Search and Matching in Structural Labour Supply Modelling

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Outline

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Introduction

- Novel integration of discrete choice frameworks with macro-search and matching models.
- Capturing both labour supply heterogeneity and demand-side constraints while allowing for frictional unemployment.
- Part of the EUROLAB project

Discrete-Choice Labour Supply Models

- Seminal works: van Soest (1995), Aaberge et al. (1995), McFadden (1974)
- Advantages in capturing heterogeneity. Deal with kink an discontinuities in the tax-schedule
- Limitations: labour demand side often only partially modelled in restrictive way.

Incorporating Labour Demand

- Existing approaches (Colombino (2013), Peichl and Siegloch (2012)).
- Important contribution of endogenising unemployment and wage adjustments in the EUROLAB model(Narazani and Colombino (2021))
  - Unemployment was here modelled as something voluntary and not a result of friction, where unemployment get's adjusted through providing more unemployment slots that individual chooses for.
  - Ooghe et al. (2025) show that the voluntary / involuntary nature really improves the welfare ranking of policy alternatives.
  - our approach here is to model unemployment as a result of friction. Advantages are
    - Closer to economic reality
    - Important for welfare evaluation
    - Modelling of firm-side explicitly also allows us to consider e.g. a productivity shock

Search and Matching Models

- Key contributions: Diamond (1982), Mortensen (1982), Pissarides (1985).
- Macro models having a micro-foundation for searching: Rogerson et al. (2005).

Model Overview

- We model the behaviour of households that we consider to be *flexible*
- Three-stage model:
  1. **Search decision**: The individual decides to search (costlessly) if there is an option out there that is preferred to being inactive
  2. **Opportunity Set Allocation**: Random process depending on characteristics of the individual
  3. **Labour Choice**: Individual chooses preferred bundle out of opportunity set.
- 3 Different models:
  - two flexible earners modelled as a unitary household
  - single flexible male model
  - single flexible female model

Utility Function

- Discrete number of hours worked possible $L = \{0, \ell^1, \dots, \ell^J\}$
- Consumption $c = w \cdot \ell - T(w \cdot \ell; z_h)$
  with $\ell \in L$,
  w denoting the hourly gross wage,
  z_h denoting the vector of household characteristics.
- Utility:

$$U(c_h, \ell_h, z_h, \epsilon_h) = u(c_h, \ell_h; z_h) + \epsilon_h^\ell,$$

with ϵ_h^ℓ: random opportunity component

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Single model

Search decision

Individual searches if

$$\exists \ell \in L \backslash \{0\} : U(c(\ell), \ell, z_h, \epsilon_h^{\ell}) > U(c(0), 0, z_h, \epsilon_h^{0})$$

Probability of an individual to search when not observing the random utility component given by

$$P(S|z_h) = 1 - \frac{e^{u(c(0),0,z_h)}}{\sum_{\ell' \in L} e^{u(c_h,\ell',z_h)}}$$

Opportunity Set Stage

- Individuals receive an opportunity set. The distribution of opportunities will depend on the vector of observable characteristics z_h
- probability of an opportunity to be present in the opportunity set is independent of the other opportunities

$$P(O|z_h) = \Pi_{\ell \in O} p(\ell|z_h) \Pi_{\ell \notin O} \left(1 - p(\ell|z_h)\right),$$

with

$$p(\ell|z_h) = \frac{e^{f(\ell|z_h,\theta)}}{1 + e^{f(\ell|z_h,\theta)}}$$

Labour choice stage

- The individual will choose the consumption labour bundle that maximises her utility.
- From the econometrician point of view, the probability that the individual will choose the (c, ℓ) bundle equals

$$\frac{e^{u(c_h,\ell,z_h)}}{\sum_{\ell' \in O} e^{u(c_h,\ell',z_h)}}$$

This is under the assumption that the random utility components differ in the search stage and the the choice stage!

Independence of random utility component

- Independence simplifies the likelihood contribution significantly.
- Otherwise one needs to keep track of Preference orderings that rationalise the supply decision and the choice made.
- Number of preference orderings will grow with the number of alternatives.

Likelihood contribution

Different specifications for the likelihood contribution:

- $\ell = 0$

$$P(NS|Z_h) + P(S|z_h) \sum_{O \in \Omega | \ell \in O} P(O|z_h) \times \frac{e^{u(c_h, \ell, z_h)}}{\sum_{\ell' \in O} e^{u(c_h, \ell', z_h)}}$$

- $\ell > 0$

$$P(S|z_h) \sum_{O \in \Omega | \ell \in O} P(O|z_h) \times \frac{e^{u(c_h, \ell, z_h)}}{\sum_{\ell' \in O} e^{u(c_h, \ell', z_h)}}$$

- unemployed

$$P(S|z_h) P(O = \{0\} | z_h)$$

Couple Model

Search Decision

- 4 possible states here: both partners might be searching or not
- Probability of observing a state where **both partners supply** is the probability that the household prefers an option **where both spouses work**
- Similarly, the probability of observing a state where **one of the partners supplies** is the probability that the household prefers one of the options where **the supplying spouse would work and the partner would not.**

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Opportunity Set Stage

- Modelled similarly to the probability of singles
- Probabilities of an opportunity with a certain number of hours being present for primary and secondary earner are independent.

Choice Stage

Given the opportunity set O, we can use the familiar multinomial logit expression to express the choice probability (conditional on the opportunity set)

Likelihood contribution

- Consider separate cases for when one of the earners is observed to be inactive or not
- We sum over all possible opportunity sets that contain the observed choice

Labour Demand: Micro side

- The demand side will affect the probability that an individual is presented with a certain labour opportunity. Through the inclusion of a constant in the specification of $p(\ell|z_h)$
- Demand side will also affect the gross hourly wage level w.

Labour demand: Macro Side

- One representative firm having the following profit function

$$\Pi = Z\,L - W\,L - \kappa\,V,$$

  where $Z\,L$ denotes production with Z being productivity and L the number of workers, W the aggregate average wage that pays per worker hired and κ the cost of posting one vacancy.
- Firm chooses number of vacancies (V)

Matching Demand and Supply: Unemployment

- Number of matches produced are a function of individuals searching (N_S) and Vacancies (V)
- Matching Function: $M(N_s, V) = AN_s^{\gamma} V^{1-\gamma}$.
- The number of unemployed is determined as $N_s - M(N_s, V)$.

Matching Demand and Supply: Wages

we follow an approach similar to that of McKay and Reis (2021) where the wage adjustment is governed by a wage shifter, ω, defined as:

$$\omega = \left(\frac{1-u}{1-\overline{u}} \right)^{\psi},$$

with $\overline{u}$ the unemployment in the baseline
and u the unemployment rate,
ψ drives elasticity of wages with respect to unemployment

Firm Behaviour

- Market tightness: $\theta = \frac{V}{N_s}$.
- The probability that a vacancy is filled $q(\theta) = \frac{M}{V}$
- Firm behaviour with respect to vacancies determined via First Order Condition of the firm's problem: $(Z - W)(1 - \gamma)q(\theta) = \kappa$

Data and Sample Selection

- Data sources: EUROMOD input data (derived from EU and National- SILC survey)
- Selection criteria: Individuals aged 18-65, excluding retirees, students, and disabled.
- Estimation for Belgium, Cyprus, Austria, and Spain.

Estimation Methodology

- Maximum likelihood estimation for the micro side.
- gross hourly wage for inactive and unemployed estimated correcting for sample selection bias
- Macro function is calibrated to when we deem economy in equilibrium
- Counterfactual are generated through EUROMOD
- Computational implementation: Python, Cython, and dynamic C-code generation.

*But that can be adjusted through ω

Computing a new equilibrium

In order to compute a new equilibrium, we use the following iterative programme.

1. Simulate counterfactuals and Calculate aggregated Labour Supply
2. Calculate new unemployment rate using matching function
3. Calculate change in wages through ω using McKay and Reis Formula
4. Repeat step 1 to 3 with wage adjustment through ω until change in Matches falls below tolerance level.
5. Adjust probabilities of receiving job opportunities proportionally through shifters in the $p(\ell|z_h)$ functions to match the unemployment level coming from the macro-structure

Parametrisation of Utility Function

- Utility function includes linear and non-linear terms.
- Heterogeneity in consumption preferences: age, number of children, migrant status.
- Calibration to match observed unemployment rate and average wages.

Reform scenario: In-Work Benefit

- Description of the reform: 20% of gross equivalised household income phased out between 50% and 70% of the median gross equivalised household income*
- Our exercise is to predict labour outcomes with and without Labour Demand adjustment

*Equivalised using OECD equivalence scales

Impact on Labour Market Outcomes

Table: Elasticities on the extensive and intensive margin

participation	hours worked	country
0.1851	0.1960	ES
0.1900	0.2294	AT
0.0762	0.0852	CY
0.0238	0.0566	BE

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Change in willing to supply labour

Table: Change in Number of individuals searching for a job

country	relative change LS (%)
ES	0.46
AT	0.55
CY	0.36
BE	0.19

Impact on Labour Market Outcomes

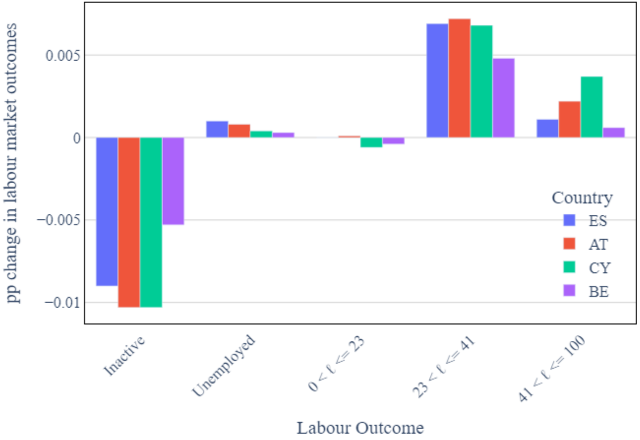

Figure: Change in Labour outcomes without Demand adjustment

Labour Demand adjustment

Table: Wage shifter for LD adjustment

country	wage shifter ω
ES	0.9978
AT	0.9987
CY	0.9997
BE	0.9997

Wage shifters for each country after Labour Demand adjustment

Changes in Labour Outcomes

Table: Change in Labour outcomes compared to baseline

Inactive	Unemployed	$0 < \ell <= 23$	$23 < \ell <= 41$	$41 < \ell <= 100$	Scenario	country
-0.0090	0.0010	0.0000	0.0069	0.0011	IWB	ES
-0.0089	0.0006	0.0001	0.0069	0.0014	IWB (LD)	ES
-0.0103	0.0008	0.0001	0.0072	0.0022	IWB	AT
-0.0102	0.0004	0.0001	0.0070	0.0027	IWB (LD)	AT
-0.0103	0.0004	-0.0006	0.0068	0.0037	IWB	CY
-0.0103	0.0002	-0.0006	0.0067	0.0040	IWB (LD)	CY
-0.0053	0.0003	-0.0004	0.0048	0.0006	IWB	BE
-0.0053	0.0001	-0.0004	0.0048	0.0009	IWB (LD)	BE

Changes in Labour outcomes (measured in pp) with and without labour demand adjustment compared to the baseline scenario.

Budgettary implications

Δc	Δ benefits	Δ taxes	Scenario	country
0.74 %	2.07 %	0 %	IWB (mechanical)	ES
0.86 %	2.11%	0.01 %	IWB	ES
0.70 %	2.12%	-0.38 %	IWB (LD adjusted)	ES
0.47 %	1.19 %	0 %	IWB (mechanical)	AT
0.56 %	1.11 %	0 %	IWB	AT
0.47 %	1.12 %	-0.33 %	IWB (LD adjusted)	AT
1.20 %	2.45 %	-6.19 %	IWB (mechanical)	CY
1.31 %	2.35 %	-6.18 %	IWB	CY
1.28 %	2.35 %	-6.25 %	IWB (LD adjusted)	CY
0.19 %	0.47%	0 %	IWB (mechanical)	BE
0.24 %	0.57 %	-0.03 %	IWB	BE
0.23 %	0.57%	-0.08 %	IWB (LD adjusted)	BE

This table portrays relative changes in aggregated disposable income (c), simulated benefits and simulated taxes compared to the baseline scenario.

Conclusions

- Summary of key findings: integration of micro and macro frameworks.
- Incorporation of search and matching framework leads to a coherent integration of frictional unemployment in structural labour supply modelling using DSM
- This is of first order importance when considering the welfare impact through money-metrics utilities for example
- Future research directions: incorporating firm heterogeneity and incorporating search intensity in the model

Thank you for your attention!

Questions and Discussion

Search Decision

- Both partners search if

$$\exists(\bar{\ell},\tilde{\ell}) \in L\backslash\{0\} \times L\backslash\{0\} : U(c(\bar{\ell},\tilde{\ell}),\bar{\ell},\tilde{\ell},z_h,\epsilon_h^{\ell}) > U(c(0),0,0,z_h,\epsilon_h^0)$$
$$\wedge\, U(c(\bar{\ell},\tilde{\ell}),\bar{\ell},\tilde{\ell},z_h,\epsilon_h^{\ell}) > U(c(0),\ell',0,z_h,\epsilon_h^0) \quad \forall \ell' \in L\backslash\{0\}$$
$$\wedge\, U(c(\bar{\ell},\tilde{\ell}),\bar{\ell},\tilde{\ell},z_h,\epsilon_h^{\ell}) > U(c(0),0,\ell'',z_h,\epsilon_h^0) \quad \forall \ell'' \in L\backslash\{0\}$$

The probability that both earners search in a household is hence the probability that one of the options were both earners provide positive utility is preferred:

$$P(\bar{S},\tilde{S}|z_h) = \sum_{(\bar{\ell},\tilde{\ell})\in L\backslash\{0\}\times L\backslash\{0\}} \frac{e^{u(c(\bar{\ell},\tilde{\ell}),\bar{\ell},\tilde{\ell})}}{\sum_{(\bar{\ell}'',\tilde{\ell}'')\in L^2} e^{u(c_h,\bar{\ell}'',\tilde{\ell}'',z_h)}}$$

Search Decision

- Only the spouse searches $(\overline{NS},\widetilde{S})$:

$$P(\overline{NS},\widetilde{S}|z_h) = \frac{\sum\limits_{\widetilde{\ell}' \in L^+} e^{u(c_h,0,\widetilde{\ell}',z_h)}}{\sum\limits_{(\overline{\ell}'',\widetilde{\ell}'')\in L^2} e^{u(c_h,\overline{\ell}'',\widetilde{\ell}'',z_h)}}$$

- Only the head searches $(\overline{S},\widetilde{NS})$:

$$P(\overline{NS},\widetilde{S}|z_h) = \frac{\sum\limits_{\overline{\ell}' \in L^+} e^{u(c_h,\overline{\ell}',0,z_h)}}{\sum\limits_{(\overline{\ell}'',\widetilde{\ell}'')\in L^2} e^{u(c_h,\overline{\ell}'',\widetilde{\ell}'',z_h)}}$$

Search Decision

- Neither member searches $(\overline{NS}, \widetilde{NS})$:

$$P(\overline{NS}, \widetilde{NS}|z_h) = 1 - P(\overline{S}, \widetilde{S}|z_h) - P(\overline{NS}, \widetilde{S}|z_h) - P(\overline{S}, \widetilde{NS}|z_h)$$
$$= \frac{e^{u(c_h,0,0,z_h)}}{\sum_{(\overline{\ell}'',\widetilde{\ell}'')\in L^2} e^{u(c_h,\overline{\ell}'',\widetilde{\ell}'',z_h)}}$$

Couple Model: Supply options

	0	U	PT	FT
0	$(NS\|S, NS\|S)$	$(NS\|S, S)$	$(NS\|S, S)$	$(NS\|S, S)$
U	$(S, NS\|S)$	(S, S)	(S, S)	(S, S)
PT	$(S, NS\|S)$	(S, S)	(S, S)	(S, S)
FT	$(S, NS\|S)$	(S, S)	(S, S)	(S, S)

PT=Part-Time,FT = Full-Time, NS= Not Supplying, S=Supplying