Asset Purchases and Heterogeneous Beliefs

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Asset Purchases (APs) in Practice and in Theory

an overview

- Finance: focus on financial markets
  - ample empirical evidence that APs lower yields, mostly narrow effects
  - theory: segmented markets (e.g. preferred habitat) and portfolio rebalancing

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- Downward-sloping aggregate asset demand $\Leftarrow$ Heterogeneous asset demand schedules
- APs and dispersed information in the primary market for sovereign debt
  - heterogeneous demand schedules + info frictions (Cole, Neuhann, Ordoñez (2022, 2024))
  - beliefs respond to policy, $\neq$ from structural heterogeneity
  - study effects on information contained in prices

▸ price discovery

This Paper

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  - Info frictions + learning from prices → asset under/over priced vs fundamentals (Albagli, Hellwig, Tsyvinski (2023))

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- *Structural* heterogeneity very $\neq \Rightarrow$ monotonic price effects, > 0 central bank gains
- Optimality: consumption-saving problem where APs undo externality from info frictions

Literature

- Irrelevant under complete info & frictionless markets
  - Wallace (81), Backus Kehoe (89)
- Central bank replaces constrained banking sector
  - Curdia Woodford (11), Gertler Karadi (11), Chen et al. (12), Cui Sterk (21)
- Segmented markets and/or limits to arbitrage
  - Vayanos Vila (21), Costain et al. (22), Gourinchas et al. (22), Fanelli Straub (21), Itskhoki Mukhin (22)
- Commitment device
  - Mussa (81), Jeanne Svensson (07), Corsetti Dedola (16), Bhattarai et al. (22)
- Information frictions (signalling or behavioural agents)
  - Mussa (81), Iovino Sergeyev (21)

$\Rightarrow$ Dispersed info absent in existing macro theories

Outline

1. The impact of APs on prices/information/profits in financial mkts
   - **quantity target**
   - price target

2. Optimal APs in a stylised consumption-saving problem

Public Sector

- Government
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    otherwise, $b_{cb} = \widetilde{S} < \mathrm{b}$, market is "passive", and we assume $Q = \theta$

Investors

- Measure one of investors
- Portfolio allocation problem

$$\max_{b_i \in [0,1]} \quad \mathbb{E}\left[b_i(\theta - Q) \mid \Omega_i\right]$$

- Agent i's information set Ω_i
  1. Private signal: $x_i = \theta + \sigma_x \xi_i$, where $\xi_i \sim N(0,1)$ (define $x_i \sim \mathcal{N}$)
  2. Equilibrium bond price: Q
  3. Asset purchases: b

Timing

1. Shocks $(\theta, \widetilde{S})$ realise, are not observed
2. Investors receive signals, submit *price-contingent* demand schedules
3. Walrasian auctioneer clears the market through equilibrium price Q
4. Payoffs are realised

Equilibrium

Definition

Given an AP policy rule, a Perfect Bayesian Equilibrium consists of

- demand schedules $b(\Omega_i)$,
- a price function $Q(\theta, \widetilde{S}, b_{\text{cb}})$,
- and posterior beliefs $\mathbb{E}[\theta \mid \Omega_i]$

such that

(i) the demand schedules solve investors' problem given their posterior beliefs;

(ii) the price function $Q(\theta, \widetilde{S}, b_{\text{cb}})$ clears the bond market;

(iii) posterior beliefs satisfy Bayes' law for all market clearing prices.

Individual Strategies

- Agent i's strategy

$$\mathbb{E}\left[\theta - Q \,|\, x_i \sim \mathcal{N},\ Q,\ \mathrm{b}\right] \begin{cases} > 0 & \text{then} \quad b_i = 1 \\ < 0 & \text{then} \quad b_i = 0 \\ = 0 & \text{then} \quad b_i \in [0, 1] \end{cases}$$

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- Discussion:
  - can extend to short-selling/leverage: $b_i \in [-\underline{b}, \overline{b}]$
  - position bounds necessary, not sufficient, for non-neutrality
  - risk neutrality buys tractability, not essential

neutrality

Market Clearing and Price Signal

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- Marginal agent's private signal $=$ function of exogenous shocks $(\theta, \widetilde{S})$

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - \mathrm{b}\right) \qquad \text{(define } x_m \sim \mathcal{M}_{\mathrm{b}}\text{)}$$

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$$Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, Q, \mathrm{b}]$$

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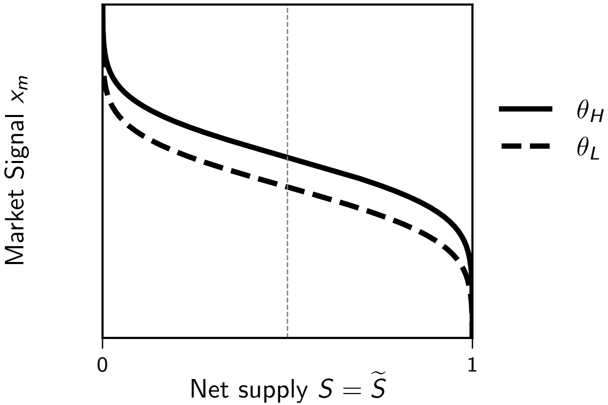
- $x_m(Q, \mathrm{b})$ is also the *price signal*. In equilibrium: $(\theta, \widetilde{S}) \overset{(\mathrm{b})}{\longleftrightarrow} x_m \overset{(\mathrm{b})}{\longleftrightarrow} Q$

Market Signal without APs $(\mathrm{b} = 0)$

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S}\right)$$



Market Signal with APs ($\mathrm{b} > 0$)

crowding out

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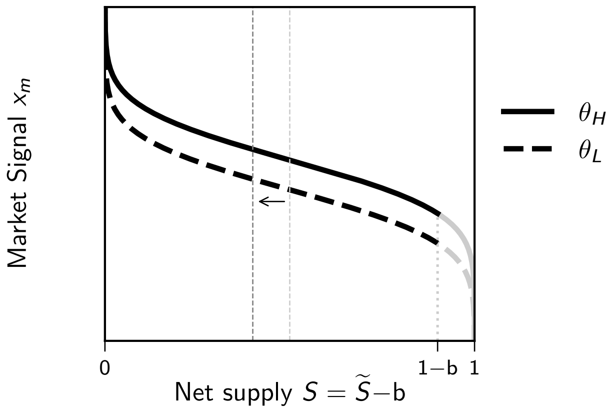

Market Signal with APs ($\mathrm{b} > 0$)

information revelation

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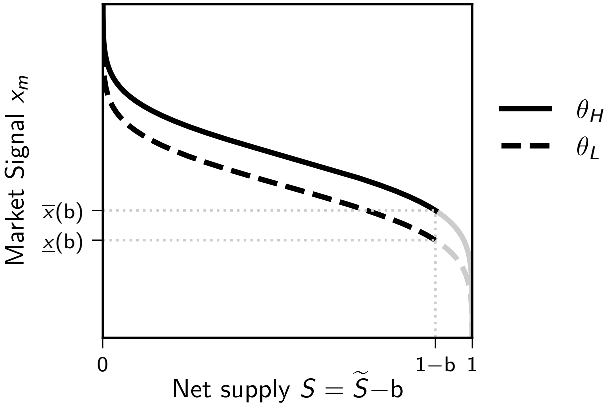

Posterior Beliefs and Equilibrium Price

- Probability of a high payoff

$$p(x_i, x_m) := P(\theta_H \mid \; x_i \sim \mathcal{N} \;, x_m \sim \mathcal{M}_{\mathrm{b}}) =$$

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- Marginal investor m's indifference condition $\Leftrightarrow$ Equilibrium price

$$Q(x_m) = \mathbb{E}[\theta \mid x_m \sim \mathcal{N}, x_m \sim \mathcal{M}_{\mathrm{b}}] \quad = p(x_m)\,\theta_H + (1 - p(x_m))\,\theta_L$$

where $p(x_m) = p(x_i, x_m)|_{x_i = x_m}$

"Bond Valuation" $\neq$ Equilibrium Price

- Condition only on public info: $x_m \sim \mathcal{M}_{\mathrm{b}}$

$$\widehat{p}(x_m) := P(\theta_H \,|\, x_m \sim \mathcal{M}_{\mathrm{b}}) = \begin{cases} \dfrac{q\,\phi\left(\dfrac{\theta_H - x_m}{\sigma_x}\right)}{\sum_j q_j\,\phi\left(\dfrac{\theta_j - x_m}{\sigma_x}\right)} & \text{if } x_m \in [\overline{x}(\mathrm{b}), +\infty) \\ 0 & \text{if } x_m \in [\underline{x}(\mathrm{b}), \overline{x}(\mathrm{b})) \end{cases}$$

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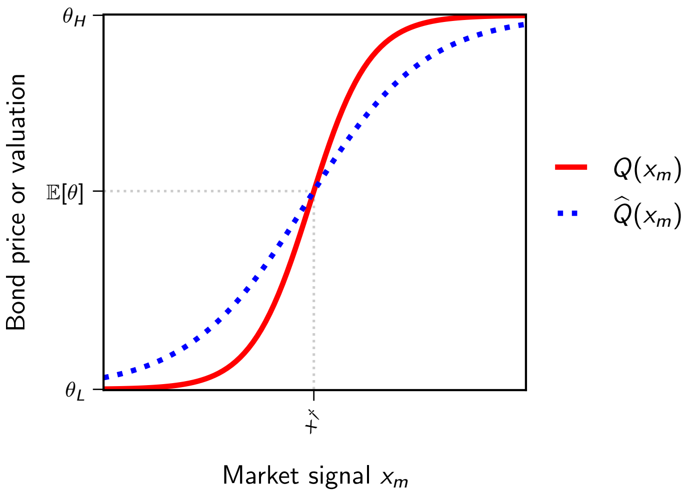
  – satisfies the L.I.E., its average is independent of APs

$$\mathbb{E}[\widehat{Q}] = \mathbb{E}[\,\mathbb{E}[\theta \,|\, x_m \sim \mathcal{M}_{\mathrm{b}}]] = \mathbb{E}[\theta] \quad \forall\, \mathrm{b}$$

The Effect of APs

without APs ($\mathrm{b} = 0$)



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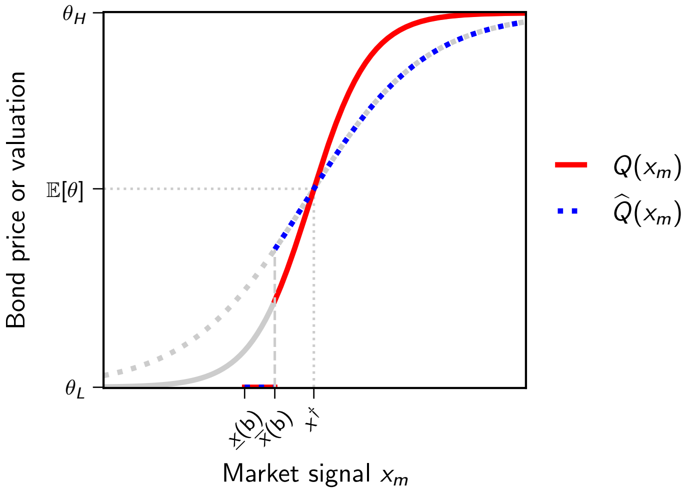

Average Prices and Returns

- The average bond valuation is independent of APs

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$$\begin{aligned}\mathcal{Q} &= \mathbb{E}[Q(x_m)] \\ &= \mathbb{E}[\theta] + \int_{\overline{\mathrm{x}}(\mathrm{b})} (Q(x_m) - \widehat{Q}(x_m))\mathrm{d}\, F_{\mathcal{M}_b}(x_m)\end{aligned}$$

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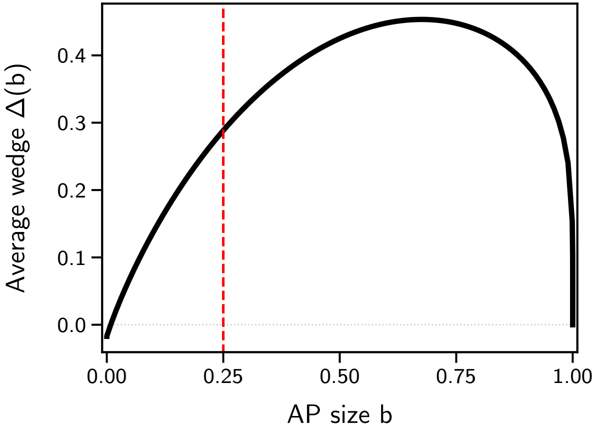
- Average wedge ≈ average bond premium

$$-\Delta(\mathrm{b}) = \mathbb{E}[\theta] - \mathcal{Q}$$

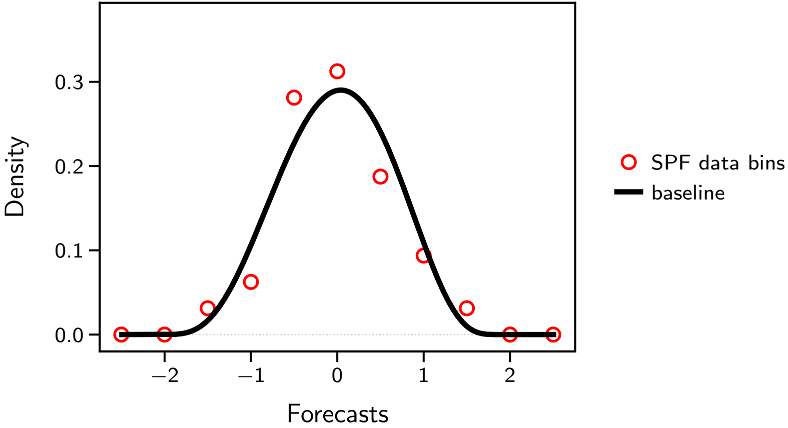
▸ interpretation

The Effect of APs on Average Prices



Dashed line = announced amount of US Treasury purchases on 18/03/2009, relative to outstanding marketable stock

The Effect of APs on Average Prices



Parametrised to match the forecast dispersion of expected real returns on 10 year US Treasuries, from the SPF of Q1-2009

▸ beliefs distribution

APs & the Distribution of Profits

- Central bank profits

$$\mathbb{E}[\pi_{\mathsf{cb}}] = \mathrm{b} \left(\widehat{\mathcal{Q}} - \mathcal{Q}\right)$$

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- Investor profits

$$\mathbb{E}[\pi_{\mathsf{inv}}] = \mathbb{E}[\widetilde{S} - \mathrm{b}]\left(\widehat{\mathcal{Q}} - \mathcal{Q}\right) + \mathsf{Cov}\left[\widetilde{S} - \mathrm{b}, \left(\theta - Q(x_m)\right)\right]$$

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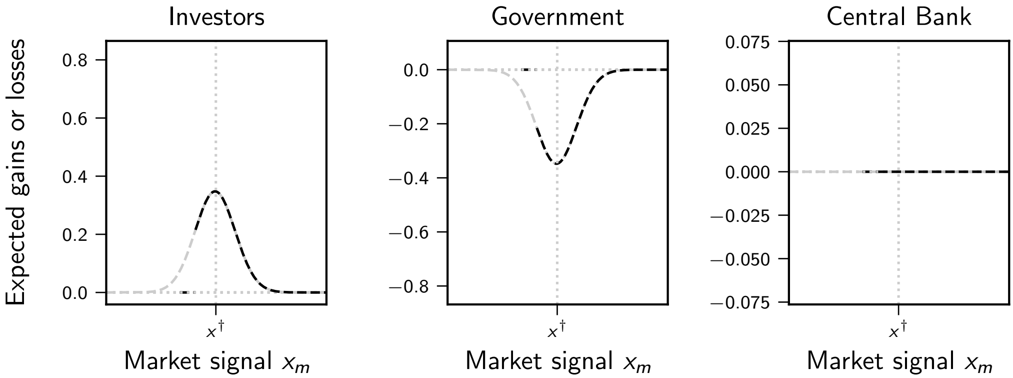
- Government profits

$$\mathbb{E}[\pi_{\text{gov}}] = -\mathbb{E}[\pi_{\text{inv}}] - \mathbb{E}[\pi_{\text{cb}}]$$

APs & the Distribution of Profits

conditional



Note: grey lines = no APs; black lines = positive APs

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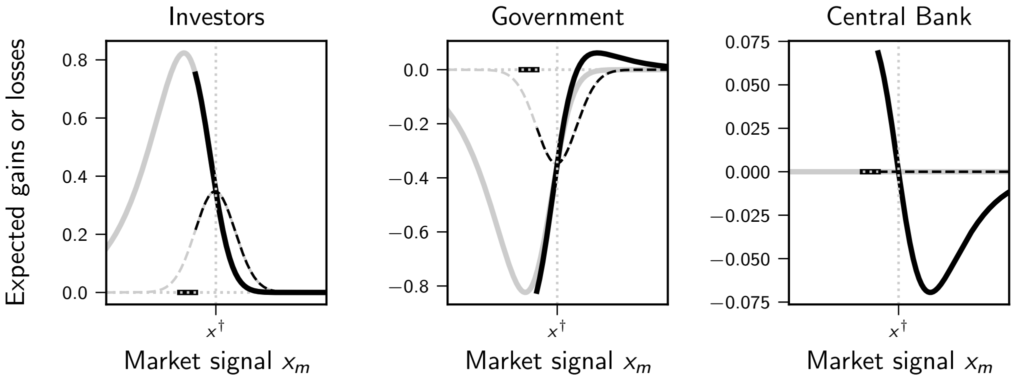

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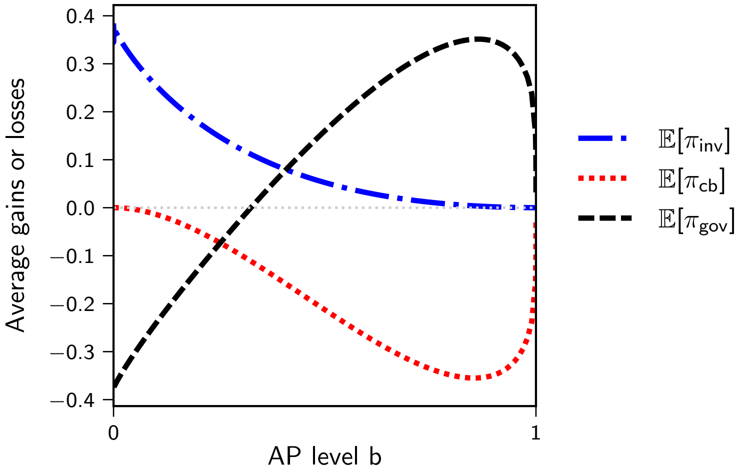

no wedge

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   - **price target** ▸ profits

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Price-targeting AP Policies

- CB *also* submits a limit order, to buy *up to* b at a price $Q \leq Q_n$
  - simultaneous to investors
  - actual APs given by $b_{cb} \in [0, \mathrm{b}]$
  - price target $Q_n \Leftrightarrow$ high-payoff probability target p_n
- $(b_{cb}, \mathrm{b}, Q_n)$ are perfectly observed by investors
- CB needs not observe $(\theta, \widetilde{S})$ to implement the policy

Price-Targeting APs

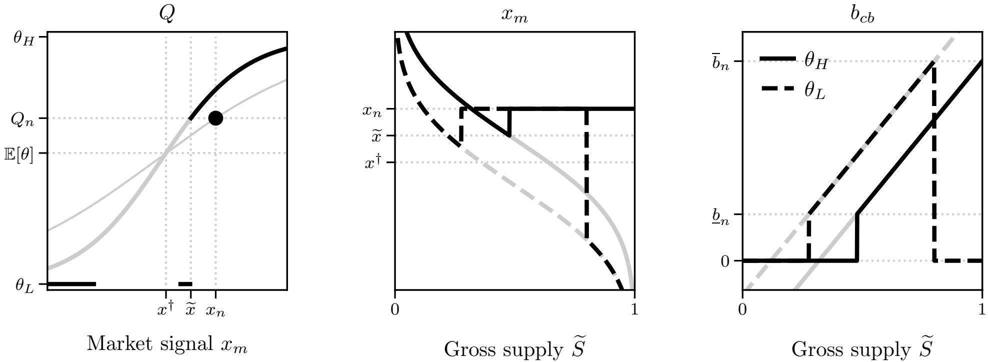
- No-APs region ($Q > Q_n$)
  - CB does not intervene, $b_{cb} = 0$
  - $Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, x_m \sim \mathcal{M}]$

- Targeted-price region ($Q = Q_n$)
  - CB intervenes and is unconstrained, $b_{cb} = \widetilde{S} - \Phi\left(\frac{\theta - x_n}{\sigma_x}\right) \in (0, \mathrm{b}]$
  - price signal Q_n is uninformative
    - CB becomes the marginal agent, Q_n inelastic to supply shocks
    - $b_{cb} \sim U$, independent from θ
    - $Q_n = \mathbb{E}[\theta \,|\, x_n \sim \mathcal{N}]$

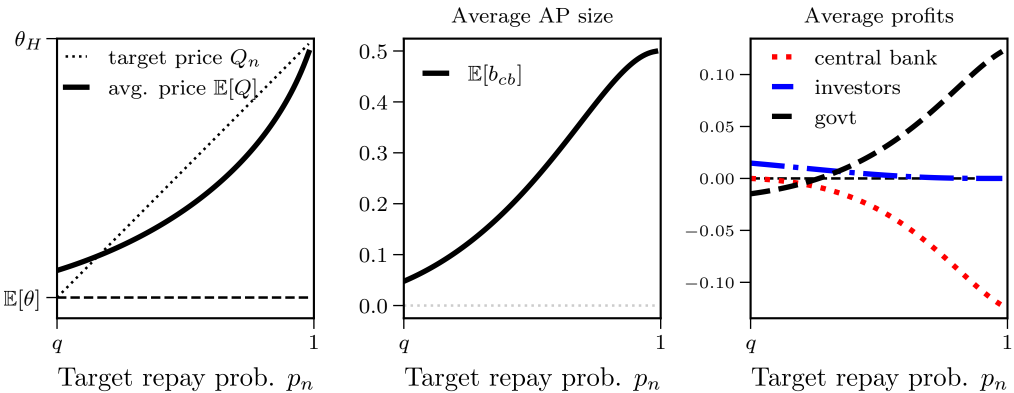
- Residual region
  - $Q < Q_n$ even if $b_{cb} = \mathrm{b}$
  - fully revealing, we assume $b_{cb} = 0$

Price-Targeting APs

$p_n > q$



Price-Targeting APs



CB average profits: $\mathbb{E}[\pi_{\text{cb}}] = P(Q = Q_n)\,\mathbb{E}[b_{cb}]\left(\mathbb{E}[\theta] - Q_n\right)$

welfare takeaways

Outline

1. The impact of APs on prices/information/profits in financial mkts
   - quantity target
   - price target

2. **Optimal APs in a stylised consumption-saving problem**

▸ takeaways

Macro Model in a Nutshell

- Two periods, no production, households + investors + government + central bank
- Households consume or deposit with investors at rate $\mathcal{R}$
  - deposit contracts signed before shocks are realised
  - investors perfectly compete for funds

$$\mathcal{R} = 1 + \frac{1}{s}\mathbb{E}[\pi_{\text{inv}}]$$

  - investors then learn and allocate funds into bonds or storage
- Social optimum: net rate of return on households' savings $= 0$

$\Rightarrow$ Welfare is increasing in the central bank quantity- or price-target insofar as

$$\mathcal{R} > 1 \quad \Leftrightarrow \quad \mathbb{E}[\pi_{\text{inv}}] > 0$$

more

Takeaways

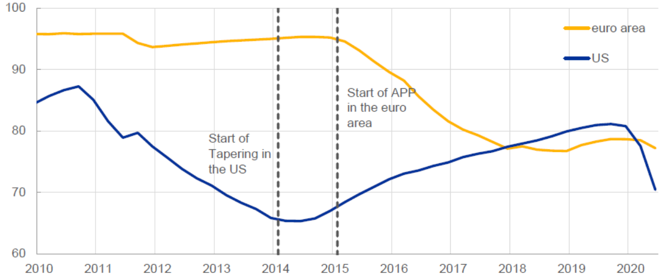
- A theory of APs with
  - dispersed info & learning from prices
  - limits to arbitrage
- Illustrate effects of (quantity/price-targeting) APs on
  - prices, and information contained therein
  - redistribution between govt, central bank and investors
- Optimality in a stylised consumption-saving model with financial intermediaries
  - limits to arbitrage create inefficiency in savings choice
  - APs reduce inefficiency via effects on learning-from-prices externality

Appendix

Asset Purchases, Information, and Asset Prices

Largest part of sovereign debt held outside of central banks, supporting price discovery

Developments in the bond free float (percent)



Sources: SHS, ECB, ECB Calculations.

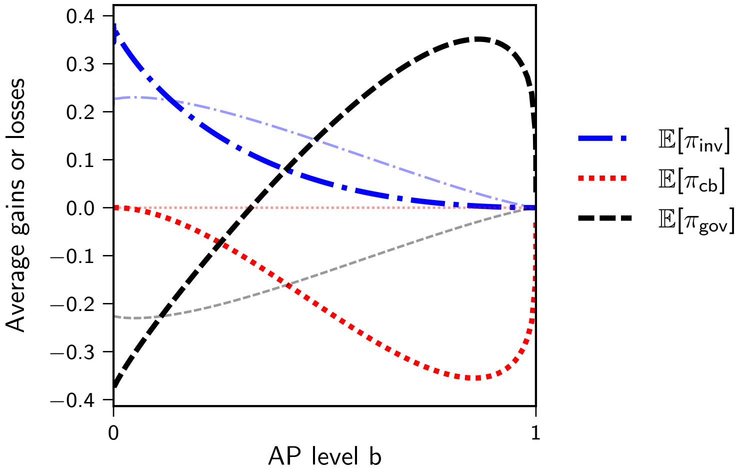
US Treasury bonds + Add to myFT

Investors struggle to hear signals from bond markets

Huge scale of Fed buying obscures once-reliable signs on the path of inflation

APs & the Distribution of Profits

unconditional



A Consumption-Saving Model with Intermediaries

Households

- In the first period, household j solves:

$$\max_{c_{j,0},c_{j,1},\{s_{j,i}\}_{i\in[0,1]}} \quad u(c_{j,0}) + u(c_{j,1})$$

$$\text{s.t.} \quad c_{j,0} = y - \int_0^1 s_{j,i}\,\mathrm{d}i \quad \text{and} \quad c_{j,1} = \int_0^1 \mathcal{R}_i s_{j,i}\,\mathrm{d}i + D - \tau$$

- Deposit contracts are signed before any shock realize: $s_{j,i} = s$ and $\mathcal{R}_i = \mathcal{R}$.

Investors and market clearing

- Investors maximize expected dividends: $\max_{b_i\in[0,1]} \quad \mathbb{E}[b_i(\theta - Q) - s_i(\mathcal{R} - 1) \mid \Omega_i]$
- *Ex ante* zero-profit condition gives $\mathcal{R} = 1 + \frac{1}{s}\mathbb{E}\left[\pi_{\text{inv}}\right]$

Government and Central Bank

- Government must consume a total of **G** in two periods:

$$g_0 = \tilde{S}Q \qquad \text{and} \qquad \mathbf{G} - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$$

- Central Bank:

$$a_{cb} = Qb_{cb} + k_{cb} \qquad \text{and} \qquad \theta b_{cb} + k_{cb} + \tau_{cb} = a_{cb}$$

- PVBCs:

$$\text{Govt: } \tau = \tau_{cb} + \mathbf{G} + \underbrace{\tilde{S}(\theta - Q)}_{\pi_{gov}} \qquad \text{and} \qquad \text{CB: } -\tau_{cb} = \underbrace{b_{cb}(\theta - Q)}_{\pi_{cb}}$$

- Consolidated public sector PVBC: $\tau - \mathbf{G} = (\tilde{S} - b_{cb})(\theta - Q)$
  - when $Q = \theta$ the budget is balanced, debt only due to time mismatch

Equilibrium and Efficiency

- Resource constraints

$$c_0 = y - s \qquad \text{and} \qquad c_1 = s - G$$

- Households' Euler equation (after using market clearing and budget identities)

$$u'(c_0) = \mathcal{R} u'(c_1)$$

- Planner problem's Euler equation

$$u'(c_0) = u'(c_1)$$

Proposition

Welfare is increasing in the central bank quantity-target b_{cb} or price-target Q_n insofar as

$$\mathcal{R} > 1 \quad \Leftrightarrow \quad \mathbb{E}\left[\pi_{inv}\right] > 0.$$

Interpretation, Beliefs Distribution

Average wedge interpretation

- Generic bond pricing model with SDF $= Z$: $Q(x_m) = \mathbb{E}[Z\theta \mid x_m]$
- Risk-free rate: $1 + r = 1/\mathbb{E}[Z \mid x_m]$
- Bond pricing equation: $\text{Cov}(Z, \theta \mid x_m) = \mathbb{E}[\theta \mid x_m] - Q(x_m)$
- Average wedge in our model: $-\Delta(\mathrm{b}) = \mathbb{E}[\theta] - \mathcal{Q}$

  $\Rightarrow$ Natural counterpart of the (average) equilibrium bond premium

average wedge

Individual expectation of the learning wedge

$$\mathbb{E}[\theta - Q(x_m) \mid x_i] = \int_{-\infty}^{+\infty} [\theta - Q(x_m)] f_{\mathcal{M} \mid \mathcal{N}}(x_m \mid x_i) \mathrm{d}x_i$$

$\Rightarrow$ Counterpart of the individual bond premium expectation: $\mathbb{E}[\text{Cov}(Z, \theta \mid x_m) \mid x_i]$

SPF

Neutrality

- Investor i's problem: $\max_{c_i, b_i} \mathbb{E}[u(c_i)|\Omega_i] \quad \text{s.t.} \quad c_i = b_i(\theta - Q) + y - \tau$
- Asset market clearing: $\int b_i \, \mathrm{d}i + b_{cb} = S$
- Budget of consolidated public sector: $\tau = (\widetilde{S} - b_{cb})(\theta - Q)$

Neutrality

- Investor i's problem: $\max_{c_i, b_i} \mathbb{E}[u(c_i)|\Omega_i] \quad \text{s.t.} \quad c_i = b_i(\theta - Q) + y - \tau$
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- Budget of consolidated public sector: $\tau = (\widetilde{S} - b_{cb})(\theta - Q)$

⇒ Rewrite budget constraint: $c_i = (b_i + b_{cb} - \widetilde{S})(\theta - Q) + y$

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- Investor i's problem: $\max_{c_i, b_i} \mathbb{E}[u(c_i)|\Omega_i] \quad \text{s.t.} \quad c_i = b_i(\theta - Q) + y - \tau$
- Asset market clearing: $\int b_i \, \mathrm{d}i + b_{cb} = S$
- Budget of consolidated public sector: $\tau = (\widetilde{S} - b_{cb})(\theta - Q)$

$\Rightarrow$ Rewrite budget constraint: $c_i = (b_i + b_{cb} - \widetilde{S})(\theta - Q) + y$

(a) Limits to arbitrage ($b_i \in [\underline{b}, \overline{b}]$) + No info frictions ($\Omega_i = \Omega$)

- RA market clearing, $c_i = c$, all agents on EE $\rightarrow$ $\mathbb{E}[u'(c)(\theta - Q) \mid \Omega] = 0$

(b) No limits to arbitrage + Info frictions

- Each i on own EE, interior solution for each i $\rightarrow$ $\mathbb{E}[u'(c_i)(\theta - Q) \mid \Omega_i] = 0$

$\Rightarrow$ **Homogeneous crowding out, APs irrelevant**

ind. strats welfare price-targeting