# FISCAL RULES AND DISCRETION WITH RISK OF DEFAULT

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# MOTIVATION

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▶ Characterization of optimal policies: spending and default rules.

- Extend previous theoretical results on spending rules.
- <u>No-default benchmark rule</u>: Amador Werning Angeletos 2006 (AWA).

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#### Gains of default vs fiscal rules an order of magnitude larger!

# A model of present-biased governments

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   β: disagreement or polarization.
- ▶  $\Rightarrow$  Quasi-hyperbolic preference. Phelps and Pollak 1968. Laibson 1997.

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Definition 1: An allocation specifies spending and default decisions:

$$\mathcal{A} = \left\{ egin{matrix} g( heta, b), \underbrace{\delta( heta, b)}_{\in \{0,1\}} 
ight\}_{ heta \in \Theta, b \in \mathbb{R}}$$

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Debt accumulation:

$$\dot{b}(\theta, b) = r(b)b + g(\theta, b) - \tau$$

## Rules-free equilibrium

▶ When not in default:

$$\rho w^{n}(\theta, b) = \underbrace{\max_{g}}_{g} \left\{ \theta u(g) + (r(b)b + g - \tau)w_{b}^{n}(\theta, b) \right\}$$

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▶ When in default:

$$\rho w^{d}(\theta) = \theta \underbrace{u(\kappa \tau)}_{\bullet} + \phi \underbrace{\left( w^{n}(\theta, \mathbf{0}) - w^{d}(\theta) \right)}_{\bullet}$$

autarky loss of insurance

regain access "clean slate"

▶ When not in default:

$$\rho w^{n}(\theta, b) = \underbrace{\max_{g} \left\{ \theta u(g) + (r(b)b + g - \tau) w_{b}^{n}(\theta, b) \right\}}_{\text{spending when in power}} + \underbrace{\lambda \underbrace{\left( \underbrace{\beta \mathbb{E} \left[ v(\theta', b) \right] - w^{n}(\theta, b) \right)}_{\text{lose power (potential default)}} }_{\text{spending when out of power}} + \lambda \underbrace{\left( \mathbb{E} \left[ v(\theta', b) \right] - w^{n}(\theta, b) \right)}_{\text{spending when out of power}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \lambda \underbrace{\left( \mathbb{E} \left[ v(\theta', b) \right] - v^{n}(\theta, b) \right)}_{\text{spending when out of power}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) + (r(b)b + g^{A}(\theta, b) - \tau) v_{b}^{n}(\theta, b)}{(\theta, b)}} + \frac{\theta u(g^{A}(\theta, b)) +$$

▶ When in default:

$$\rho w^{d}(\theta) = \underbrace{\theta \underbrace{u(\kappa\tau)}_{\text{autarky}} + \underbrace{\phi}_{\substack{\text{(}w^{n}(\theta, \mathbf{0}) - w^{d}(\theta))}_{\text{regain access}} + \lambda \underbrace{\left(\underbrace{\beta \mathbb{E}\left[v^{d}(\theta')\right] - w^{d}(\theta)\right)}_{\text{lose power}}}_{\text{lose power}}$$
$$\rho v^{d}(\theta) = \theta u(\kappa\tau) + \oint \left(v^{n}(\theta, \mathbf{0}) - v^{d}(\theta)\right) + \lambda \left(\mathbb{E}\left[v^{d}(\theta')\right] - v^{d}(\theta)\right)$$

• Government has **discretion to spend** whenever:

$$\theta u'\left(g^{A}\left(\theta,b\right)\right) = -w_{b}^{n}\left(\theta,b\right)$$

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debt threshold

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- Budget effects: debt repayment burden versus output loss.
- Dynamic effects:
  - Cost: loss of insurance in autarky.
  - Benefit: a "clean slate" upon re-access.

### IF MILD PRESENT BIAS







9/22







#### BUT IF RE-ACCESS IS "EASY"

 $\Rightarrow$  CLEAN-SLATE EFFECT DOMINATES







**Proposition 1** (Default bias). Consider type  $\theta \in \Theta$ ,  $\bullet$  Gaps in values

(I) If  $\frac{\partial b^{A}(\theta)}{\partial \theta} < 0$ , type  $\theta$  over-defaults at  $b^{A}(\theta)$ :  $v^{n}\left(\theta, b^{A}(\theta)\right) > v^{d}\left(\theta\right)$ .



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## **Optimal rules**

▶ Principal (planner, rule-writer): government type  $\theta$  is noncontractible.

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Principal chooses allocation  $\mathcal{A} = \{g(\theta, b), \delta(\theta, b)\}_{\theta \in \Theta, b \in \mathbb{R}}$ :

$$\max_{\mathcal{A}} \int_{\underline{\theta}}^{\overline{\theta}} v(\theta, b; \mathcal{A}) dF(\theta)$$

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s.t. 
$$w(\theta, \theta, b; \mathcal{A}) \ge w(\tilde{\theta}, \theta, b; \mathcal{A}), \quad \forall \theta, \tilde{\theta} \in \Theta$$
 • Report

market interest rate  $r(b; \mathcal{A})$ 

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Delegation problem.

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▶ Delegation problem.

▶ Mechanism is *static*. Sequentially Optimal (no commitment required).

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### CONSTRAINED EFFICIENCY

Proposition 3. There exists upper and lower bounds of debt

$$0 \leq \underline{\mathbf{b}} \leq \overline{\mathbf{b}} < \frac{\tau}{r_f}$$

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 $\boldsymbol{\theta^s(b)} \in \Theta, \forall b \leq \overline{b}$ 

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I) Spending rule: agents with low needs have discretion to spend.

$$g(\theta, b) = \begin{cases} g^{A}(\theta, b), & \forall \theta \leq \theta^{s}(b) \\ g^{A}(\theta^{s}(b), b), & \forall \theta \geq \theta^{s}(b); \end{cases}$$

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II) Default rule: for intermediate debt levels, full discretion to default.

$$\delta\left(\theta,b\right) = \begin{cases} 0, & \forall b \leq \underline{b} \\ \delta^{A}(\theta,b), & \forall b \in (\underline{b},\overline{b}] \\ 1, & \forall b > \overline{b}, \end{cases}$$

### How to fix overspending bias?



### How to fix overspending bias?



### How to fix overspending bias?



# How to fix default bias? Over-default



# How to fix default bias? Over-default



# How to fix default bias? Over-default



# How to fix default bias? Under-default



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▶ Implement the spending rule with a debt-dependent deficit limit.

• <u>European Fiscal Compact</u>: if debt>60% GDP, a "debt-brake-rule" is triggered, deficit limit changes from 3% of GDP to 1% surplus.

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- ▶ To prevent default: debt services must be a budget priority.
  - <u>Bulgaria's Public Finance Act</u>: "any interest and principle payables related to government debt shall constitute a priority liability for the state budget."

▶ To force default: debt limit. But debt over the limit must be defaulted.

• Ex. Special law stating debt above the limit is not recognized as a legitimate obligation.

### Spending rule: No-default AWA rule



### Spending rule: with default risk



### Spending Rule: with default risk



### Spending Rule: with default risk



### QUANTITATIVE: DOMINANT INSURANCE EXTERNALITY



No spending rules in default-risk region.

▶ Default manipulation: disincentive default.

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### QUANTITATIVE: DOMINANT INSURANCE EXTERNALITY



No spending rules in default-risk region.

▶ Default manipulation: disincentive default. *Forbid default* if possible!

### QUANTITATIVE: DOMINANT CLEAN-SLATE EXTERNALITY



No discretionary spending in high default-risk area.

### QUANTITATIVE: DOMINANT CLEAN-SLATE EXTERNALITY



No discretionary spending in high default-risk area.

▶ Default manipulation: incentive default.

### QUANTITATIVE: DOMINANT CLEAN-SLATE EXTERNALITY



#### No discretionary spending in high default-risk area.

- ▶ Default manipulation: incentive default. *Force default* if possible!
- ▶ Without default rules, impose a draconian spending rule to force default.

#### DECOMPOSITION OF WELFARE GAINS



• Gains from **fine-tuning spending rules** are minuscule.

▶ Gains from **default rules** are orders of magnitude larger.

### CONCLUSION

- ▶ Default risk is a key consideration on the debate about fiscal rules.
- ▶ Must be studied together, they come hand in hand.
- ▶  $\Rightarrow$  Need for debt limits and debt-dependent spending rules.
- Some takeaways:
  - Even with spending rules in place, it could be optimal to forbid default when debt is low.
  - There may be too little default; it could be optimal to force it.
- Default rules are less common but should be considered.
- ▶ How? Some rudimentary thoughts on implementation.

# Appendix slides

### MARKOV EQUILIBRIUM • BACK

- ▶ Due to time inconsistency, a tough cookie!
  - **?** Equilibrium existence? Uniqueness? Continuity?
    - Krusell Smith 2003: when it exists, everything is possible.
    - Harris Laibson 2013: "immediate" gratification,  $\lambda \to \infty$ .
    - Chatterjee Eyigungor 2016: need lotteries for continuous equilibria.
- Our approach: Piguillem Shi 2024
  - Equivalent to the problem of a *time-consistent* agent with a *distorted "distribution"* of shocks.
  - $\bigcirc$  Intuition: *myopia* is behavioral equivalent to *optimism* about the future.
  - $\heartsuit$  Use standard tools to characterize the equilibrium.

### A USEFUL RELATION $\bullet$ Back

Lemma A1 (Value functions). The value functions satisfy:

$$\beta v^{n}(\theta, b) = w^{n}(\theta, b) - (1 - \beta) \theta w^{n}_{\theta}(\theta, b)$$
$$\beta v^{d}(\theta) = w^{d}(\theta) - (1 - \beta) \theta w^{d}_{\theta}(\theta)$$

#### A USEFUL RELATION $\bullet$ Back

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$$\beta v^{d}(\theta) = w^{d}(\theta) - (1 - \beta) \theta w^{d}_{\theta}(\theta)$$

Default bias tied to default pattern:

$$\beta \left[ v^n \left( \theta, b^A(\theta) \right) - v^d \left( \theta \right) \right] = \underbrace{\left[ w^n \left( \theta, b^A(\theta) \right) - w^d \left( \theta \right) \right]}_{=0} - (1 - \beta) \, \theta \underbrace{\left[ w^n_\theta \left( \theta, b^A(\theta) \right) - w^d_\theta \left( \theta \right) \right]}_{\text{default pattern?}}$$

#### EQUIVALENCE • BACK

**Proposition A1 (Equivalence)**. The biased agent's problem can be equivalently represented as the problem of the following agent:  $\forall \theta \in \Theta, b \geq \underline{b}$ ,

$$(\rho + \lambda) w^{n}(\theta, b) = \max_{g} \left\{ \theta u(g) + (r(b)b + g - \tau) w_{b}^{n}(\theta, b) + \lambda \int_{\underline{\theta}}^{\overline{\theta}} w(\theta', b) dG(\theta') \right\}$$
$$(\rho + \lambda + \phi) w^{d}(\theta) = \theta u(\kappa\tau) + \phi w^{n}(\theta, 0) + \lambda \int_{\underline{\theta}}^{\overline{\theta}} w^{d}(\theta') dG(\theta'),$$

where the time-consistent agent assigns "expectation" to the taste shocks:

$$G(\theta) = \begin{cases} F(\theta) + (1 - \beta) \, \theta f(\theta), & \underline{\theta} \le \theta < \overline{\theta} \\ 1, & \theta = \overline{\theta}. \end{cases}$$

The adjusted distribution imply an average  $\int_{\theta}^{\bar{\theta}} \theta dG(\theta) = \beta$ .

Plus transversality condition  $\Rightarrow$  Viscosity solution.

**Lemma 1 (Spending pattern)**. Suppose CRRA with risk aversion  $\gamma$ . When not in default, the spending growth rate is

$$\frac{\dot{g^{A}}\left(\theta,b\right)}{g^{A}\left(\theta,b\right)} = \frac{1}{\gamma} \left( \frac{\partial (r\left(b\right)b)}{\partial b} - \rho - \lambda + \lambda \beta \frac{-\frac{\partial}{\partial b} \mathbb{E}\left[v\left(\theta',b\right)\right]}{\theta u'\left(g^{A}\left(\theta,b\right)\right)} \right)$$

▶ Present bias leads to overspending and debt overaccumulation.

#### DEFAULT PATTERN PRACE

#### Lemma 3 (High types default first). If permanent exclusion $\phi = 0$ ,

- Default threshold  $b^A(\theta)$  is decreasing in  $\theta$ :  $\frac{\partial b^A(\theta)}{\partial \theta} \leq 0$ .
- ▶ If there exists savers, the threshold is strictly decreasing:  $\frac{\partial b^A(\theta)}{\partial \theta} < 0, \forall \theta.$



• If type  $\theta$  reports  $\tilde{\theta}$ , follow allocation  $\left\{g\left(\tilde{\theta},b\right),\delta\left(\tilde{\theta},b\right)\right\}$  and obtain payoff:

$$w(\tilde{\theta}, \theta, b; \mathcal{A}) = \left(1 - \delta\left(\tilde{\theta}, b\right)\right) w^{n}(\tilde{\theta}, \theta, b; \mathcal{A}) + \delta\left(\tilde{\theta}, b\right) w^{d}(\tilde{\theta}, \theta; \mathcal{A}),$$

where

$$(\rho + \lambda) w^{n}(\tilde{\theta}, \theta, b; \mathcal{A}) = \theta u \left( g \left( \tilde{\theta}, b \right) \right) + \dot{b} \left( \tilde{\theta}, b \right) w_{b}^{n}(\tilde{\theta}, \theta, b; \mathcal{A}) + \lambda \beta \mathbb{E}[v(\theta', b; \mathcal{A})]$$
$$(\rho + \lambda + \phi) w^{d}(\tilde{\theta}, \theta; \mathcal{A}) = \theta u \left( \kappa \tau \right) + \phi w^{n}(\tilde{\theta}, \theta, 0; \mathcal{A}) + \lambda \beta \mathbb{E}[v^{d}(\theta'; \mathcal{A})]$$


#### Assumption 1 (Type distribution). $F(\theta)$ admits a differentiable density $f(\theta)$ :

$$\frac{\theta f^{\prime}\left(\theta\right)}{f\left(\theta\right)}\geq-\frac{2-\beta}{1-\beta};\ \, \forall\theta\in\Theta$$

- Similiar to Amador Werning Angeletos 2006.
- Ensures  $G(\theta) = F(\theta) + (1 \beta) \theta f(\theta)$  non-decreasing in  $[\underline{\theta}, \overline{\theta})$

## INTEREST RATE EXTERNALITY • BACK

**Lemma 4 (Interest rate externality)**. If the principal had perfect information, it would default when debt exceeds  $b^{P}(\theta)$ , which satisfies:

$$v^{n}(\theta, b^{P}(\theta)) = v^{d}(\theta) + \lambda \mathbb{E}\left[\frac{\partial v^{n}(\theta', b^{P}(\theta))}{\partial r(b^{P}(\theta))}\left(1 - \delta^{P}(\theta', b^{P}(\theta))\right)\right]$$

#### DEFAULT RULES • BACK

#### **Proposition 5 (Forbid or force default?).** Suppose $\beta \in (0, 1)$ ,

(I) If discretionary default  $b^A(\theta)$  is monotone decreasing, the debt bounds:

$$\underline{b} > \underline{b}^A$$
 and  $\overline{b} = \overline{b}^A$ .

 $\forall b \in [\underline{b}^A, \underline{b}), \, \text{discretionary default } \mathbb{E}\left[v(\theta, b)\right] < \mathbb{E}\left[v^n(\theta, b)\right] \Rightarrow \text{forbid default.}$ 

(II) If discretionary default  $b^{A}(\theta)$  is monotone increasing, the debt bounds:

$$\underline{b} = \underline{b}^A \quad and \quad \overline{b} < \overline{b}^A.$$

 $\forall b \in (\bar{b}, \bar{b}^A]$ , discretionary default  $\mathbb{E}[v(\theta, b)] < \mathbb{E}[v^d(\theta)] \Rightarrow$  force default.

# NO-DEFAULT BENCHMARK: AWA RULE PRACE

**Lemma 5 (No default)**. When default is not possible, for all debt levels  $b < \frac{\tau}{r_f}$ , a debt-independent spending threshold  $\theta^{s*}$ :

(I) (Severe present bias) If  $\beta \leq \underline{\theta}$ , all discretion is taken away.

(II) (Mild present bias) If  $\beta > \underline{\theta}$ , the spending threshold:

 $\theta^{s*} = \beta \mathbb{E} \left[ \theta | \theta \ge \theta^{s*} \right].$ 

### DIFFERENTIABILITY, JUMPS & KINKS • BACK

**Lemma A4**. If the rule forbids default, i.e.,  $\underline{b}^A < \underline{b} < \overline{b}$ , when debt surpasses  $\underline{b}$ : (I) Interest rate r(b) jumps upward:  $r(\underline{b}) < \lim_{b \downarrow \underline{b}} r(b)$ .

(II) Discretionary spending  $g(\theta, b)$  jumps downward:  $g(\theta, \underline{b}) > \lim_{b \downarrow \underline{b}} g(\theta, b)$ .

(III) Value functions display a kink and are piecewise differentiable:

 $w_b^n(\theta,\underline{b}) > \lim_{b \downarrow \underline{b}} w_b^n(\theta,b) \quad \text{and} \quad v_b^n(\theta,\underline{b}) > \lim_{b \downarrow \underline{b}} v_b^n\left(\theta,b\right).$ 



# EXTENSION: BUSINESS CYCLES • BACK

• Revenue  $\tau$  is observable and contractible and follows:

$$d\log(\tau) = \nu \log\left(\frac{\bar{\tau}}{\tau}\right) dt + \sigma_{\tau} dW$$

► Allocation:  $\{g(\theta, b, \tau), \delta(\theta, b, \tau)\}$ . Interest rate:

$$r(b, \tau) = r_f + \lambda \mathbb{E} \left[ \delta(\theta, b, \tau) \right]$$

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$$r(b,\tau) = r_f + \lambda \mathbb{E} \left[ \delta(\theta, b, \tau) \right]$$

Proposition B1. There exist state-dependent bounds of debt

 $0 \leq \underline{b}(\tau) < \overline{b}(\tau)$ 

and a state-dependent spending threshold

 $\boldsymbol{\theta}^{s}(\boldsymbol{b},\boldsymbol{\tau}) \in \Theta, \ \forall \boldsymbol{b} \leq \bar{\boldsymbol{b}}(\boldsymbol{\tau})$ 

I) Types  $\theta \leq \theta^s(b, \tau)$  have discretion to spend; above abide by the rule.

II) If  $b \leq \underline{b}(\tau)$ , forbid default; if  $b > \overline{b}(\tau)$ , force default; in between, discretion.

## EXTENSION: BUSINESS CYCLES • BACK

▶ When not in default, value functions satisfy:

$$\begin{aligned} (\rho+\lambda)w^{n}(\theta,b,\tau) &= \max_{g} \left\{ \theta u(g) + (r(b,\tau)b + g - \tau)w_{b}^{n}(\theta,b,\tau) \right\} + \lambda\beta \mathbb{E}[v(\theta',b,\tau) \\ &+ \nu \log\left(\frac{\bar{\tau}}{\tau}\right)\tau w_{\tau}^{n}(\theta,b,\tau) + \frac{1}{2}\sigma_{\tau}^{2}\tau^{2}w_{\tau\tau}^{n}(\theta,b,\tau) \\ (\rho+\lambda)v^{n}(\theta,b,\tau) &= \theta u(g^{*}) + (r(b,\tau)b + g^{*} - \tau)v_{b}^{n}(\theta,b,\tau) + \lambda \mathbb{E}[v(\theta',b,\tau)] \\ &+ \nu \log\left(\frac{\bar{\tau}}{\tau}\right)\tau v_{\tau}^{n}(\theta,b,\tau) + \frac{1}{2}\sigma_{\tau}^{2}\tau^{2}v_{\tau\tau}^{n}(\theta,b,\tau) \end{aligned}$$

▶ When in default, value functions satisfy:

$$\begin{split} (\rho + \lambda + \phi) w^d(\theta, \tau) &= \theta u(\kappa \tau) + \phi w^n(\theta, 0, \tau) + \lambda \beta \mathbb{E}[v^d(\theta', \tau)] \\ &+ \nu \log\left(\frac{\bar{\tau}}{\tau}\right) \tau w^d_\tau(\theta, \tau) + \frac{1}{2} \sigma^2_\tau \tau^2 w^d_{\tau\tau}(\theta, \tau) \\ (\rho + \lambda + \phi) v^d(\theta, \tau) &= \theta u(\kappa \tau) + \phi v^n(\theta, 0, \tau) + \lambda \mathbb{E}[v^d(\theta', \tau)] \\ &+ \nu \log\left(\frac{\bar{\tau}}{\tau}\right) \tau v^d_\tau(\theta, \tau) + \frac{1}{2} \sigma^2_\tau \tau^2 v^d_{\tau\tau}(\theta, \tau) \end{split}$$