Optimal (Un)Conventional Monetary Policy

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Motivation

- **Central Banking** with **many policy instruments**:
  - Interest rates
    - on excess reserves i
    - on required reserves $\underline{i}$
    - reserve requirements $\underline{\theta}$
  - Balance sheet management
    - purchases/sales of long-term gov. bonds for reserves
- Generalized policy rule:
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- Generalized policy rule:
  $\textit{state variables} \mapsto \{i, \underline{i}, \underline{\theta}, \textit{CB balance sheet (maturity)}\}$
- What role does each instrument play? How do they **interact**?
- What is the welfare-maximizing **policy mix**?
- What are the implications for the **yield curve** and duration dynamics?

App

Overview of Main Results

- Macro model with financial sector, aggregate & idiosyncratic risk, sticky prices
  Brunnermeier & Sannikov (2016), Li & Merkel (2025), Merkel (2020)

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- Adding **distributional efficiency** requires dynamic balance sheet management
- **Yield curve** implications

Related Literature (incomplete)

- Macro model with financial sector, aggregate & idiosyncratic risk, sticky prices
  Brunnermeier & Sannikov (2016), Li & Merkel (2025), Merkel (2020), Sims (2011)
  He & Krishnamurthy, Elenev et al. (2021)
- Roles of balance sheet management (QE/QT) in existing literature:
  1. Relaxes intermediaries' constraints

     Gertler & Karadi (2011), Karadi & Nakov (2021), Eren, Jackson & Lombardo (2024), Nisticó & Seccareccia (2025), ...
  2. Moves asset prices and term premia

     Vayanos & Vila (2021), Kekre & Lenel (2025), Kamdar & Ray (2025), ...
  3. Signals future interest rates

     Eggertsson & Woodford (2003), Bhattarai, Eggertsson, & Gafarov (2023), ...
- Convenience yield on bank deposits
  Transaction cost models, Begenau (2020), ...

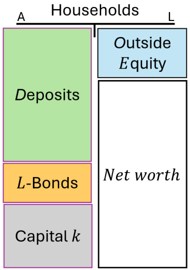
NK Framework with Financial Sector — Risk Focus

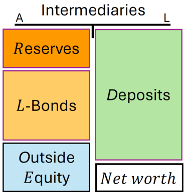
- **Households/Entrepreneurs:** HH
  - Hold capital, utilize it in production ($y_t = a v_t k_t$) and invest (ι_t)
  - Capital accumulation is subject to uninsurable idiosyncratic risk:

$$dk_t = \left(1/\phi \log(1+\phi\iota_t) - \delta\right) k_t dt + \tilde{\sigma}_t k_t d\tilde{Z}_t$$

$$\text{e.g.} \quad d\tilde{\sigma}_t^2 = -b_s(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt + \sigma\tilde{\sigma}_t^2 dZ_t$$

  - Hold long-term bonds and deposits
    - More deposits $\Rightarrow$ lower velocity ν_t and transaction cost $\mathfrak{t}(\nu_t)$
  - Issue risky claims to intermediaries, passing on fraction χ_t of risk
- **Intermediaries:** II
  - Hold HHs' risky claims, reserves, long-term bonds; issue deposits
  - Diversify idiosyncratic risk to fraction $\varphi \in (0,1)$
- Sticky prices (Rotemberg) Firms
  - NK-monopolistic competition for good-differentiators





Optimal Policy: Roadmap

1. Study a *relaxed* Ramsey problem (with dt and dZ_t taxes) allowing for **analytical** characterization of efficient **allocations**
2. Compare efficient outcomes to laissez-faire
3. Consider a *realistic* government and study:
    - Under which conditions efficient allocations are achievable
    - The corresponding **optimal policy mix** (interest rate + balance sheet management)

Efficiency (Relaxed Problem)

- Suppose the government raises taxes over time (flow/dt taxes) and in response to aggregate shocks (loading on dZ_t), but not to idios. shocks (not on $d\tilde{Z}_t$)

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- Behaves as a utilitarian planner that can directly set:

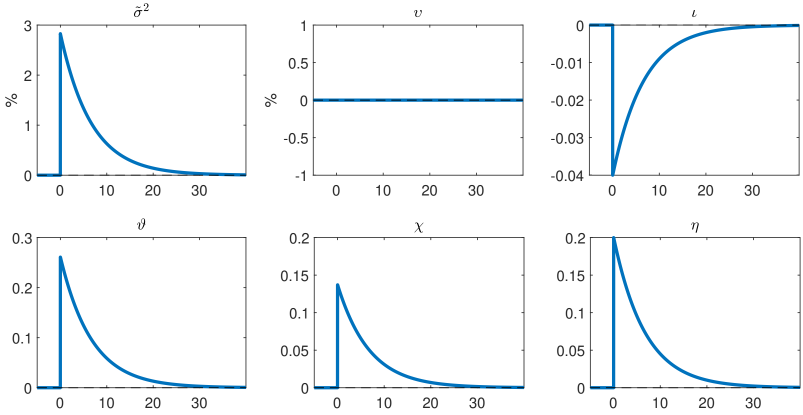
  Aggregate efficiency (static/dynamic output gap)
  - Capital utilization rate υ_t
  - Capital investment rate ι_t

  Distributional efficiency
  - Distribution of idiosyncratic risk exposure χ_t
  - Distribution of wealth/consumption across sectors $\eta_t = N_t^I / N_t$,
  - Distribution of wealth across assets $\vartheta_t = \mathcal{B}_t / (\mathcal{P}_t N_t) = (\mathcal{R}_t + P_t^L L_t)/(\mathcal{P}_t N_t)$

App 1 App 2 App 3

IRF Planner's Solution after $\tilde{\sigma}_t$-Shock

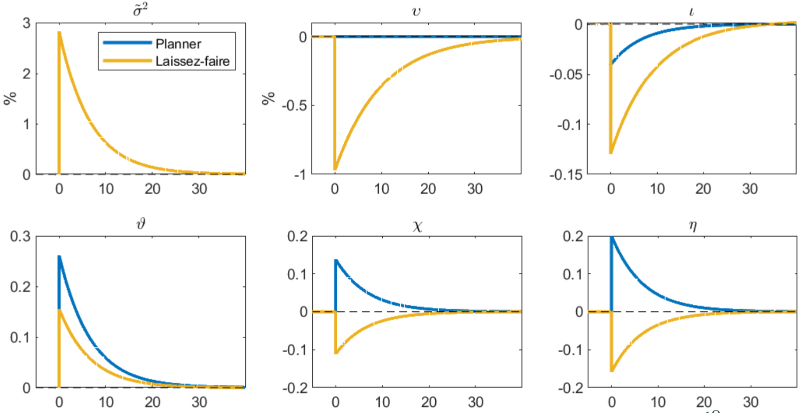
Aggregate Eff: Constant utilization ($\upsilon_t =$) but capital disinvestment ($\iota_t \downarrow$)



Distributional Eff: Flight-to-safety ($\vartheta_t \uparrow$) + redistribution of risk, wealth to bankers ($\chi_t, \eta_t \uparrow$)

Laissez-faire vs Planner's IRF

Aggregate Eff: Output gap $\log \hat{Y}_t \equiv \log \frac{a\upsilon_t K_t}{a\upsilon_t^* K_t^*} = \overbrace{(\log \upsilon_t - \log \upsilon_t^*)}^{\text{instantaneous}} + \overbrace{(\log K_t - \log K_t^*)}^{\text{dynamic}}$



Distributional Eff: Flight-to-safety lowers bankers' wealth, risk share: $\overbrace{\eta_t \sigma_t^\eta}^{<0} = \overbrace{(\eta_t - \chi_t)}^{<0} \overbrace{\sigma_t^\vartheta}^{>0}$

Externalities

- Sticky prices $\Rightarrow$ instantaneous output gap (inefficient utilization υ_t)
- 'Aggregate' pecuniary externality:
  - Agents' demand for capital affects capital price q_t^K, which in turn drives investment decision ι_t (Tobin's q)
- 'Distributional' pecuniary externality:
  - Aggregate wealth/consumption distribution η_t affects intermediaries' risk-bearing capacity, driving idiosyncratic risk distribution χ_t

Realistic Government

- **Central bank:**
  - Sets interest rates $\underline{i}_t$ and i_t, reserve requirements $\underline{\theta}_t^R$
  - Issues reserves $\frac{d\mathcal{R}_t}{\mathcal{R}_t} = \mu_t^{\mathcal{R}} dt + \sigma_t^{\mathcal{R}} dZ_t$
  - Holds bonds $\frac{dL_t^{CB}}{L_t^{CB}} = \mu_t^{L,CB} dt + \sigma_t^{L,CB} dZ_t$
- **Fiscal authority:**
  - Issues long-term bonds $dL_t^F = \mu_t^{L,F} L_t^F dt$ paying interest i^L, nominal price P_t^L
  - Levies a range of ‘flow’ taxes (intermediation, wealth, capital)
- Motivation: bonds are issued at auctions, CB bond purchases/sales are OMO

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- Motivation: bonds are issued at auctions, CB bond purchases/sales are OMO
- Long-term bond holdings of private agents are $L_t^I + L_t^H = L_t = L_t^F - L_t^{CB}$
- Distribution across sectors $\eta_t^L = L_t^I / L_t$ is endogenous

App

Policy

- Focus on **interest rate** and **balance sheet policy**
- Fiscal policy operates in the background
- Interest rate controls sensitivity of bond price to aggregate shocks, (Sims 2011):

$$\sigma_t^{P^L} = \frac{\partial \log P_t^L}{\partial \log \tilde{\sigma}_t^2}\sigma, \qquad \frac{\mathbb{E}\left[dP_t^L/dt\right]}{P_t^L} = i_t - \frac{i^L}{P_t^L} + \varsigma_t^I \sigma_t^{P^L}$$

- Balance sheet policy controls the **share** of long-term bonds in nominal wealth = total **duration** exposure in private hands:

$$\vartheta_t^L = \frac{P_t^L L_t}{\mathcal{R}_t + P_t^L L_t} = \frac{P_t^L L_t}{\mathcal{B}_t}$$

- **Total duration risk**: $\sigma_t^{\mathcal{B}} = \vartheta_t^L \sigma_t^{P^L}$

Passive Balance Sheet Management

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Proposition 2. Fiscal+interest rate policy can implement **aggregate efficiency**

$$a\upsilon_t^* = (1+\rho\phi)\iota_t^* + \rho + \rho\frac{\mathcal{B}_t}{\mathcal{P}_t K_t}$$

$$\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t \qquad \underbrace{\sigma_t^{\mathcal{B}}}_{\text{total duration risk}} = \vartheta^L \sigma_t^{P^L} = \underbrace{\frac{P_t^L L_t}{\mathcal{R}_t + P_t^L L_t}}_{\text{exposure } \vartheta^L} \underbrace{\frac{\partial \log P_t^L}{\partial \log \tilde{\sigma}_t^2}}_{\text{driven by } i_t} \sigma$$

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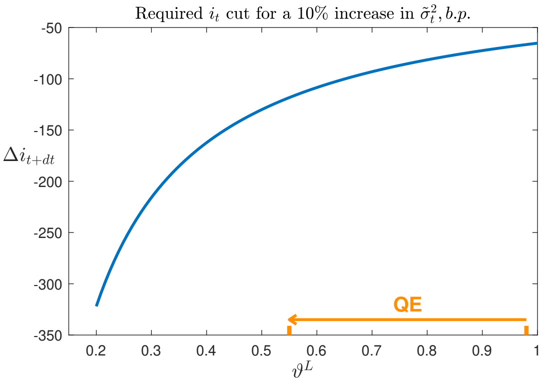
- Optimal interest rate policy *depends* on the balance sheet:
  - CB balance sheet **expansion** (smaller ϑ^L) $\Longrightarrow$ **more aggressive** interest rate policy

Passive Balance Sheet Management



- 120bp cut after QE $\sim$ 65bp cut before QE
- Near ZLB, balance sheet expansion shrinks ‘effective’ policy space

Active Balance Sheet Management - "preparatory"

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- From intermediaries' balance sheet:

$$\eta_t^* \sigma_t^{\eta,*} = \underbrace{-(\chi_t^* - \eta_t^*)\sigma_t^{\vartheta,*}}_{\text{Flight to safety}} + \underbrace{(\eta_t^L - \eta_t^*)\vartheta_t^* \vartheta^L \sigma_t^{PL}}_{\text{Direct effect}} + \underbrace{(\chi_t^* - \eta_t^*)(1 - \vartheta_t^*)\vartheta^L \sigma_t^{PL}}_{\text{Indirect effect}}$$

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- Challenge: endogenous bond distribution η_t^L
- Intermediaries scale down on bonds if they get more volatile ($\uparrow \sigma_t^{PL} \Rightarrow \eta_t^L \downarrow$)
- By optimizing ϑ^L, CB can ensure efficient duration risk distribution
  $\Rightarrow$ requires **dynamic** balance sheet management

App

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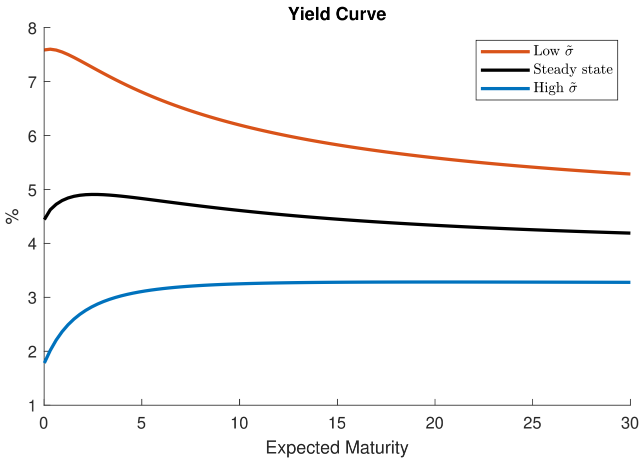
2. With full segmentation or short sell constraints on gov. bonds:
   $\Rightarrow$ CB is constrained in manipulating wealth distribution
   $\Rightarrow$ Competitive equilibrium is generically inefficient with full segmentation

Yield Curve Implications



- $y_t^\lambda = \frac{i^L}{P_t^L} + \lambda\left(\frac{1-P_t^L}{P_t^L}\right)$
- **term spread**:
  positive in recession
  negative in boom
- **hump** shape is optimal $\Rightarrow$ policy should not undo hump

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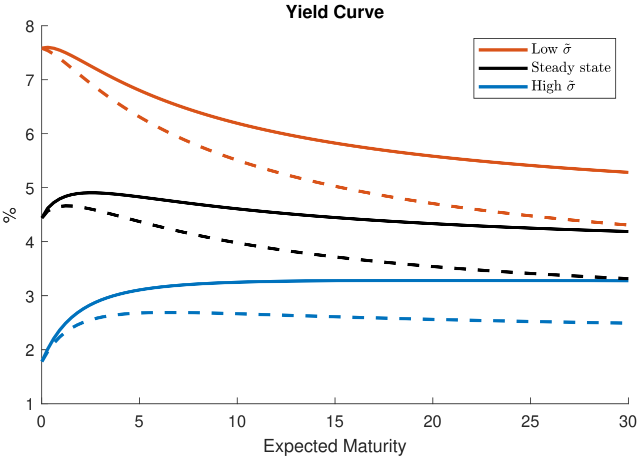
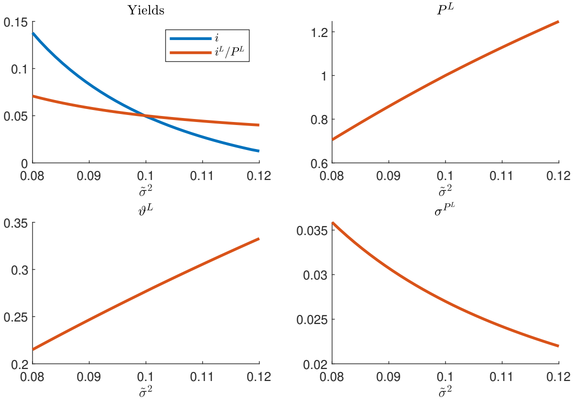


- **hump** shape also if price of risk for intermediaries is zero $\varsigma_t^I = 0$
- expectations hypothesis

Optimal Policy Over the Cycle

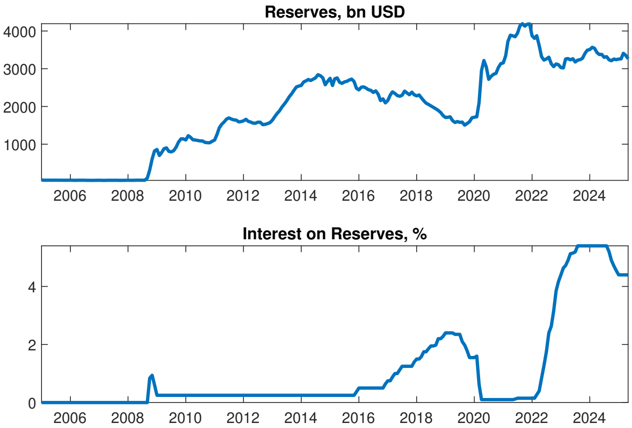
- **Proposition 5.** Let $\tilde{\sigma}_t^2$ be small. Then, an increase in $\tilde{\sigma}_t^2$ leads to:
  - a cut in the interest rate i_t
  - a milder interest rate policy going forward (lower $\sigma_t^{P^L}$)
  - a rebalancing towards long-term bonds (higher ϑ_t^L)



Summary

- Macro model with financial sector to study optimal policy mix
  - Role for aggregate and distributional efficiencies
- Larger CB balance sheet requires more aggressive interest rate policy subsequently
- Joint aggregate and distributional efficiency requires active balance sheet management over the cycle
- Optimal usage of balance sheet management is preparatory:
  - Efficient exposure to future interest rate moves
- Yield curve can be optimally humped shaped

Motivation



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Household's Problem

$$\max_{c_t^H,\upsilon_t,\iota_t,\nu_t,\theta_t^{D,H},\theta_t^{L,H},\theta_t^K,\chi_t} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log(c_t^H)-b(\upsilon_t)\right)dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H}dt + \theta_t^{D,H}dr_t^D + \theta_t^{L,H}dr_t^L + \theta_t^K\left(dr_t^K(\upsilon_t,\iota_t,\nu_t) - \chi_t dr_t^{x,H}\right) + \tau_t^H dt$$

$$1 = \theta_t^{D,H} + \theta_t^{L,H} + \theta_t^K(1-\chi_t) \qquad\qquad \nu_t\theta_t^{D,H} = \theta_t^K$$

- $dr_t^K = \left[\frac{p_t a\upsilon_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}_t(\nu_t) + \mu_t^{q^K} + g(\iota_t)\right]dt + \sigma_t^{q^K}dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^{x,H} = r_t^x dt + \sigma_t^{q^K}dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[i_t^D - \pi_t\right]dt$
- $dr_t^L = \frac{i^L}{P_t^L}dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right]dt + \sigma_t^{P^L}dZ_t$

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Intermediary's Problem

$$\max_{c_t^I,\theta_t^{\mathcal{R}},\theta_t^{L,I},\theta_t^{D,I},\theta_t^{x,I}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_t^I)dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^I}{n_t^I} = -\frac{c_t^I}{n_t^I}dt + \theta_t^{\mathcal{R}} dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) + \theta_t^{D,I} dr_t^D + \theta_t^{L,I} dr_t^L + \theta_t^{x,I} dr_t^{x,I} + \tau_t^I dt$$

$$1 = \theta_t^{\mathcal{R}} + \theta_t^{D,I} + \theta_t^{L,I} + \theta_t^{x,I} \qquad \theta_t^{\mathcal{R}} \geq \underline{\theta}_t^{\mathcal{R}}$$

- $dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) = i(\theta_t^{\mathcal{R}})dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[\frac{\underline{\theta}_t^{\mathcal{R}}\underline{i}_t + (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}})i_t}{\theta_t^{\mathcal{R}}} - \pi_t\right] dt$
- $dr_t^{x,I} = (r_t^x + \tau_t^x)\, dt + \sigma_t^{q^K} dZ_t + \varphi\tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[i_t^D - \pi_t\right] dt$
- $dr_t^L = \frac{i^L}{P_t^L}dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right] dt + \sigma_t^{P^L} dZ_t$

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Monopolistic Firms

- Monopolistic producers add variety to a common good produced by HH:
  - Linear technology: $Y_t^j = y_t^j$, set prices P_t^j s.t. Rotemberg frictions:

$$\int_0^\infty \Xi_t^H \left[\left(\frac{P_t^j}{P_t} \right)^{1-\varepsilon} - p_t(1-\tau^F) \left(\frac{P_t^j}{P_t} \right)^{-\varepsilon} - \frac{\kappa}{2} \left(\pi_t^j \right)^2 - T_t^F \right] Y_t dt$$

- Perfectly competitive final good producers
  - Bundle varieties into consumption good using CES aggregator
- NKPC:

$$\frac{\mathbb{E}\left[d\pi_t\right]}{dt} = \left(r_t^{f,H} - \frac{\mathbb{E}\left[dY_t\right]}{Y_t dt} + \varsigma_t^{C,H} \sigma_t^Y \right) \pi_t - \frac{\varepsilon}{\kappa} \left(p_t(1-\tau) - \frac{\varepsilon - 1}{\varepsilon} \right)$$

$$\pi_t = \frac{\varepsilon}{\kappa Y_t} \mathbb{E}_t \int_t^\infty e^{-\int_t^s r_\tau^f d\tau} Y_s \left(p_s(1-\tau) - \frac{\varepsilon - 1}{\varepsilon} \right) ds$$

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Equilibrium

- Key variables: $\tilde{\sigma}_t$, η_t, υ_t, ϑ_t, P_t^L, π_t
- Markovian equilibrium with state variables $S \equiv \{\tilde{\sigma}, \eta, \upsilon\}$:
  - Laws of motion for S:

$$d\tilde{\sigma}_t^2 = -b_s(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt + \sigma\tilde{\sigma}_t^2 dZ_t$$

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ_t$$

$$\frac{d\upsilon_t}{\upsilon_t} = \mu_t^\upsilon dt + \sigma_t^\upsilon dZ_t$$

  - Policy variables $\underline{i}(S)$, $i(S)$, $\vartheta^L(S)$, $\underline{\theta}^{\mathcal{R}}(S)$, $\tau^I(S)$, $\tau^x(S)$, $\tau^K(S)$
  - Mappings $\vartheta(S), P^L(S), \pi(S)$

satisfying agents' optimality and market clearing

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Forward-Looking Equations

- Nominal long-term bond price

$$\frac{\mathbb{E}\left[dP_t^L\right]}{P_t^L} = \mu_t^{P^L} = i_t^m - \frac{i^L}{P_t^L} + \sigma_t^{N^I}\sigma_t^{P^L}$$

- Money valuation equation

$$\frac{\mathbb{E}\left[d\vartheta_t\right]}{\vartheta_t} = \mu_t^{\vartheta} = \rho - s_t - (1-\vartheta_t)\Big((1-\vartheta_t^L)\Big(i_t^m - i_t^a + \nu_t^2\mathfrak{t}'(\nu_t)\Big) + \tilde{\sigma}_t^{n^H}\tilde{\sigma}_t\Big)$$
$$- (1-\eta_t)\theta_t^{D,H}\nu_t^2\mathfrak{t}'(\nu_t) + \sigma_t^{1-\eta}\sigma_t^{\vartheta}$$

- NKPC:

$$\frac{\mathbb{E}\left[d\pi_t\right]}{dt} = \left(r_t^{f,H} - \frac{\mathbb{E}\left[dY_t\right]}{Y_t dt} + \varsigma_t^{C,H}\sigma_t^Y\right)\pi_t - \frac{\varepsilon}{\kappa}\left(p_t(1-\tau_t) - \frac{\varepsilon-1}{\varepsilon}\right)$$

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Efficient Consumption & Risk Allocation Only: Implementation

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*)\sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^*(\eta_t^L - \chi_t^*))\vartheta_t^L \sigma_t^{PL}$$

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*}\sigma_t^{PL} = \nu_t^2 \mathfrak{t}'(\nu_t)$$

$$\nu_t \left[\chi_t^* - \eta_t^* + \vartheta_t^*(1 - \chi_t^*) - (1 - \eta_t^L)\vartheta_t^L \vartheta_t^*\right] = 1 - \vartheta_t^*$$

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Efficiency (Relaxed Problem)

- Planner's objective:

$$\max_{\{\upsilon_t,\iota_t,\eta_t,\chi_t,\vartheta_t\}_{t=0}^{\infty}} \lambda\mathbb{E}\int_0^{\infty} e^{-\rho t}\log(\tilde{\eta}_t^I\eta_t c_t K_t)dt + (1{-}\lambda)\mathbb{E}\int_0^{\infty} e^{-\rho t}\Big(\log(\tilde{\eta}_t^H(1{-}\eta_t)c_t K_t) - b(\upsilon_t)\Big)\,dt$$

$$\text{s.t. } c_t K_t = a\upsilon_t K_t - \iota_t K_t = \rho\frac{q_t^K}{1-\vartheta_t}K_t, \qquad q_t^K = (1+\phi\iota_t) \quad \text{Tobin's } q$$

$$\frac{d\tilde{\eta}_t^I}{\tilde{\eta}_t^I} = \chi_t\frac{1-\vartheta_t}{\eta_t}\varphi\tilde{\sigma}_t d\tilde{Z}_t, \qquad \frac{d\tilde{\eta}_t^H}{\tilde{\eta}_t^H} = (1-\chi_t)\frac{1-\vartheta_t}{1-\eta_t}\tilde{\sigma}_t d\tilde{Z}_t$$

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Efficiency (Relaxed Problem)

- Planner's objective can be simplified to a static one

$$\max_{\upsilon_t,\iota_t,\eta_t,\chi_t,\vartheta_t} W_t = \overbrace{\log(a\upsilon_t - \iota_t) - (1-\lambda)b(\upsilon_t) + \frac{1}{\rho}\left(\frac{1}{\phi}\log(1+\phi\iota_t) - \delta\right)}^{\text{aggregate efficiency}_t(\upsilon_t,\iota_t)}$$
$$\underbrace{+\lambda\log(\eta_t) + (1-\lambda)\log(1-\eta_t) - \frac{\tilde{\sigma}_t^2}{2\rho}\left[\lambda\frac{\chi_t^2}{\eta_t^2}\varphi^2 + (1-\lambda)\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right](1-\vartheta_t)^2}_{\text{distributional efficiency}_t(\chi_t,\eta_t,\vartheta_t)}$$

- Constrained efficient allocation $\upsilon^*(\tilde{\sigma})$, $\iota^*(\tilde{\sigma})$, $\chi^*(\tilde{\sigma})$, $\eta^*(\tilde{\sigma})$, $\vartheta^*(\tilde{\sigma})$,
- Optimal allocation: $1 > \chi^*(\tilde{\sigma}) > \eta^*(\tilde{\sigma}) > \lambda$

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Efficiency (Relaxed Problem)

- **Proposition 1.** Under some assumption on $\{\lambda, \varphi\}$, there exists a unique solution to the planner's problem for $\eta^*(\tilde{\sigma}_t) > \lambda$ and the constrained efficient allocation has the following properties:
  - **Aggregate Efficiency** (Output gap)
    - Capital utilization $\upsilon^*(\tilde{\sigma}_t)$ is constant ($\mu_t^{\upsilon,*} = \sigma_t^{\upsilon,*} = 0$)
    - Investment rate $\iota^*(\tilde{\sigma}_t)$ is decreasing in $\tilde{\sigma}_t$
  - **Distributional Efficiency**
    - Intermediaries' wealth and risk shares $\eta^*(\tilde{\sigma}_t)$ and $\chi^*(\tilde{\sigma}_t)$ are increasing in $\tilde{\sigma}_t$
    - Nominal wealth share $\vartheta^*(\tilde{\sigma}_t)$ is increasing in $\tilde{\sigma}_t$

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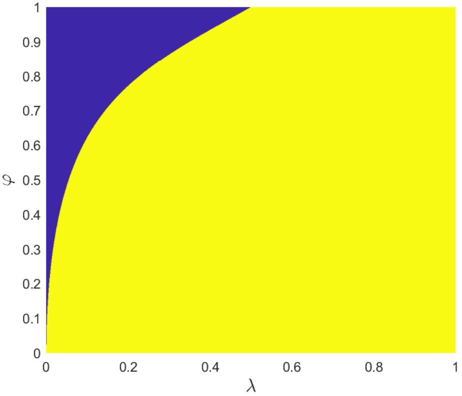
Constrained Efficiency: Properties

$$6\lambda(1-\lambda)(1-\varphi^2)(1-\lambda+\lambda\varphi^2)-(1-2\lambda)\varphi^2 \geq 0$$



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Consolidated Government

$$\mu_t^{\mathcal{R}}\mathcal{R}_t + P_t^L \mu_t^L L_t + \mathcal{P}_t \tau_t^K K_t = \underline{i}_t \underline{\mathcal{R}}_t + i_t(\mathcal{R}_t - \underline{\mathcal{R}}_t) + i^L L_t - \sigma_t^{P^L} \sigma_t^L P_t^L L_t$$
$$\sigma_t^{\mathcal{R}}\mathcal{R}_t + P_t^L \sigma_t^L L_t = 0$$

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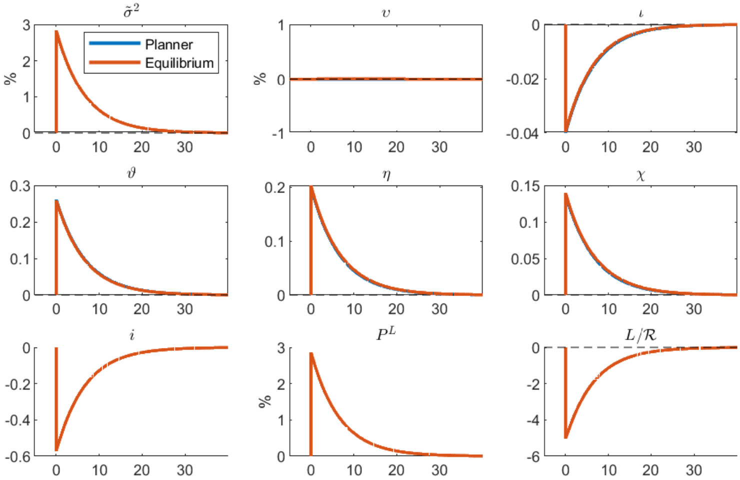
IRF under Full Efficiency



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Production Efficiency Only: Implementation

- ϑ_t is an equilibrium 'mapping'
  $\Rightarrow$ implement ϑ_t^* by appropriate capital taxes τ_t^K along the equilibrium path
- υ_t is a state variable $\Rightarrow$ need to ensure $\mu_t^\upsilon = \sigma_t^\upsilon = 0 \; \forall t$
  - Drift is targeted by $\underline{i}_t$
  - Volatility loading is targeted by i_t and ϑ_t^L
- From goods market clearing:

$$a\upsilon_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \iota_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \frac{q_t^K - 1}{\phi} = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \frac{q_t^{\mathcal{B}}(1 - \vartheta_t)}{\phi \vartheta_t} - \frac{1}{\phi}$$

$$q_t^{\mathcal{B}} = \frac{\mathcal{B}_t}{\mathcal{P}_t K_t} \qquad \mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$$

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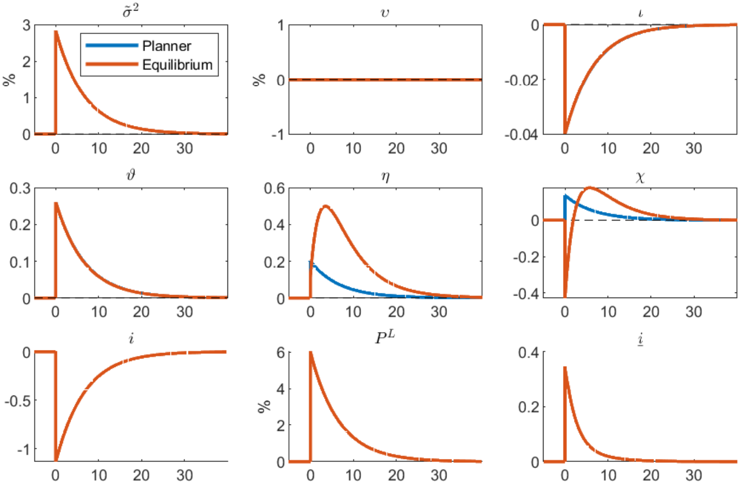
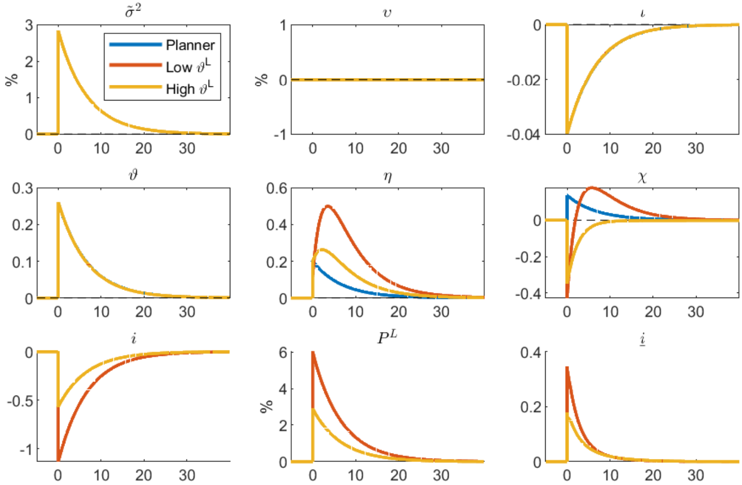
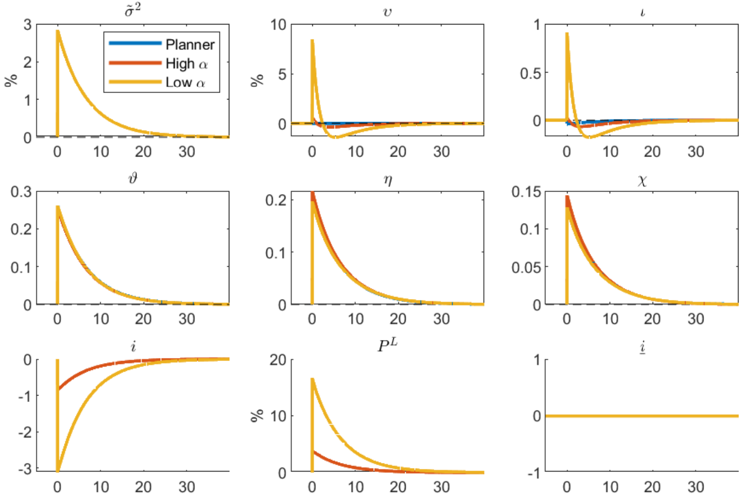
IRF under Production Efficiency



IRF under Production Efficiency: Equivalence



IRF under Allocative Efficiency: Multiplicity



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