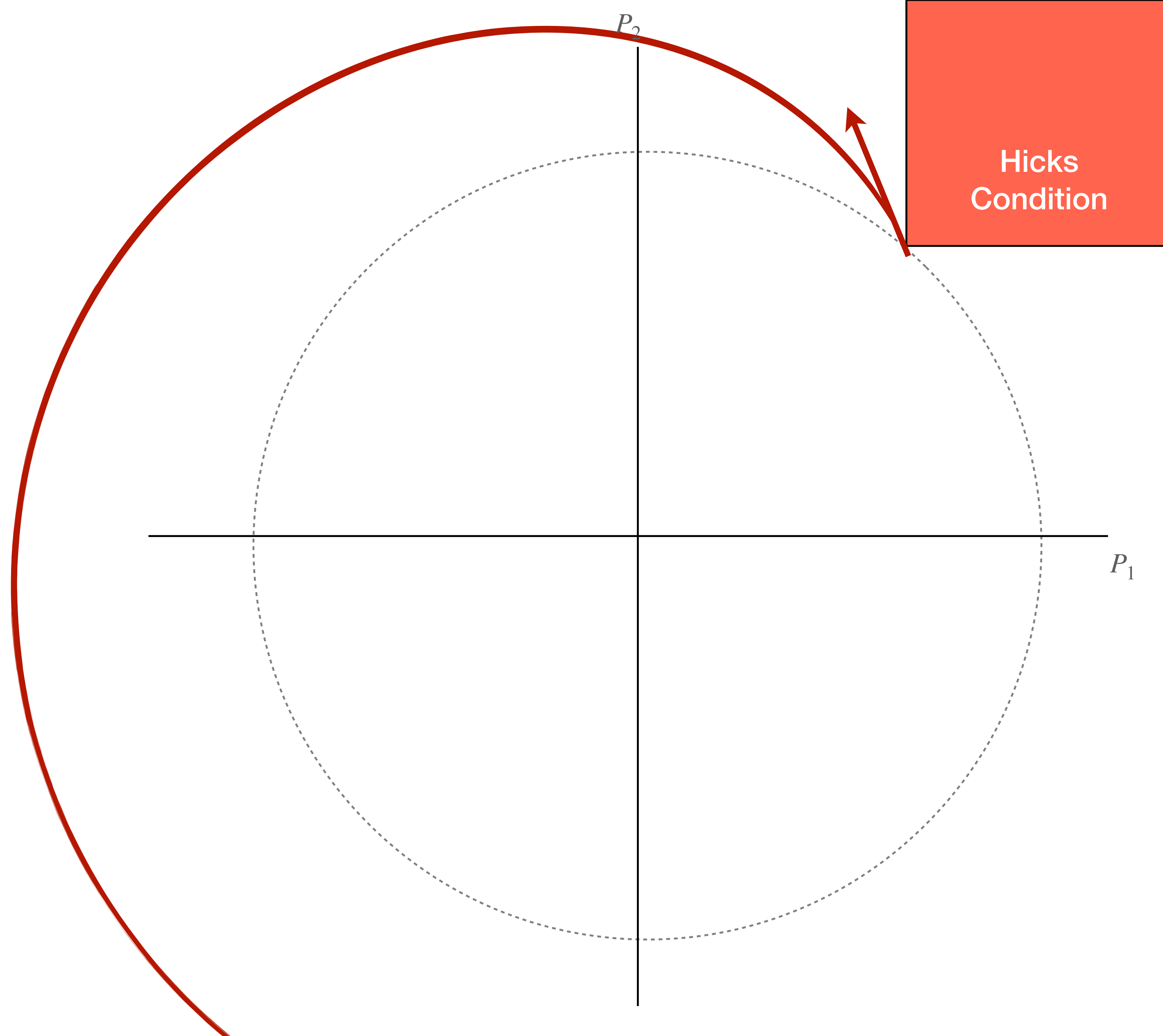
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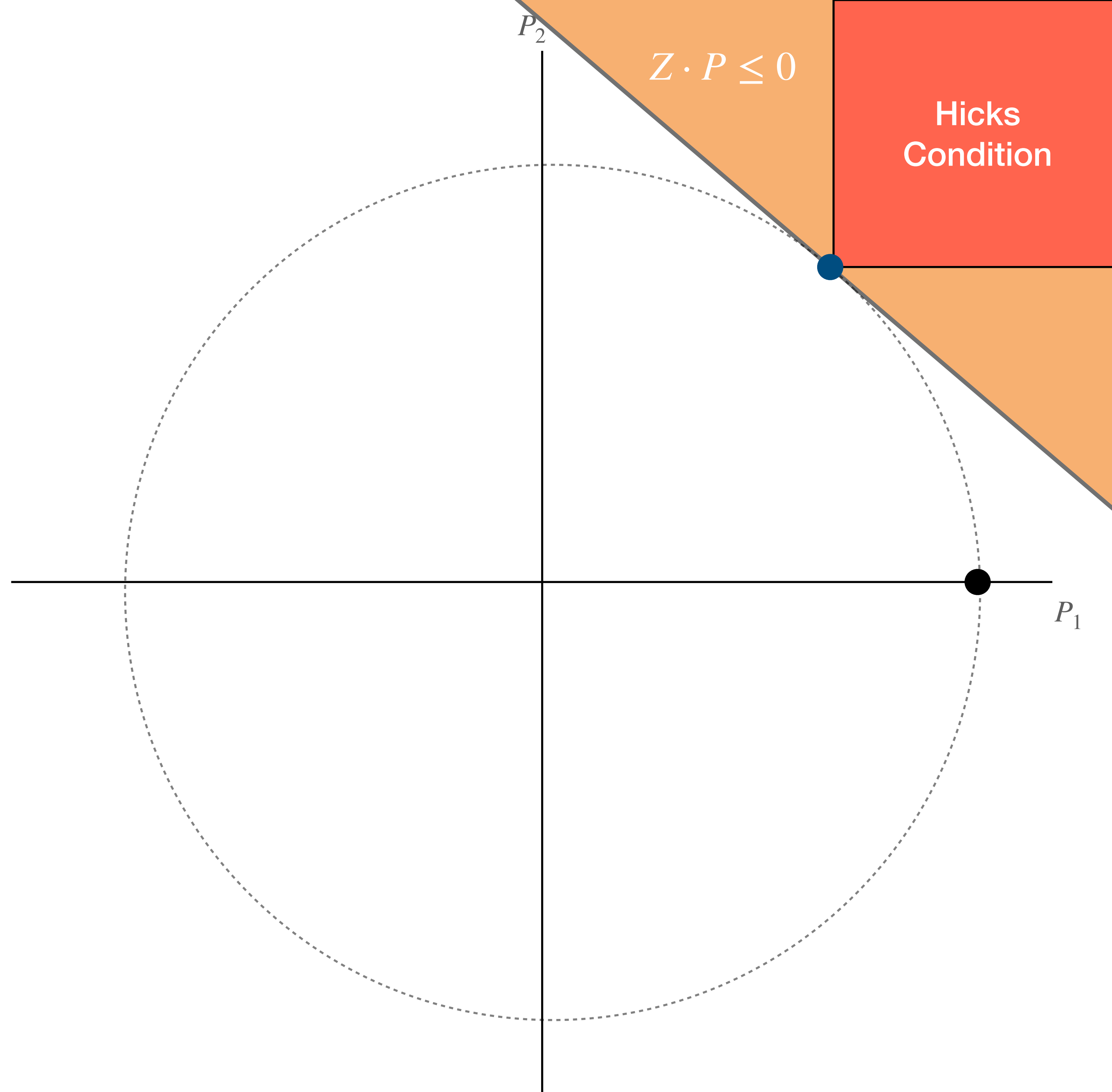
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■ Two ways to **diverge**
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- real → Hicks rules out!
- complex
→ strong law Demand
rules out!



Production Economies

Production Economies

■ Production economy

$$N = N_c + N_\ell$$

► N_c final + intermediate goods

► N_ℓ labor types

■ Production is CRS (w.l.o.g.)

■ Pricing: monopolistic...

► firms set price of goods p_c

► households set wages p_ℓ

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Proposition.

Stability if...

1. labor sufficiently elastic
2. prices (vs wage) very flexible

Partitioned Production Economies

- Partitioned production economy...

- ▶ N_c partitions

- ▶ each partition $\rightarrow$ single final good

Partitioned Production Economies

■ Partitioned production economy...

▶ N_c partitions

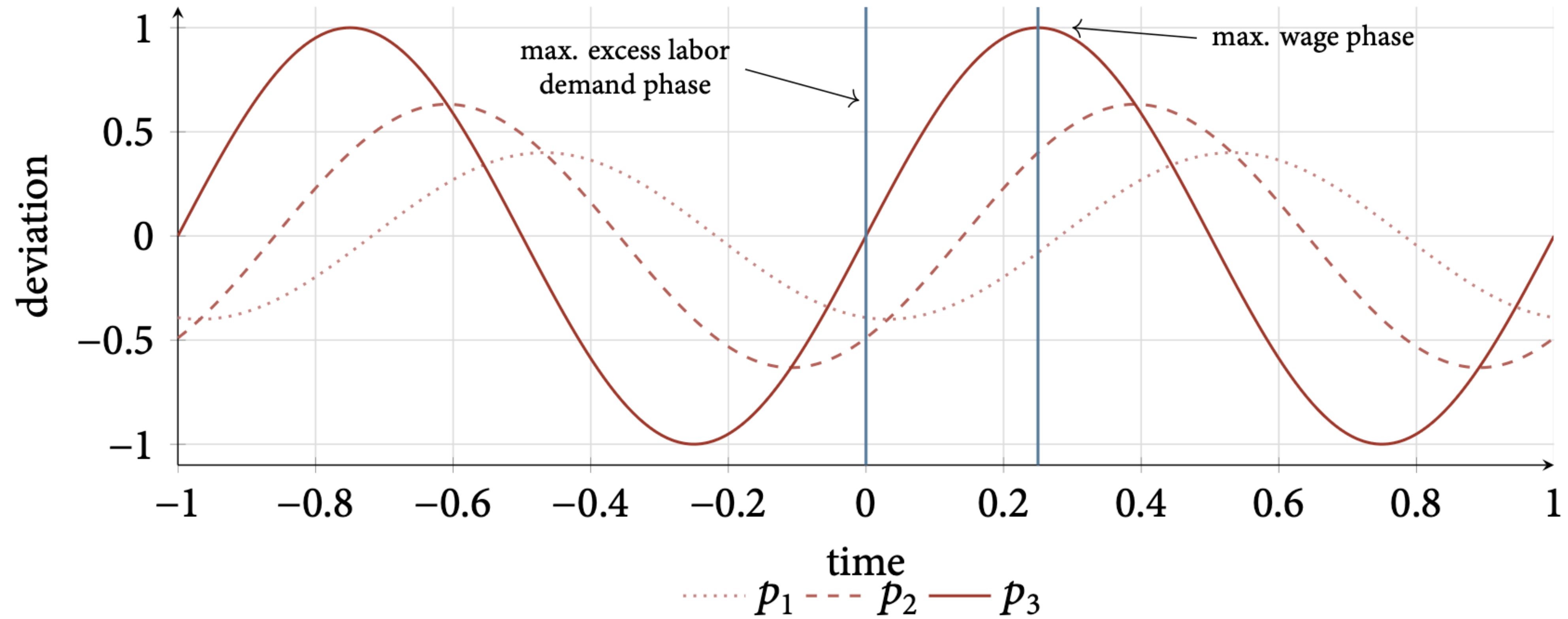
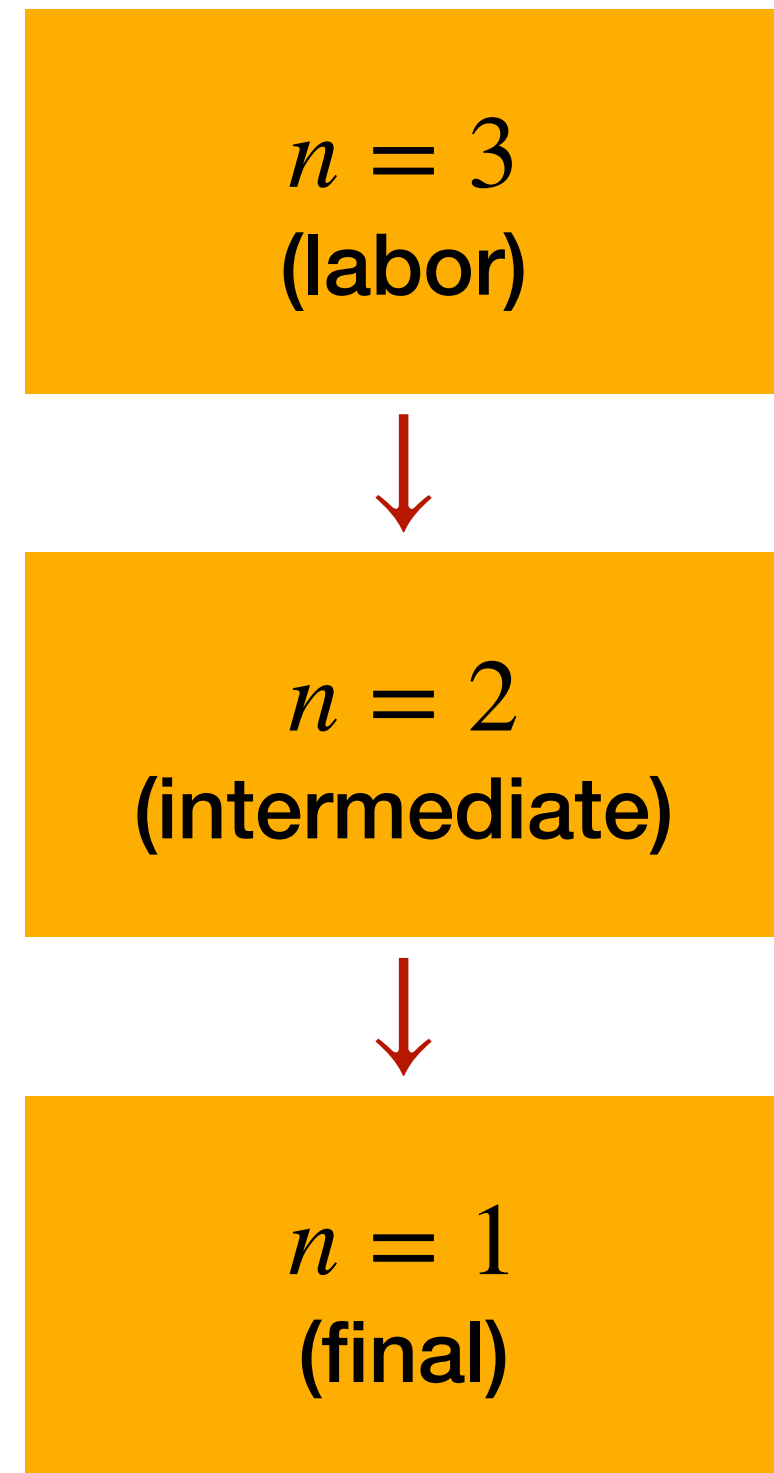
▶ each partition $\rightarrow$ single final good

Proposition.

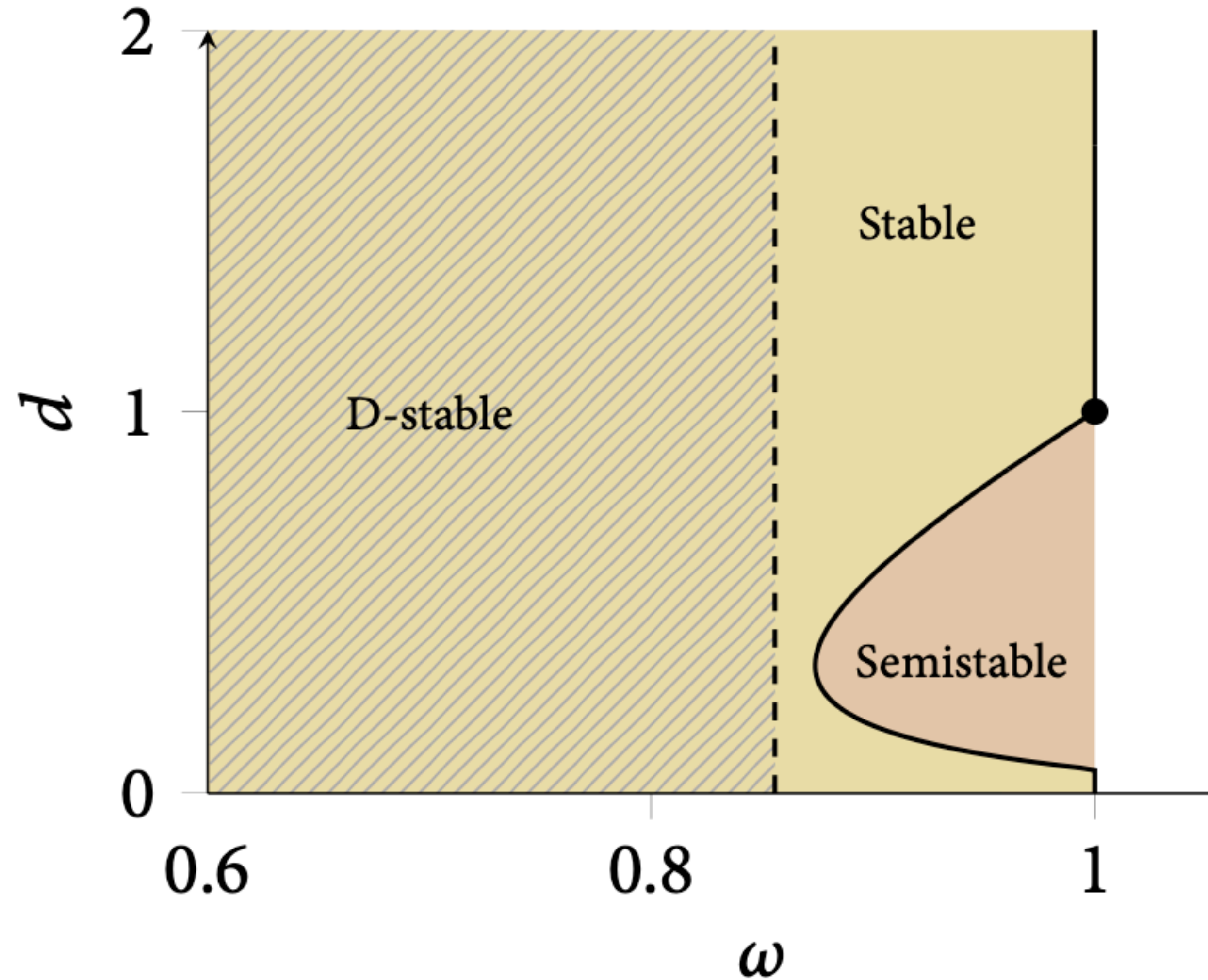
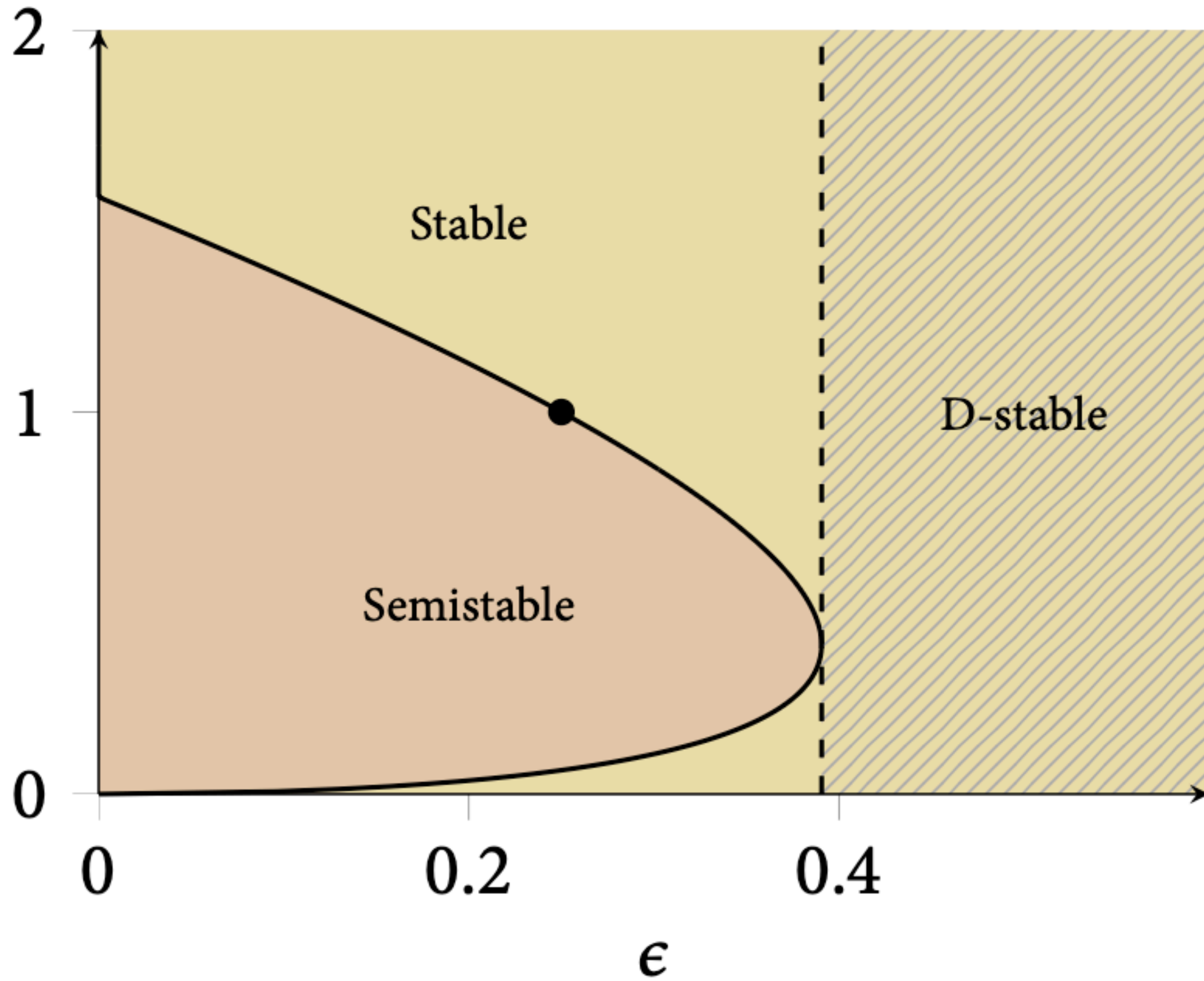
Within each partition....

1. No input-output $\rightarrow$ Stable
2. Dedicated exclusive labor *or* a Leontief input-output
 $\rightarrow$ stable or oscillatory instability (no explosive)

Production Chain Cycle: 3 goods



Example of Cycle: 3 goods



Forward-Looking Pricing

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■ Forward Looking: 2nd order ODE...

$$\dot{p} = \bar{d}Cp + \frac{1}{\rho}\ddot{p}$$

Forward-Looking Pricing

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Myopic
Stable



Stable!

Forward-Looking Pricing

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Stable!

Myopic
Oscillatory
Unstable

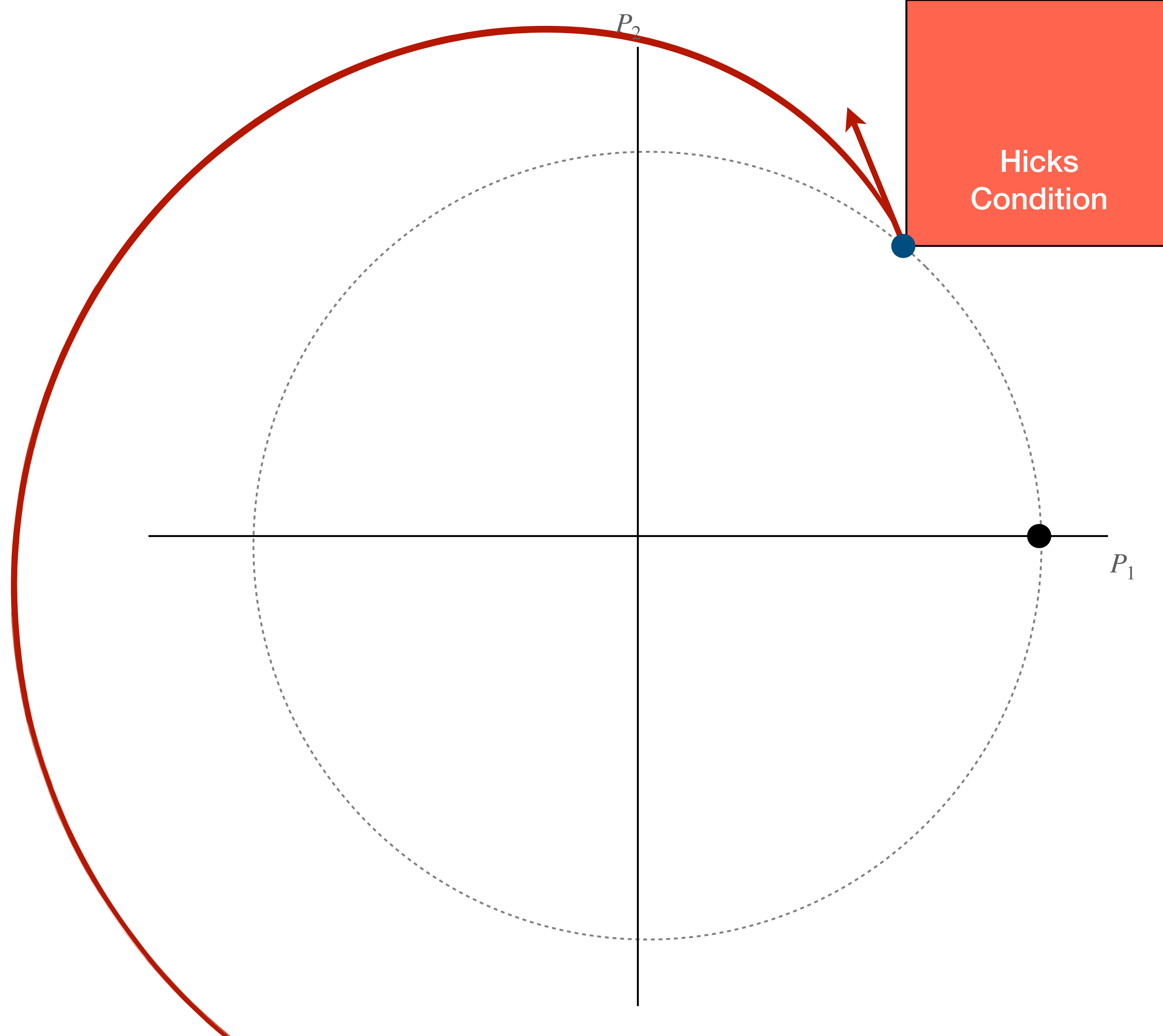


Stable for...

$$\bar{d}/\rho \geq \hat{d}_C$$

■ Two ways to **diverge**
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 - strong law Demand rules out! (myopic)
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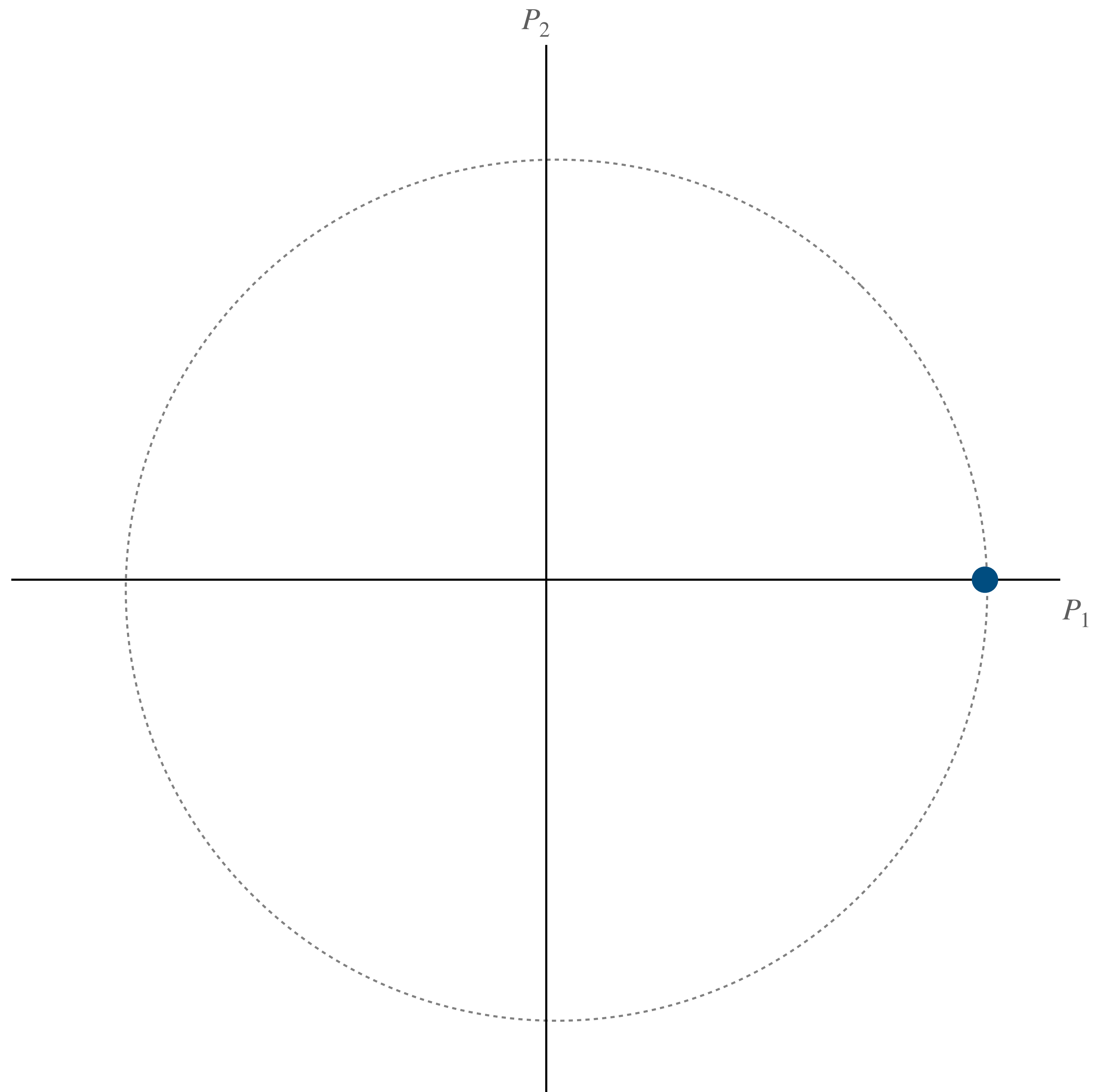


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- start initial belief:
myopic borderline (“circle”)
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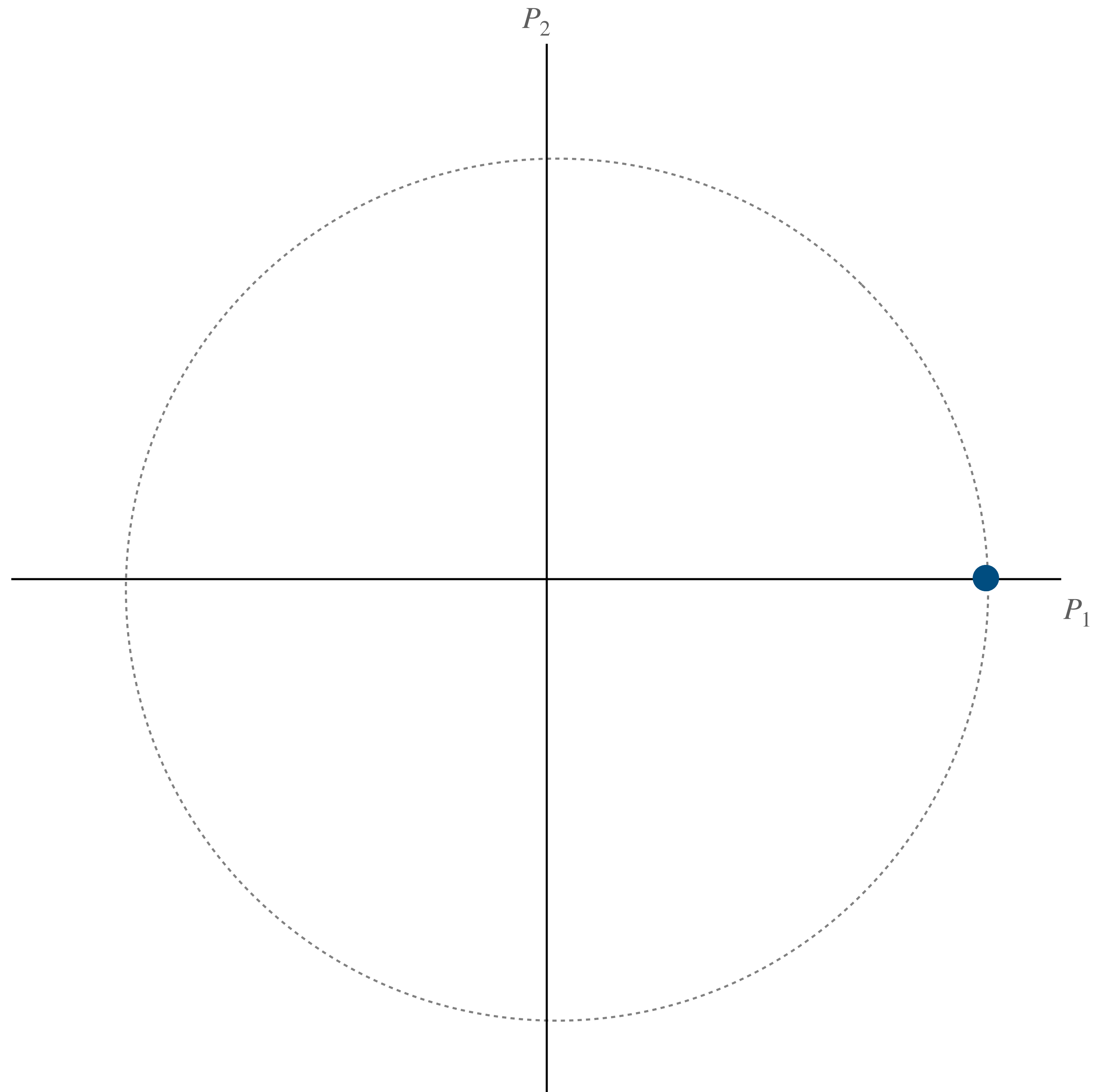


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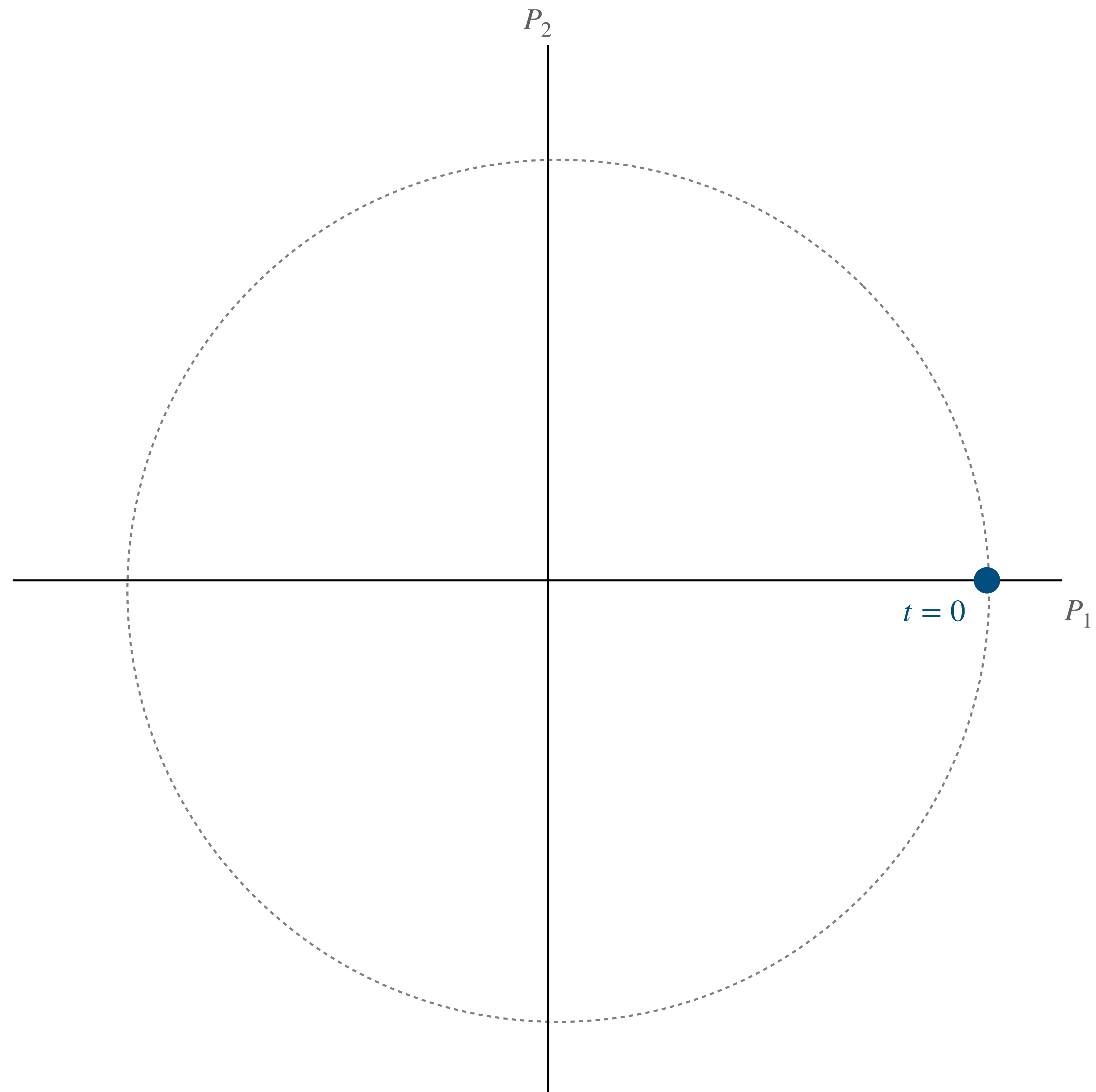


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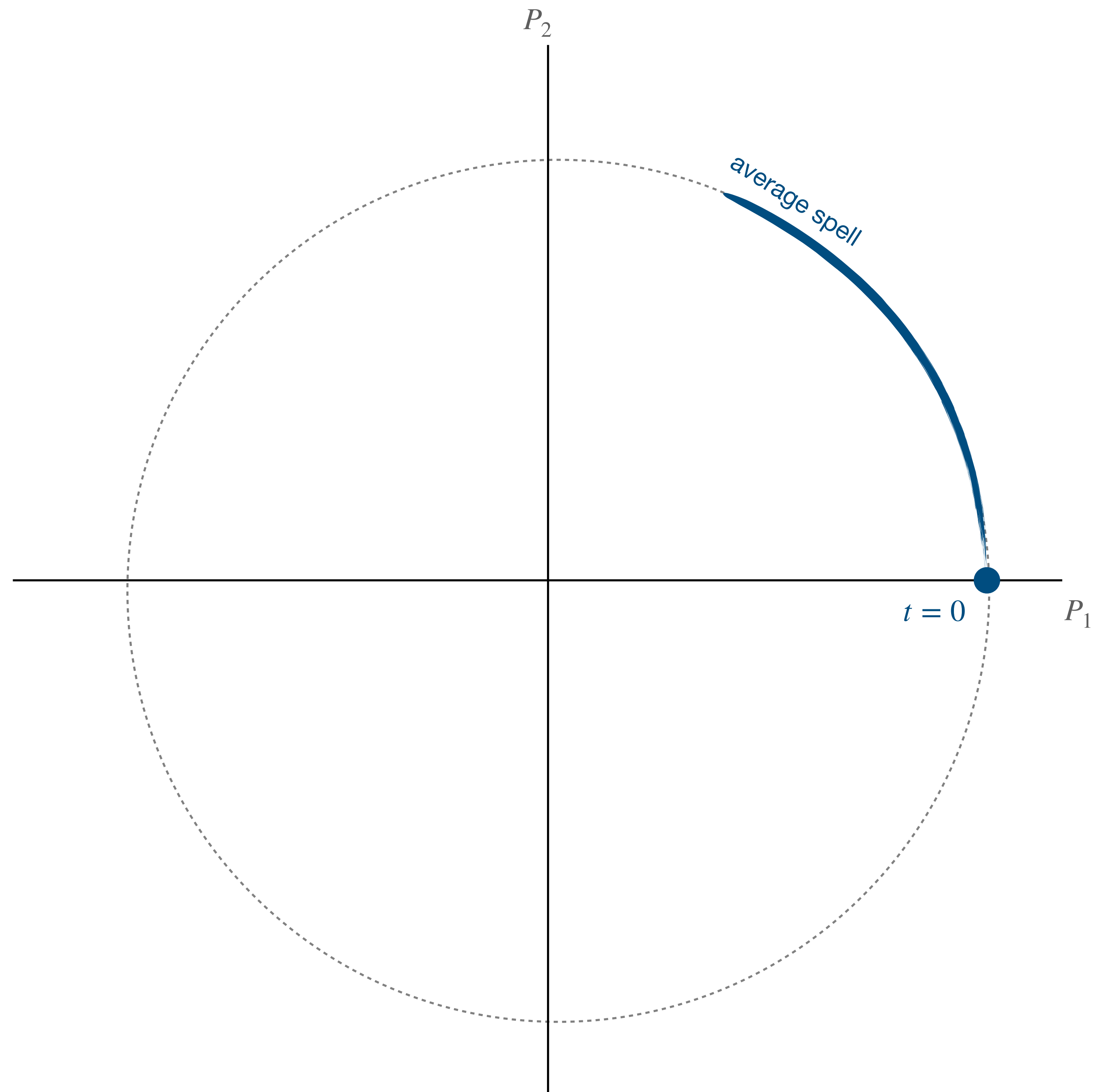


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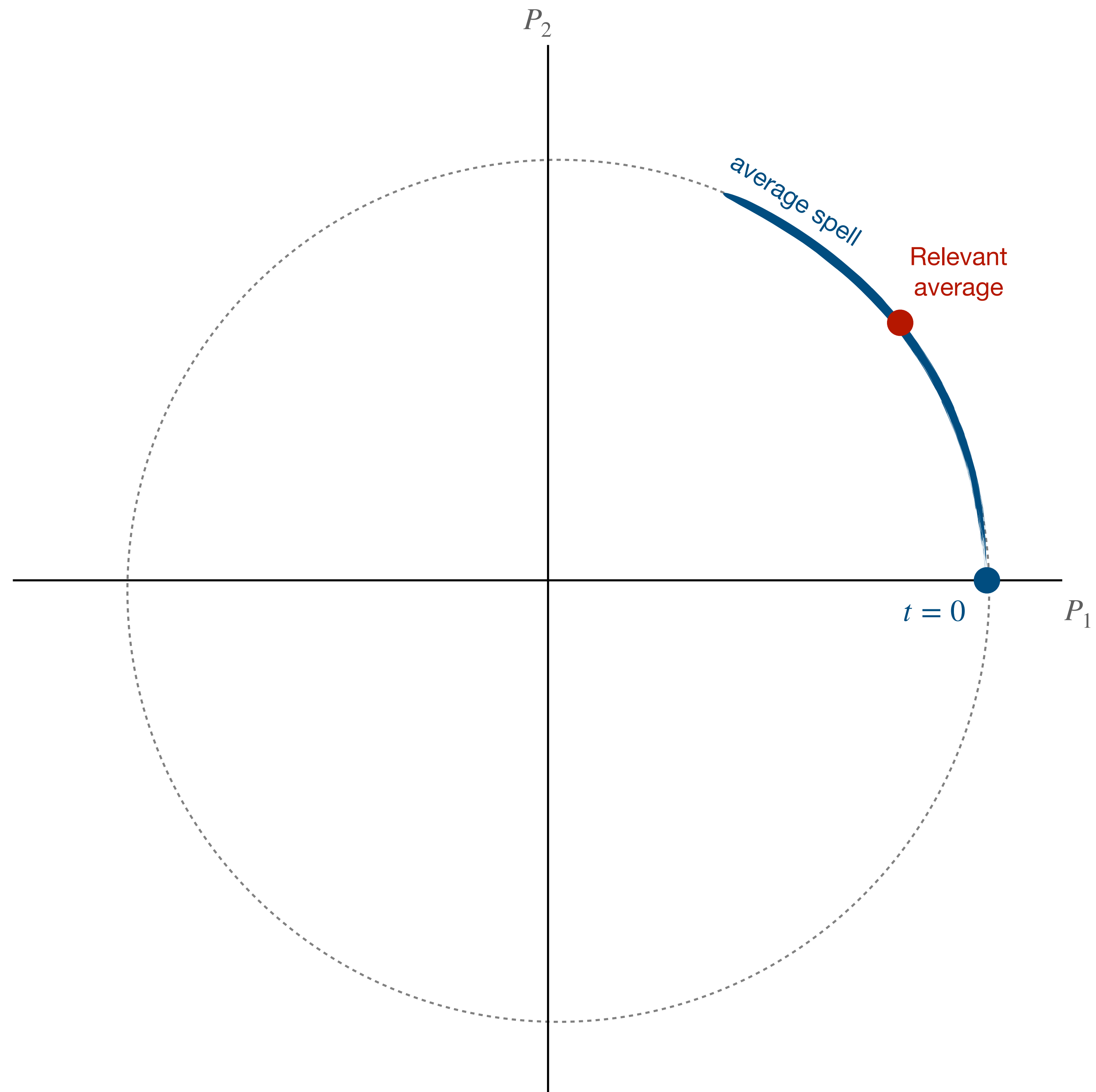


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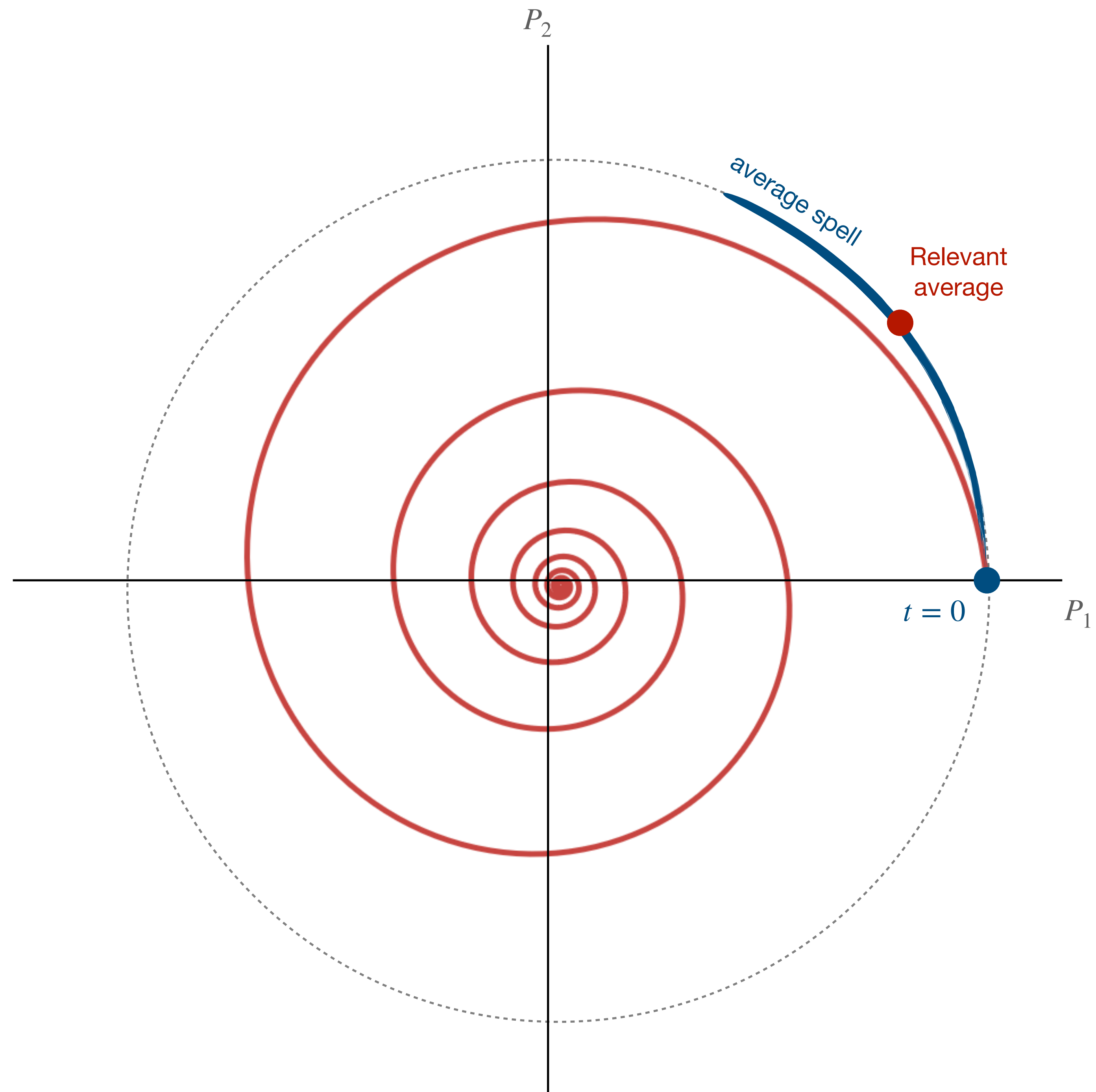


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 - **stable spiral!**



Proposition.

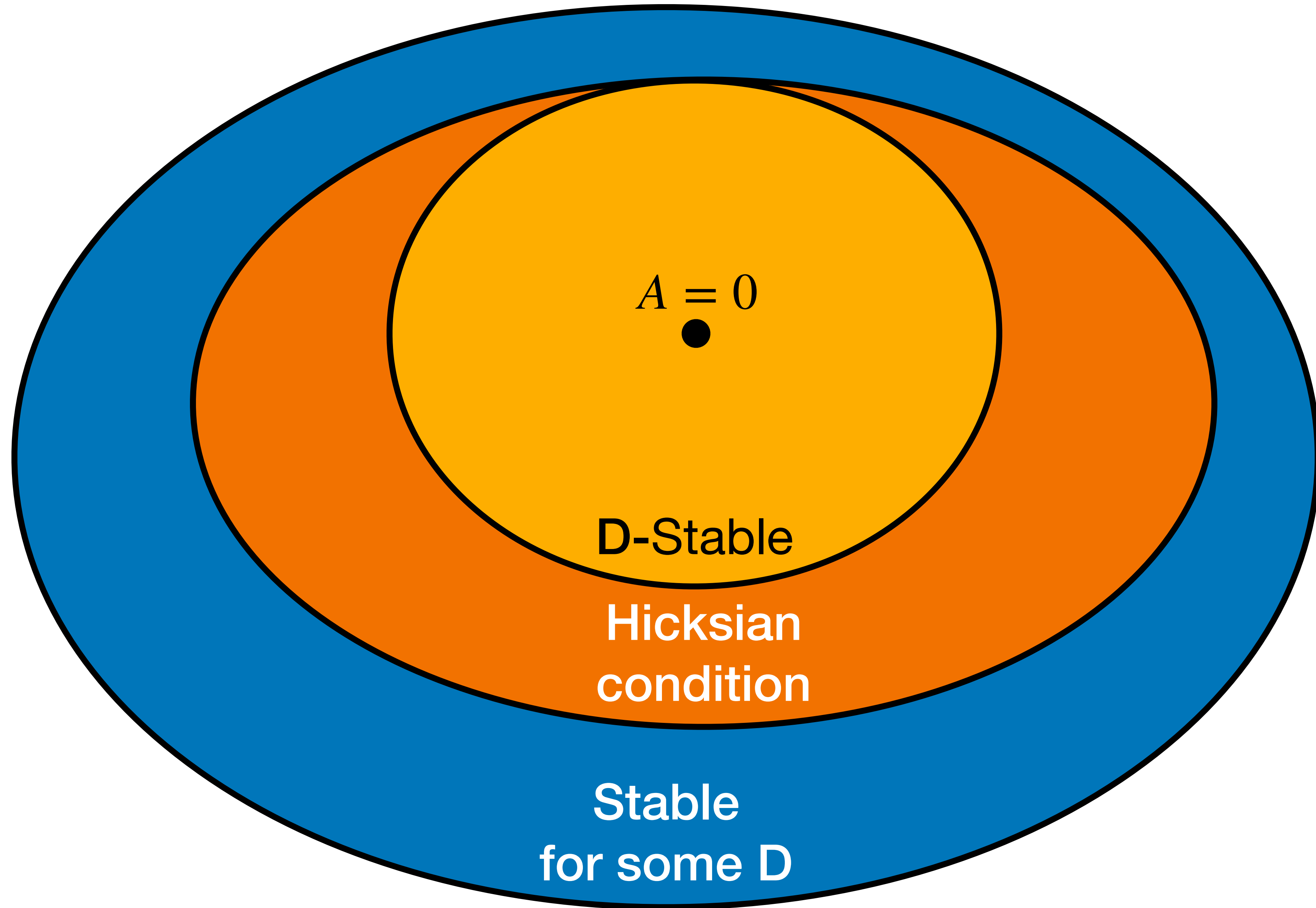
B is Hicks



Forward-Looking
D-Stable

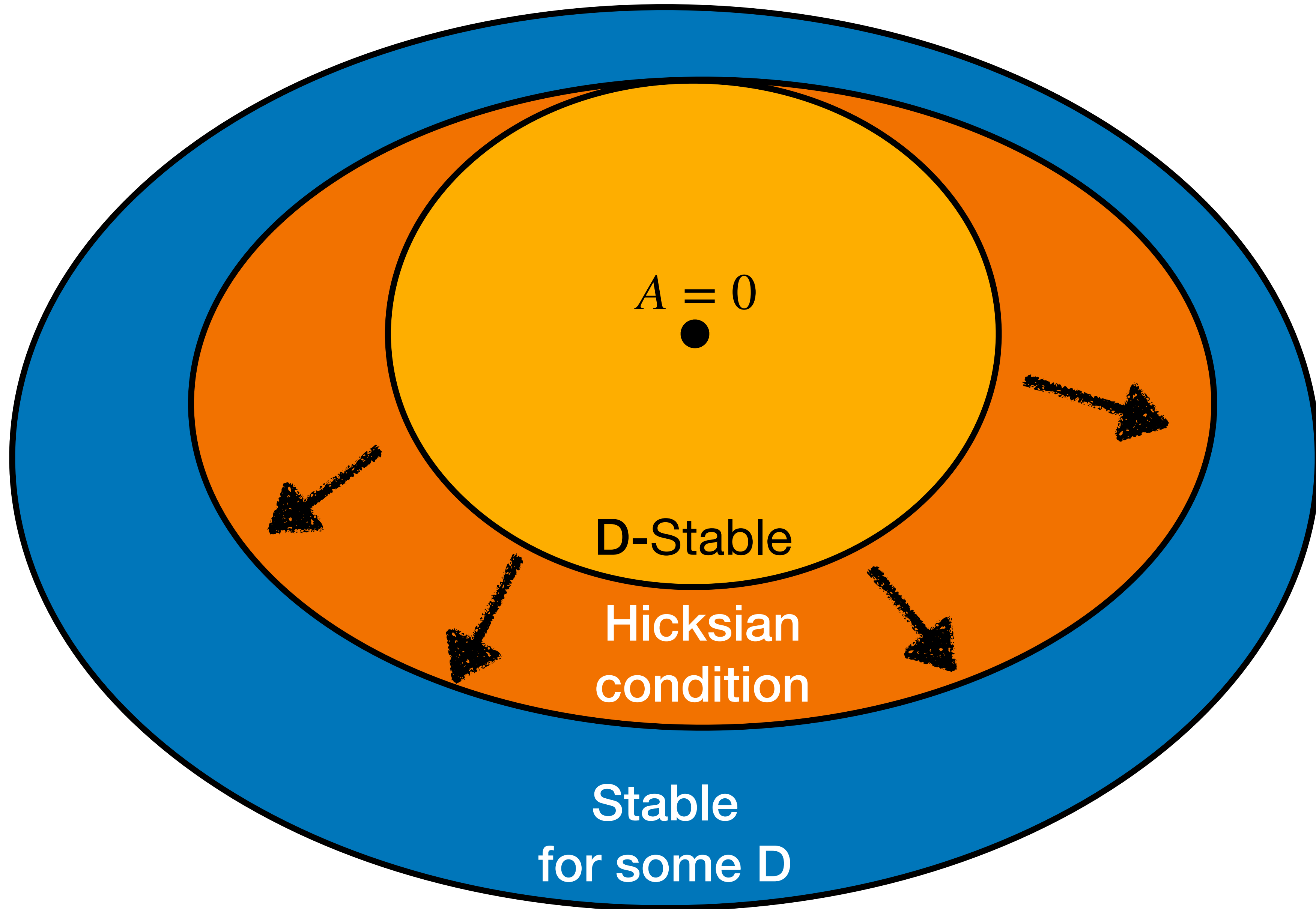
$$d_n/\rho \geq \hat{\delta}_B$$

Hicks is Back!



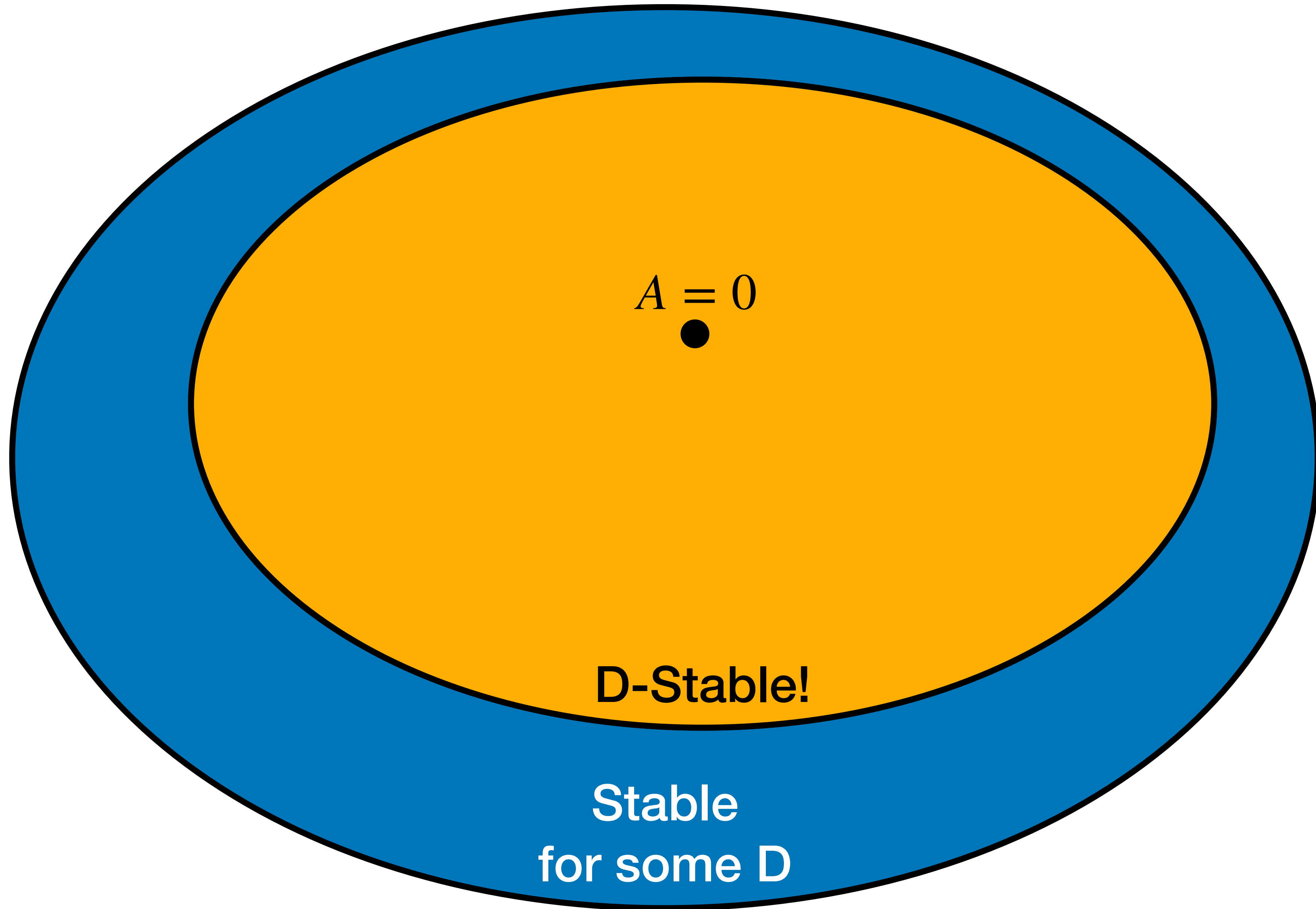
Unstable
for all D

Hicks is Back!



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Hicks is Back!



Unstable
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Part #1: Conclusions



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■ 1. Foundations...

► Provide **first foundation for tâtonnement** $\dot{p} = DZ(p)$



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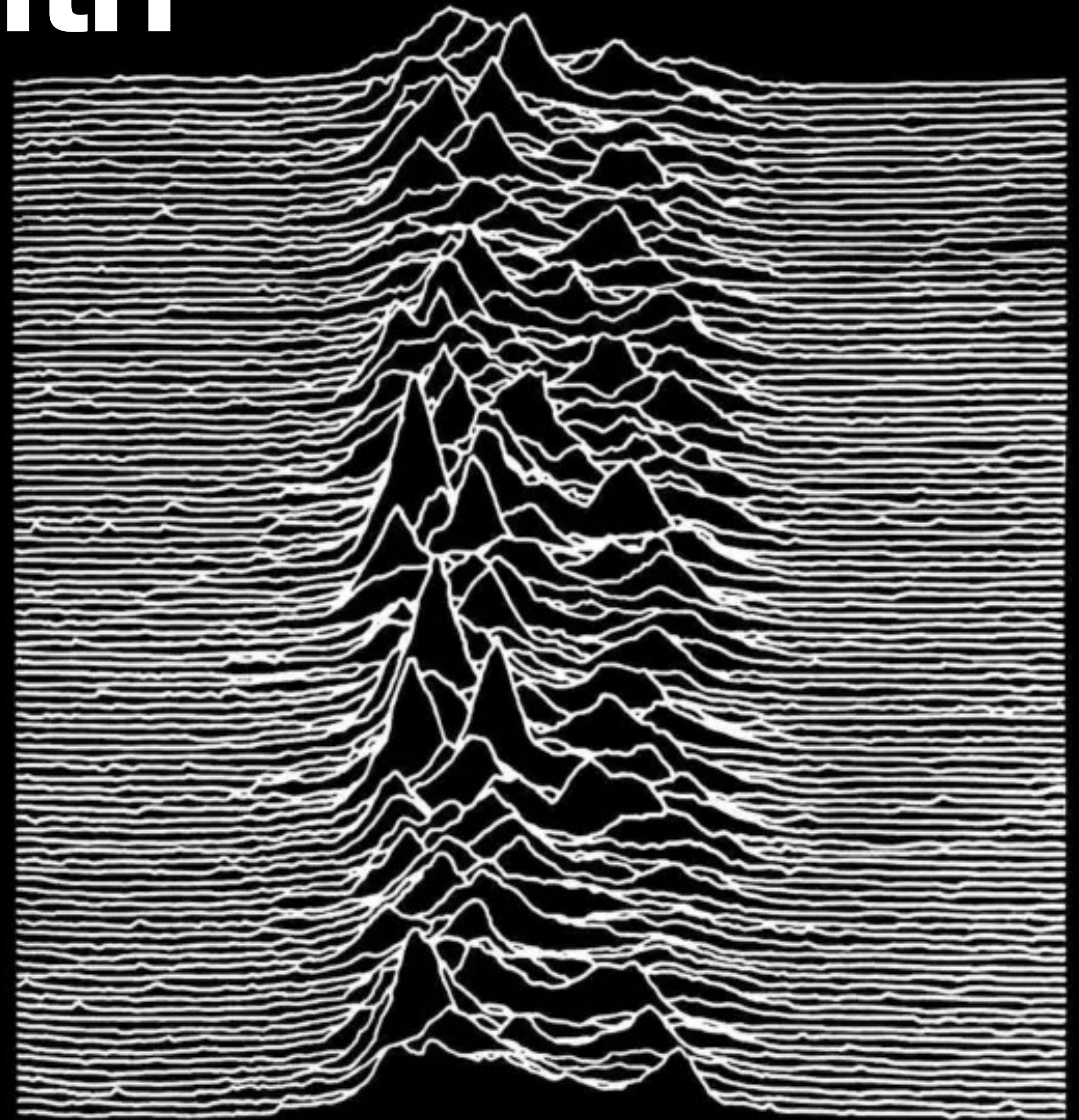
■ 3. Method: Macro with Excess Demands



Part #2

Macroeconomics with Excess Demands

Guido Lorenzoni and Iván Werning



Macro with Excess Demands...

- Excess Demand $\neq$ Gap
- Monetary Policy Rules + Determinacy
- ZLB
- Shocks: Primitives + Policy
- Government Spending example (g_t)
- Extensions...
 - ▶ Non-Ricardian consumers
 - ▶ non-Calvo pricing
 - ▶ non-rational expectations, e.g. learning

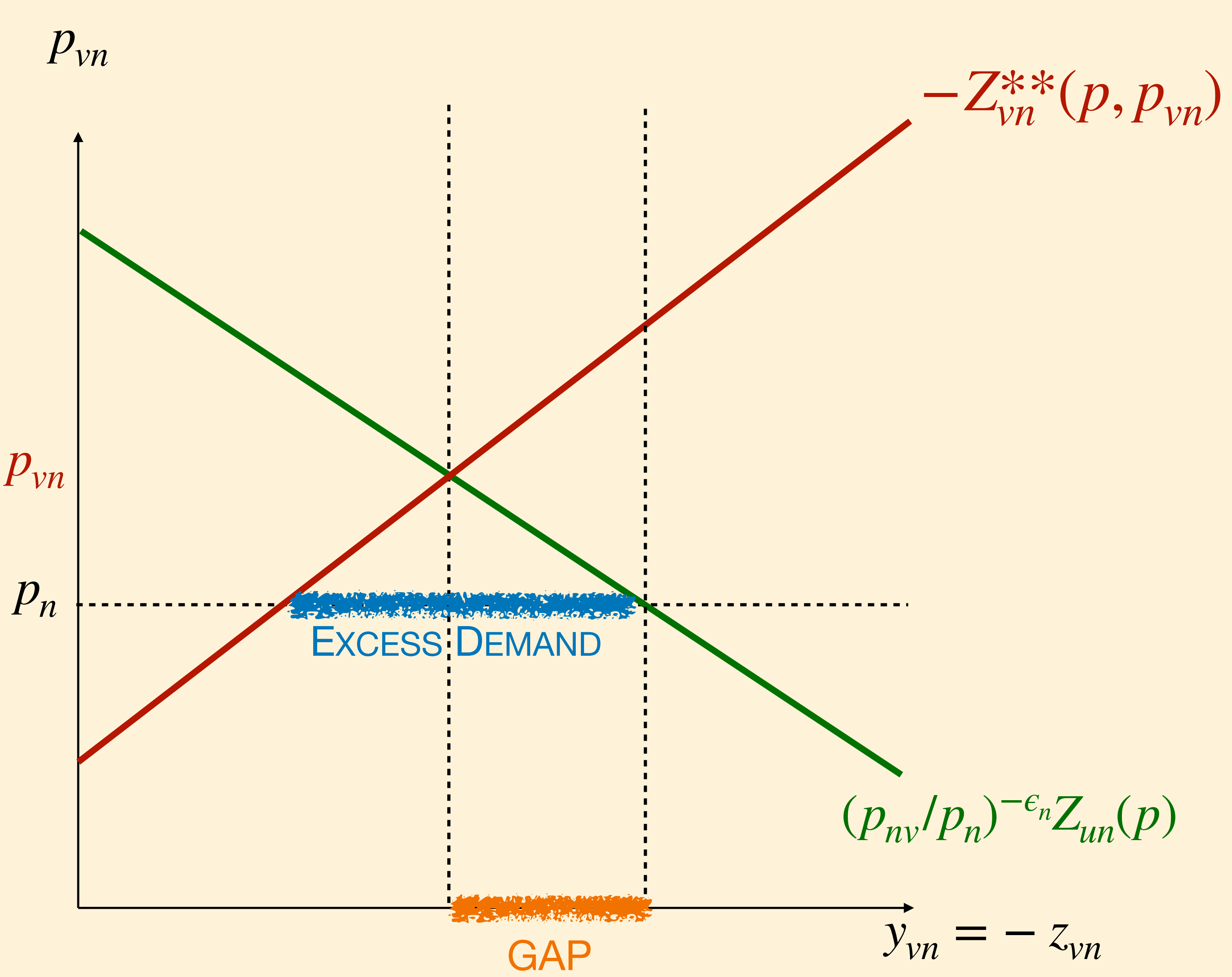
Excess Demand $\neq$ Output Gap

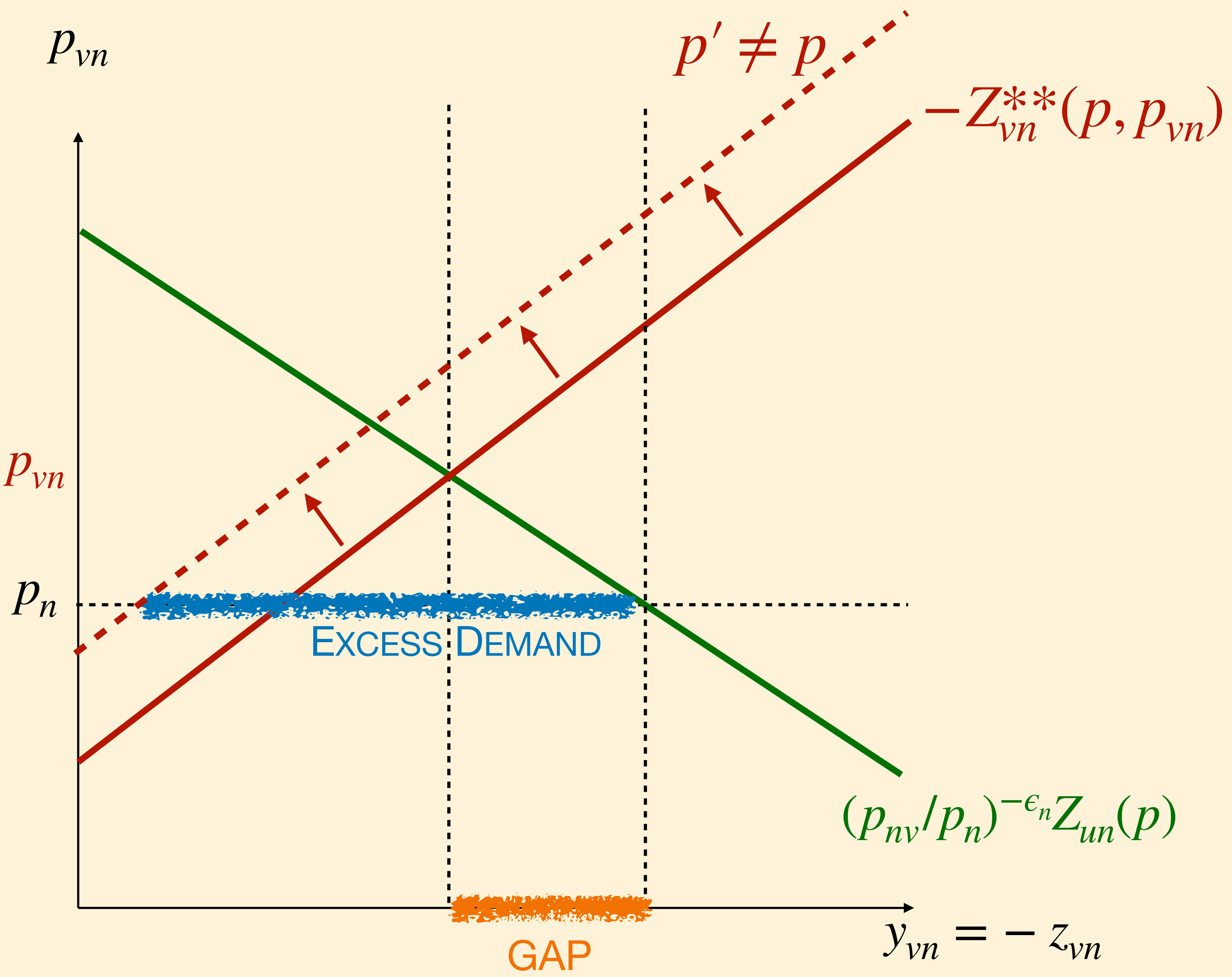
■ Gap...

- ▶ $N = 2$ textbook single final good + flex wages
→ output gap $\sim$ excess demand
- ▶ $N \geq 3$ “output gap” as aggregate measure (e.g. Rubbo, 2023)
→ special assumptions (homothetic + identical preferences+ CRS)
- ▶ $N \geq 3$ output gap in each market?

$$z_n^{\text{gap}} = z_{un} - \bar{z}_{un} \neq \tilde{z}_n$$

- invert $z^{\text{gap}} \rightarrow \tilde{z}$? but then price pressure not gap in own market!
also.... there is an invertibility problem (skip)
- not equal to excess demand! (next figure...)





Monetary Policy

$$z_t = Z(p_t)$$

$$\hat{Q}_t = Q_t e^{\rho t} = 1$$

$$\bar{\mu} = 1$$

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M matrix = Monetary Policy $\rightarrow$ Rank 1!

Proposition. [Monetary Policy $\rightarrow$ Determinacy]

Exists ω and $\phi > 1$

$\hat{q}_t = -\phi \sum_n \omega_n p_{nt} \rightarrow N - 1$ roots real and stable

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($N-1$ eigenvalues, just as in classical ad hoc Tattonement analyses)
- **Extra:** no convergence by cycles! No complex eigenvalues → “almost” monotone
- What about ω ...?
 - ▶ it is the left eigenvector of the targeted eigenvalue; no other eigenvalues are changed!
 - ▶ adds up to 1, but may have some negative components (gross substitutes gives $\omega > 0$)

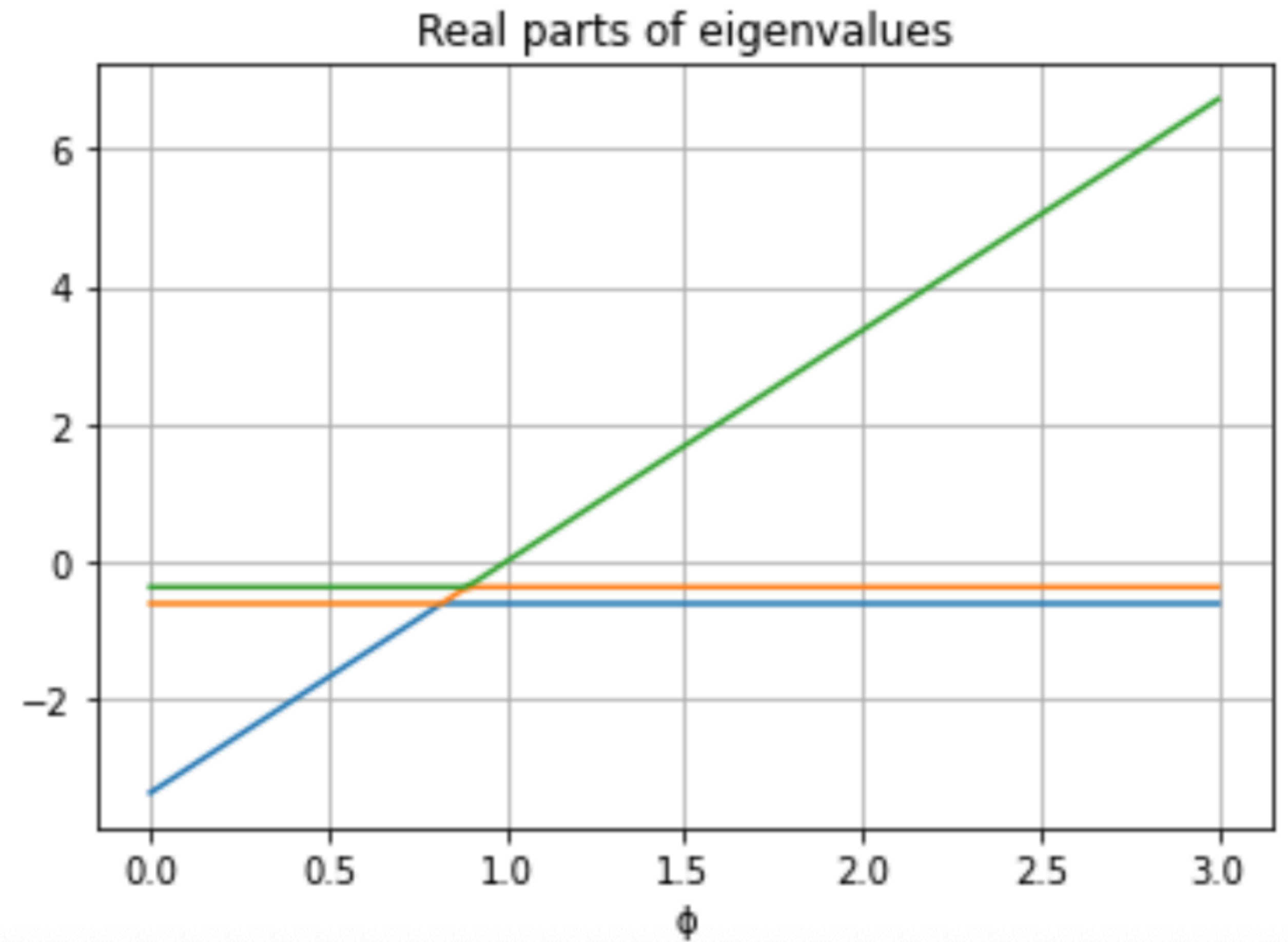
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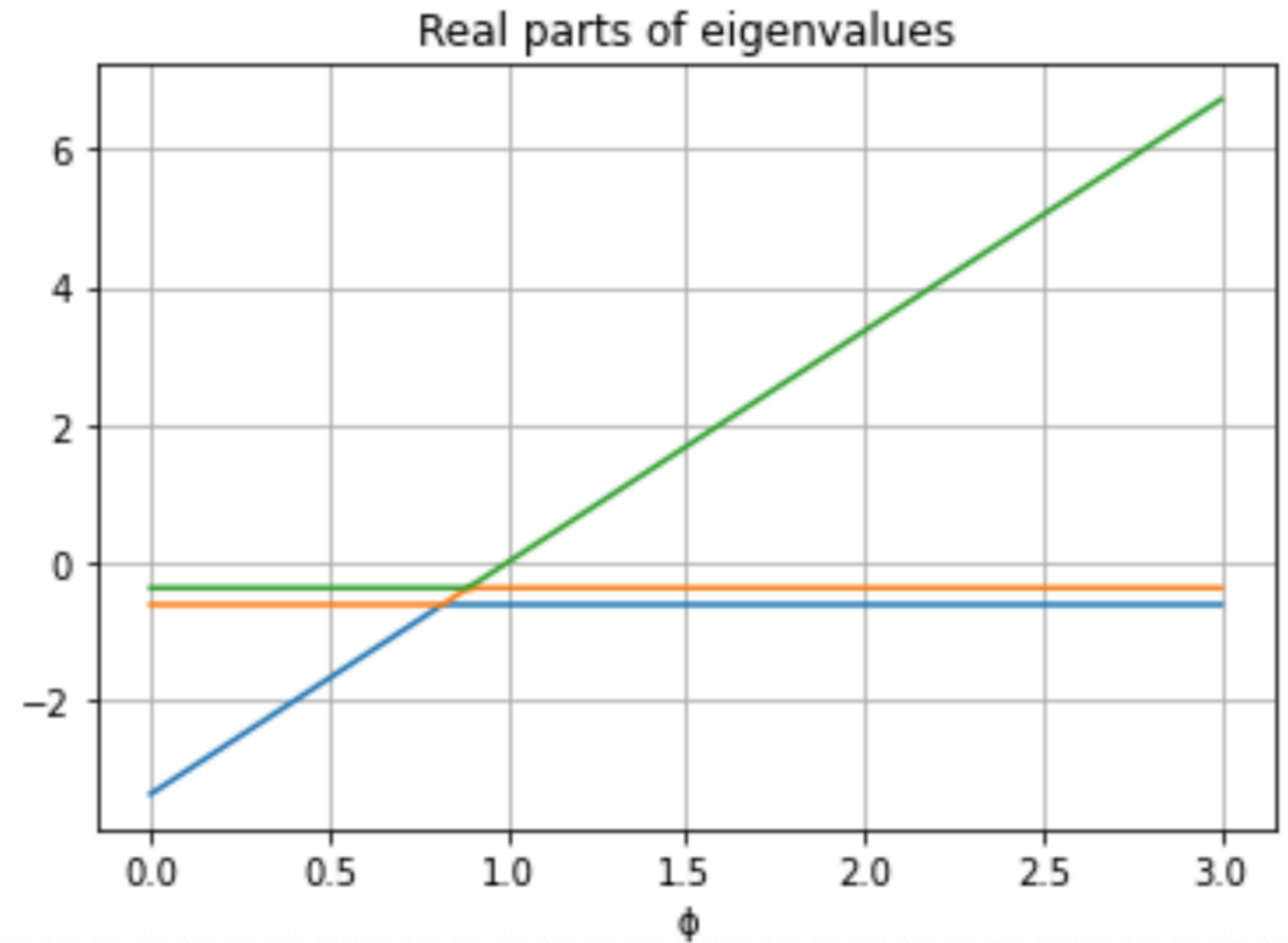
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Example: Three good economy



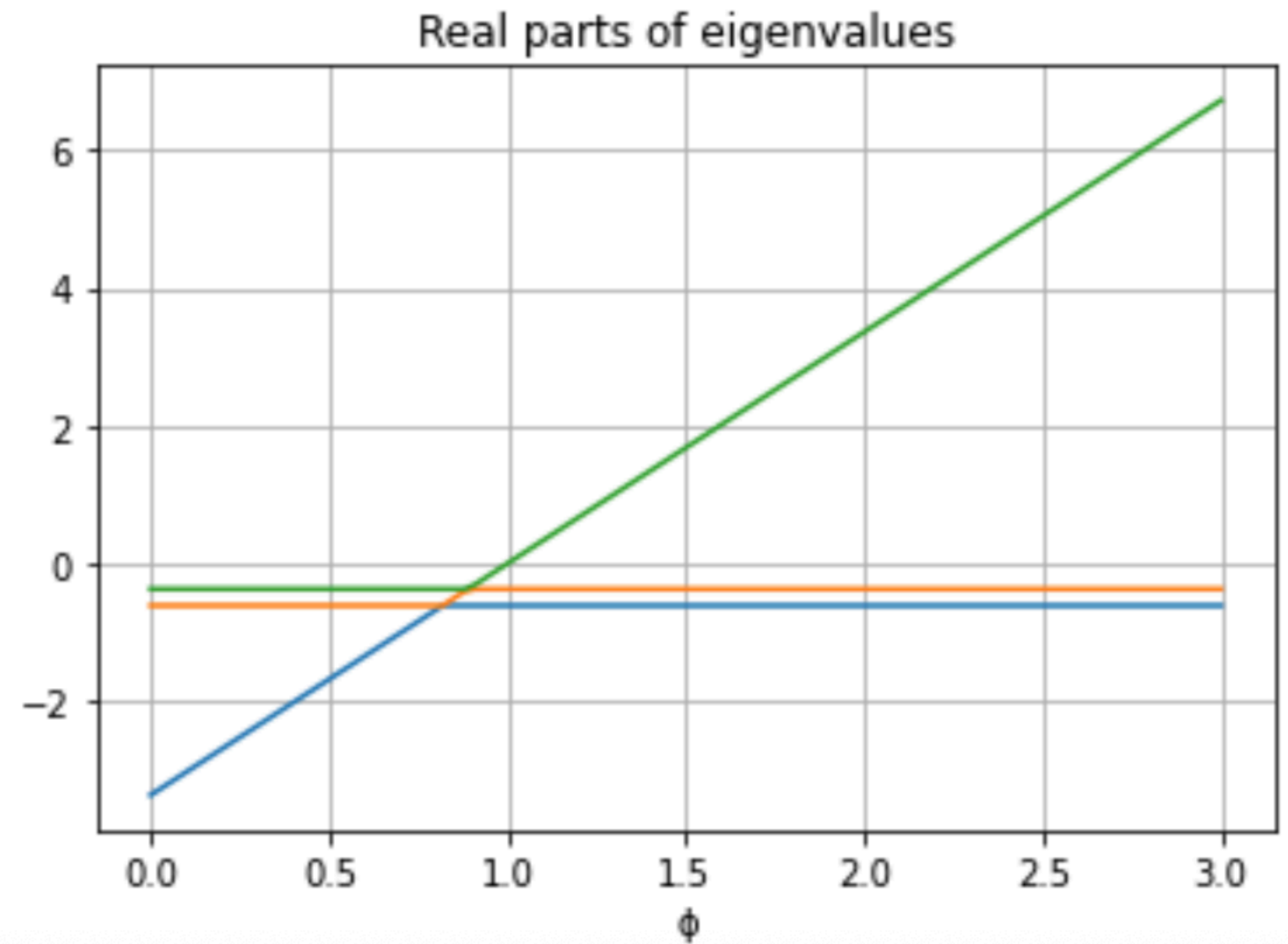
Example: Three good economy

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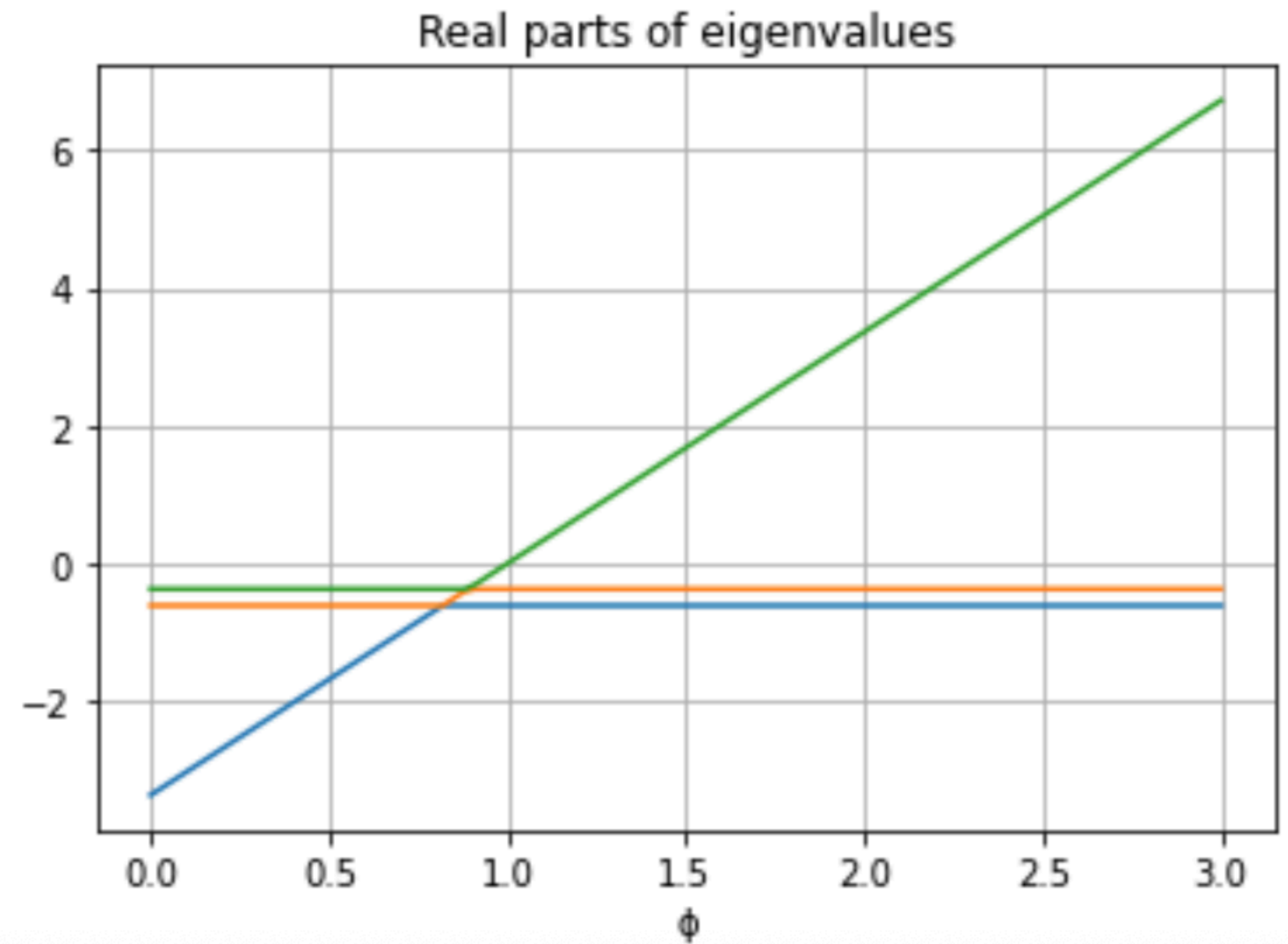
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- Taylor rule with weights = left eigenvector
- At $\phi = 0$: three real and negative eigenvalues



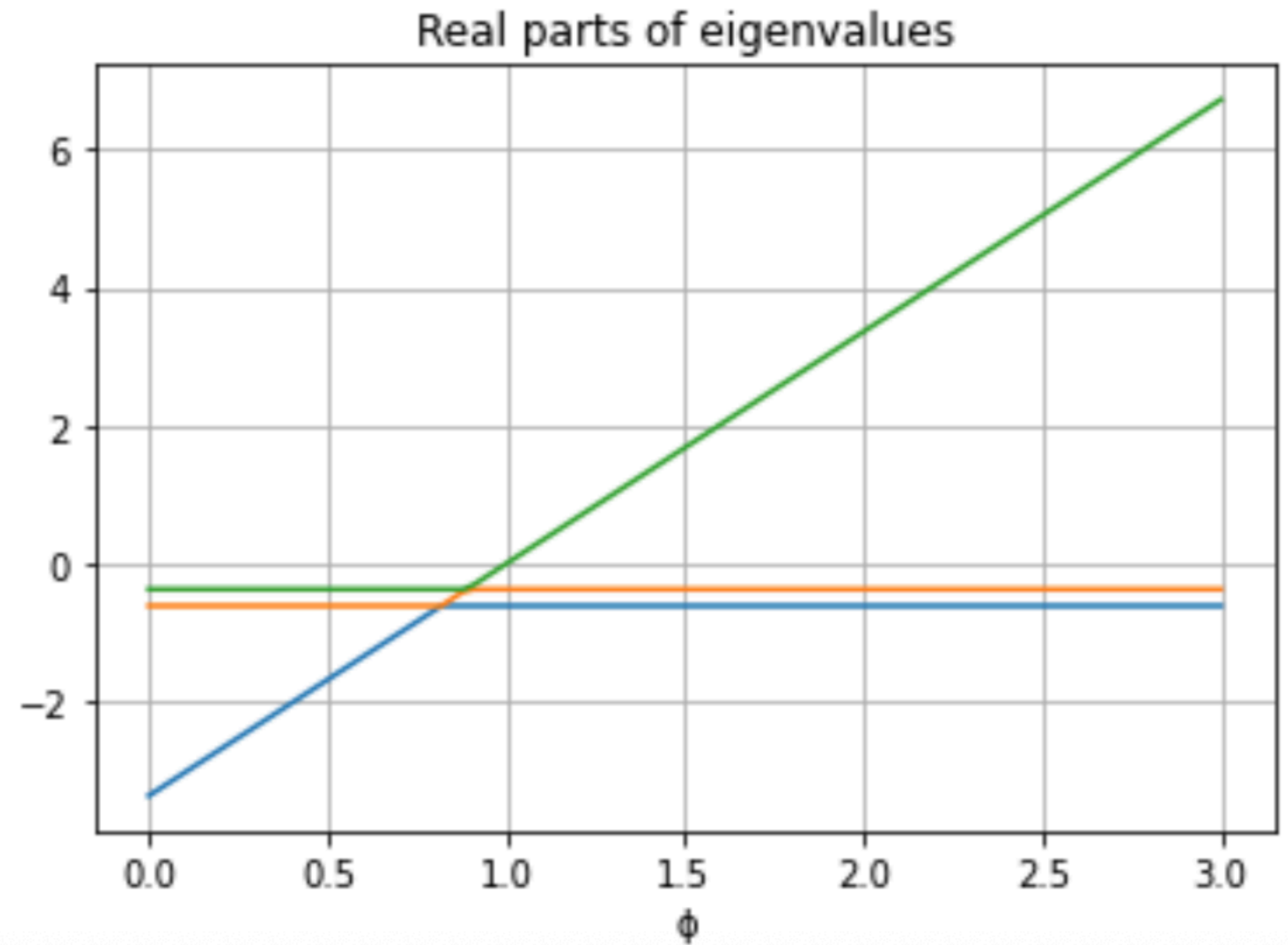
Example: Three good economy

- Taylor rule with weights = left eigenvector
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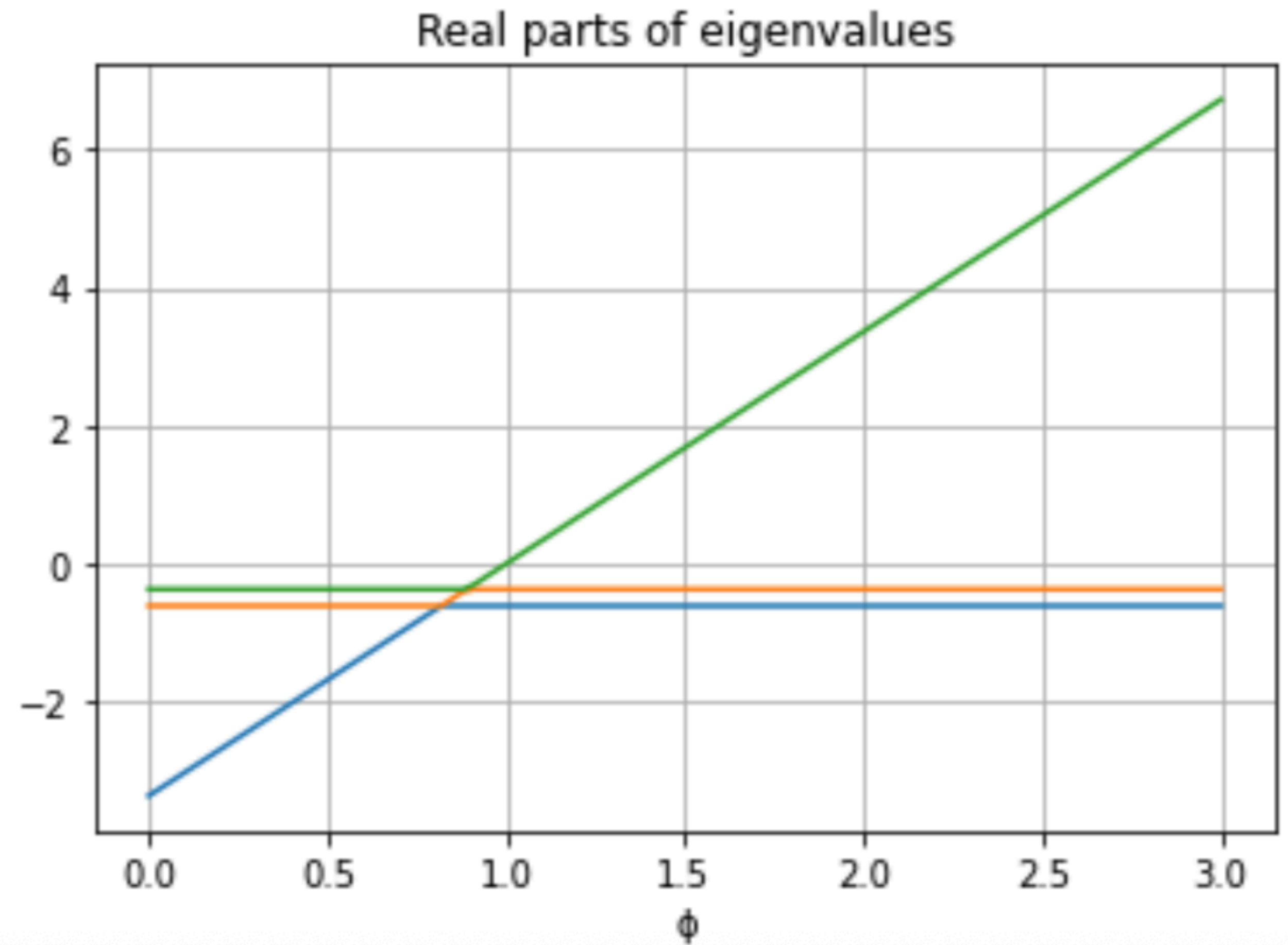
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 - two eigenvalues remain unchanged

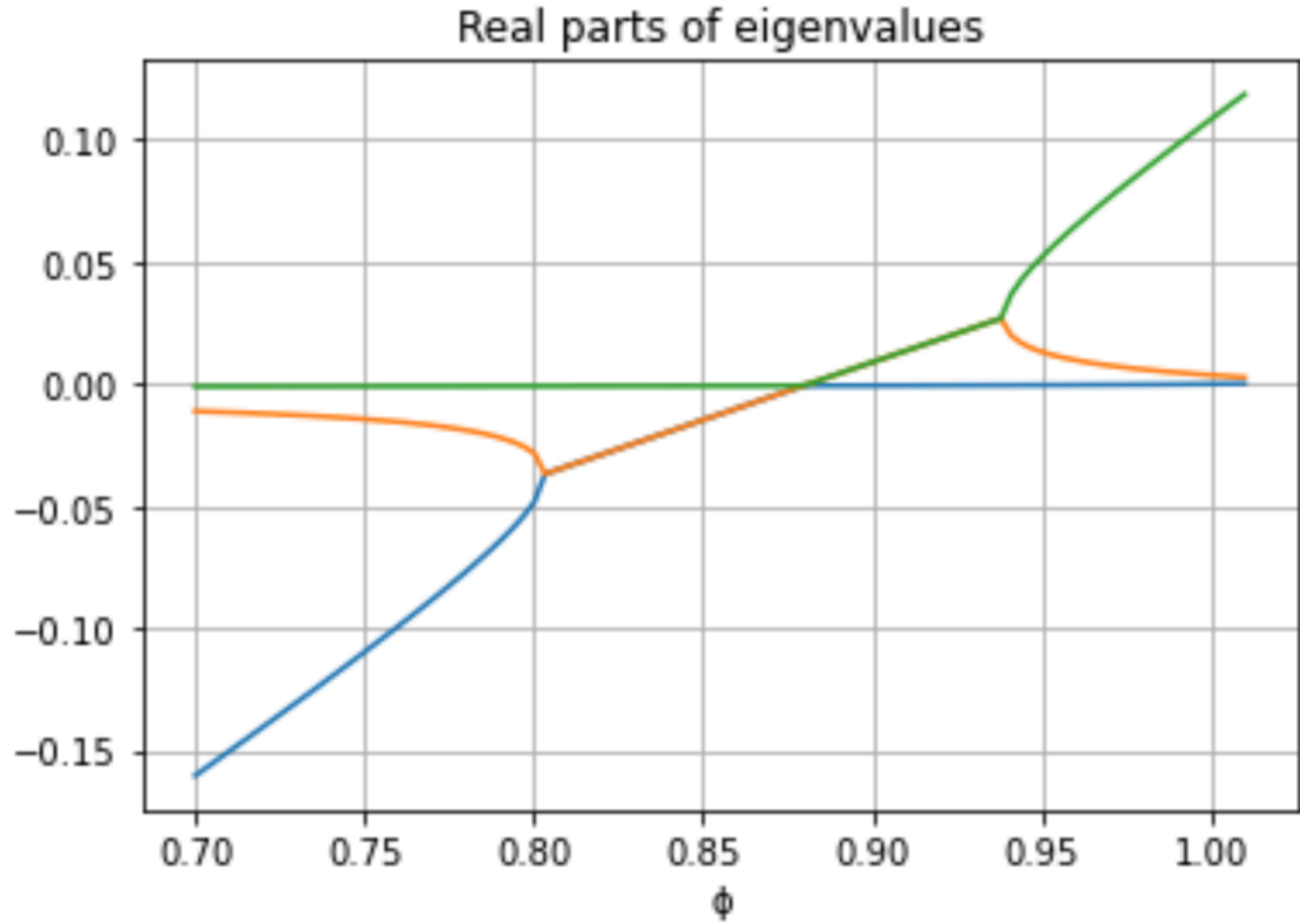


Example: Three good economy

- Taylor rule with weights = left eigenvector
- At $\phi = 0$: three real and negative eigenvalues
- As ϕ rises...
 - ▶ two eigenvalues remain unchanged
 - ▶ other rises and crosses 0 at $\phi = 1$! 😎

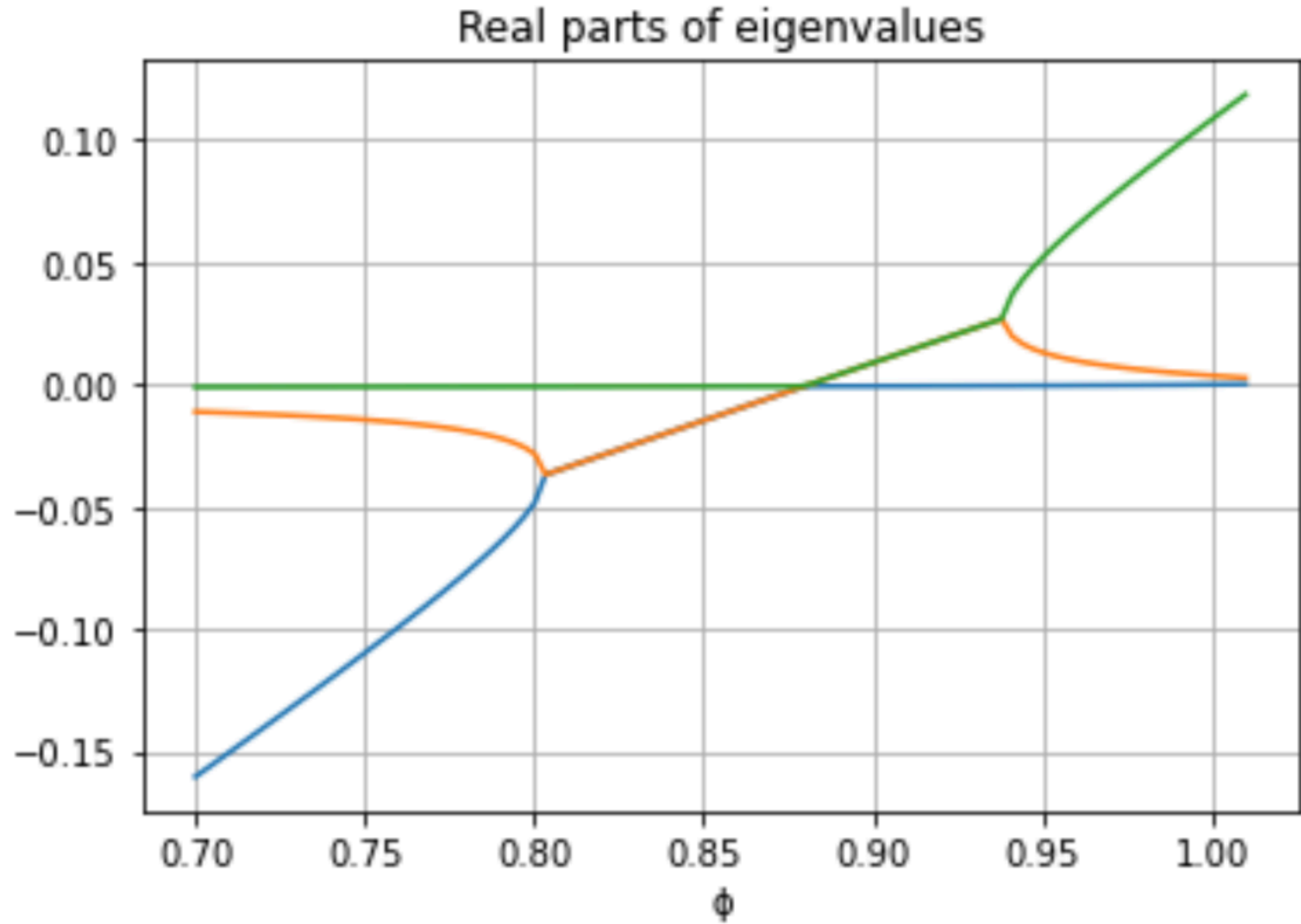


Three good economy example



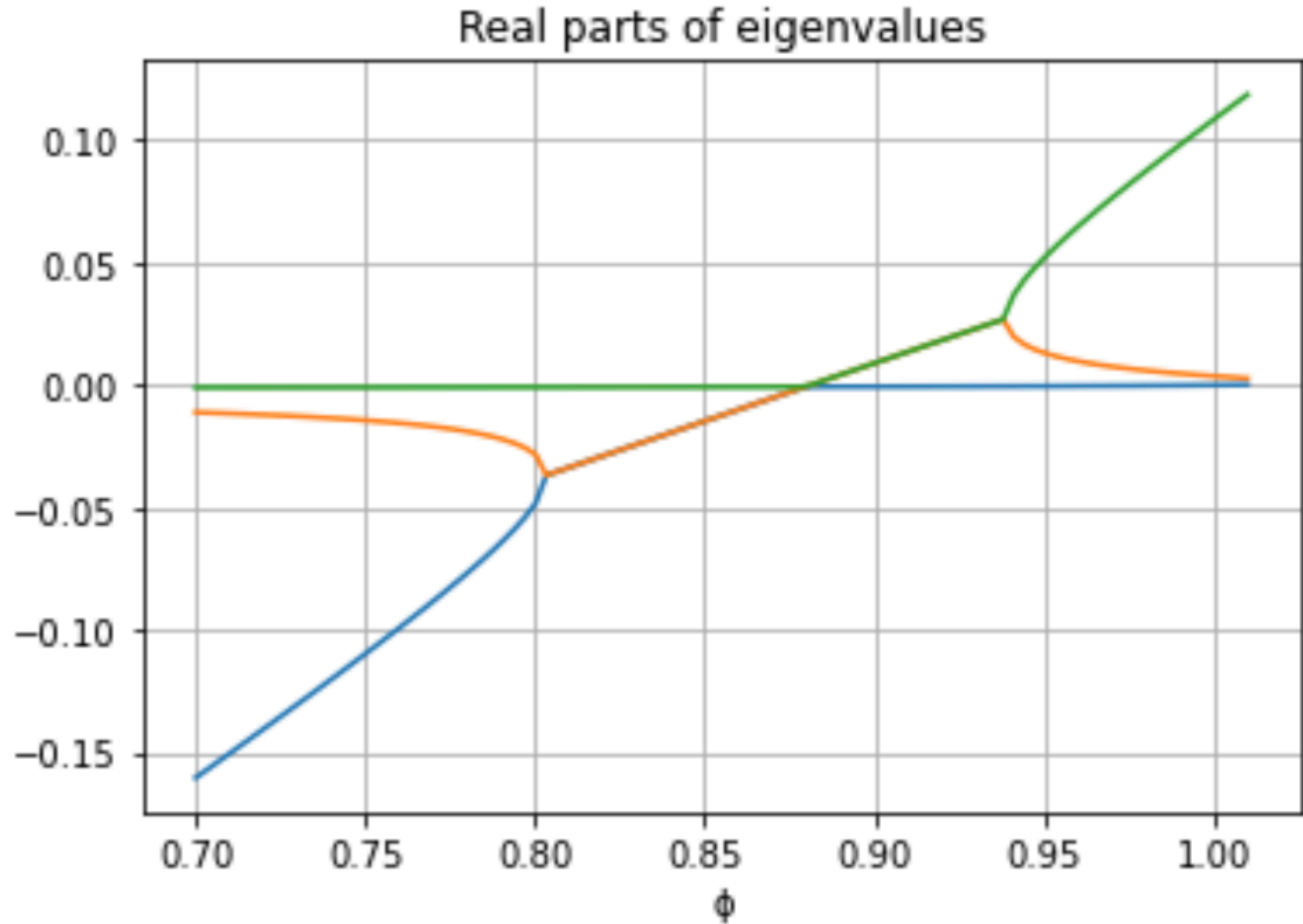
Three good economy example

- Taylor rule with weights $\neq$ left eigenvector



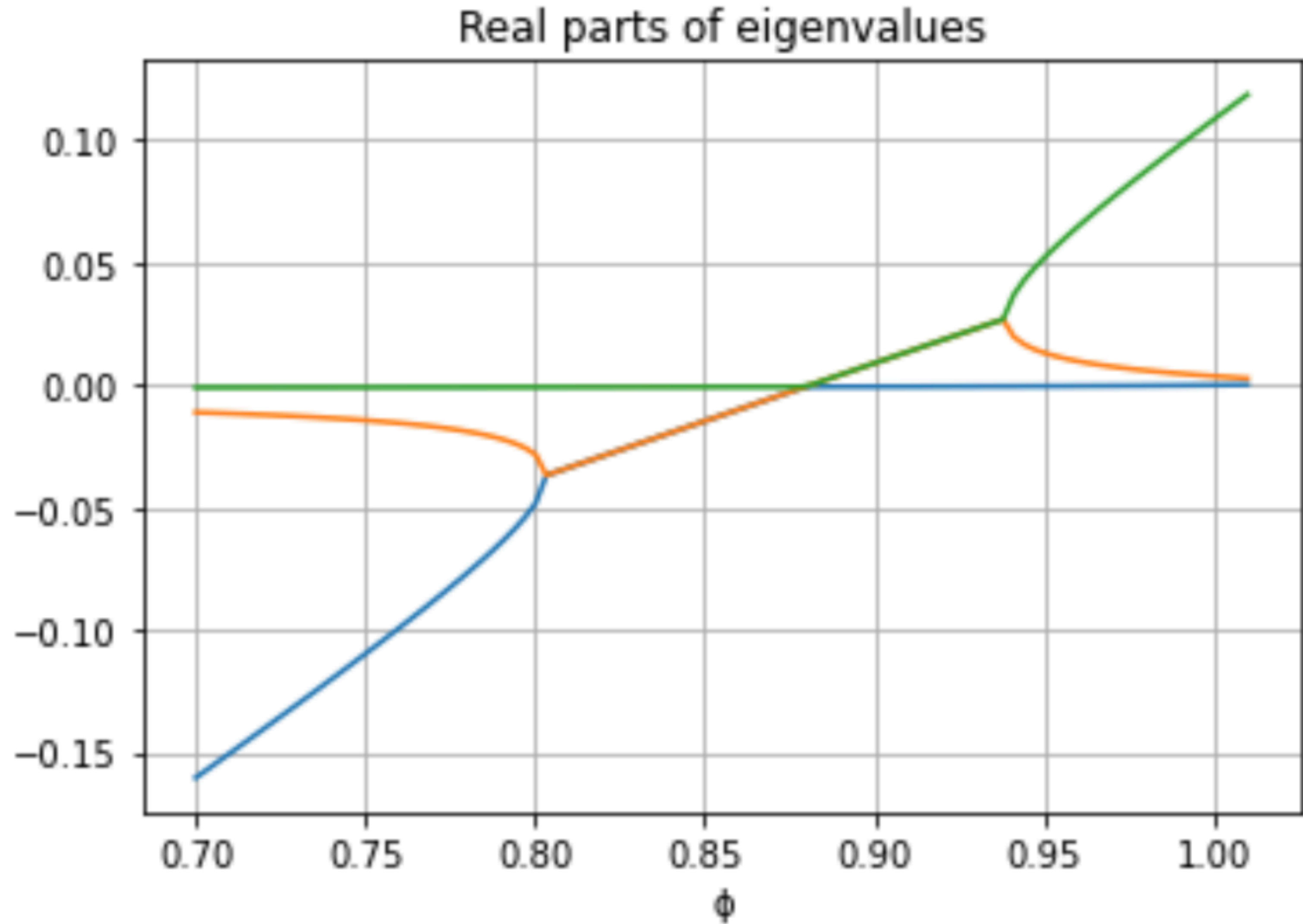
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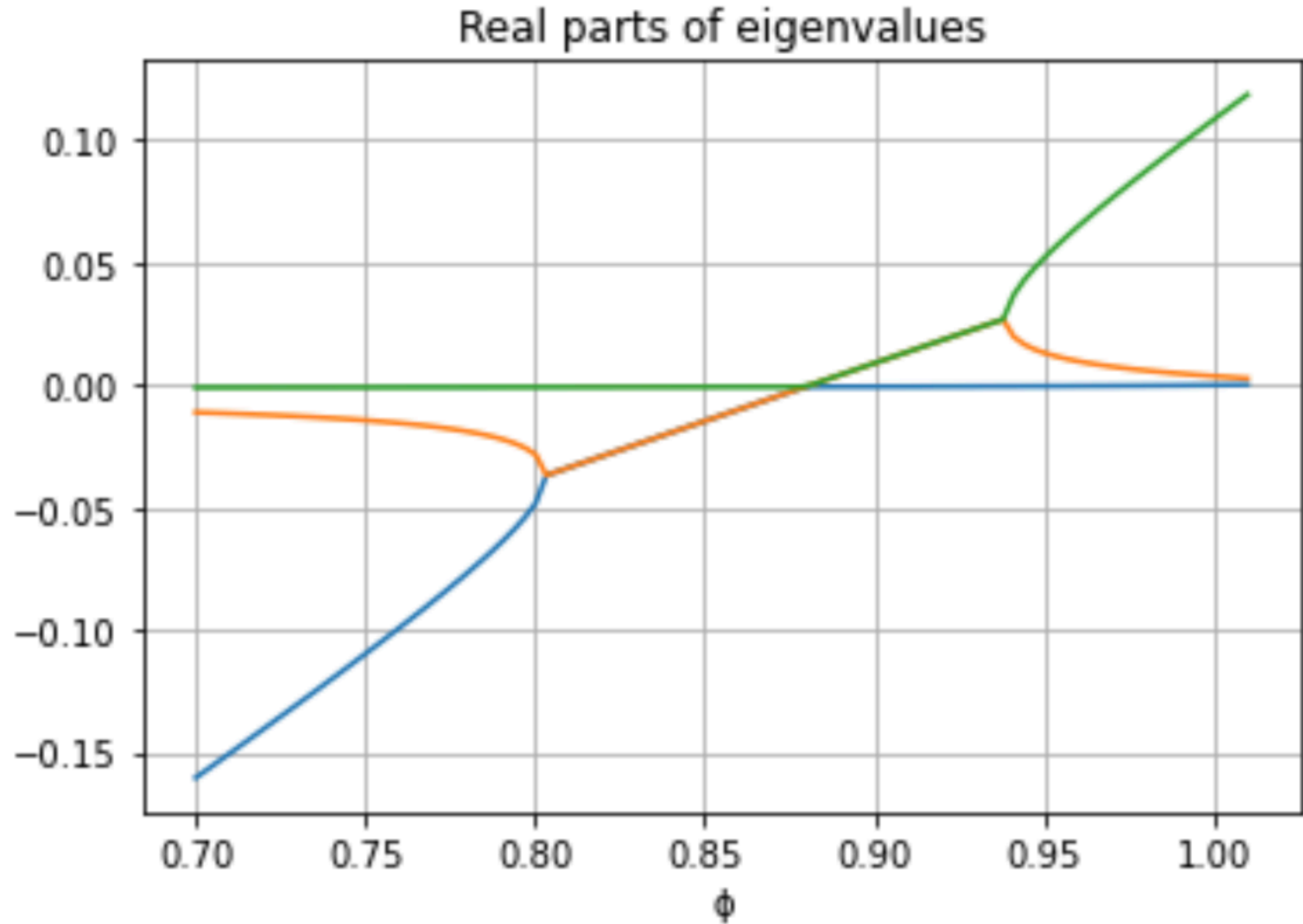
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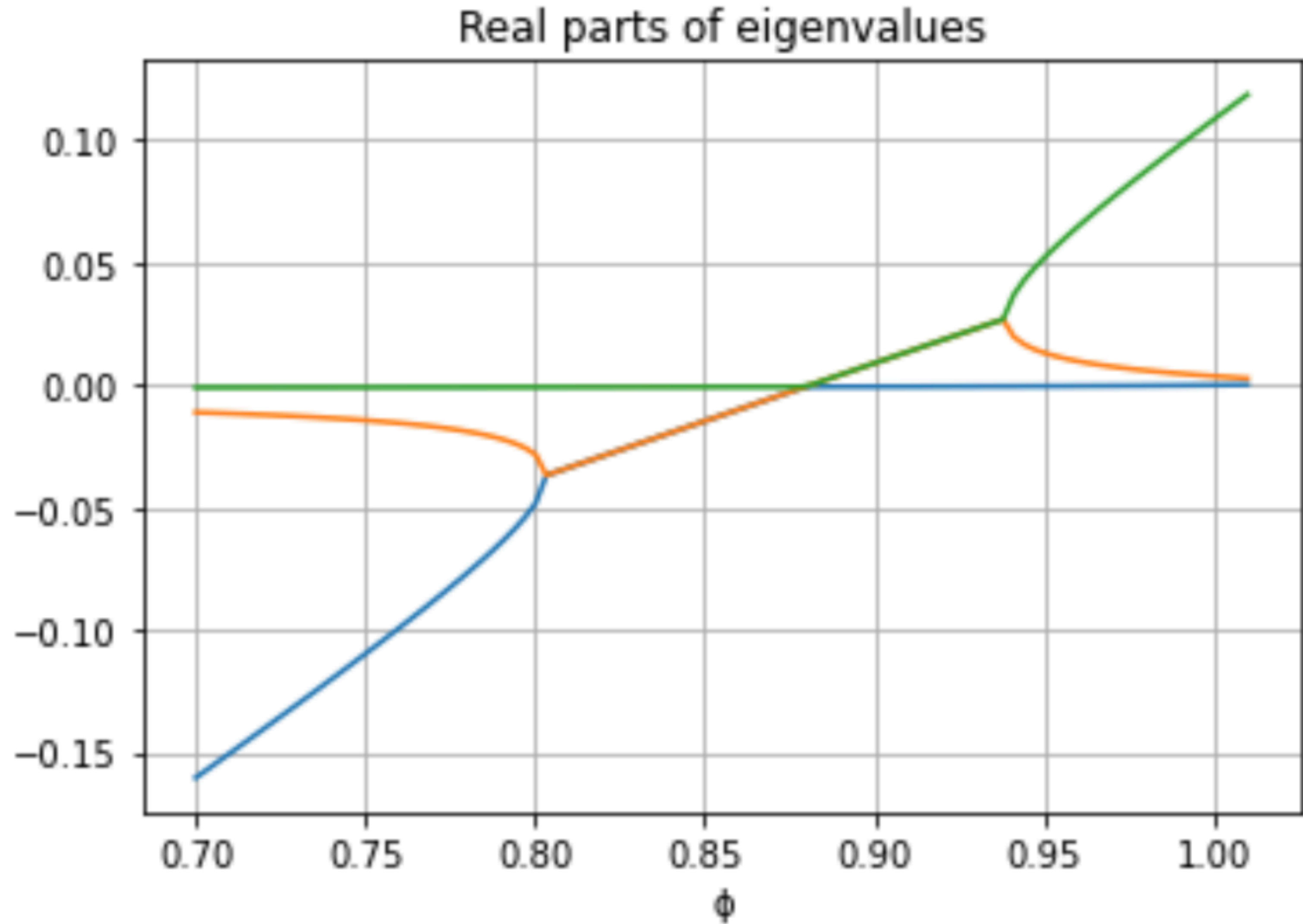
Three good economy example

- Taylor rule with weights $\neq$ left eigenvector
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- As ϕ rises:
 - all eigenvalues affected



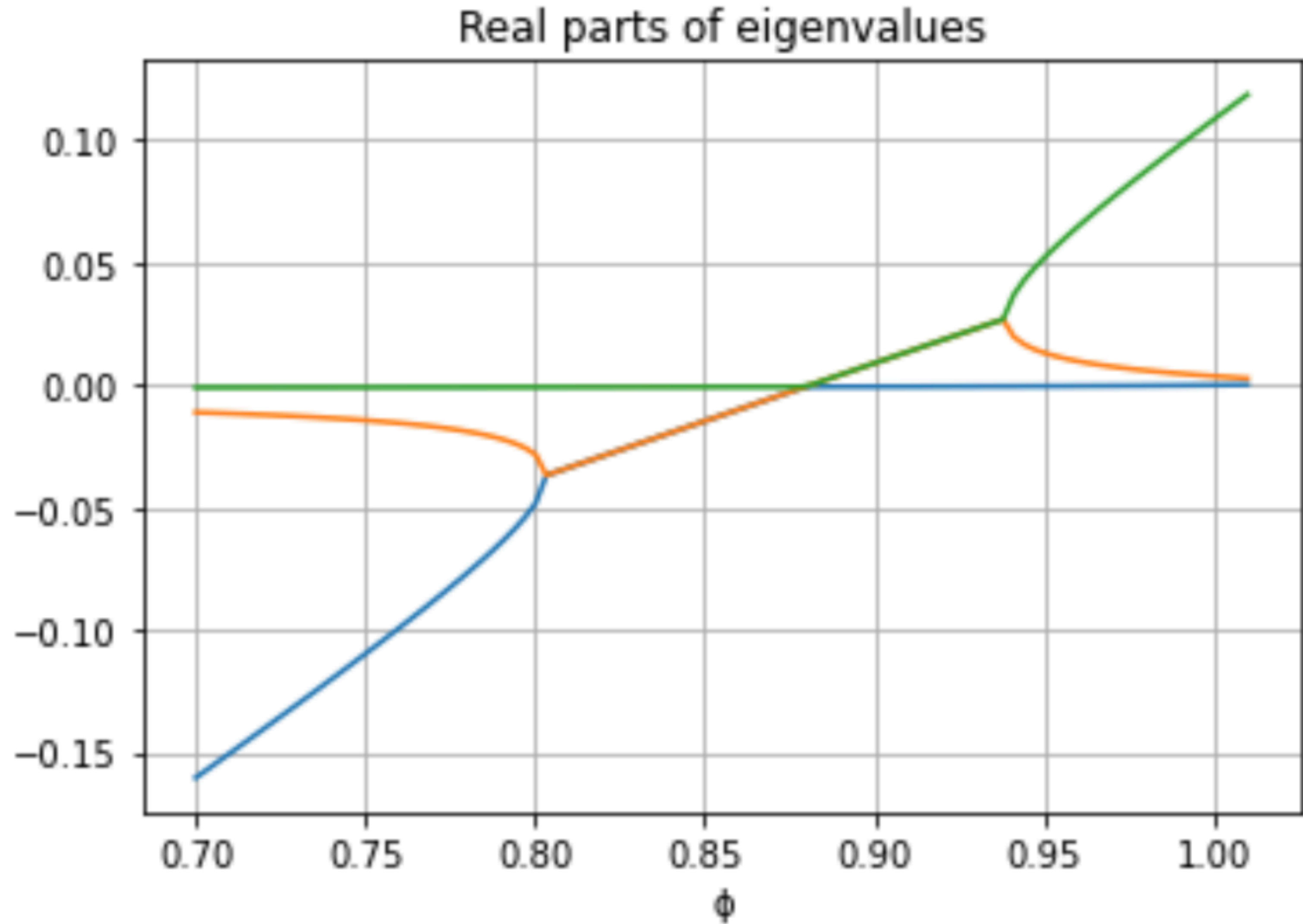
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 - complex conjugate pair of eigenvalues emerge



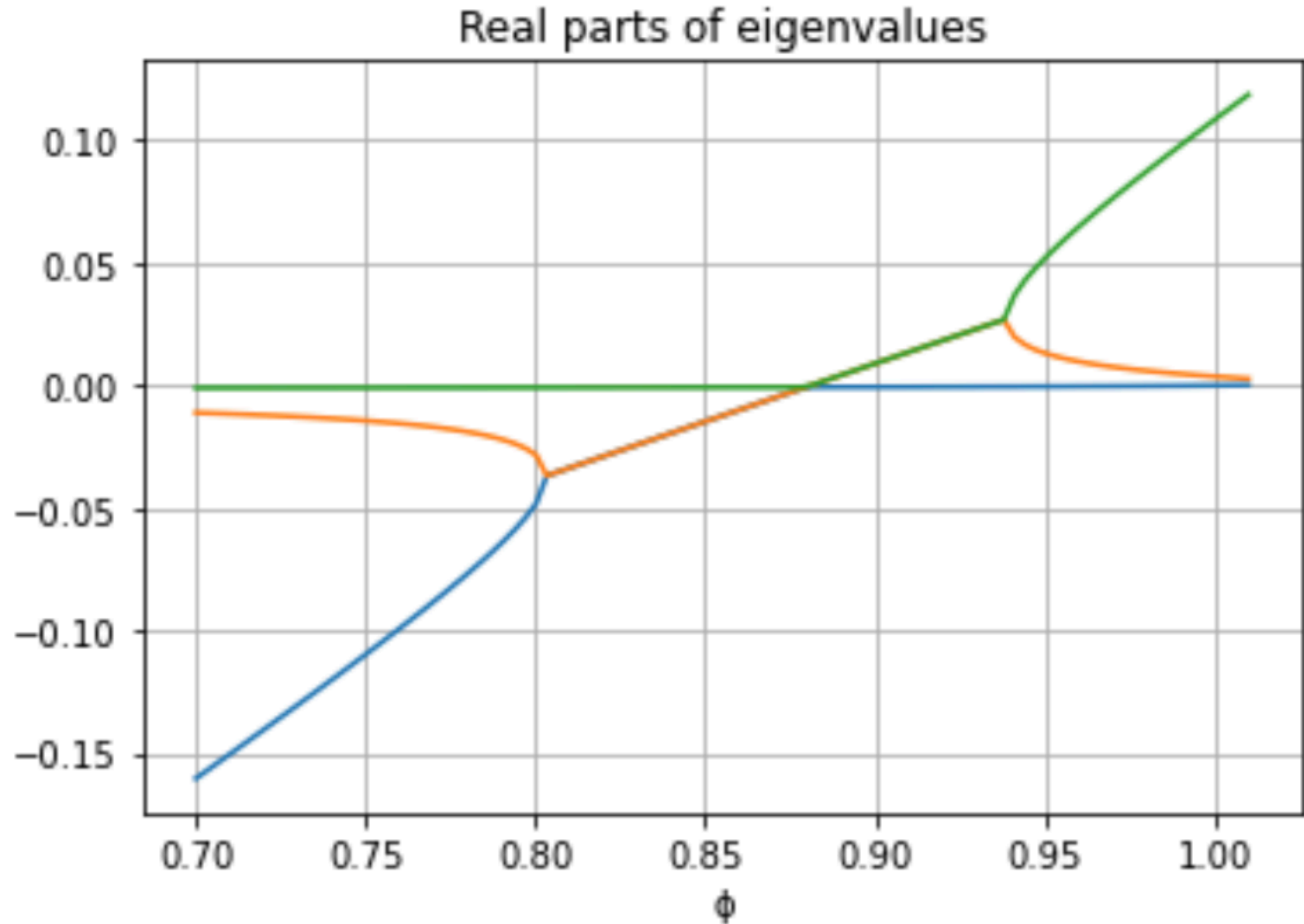
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 - ▶ common real part pair crosses 0 around $\phi = 0.857$



Three good economy example

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- At $\phi = 0$: again three real and negative eigenvalues
- As ϕ rises:
 - ▶ all eigenvalues affected
 - ▶ complex conjugate pair of eigenvalues emerge
 - ▶ common real part pair crosses 0 around $\phi = 0.857$
- No ϕ with two negative and one positive eigenvalues! 😓



■ Monetary Policy $Q_t[p_t]$ time dependent

► for $t \in [0, T]$ $\rightarrow Q_t[]$ flat

► for $t > T$ $\rightarrow Q_t[]$ Taylor Rule (determinacy)

■ Solve backwards...

$$\pi_t = Z_t(p_t) + \beta \mathbb{E} \pi_{t+1}$$

$$(\pi_t = DB(I - A)^{-1} SM_t p_t + \beta \mathbb{E} \pi_{t+1})$$

Demand Shocks

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$$z_t = Z(p_t; \theta_t)$$

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e.g. sectoral reallocation Guerrieri-Lorenzoni-Straub-Werning (2020)

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$$\tilde{z}_t = Z^{**}(p_t) + g_t + A\tilde{z}_t$$

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Generalized
Keynesian
Cross

Aggregation

■ Aggregation...

▶ standard analysis → special assumptions (homothetic + identical preferences + CRS)

state of the art: Rubbo (2023)

▶ our approach → general + immediate!

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Endogenous “aggregate output gap”!

Extensions

Extensions

- Non-Ricardian consumers...
 - ▶ Borrowing constraints...
 - ▶ TANK: fraction λ Hand-to-Mouth...

$$\pi_{n,t} = d_n \tilde{z}_t + \beta \mathbb{E} \pi_{n,t+1}$$

Extensions

- Non-Ricardian consumers...

- ▶ Borrowing constraints...

- ▶ TANK: fraction λ Hand-to-Mouth...

$$Z(p_t) = (1 - \lambda)Z^F(p_t) + \lambda Z^M(p_t)$$

- ▶ HANK

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$$\tilde{z}_t = Z_t(p_t, \text{wealth distribution}, Q_t, \dots)$$

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Excess Demand Approach Applies

Extensions

Extensions

■ Non-Calvo...

- ▶ time dependent (e.g. Taylor): vintages of past prices
- ▶ menu cost: distribution of prices
- ▶ as usual → more complicated

Extensions

■ Non-Calvo...

- ▶ time dependent (e.g. Taylor): vintages of past prices
- ▶ menu cost: distribution of prices
- ▶ as usual → more complicated

■ Excess demand approach:

- ▶ as forcing variable $\{\tilde{z}_t\}$
- ▶ linearized approximations

Extensions

- Non-rational expectations...
 - ▶ households
 - ▶ price setters
- Excess Demand approach unchanged!
- Already did for price setters → myopic!
- Also doing some forms of learning

Conclusions

**Keynes
&
ISLM**

**General
Equilibrium**
(with Money)

Disequilibrium
(Fixed Price)

**New
Keynesian**

1930-40s

1950s & 1960s

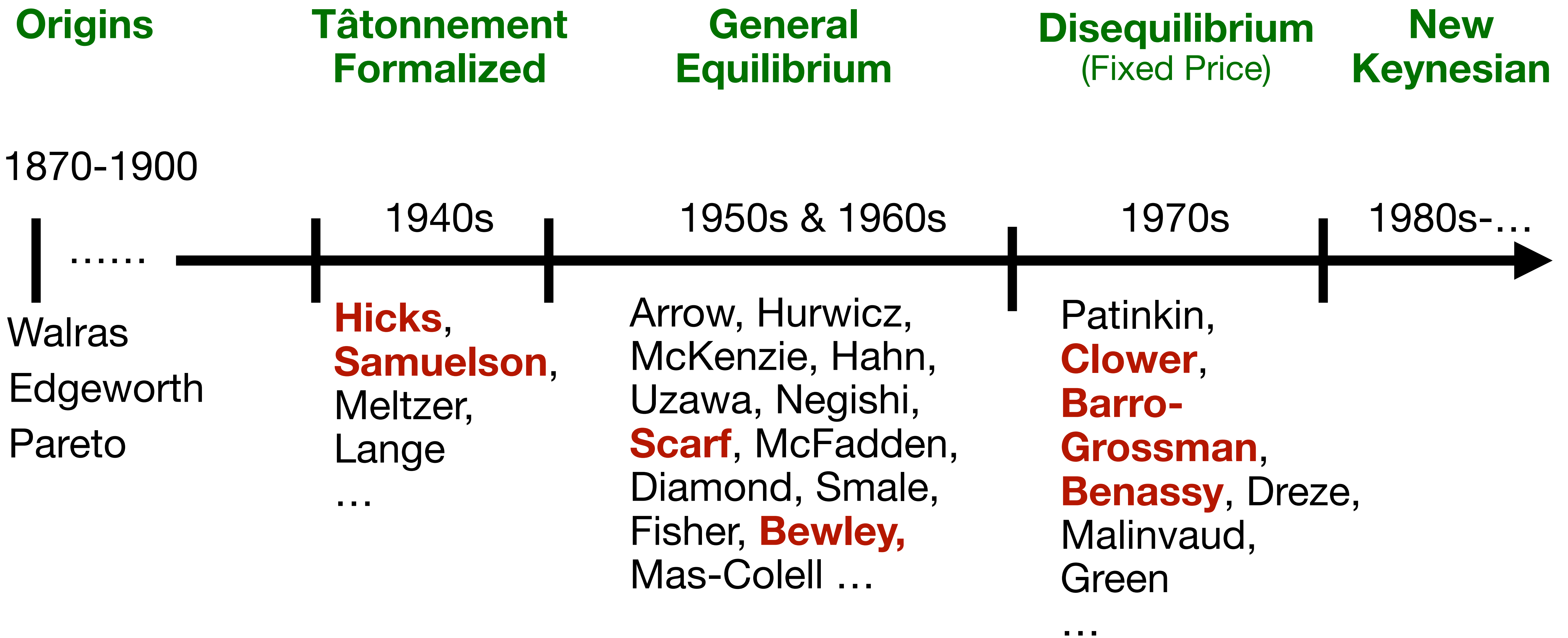
1970s

1980s-present

Keynes
Hicks

Patinkin
Clower

Patinkin,
Clower,
Barro-Grossman,
Benassy, Dreze,
Malinvaud,
Green
...



MICROECONOMIC THEORY

ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN

GE $\leftrightarrow$ Macro

Michael Woodford

INTEREST & PRICES

FOUNDATIONS OF A THEORY
OF MONETARY POLICY

MICROECONOMIC THEORY

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Michael Woodford

INTEREST
& PRICES

Part #1. “Tâtonnement and Price Setting in General Equilibrium”

$$\{\pi_{n,t}\} \leftrightarrow \{z_{n,t}\}$$

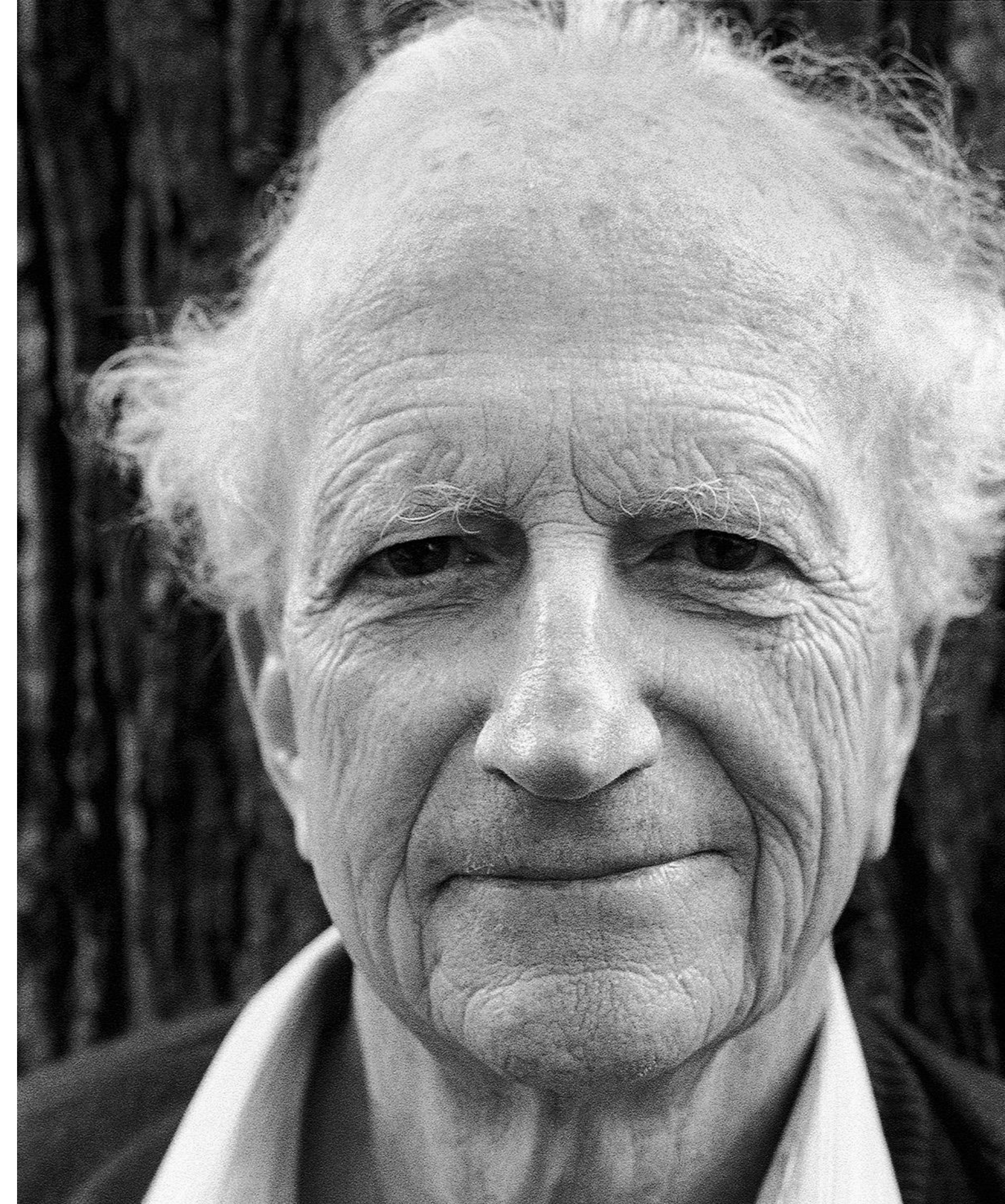
(excess demand $\neq$ output gap)

$$\pi_{n,t} = \kappa_n Z_n(p_t) + \beta \mathbb{E}_t \pi_{n,t+1}$$

more general $\rightarrow$ more intuitive!

Part #2.

“Macroeconomics with Excess Demands”



Micro 101...
“Prices Rise with
Excess Demand”



THANK
YOU!!



Appendix

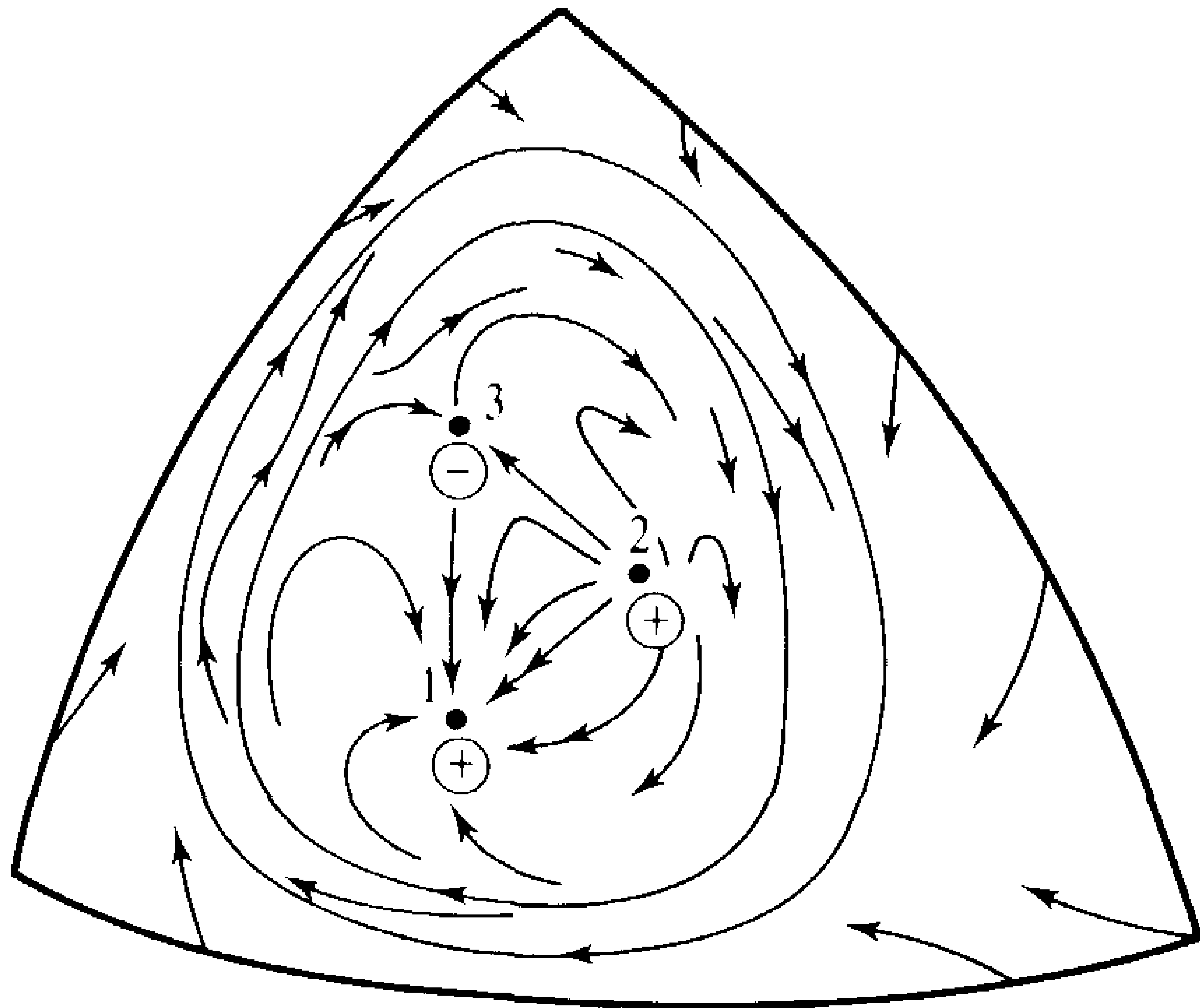
Appendix

Appendix

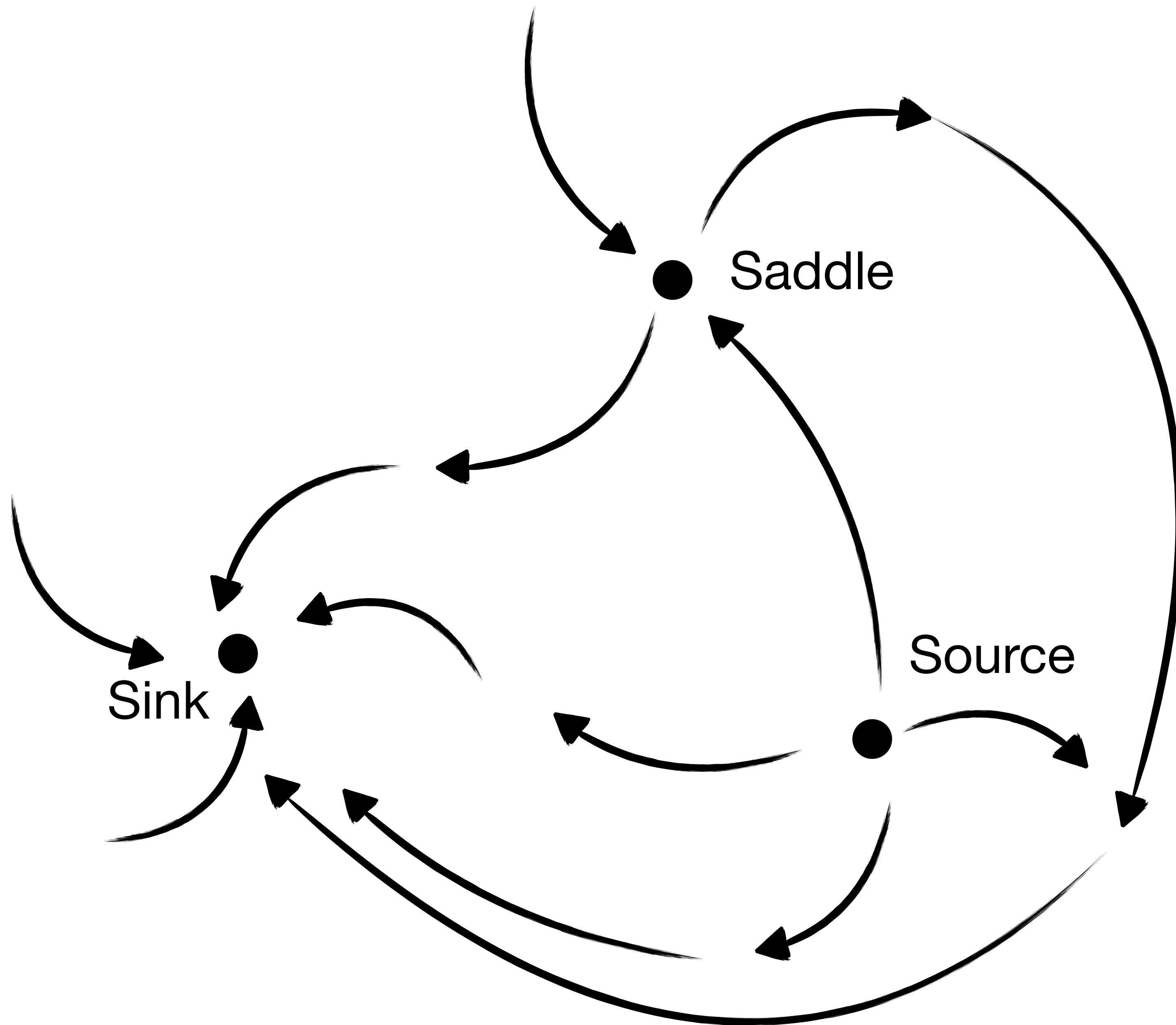
- Global and local convergence...

- ▶ Poincaré-Hopf impossibilities?

- ▶ differences in dynamical system; role of marginal utilities $\{\mu^h\}$



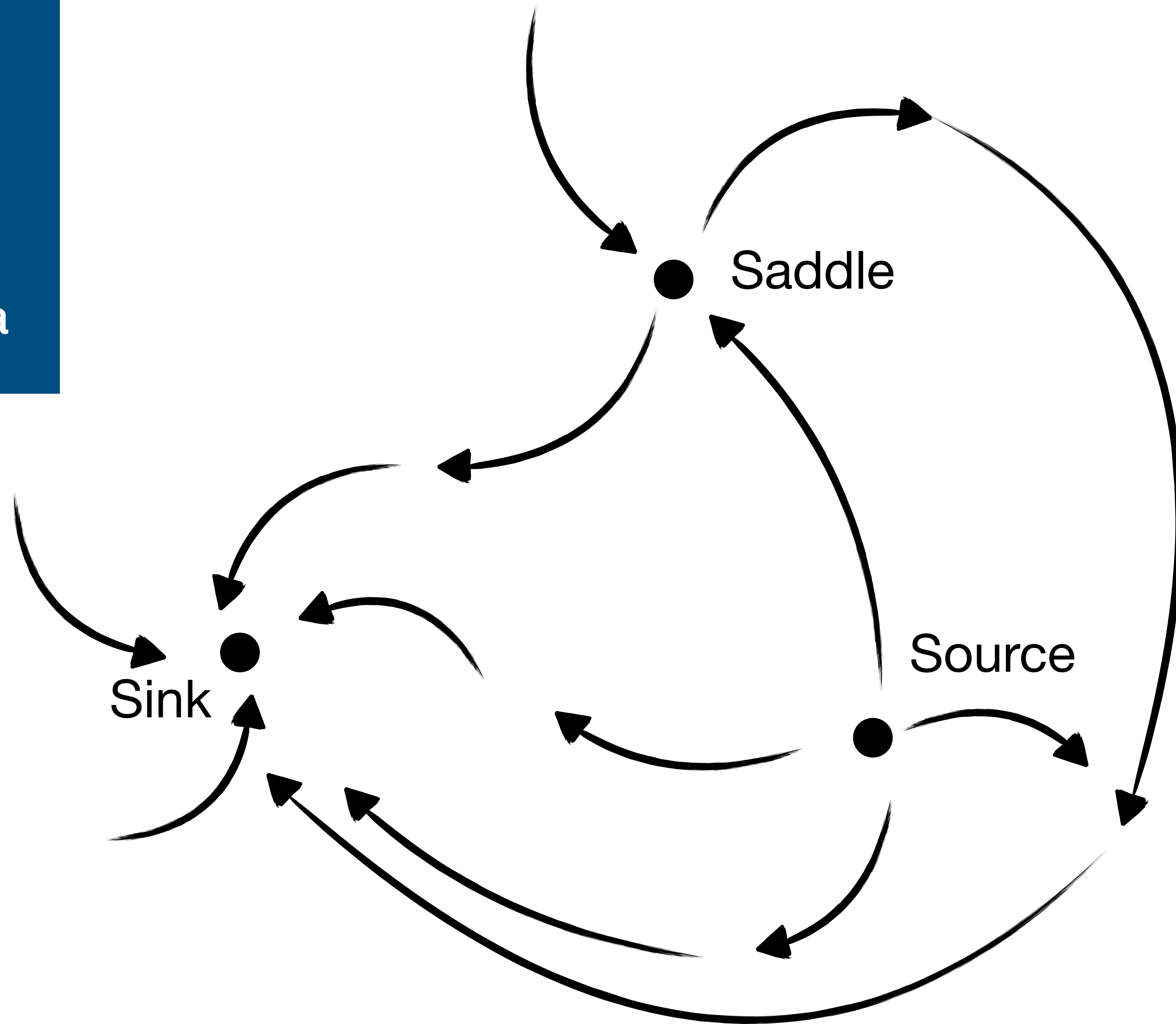
$$\dot{P}_t = F(P_t)$$



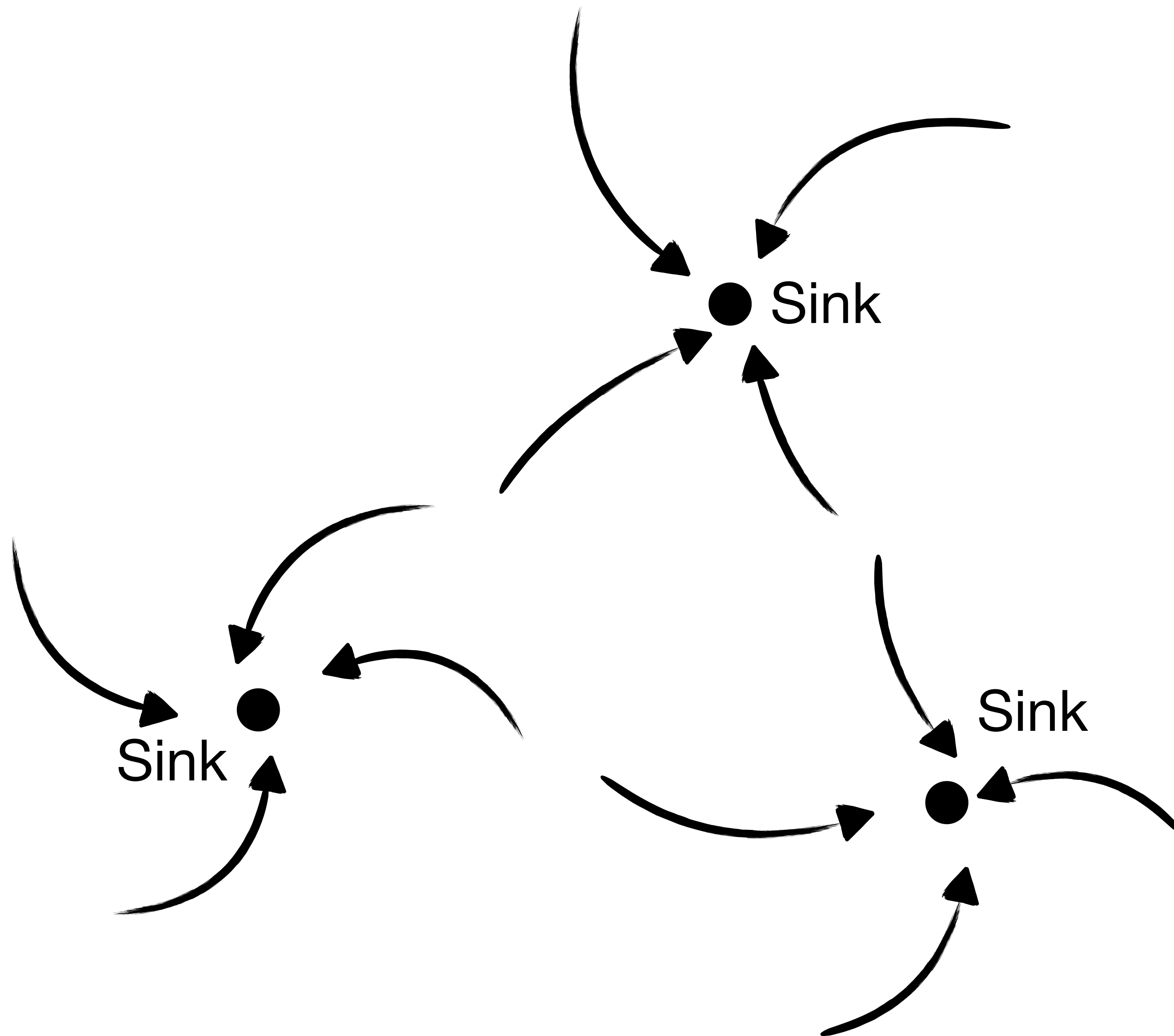
$$\dot{P}_t = F(P_t)$$

An example

Globally Stable
+
Multiple Equilibria



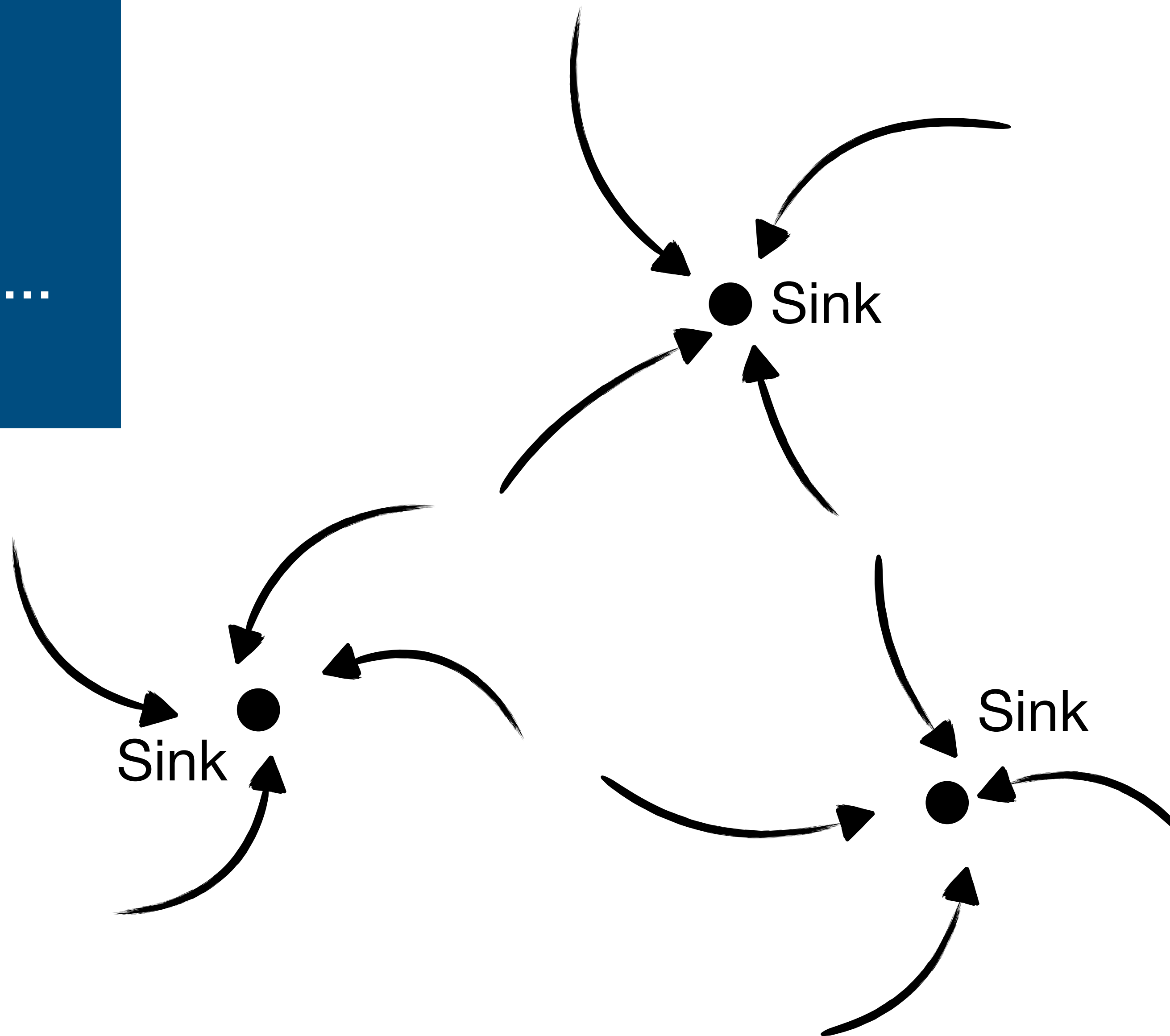
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An example (?)

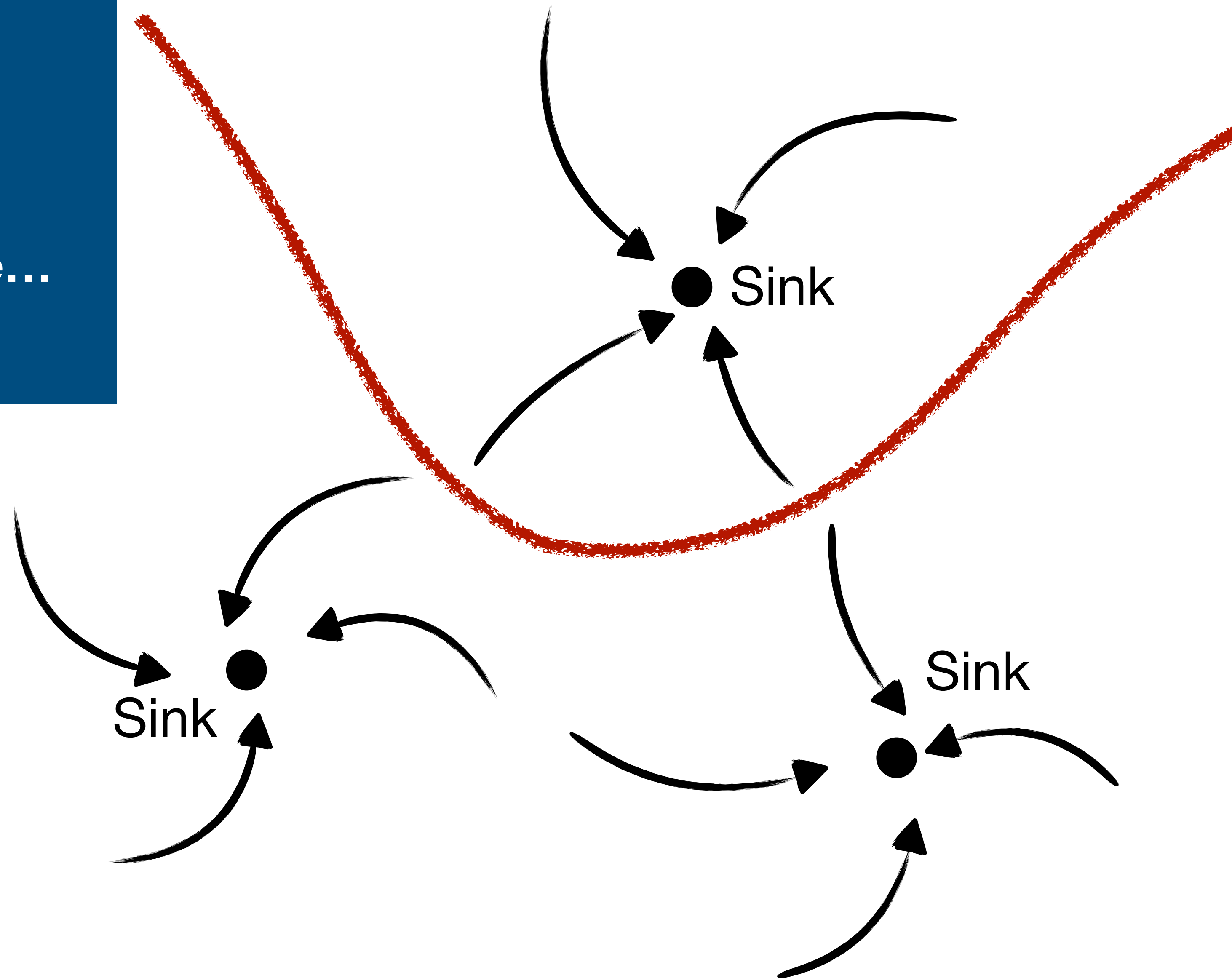
All Locally Stable...



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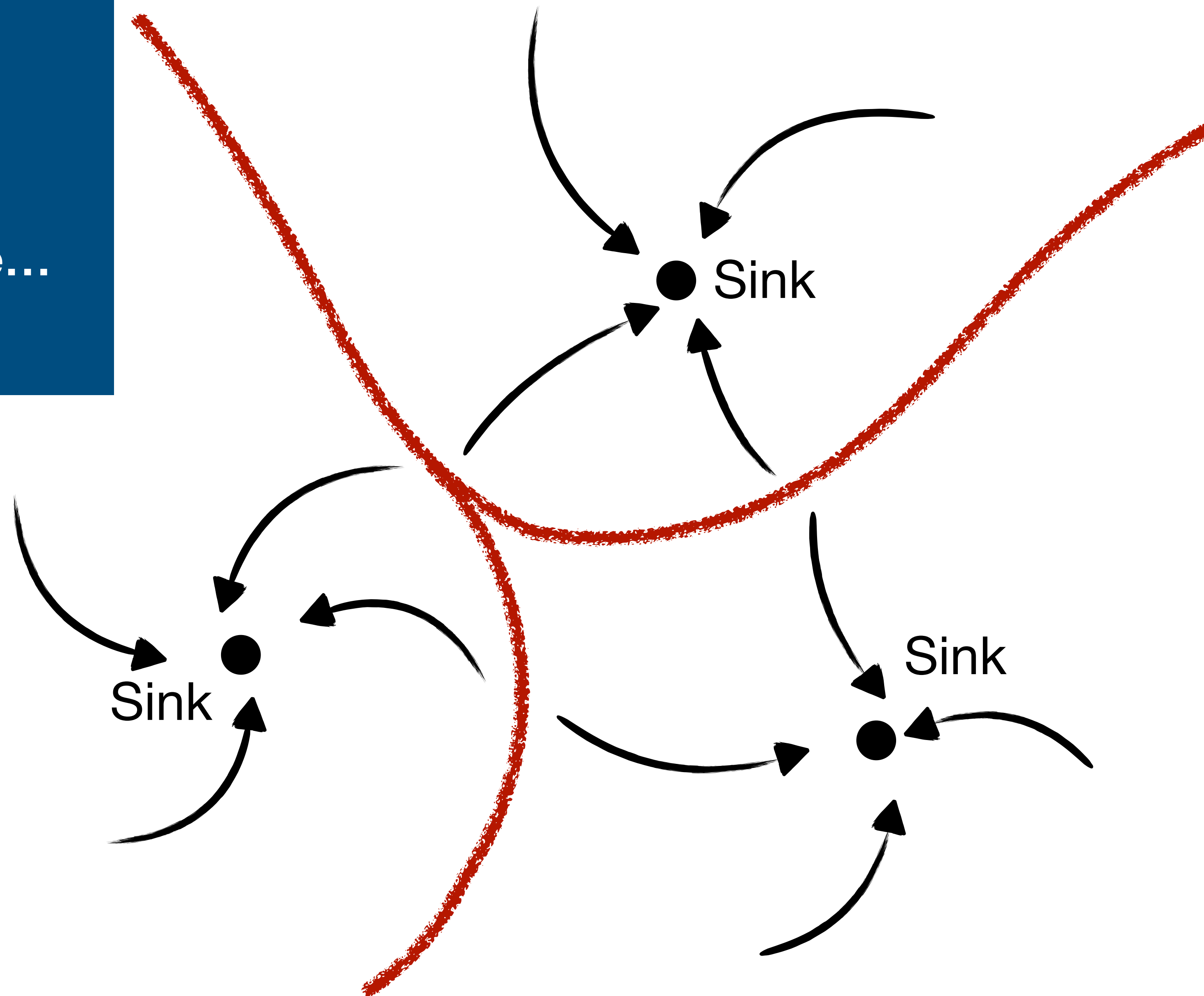
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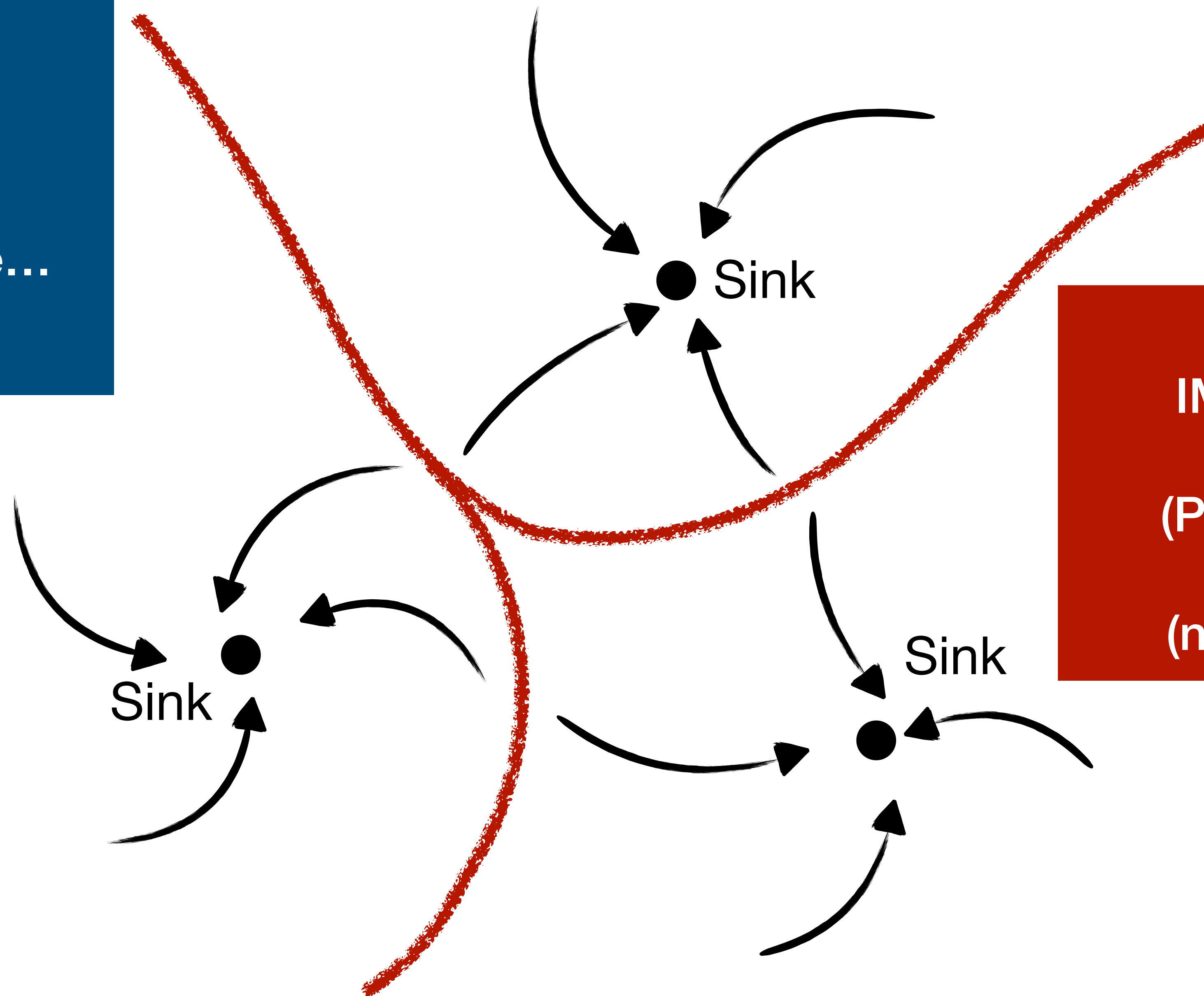
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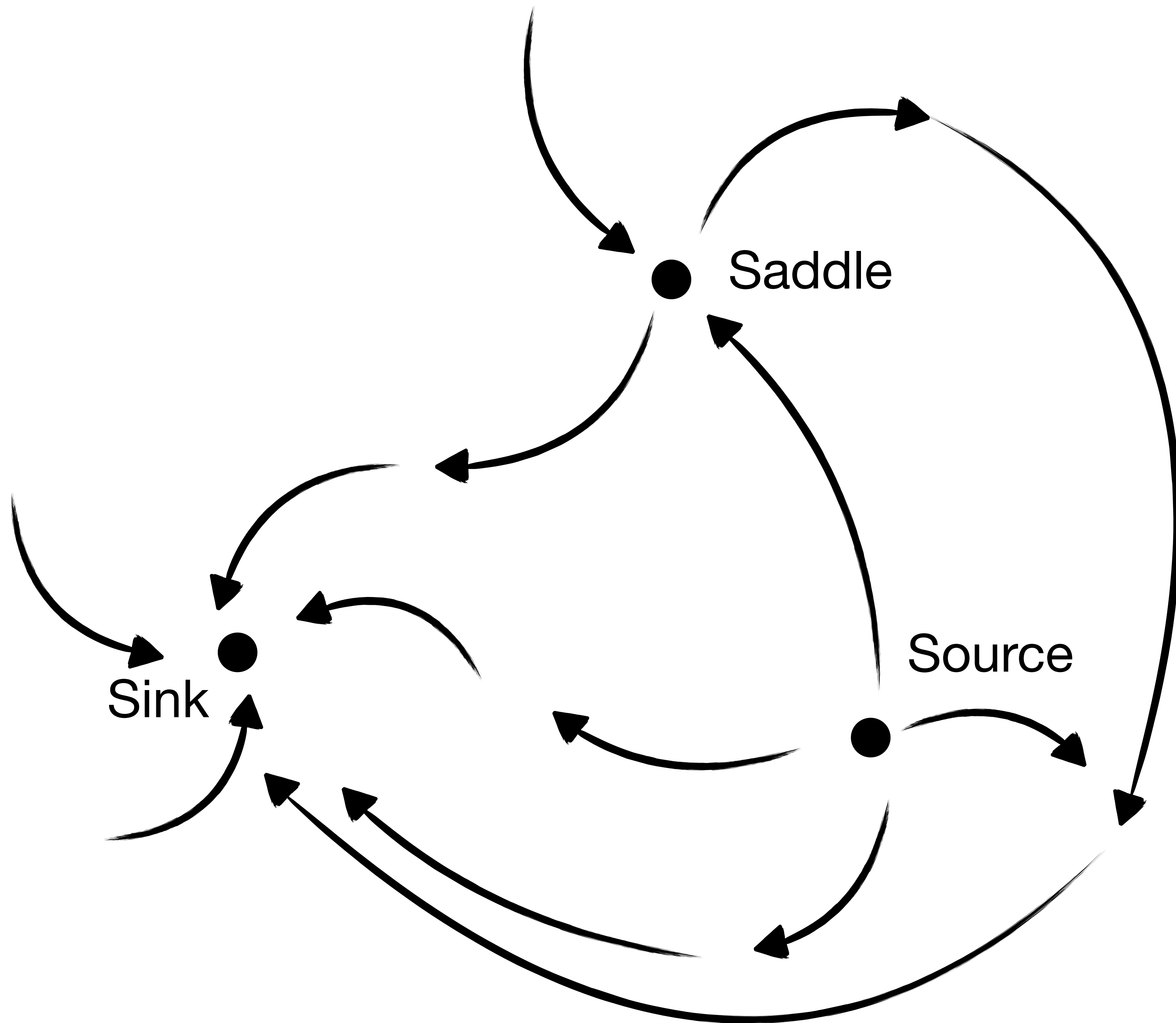
An example (?)

All Locally Stable...



... NO!!
IMPOSSIBLE!!
(Poincaré-Hopf)
(need saddles!)

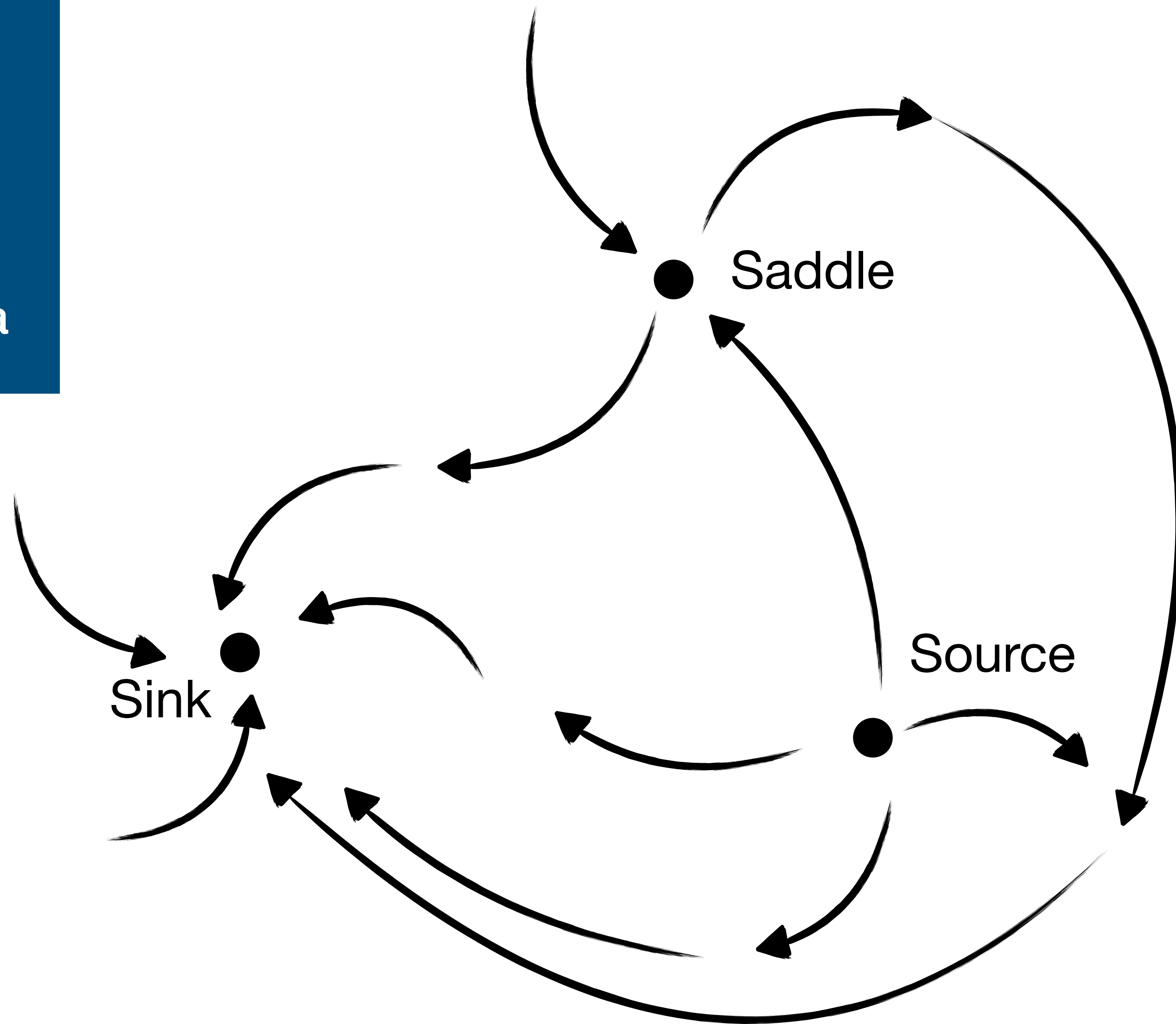
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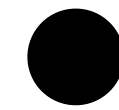
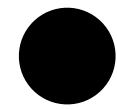
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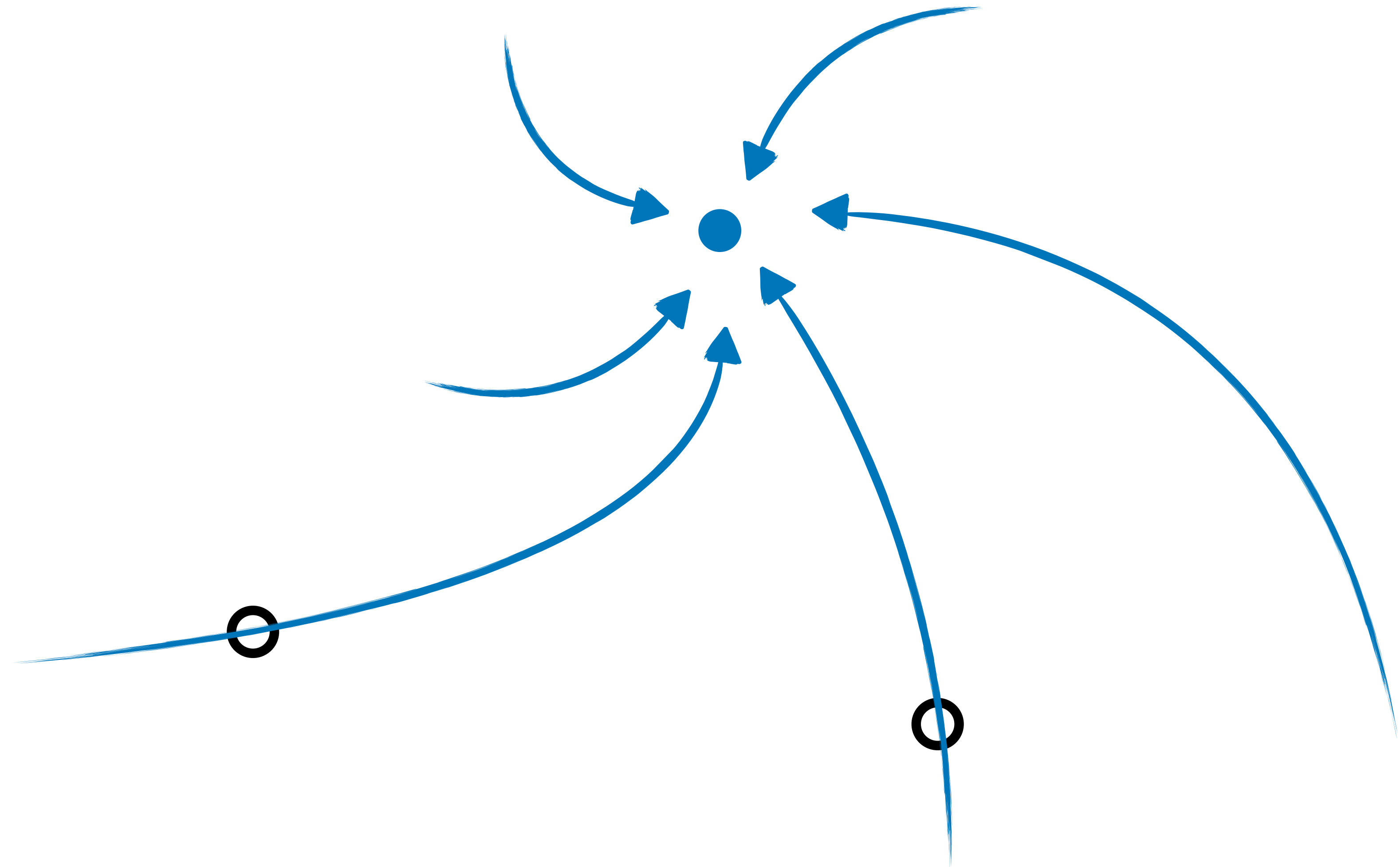
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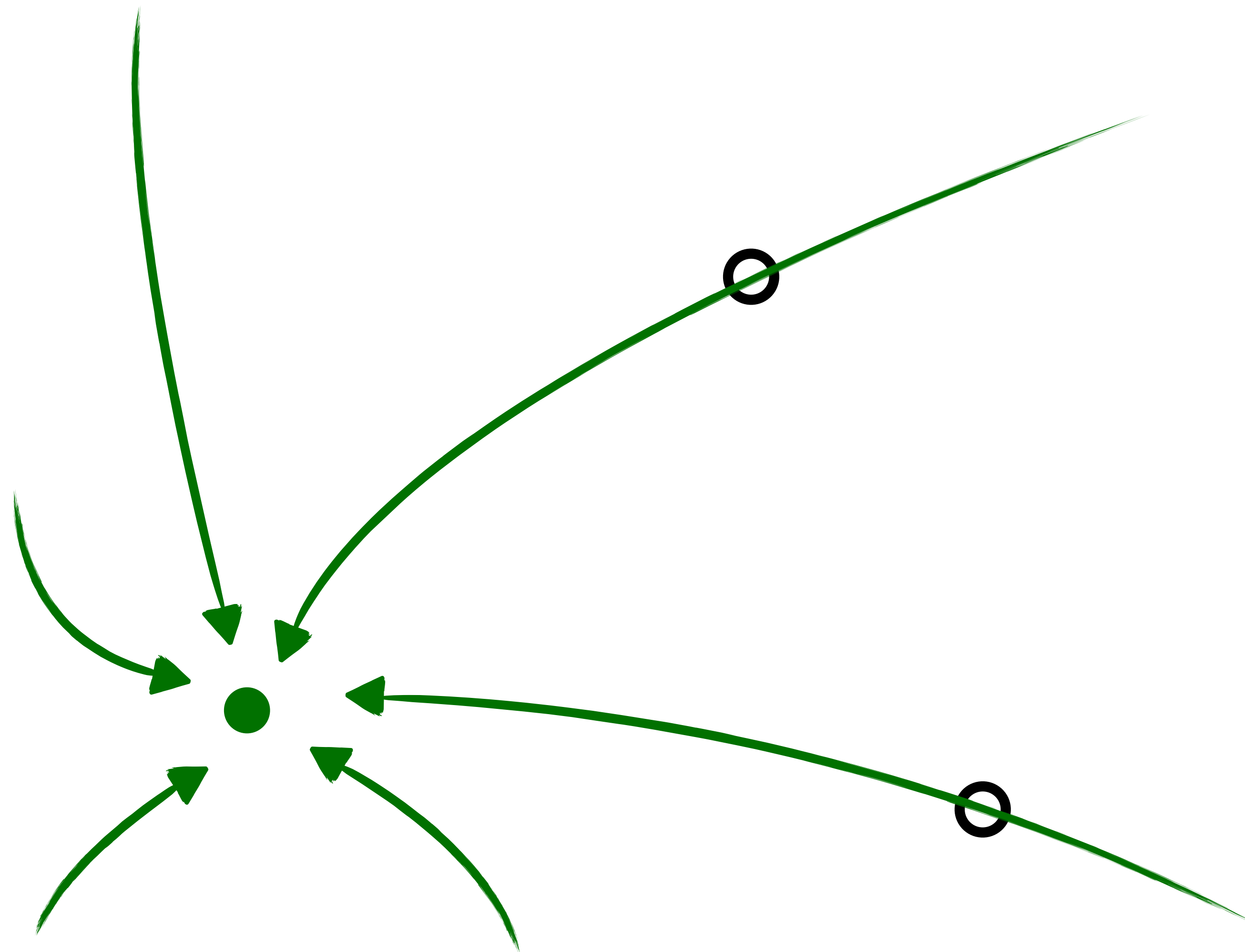
$$\dot{P}_t = F(P_t, \mu)$$



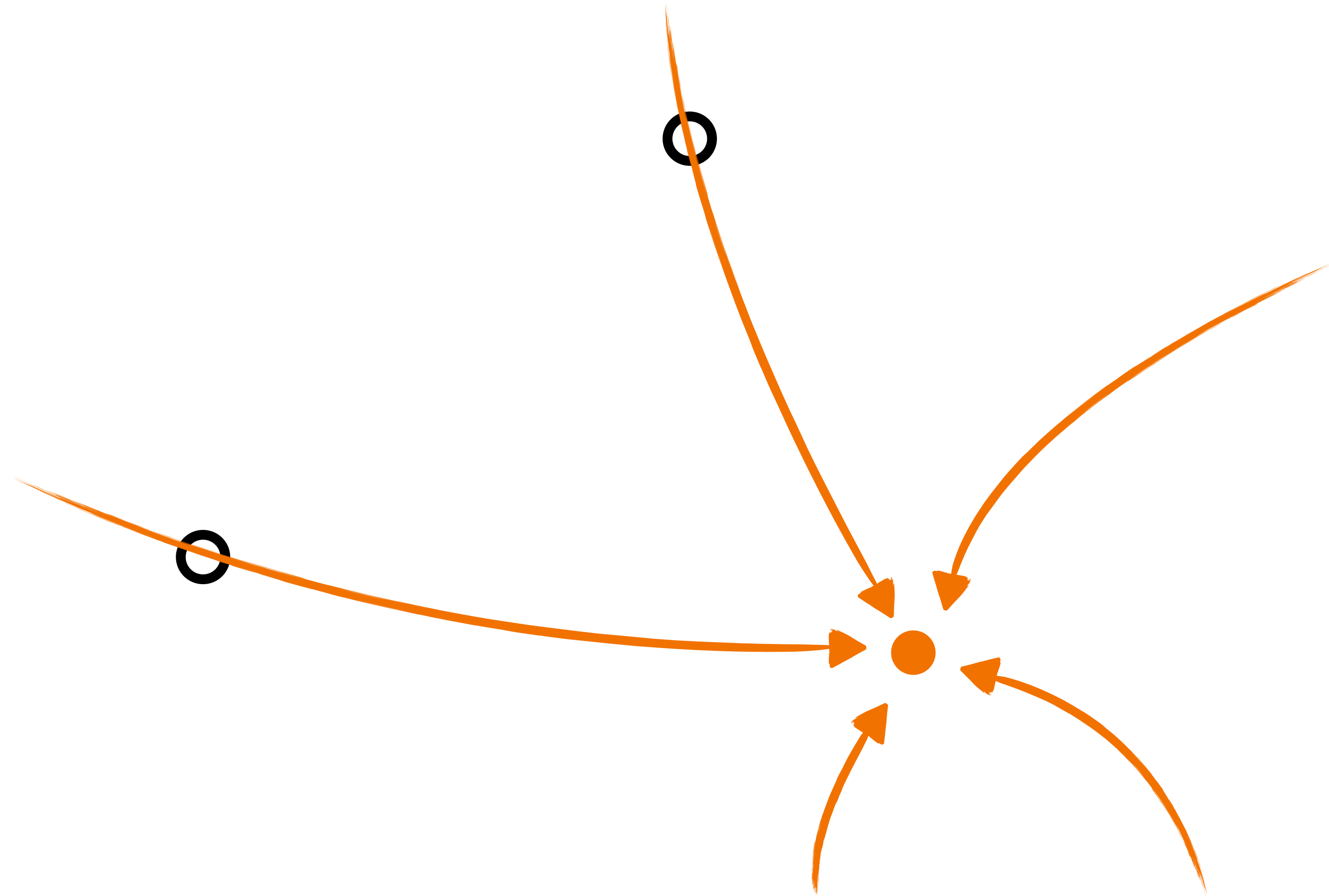
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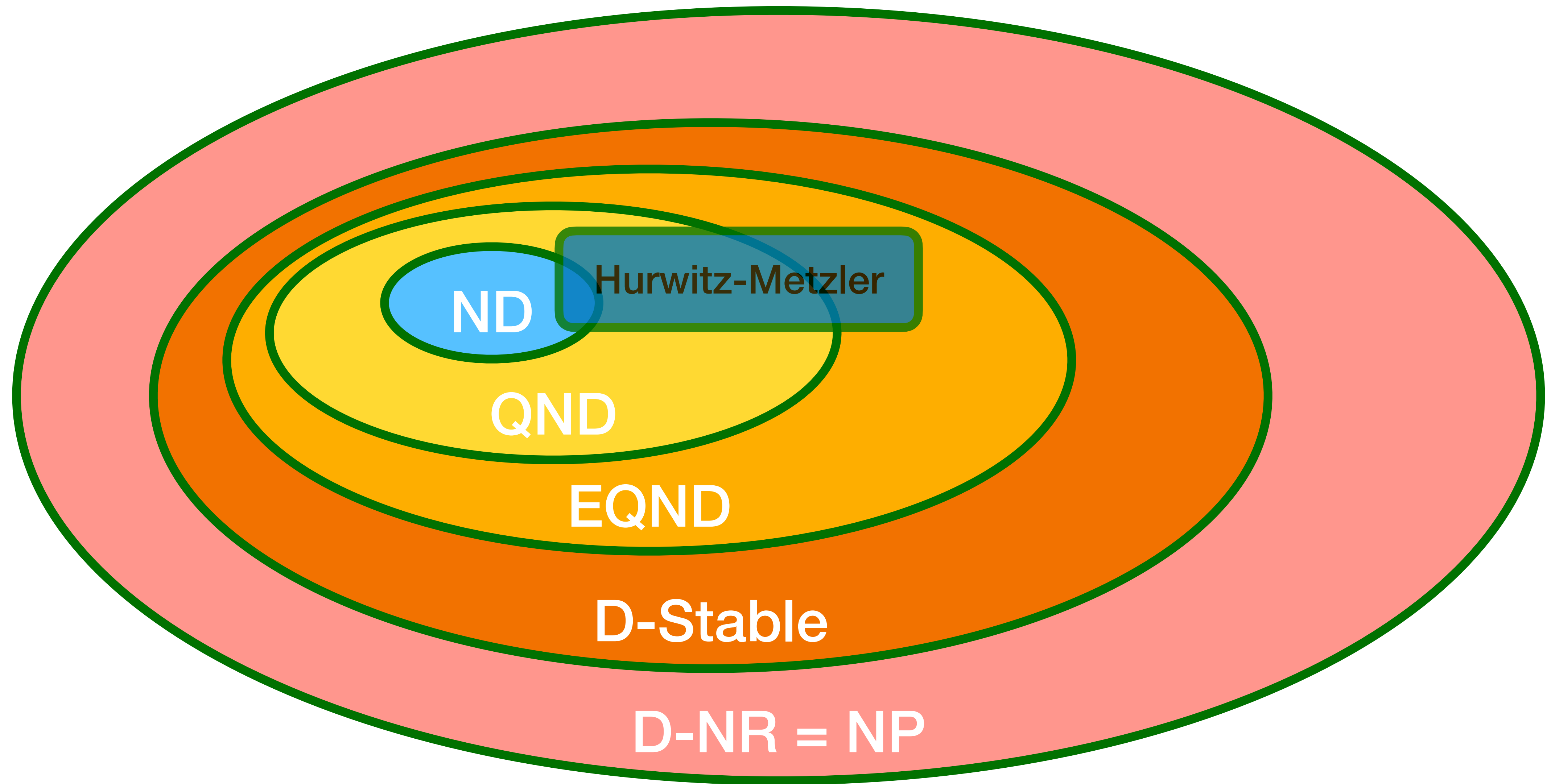


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↓ (linearized)

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No. Not really → Conceptual problems!

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study eigenvalues of M

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...may come back to this...

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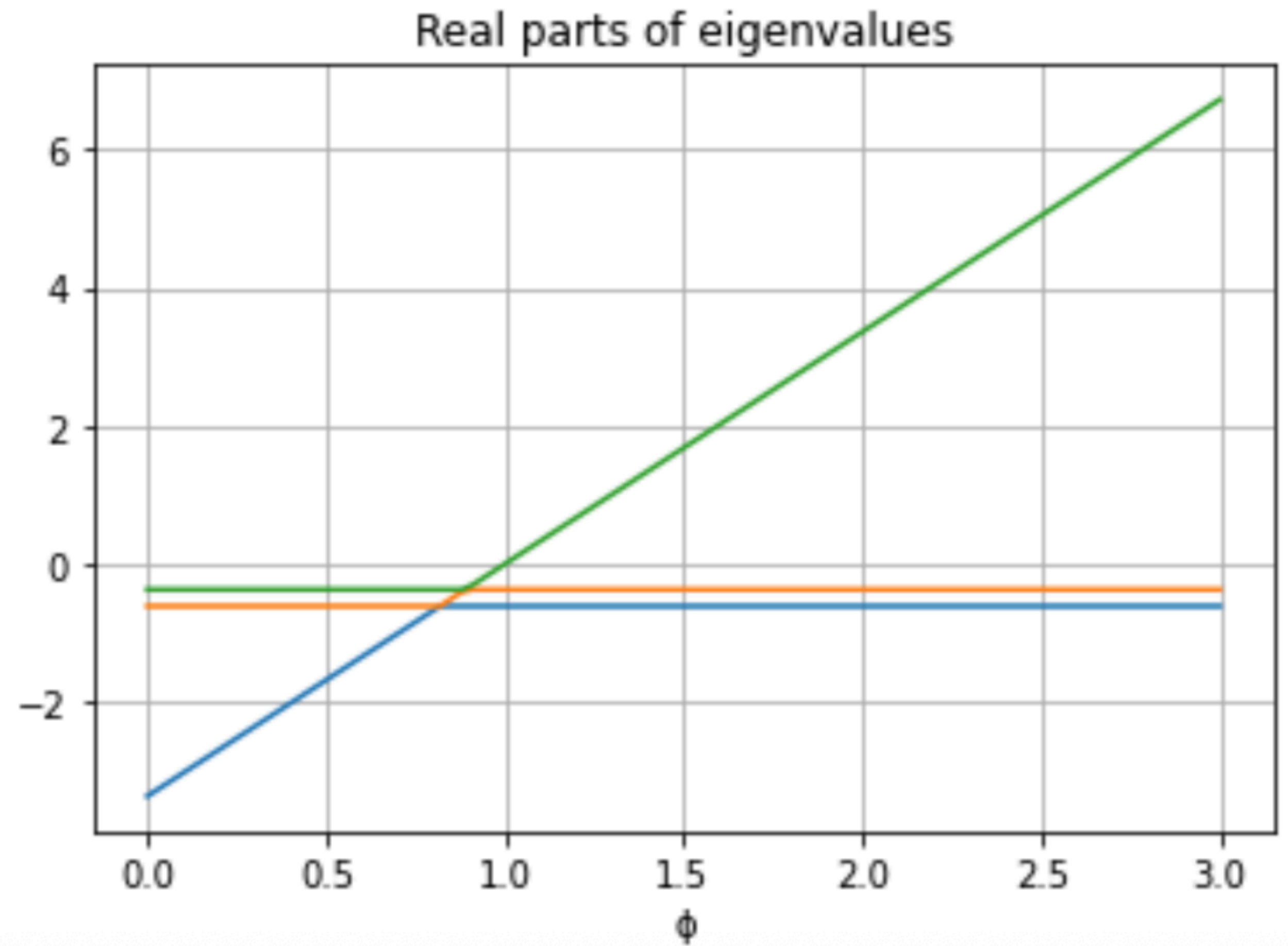
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 - ▶ less than $N - 1$ stable
 - ▶ complex roots

Example 1: Three good economy

```
plt.xlabel(" $\phi$ ")
```

```
print(vl)
```

```
[0.149198 0.504783 0.346019]
```



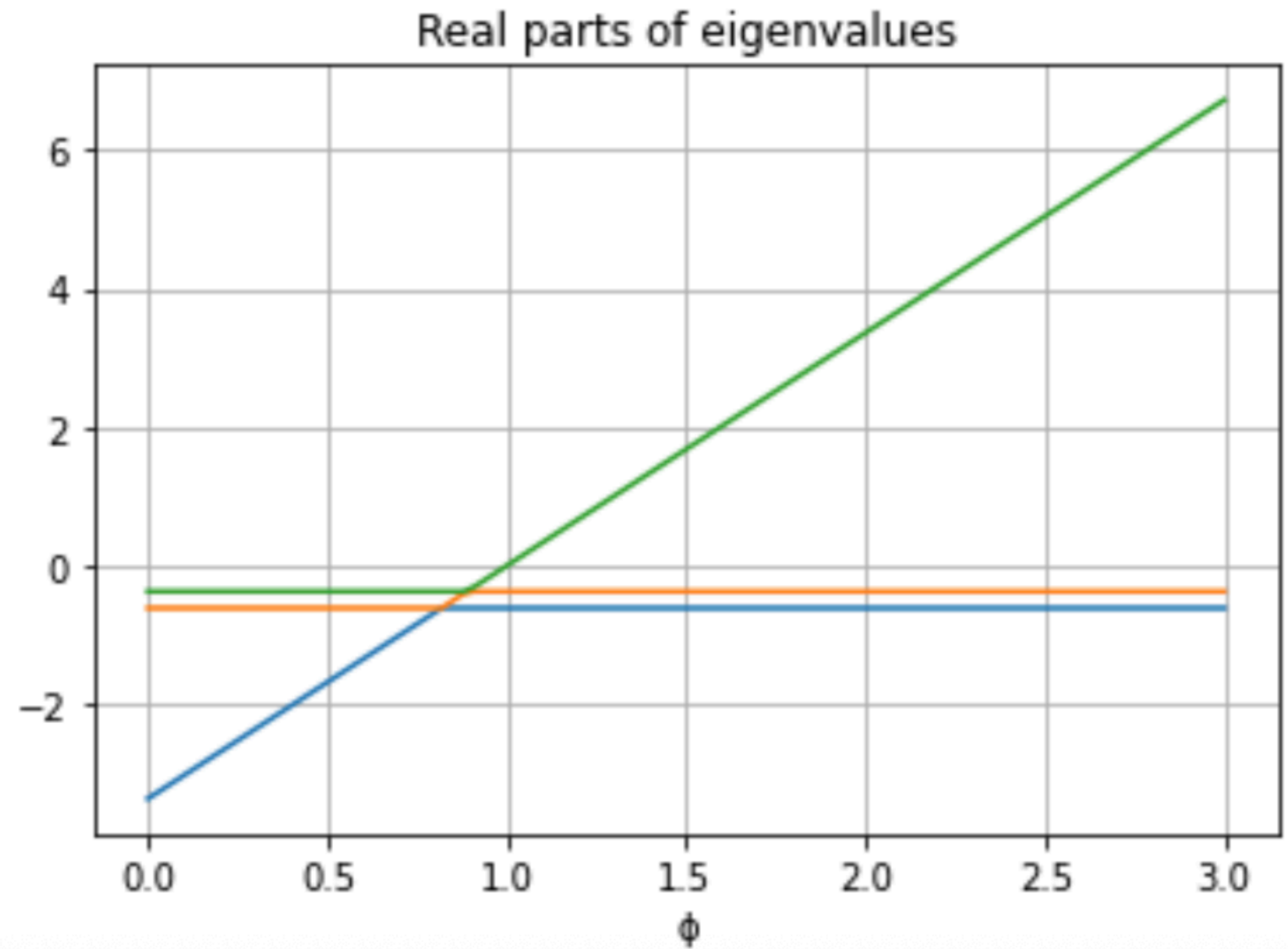
Example 1: Three good economy

- Taylor rule with weights = left eigenvector

```
plt.xlabel(" $\phi$ ")
```

```
print(vl)
```

```
[0.149198 0.504783 0.346019]
```



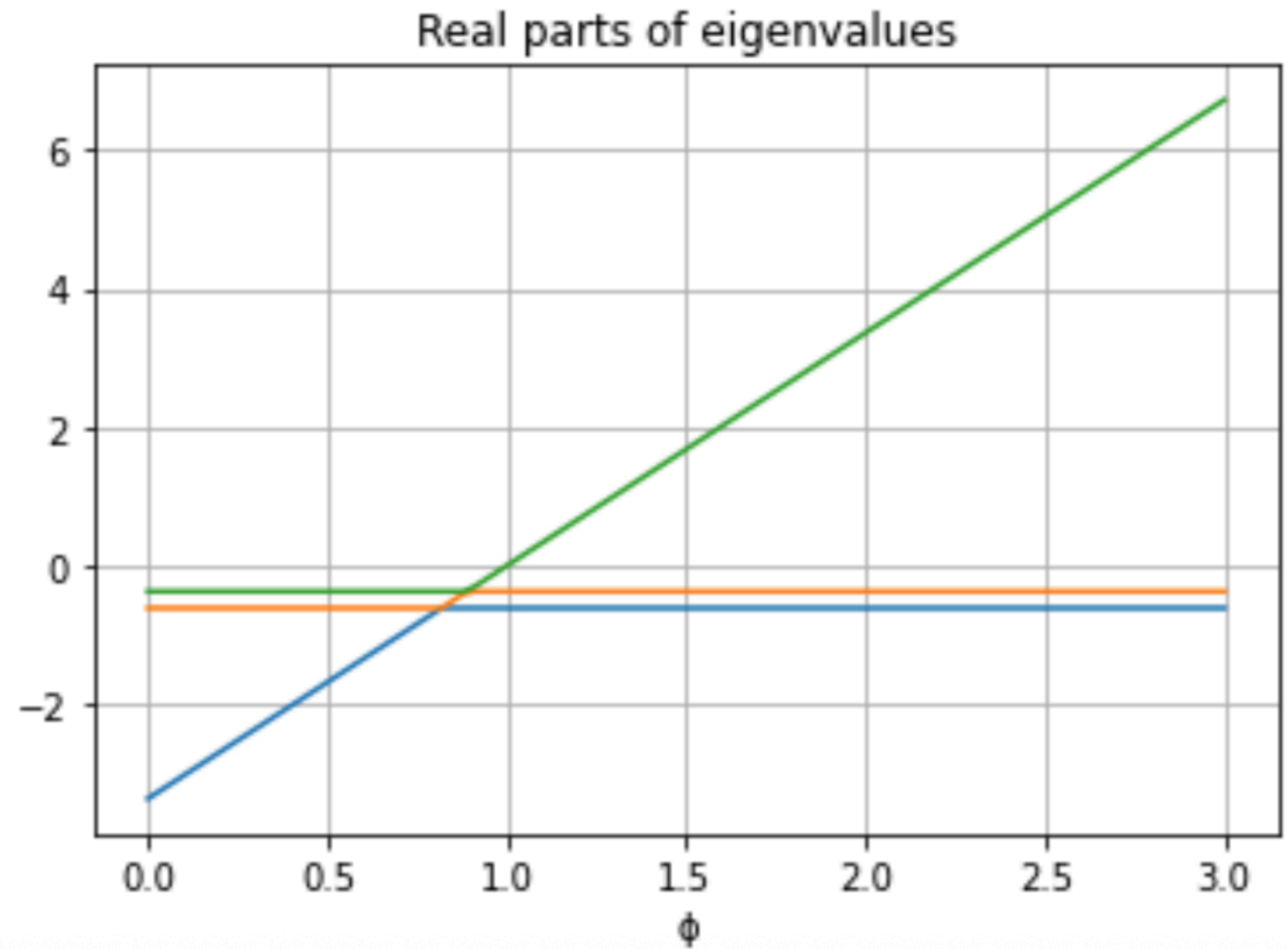
Example 1: Three good economy

- Taylor rule with weights = left eigenvector
- At $\phi = 0$: three real and negative eigenvalues

```
plt.xlabel(" $\phi$ ")
```

```
print(v1)
```

```
[0.149198  0.504783  0.346019]
```



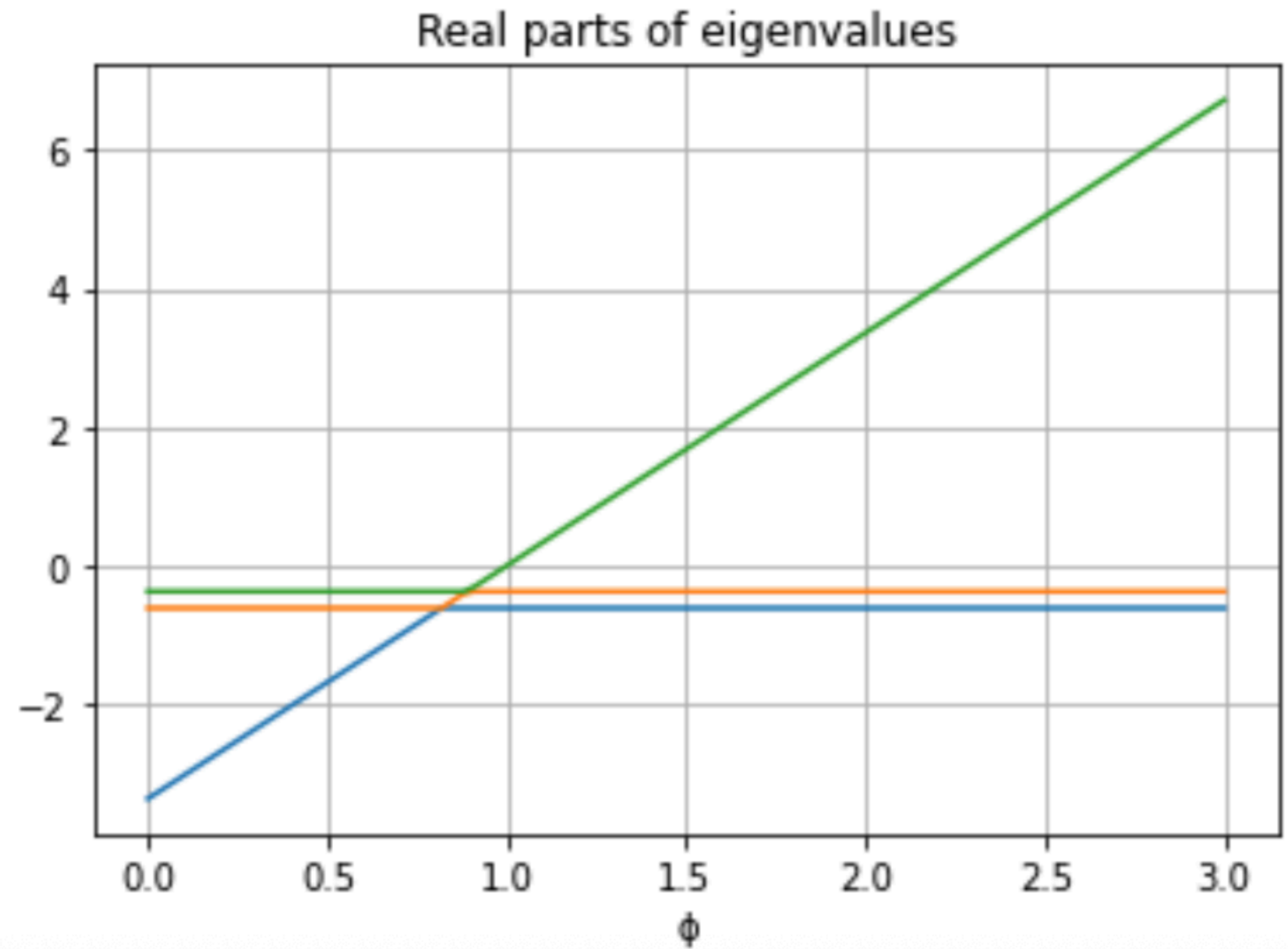
Example 1: Three good economy

- Taylor rule with weights = left eigenvector
- At $\phi = 0$: three real and negative eigenvalues
- As ϕ rises

```
plt.xlabel(" $\phi$ ")
```

```
print(v1)
```

```
[0.149198  0.504783  0.346019]
```



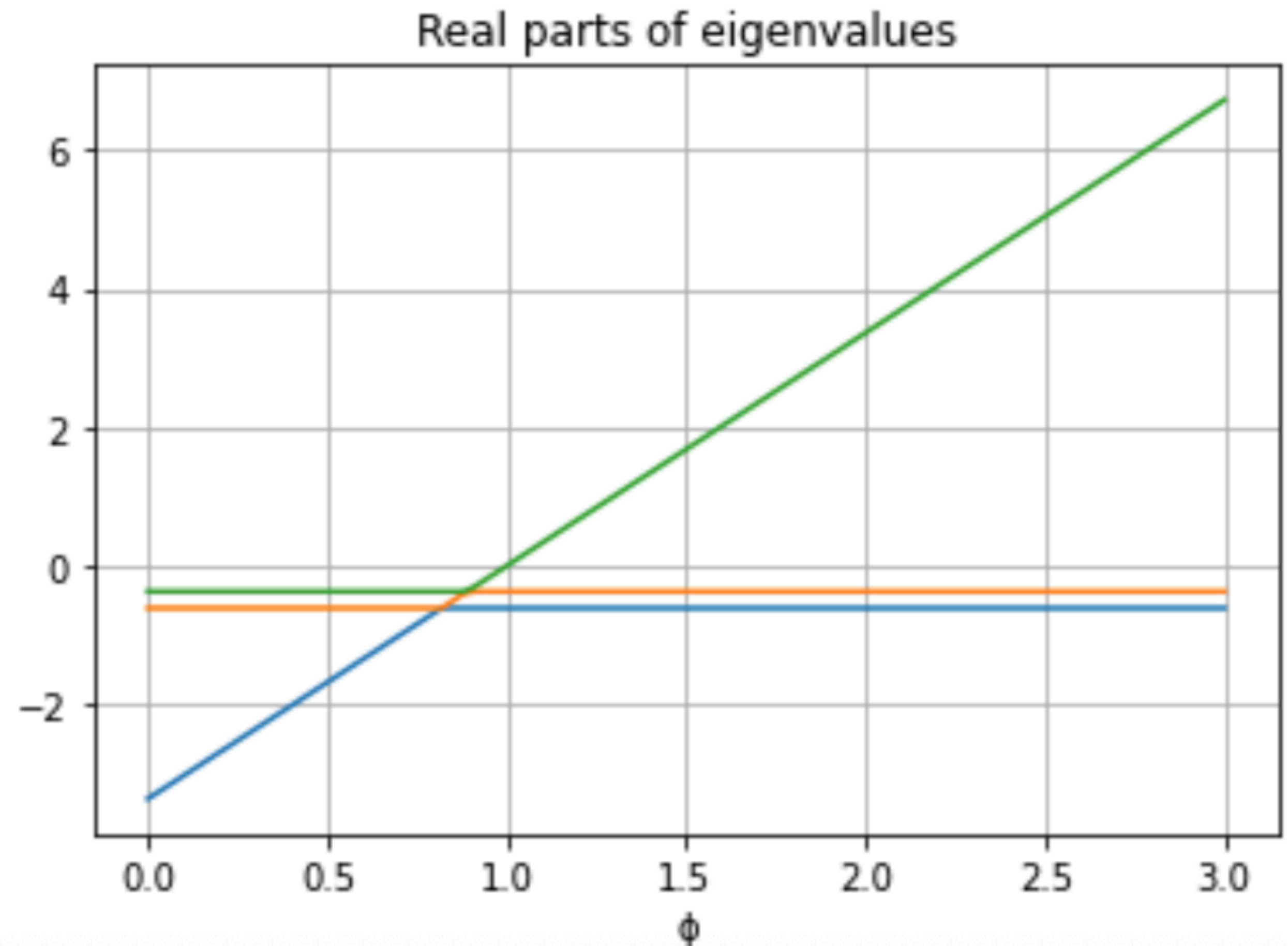
Example 1: Three good economy

- Taylor rule with weights = left eigenvector
- At $\phi = 0$: three real and negative eigenvalues
- As ϕ rises
 - ▶ two eigenvalues remain unchanged

```
plt.xlabel(" $\phi$ ")
```

```
print(v1)
```

```
[0.149198 0.504783 0.346019]
```



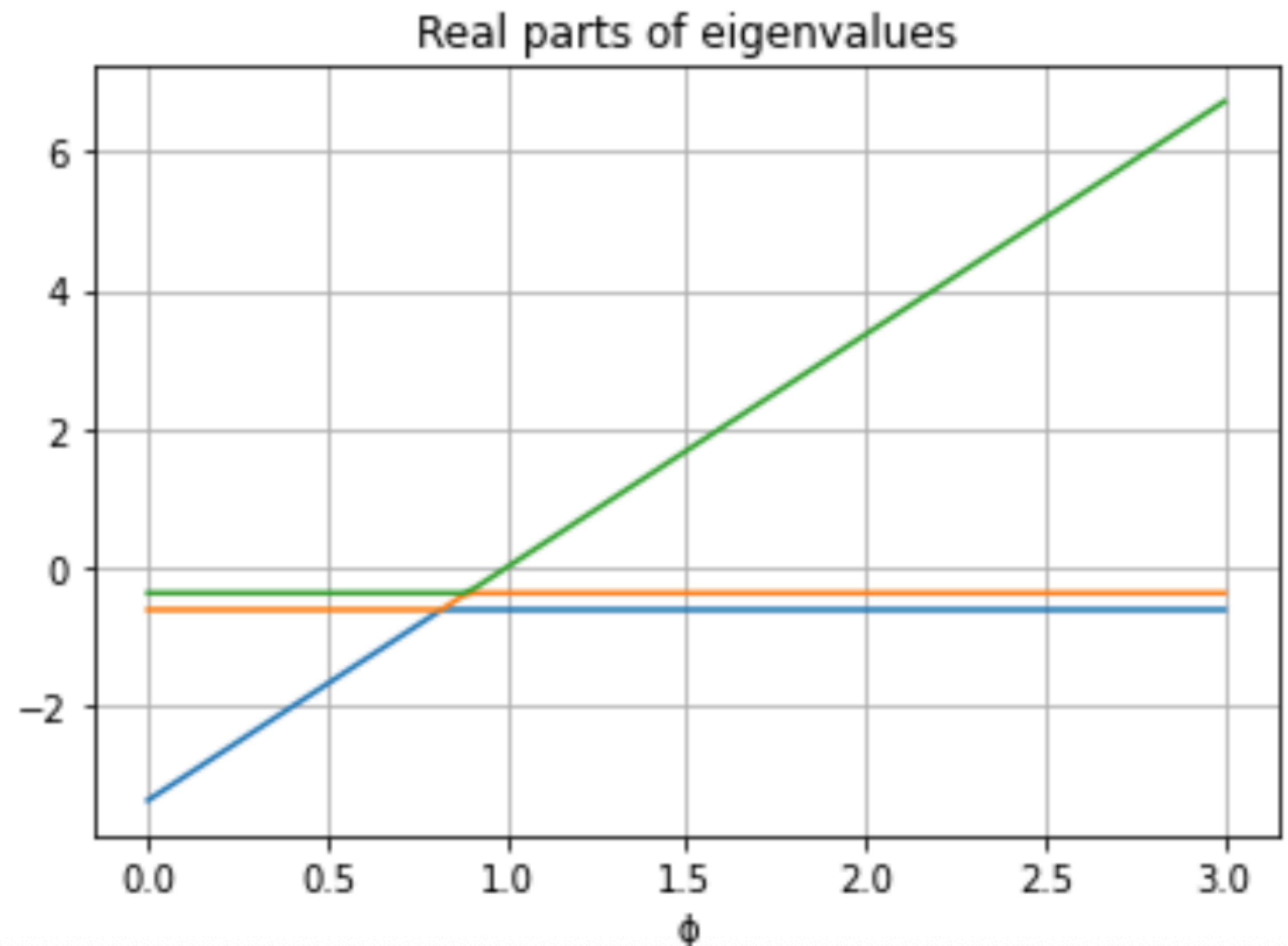
Example 1: Three good economy

- Taylor rule with weights = left eigenvector
- At $\phi = 0$: three real and negative eigenvalues
- As ϕ rises
 - ▶ two eigenvalues remain unchanged
 - ▶ other rises and crosses 0 at $\phi = 1$! 😎

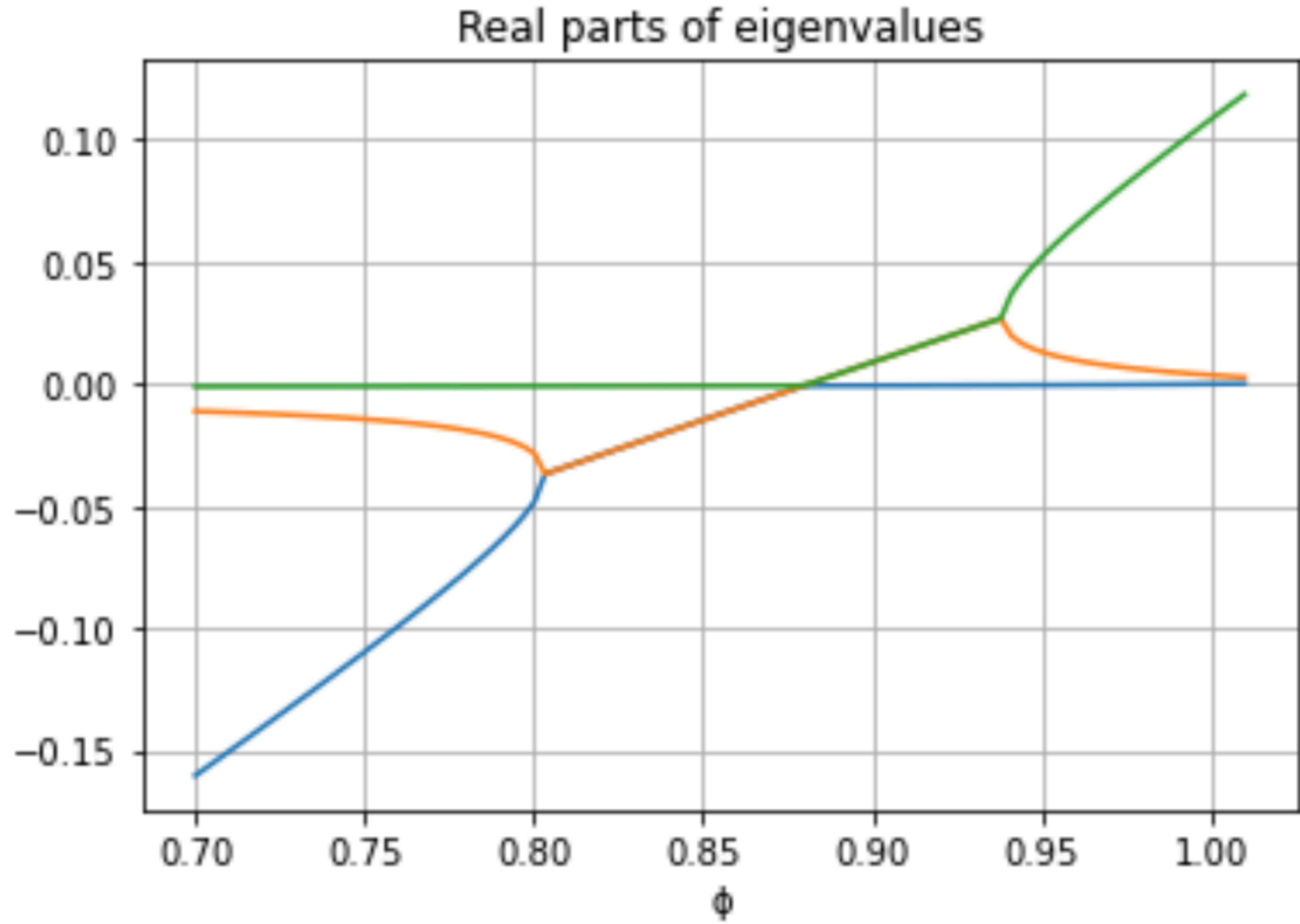
```
plt.xlabel(" $\phi$ ")
```

```
print(v1)
```

```
[0.149198 0.504783 0.346019]
```

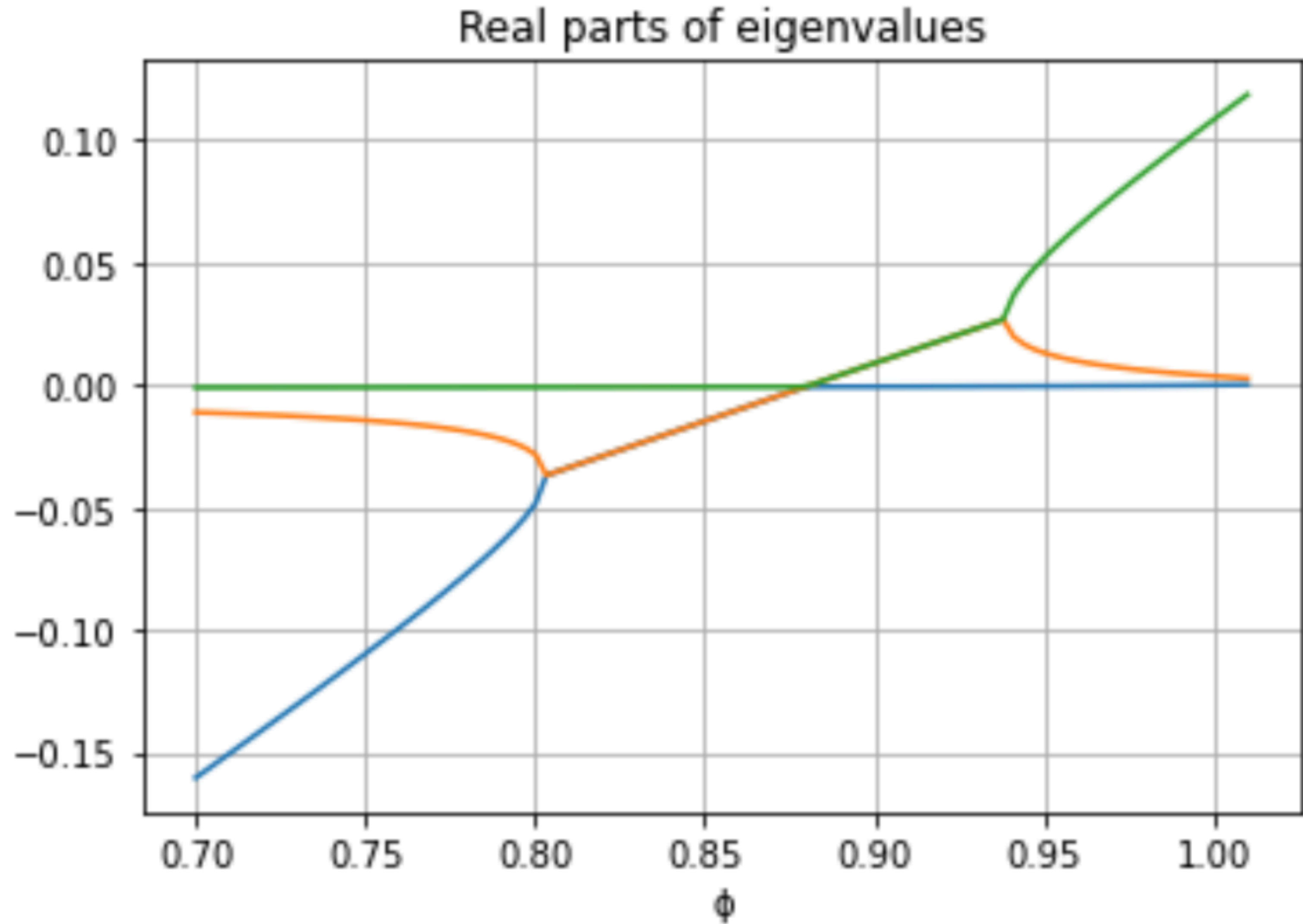


Example 2: Three good economy again



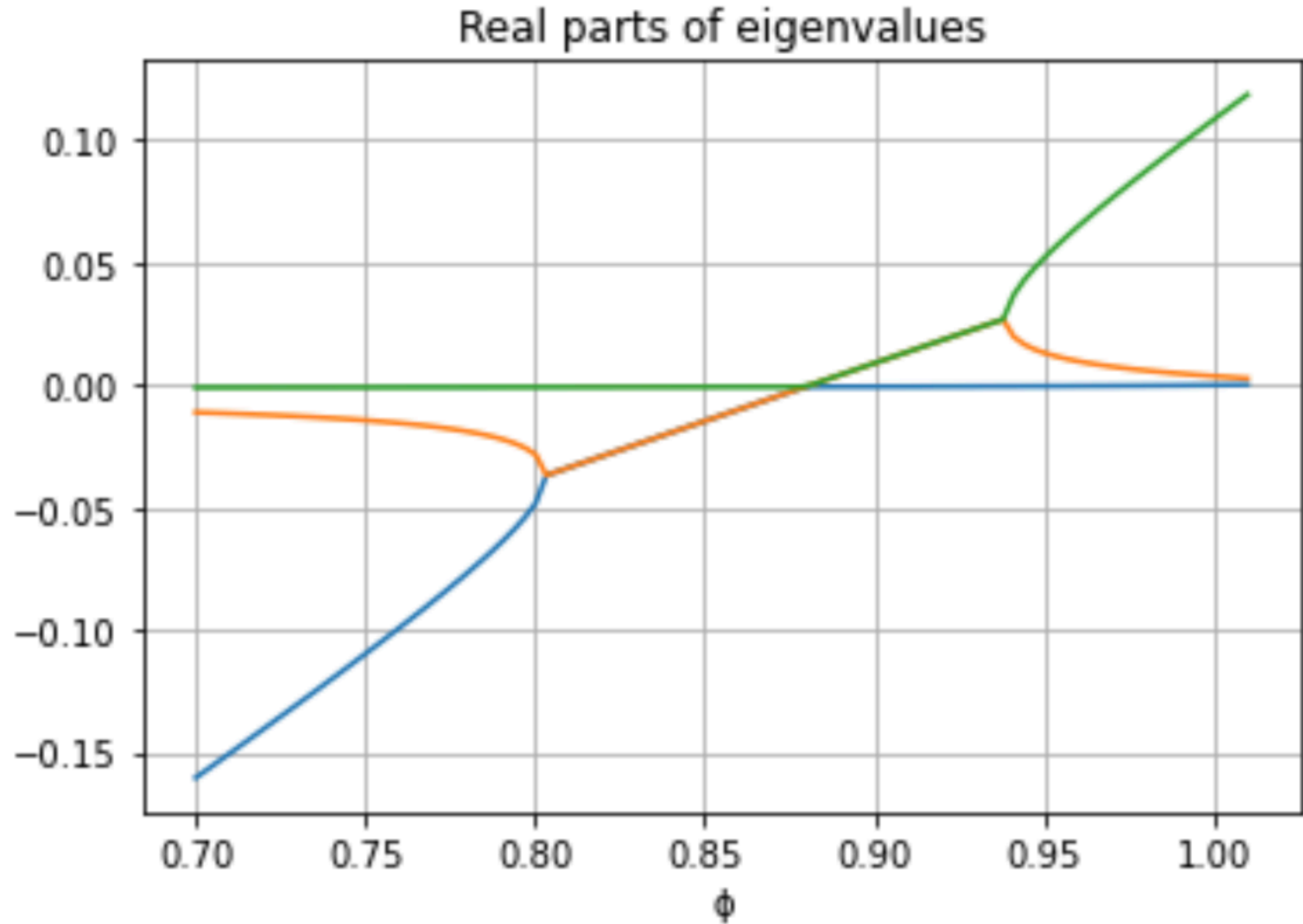
Example 2: Three good economy again

- Taylor rule with weights $\neq$ left eigenvector



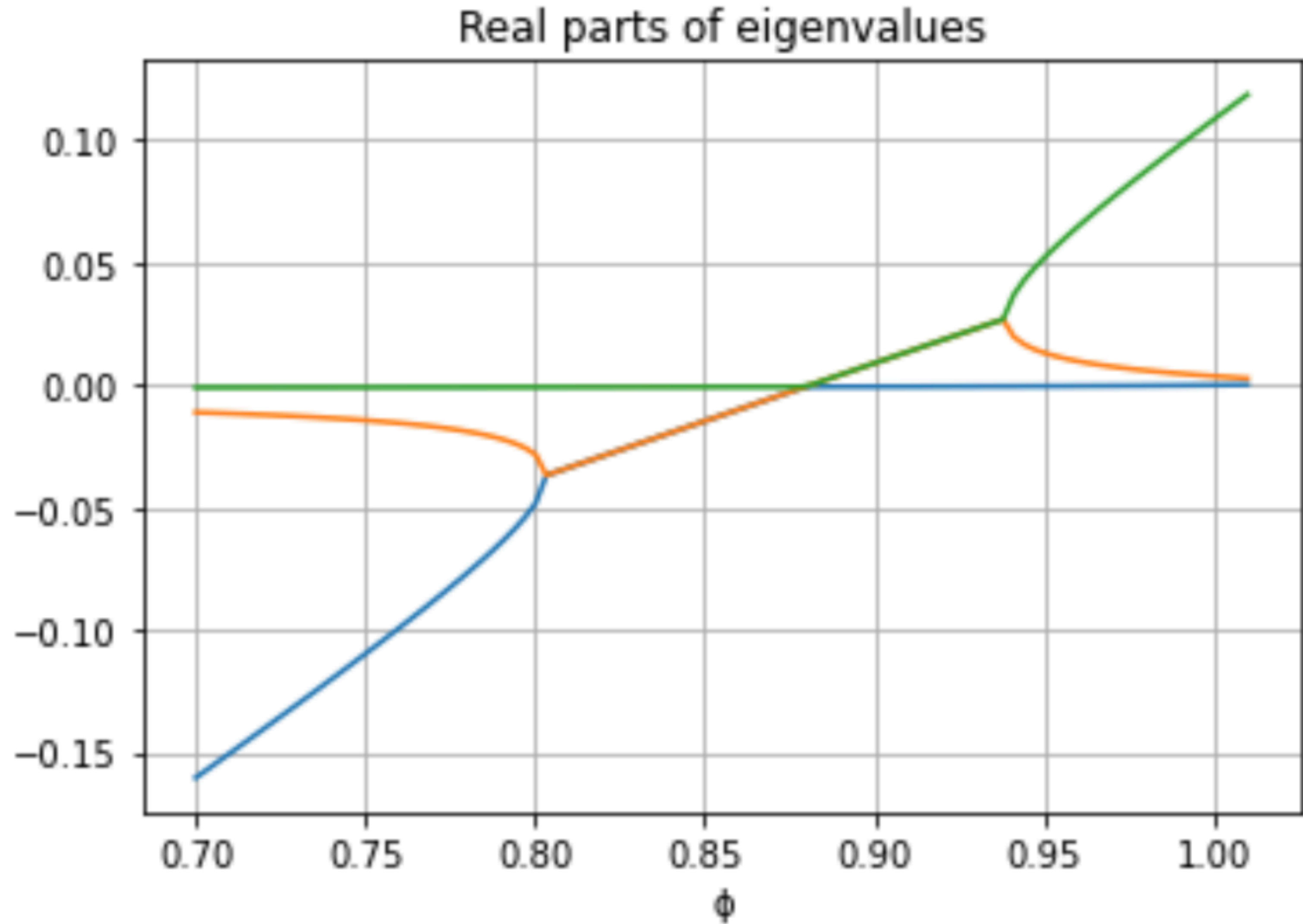
Example 2: Three good economy again

- Taylor rule with weights $\neq$ left eigenvector
- At $\phi = 0$: again three real and negative eigenvalues



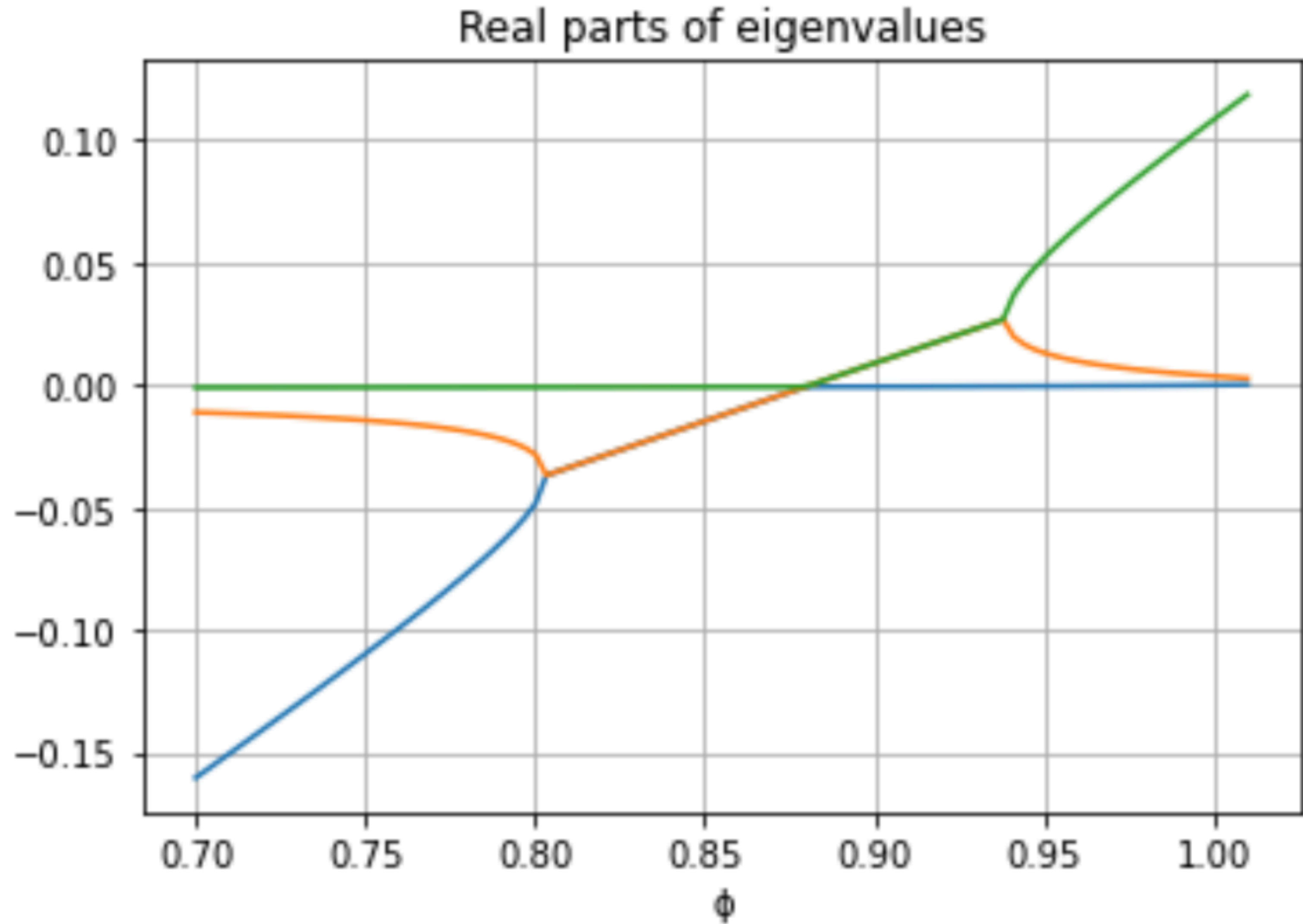
Example 2: Three good economy again

- Taylor rule with weights $\neq$ left eigenvector
- At $\phi = 0$: again three real and negative eigenvalues
- As ϕ rises:



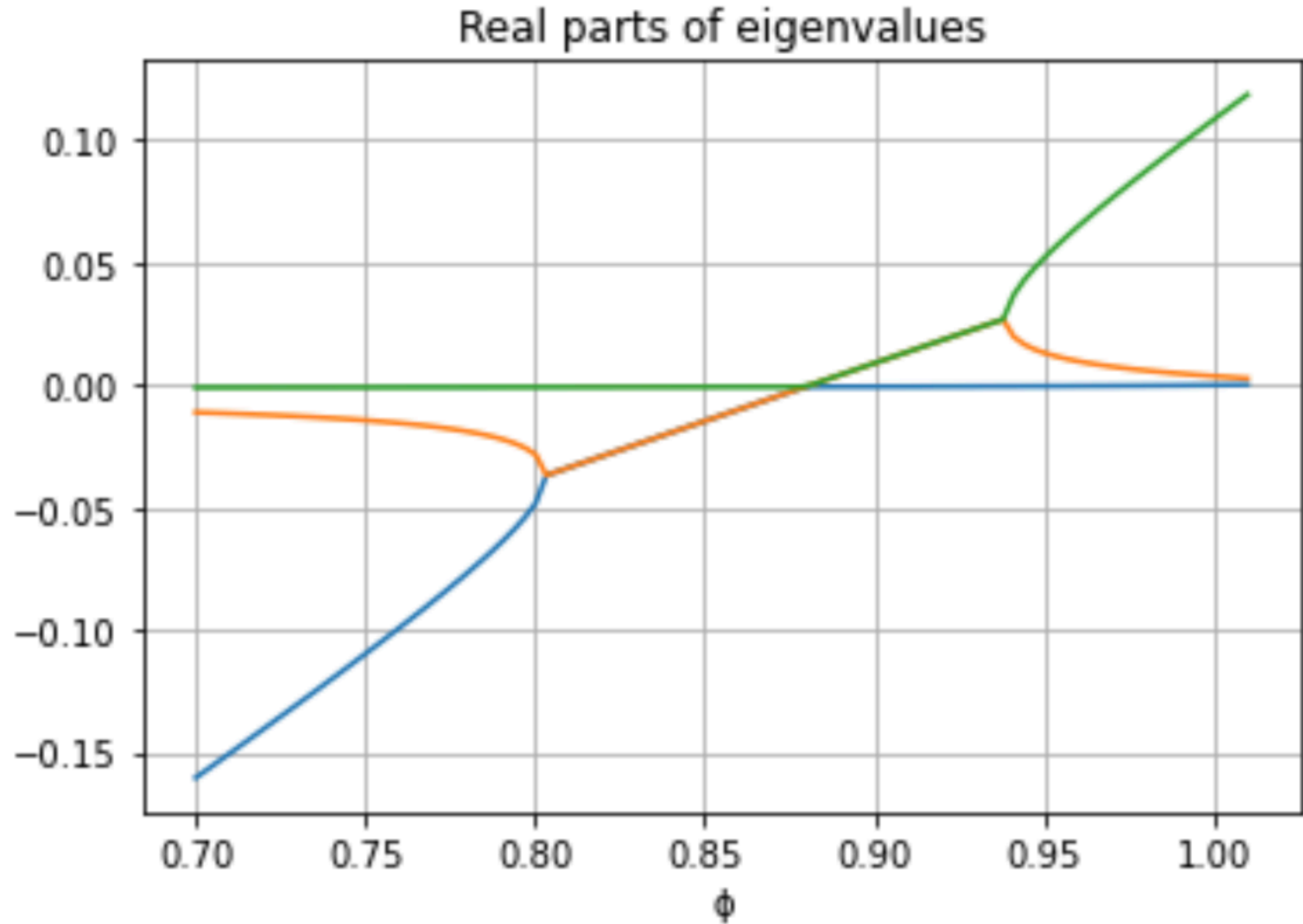
Example 2: Three good economy again

- Taylor rule with weights $\neq$ left eigenvector
- At $\phi = 0$: again three real and negative eigenvalues
- As ϕ rises:
 - all eigenvalues affected



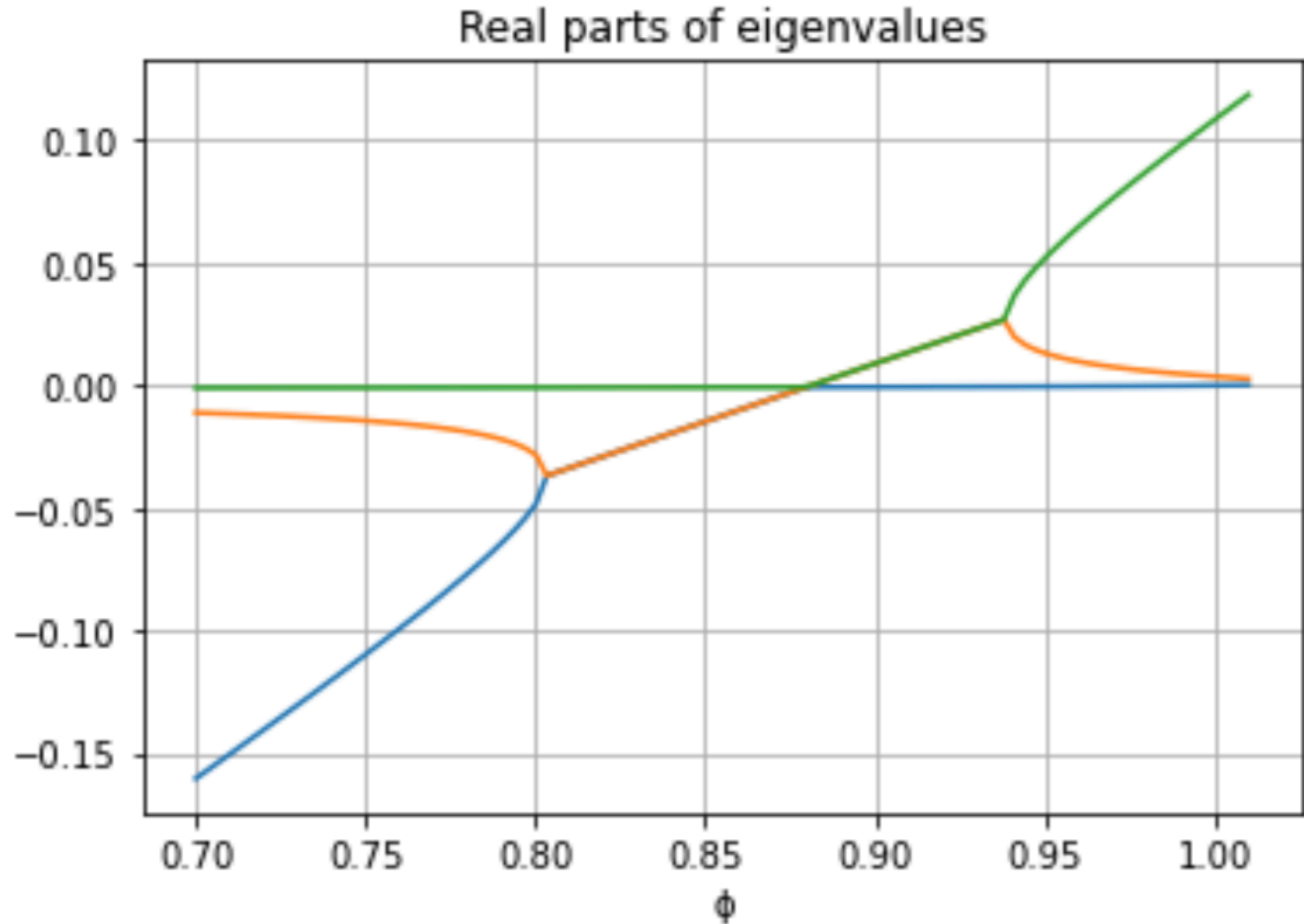
Example 2: Three good economy again

- Taylor rule with weights $\neq$ left eigenvector
- At $\phi = 0$: again three real and negative eigenvalues
- As ϕ rises:
 - all eigenvalues affected
 - complex conjugate pair of eigenvalues emerge



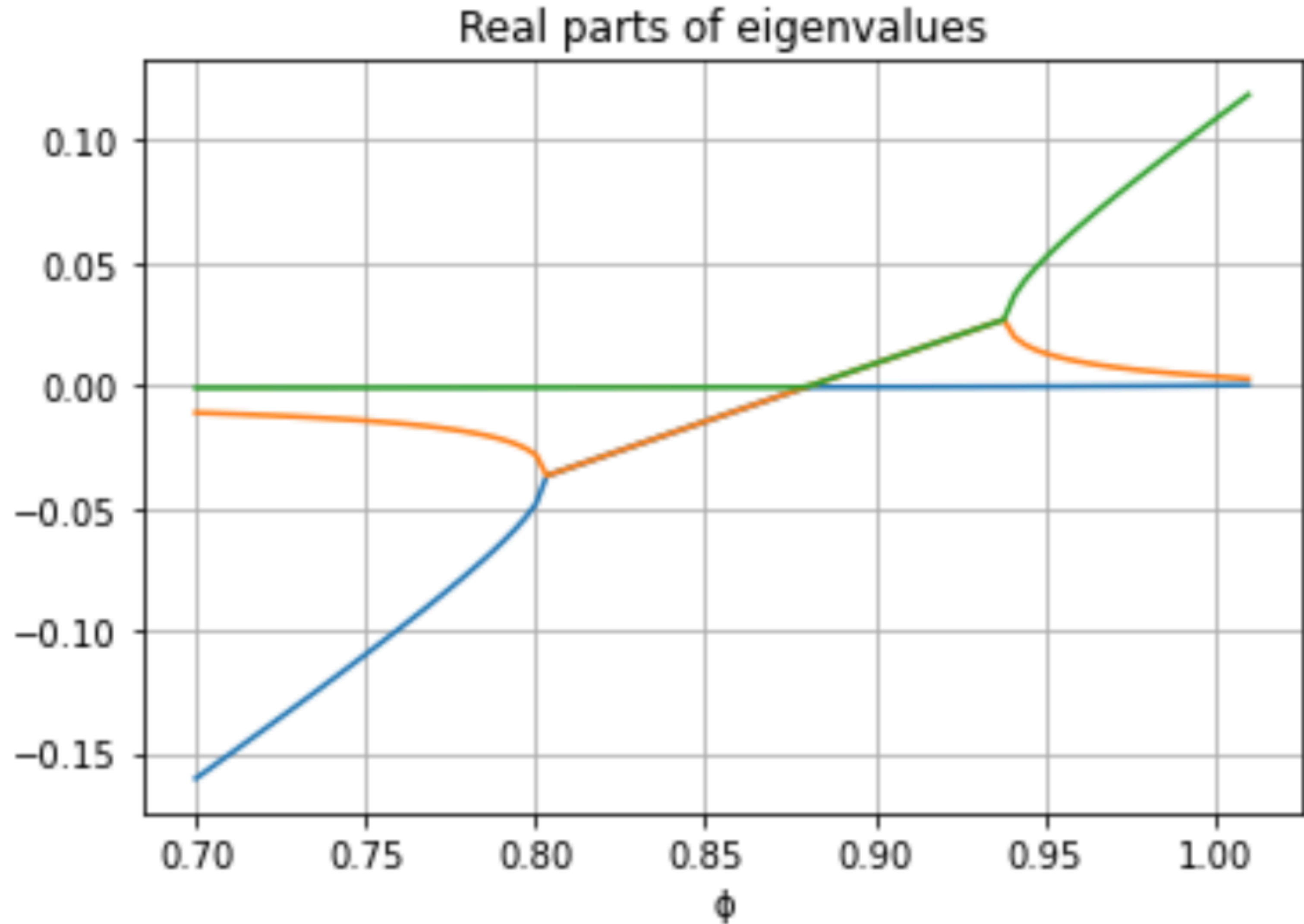
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 - ▶ common real part pair crosses 0 around $\phi = 0.857$



Example 2: Three good economy again

- Taylor rule with weights $\neq$ left eigenvector
- At $\phi = 0$: again three real and negative eigenvalues
- As ϕ rises:
 - ▶ all eigenvalues affected
 - ▶ complex conjugate pair of eigenvalues emerge
 - ▶ common real part pair crosses 0 around $\phi = 0.857$
- No ϕ with two negative and one positive eigenvalues! 😞



Adding Market Power

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 Each market n ...

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■ Each market n ...

▶ **undifferentiated side (u)** vs **differentiated side (v = varieties)**

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$$P_{nu} = \left(\int P_{nv}^{1-\epsilon_n} dv \right)^{\frac{1}{1-\epsilon_n}}$$

Adding Market Power

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$$\bar{z}_u + \bar{z}_v = 0$$

$$P_{nu} = \left(\int P_{nv}^{1-\epsilon_n} dv \right)^{\frac{1}{1-\epsilon_n}}$$

Adding Market Power

■ Each market n ...

► **undifferentiated side (u)** vs **differentiated side (v = varieties)**

► differentiated side → sets price P_{nv} either...

□ monopolistic supply → $z_{nv} = -y_{nv}$

□ monopsonistic demand → $z_{nv} = x_{nv}$

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► differentiated side → assigned to agent type j

■ Assignment: $n \leftrightarrow j$ (market ↔ agent)

$$\bar{z}_u + \bar{z}_v = 0$$

$$P_{nu} = \left(\int P_{nv}^{1-\epsilon_n} dv \right)^{\frac{1}{1-\epsilon_n}}$$

Adding Market Power

■ Each market n ...

► **undifferentiated side (u)** vs **differentiated side (v = varieties)**

► differentiated side → sets price P_{nv} either...

□ monopolistic supply → $z_{nv} = -y_{nv}$

□ monopsonistic demand → $z_{nv} = x_{nv}$

► differentiated side → assigned to agent type j

■ Assignment: $n \leftrightarrow j$ (market ↔ agent)

► without loss → single agent j in each market n

► simplify → (at most) one market n for each agent j
i.e. agent j sets (at most) one price (possibly none)

$$\bar{z}_u + \bar{z}_v = 0$$

$$P_{nu} = \left(\int P_{nv}^{1-\epsilon_n} dv \right)^{\frac{1}{1-\epsilon_n}}$$

Invertible

The diagram consists of three nested rectangles. The outermost rectangle is red and labeled 'Invertible'. Inside it is an orange rectangle labeled 'P (D-Real-Stable)'. Inside the orange rectangle is a yellow rectangle labeled 'D-Stable'. At the bottom of the yellow rectangle is a smaller dashed rectangle labeled 'CES?'. The labels are in bold black font, except for 'CES?' which is in a lighter font.

P (D-Real-Stable)

D-Stable

CES?

Invertible

**Hicks
not D-Stable**

D-Stable + Hicks

CES?

