The Optimal Supply of Central Bank Reserves under Uncertainty

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1st Banca d'Italia Annual Research Conference on Monetary Policy

June 12, 2025

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Motivation

- What is the optimal supply of reserves the central bank should provide?
- Central banks have multiple goals in supplying reserves, including
  - ◇ Targeting policy rate
  - ◇ Managing balance-sheet size
- Different jurisdictions have different implementation frameworks
- Banks' demand for reserves is highly nonlinear and uncertain

This Paper

- Tractable analytical framework for optimal supply of reserves
- Demand for reserves is nonlinear and uncertain
  - Regions: abundant (zero slope), ample (gentle slope), scarce (steep slope)
  - Uncertainty: horizontal shocks, vertical shocks, and slope shocks
- Central bank targets: interbank market rate & aggregate reserve level
- Central bank controls reserve supply
  - Extension: lending facility rate

Our Main Results

- Characterizes optimal supply of reserves and associated equilibrium

1) Optimal supply under uncertainty exceeds that absent risk

2) *Abundant* reserves are optimal with sufficient degree of uncertainty
   - Even if central bank prefers ample reserves absent uncertainty

3) Optimal mean policy rate may be higher or lower than absent uncertainty

4) Lending facility reduces optimal reserve supply (if rate is chosen optimally)

Reserve Demand Curve

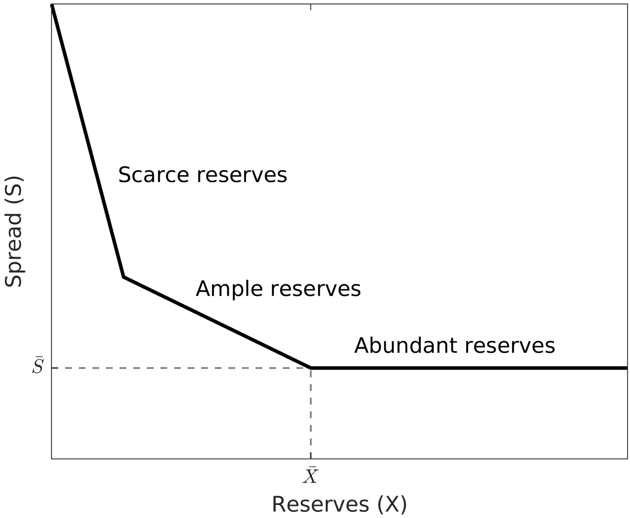
- Price at which banks are willing to trade reserves as a function of total reserve
- Banks receive benefits from holding reserves but also incur (opportunity) costs
  - ◇ Benefit: buffer against liquidity shocks to meet liquidity targets
  - ◇ Cost: spread between market rate and interest rate on reserve balances
- Marginal net benefits decline with quantity of reserves held

⇒ Satiation point: curve becomes flat as reserves become sufficiently large

Reserve Demand Curve

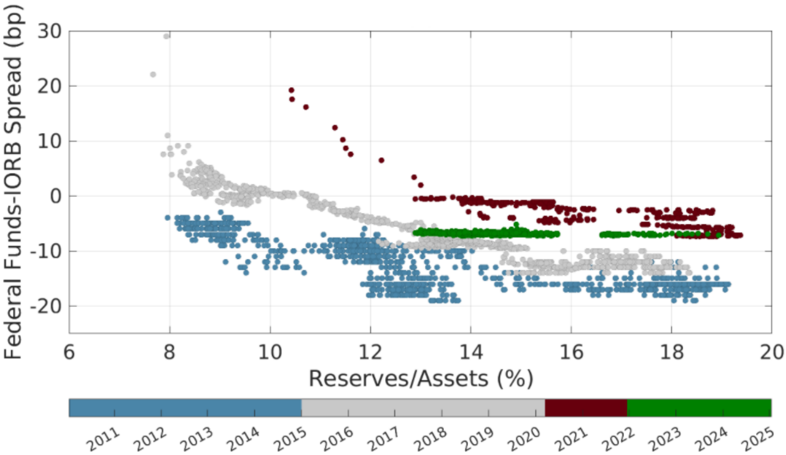
- The demand for reserves is nonlinear:
  - ◇ At high levels of reserves, demand curve is flat: *abundant reserves*
  - ◇ At intermediate levels, demand curve is gently sloped: *ample reserves*
  - ◇ At low levels, demand curve is steeply sloped: *scarce reserves*
- Reserve ampleness based on slope of reserve demand curve

Reserve Demand Curve



- $\bar{X}$: satiation point (banks' reserve targets + distribution of liquidity shocks)
- $\bar{S}$: lower asymptote (banks' balance-sheet costs + outside option)

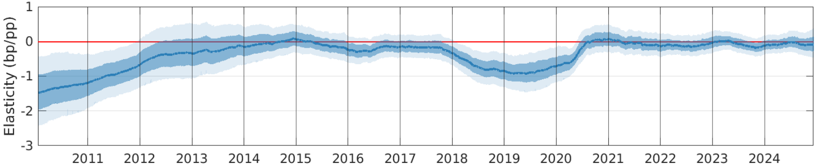
Empirical Evidence: Nonlinear and Uncertain Demand



- Afonso, Giannone, La Spada, and Williams (2022)
  - ◇ Evidence of both horizontal and vertical shifts

Empirical Evidence: Estimation Uncertainty about Slope

- Afonso, Giannone, La Spada, and Williams (2022)
  - ◇ Time-varying IV estimate of the slope moving along the curve
  - ◇ Slope: zero above satiation & increasingly negative below
  - ◇ Satiation point: 12% of bank assets in 2010–2014 → 13% in 2015–2020



Simple Model with Two Regions

- Spread between market rate and interest on reserves

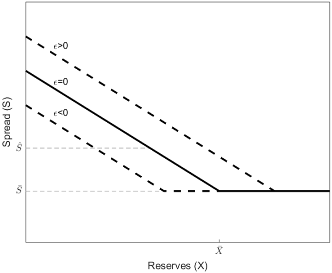
$$S(X) = \begin{cases} \bar{S} + \nu - (\alpha + \eta)(X - \bar{X} - \epsilon) & \text{if } \epsilon > X - \bar{X} \\ \bar{S} + \nu & \text{else} \end{cases}$$

- X: reserves; $\alpha > 0$: slope in the ample region

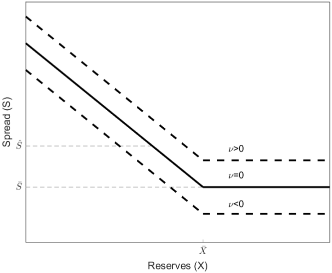
1) Horizontal shock: ϵ shifts the kink in the demand curve ($\bar{X}$)
2) Vertical shock: ν shifts the demand curve up and down ($\bar{S}$)
3) Slope shock: η changes the slope of the demand curve (α)

- ϵ, ν, and η: independent mean-zero random variables with known variances
- ϵ: pdf $g(\cdot)$ and cdf $G(\cdot)$

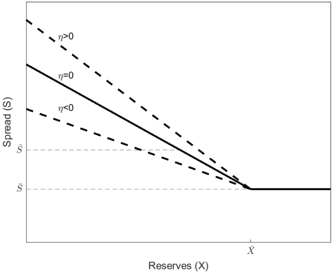
A Graphical Representation of Uncertainty



Horizontal ϵ shocks



Vertical ν shocks



Slope η shocks

Central Bank Optimization

$$\mathscr{L} = \min_{X} \frac{1}{2}\mathbb{E}\left[(S - \hat{S})^2 + \lambda(X - \hat{X})^2\right]$$

- $S = S(X)$ piecewise-linear demand for reserves described above
- Central bank minimizes weighted sum of expected squared deviations ($\lambda \geq 0$):
  1) distance between realized spread S and target spread $\hat{S}$
  2) distance between level of reserves X and target level $\hat{X}$
- $\hat{S}$: efficiency goals based on prices (Friedman rule: $\hat{S} = \bar{S}$. Frictions $\Rightarrow \hat{S} > \bar{S}$)
- $\hat{X}$: welfare goals based on quantity of public liquidity (e.g., financial stability)

Assumptions

1) X is chosen before the realization of shocks

2) No ability to learn

3) Ample reserves are optimal absent uncertainty: $\hat{S} > \bar{S}$ and $\hat{X} < \bar{X}$

- Loss is equivalent to

$$\mathscr{L} = \min_X \frac{1}{2} \left((\mathbb{E}S - \hat{S})^2 + \mathrm{Var}[S] + \lambda (X - \hat{X})^2 \right),$$

  - Minimize squared distance between expected spread & target spread
  - Minimize spread variability

Optimal Reserves Absent Uncertainty

- The first-order condition (FOC) is:

$$-\alpha(S - \hat{S}) + \lambda(X - \hat{X}) = 0$$

- Using demand curve $\rightarrow$ optimal deterministic supply of reserves:

$$X^* = \bar{X} - \frac{\lambda}{\alpha^2 + \lambda}\left(\bar{X} - \hat{X}\right) - \frac{\alpha}{\alpha^2 + \lambda}(\hat{S} - \bar{S}).$$

  - $\diamond$ X^* increases less than one-for-one with the target level of reserves
  - $\diamond$ X^* decreases with the target spread

- Resulting optimal spread S^*:

$$S^* = \hat{S} - \frac{\lambda}{\alpha^2 + \lambda}(\hat{S} - \bar{S}) + \frac{\alpha\lambda}{\alpha^2 + \lambda}(\bar{X} - \hat{X})$$

Model Implications under Uncertainty

- Define cutoff level of the shock when reserves become ample as

$$\bar{\epsilon}(X) = X - \bar{X}$$

- The mean spread is greater than absent uncertainty for every reserve level

$$\mathbb{E}S = \bar{S} - \alpha(X - \bar{X}) + \alpha\mathscr{G}(\bar{\epsilon})$$

- $\mathscr{G}(\cdot)$ is the super-cumulative distribution function

$$\mathscr{G}(\bar{\epsilon}) = \int_{-\infty}^{\bar{\epsilon}} G(\epsilon)d\epsilon > 0$$

Optimal Supply of Reserves under Uncertainty

- FOC determines implicit function for X^{**}

$$X^{**} = X^{*} + \frac{\alpha^2 + \sigma_\eta^2}{\alpha^2 + \sigma_\eta^2 + \lambda}\mathscr{G}(\bar{\epsilon}) + \frac{\alpha}{\alpha^2 + \lambda}(\hat{S} - \bar{S})G(\bar{\epsilon})$$
$$+ \frac{\sigma_\eta^2}{(\alpha^2 + \lambda)(\alpha^2 + \sigma_\eta^2 + \lambda)}\Big(\alpha(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \lambda(\bar{X} - \hat{X})\Big)$$

1) Uncertainty about vertical shifts *has no effect* on optimal reserves

2) Uncertainty about horizontal shifts *increases* optimal reserves

3) Uncertainty about the slope of demand *increases* optimal reserves

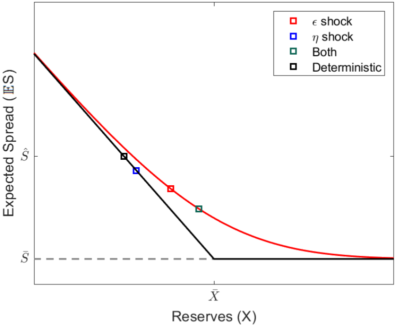
The Optimal Mean Spread

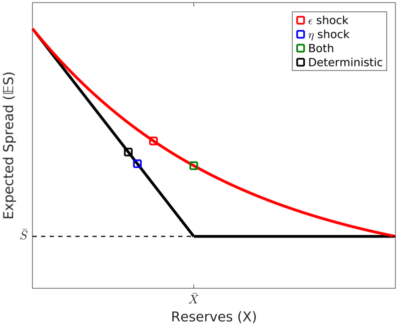
- Uncertainty may increase or decrease the optimal mean spread

$$\mathbb{E}S^{**} = \bar{S} + \left(\frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}\right)\left(\lambda(\bar{X} - \hat{X}) + \lambda\mathscr{G}(\bar{\epsilon}) - (1 - G(\bar{\epsilon}))\,(\hat{S} - \bar{S})\right) \gtrless S^*$$

- Two effects on the mean spread work in opposite directions
  - ◇ Positive (direct) effect of higher uncertainty for a given value of X
  - ◇ Negative (indirect) effect of a higher optimal level of X.
- Optimality cannot be judeged by the average level of the spread

The Optimal Mean Spread





When is Abundant Supply of Reserves Optimal?

- Assumption: ample reserves are optimal absent uncertainty $\left(\hat{X} < \bar{X}\right)$

- Abundant reserves are optimal if uncertainty is sufficiently large
  - ◇ Relative to penalty on deviations from target reserve level λ

- $X^{**} > \bar{X}$ is optimal if

$$(\alpha^2 + \sigma_\eta^2)\mathscr{G}(\bar{\epsilon}) - \alpha\left(1 - G(\bar{\epsilon})\right)(\hat{S} - \bar{S}) > \lambda\left(\bar{X} - \hat{X}\right)$$

  - ◇ Higher λ (e.g., high cost of large balance sheet) $\Rightarrow$ Harder to meet condition

Extension to a Third Region: Scarce Reserves

- Scarce reserves: $X < \bar{X}_2$ (steep slope)
  - Ample: $\bar{X}_2 \leq X < \bar{X}_1$ (gentle slope). Abundant: $X \geq \bar{X}_1$ (zero slope)
- Optimal solution in three-region model is

$$X^{***} = X^{**} + \frac{\beta(1 - G(\bar{\epsilon}_2))}{\alpha^2 + \sigma_\eta^2 + \lambda}\left(\bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) - \hat{S}\right) > X^{**}$$

- Incorporating scarce reserves region increases optimal supply of reserves
  - Presence of third region relaxes condition for optimality of abundant supply

Lending Facility

- Central bank offers frictionless lending facility at fixed spread S^B

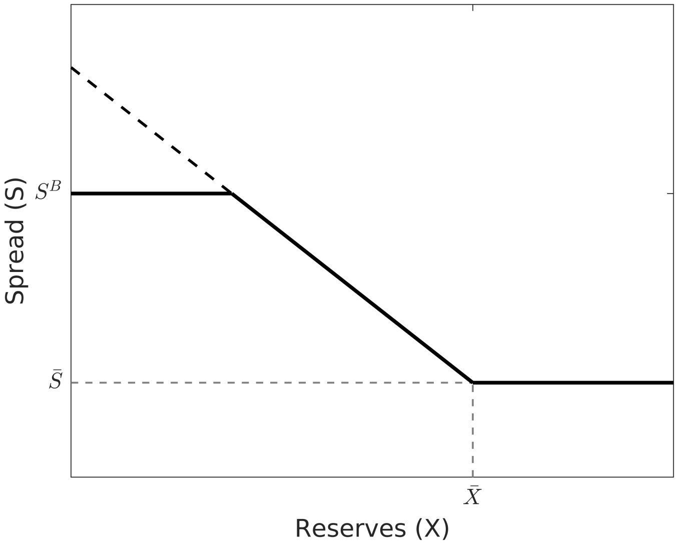
- Demand above S^B is fully met by borrowing from the facility

$$B = \begin{cases} -\frac{1}{\alpha}\left(S^B - \bar{S}\right) - (X - \bar{X} - \epsilon) & \text{if} \quad \bar{S} - \alpha(X - \bar{X} - \epsilon) > S^B \\ 0 & \text{else} \end{cases}$$

- $\bar{\epsilon}^B = X - \bar{X} + \dfrac{1}{\alpha}(S^B - \bar{S})$: cutoff demand shock for facility utilization

- Assumption: only horizontal demand shocks

Visual Representation of Demand with Lending Facility



Central Bank Optimization with Lending Facility

$$\mathscr{L} = \min_{X, S^B} \frac{1}{2} \left\{ \mathbb{E} \left[(S - \hat{S})^2 + \lambda (X - \hat{X})^2 + \lambda^B B \right] \right\},$$

- Assumption: central bank does not like large facility utilization
  - Penalty term: $\lambda^B B$ with $\lambda^B \geq 0$
- $B = B(S^B, \epsilon)$ depends on facility spread and demand shock
- Exogenous lending spread: $X^{**B} \gtrless X^{**}$
  - Lending facility $\Rightarrow$ cap on rates $\Rightarrow$ lower optimal reserves
  - Cost of facility usage $\Rightarrow$ reduce borrowing $\Rightarrow$ higher optimal reserves

Optimal Reserve Supply with (Optimal) Lending Facility

- Optimal lending spread is above target: $S^B = \hat{S} + \frac{\lambda^B}{\alpha} \geq \hat{S}$
- Optimal reserve supply is lower than absent the facility: $X^{**B} < X^{**}$
- Optimal lending spread increases with λ^B: $\frac{dS^B}{d\lambda^B} = \alpha^{-1} > 0$
- Optimal supply of reserves increases with λ^B: $\frac{dX^{**B}}{d\lambda^B} = \frac{1 - G(\bar{\epsilon}^B)}{\Gamma} \geq 0$
  - ⋄ $\Gamma > 0$ is the SOC for X
- Optimal lending-facility rate: expected cost of utilization ↓ but cap on rates ↑

Conclusion

- We study the optimal supply of central bank reserves under uncertainty
- Analytically tractable framework that can be adjusted to different jurisdictions
- Uncertainty about demand for reserves implies larger optimal supply
- Abundant reserves can be optimal even if cental bank prefers ample reserves
- Lending facility lowers optimal supply of reserves (if rate chosen optimally)