Moral Hazard with Risk-Sharing and Safe Debt

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Motivation

- In models of risk-sharing, moral hazard is usually a concern:
  - With effort the agent can improve its risk-profile.
  - Effort is not contractible and the principal must infer from observable outcomes.
- Solution: Mechanism design with incentive compatibility constraints (ICC).
  - Reward and punishment based on observed performance (Holmstrom 1979)

$\Longrightarrow$ *Trade-off* between efficiency and risk-sharing.

Research Questions

- Can we design constrained efficient contracts that minimize distortions to risk sharing?
- What is the provision of incentives in such contracts?
- What are the underlying welfare properties?

This Paper

1. Optimal design of Financial Stability Fund:
   - Contract between risk averse sovereign (agent) and risk neutral Fund (principal).
   - Two sided limited enforcement (LE) constraints + moral hazard (MH).
2. Two specifications of moral hazard:
   - Generalize the flexible moral hazard à la Georgiadis et al. (2024) to dynamic contracts.
   - Contrast with canonical dynamic moral hazard à la Atkeson and Lucas (1992).

Canonical and Flexible Moral Hazard

- Canonical moral hazard:
  - Effort translates into stochastic dominance over ex-ante given distributions.
  - *Contracting principle*: Reward and punish based on observed performance.
- Flexible moral hazard:
  - Agent can choose in advance any ex-post distribution directly, each with different costs.
  - *Contracting principle*: Reward marginal cost beyond minimum performance.

Main results

Literature

1. We find that the Fund under flexible vs canonical MH:
   - Rewards based on cost of choosing distribution instead of observed performance.
   - No disruption of risk sharing.
   - Bliss as opposed to immiseration in Atkeson and Lucas (1992).
2. Bridging the two approaches
   - In canonical MH back-loading incentives dampen disruption on risk sharing.
   - Propose a notion of restricted flexible moral hazard.
3. Quantitative implications of ranking different approaches
   - Difference not too big but interaction with limited enforcement constraint may change

Outline

General Setting

- Small open economy in infinite discrete time:

  1. One risk neutral lender (i.e. the Fund) with discounting $\frac{1}{1+r}$.
  2. Risk averse sovereign borrower with discounting $\beta \leq \frac{1}{1+r}$ and additive separable utility.

- Sovereign borrower is a benevolent government:

  – Production technology $y = \theta f(n)$ where θ follows a Markov chain of order 1, $\pi(\theta'|\theta)$.

  – Stochastic expenditure $g \in G$ with $\overline{g} = \max\{G\}$ and $\underline{g} = \min\{G\}$.

- Exogenous state vector is $s \equiv (\theta, -g)$.

Outline

Incentive Compatibility Constraint I

- Borrower can choose any distribution μ over G
- Convex cost of generating a specific distribution μ is $\nu(\mu) = K\left[\int(\overline{g} - g)\mu(\mathrm{d}g)\right]$

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Assumptions:

1. $v(\mu)$ is Gateaux twice differentiable.
2. If μ first-order stochastically dominates μ' then $v(\mu) \geq v(\mu')$.
3. $v_\mu(\overline{g}) = 0$.

This enables us to adapt the first-order approach of Rogerson (1988) to our setting.

Incentive Compatibility Constraint II

Given $c(s^t)$ and $n(s^t)$ the distribution μ_{t+1} solves

$$\mu_{t+1} = \underset{\tilde{\mu}}{\text{argmax}} \overbrace{u(c(s^t)) + h(1 - n(s^t)) - v(\tilde{\mu})}^{U(c(s^t), n(s^t), \mu_{t+1})} + \beta \sum_{\theta' | \theta} \pi(\theta' | \theta) \left[\int V^b(s^{t+1}) \tilde{\mu}(\mathrm{d}g^{t+1}) \right],$$

Incentive Compatibility Constraint II

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if, and only if, it also solves

$$\mu_{t+1} = \underset{\tilde{\mu}}{\operatorname{argmax}} \; u(c(s^t)) + h(1 - n(s^t)) + \sum_{\theta'|\theta} \pi(\theta'|\theta) \left[\int \left[\beta V^b(s^{t+1}) - \frac{v_{\mu_{t+1}}(g^{t+1}) + m(s^{t+1})}{\pi(\theta'|\theta)} \right] \tilde{\mu}(\mathrm{d}g^{t+1}) \right].$$

Incentive Compatibility Constraint III

- For any s^{t+1}, $t \geq 0$,

$$v_{\mu_{t+1}}(g^{t+1}) = \beta\pi(\theta' \mid \theta) V^b(s^{t+1}) - m(s^{t+1}). \tag{ICC}$$

Incentive Compatibility Constraint III

- For any $s^{t+1}, t \geq 0$,

$$v_{\mu_{t+1}}(g^{t+1}) = \beta\pi(\theta' \mid \theta)V^b(s^{t+1}) - m(s^{t+1}). \tag{ICC}$$

- Since $v_{\mu_{t+1}}(\bar{g}) = 0$,

$$v_{\mu_{t+1}}(g^{t+1}) = \beta\pi(\theta' \mid \theta) \underbrace{\left[V^b(s^{t+1}) - V^b(\{\theta^{t+1}, -\bar{g}\})\right]}_{\text{Reward above minimum performance}}$$

$\Longrightarrow$ Incentive towards compensation marginal cost of reducing expenditure below $\bar{g}$

Participation Constraints

- Participation constraint of the borrower:

$$\mathbb{E}\Big[\sum_{j=t}^{\infty} \beta^{j-t} U(c(s^j), n(s^j), \mu_{j+1}) \Big| s^t\Big] \geq \underbrace{V^o(s^t)}_{\text{Value under default}}. \tag{PCb}$$

- Participation constraint of the lender:

$$\mathbb{E}\Big[\sum_{j=t}^{\infty} \big(\frac{1}{1+r}\big)^{j-t} \big[\underbrace{\theta_t f(n(s^t)) - c(s^t) - g_t}_{\text{Primary surplus}}\big] \Big| s^t\Big] \geq Z. \tag{PCl}$$

Fund Contract

Details

The Fund contract in sequential form solves:

$$\max_{\{c(s^t),n(s^t),\mu_{t+1}\}_{t=0}^{\infty}} \mathbb{E}\Big[\alpha_{b,0} \overbrace{\sum_{t=0}^{\infty} \beta^t U(c(s^t), n(s^t), \mu_{t+1})}^{\text{Value of sovereign}} + \alpha_{l,0} \overbrace{\sum_{t=0}^{\infty} \Big(\frac{1}{1+r}\Big)^t \big[\theta_t f(n(s^t)) - c(s^t) - g_t\big]}^{\text{Value of lender}}\Big]$$

$$\text{s.t. (PCb), (PCl) and (ICC) for all } (t, s^t), t \geq 0.$$

Lagrange multiplier attached to (ICC) is $\varrho_\mu(s')$.

Fund Contract: recursive form

Details

Following Marcet and Marimon (2019), the Fund contract in recursive form solves:

$$FV(x,s) = \mathrm{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,\mu\}} \Bigg\{ \boldsymbol{x}\Big[(1+\nu_b)U(c,n,\mu) - \nu_b V^o(s)\Big]$$
$$+ \Big[(1+\nu_l)[\theta f(n) - c - g] - \nu_l Z\Big]$$
$$+ \sum_{\theta'|\theta} \pi(\theta'|\theta)\Bigg[\int \Bigg[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - \boldsymbol{x}\varrho_\mu(s')\frac{v_\mu(g') - m(s')}{\pi(\theta'|\theta)}\Bigg]\mu(\mathrm{d}g')\Bigg]\Bigg\}$$

$$\text{s.t.} \quad \boldsymbol{x}'(s') = \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varrho_\mu(s')}{1+\nu_l}\right]\eta \boldsymbol{x}.$$

ν_b is Lagrange multiplier to (PCb), ν_l to (PCl) and $\varrho_\mu(s')$ to (ICC).

Optimal Distribution

- Proposition: if third Gateaux derivative is zero, optimal μ has only one g' in support.

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- Lemma: when $g' = \overline{g}$, then $\varrho_\mu(s') = 0$. Otherwise, $\varrho_\mu(s') > 0$.
- The expected next-period Pareto weight:

$$\mathbb{E}_t x_{t+1}(s^{t+1}) \equiv \mathbb{E}_t \left[\bar{x}_{t+1}(s^t) + \hat{x}_{t+1}(s^{t+1})\right] = \mathbb{E}_t \left[\frac{1+\nu_{b,t}(s^t)}{1+\nu_{l,t}(s^t)} x(s^t) + \frac{\varrho_{\mu_{t+1}}(s^{t+1}|s^t)}{1+\nu_{l,t}(s^t)} x(s^t)\right] \eta.$$

Optimal Distribution

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Proposition

Without (PCI) *and with* $\eta \equiv \beta(1+r) = 1$, $\mathbb{E}_t x_{t+1}(s^{t+1}) \geq x(s^t)$.

Ex-post value of the sovereign converges (as a submartingale) to Bliss

Outline

Distribution and Effort Cost

- Borrower can choose among a family of distribution $Q = \zeta(e)Q_H + (1 - \zeta(e))Q_L$.
  - Effort $e \in [0, 1]$ with convex cost $\hat{v}(e)$ and weighting function $\zeta(e)$.
  - Joint distribution of θ and g is $\Pi(s'|s, e)$.
- Assumptions:
  1. $F_j(e, s) = \sum_{i=1}^{j} \Pi(\{\theta_i, g'\}|s, e)$ is twice differentiable in e.
  2. Q_H first-order stochastically dominates Q_L.
  3. $\hat{v}(0) = 0$.

Limited Enforcement and Moral Hazard

- Up to the redefinition of the utility function, (PCb) and (PCl) remain the same.
- Optimal choice of effort given by

$$e(s^t) = \underset{\tilde{e}}{\text{argmax}} \;\; \hat{U}(c(s^t), n(s^t), \tilde{e}) + \beta \sum_{s^{t+1}|s^t} \Pi(s^{t+1}|s_t, \tilde{e}) \hat{V}^b(s^{t+1}).$$

The (ICC) is therefore

$$\hat{v}_e(e(s^t)) = \beta \sum_{g^{t+1}|g^t} \frac{\partial \Pi(g^{t+1}|g_t, e)}{\partial e} \hat{V}^b(s^{t+1}). \qquad \text{(ICC)}$$

Optimal Effort

- Optimal effort is interior, i.e. $e(s^t) \in (0,1)$.
- With $\partial_e \Pi(s^{t+1}|s^t, e) \gtreqless 0$, the law of motion of the relative Pareto weight is

$$x_{t+1}(s^{t+1}) \equiv \left[\bar{x}_{t+1}(s^t) + \hat{x}_{t+1}(s^{t+1})\right] = \left[\frac{1+\nu_{b,t}(s^t)}{1+\nu_{l,t}(s^t)} x(s^t) + \frac{\varrho \frac{\partial_e \Pi(s^{t+1}|s^t,e)}{\Pi(s^{t+1}|s^t,e)}}{1+\nu_{l,t}(s^t)} x(s^t)\right] \eta.$$

- As we have that $\mathbb{E}_t \frac{\partial_e \Pi(s^{t+1}|s^t,e)}{\Pi(s^{t+1}|s^t,e)} = 0$,

$$\mathbb{E}_t x_{t+1}(s^{t+1}) = \bar{x}_{t+1}(s^t).$$

Outline

Back Loading

Details

- We split the contract into sequence of subprograms with perfect risk sharing.
- A new subprogram starts when one of the participation constraints binds.
    - Need to avoid *exit*.
    - When (PCb) binds only reward is possible (close to flexible MH).
- Within subprogram:
    - Consumption decays at rate $\eta \leq 1$.
    - Incentives are recorded by a latent multiplier $\bar{x}$.

Restricted Flexible Moral Hazard

- The flexible MH can be restricted to the family of distribution Q.

$$v(Q_e) = K\left[\int (\overline{g} - g) Q_e(\mathrm{d}g)\right] \quad \text{where} \quad Q_{\tilde{e}} = \overline{\zeta}(\tilde{e}) Q_L + (1 - \overline{\zeta}(\tilde{e})) Q_H.$$

- If Q is convex set and locally flexible, our characterization continues to hold true:
  - Corner effort, i.e. $e \in \{0, 1\}$.
  - Bliss instead of immiseration.

Outline

Parametrization

Details

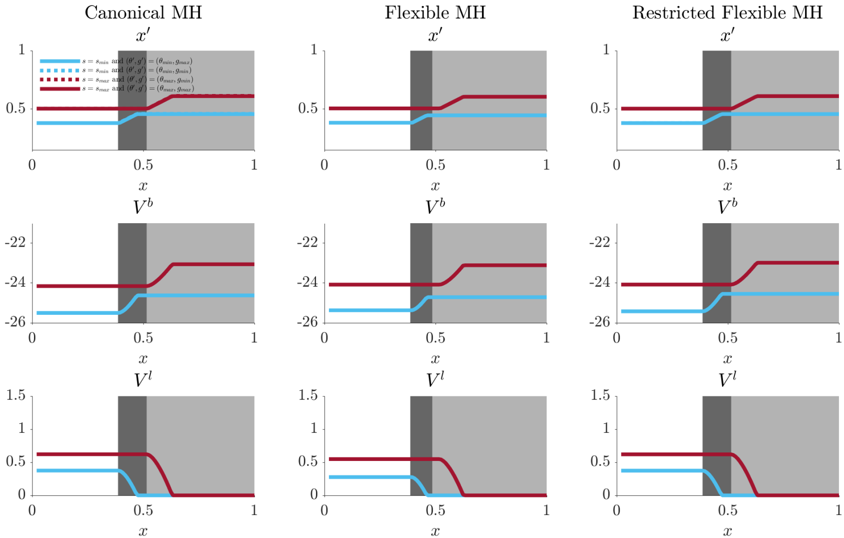
We parametrize the model following Ábrahám et al. (2022):

α	β	σ	γ	r	λ	ψ	δ	ω	Z
0.566	0.945	0.6887	1.4	0.0248	0.1	0.8099	0	0.1	0

Productivity and expenditure vectors are $\theta \in \{0.81, 1.01, 1.12\}$ and $g \in \{0.0785, 0.0415, 0.0185\}$ with

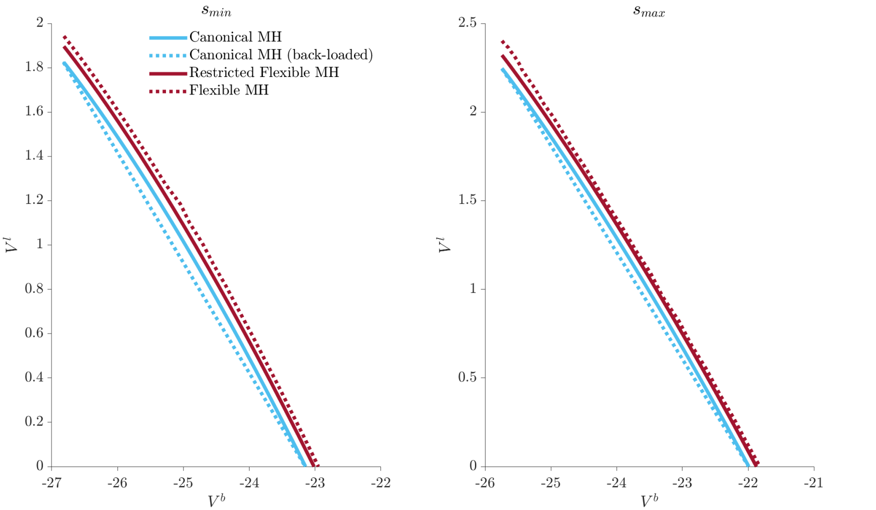
$$\pi = \begin{bmatrix} 0.980 & 0.015 & 0.005 \\ 0.005 & 0.975 & 0.020 \\ 0.015 & 0.025 & 0.960 \end{bmatrix}, \; Q_L = \begin{bmatrix} 0.93 & 0.0466 & 0.0234 \\ 0 & 0.99 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}, \; Q_H = \begin{bmatrix} 1 & 0 & 0 \\ 0.06 & 0.94 & 0 \\ 0.03 & 0.04 & 0.93 \end{bmatrix}$$

Policy Functions



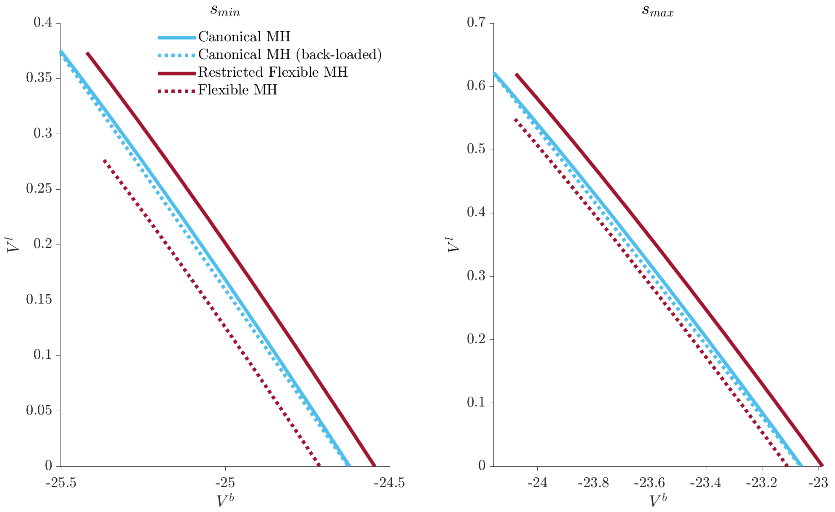
Pareto Frontiers

Same outside option



Pareto Frontiers

IMD outside option



Conclusion

- Optimal design of Financial Stability Fund under different MH specifications.
  - Flexible: full control over distribution of g'.
  - Canonical: partial control over distribution of g'.
- Flexible MH does not disrupt risk-sharing: Bliss as opposed to immiseration
- Canonical moral hazard disrupts risk sharing.
- We can dampen this effect by back-loading incentives.

Thanks for your attention!

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Literature

Go back

- Role and design of Financial Stability Fund:
  - **Ábrahám et al. (2022)**, **Liu et al. (2023)**,Dovis and Kirpalani (2023), Callegari et al. (2023).

  ⇒ Different IC constraints and how interact with LE under Canonical and Flexible MH.
- Dynamic contracts:
  - Thomas and Worrall (1994), Kehoe and Levine (1993, 2001), Kehoe and Perri (2002), Müller et al. (2019), Dovis (2019) and **Marcet and Marimon (2019)**.

  ⇒ Bring flexible moral hazard to dynamic contracts.
- Moral hazard in dynamic models:
  - **Prescott and Townsend (1984)**, Atkeson (1991), Atkeson and Lucas (1992),Tsyrennikov (2013).

  ⇒ Revisit Trade-off between Incentive compatibility and Risk sharing.

Following Marcet and Marimon (2019), the Fund contract in recursive form solves:

$$FV(x,s) = \mathrm{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,\mu\}} \Big\{ \boldsymbol{x}\Big[(1+\nu_b)U(c,n,\mu) - \nu_b V^o(s)\Big]$$

$$+ \Big[(1+\nu_l)[\theta f(n) - c - g] - \nu_l Z\Big]$$

$$+ \sum_{\theta'|\theta} \pi(\theta'|\theta)\Big[\int \Big[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - \boldsymbol{x}\varrho_\mu(s')\frac{v_\mu(g') - m(s')}{\pi(\theta'|\theta)}\Big]\mu(\mathrm{d}g')\Big]\Big\}$$

$$\text{s.t.} \quad \boldsymbol{x'}(s') = \Big[\frac{1+\nu_b}{1+\nu_l} + \frac{\varrho_\mu(s')}{1+\nu_l}\Big]\eta\boldsymbol{x}.$$

ν_b is Lagrange multiplier to (PCb), ν_l to (PCl) and $\varrho_\mu(s')$ to (ICC).

Appendix

Fund contract under flexible moral hazard

Go back

We normalize the multipliers as:

$$\nu_b(s^t) = \frac{\gamma_b(s^t)}{\alpha_{b,t}(s^t)},\ \nu_l(s^t) = \frac{\gamma_l(s^t)}{\alpha_{l,t}(s^t)} \text{ and } \varrho_{\mu_{t+1}}(s^{t+1}) = \frac{\xi_{\mu_{t+1}}(s^{t+1})}{\alpha_{b,t}(s^t)}.$$

The Fund's value functions can be decomposed as:

$$FV(x,s) = xV^b(x,s) + V^l(x,s), \text{ with}$$

$$V^l(x,s) = \theta f(n) - c - g + \frac{1}{1+r}\mathbb{E}_{\theta'|\theta}\left[\int \left[V^l(x'(s'),s')\right]\mu(\mathrm{d}g')\right],$$

$$V^b(x,s) = U(c,n,\mu) + \beta\mathbb{E}_{\theta'|\theta}\left[\int \left[V^b(x'(s'),s')\right]\mu(\mathrm{d}g')\right]$$

Appendix

Back-loaded canonical MH

Go back

Express the Fund contract as the solution to a sequence of recursive problems:

$$\hat{FV}(x,s) = \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ x\big[(1+\nu_b)(u(c)+h(1-n)-\hat{v}(e)) - \nu_b V^0(s) - \varrho v'(e)\big]$$
$$+ [(1+\nu_l)(\theta f(n) - c - g) - \nu_l Z]$$
$$+ \frac{1+\nu_l}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s')\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s')\}})\overline{FV}(x',s';\bar{x}') \mid s,e\right]\Big\}$$
$$\text{s.t.} \quad x'(s') = \eta x \frac{1+\nu_b+\psi(s' \mid s,e)}{1+\nu_l}, \quad \bar{x}' = \eta x \frac{1+\nu_b}{1+\nu_l}$$

Then within the subprogram:

$$\overline{FV}(x,s;\bar{x}) = \min_{\{\varrho\}} \max_{\{c,n,e\}} \Big\{ \bar{x}\,[u(c)+h(1-n)] - x\big[\hat{v}(e) + \varrho v'(e)\big] + (\theta f(n) - c - g)$$
$$+ \frac{1}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s')\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s')\}})\overline{FV}(x',s';\bar{x}') \mid s,e\right]\Big\}$$
$$\text{s.t.} \quad x' = \eta x\left[1+\psi(s' \mid s,e)\right], \quad \bar{x}' = \eta \bar{x},$$

Appendix

Parametrization

Go back

- Production function: $f(n) = n^{\alpha}$.
- Utility of consumption and leisure: $u(c) = \log(c)$ and $h(1-n) = \gamma \frac{(1-n)^{1-\sigma}-1}{1-\sigma}$.
- Disutility of effort: $v(\mu) = \omega(\int [\overline{g} - g]\mu(\mathrm{d}g))^2$ and $\hat{v}(e) = \omega e^2$.
- Dstribution: $Q = \zeta(e)Q_L + (1-\zeta(e))Q_H$ with $\zeta(e) = (e-1)^2$.