A Theory of Safe Asset Creation, Systemic Risk, and Aggregate Demand

Levent Altinoglu*

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Abstract

This paper presents a theory in which the creation of safe assets leads to demand-driven fluctuations in output. The model features a two-way feedback between high systemic risk and depressed aggregate demand: The creation of safe assets by financial intermediaries generates a risk of future crisis (systemic risk), which in turn creates a precautionary demand for safe assets ex ante. The natural rate of interest is therefore determined by the level of systemic risk. If systemic risk is sufficiently high, the natural rate falls below the effective lower bound on monetary policy, leading to a demand-driven recession. The economy can enter a risk-driven stagnation trap in which persistently low output growth arises due to excessive systemic risk. Government purchases of risky assets can stimulate aggregate demand by transferring risk from bank balance sheets to that of the government. By contrast, purchases of safe assets may be ineffective. Macroprudential policy can actively stimulate aggregate demand when monetary policy is constrained.

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1 Introduction

Persistent macroeconomic slumps, such as the stagnant growth in the United States and Eurozone following the Global Financial Crisis and the economic slump in Japan from the early 1990s until 2020, typically feature elevated risk premia and deleveraging in the financial system. In order to counteract these effects, central banks have often resorted to highly unorthodox policies such as the purchase of *risky* assets, including mortgage-backed securities, high-yield corporate bonds, and even equities, to stimulate the economy.[1] Such interventions proved controversial because they expose the central bank to the risks associated with these assets, contrary to the traditional conduct of monetary policy. Overall, these episodes point to interactions between persistent demand-driven fluctuations in the real economy and the buildup of risks in the financial system.

The literature has linked these episodes to the supply of and demand for safe financial assets – liabilities issued by financial intermediaries or the government which are relatively safer than the assets which back them.[2] In particular, the literature on financial leverage has highlighted how the supply of safe assets generates financial instability, as it requires intermediaries to take leverage and bear the risk associated with their assets (e.g. Bocola and Lorenzoni (2023) and Caramp (2023)). The literature on financial risk and aggregate demand has also showed how the demand for safe assets leads to fluctuations in aggregate demand (e.g. Caballero and Farhi (2018), Caballero and Simsek (2020), and Caballero and Simsek (2021)).

But often these supply- and demand-side stories are modeled in isolation, producing limited interactions between aggregate demand and financial risk.[3] As a result, several open questions remain: How are persistent demand-driven slumps related to the buildup of financial vulnerabilities? How does the natural rate of interest – the interest rate at which output would be at potential – depend on financial risks? When should the central bank purchase risky assets from the private sector rather than safe ones?

In this paper, I present a theory which shows that the creation of safe assets by the financial sector can generate demand-driven fluctuations in output through a rise in systemic risk. In the model, fluctuations in output arise due to general equilibrium interactions between systemic risk and aggregate demand. In light of these interactions, I then study the transmission of central bank asset purchases and macroprudential policy and show that they can differ qualitatively from those in the related literature, such as Benigno and Fornaro (2018), Caballero and Farhi (2018), Farhi

[1] Examples are the Bank of Japan's purchases of equity ETFs, the Federal Reserve's QE1, and the ECB's LTRO.

[2] Private safe assets include bank deposits, money market mutual fund liabilities, and asset backed-securities, while public safe assets are those issued by the consolidated government, such as government bonds and central bank reserves. In my model, a safe asset will be an uncontingent bond.

[3] Much of the literature either focuses on the determination of aggregate demand in models without financial frictions or in which the supply of safe assets is pinned down by an exogenous constraint; or uses a real model in which aggregate demand does not play a meaningful role in determining output. An exception is Boissay et al. (2023).

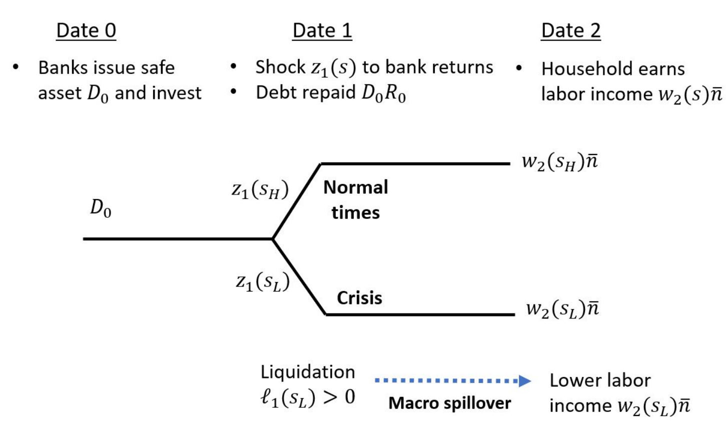
and Werning (2016), Gorton and Ordonez (2022), and Korinek and Simsek (2016).

I build a three-period, New Keynesian model with capital and financial frictions. The basic timeline is depicted in Figure 1. Risk-neutral banks issue non-defaultable bonds, D_0, to risk-averse households in order to invest in risky capital at date 0. Because bank debt is non-contingent, it is a safe asset from the perspective of individual households. At date 1, banks experience a shock to the return on their capital holdings, denoted $z_1(s)$. Therefore, the creation of private safe assets is effectively a risk sharing arrangement in which banks insure households against the shock to the return on capital at date 1.

The core mechanism is built around two basic premises. The first premise is that the creation of private safe assets (i.e., the issuance of safe bank debt at date 0) generates a *risk of a future crisis* at date 1. At date 1, if the shock to the return on banks' capital investments is low, banks are faced with large losses and are therefore forced to liquidate some of their capital holdings, $\ell_1(s)$, at a cost in order to repay their debt to the household. I refer to this situation as a 'crisis', as it entails a deadweight loss which captures the inefficiencies associated with financial crises.[4] The more safe assets that banks create at date 0 (i.e., the higher their leverage), the more capital they must liquidate in the bad state at date 1, and therefore the more severe the future crisis.

The second premise is that such crises entail *macroeconomic spillovers* which reduce households' future labor income, $w_2 n$, similar to that in Bocola and Lorenzoni (2023). When banks liquidate capital to repay their debt in bad states of the world at date 1, they reduce the future stock of capital at date 2. The lower future stock of capital, in turn, reduces the future wage to be earned by households at date 2, owing to complementarities between capital and labor in the production of goods. As a result, the cost of crises is shared in general equilibrium by households in the form of lower future labor income. This spillover, illustrated by the bottom arrow in Figure 1, captures the broader externalities that financial crises have on the economy.

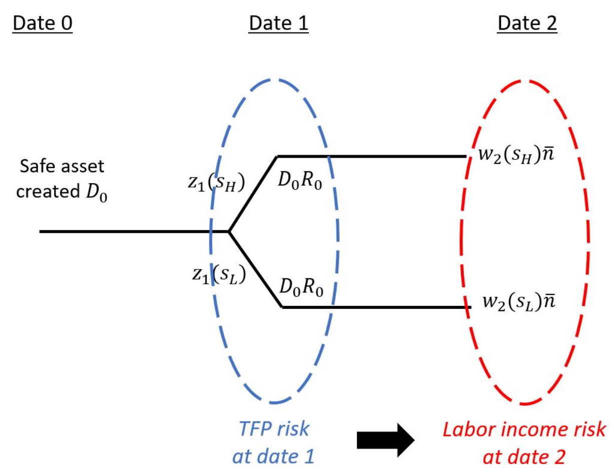
Figure 1: Crises entail macroeconomic spillovers



[4]This is a reduced-form way to capture the inefficiencies associated with the fire sales, as in Lorenzoni (2008). Similar effects obtain in models of occasionally binding collateral constraints, like Bianchi and Mendoza (2018).

Together, these two premises imply that the creation of safe assets by financial institutions entails a kind of *risk transformation*, illustrated in Figure 2. The issuance of bank debt insures households against the risk associated with investment in capital, but in general equilibrium, this generates future labor income risk through the macroeconomic spillover. Bank debt insures households against the shock to the return to capital at date 1, as the household's date 1 interest income $R_0^D D_0$ is constant across states. But in general equilibrium, this *increases* the household's labor income risk at date 2 due to liquidation at date 1 and the macroeconomic spillover. Thus, safe asset creation doesn't eliminate fundamental risk – it just reallocates it. Moreover, the creation of private safe assets actually *amplifies* aggregate risk in general equilibrium: The creation of safe assets generates deadweight losses from liquidation in the bad state, increasing the the losses that must be absorbed by agents.

Figure 2: Safe asset creation entails risk transformation



The anticipation of a future crisis at date 1 generates a precautionary saving demand for safe assets on the part of the household at date 0 in order to smooth consumption across future dates and states. As a result, the model features a *paradox of safety* in which the demand for insurance against aggregate risk further increases aggregate risk through the creation of safe assets and its effect on future crises.[5] However, because they take future wages as given, individual households do not internalize how their saving decisions at date 0 affect their future labor income in general equilibrium.

Moreover, the household's demand for safe assets depresses aggregate demand for goods at date 0. Therefore, the natural rate of interest at date 0 – that is, the interest rate at which output

[5]This is related to the well-known paradox of thrift, in which an increase in saving lowers aggregate demand and output, ultimately reducing total saving. However, the paradox of safety works through the endogenous creation of aggregate risk, and the desire for insurance against this risk.

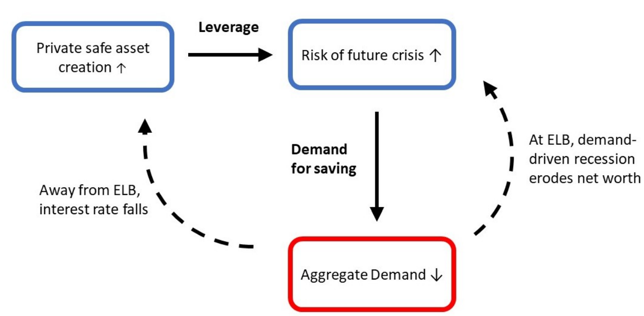
would be at potential – depends in equilibrium on the level of systemic risk (the severity of future crises). A higher risk of future crises increases the household's labor income risk, which increases its precautionary saving and depresses aggregate demand ex ante. This lowers the natural rate of interest at date 0.

When monetary policy is unconstrained by the effective lower bound, the monetary authority can reduce the nominal interest rate to ensure output remains at potential. But when crisis risk (the severity of a future crisis) is sufficiently high, the natural rate of interest may fall below the effective lower bound on monetary policy, giving rise to a demand-driven recession. In this situation, a fall in output is needed to clear the market for goods through a fall in the utilization of resources, similar to Caballero and Farhi (2018).

The demand-driven recession at date 0 occurs because aggregate risk is too high relative to the capacity of the economy to absorb this risk. Given the labor income risk it faces, the household wants to save more, which lowers consumption demand at date 0. However, banks are unwilling to issue more safe assets at date 0 because of the high liquidation risk they face, which reduces investment demand. The equilibrium can only be reached through a fall in utilization and output at date 0, which reduces total investment and the amount of aggregate risk that agents must absorb.[6]

At the effective lower bound, there is a two-way feedback between high systemic risk and depressed aggregate demand at date 0, stylistically illustrated in Figure 3. High systemic risk depresses aggregate demand by increasing households' precautionary saving, resulting in a demand-driven recession at date 0. In turn, the recession erodes the net worth of banks, further increasing systemic risk at date 0. The intensity of this feedback depends on elasticities controlling the strength of the macroeconomic spillover and the precautionary saving effect.

Figure 3: Interaction between crisis risk and aggregate demand



When the two-way feedback between high systemic risk and depressed aggregate demand is sufficiently strong, the creation of safe assets can give rise to persistent slumps driven by excessive systemic risk – a situation I call *risk-driven stagnation trap*. In this situation, the creation of

[6]This is related to Caballero and Simsek (2020). See the literature review.

private safe assets by banks leads to a high level of systemic risk. In turn, this depresses aggregate demand and forces the natural rate of interest to fall below the effective lower bound. The resulting demand-driven recession lowers date 0 output and reduces the resources available for investment. Therefore, even though a higher share of output goes to investment, the level of investment can fall. As a result, the future capital stock is lower at date 1, and so is future output in all states of the world. Furthermore, the lower expected future output reduces banks' expected future earnings, thereby increasing the banks' burden of debt and further increasing systemic risk ex ante.

The risk-driven stagnation trap is similar in spirit to the stagnation trap first identified in the seminal work by Benigno and Fornaro (2018). However, the trap in my paper derives from high systemic risk rather than self-fulfilling expectations of low future growth. Therefore, in contrast to that paper, policies designed to stimulate investment may be counterproductive to the extent they further incentivize bank leverage. The trap in this model is also similar to the safety trap first identified in Caballero and Farhi (2018) in which a demand-driven recession arises due to a shortage of safe assets. However, the trap here derives in part from an *oversupply* of private safe assets. Therefore, policies designed to increase the supply of private safe assets would only worsen the demand-driven recession.

When monetary policy is constrained by the effective lower bound, central bank purchases of *risky* assets can stimulate demand through a *risk absorption channel.* Suppose that, at date 0, the economy is in a demand-driven recession at the effective lower bound – that is, before a crisis occurs at date 1, output is depressed at date 0 because of an excessive amount of aggregate risk required to be absorbed by agents. Suppose further that, at date 0 the government issues debt to households to purchases capital (risky assets) from banks. At date 1, the government earns rental income from its holdings of capital and repays its debt to the household out of this rental income. If its rental income is insufficient to pay its debt, the government can levy a lump-sum tax on households to cover the difference.

From an ex post perspective (that is, at date 1 and 2), this policy reduces the severity of crises, ceteris paribus. At date 1, the government can always repay its debt without liquidating capital because of its power to tax. As a result, there is less liquidation of capital in the bad state, which reduces the deadweight losses from crises. Moreover, this reduces the macroeconomic spillovers of crises, boosting the household's labor income in the bad state at date 2. From an ex ante perspective (at date 0), the household faces less labor income risk as a result of the policy. Therefore, the household reduces its precautionary saving at date 0, which stimulates aggregate demand and output at date 0.

Intuitively, risky asset purchases can stimulate output because the government has a comparative advantage at bearing aggregate risk due to its power to tax. While private safe assets are backed by the liquidation value of capital, public safe assets are implicitly backed by future tax revenue, which the government can generate without inefficient liquidation of capital. As a result, unlike the creation of private safe assets by banks, public safe asset creation by the government does not generate systemic risk. For that reason, public and private safe assets are *not* equivalent from a social perspective. By transferring risky assets from bank balance sheets to that of the government, risky asset purchases *reduce* aggregate risk by reducing the severity of crises and their associated deadweight losses. In turn, this stimulates aggregate demand ex ante.[7]

Thus, the model can help rationalize central banks' unconventional purchases of risky assets to stimulate output during persistent slumps. By contrast, central bank purchases of safe assets, such as the purchase of investment grade bonds or government securities, are not as effective at stimulating output when the demand-recession is a result of excessive risk: Government purchases of safe assets do not reallocate the risk associated with investment from banks to households, and therefore do not reduce the aggregate risk borne by private agents.

Nevertheless, there is a social tradeoff associated with quantitative easing, as risky asset purchases can crowd-in private safe asset issuance and investment in general equilibrium, leading to suboptimal levels of investment at date 0. As a result, it is not, in general, socially optimal for the government to maximize its purchases of capital from the bank.

In this environment, macroprudential policy can serve as a tool for the active management of aggregate demand. Typically, policymakers view macroprudential tools, such as bank regulation as being divorced from the monetary policy toolkit, designed for the management of aggregate demand. In contrast, this model illustrates how the distinction is blurred by the interaction between systemic risk and aggregate demand. When the economy is in a demand-determined regime with interest rates at the effective lower bound, tighter macroprudential regulation, such as Pigouvian taxes which reduce bank leverage, can stimulate output ex ante by reducing systemic risk and the precautionary demand for saving. Moreover, such policies boost the natural rate of interest, alleviating the burden on monetary policy to stimulate output.

This paper thus highlights a distinct role for macroprudential policy relative to the related literature (such as Farhi and Werning (2016) and Korinek and Simsek (2016)) which showed how such policies can reduce the severity of *future* recessions by boosting demand in bad future states of the world. Here, tighter macroprudential policy in this model can be used to actively stimulate *current* output when demand is depressed due to systemic risk.

The allocative inefficiency associated with the creation of private safe assets derives in part from inefficient risk sharing between households and banks: Households bear too little of the risk associated with banks' investments in capital.[8] Banks ensure households against this risk by

[7]Intuitively, this policy forces household to bear losses associated with bank assets through taxation, and thus improves risk sharing between households and banks in general equilibrium.

[8]Inefficiencies are driven in part by incomplete markets. Moreover, there is an aggregate demand externality owing to the nominal rigidity and the effective lower bound.

liquidating capital in the bad state. But this forces the household to bear greater future labor income risk in general equilibrium. This risk sharing problem does not obtain for the creation of public safe assets. In creating public safe assets, the government forces households to bear risk associated with capital investment through lump-sum taxation.

This inefficient risk sharing results in bank leverage being too high relative to what a Ramsey planner would choose. Equivalently, households hold too little public safe assets and too much private safe assets. Macroprudential policy and quantitative easing can improve welfare by distorting this margin.[9]

1.1 Literature review

The seminal paper of Caballero and Farhi (2018) established the notion of the safety trap in which demand-driven recessions arise due to a general shortage of safe assets. The main innovation in my model is the dynamic interplay between aggregate demand and systemic risk, which stems chiefly from how I model the model the supply of safe assets. In the baseline case of Caballero and Farhi (2018), the supply of (private) safe assets is pinned down by an exogenous collateral constraint, and as a result, the supply of safe assets is not affected by macroeconomic conditions. While there is a social benefit to issuing safe assets, there is no social cost and therefore the supply of safe assets is an aggregate demand shifter: increase in the supply of safe asset only ever boosts aggregate demand.[10]

In this paper, by contrast, the dynamic interaction between aggregate demand and systemic risk gives rise to a social cost of issuing private safe assets. This is because (private) safe asset creation endogenously generates the risk of future crisis. In turn, crisis risk affects the demand for safe assets and aggregate demand ex ante due to a macroeconomic spillover from crises to future labor income. However this social cost is not internalized by agents ex ante because the macroeconomic spillover materializes only in general equilibrium. As a result of these interactions, the response of economy to shocks and to various policies are qualitatively different in this model. This paper also implies a sharp distinction between public and private safe assets with regard to their macroeconomic consequences. Therefore, in this model, aggregate demand (and the level of systemic risk) is determined not only by the total supply of safe assets, but also by the *composition* of safe assets between private and public.[11] Indeed, safety traps can arise from an *oversupply* of

[9]Normative considerations are work in progress.

[10]In an extension of their baseline model, Caballero and Farhi (2018) allow for agents to relax the collateral constraint and increase the safe asset supply subject to a convex cost. However, this cost is private while the benefit of supplying safe assets in a safety trap is social. Hence, the extension still features a general under-provision of safe assets.

[11]While the supply of public safe assets acts as an aggregate demand shifter, as in Caballero and Farhi (2018), this is not generally the case for private safe assets: The risk sharing externality in my paper implies that private safe asset

private safe assets, which can drastically alter the policy implications.

The macroeconomic spillover first modeled in Bocola and Lorenzoni (2023) plays a similar role in my model. However, their focus is on the risk-sharing problem between consumers and banks, and so these agents can trade full set of state-contingent claims. By contrast, I take as given market incompleteness by assuming that households and banks trade fixed rate bonds, and as a result, inefficient risk-sharing manifests as excessive private safe asset creation. Moreover, while Bocola and Lorenzoni (2023) use a real model whereas my focus is on aggregate demand and its interaction with systemic risk. Finally, the contrast between how the public and private sectors absorb risk is central to my results.

This paper also relates to the theory of risk-centric perspective of demand recessions of Caballero and Simsek (2020) and Caballero and Simsek (2021), in which the ability of the economy to absorb the risk associated with investment interacts with aggregate demand and output. Similar to these papers, the demand recession arises fundamentally due to an excessive amount of risk and constraints on monetary policy. The crucial difference is that aggregate risk arises endogenously in my model through the creation safe assets. Households' demand for safe assets as insurance against aggregate risk can itself create further aggregate risk. This is the paradox of safety at the heart of this paper. Methodologically, while financial frictions play an important role in generating my mechanism, these papers abstract from financial frictions and focus on how asset prices affect the distribution of wealth across agents who vary in their beliefs or risk tolerance.

Using a similar framework to these papers, Goldberg and Lopez-Salido (2023) identify new channels through which monetary policy may affect the severity of speculative booms and demand-driven recessions, which informs the debate surrounding the macroprudential use of monetary policy (see Ajello et al. (2019)). Boissay et al. (2023) and Collard et al. (2017) also analyze optimal monetary and macroprudential policies when monetary policy affects financial stability.

Benigno and Fornaro (2018) is the first paper to formalize the notion of a stagnation trap in which deficient demand results in persistently low economic growth. While in that paper, the stagnation trap arises due to an endogenous fall in investment which reduces innovation and future productivity, in my paper, productivity is exogenous; the fall in expected future output arises due to a fall in investment and the future stock of capital. In addition, while self-fulfilling expectations of low growth and multiplicity of equilibria are central to the stagnation trap of Benigno and Fornaro (2018), in my model, the two-way interaction between systemic risk and aggregate demand is what sustains low investment and growth in equilibrium. Therefore, policies designed to incentivize investment may be counterproductive to the extent that they lead to higher financial leverage. Other papers which study secular stagnation include Eggertson and Krugman (2012), Cuba-Borda and Singh (2021), and Xavier (2023).

creation may be excessively high in a safety trap.

Korinek and Simsek (2016) and Farhi and Werning (2016) have a similar role for macroprudential policy to address an aggregate demand externality. While in Korinek and Simsek (2016), output is always supply-determined ex ante and demand-determined in bad states ex post, the reverse is true in my model. Therefore, in contrast to that paper, macroprudential policy can be used to stimulate current aggregate demand, while fiscal or monetary policy can be beneficial ex ante both by stimulating demand and by reducing systemic risk.

The paper incorporates several ingredients common to New Keynesian models of the financial acclerator, including Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2014). In those models, the interactions between aggregate demand and financial risks are muted because the absence of nonlinearities associated with fire sales or occasionally-binding constraints limits the macroeconomic spillovers associated with idiosyncratic default.[12] More relevant is the notable paper Cao, Luo and Nie (2023), which incorporates both the effective lower bound on monetary policy and occasionally-binding collateral constraints. The presence of both constraints implies that some of the forces at heart of my paper are present in the background of that model. In my paper, I bring this mechanism to the foreground, characterize it analytically, explicitly focus on its implications for persistent slumps, and examine its implications for policy.

This paper is related to Acharya, Dogra and Singh (2022), who use a real model without aggregate risk to show that the supply of private safe assets may create its own demand. While they abstract from uncertainty to focus on the multiplicity of equilibria, I instead abstract from multiplicity to focus on how safe asset creation amplifies uncertainty through macroeconomic spillovers, and its interactions with aggregate demand. Gorton and Ordonez (2022) studies the tradeoff between provision of public and private safe assets from the perspective of information production and use as collateral. My paper take a different but complementary approach, and studies how the creation of private versus public safe assets entail different macroeconomic externalities, with a focus on aggregate demand. Benigno and Robatto (2019) and Infante and Ordonez (2021) examine the optimal supply of private and public liquidity. The latter paper focuses comparing the ability of public and private safe assets to facilitate the sharing of idiosyncratic liquidity risk through their use as collateral in light of their different exposure to aggregate risk. Angeletos, Collard and Dellas (Forthcoming) also examine the optimal supply of public debt in the presence of tradeoff between reducing the severity of financial frictions and reducing fiscal space. Relative to these papers, I abstract from liquidity benefits of public safe assets to focus on how public versus private safe assets generate systemic risk. Since agents in my model do not internalize that private safe assets generate more aggregate risk in general equilibrium, the convenience yield on public debt is

[12] In those papers, financial leverage entails idiosyncratic default, which entails some deadweight loss. But the cost of default is primarily borne by the entrepreneur or bank; the general equilibrium spillover from default to the household's future income is quantitatively small.

inefficiently low.

This paper is also related to several recent papers on safe assets and crises, such as Diamond (2020), Lenel (2023), Luck and Schempp (2023), Ross (2023), and Segura and Villacorta (2023) which imply that the production of safe assets generates financial externalities. However, these papers abstract from macroeconomic dynamics such as the role of aggregate demand, or are in partial equilibrium rather than general equilibrium, both of which are central to my mechanism. Agarwal (2022) also analyzes the macroeconomic implications of safe asset demand, but the focus is on the effect of shocks to precautionary saving on aggregate demand rather than endogenous creation of aggregate risk. Ebrahimy (2023) studies theoretically what makes government debt a safe asset. Finally, Azzimonti and Yared (2019) study optimal provision of public versus private safe assets in a model without aggregate risk, and Benigno and Nistico (2017) study optimal monetary policy in a model with a kind of cash-in-advance constraint for private and public safe assets.

2 Model

2.1 Overview of setup

There are three periods: dates 0, 1, and 2. There are two types of agents who consume a consumption good: a measure one of identical, risk-averse households, and a measure one of identical, risk-neutral banks. Capital (a risky asset) can be held and produced by banks only. In addition, there are New Keynesian firms who are owned by the household and have rigid prices, and there is a government which, for now, plays a passive role. There are two types of safe assets in which the household can invest at date 0: non-contingent debt issued by the banks (a private safe asset), and non-contingent debt issued by the government (a public safe asset).

Within each period, the New Keynesian firms hire labor from the household and rent capital from banks to produce the consumption good. At date 0, banks can invest in new capital at date 0. To finance investment in new capital at date 0, the bank can issue nominally safe, one-period debt D_0 to the household in a competitive market at date 0. This private bond pays a nominal gross rate of return denoted R_0^D at date 1 which is not contingent on the state of the world. [13]

At date 1, there is a shock to the total factor productivity z_1 of the New Keynesian firms, which can take a high or low value: $z_1 \in \{z_1^H, z_1^L\}$ where $z_1^H > z_1^L$. This shock affects the marginal productivity of capital and therefore the banks' rental income at date 1. Thus, bank assets (capital holdings) are risky from a date 0 point-of-view, while their liabilities are safe. Figure 1 summarizes the sequence of key events.

[13] I abstract from default and assume that borrowers have full commitment to repay debt, but relaxing this assumption would not alter central insights of the model.

2.2 Households

The representative, risk-averse household has log utility over consumption c_t and is endowed with e_0 units of the consumption good at date 0. Each period, the household supplies labor $\bar{n}$ in inelastically for a wage w_t, and solves consumption-saving decision. At date 0 the household has two assets to save in: private bonds D_0 and public bonds B_0. Households cannot hold capital directly.

$$\max_{c_0,c_1,c_2,D_0,B_0,B_1} \quad \log c_0 + E_0 \left[\log c_1 + \log c_2\right]$$

Each date t, the household earns labor income $w_t\bar{n}$, income on its bond holdings $R_0^D D_0$, $R_0^B B_0$, and any dividends d_t^F by the New Keynesian firms it owns, and pays lump-sum taxes T_t. P_t denotes the price level (which will be fixed and normalized to 1). At date 1, after uncertainty is resolved, the household has access to a riskless storage technology B_1 with return $R_1^B = 1$.[14] Therefore, the household's budget constraints for dates 0, 1, and 2, respectively, are

$$c_0 + \frac{D_0}{P_0} + \frac{B_0}{P_0} \leq e_0 - T_0 + d_0^F + w_0\bar{n} \tag{1}$$

$$c_1(s) + B_1(s) \leq \frac{R_0^D D_0}{P_1(s)} + \frac{R_0^B B_0}{P_1(s)} + d_1^F(s) - T_1 + w_1(s)\bar{n} \tag{2}$$

$$c_2(s) \leq d_2^F(s) + R_1^B B_1(s) - T_2 + w_2(s)\bar{n} \tag{3}$$

The household's optimality conditions, derived in Online Appendix 1, imply that the nominal rates of the return on the private and public bonds must be equalized in equilibrium, $R_0^D = R_0^B$, as the private and public bond are equivalent assets from the perspective of private agents as they both offer a nominally risk-free, state-uncontingent return in date 1 (although the two assets are not equivalent from a social perspective). I henceforth use $R_0 \equiv R_0^D = R_0^B$ to denote the nominal interest rate. The household's date 0 consumption-saving decision is governed by the Euler equation $\frac{1}{c_0} = R_0 E_0\left[\frac{1}{c_1(s)}\right]$, which pins down the household's demand for total saving (i.e., its demand for both types of bonds together). Moreover, the household perfectly smooths its consumption between dates 1 and 2 so that $c_2(s) = c_1(s)$.

[14] The storage technology is not critical for the qualitative results but improves the tractability of the model and also ensures that the natural rate of interest is equal to 1 in all states at dates 1 and 2 so that the effective lower bound never binds at those dates. If I instead assumed that the household could invest in government bonds at date 1, the results would be similar – I would simply restrict my analysis to the cases in which the natural rate is weakly greater than 1 at dates 1 and 2, ensuring that output is at its potential at these dates.

2.3 Government

At date 0, the government issues to the household one-period debt on a competitive market, where B_0 denotes the nominal face value of the debt (the public safe asset), which yields a state-uncontingent nominal gross rate of return R_0^B. The government can also levy lump-sum taxes on agents each period, where T_t and T_t^B respectively denote a lump-sum tax on the household and a lump-sum transfer to the bank in period t. For now, I leave aside government asset purchases and distortionary taxation, and I take the government's behavior as given. (In the normative section, I will characterize the government's optimal behavior.) Therefore, the government's budget constraints at dates 0, 1, and 2, respectively, are

$$\frac{B_0}{P_0} + T_0 = T_0^B \tag{4}$$

$$T_1 - T_1^B = R_0^B \frac{B_0}{P_0} \frac{P_0}{P_1} \tag{5}$$

$$T_2 - T_2^B = 0. \tag{6}$$

2.4 New Keynesian block

The New Keynesian block of the economy is fairly standard and is expounded upon in Online Appendix 3. There is a continuum of monopolistically competitive firms, indexed by ν, who hire labor and rent capital in competitive markets and produce a variety according to a Cobb-Douglas production function with productivity $z_t(s)$.

$$y_t(\nu) = z_t \left(u_t(\nu) k_t(\nu)\right)^{\alpha} n_t(\nu)^{1-\alpha} \tag{7}$$

These firms have variable capital utilization, $u_t(\nu) \in [0,1]$, which is costless between 0 and 1 and infinitely costly above 1. These firms have pre-set nominal prices at date 0, normalized to $P_t(\nu) = 1$, which are fixed forever. Inflation is therefore 0 in all dates and states. For simplicity, I assume that the government taxes in lump-sum fashion each firms' monopoly profits and redistributes the proceeds of this tax back to firms in the form of a linear subsidy to capital, which ensures that profits are 0 in equilibrium.

The monopolistically competitive firms sell their goods to a competitive sector of final good producers, who aggregate the intermediate goods according to a CES production technology $y_t = \left(\int_0^1 y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\frac{\varepsilon}{\varepsilon-1}}$ with $\varepsilon > 1$, where $P_t = \left(\int_0^1 P_t(\nu)^{1-\varepsilon} d\nu\right)^{\frac{1}{1-\varepsilon}} = 1$ is the ideal nominal price index. Utilization is determined to meet the demand faced by the firm $y_t^d(\nu)$ by competitive final goods producers. The equilibrium is symmetric implying that, for all ν,

$$u_t(\nu) = \min\left\{1,\, \frac{1}{k_t}\left[\frac{y_t^d}{z_t n_t^{1-\alpha}}\right]^{1/\alpha}\right\} \tag{8}$$

Moreover, the rental rate of capital and the wage are equal to the factor shares of income: $r_t^k = \alpha \frac{y_t}{k_t}$ and $w_t = (1-\alpha)\frac{y_t}{n_t}$.

Monetary authority Since prices are fully sticky, the real interest rate is equal to the nominal interest rate, which is controlled by the monetary authority.[15] I assume that the monetary authority attempts to target potential output in each period, defined as the level of output given full utilization of resources ($u_t = 1$), i.e. $y_t = z_t k_t^{\alpha} \bar{n}^{1-\alpha}$. However, there is a lower bound constraint on the gross nominal interest rate, implying $R_t^B \geq 1$. Thus, the monetary authority sets the nominal interest rate according to $R_t^B = \max\{R_t^*, 1\}$, where R_t^* is the natural rate of interest rate – the interest rate which ensures output is at potential. For simplicity, I assume that the natural interest rate weakly exceeds 1 in all states at date 1 and 2 so that output is always at potential at these dates.[16]

Takeaways from the New Keynesian block There are two main takeaways from this section. First, output is at potential (that is, $u_0 = 1$) at date 0 when the natural rate of interest exceeds 1. Second, when the natural rate is below 1 at date 0, output is demand-determined, requiring $u_0 < 1$. Thus, output can be below its potential level at date 0 due to a shortage of aggregate demand and a binding effective lower bound on monetary policy.

2.5 Banks

The representative, risk-neutral bank has linear utility $v(c_2^B) = c_2^B$ and consumes only at date 2, where c_2^B denotes the bank's date 2 consumption. The bank is endowed with k_0 units of capital at date 0. Within each period, the bank rents its capital holdings to intermediate goods producers in a competitive market and receives a real gross rental rate of capital $r_t^k(s)$. Banks also have access to an investment technology to produce new capital between periods. While this investment technology is itself risk-free, the bank's date 0 investment is subject to aggregate risk due to the productivity shock at date 1 which affects the date 1 rental rate of capital $r_1^k(s)$.

The bank can finance investment at date 0 out of its rental income $r_0^k k_0$ and government transfers T_0^B, or by issuing debt to the household in a competitive market at date 0. Bank debt is an uncontingent bond of face value D_0 pays a gross nominal rate of return R_0 at date 1 in all states

[15] In equilibrium at date 0, the policy rate set by the monetary authority will be equal to the nominal rate on government bonds $R_0^{MP} = R_0^B$. Therefore, I henceforth ignore the notation R_0^{MP} and instead refer to R_0^B as both the rate on government bonds and the monetary policy rate.

[16] This is reminiscent of the simplifying assumptions employed in Caballero and Simsek (2021) and Caballero and Farhi (2018).

of the world. Bank borrowing is subject to a natural borrowing limit which ensures banks can credibly commit to honoring their obligations in all states, ruling out default. Imposing $P_t = 1$, the bank's date 0 budget constraint at date 0 is

$$D_0 + r_0^k k_0 + T_0^B \geq i_0. \tag{9}$$

The bank's stock of capital evolves between dates 0 and 1 according to

$$k_1 = i_0 + k_0 \tag{10}$$

At date 1, banks have to meet their obligations $D_0 R_0$ and can invest in new capital at date 1, subject to a non-negativity constraint on investment $i_1(s) \geq 0$. They can finance these expenditures out of their earnings from renting capital at date 1 $r_1^k(s)k_1$ or out of any lump-sum government transfers T_1^B. Alternatively, the bank can raise funds at date 1 by liquidating a fraction $\ell_1(s)$ of their capital holdings at date 1, which converts a unit of capital into a unit of the consumption good within the period. The bank's date 1 budget constraint is therefore

$$r_1^k(s)k_1 + \ell_1(s)k_1 + T_1^B \geq i_1(s) + D_0 R_0 \tag{11}$$

Liquidation is costly and involves a loss of capital given by the strictly convex function $\phi(\ell_1) = \ell_1^{\eta}$, where $\eta > 1$.[17] [18] Therefore, the bank's date 2 capital holdings evolves according to

$$k_2(s) = i_1(s) + (1 - \ell_1(s) - \phi(\ell_1(s)))\, k_1(s) \tag{12}$$

At date 2, the bank rents its capital stock to intermediate goods firms and it consumes, and capital fully depreciates at date 2 after being used in production. The bank's date 2 budget constraint is

$$r_2^k(s)k_2(s) + T_2^B \leq c_2^B(s). \tag{13}$$

Bank's optimality conditions The problem of the bank is to choose in each period and each state how much to invest in new capital i_0, i_1, how much of its holdings of capital to liquidate at date 1 ℓ_1, and how much date 0 debt to issue D_0 in order to maximize its expected date 2 consumption $E_0\left[c_2^B\right]$, subject to its budget constraints, the natural borrowing limit, the non-negativity constraint on date 1 investment, and the law of motions for capital. The full problem and optimality condition are given in Online Appendix 2.

[17] This liquidation cost is a reduced form way to capture endogenous fire sales similar to Lorenzoni (2008). While this reduced-form representation yields very similar dynamics, it makes the model considerably more tractable.

[18] Thus, at date 1, the bank is potentially liquidity constrained for three reasons: It cannot issue state-contingent debt ex ante, it cannot raise new debt to finance repayment ex post, and capital is partially illiquid at date 1.

Assumption 1 below ensures that the bank's date 0 natural borrowing limit on borrowing D_0 is never binding in equilibrium, and that liquidation occurs only in the bad state, $\ell_1(s_L) > 0$, $\ell_1(s_H) = 0$. Appendix 3 also shows that these restrictions are satisfied for a non-empty set of parameters.

Assumption 1:
A) $z_1(s_L) \in \left(\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha (\tilde{k}_1)^{\alpha-1} \bar{n}^{1-\alpha}}, \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha (\tilde{k}_1)^{\alpha-1} \bar{n}^{1-\alpha}} \right)$ and $z_1(s_H) \geq \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha (\tilde{k}_1)^{\alpha-1} \bar{n}^{1-\alpha}}$
B) $T_1^B < k_1$

Date 1 In the good state at date 1 ($z_1 = z_1^H$), the bank's rental income $r_1^k(s_H)k_1$ is high, and the bank can repay its debt and finance investment without liquidating any of its capital holdings, so that $\ell_1(s_H) = 0$. I refer to this situation as a 'normal times'. On the other hand, in the bad state at date 1 ($z_1 = z_1^L$), the bank's rental income $r_1^k(s_L)k_1$ is insufficient to meet debt repayments, so the bank liquidates capital to cover the difference. I refer to this situation as a 'crisis' at date 1.[19] More precisely, the bank forgoes investment, $i_1(s_L) = 0$, and liquidates just enough capital to repay its debt, so that $\ell_1(s_L)$ is pinned down by the binding non-negativity constraint on date 1 investment,

$$\ell_1(s_L) = Lev_0 - r_1^k(s_L) \tag{14}$$

where I have defined the leverage of the bank at date 0 as the ratio of the bank's effective liabilities net of lump-sum transfers to its assets, $Lev_0 := \frac{D_0 R_0 - T_1^B}{k_1}$. Thus, there is a kind of pecking order in how the bank finances its debt repayment: It first uses its rental income to repay its debt to the extent possible, and then covers any remaining obligations by liquidating capital.

Equation (14), which I refer to as the *Crisis Risk curve*, is a key equation which links bank leverage (and the supply of safe assets) to crisis risk.[20] It follows directly from this expression that the higher the bank's date 0 leverage, the more of its capital holdings the bank must liquidate in the bad state at date 1. Thus, higher private safe asset creation at date 0 leads to more severe crises in the bad state at date 1. This is shown formally in Appendix 4.

Date 0 The bank's desired leverage ratio at date 0 is determined by its choice of D_0 , which balances the marginal benefit of debt with the marginal cost.

$$\underbrace{E\left[r_2^k\left(r_1^k(s)+1\right)\right] + \lambda_1(s_L)\left[r_1^k(s_L) + \ell_1(s_L)\right]}_{marginal\ benefit} = \underbrace{E\left[r_2^k\left(R_0 + \phi\left(\ell_1(s)\right)\right)\right] + \lambda_1(s_L)R_0}_{marginal\ cost} \tag{15}$$

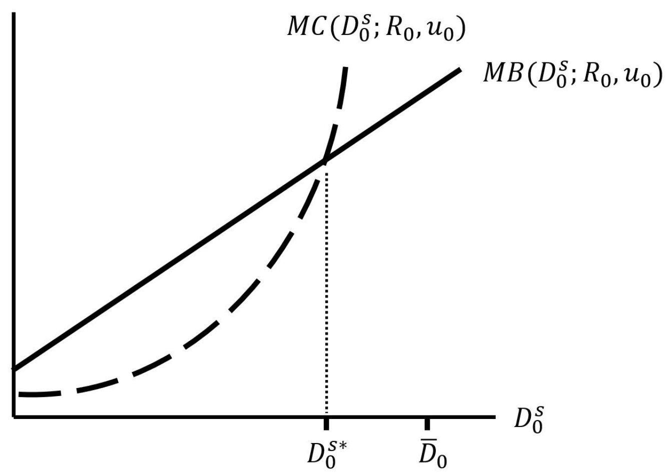
[19] The term 'crisis' here refers to the liquidation costs and associated deadweight loss associated with liquidation when the banking sector is faced with large losses. These costs are intended to capture the inefficiencies associated with financial crises and fire sales when losses in the financial sector are large.

[20] As will become clear in section 3, the severity of crises $\ell_1(s_L)$ is directly related to the household's expected future labor income and consumption $E_0\left[\frac{1}{c_1(s)}\right]$ via the macroeconomic spillover from liquidation to the household's labor income.

The marginal benefit of date 0 debt is given by the expected return to capital across dates 1 and 2 and the value of relaxing the non-negativity constraint on investment in the bad state (the second term on the left), where $\lambda_1(s_L)$ reflects the shadow price of funds in the bad state. The marginal cost of debt is given by the interest rate R_0 and the expected cost of liquidating extra capital in the bad state. Moreover, each unit of debt issued at date 0 tightens the bank's non-negativity constraint on date 1 investment in the bad state by the interest rate R_0, which has a cost given by the shadow value of liquid funds in that state, $\lambda_1(s_L)$.

The convex liquidation cost $\phi(\ell_1)$ introduces a concavity in the bank's date 1 payoff function (as a function of D_0) which makes the bank behave at date 0 as if it is risk averse. I show in Appendix 1 that, as long as the liquidation cost function $\phi(\cdot)$ is sufficiently convex, the bank is at an interior optimum for his choice of D_0 and the natural borrowing limit $\overline{D}_0$ is non-binding in equilibrium. This is illustrated in Figure 4, where $MB(D_0;R_0)$ and $MC(D_0;R_0)$ denote the marginal benefit and marginal cost, respectively, of date 0 private debt D_0. The bank's choice of D_0^{s*}, given the interest rate, defines the supply curve of private safe assets.

Figure 4: Bank's leverage choice



Risk premium In equilibrium, there is a risk premium on capital at date 0, which is defined as the difference in the expected (gross) rate of return to capital, $1+\frac{E\left[r_2^k(s)r_1^k(s)\right]}{E\left[r_2^k(s)\right]}$, and the effective risk-free rate of return, $\tau_0^R R_0$. Online Appendix 12 shows that this risk premium is strictly positive due to the liquidation cost of capital $\ell_1(s)>0$.[21]

$$RP_0 := 1+\frac{E\left[r_2^k(s)r_1^k(s)\right]}{E\left[r_2^k(s)\right]}-R_0>0 \tag{16}$$

[21] Thus, the liquidation cost in this model lays a similar role in generating a risk premium that the collateral constraint plays in Caballero and Farhi (2018).

3 Equilibrium

The equilibrium is a set of processes for allocations, prices, and returns such that households and banks maximize expected utility, firms maximize profits, capital evolves according to its laws of motion, the nominal interest rate follows the rule described in section 2.4, and the markets for labor, capital, and private and public bonds clear. Recall that supply of public bonds B_0 is taken as exogenous until the normative section. I solve for the equilibrium recursively.

3.1 Equilibrium at dates 1 and 2

At dates 1 and 2, the monetary policy rule ensures that output is at potential in both states of the world: $y_1(s) = z_1(s)k_1(s)^{\alpha}\overline{n}^{1-\alpha}$ and $y_2(s) = z_2k_2(s)^{\alpha}\overline{n}^{1-\alpha}$. I solve for the date 2 equilibrium in Online Appendix 4.

The equilibrium at date 1 features two regimes, depending on whether the bank liquidates capital or not, as shown in Online Appendix 5. In the *normal regime*, which obtains when productivity is high $z_1 = z_1^H$, the bank's non-negativity constraint on date 1 investment is non-binding and the bank does not liquidate any of its capital holdings so that $\ell_1(s) = 0$, $\lambda_1(s) = 0$. In this regime, the bank's date 1 return from rental income is sufficient to meet its debt obligations D_0R_0. In the *crisis regime*, which obtains when productivity is low $z_1 = z_1^L$, the banks' date 1 income is insufficient to cover its debt obligations, forcing the bank to liquidate some portion $\ell_1(s) > 0$ of its capital holdings, which is pinned down by its binding non-negativity constraint on date 1 investment. The lower that productivity is $z_1(s_L)$ at date 1, the more severe the crisis.[22]

Crises entail a macroeconomic spillover Crises at date 1 entail *macroeconomic spillovers* on the household's future labor income owing to complementarities between capital and labor. This is illustrated by the dashed arrow in Figure 1. In particular, liquidation at date 1 $\ell_1(s_L)$ lowers the future capital stock $k_2(s_L)$ given by (12), both directly and due to the liquidation cost $\phi(\ell_1)$. Since labor supply is fixed $\overline{n}$, the fall in the date 2 capital stock reduces date 2 output $y_2(s_L)$, and thereby lowers the labor income earned by the household at date 2, $w_2(s_L)\overline{n}$.[23] Therefore, $\frac{dw_2(s_L)\overline{n}}{d\ell_1(s_L)} < 0$.

Lemma 1: *Crises entail a macroeconomic spillover* Liquidation in the bad state at date 1 reduces the household's labor income in the bad state at date 2, $\frac{dw_2(s_L)\overline{n}}{d\ell_1(s_L)} < 0$.

Proof: This follows directly from the equilibrium expression for the date 2 wage $w_2(s)\overline{n} = (1-\alpha)y_2(s)$, the date 2 production function for $y_2(s)$, and the law of motion for $k_2(s)$, after imposing the result that $i_1(s_L) = 0$ in equilibrium.

[22]Formally, $\frac{d\ell_1}{dz_1} = -\frac{dr_1^k}{dz_1} = -\frac{\alpha}{\bar{k}_t}\frac{y_1}{z_1} < 0$ because k_1 is predetermined at date 1 and labor is in fixed supply.

[23]More specifically, the lower capital stock at date 2 reduces the marginal product of labor in that date, and hence the wage.

Intuitively, the macroeconomic spillover captures in a simple way the broad and persistent macroeconomic effects of financial crises. When their losses are large, banks incur liquidation costs which capture the deadweight losses associated with financial crises. These losses result in lower investment, which, in equilibrium, reduces the future labor income for the household.[24]

Safe asset creation entails risk transformation The macroeconomic spillover implies that safe asset creation entails *risk transformation*, the key mechanism of the model. As illustrated in Figure 2, private safe assets (bank debt) insure individual households against the productivity shock at date 1, as the household's date 1 interest income $R_0^D D_0$ is constant across states. But in general equilibrium, safe asset creation (bank leverage at date 0) *increases* the household's labor income risk at date 2 in general equilibrium due to liquidation at date 1 and the macroeconomic spillover. This is showed formally in Appendix 2.[25] Thus, safe asset creation doesn't eliminate fundamental risk – it just *reallocates* it.

This risk transformation highlights a contradictory aspect of safe asset creation: The creation of private safe assets actually *amplifies* aggregate risk in general equilibrium. This is because the creation of safe assets generates deadweight losses from liquidation, increasing the the losses that must be absorbed by agents in the bad state.

However, individual households do not internalize how their saving decisions at date 0 affect their future labor income in general equilibrium, since they take future wages $w_2(s_L)$ as given. This will be a key source of inefficiency of the competitive equilibrium and will generate scope for monetary and macroprudential policies to improve efficiency.

3.2 Date 0 equilibrium

Household's demand for safe assets at date 0 The anticipation of a future crisis at date 1 generates a precautionary saving demand for safe assets on the part of the household at date 0, which is illustrated by the red dashed arrow in Figure 5. To see this, note that the household's demand at date 0 for private safe assets (bank debt) can be expressed by combining the household's Euler equation with its date 0 budget constraint, which I refer to as the *Saving Demand* curve. This equation is downward-sloping in the interest rate R_0 and depends on the level of utilization u_0 in

[24] Of course, liquidation would result in lower labor income $w_2(s_L)\overline{n}$ even in the absence of the liquidation costs $\phi(\ell_1)$. But these costs entail a deadweight loss which have normative implications to be explored later.

[25] Formally, Appendix 2 shows that a marginal increase in the supply of safe assets increases the severity of crises and therefore increases the household's expected marginal utility of consumption at dates 1 and 2, $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} \frac{d\ell_1(s_L)}{dD_0^s} > 0$, as long the government's holdings of capital k_2^G are sufficiently small that $\frac{k_2^G}{k_2+k_2^G} < \frac{1}{\overline{n}}$ (which holds since I have thus far assumed that $k_2^G = 0$).

general equilibrium.[26] Note also that, because only the household consumes at date 0, D_0^d inversely reflects aggregate consumption demand at date 0. As a result, the quantity of public bonds supplied by the government at date 0 acts as a consumption-demand shifter.

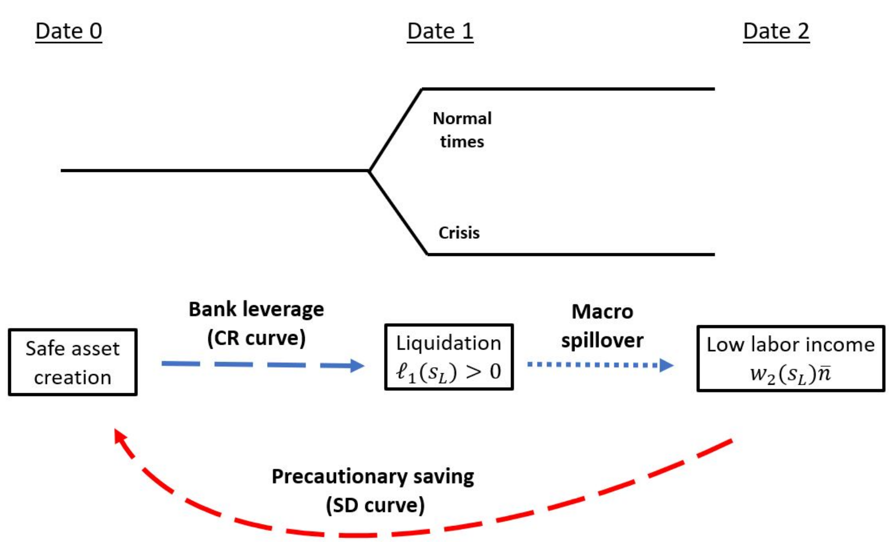
$$\underbrace{D_0^d\left(R_0, B_0; u_0\right)}_{demand\ for\ private\ safe\ assets} = \underbrace{w_0\bar{n} + e_0 - T_0 + d_0^F}_{income} - \underbrace{\frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1}}_{consumption\ demand} - \underbrace{B_0}_{supply\ of\ public\ assets} \tag{17}$$

One can see from this equation that a higher expected future marginal utility of consumption $E_0\left[\frac{1}{c_1(s)}\right]$, ceteris paribus, increases the household's demand for private safe assets at date 0. This equation captures the equilibrium relationship between the demand for safe assets and crisis risk. The lemma below shows that greater crisis risk increases the household's demand for safe assets

Lemma 2: *Crisis risk increases the demand for safe assets* In equilibrium we have $\frac{dD_0^d}{d\ell_1(\underline{s})} > 0$.
Proof: This is shown formally in Appendix 5.

Intuitively, as systemic risk increases, the household would like to save more at date 0 both to smooth consumption (in response to lower expected future consumption) and for precautionary motives (in response to higher future consumption risk).

Figure 5: Precautionary demand for safe assets



[26]The negative relationship between the household's demand for private bonds D_0^d and the government's supply of public bonds B_0 reflects that a higher supply of public bonds crowds out the household's demand for private bonds, since the bonds are perfect substitutes from the perspective of the household.

Paradox of Safety Together, the Crisis Risk curve (14), the macroeconomic spillover, and Saving Demand curve (17) imply that creation of safe assets generates a *two-way feedback* between crisis risk and the demand for safe assets, illustrated in Figure 5. In particular, safe asset creation by banks at date 0 leads to liquidation of capital in the bad state at date 1 (via the Crisis Risk curve), which in turn lowers the household's future (date 2) labor income due to the macroeconomic spillover. In anticipation of this, households at date 0 desire greater saving precautionary saving ex ante (via the Saving Demand curve). In turn, the higher demand for safe assets induces banks to create more safe assets at date 0, which further increases liquidation in the bad state. Moreover, the supply and demand for safe assets at date 0 will determine aggregate demand, which will have important implications at the effective lower bound, which is an issue I will return to later.

This two-way feedback between crisis risk and the demand for safe assets gives rise to a *paradox of safety* in which the demand for insurance against systemic risk *further increases* systemic risk through the creation of private safe assets.[27] This paradox is at the heart of the model's mechanism, and arises because households do not internalize how their demand for safe assets increases their labor income risk in general equilibrium.

3.2.1 General Equilibrium at Date 0

The interest rate and utilization rate R_0, u_0 are determined in general equilibrium to clear the market for safe assets $D_0^{d*}(R_0; u_0) = D_0^{s*}(R_0; u_0)$, where the bank's supply curve of safe assets is given by the implicit solution to (15) and the household's demand for safe assets is given by the Saving Demand curve (17). Whether R_0 or u_0 adjusts to clear the market for safe assets depends on whether monetary policy is constrained in equilibrium by the effective lower bound.

Natural rate of interest The natural rate of interest at date 0, R_0^*, is defined as the interest rate which would clear the market for safe assets, $D_0^{d*}(R_0; u_0) = D_0^{s*}(R_0; u_0)$, when resources are fully utilized $u_0 = 1$. As a result of the dependence of the household's demand for safe assets $D_0^{d*}(R_0; u_0)$ on the severity of future crises $\ell_1(s_L)$, the natural rate of interest depends on the level of systemic risk. One way to see this is to rearrange the Euler equation to express the interest rate as a function of the household's date 1 consumption.

$$R_0^* = \frac{u'(c_0)}{E_0\left[u'\left(c_1(s)\right)\right]}\ , \quad c_1(s) = \gamma \underbrace{\left[R_0\left(D_0 + B_0\right) + w_1(s)\bar{n} - T_1\right]}_{date\ 1\ net\ income} + (1-\gamma)\underbrace{\left[w_2(s)\bar{n} - T_2\right]}_{date\ 2\ net\ income}$$

Note that equilibrium consumption at date 1 reflects not only the household's date 1 income, but

[27] This is related to the well-known paradox of thrift, in which an increase in saving lowers aggregate demand and output, ultimately reducing total saving. However, the paradox of safety works through the endogenous creation of aggregate risk, and the desire for insurance against this risk.

also its date 2 income due to the household's consumption smoothing between dates 1 and 2.[28] A higher risk of crisis $\ell_1(s_L)$ reduces the household's future labor income $w_2(s_L)\bar{n}$ in the bad state, increasing the household's precautionary demand for safe assets D_0^{d*}ex ante. This depresses the natural rate at date 0. Thus, when crisis risk is sufficiently high, the natural rate can fall below the effective lower bound on monetary policy, giving rise to a demand-driven recession discussed below.

Two regimes at date 0 When crisis risk $\ell_1(s_L)$ is sufficiently high in equilibrium, the natural rate is below the effective lower bound, $R_0^* < 1$. Thus, there are two equilibrium regimes at date 0 depending on whether or not the effective lower bound is binding in equilibrium.

Lemma 3: There are two regimes at date 0: a supply-determined regime in which the effective lower bound on the nominal interest is non-binding $R_0^B > 1$ and capital is fully utilized $u_0 = 1$; and demand-determined regime in which the effective lower bound is binding $R_0^B = 1$ and capital is under-utilized $u_0 < 1$.

Proof: See Online Appendix 6.

If the natural rate $R_0^* \geq 1$, then the aggregate *supply-determined regime* prevails, in which case the effective lower bound is not binding and monetary policy ensures that output is at potential. The nominal interest rate R_0 is able to adjust to equilibrate the demand for saving at date 0 and is determined such that the bank's optimality condition for borrowing holds. If the natural rate $R_0^* < 1$, the effective lower bound is binding $R_0 = 1$ and the aggregate *demand-determined regime* obtains. In this case, output is demand determined, a fall in utilization must clear the market giving rise to a **demand-driven recession** in which output is below potential. In Online Appendix 7, I show how the adjustment in R_0 or u_0 equilibrates the the economy in each regime.[29]

3.2.2 Depressed Demand and High Systemic Risk Reinforce One Another

Demand-driven recession is caused by excessive systemic risk The demand-driven recession at date 0 occurs because aggregate risk is too high relative to the capacity of the economy to

[28]As shown in Online Appendix 6, $\gamma = 1/2$ due to the household's perfect consumption smoothing between dates 1 and 2.

[29]Essentially, which regime prevails at date 0 depends on how the economy adjusts to clear an excess demand for saving, denoted by $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$. Recall that I had showed above that D_0^d reflects aggregate consumption demand while D_0^s reflects aggregate investment demand at date 0. Thus, aggregate demand relative to potential output at date 0 is captured by the excess demand for saving at date 0, $D_0^d(R_0, B_0; u_0) - D_0^s(R_0)$. (Note that the expression for excess demand for private debt already embeds the market clearing condition in the market for public bonds, and reflects that the government's exogenous supply of public bonds affects household's demand for private bonds through $D_0^d(R_0, B_0; u_0)$.) That is, if there is an excess demand for safe assets, given by $D_0^d - D_0^s > 0$, aggregate demand is below total output. In the supply-determined regime, when the lower bound on the nominal interest is not binding, monetary policy can fall to be equal to the natural rate of interest ensuring that aggregate demand for output equals potential (that, there is no excess demand for saving, $D_0^d - D_0^s = 0$).

absorb this risk. Given the labor income risk it faces, the household wants to save more, which lowers consumption demand at date 0. However, banks are unwilling to issue more safe assets at date 0 because of the high liquidation risk they face, which reduces investment demand. Hence, investment demand does not compensate for the fall in consumption demand, and so the high level of risk depresses aggregate demand at date 0. At the effective lower bound, the high interest rate not only increases the household's desire for saving; it also increases the bank's burden of debt, and hence, the severity of a future crisis. The equilibrium can only be reached through a fall in utilization and output, which reduces total investment and the amount of aggregate risk that agents must absorb.[30]

Recession reinforces systemic risk In turn, the demand-driven recession at date 0 reinforces systemic risk and the shortage of safe assets. To see this, note that the recession at lowers the banks date 0 rental income $r_0^k k_0 = \alpha y_0$ by reducing date 0 output. The fall in rental income erodes the bank's net worth at date 0, further increasing leverage at a given level of debt. (To see this, note that we can express the bank's leverage ratio as $Lev_0 = \frac{D_0 R_0 - T_1^E}{D_0 + \left(r_0^k + 1\right) k_0 + T_0^E}$ using its date 0 budget constraint and the law of motion for k_1.) In order to manage its leverage ratio, the bank also reduces its supply of safe assets somewhat, $D_0^{s*}(R_0; u_0)$. This exacerbates excess demand for safe assets $D_0^{d*}(R_0; u_0) - D_0^{s*}(R_0; u_0)$, requiring deeper recession to reach equilibrium.

However, the bank also accomodates part of the increase in its leverage, resulting in higher systemic risk in equilibrium: Crises at date 1 become more severe ($\ell_1(s_L)$ rises), which lowers the household's date 2 labor in the bad state. Thus, two-way feedback between low aggregate demand and high systemic risk. These effects are formalized in Online Appendix 14.

3.3 Risk-Driven Stagnation Trap

When the feedback loop between aggregate demand and systemic risk is sufficiently severe, the economy can enter a *risk-driven stagnation trap* in which (economic stagnation arises due to excessive systemic risk) high systemic risk results in persistently low future output growth.

To exposit the economic forces behind the trap, I consider the response of the economy to an exogenous increase in systemic risk in the demand-determined regime at date 0. In particular, I trace out the effects of an unanticipated (MIT) shock to future TFP in the bad state, detailed in Online Appendix 10. Agents learn at date 0 that TFP in the bad state at date 1, $z_1(s_L)$, is lower than

[30] The role played by aggregate risk is similar in spirit to the risk-centric perspective of demand recessions in Caballero and Simsek (2020). Here, as in that paper, the demand recession arises fundamentally due to an excessive amount of risk and constraints on monetary policy. The crucial difference of the mechanism compared to that paper is that aggregate risk arises endogenously through the creation safe assets. Households' demand for safe assets as insurance against aggregate risk can itself create further aggregate risk. This is the paradox of safety at the heart of this paper. Methodologically, financial frictions play an important role in generating this mechanism.

initially thought – essentially an adverse news shock about future productivity. This shock reduces the bank's rental income in the bad state at date 1, and therefore increases the level of liquidation in that state, $\ell_1(s_L)$. Since the shock directly increases the severity of crises in the bad state at date 1, it can be interpreted as an exogenous increase the systemic risk faced by agents at date 0.[31]

The macroeconomic forces outlined in sections 3.1 and 3.2 (that is, the macroeconomic spillover and the precautionary demand for safe assets at date 0) imply that the shock to systemic risk reduces output at date 0, y_0. A more severe crisis at date 1 lowers the household's future labor income at date 2 due to the macroeconomic spillover (Lemma 1). The prospect of more severe increases the household's precautionary saving at date 0 (Lemma 2). In turn, the higher saving demand at date 0 lowers aggregate demand and the natural rate of interest, requiring a larger drop in output at date 0 to reach equilibrium.[32]

Recession may reduce capital accumulation and lead to persistently low output growth If these forces are sufficiently strong, the shock can lead to trap in which the increase in systemic risk leads to persistently lower output growth. In particular, the recession at date 0 has two effects on date 0 investment, $i_0 = D_0 + r_0^k k_0 + B_0$, in the demand-determined regime. First, the shock and recession have an ambiguous effect on the equilibrium amount of private debt issued, D_0. Second, the recession reduces output and the bank's rental income $r_0^k k_0$ at date 0. If the latter effect of the recession on date 0 income is sufficiently severe, then this latter effect dominates and investment at date 0 falls in response to the shock, $\frac{di_0}{dz_1(s_L)} > 0$. Moreover, if the fall in investment is sufficiently large, then the shock may have persistently negative effects on future output growth at dates 1 and 2. This is summarized in the following proposition.

Proposition 1: *An increase in systemic risk can lead to stagnation:*

A) If the date 0 recession is sufficiently severe, then date 0 investment falls, $\frac{di_0}{du_0} > 0$. The recession is more severe in response to the shock to systemic risk if the macroeconomic spillover ($\frac{dw_2(s_L)\bar{n}}{d\ell_1(s_L)}$) and the effect of crisis risk on precautionary saving ($\frac{dD_0^d}{d\ell_1(\underline{s})}$) are sufficiently strong.

B) If the fall in date 0 investment is sufficiently large, then the shock can lead to persistently low output growth such that $\frac{dy_1(s)}{dz_1(s_L)}, \frac{dy_2(s)}{dz_1(s_L)} > 0$ in each state.

Proof: See Appendix 6.

Intuitively, a severe recession at date 0 erodes the resources available for investment at dates 0 and 1, which reduces capital accumulation over time k_1,$k_2(s)$, and therefore lowers future output $y_1(s)$, $y_2(s)$ in all states at dates 1 and 2. Thus, a rise in systemic risk can result in persistently

[31] The response of the economy, and hence the channels I outline below, will be similar across any shock which causes an excess demand for safe assets at date 0.

[32] I restrict analysis to the case in which $\ell_1(s_L) > \left(\frac{1}{\eta}\right)^{\frac{1}{\eta-1}}$ holds in equilibrium, which, as I show in Online Appendix 12, is a sufficient condition to ensure the shock worsens the recession.

low output growth, by triggering a demand-driven recession ex ante and reducing investment and capital accumulation.[33]

The combination of high systemic risk and low future output growth has the features of a trap. The trap arises because households have a demand for insurance against aggregate risk at date 0, and for consumption smoothing. But creating safe assets requires banks to take leverage, which would only increase systemic risk borne by households at date 0. Thus, the risk-driven stagnation trap arises because both the creation of safe assets, and investment in capital, requires that banks take more leverage, which only serves to increase systemic risk and worsen the demand-driven recession ex ante.

The risk-driven stagnation trap described here is closely related to notable papers in the literature, but the different nature and origin of the trap here leads to qualitatively different implications. In particular, the trap described here is similar to the stagnation trap of Benigno and Fornaro (2018), but arises from the endogenous creation of aggregate (systemic) risk rather than self-fulfilling expectations of low future growth. As a result, policies designed to incentivize investment – such as subsidies to private investment or subsidies for the creation of private safe assets – while useful in that paper, would be counterproductive here: Banks are already too highly levered, and increasing investment or the supply of private safe assets would only further increase their leverage and aggregate risk. The risk-driven stagnation trap is also similar to the safety trap of Caballero and Farhi (2018). But here, the trap derives from an oversupply of private safe assets rather than a shortage of such assets, as in that paper. Therefore, unlike in that paper, policies designed to increase the supply of private safe assets will be counterproductive here, as they would serve to increase bank leverage and the aggregate risk required to be absorbed by agents.

4 Policy Implications

I now explore the policy implications of the model, taking the monetary policy rule laid out in section 2.4. as given.

4.1 Quantitative Easing

Persistent slumps often feature low expected output growth, high risk premia, deleveraging in the financial sector. These periods have featured highly unorthodox policy interventions in which the

[33] In Online Appendix 12, I also show that this stagnation is more severe when the macroeconomic spillover is larger (i.e., the elasticity $\frac{dw_2(s_L)\bar{n}}{d\ell_1(s_L)}$ is more negative), and when the effect of crisis risk on on precautionary saving is stronger (i.e., the elasticity $\frac{dD_0^d}{d\ell_1(\underline{s})}$ is larger).

central bank bought risky assets from the private sector.[34] Such interventions proved controversial amongst academics and policymakers, as they involved the government taking on the risk associated with these assets. An open question is why policymakers resorted to such unconventional policies, and why government purchases of safe assets, such as government bonds and investment grade corporate bonds, were deemed inadequate?

Motivated by this question, I now consider the effect of government purchases of risky assets when monetary policy is at the effective lower bound. Recall from section 3.2.2. that, in this setting, demand-driven recessions arise because of an excess amount of risk being borne by banks and households. Purchases of risky assets by the government stimulate demand by transferring risk from bank balance sheets to the government, in the process reducing the severity of crises and stimulating aggregate demand ex ante.[35]

4.1.1 Government purchases of risky assets

At date 0 – that is, *before* a financial crisis at date 1 – the government can purchase capital (the risky asset) from banks in a competitive spot market at price q_0, where k_1^G denotes the quantity of capital that the government purchases.[36] [37] The government finances these purchases by issuing public debt to households. Therefore, the government's date 0 budget constraint (4) is modified to allow for asset purchases (and for ease of exposition, I set other transfers to 0).

$$B_0 = \underbrace{q_0 k_1^G}_{\textit{risky asset purchases}} \tag{18}$$

The bank's date 0 budget constraint is also modified to incorporate income from sales of capital to the government $q_0 k_1^G$.[38]

$$D_0 + r_0^k k_0 + \underbrace{q_0 k_1^G}_{\textit{asset sales}} = i_0 \tag{19}$$

[34] For example, during the long slump in Japan (1995-2020), the Bank of Japan resorted to buying equities to stimulate aggregate demand, while central banks in the US and the Eurozone purchased mortgage-backed securities (QE1), corporate bonds, and risky sovereign debt (Long-Term Refinancing Operations) during the period of anemic growth and low interest rates following the Global Financial Crisis.

[35] The literature has offered other channels through which asset purchases may operate, such as through risk premia, asset prices, and availability of credit to riskier borrowers, etc. I view the channels outlined in this paper as complementary to these alternative explanations.

[36] The government here is the consolidated fiscal and monetary authority.

[37] Rather than buying capital directly, I could equivalently assume that the government buys at date 0 a claim on the date 1 rental rate of capital, and issues a lump-sum transfer to the bank in the bad state at date 1 to prevent it from liquidating its capital holdings. This alternative would yield the same allocation.

[38] Formally, the bank chooses at date 0 how much of its date 0 capital holdings to sell in the spot market k_1^{QE} given the prevailing spot price. For simplicity, I have imposed here the market clearing condition that $k_1^G = k_1^{QE}$, but Online Appendix 6 formalizes the problem.

The bank can immediately reinvest any proceeds from asset sales in the creation of new capital. The bank's stock of capital evolves between dates 0 and 1 according to $k_1 = i_0 + k_0 - k_1^G$. In equilibrium, the price of capital is constant $q_0 = 1$.

At date 1, the government earns rental income on its capital holdings, $r_1^k(s)k_1^G$, and has to repay its debt $R_0^B B_0$.[39]

$$\underbrace{R_0^B B_0}_{debt\ repayment} = \underbrace{r_1^k(s)k_1^G}_{rental\ income} + \underbrace{T_1(s) - T_1^B(s)}_{lump-sum\ taxes\ and\ transfers} \tag{20}$$

I assume that the government covers any net losses at date 1 by levying lump-sum taxes on the household, and it transfers any profits to banks in lump-sum fashion. More precisely, I assume that lump-sum taxes on the household are $T_1(s_H) = 0$ and $T_1(s_L) = R_0 B_0$.[40]

The government stores its date 1 capital holdings through date 2 without depreciation, so that $k_2^G = k_1^G$. At date 2, the government earns rental income on its holdings of capital and keeps a balanced budget.

$$T_2 - T_2^B + r_2^k k_2^G = 0 \tag{21}$$

For now, I take the government's behavior as given.

In this environment, asset purchases affect real variables through broadly two channels. First, QE works conventionally by affecting interest rates/spreads or asset prices, in line with channels explore in previous literature (e.g.). Second, QE affects real outcomes through a new *risk-absorption channel* in which government purchases of risky assets stimulate by reallocating aggregate risk from bank to government balance sheets (?).

Conventional channels of quantitative easing QE has conventional effects on the risk premium and aggregate demand. In particular, QE increases the aggregate capital stock which reduces lowers future rental rate $r_1^k(s)$ in all states. This reduces the risk premium $1 + \frac{E\left[r_2^k(s) r_1^k(s)\right]}{E\left[r_2^k(s)\right]} - R_0 > 0$.[41] Moreover, QE increases aggregate investment demand because the government buys capital from the bank, financed by resources borrowed from the household.[42] In turn, the rise in

[39] Because the government rents its holdings of capital to the monopolistically competitive firms at dates 1 and 2, output at those dates depends on the aggregate capital stock $\tilde{k}_t := k_t + k_t^G$.

[40] This implies that the household is fully bailed-in on its holdings of B_0 via a lump-sum tax. The government's date 1 budget constraint then implies that the lump-sum transfers to the bank are $T_1^B(s_H) = r_1^k(s_H)k_1^G - R_0^B B_0$ and $T_1^B(s_L) = r_1^k(s_L)k_1^G$.

[41] QE does not affect price of capital q_0, nor does it create a spread between public and private safe assets. This is because the price of capital is pinned down by arbitrage by the one-to-one conversion rate of the consumption good to capital. Moreover, public and private bonds are equivalent from persepective of individual market participants, so QE cannot insert a wedge between their rates of return. Also, QE does not directly affect the interest rate, but affects it only in general equilibrium through the Euler equation by affectin the marginal utilities of current versus future consumption.

[42] Since Ricardian equivalence does not hold in this model, the larger public debt is not fully offset by a fall in

investment demand boosts aggregate demand, which lifts output and the natural rate.

4.1.2 Risk-absorption channel of quantitative easing

Government purchases of risky assets affects date 0 output through a new channel in which QE transfers risk from private balance sheets to that of the government. To understand the effects of QE on risk borne by households, we can divide the effects into ex post effects (on date 1 and 2 variables) and ex ante effects (effect on date 0 variables).

First consider the ex post effects at date 1. One can see from the government's date 1 budget constraint (20) that the government can always repay its debt $B_0 R_0^B$ *without liquidating capital*, due to its power to tax. If its rental income is insufficient to pay its debt at date 1, the government levies lump-sum taxes on the household to cover the losses. Effectively, this bails-in the household on losses incurred on capital. Therefore, in the bad state at date 1, none of the government's holdings of capital k_1^G is liquidated. As a result, there are smaller deadweight losses from liquidation, ceteris paribus. This mitigates the macroeconomic spillover to date 2 and boosts the household's date 2 labor income $w_2(s_L)\overline{n}$ (see Lemma 1).

From an ex ante perspective, in anticipation of its effects at dates 1 and 2 effect, the government asset purchases thus lowers the labor income risk borne by the household. This lowers precautionary saving ex ante via the Saving Demand curve (17), which increases consumption demand and output at date 0 (see Lemma 2).

This risk-absorption channel, through which government purchases of risky capital lower the household's demand for safe assets, is formalized in Part A of Proposition 2. The channel follows from the macroeconomic spillover and its effect on precautionary saving ex ante (Lemmas 1 and 2): Less liquidation of capital at date 1 (less severe crises) reduces the labor income risk borne by household, and therefore mitigates precautionary saving ex ante. Ceteris paribus, this reduces the excess demand for safe assets at date 0, $D_0^d - D_0^s$, which leads to a higher utilization rate u_0 and output y_0.

Proposition 2:

A) Risk-absorption channel of QE Government purchases of capital at date 0 reduce the household's demand for safe assets at date 0 by reducing its labor income risk, $\frac{dD_0^d}{dk_1^G} > 0$. Through this channel, the asset purchases alleviate an excess demand for safe assets at the effective lower bound, ceteris paribus. This channel is stronger when the macroeconomic spillover ($\frac{dw_2(s_L)\overline{n}}{d\ell_1(s_L)}$) and precautionary saving effect ($\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}$) are stronger.

B) These asset purchases stimulate date 0 output at the effective lower bound, $\frac{du_0}{dk_1^G} > 0$, as long

consumption demand by the household.

as government purchases do not crowd out private borrowing D_0^s and investment too much. A sufficient condition is that liquidation costs are sufficiently large relative to the date 1 rental rate in the bad state, $\phi\left(\ell_1(s_L)\right) > r_1^k(s_L)$.

Proof: See Appendix 7.

Intuitively, government purchases of capital k_1^G from banks reallocates aggregate risk associated with the date 1 productivity shock, $r_1^k(s)k_1^G$, from bank balance sheets to that of the government. This reallocation of risk has real effects because the government has a comparative advantage at bearing aggregate risk as a result of its power to tax. (Indeed, the power to tax is the only capability the government has that the private sector does not, in the model.) This power to tax allows the government to smooth losses across agents and time in contrast to banks, who are forced to finance losses at date 1 by liquidating capital.

The reallocation of aggregate risk from bank balance sheets to the government, in turn, has two effects. First, it improves risk sharing between banks and households in general equilibrium.[43] In particular, it forces households to bear losses on the rental rate of capital in the bad state (in the form of taxes exacted to cover the government's losses), and therefore ex ante, the household bears some of the risk associated with investment in capital. Put differently, QE reduces the risk transformation that results from the creation of private safe assets, discussed in section 3.1.

Second, this improved risk sharing *reduces* aggregate risk endogenously: By forcing households to take losses in the bad state at date 1, the government's asset purchases reduce the deadweight loss incurred from liquidation and thus reduces the aggregate losses needed to be absorbed by agents in the bad state. In turn, the fall in risk reduces precautionary saving, stimulating aggregate demand and output at date 0.

Safe asset purchases inadequate As discussed in section 3.2.2., the demand recession here arises ultimately because there is too much risk associated with production and investment relative to the capacity of agents to absorb it. Government purchases of risky assets reduce the risk borne by the household by transferring risky assets from bank balance sheets to that of government, reducing crisis risk in the process. In contrast, government purchases of safe assets (here, private bonds issued by banks) would be inadequate to stimulate output in this setting, as it would not reduce the risk borne by banks nor the labor income risk borne by households.[44]

The model may thus help rationalize the extraordinary government interventions observed in advanced economies mentioned at the beginning of this section. Viewed through the lens of this model, output is depressed in these situations due to an excessive amount of risk borne by the

[43] Effectively, QE changes the composition of the assets which implicitly back safe assets, as a higher fraction is backed by the government's tax power as opposed to the liquidation value of capital.

[44] Government purchases of safe assets would stimulate output here only if coupled with government guarantees of bank assets or an announcement of a bailout of banks in bad states. But even then, such a policy would be useful only to the extent that it reallocates risk from banks to the government, and would be equivalent to risky asset purchases.

private sector. Asset purchases can stimulate only by reallocating this risk from private sector to government, which has greater capacity to bear this risk. This is precisely what is achieved by risky asset purchases.

Part (B) of Proposition 2 outlines the conditions under which risky asset purchases increase output on net. Intuitively, while asset purchases reduce demand for safe assets D_0^d through the risk absorption channel, they also have an effect on the bank's supply of safe assets D_0^s. The response of D_0^s to QE depends on two effects. On the one hand, QE reduces the bank's liquidation of capital and therefore lowers the expected marginal cost of issuing debt for the bank. On the other hand, it may increase the aggregate capital stock, which lowers the expected rental rate of capital at date 2 and lowers the marginal benefit of issuing debt. As long as the net effect is to increase D_0^s by crowding in private debt, or D_0^s is not crowded out too much, then QE will reduce excess demand for safe assets, stimulating output.

4.1.3 Social tradeoff associated with risky asset purchases

Recall from section 2.2. that private and public safe bonds are equivalent assets from the perspective on an individual household. However, they are not equivalent from a social perspective. The creation of private safe assets by banks entails the creation of crisis risk in general equilibrium by increasing liquidation and its associated deadweight costs. By contrast, public safe asset creation does not generate systemic risk in general equilibrium, as the government never needs to liquidate capital to finance its debt.[45]

Nevertheless, there is a social tradeoff associated with quantitative easing, implying that it is *not*, in general, socially optimal for the government to maximize its purchases of capital from the bank. To see this, note that by increasing date 0 output in the demand-determined regime, asset purchases may also crowd-in private debt issuance and investment, leading to a larger aggregate stock of future capital $\tilde{k}_1$. (Equivalently, QE may increase the size of banking sector in aggregate.) Since there are decreasing returns to investment, this may lead to suboptimal investment levels at date 0. This distorts the allocation of household consumption across time through the household's Euler equation.

Thus, QE involves a social tradeoff between boosting date 0 output and distorting the household's consumption-saving margin. As a result, a Ramsey planner who takes the behavior of agents and the determination of prices as given would, in general, want to calibrate the government's risky asset purchases in light of both margins of distortion to maximize welfare. Therefore, in general, it is not socially optimal for public safe assets to be the only type of debt held by households: The socially optimal level of QE should be at an interior solution rather than a corner.[46]

[45] This implies that public and private safe assets are *not* equivalent from a social perspective.

[46] Note that I have ruled out other distortions caused by government intervention, such as distortionary taxation.

4.2 Macroprudential Policy

Policymakers traditionally view macroprudential tools, such as bank capital regulation, as having a fundamentally distinct role from the monetary policy toolkit, with the former being useful tamping down on the buildup of financial vulnerabilities in the financial system and the latter for the management of aggregate demand. This paper shows that, due to the dynamic interplay between systemic risk and aggregate demand, macroprudential policies can help actively manage aggregate demand, particularly when monetary policy is constrained. Therefore, the paper introduces a new role for macroprudential policy – one of active management of aggregate demand.

To illustrate this, I consider the response of the economy (in the demand-determined regime) to a Pigouvian tax $\tau_0^R \geq 1$ on bank borrowing in which, at date 1, the bank pays to the government a tax $\left(\tau_0^R - 1\right) R_0^D D_0$ proportional to its date 0 borrowing.[47] The government's date 1 budget constraint modified to account for this additional tax revenue. The bank's date 1 budget constraint is also modified to reflect that the bank's total interest payments are $\tau_0^R D_0 R_0^D$, of which $D_0 R_0^D$ is paid to household creditors.

$$r_1^k(s)k_1 + \ell_1(s)k_1 + T_1^B(s) = i_1(s) + \tau_0^R D_0 R_0^D \tag{22}$$

This tax reduces bank borrowing at the margin by increasing its marginal cost, as can be seen in the bank's optimality condition for leverage.[48]

$$\underbrace{E\left[r_2^k\left(r_1^k(s)+1\right)\right] + \lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]}_{marginal\ benefit} = \underbrace{E\left[r_2^k\left(\tau_0^D R_0 + \phi\left(\ell_1(s)\right)\right)\right] + \lambda_1(s_L)\tau_0^D R_0}_{marginal\ cost} \tag{23}$$

By reducing private safe asset creation, the policy reduces the severity of future crises, $\frac{d\ell_1(s_L)}{d\tau_0^R} < 0$.[49] In doing so, the policy mitigates the macroeconomic spillover and thereby raises the household's labor income at date 2 $w_2(s_L)\overline{n}$ (by Lemma 1). Faced with less labor income risk, the

Nevertheless, one could easily include such distortions. This would not add much qualitative insight other than to introduce another social cost of public safe creation, in addition to the one mentioned above. Of course, any quantitative analysis of the benefits of QE would need to account for such costs of government intervention.

[47] One can interpret these taxes as macroprudential regulations, such as bank capital requirements, which limit the leverage of financial intermediaries.

[48] An alternative implementation of macroprudential policy which would have very similar effects is a tax on the household's holdings of bank debt. Since the supply of public bonds is fixed exogenously by the fiscal authority, macroprudential policy cannot lead to more public bonds. However, such a tax would increase the bank's cost of borrowing by introducing a wedge between the effective interest rate on public and private bonds (a kind of convenience yield), and therefore lower bank leverage, similarly to the effect of τ_0^R. Thus, macroprudential tax could reduce bank leverage and crisis risk either through a tax on bank borrowing or on the household's holdings of bank debt.

[49] There are two opposing effects of an increase in τ_0^R on $\ell_1(s_L)$. On the one hand, it reduces D_0, which reduces leverage and the amount of liquidation at date 1. On the other hand, it increase interest expenses at any level of D_0, which increase liquidation. For ease of exposition, I restrict analysis to the case in which the former effect dominates, outlined in Appendix 8.

household reduces its precautionary saving D_0^d at date 0 (by Lemma 2), stimulating aggregate demand. Thus, the dynamic interplay between aggregate demand and systemic risk implies that, when monetary policy is constrained, tighter macroprudential policy can stimulate aggregate demand and output ex ante. Such effects are larger when the macroeconomic spillover, $\frac{dw_2(s_L)\bar{n}}{d\ell_1(s_L)}$, and the precautionary saving effect, $\frac{dD_0^d}{d\ell_1(\underline{s})}$, are stronger.

The effect of tighter macroprudential regulation on output at date 0 depends on how it affects the excess demand for saving $D_0^d - D_0^s$ at the effective lower bound. There are two opposing forces. On the one hand, the reduction in the issuance of private safe assets $\frac{dD_0^s}{d\tau_0^R} < 0$ directly increases the excess demand for safe assets. On the other hand, the fall in systemic risk $\frac{d\ell_1(s_L)}{d\tau_0^R} < 0$, and by extension, the fall in precautionary saving $\frac{dD_0^d}{d\tau_0^R} < 0$ reduces the excess demand for safe assets. The following proposition establishes that this latter effect dominates when the macroeconomic spillover $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}$ and precautionary saving effect $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}$ are sufficiently strong.

Proposition 3: *Tighter macroprudential regulation can stimulate output ex ante:* Tighter macroprudential regulation (a higher τ_0^R) increases date 0 output in the demand-determined regime the reduction in the demand for safe assets due to a fall in crisis risk is larger than the fall in the supply of private safe assets. This occurs when the macroeconomic spillover and precautionary saving effect are sufficiently strong, formalized below.

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \tau_0^R} + \underbrace{\overbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}}^{\textit{precautionary saving}} \overbrace{\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}^{\textit{macro spillover}}}_{>0} \underbrace{\overbrace{\frac{d\ell_1(s_L)}{d\tau_0^R}}^{\textit{crisis risk}}}_{<0} < \underbrace{\overbrace{\frac{dD_0^s}{d\tau_0^R}}^{\textit{supply of private safe assets}}}_{<0}$$

Proof: See Appendix 8.

Hence, when the dynamic interplay between systemic risk and aggregate demand is strong, policies which restrict the supply of private safe assets, such as bank capital regulation, may counterintuitively mitigate safety traps and stimulate output ex ante.

Relative to the literature, macroprudential policy plays a qualitatively different role in managing the business cycle here. In the seminal work by Farhi and Werning (2016) and Korinek and Simsek (2016), in the presence of aggregate demand externalities, macroprudential policy can reduce the severity of *future* recessions by boosting future output in bad future states of the world. By contrast, macroprudential policy in this model can stimulate *current* output. Hence, the model points to a role for macroprudential policy in the active management of aggregate demand when demand is depressed due to excessive systemic risk. By lowering systemic risk and, as a result, precautionary saving, macroprudential policy can raise the natural rate of interest. In that sense, the model shows that macroprudential policies such as bank capital regulation can substitute for monetary policy to stimulate aggregate demand at the effective lower bound. This insight emerges precisely from dynamic interplay between systemic risk and aggregate demand at the heart of the model.

5 Normative Implications

This section is work in progress. In this section, I will examine the normative and policy implications of the model. To formalize these insights, I will solve the social planner's problem taking as given the government's behavior, to elucidate the externalities at play. Then I will solve the Ramsey problem to characterize optimal policy. and understand how different policy interventions can improve the allocation at the margin.

Pecuniary externality: The competitive equilibrium is in general constrained inefficient due to the presence of two externalities. The first is a pecuniary externality through the wage at date 2. Households do not internalize how their demand for safe assets at date 0 lowers their date 2 labor income in the bad state of the world due to the macroeconomic spillover in which a crisis at date 1 lowers date 2 labor income. Note that the effect of debt issuance on the severity of crises (the size of $\ell_1(s_L)$) is priced in the interest rate (the banks' cost of borrowing). However, the effect that liquidation has on the household's date 2 labor income in general equilibrium is not priced in, and hence there is an externality.

Relative to the social optimum, banks bears too much of the risk associated with investment at date 0 while the households bear too little. In order to insure households against investment risk at date 0, banks preserve the safety of their liabilities by liquidating capital in the bad state at date 1. However, due to the deadweight loss and macroeconomic spillover associated with liquidation, this forces the household to bear losses at date 2 in the form of labor income. Thus, by preventing the household from bearing investment risk at date 1, private safe asset creation forces the household to bear labor income risk at date 2 in general equilibrium, which is not internalized by the household.

The inefficient risk-sharing associated with private safe assets does not obtain for public safe assets. From the point-of-view of individual savers, these are equivalent instruments to smooth consumption as both the private and public bonds promise the same payoff profile at date 1. However, these are not equivalent means of smoothing consumption from a social perspective. Public safe assets are implicitly backed in part by the state-contingent stream of future tax revenue, which the government can generate without liquidating capital inefficiently.

Indeed, quantitative easing can improve welfare by forcing the household to bear some of the risk associated with investment, thus reducing systemic risk. Alternatively, macroprudential policies which tax bank debt or incentivize households to hold public debt rather than private debt, both can improve welfare by reducing bank leverage and systemic risk. This would increase the convenience yield on public debt.

Aggregate demand externality: The second externality, which obtains only in the demand-determined regime, is an aggregate demand externality in which households do not internalize how, at the effective lower bound, their date 0 spending affects boosts date 0 and therefore other households' and banks' income.[50] Moreover, because of the two-way interaction between aggregate demand and systemic risk, agents' date 0 spending decisions (aggregate demand) are also associated with the risk-sharing externality as date 0 output affects banks' net worth, and hence the severity of crises and the households' date 2 labor income.

6 Conclusion

This paper introduces a theory in which the creation of safe assets generates a two-way interaction between aggregate demand and the risk of future crises. The model highlights the role played by the composition of safe assets between public and private safe assets in the determination of economic activity, systemic risk, and growth. The paper also showed that monetary and macroprudential policy interventions can operate through additional channels once one accounts for the interactions between systemic risk and aggregate demand. The paper thus sheds light on the nature of safe asset shortages, and persistent slumps, and provides insight on how a range of policy interventions can influence these episodes.

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[50]While in similar spirit to that in Korinek and Simsek (2016), in that private leverage creates risk of demand-driven recessions, here the nature of the aggregate demand externality is meaningfully different. I discuss this further in the literature review.

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Appendices

APPENDIX 1: Bank behaves as if it is risk-averse

In this appendix, I show that under Condition 1, although the bank is risk neutral, the convex liquidation cost $\phi(\ell_1)$ introduces a convexity in the bank's payoff function (as a function of D_0). This convexity makes the bank behave at date 0 as if it's risk averse. First, let us define the marginal cost MC and marginal benefit MB of borrowing to the bank from its optimality condition for D_0.

$$MC \equiv E\left[v' r_2^k \left(\phi\left(\ell_1(s)\right)\left(1-\tau_0^D\right)+\tau_0^R R_0\right)\right]+\lambda_1(s_L)\tau_0^R R_0$$

$$MB \equiv \left(1-\tau_0^D\right)E\left[v' r_2^k\left(r_1^k(s)+1\right)\right]+\left(1-\tau_0^D\right)\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]$$

To show this, I will show that, although E's marginal benefit MB and marginal cost MC from borrowing are increasing in D_0, at the equilibrium value of D_0 (such that $MB = MC$), the marginal cost is increasing at a faster rate than the marginal benefit. That is, $\frac{\partial MC}{\partial D_0}|_{D_0^*} > \frac{\partial MB}{\partial D_0}|_{D_0^*}$, in partial equilibrium (i.e. taking prices as given). This is because of our assumption that the cost of liquidating capital in the bad state at date 1 is convex, $\phi(\ell_1) \geq 0$, $\phi'(\ell_1) \geq 0$, and $\phi''(\ell_1) \geq 0$, where the inequalities hold strictly for all $\ell_1 > 0$.For this reason, in partial equilibrium (i.e. taking prices as given), the bank can reach an interior optimum for his choice of D_0 – that is, he doesn't necessarily try to maximize borrowing, and so I can have a situation in which his

To show this, first recall from the bank's non-negativity constraint on date 1 investment, and the expression for ℓ_1 in the low state that, I have

$$\ell_1(s) = \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1} - r_1^k(s) = Lev_0 - r_1^k(s) \tag{24}$$

where $Lev_0 := \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1}$. Moreover, from the expression for the Lagrange multiplier, the extent to which the constraint binds depends also on ℓ_1, and hence leverage.

$$\lambda_1(s) = v' r_2^k(s)\phi'\left(\ell_1(s)\right) \tag{25}$$

$$\frac{\partial \lambda_1(\underline{s})}{\partial D_0} = v' r_2^k(\underline{s})\phi''\left(\ell_1(\underline{s})\right)\frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

Under what conditions do I have $\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$? First note that, since the bank takes prices as given in making its leverage decision, I are interested in the derivatives $\frac{\partial MC}{\partial D_0}, \frac{\partial MB}{\partial D_0}$ in partial equilibrium, i.e. leaving prices fixed. (While the bank's leverage decision will affect prices in general equilibrium, these effects are not internalized at the margin by the bank. Since here I are interested in characterizing the bank's marginal decisions in partial equilibrium, I hold prices fixed. Therefore, I assume that, while the bank takes prices (including the rental rates) as given, it internalizes how its decision to borrow affects $\ell_1(s_L)$ and the shadow value of funds at date 1 $\lambda_1(s)$.) Therefore, from the definitions of MC and MB above, I have

$$\frac{\partial MC}{\partial D_0} = \left[\pi(\underline{s})\left(1-\tau_0^D\right)\phi'\left(\ell_1(\underline{s})\right) + \tau_0^R R_0 \phi''\left(\ell_1(\underline{s})\right)\right] v' r_2^k(\underline{s}) \frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

And

$$\frac{\partial MB}{\partial D_0} = \left(1-\tau_0^D\right)\left[r_1^k(s_L) + \ell_1(s_L)\right] v' r_2^k(\underline{s})\phi''\left(\ell_1(\underline{s})\right)\frac{\partial \ell_1(\underline{s})}{\partial D_0} + \left(1-\tau_0^D\right)\lambda_1(s_L)\frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

Imposing the equilibrium result that $\underline{s} = s_L$.

$$= \left(1-\tau_0^D\right)\left[\left(r_1^k(s_L) + \ell_1(s_L)\right)\phi''\left(\ell_1(s_L)\right) + \phi'\left(\ell_1(s_L)\right)\right] v' r_2^k(s_L)\frac{\partial \ell_1(\underline{s})}{\partial D_0}$$

Therefore, $\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$ iff

$$\left[\pi(\underline{s})\left(1-\tau_0^D\right)\phi'\left(\ell_1(\underline{s})\right) + \tau_0^R R_0 \phi''\left(\ell_1(s_L)\right)\right] v' r_2^k(s_L)\frac{\partial \ell_1(s_L)}{\partial D_0} > \left[\left(r_1^k(s_L) + \ell_1(s_L)\right)\phi''\left(\ell_1(s_L)\right) + \phi'\left(\ell_1(s_L)\right)\right]\left(1-\right.$$

$$0 > \left[\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0\right]\phi''\left(\ell_1(s_L)\right) + \left(1-\pi(s_L)\right)\left(1-\tau_0^D\right)\phi'\left(\ell_1(s_L)\right) \quad (26)$$

What is the sign of $\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0$ when evaluated at the equilibrium D_0^*? Recall that the non-negativity constraint on date 1 investment binds in the bad state, so that

$$r_1^k(s_L) + \ell_1(s_L) = \tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^E}{k_1}$$

So I can write $\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0$ as

$$\left(1-\tau_0^D\right)\left(r_1^k(s_L) + \ell_1(s_L)\right) - \tau_0^R R_0 = \left(1-\tau_0^D\right)\left(\tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^E}{k_1}\right) - \tau_0^R R_0$$

As long as T_0^B and T_1^B are not significantly negative (indeed, I will assume they are both weakly positive), I have that $\left(1-\tau_0^D\right)\left(\tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^B}{k_1}\right) - \tau_0^R R_0 < 0$. To see this

$$\left(1-\tau_0^D\right)\left(\tau_0^R R_0 \frac{D_0}{k_1} - \frac{T_1^B}{k_1}\right) <? \tau_0^R R_0$$

Note that $\left(1-\tau_0^D\right) D_0 - k_1$ can be expressed as

$$\left(1-\tau_0^D\right)D_0-k_1=-\left(r_0^k+1\right)k_0-T_0^B$$

Replace this in the above inequality

$$-\left[\left(r_0^k+1\right)k_0+T_0^B\right]\tau_0^R R_0<?\left(1-\tau_0^D\right)T_1^B$$

This holds as long as as T_0^B and T_1^B are not significantly negative. (A sufficient condition is that $T_0^B, T_1^B \geq 0$.) Thus the sign of $\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)-\tau_0^R R_0$ is $\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)-\tau_0^R R_0<0$.

Therefore, I can write condition (26) as

$$\left[\tau_0^R R_0-\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]\phi''\left(\ell_1(s_L)\right)>\left(1-\pi(s_L)\right)\left(1-\tau_0^D\right)\phi'\left(\ell_1(s_L)\right) \tag{27}$$

where both the right-hand and left-hand sides are strictly positive. I can rewrite this condition as

$$\frac{\phi''\left(\ell_1(s_L)\right)}{\phi'\left(\ell_1(s_L)\right)}>\left(1-\pi(s_L)\right)\frac{\left(1-\tau_0^D\right)}{\left[\tau_0^R R_0-\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]} \tag{28}$$

Thus, I have that the banks behaves as if it is risk averse ($\frac{\partial MC}{\partial D_0}>\frac{\partial MB}{\partial D_0}$) if and only if this condition holds. To interpret this condition, there are three terms which affect it. First, if the liquidation cost function is sufficiently convex (so that ϕ'' is sufficiently large relative to ϕ') then this condition is more likely to hold. This is is because then, at the margin, higher leverage will be associated with a higher liquidation cost. Second, if the bank's losses $\tau_0^R R_0-r_1^k(s_L)-\ell_1(s_L)$ (i.e. the difference between its repayment and its revenue plus liquidation value) is sufficiently high, then this condition is more likely to hold. This is again because higher losses in the bad state make the cost of borrowing larger at the margin. And third, if the probability of the bad state $\pi(\underline{s})$ is sufficiently high, then this condition is more likely to hold. This is because the bank incurs losses in the bad state, making borrowing more costly.

Let us break down this condition further. Since I have $\phi\left(\ell_1\right)=\ell_1^\eta$ and $\phi'\left(\ell_1\right)=\eta\ell_1^{\eta-1}$ and $\phi''\left(\ell_1\right)=\eta\left(\eta-1\right)\ell_1^{\eta-2}$, where $\eta>1$, this can condition can be written as

$$\phi''\left(\ell_1(s_L)\right)\left[\tau_0^R R_0-\left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]>\left(1-\pi(s_L)\right)\left(1-\tau_0^D\right)\phi'\left(\ell_1(s_L)\right) \tag{29}$$

$$\ell_1(s_L)<\left[\tau_0^R R_0-\left(1-\tau_0^D\right)r_1^k(s_L)\right]\frac{(\eta-1)}{\left(\eta-\pi(s_L)\right)\left(1-\tau_0^D\right)} \tag{30}$$

And recall that $\ell_1(s_L) = Lev_0 - r_1^k(s)$. So this condition is

$$Lev_0 < \left[\tau_0^R R_0 - \left(1-\tau_0^D\right) r_1^k(s_L)\right] \frac{(\eta-1)}{(\eta - \pi(s_L))\left(1-\tau_0^D\right)} + r_1^k(s) \tag{31}$$

where the right-hand side is strictly positive. Thus, as long as, in equilibrium, Lev_0 is sufficiently small then I have $\frac{\partial MC}{\partial D_0} > \frac{\partial MB}{\partial D_0}$. In that case, a marginally higher D_0 will make MC higher than MB.

Therefore, I henceforth assume the following condition holds in equilibrium.

Condition 1:
A) $\frac{\phi''(\ell_1(s_L))}{\phi'(\ell_1(s_L))} > (1-\pi(s_L)) \frac{\left(1-\tau_0^D\right)}{\left[\tau_0^R R_0 - \left(1-\tau_0^D\right)\left(r_1^k(s_L)+\ell_1(s_L)\right)\right]}$
B) $T_0^B, T_1^B(s) \geq 0$

As I show in Appendix 3, this condition ensures that the bank's expected date 1 losses from borrowing at date 0 are sufficiently high that the bank behaves at date 0 as if it is risk-averse. The condition is benign and amounts to saying that the liquidation cost function $\phi(\cdot)$ is sufficiently convex, the probability of the bad state is sufficiently high, and the bank's losses in bad state are sufficiently high. Moreover, note that $\frac{\phi''(\ell_1(s_L))}{\phi'(\ell_1(s_L))} = \frac{(\eta-1)\eta\ell_1^{\eta-2}}{\eta\ell_1^{\eta-1}} = \frac{(\eta-1)\ell_1}{\ell_1^2} = \frac{(\eta-1)}{\ell_1}$. Therefore, since the only restriction on η is that $\eta > 1$, it is otherwise a free parameter which can always make sufficiently large that this condition holds.

APPENDIX 2: Effects of safe asset creation and crises on precautionary saving

Here, I show that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} > 0$ and $\frac{d\ell_1(s_L)}{dD_0^s} > 0$, under assumptions that lump-sum transfers are sufficiently small, and assumption that $\frac{k_2^G}{\bar{k}_2} < \frac{1}{\bar{n}}$. Start with Channel 2:

Claim: $\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} > 0$

Proof: Note from the Saving Demand curve that

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})}.$$

I proceed by showing first that $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$, and then that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$.

Claim: $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$

Proof: Note that

$$\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} = -\frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})}$$

where
$$c_1(s) = c_2(s) = \frac{1}{2}\left[(w_1 + w_2)\,\bar{n} + R_0\,(D_0 + B_0) - T_1 - T_2\right]$$

So

$$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\bar{n}}{2}\left(\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} + \frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})} - \frac{\partial T_1}{\partial \ell_1(\underline{s})} - \frac{\partial T_2}{\partial \ell_1(\underline{s})}\right)$$

Note that

$$w_1 = (1-\alpha)\,\frac{y_1}{\bar{n}} \tag{32}$$

$$w_2 = (1-\alpha)\,\frac{y_2}{\bar{n}} \tag{33}$$

So

$$\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_1(\underline{s})}{\partial \ell_1(\underline{s})} = 0$$

and
$$\frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

And since

$$y_2 = z_2\left(\tilde{k}_2\right)^{\alpha}\bar{n}^{1-\alpha} \tag{34}$$

where $\tilde{k}_2 = k_2 + k_2^G$, then

$$\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\partial y_2(\underline{s})}{\partial \tilde{k}_2(\underline{s})}\frac{\partial \tilde{k}_2(\underline{s})}{\partial k_2(\underline{s})}\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = \alpha\frac{y_2}{\tilde{k}_2}\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

and $k_2(s) = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right]k_1(D_0) - \tau_0^R D_0 R_0 + T_1^B$ so

$$\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1(\underline{s})\phi' = -\eta k_1(\underline{s})\ell_1^{\eta-1}(\underline{s}) < 0$$

The last inequality follows from the fact that $\ell_1(\underline{s}) > 0$ in the bad state.

Note also that

$$T_1 = R_0^B B_0 - \tau_1 + T_1^B - r_1^k(s)k_1^G \tag{35}$$

$$T_2 = T_2^E - \tau_2 - r_2^k k_2^G \tag{36}$$

So

$$\frac{\partial T_1}{\partial \ell_1(\underline{s})} = -k_1^G \frac{\partial r_1^k(\underline{s})}{\partial \ell_1(\underline{s})} = 0$$

where the last equality holds since $r_1^k = \alpha \frac{y_1}{\tilde{k}_1}$ and $\frac{\partial \tilde{k}_1}{\partial \ell_1(\underline{s})} = 0$ since $\tilde{k}_1 = k_1^G + k_1$ and $k_1 = D_0 + T_0^E + \left(r_0^k + 1\right) k_0$. Moreover

$$\frac{\partial T_1}{\partial \ell_1(\underline{s})} = -k_2^G \frac{\partial r_2^k(\underline{s})}{\partial \ell_1(\underline{s})}$$

Note that, since $r_2^k = \alpha \frac{y_2}{k_2 + k_2^G}$,

$$\frac{\partial r_2^k}{\partial \ell_1(\underline{s})} = \frac{\alpha}{k_2 + k_2^G} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} - \alpha \frac{y_2}{\left(k_2 + k_2^G\right)^2} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

And since $k_2(s) = \left[1 + r_1^k(s) - \phi\left(\ell_1(s)\right)\right] k_1(D_0) - \tau_0^R D_0 R_0 + T_1^B$, I have

$$\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1 \phi'\left(\ell_1(s)\right)$$

So

$$\frac{\partial r_2^k}{\partial \ell_1(\underline{s})} = \frac{\alpha}{k_2 + k_2^G} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} + \alpha \frac{y_2}{\left(k_2 + k_2^G\right)^2} k_1 \phi'\left(\ell_1(s)\right)$$

Plugging these expressions into the equation of $\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})}$.

$$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\bar{n}}{2} \left(\frac{(1-\alpha)}{\bar{n}} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} + k_2^G \frac{\alpha}{k_2 + k_2^G} \frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} + k_2^G \alpha \frac{y_2}{\left(k_2 + k_2^G\right)^2} k_1 \phi'\left(\ell_1(s)\right) \right)$$

and since I showed that $\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} = \alpha \frac{y_2}{\tilde{k}_2} \frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$ and $\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1(\underline{s})\phi' < 0$, this is

$$= \frac{\bar{n}}{2} \left(-\left[\frac{(1-\alpha)}{\bar{n}} + k_2^G \frac{\alpha}{k_2 + k_2^G} \right] \alpha \frac{y_2}{\tilde{k}_2} k_1(\underline{s}) \phi' + k_2^G \alpha \frac{y_2}{\left(\tilde{k}_2\right)^2} k_1(\underline{s}) \phi' \right)$$

Thus, a rise in liquidation in the bad state causes future consumption to fall through a decrease in the wage, but also pushes up consumption through a possible rise in the rental rate of capital. In what follows, I find a condition which ensures that this latter effect does not dominate. I have

$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} < 0$ if and only if

$$-\left[\frac{(1-\alpha)}{\bar{n}} + k_2^G \frac{\alpha}{k_2 + k_2^G}\right] \alpha \frac{y_2}{\tilde{k}_2} k_1(\underline{s}) \phi' + k_2^G \alpha \frac{y_2}{(\tilde{k}_2)^2} k_1(\underline{s}) \phi' < 0$$

which reduces to

$$\frac{k_2^G}{\tilde{k}_2} < \frac{1}{\bar{n}}.$$

This holds if labor supply is sufficiently small or government's share of date 2 capital stock is sufficiently small.

Suppose then that $\frac{k_2^G}{\tilde{k}_2} < \frac{1}{\bar{n}}$. Then it follows that $\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} < 0$. Therefore,

$$\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} = -\frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} > 0$$

Q.E.D.

Claim: $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$.

Proof: Recall the household's demand function $D_0^d(R_0, B_0) = e_0 - T_0 + d_0^F + w_0 \bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1}$. This implies that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} = \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2} > 0$

Thus, since both $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} > 0$ and $\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$, I have

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$$

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{1}{R_0\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^2} \frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\bar{n}}{2} \frac{(1-\alpha)}{\bar{n}} \alpha \frac{y_2}{\tilde{k}_2} \eta k_1(\underline{s}) \ell_1^{\eta-1}(\underline{s}) > 0$$

Thus Channel 2: $\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]} \frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} > 0$. Q.E.D.

To summarize, I have

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = -\frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2} \frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\bar{n}}{2}\left(-\left[\frac{(1-\alpha)}{\bar{n}} + k_2^G \frac{\alpha}{k_2 + k_2^G}\right] \alpha \frac{y_2}{\tilde{k}_2} k_1(\underline{s}) \phi' + k_2^G \alpha \frac{y_2}{(\tilde{k}_2)^2} k_1(\underline{s}) \phi'\right) > 0$$

APPENDIX 3: Proofs regarding Assumption 1

In this appendix, I show that Assumption 1 ensures that the bank's date 0 natural borrowing limit on borrowing D_0 is never binding in equilibrium, and that liquidation occurs only in the bad state, $\ell_1(s_L) > 0$, $\ell_1(s_H) = 0$. I also show that these restrictions are satisfied by a non-empty set of parameters satisfies these restrictions.

Claim 1: $z_1(s_L) < \frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}$ implies that $\ell_1(s_L) > 0$.

Proof: I have $\ell_1(s_L) > 0$ if and only if

$$\ell_1(s_L) = \frac{\tau_0^R D_0 R_0}{k_1} - r_1^k(s_L) - \frac{T_1^E}{k_1} > 0 \tag{37}$$

i.e.

$$\frac{\tau_0^R D_0 R_0 - T_1^E}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} > z_1(s_L) \tag{38}$$

Q.E.D.

Claim 2: $z_1(s_H) \geq \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}$ implies that $\ell_1(s_H) = 0$.

Proof: From the expression for ℓ_1, it follows that $\ell_1(s_H) = 0$ if and only if

$$\frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \leq z_1(s_H) \tag{39}$$

I can set $z_1(s_H)$ arbitrarily high to ensure this is satisfied. Q.E.D.

Claim 3: The natural borrowing limit is non-binding.

Proof: Recall that natural borrowing limit is $\tau_0^D R_0 D_0 \leq P_1(\underline{s}) \left(r_1^k(\underline{s}) k_1 + \bar{\ell}_1 k_1\right)$ where $\bar{\ell}_1$, the maximum fraction of its capital that the bank can liquidate without violating the non-negativity constraint on k_2, solves $k_2(s) = 0$ when $i_1 = 0$:

$$k_2(s) = i_1 + (1 - \ell_1 - \phi(\ell_1(s))) k_1(s) \tag{40}$$

$$1 = \ell_1 + \ell_1^\eta \tag{41}$$

So the natural borrowing limit is non-binding if and only if

$$\tau_0^D R_0 D_0 < P_1(\underline{s}) \left(r_1^k(\underline{s}) k_1 + \bar{\ell}_1 k_1\right)$$

$$\frac{\tau_0^D R_0 D_0}{k_1} - r_1^k(s_L) < \bar{\ell}_1$$

Note that since $\bar{\ell}_1$ solves $1 = \ell_1 + \ell_1^\eta$, as $\eta > 1$ approaches infinity, $\bar{\ell}_1 > 0$ approaches 1. Therefore, making η arbitrarily large would by itself not suffice to ensure the natural borrowing limit is always non-binding. But it would suffice if it is also the case that

$$\frac{\tau_0^D R_0 D_0}{k_1} - r_1^k(s_L) < 1$$

i.e.

$$\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} < z_1(s_L)$$

Thus, I can ensure that the natural borrowing limit is non-binding by simultaneously making η arbitrarily large and $z_1(s_L)$ is not too small, so that it satisfies the above inequality. Q.E.D.

Claim 4: There is a non-empty set of parameters which satisfy these assumptions.

Proof: First recall that I can make $z_1(s_H)$ arbitrarily large to ensure that $\frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \leq z_1(s_H)$.

To simultaneously ensure that both the natural borrowing limit is non-binding and that $\ell_1(s_L) > 0$, I must jointly assume that $z_1(s_L)$ satisfies

$$z_1(s_L) \in \left(\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}, \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} \right)$$

Note that such a $z_1(s_L)$ exists as long as

$$\frac{\tau_0^D R_0 D_0 - k_1}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}} < \frac{\tau_0^R D_0 R_0 - T_1^B}{k_1 \alpha \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha}}$$

i.e.

$$T_1^B < k_1$$

Thus, part (B) of Assumption (1) ensures such a $z_1(s_L)$ exists. Q.E.D.

APPENDIX 4: Effect of safe asset creation on severity of crises

Here, I show that higher date 0 borrowing increases the severity of a crises, conditional on a crisis occurring. Formally, in equilibrium I have $\frac{d\ell_1(s)}{dD_0} > 0$ if and only if $\ell_1(s) > 0$, and $\frac{d\ell_1(s)}{dD_0} = 0$ otherwise.

Claim: As long as lump-sum transfers are sufficiently small, I have $\frac{d\ell_1(s_L)}{dD_0} > 0$ and $\frac{d\ell_1(s_H)}{dD_0} = 0$.

Proof: I already showed that, in the good state $s = s_H$, I are in the normal regime so that $\frac{d\ell_1(s_H)}{dD_0} = 0$. In the bad state $s = s_L$, the variable $\ell_1(s)$ is pinned down by the binding non-negativity constraint on date 1 investment.

$$\ell_1(s) = \frac{\tau_0^R D_0 R_0}{k_1} - r_1^k(s) - \frac{T_1^B}{k_1}$$

Recall the law of motion for k_1.

$$k_1 = i_0 + k_0 - k_1^{QE} \tag{42}$$

Replacing i_0 with the bank's binding date 0 budget constraint yields

$$k_1 = \frac{D_0}{P_0} + \left(r_0^k + 1\right) k_0 + T_0^B \tag{43}$$

So

$$\frac{dk_1}{dD_0} = \frac{1}{P_0} = 1$$

Therefore, I can express the derivative $\frac{d\ell_1(s)}{dD_0}$ as

$$\frac{d\ell_1(s)}{dD_0} = \frac{\tau_0^R R_0}{k_1} - \frac{dr_1^k(s)}{dD_0} + \frac{T_1^E}{(k_1)^2} - \frac{\tau_0^R D_0 R_0}{(k_1)^2}$$

Recall

$$r_1^k = \alpha \frac{y_1}{\tilde{k}_1} = \alpha z_1 \left(\tilde{k}_1\right)^{\alpha-1} \bar{n}^{1-\alpha} \tag{44}$$

So

$$\frac{dr_1^k(s)}{dD_0} = \alpha(\alpha-1) z_1 \left(\tilde{k}_1\right)^{\alpha-2} \bar{n}^{1-\alpha} < 0$$

Since $\frac{dr_1^k(s)}{dD_0} < 0$, a sufficient condition for $\frac{d\ell_1(s)}{dD_0} > 0$ is

$$\frac{\tau_0^R R_0}{k_1} + \frac{T_1^B}{(k_1)^2} - \frac{\tau_0^R D_0 R_0}{(k_1)^2} > 0$$

i.e.

$$\tau_0^R R_0 (k_1 - D_0) > T_1^B$$

Recall.

$$k_1 = \frac{D_0}{P_0} + \left(r_0^k + 1\right) k_0 + T_0^E > D_0 \tag{45}$$

Thus, as long as T_1^B is sufficiently small, I have $\frac{d\ell_1(s)}{dD_0} > 0$ in the bad state. Henceforth, I assume that T_1^B, which for now I take as exogenous, satisfies this condition. Note that, for the baseline model, I set transfers to $T_1^B = 0$. Then this condition would reduce to

$$k_1 > D_0.$$

This condition holds because the bank's date 0 net worth is strictly positive.

APPENDIX 5: Effect of crisis risk on precautionary saving

Here, I show that $\frac{\partial D_0^d}{\partial \ell_1(\text{s})} > 0$. First, let's reduce the dynamic system of equations . Note that I can get rid of both c_1 and B_1. The first order condition for B_1 implies $c_1 = c_2$. And from the expressions for c_1 and c_2, I can compute the sum of c_1 and c_2 as

$$c_1(s) + c_2(s) = B_1(s) - T_2 + w_2\bar{n} + R_0 D_0 + R_0^B B_0 - T_1 + w_1\bar{n} - B_1(s)$$

$$= (w_1 + w_2)\,\bar{n} + R_0\,(D_0 + B_0) - T_1 - T_2$$

And since $c_1 = c_2$, this means I can ignore both c_1 and B_1, and capture how shocks affect total consumption after date 0:

$$c_2(s) = \frac{1}{2}\left[(w_1 + w_2)\,\bar{n} + R_0\,(D_0 + B_0) - T_1 - T_2\right]$$

Intuitively, the household uses the storage technology $B_1(s)$ (with $R_1^B = 1$) to fully smooth consumption between dates 1 and 2. So I only need to keep track of the household's total income at dates 1 and 2 - its divided optimally via $B_1(s)$.

Recall the household's FOC for D_0^d:

$$\frac{1}{c_0} = R_0 E_0\left[\frac{1}{c_1(s)}\right] \tag{46}$$

Plug this into the household's date 0 budget constraint:

$$c_0 + D_0^d + B_0 = e_0 - T_0 + d_0^F + w_0\bar{n} \tag{47}$$

$$D_0^d = e_0 - T_0 + w_0\bar{n} - B_0 - \frac{1}{R_0 E_0\left[\frac{1}{c_1(s)}\right]} \tag{48}$$

So

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{1}{R_0\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^2}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})}$$

where

$$\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(\underline{s})} = -\frac{\pi(\underline{s})}{(c_1(\underline{s}))^2}\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})}$$

where

$$c_1(s) = c_2(s) = \frac{1}{2}\left[(w_1+w_2)\bar{n} + R_0(D_0+B_0) - T_1 - T_2\right]$$

So

$$\frac{\partial c_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\bar{n}}{2}\left(\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} + \frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})}\right)$$

Note that

$$w_1 = (1-\alpha)\frac{y_1}{\bar{n}} \tag{49}$$

$$w_2 = (1-\alpha)\frac{y_2}{\bar{n}} \tag{50}$$

So

$$\frac{\partial w_1(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_1(\underline{s})}{\partial \ell_1(\underline{s})} = 0$$

and

$$\frac{\partial w_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{(1-\alpha)}{\bar{n}}\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

And since

$$y_2 = z_2\left(\tilde{k}_2\right)^{\alpha}\bar{n}^{1-\alpha} \tag{51}$$

where $\tilde{k}_2 = k_2 + k_2^G$, then

$$\frac{\partial y_2(\underline{s})}{\partial \ell_1(\underline{s})} = \frac{\partial y_2(\underline{s})}{\partial \tilde{k}_2(\underline{s})}\frac{\partial \tilde{k}_2(\underline{s})}{\partial k_2(\underline{s})}\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = \alpha\frac{y_2}{\tilde{k}_2}\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})}$$

and $k_2(s) = i_1 + (1-\phi(\ell_1(s)))\, k_1(s)$, where $i_1 = 0$ in the bad state, so

$$\frac{\partial k_2(\underline{s})}{\partial \ell_1(\underline{s})} = -k_1(\underline{s})\phi' = -\eta k_1(\underline{s})\ell_1^{\eta-1}(\underline{s}) < 0$$

The last inequality follows from the fact that $\ell_1(\underline{s}) > 0$ in the bad state. Plugging this into our expression for $\frac{\partial D_0^d}{\partial \ell_1(\underline{s})}$ yields

$$\frac{\partial D_0^d}{\partial \ell_1(\underline{s})} = \frac{1}{R_0\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^2} \frac{\pi(\underline{s})}{(c_1(\underline{s}))^2} \frac{\bar{n}}{2} \frac{(1-\alpha)}{\bar{n}} \alpha \frac{y_2}{\tilde{k}_2} \eta k_1(\underline{s})\ell_1^{\eta-1}(\underline{s}) > 0$$

APPENDIX 6: Risk-driven stagnation trap

Effect of shock on severity of recession How does the MIT shock to $z_1(s_L)$ affect he recession in the demand-determined regime at date 0? This is given by the response of u_0, which is determined to clear the market for safe assets.

$$D_0^s(R_0 = 1; u_0^*, z_1(s_L)) = D_0^d(R_0 = 1; u_0^*, z_1(s_L))$$

First begin with the effect of the shock on the demand curve m $\frac{dD_0^d}{dz_1(s_L)}$. The demand for safe assets D_0^d is expressed explicitly via the Saving Demand curve, $D_0^d(R_0, u_0, B_0) = e_0 - T_0 + d_0^F + w_0\bar{n} - B_0 - \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-1}$. Therefore, we have

$$\frac{dD_0^d|_{R_0=1,u_0^*}}{dz_1(s_L)} = \underbrace{\frac{\partial D_0^d|_{R_0=1,u_0^*}}{\partial u_0}\frac{du_0}{dz_1(s_L)}}_{GE\,feedback} + \underbrace{\frac{\partial D_0^d|_{R_0=1,u_0^*}}{\partial z_1(s_L)}}_{direct\,effect}$$

where $D_0^d|_{R_0=1,u_0^*}$ is the level of demand for safe assets at $R_0 = 1$ and conditional on the initial (pre-shock) level of utilization u_0^*, and the initial level of date 1 productivity (before the shock). (I will show below that the general equilibrium feedback effect $\frac{\partial D_0^d|_{R_0=1,u_0^*}}{\partial u_0}\frac{du_0}{dz_1(s_L)}$, which captures the marginal effect of the recession on the demand for saving, is 0 in the demand-determined regime.)

Note that the general equilibrium feedback effect $\frac{\partial D_0^d|_{R_0=1,u_0^*}}{\partial u_0}\frac{du_0}{dz_1(s_L)}$, which captures the marginal effect of the recession on the demand for saving, is 0 in the demand-determined regime. This is because $w_0 = 0$ in the demand-determined regime, and so we have $\frac{\partial D_0^d|_{R_0=1,u_0^*}}{\partial u_0} = 0$.

$$\frac{dD_0^d|_{R_0=1,u_0^*}}{dz_1(s_L)} = \underbrace{\frac{\partial D_0^d|_{R_0=1,u_0^*}}{\partial z_1(s_L)}}_{direct\,effect} < 0$$

The sign of this term is negative as the fall in $z_1(s_L)$ lowers the household's expected future consumption directly and through a higher value of $\ell_1(s_L)$, both of which lead to greater demand for saving at a given R_0 and u_0.

Consider the effect on D_0^s. The bank's supply of safe assets D_0^s is determined implicitly to equate the bank's date 0 marginal cost and marginal benefit of debt, as discussed in the optimality conditions $MB\left(D_0^s, z_1(s_L)\right) = MC\left(D_0^s, z_1(s_L)\right)$, i.e.

$$\underbrace{E\left[r_2^k\left(r_1^k(s)+1\right)\right] + \lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]}_{marginal\ benefit} = \underbrace{E\left[r_2^k\left(\phi\left(\ell_1(s)\right)+\tau_0^R R_0^D\right)\right] + \lambda_1(s_L)\tau_0^R R_0^D}_{marginal\ cost} \quad (52)$$

We can implicitly derive this equation with respect to $z_1(s_L)$

$$\frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial z_1(s_L)} + \frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial D_0^s}\frac{dD_0^s}{dz_1(s_L)} = \frac{\partial MC\left(D_0^s, z_1(s_L)\right)}{\partial z_1(s_L)} + \frac{\partial MC\left(D_0^s, z_1(s_L)\right)}{\partial D_0^s}\frac{dD_0^s}{\partial z_1(s_L)}$$

Solving for $\frac{dD_0^s}{\partial z_1(s_L)}$ yields the first-order effect of the shock on the supply of bank debt.

$$\frac{dD_0^s}{dz_1(s_L)} = \frac{\frac{\partial MC}{\partial z_1(s_L)} - \frac{\partial MB}{\partial z_1(s_L)}}{\frac{\partial MB}{\partial D_0^s} - \frac{\partial MC}{\partial D_0^s}}$$

We can express

$$MC \equiv E\left[r_2^k\left[\left(1-\tau_0^D\right)\phi\left(\ell_1\right)+\tau_0^R R_0\right]\right] + \lambda_1(s_L)\tau_0^R R_0$$

$$MB \equiv \left(1-\tau_0^D\right)E\left[r_2^k\left(r_1^k(s)+1\right)\right] + \left(1-\tau_0^D\right)\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]$$

where $\ell_1(s) = \frac{\tau_0^R D_0 R_0 - T_1^E}{k_1} - r_1^k(s)$. So we can rewrite

$$MB = \left(1-\tau_0^D\right)E\left[r_2^k\left(r_1^k(s)+1\right)\right] + \left(1-\tau_0^D\right)\lambda_1(s_L)\frac{\tau_0^R D_0 R_0 - T_1^E}{k_1}$$

Then the partials of MC and MB with respect to $z_1(s_L)$ (via $r_1^k(s) = \alpha \frac{y_1(s)}{k_1}$) are

$$\frac{\partial MC\left(D_0^s, z_1(s_L)\right)}{\partial z_1(s_L)} = \frac{\partial MC\left(D_0^s, z_1(s_L)\right)}{\partial \ell_1(s_L)} \frac{\partial \ell_1(s_L)}{\partial r_1^k(s_L)} \frac{\partial r_1^k(s_L)}{\partial z_1(s_L)} = -\pi^L r_2^k(s_L)\left(1-\tau_0^D\right)\phi' \alpha \frac{y_1(s_L)}{z_1(s_L)k_1} < 0$$

and

$$\frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial z_1(s_L)} = \frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial r_1^k(s_L)} \frac{\partial r_1^k(s_L)}{\partial z_1(s_L)} + \frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial \ell_1(s_L)} \frac{\partial \ell_1(s_L)}{\partial r_1^k(s_L)} \frac{\partial r_1^k(s_L)}{\partial z_1(s_L)} = \pi^L\left(1-\tau_0^D\right) r_2^k(s_L) \alpha \frac{y_1(s_L)}{z_1(s_L)k_1}$$

since $\frac{\partial r_1^k(s_L)}{\partial z_1(s_L)} = \alpha \frac{y_1(s_L)}{z_1(s_L)k_1}$. Note that, this implies $\frac{\partial MC}{\partial z_1(s_L)} - \frac{\partial MB}{\partial z_1(s_L)} = -\pi^L r_2^k(s_L)\left(1-\tau_0^D\right)\alpha \frac{y_1(s_L)}{z_1(s_L)k_1}[\phi' - 1]$.

$$\frac{\partial MC}{\partial z_1(s_L)} - \frac{\partial MB}{\partial z_1(s_L)} = -\pi^L r_2^k(s_L)\left(1-\tau_0^D\right)\phi' \alpha \frac{y_1(s_L)}{z_1(s_L)k_1} - \pi^L\left(1-\tau_0^D\right) r_2^k(s_L)\alpha \frac{y_1(s_L)}{z_1(s_L)k_1}$$

$$= -\pi^L r_2^k(s_L)\left(1-\tau_0^D\right)\alpha \frac{y_1(s_L)}{z_1(s_L)k_1}\left(\phi' - 1\right)$$

Note that $\phi'(\ell_1(s_L)) = \eta \ell_1^{\eta-1}(s_L)$. Hence, we have $\frac{\partial MC}{\partial z_1(s_L)} - \frac{\partial MB}{\partial z_1(s_L)} > 0$ if and only if $\ell_1(s_L) < \left(\frac{1}{\eta}\right)^{\frac{1}{\eta-1}}$. I restrict analysis to the case in which this holds in equilibrium. (Note that, since $\eta > 1$, this is satisfied as long as banks liquidate less than $\ell_1(s_L) < 0.38$ (that is 38%) of their capital holdings.)

Taking stock, we can combine the derivatives (and recall from above that we restricted analysis to the case in which $\ell_1(s_L) < \left(\frac{1}{\eta}\right)^{\frac{1}{\eta-1}}$) to obtain

$$\frac{dD_0^s}{dz_1(s_L)} = \frac{\frac{\partial MC}{\partial z_1(s_L)} - \frac{\partial MB}{\partial z_1(s_L)}}{\frac{\partial MB}{\partial D_0^s} - \frac{\partial MC}{\partial D_0^s}} = \frac{-\pi^L r_2^k(s_L)\left(1-\tau_0^D\right)\alpha \frac{y_1(s_L)}{z_1(s_L)k_1}\left(\phi'(\ell_1(s_L)) - 1\right)}{\left(1-\tau_0^D\right)\frac{\tau_0^R R_0}{k_1}\lambda_1(s_L)\left[1-\pi^L\right]} > 0.$$

Thus, we have $\frac{dD_0^d(u_0^*)}{dz_1(s_L)} - \frac{dD_0^s(u_0^*)}{dz_1(s_L)} < 0$, so the fall in $z_1(s_L)$ creates a higher excess demand for safe assets. Hence, the shock increases the excess demand for safe assets, requiring a deeper recession, that is a lower u_0 and y_0 to reach equilibrium.

Effect of shock on saving What is the effect of the shock on the equilibrium value of saving and investment, $\frac{dD_0^*}{dz_1(s_L)}$? Given the equilibrium value of u_0^*, the equilibrium value of debt, based on the market clearing condition for safe assets, is $D_0^* = D_0^d(u_0^*) = D_0^s(u_0^*)$. The effect of the shock on the equilibrium level of debt depends on the direct effect of the shock, and the effect of the adjustment in u_0^*, where we showed above that $\frac{du_0^*}{dz_1(s_L)} > 0$.

$$\frac{dD_0^*}{dz_1(s_L)} = \frac{\partial D_0^*}{\partial z_1(s_L)} + \frac{\partial D_0^*}{\partial u_0^*} \underbrace{\frac{du_0^*}{dz_1(s_L)}}_{>0} \tag{53}$$

Effect of shock on date 0 investment How does this affect i_0? First note from above that

$$i_0 = D_0 + B_0 + r_0^k k_0$$

So

$$\frac{di_0}{dz_1(s_L)} = \underbrace{\frac{\partial i_0}{\partial D_0} \frac{dD_0}{dz_1(s_L)}}_{effect\,on\,inv\,via\,saving} + \underbrace{\frac{\partial i_0}{\partial r_0^k k_0} \frac{dr_0^k k_0}{dz_1(s_L)}}_{effect\,on\,inv\,via\,recession\,and\,income}$$

Note also that

$$\frac{dr_0^k k_0}{dz_1(s_L)} = \frac{dr_0^k k_0}{dy_0} \frac{\partial y_0}{\partial u_0} \frac{du_0}{dz_1(s_L)}$$

where $\frac{du_0}{dz_1(s_L)} > 0$ both through ell1 ($\frac{\partial u_0}{\partial \ell_1(s_L)} \frac{\partial \ell_1(s_L)}{\partial z_1(s_L)} > 0$) and because the household increases its saving because future output is lower directly due to the shock.

Combining these we have

$$\frac{di_0}{dz_1(s_L)} = \frac{\partial i_0}{\partial D_0} \left[\frac{\partial D_0^*}{\partial z_1(s_L)} + \frac{\partial D_0^*}{\partial u_0^*} \frac{du_0^*}{dz_1(s_L)} \right] + \frac{\partial i_0}{\partial r_0^k k_0} \frac{dr_0^k k_0}{dy_0} \frac{\partial y_0}{\partial u_0} \frac{du_0}{dz_1(s_L)} \tag{54}$$

Note that since $\ell_1(s_L) = \frac{D_0 R_0 - T_1^E}{k_1} - r_1^k(s_L)$, we have

$$\frac{\partial \ell_1(s_L)}{\partial z_1(s_L)} = -\frac{\partial r_1^k(s_L)}{\partial z_1(s_L)} = -\frac{\partial}{\partial z_1(s_L)} \left(\alpha \frac{y_1(s_L)}{k_1} \right) = -\frac{\partial}{\partial z_1(s_L)} \left(\alpha \frac{z_1(s_L) k_1^\alpha \overline{n}^{1-\alpha}}{k_1} \right) = -\alpha \frac{k_1^\alpha \overline{n}^{1-\alpha}}{k_1} = -\alpha k_1^{\alpha-1} \overline{n}^{1-\alpha} =$$

Also we have

$$\frac{\partial y_0}{\partial u_0} = \alpha \frac{y_0}{u_0} > 0$$

Finally, note that Appendix 5 already showed that $\frac{\partial D_0^d}{\partial \ell_1(s_L)} > 0$ while Online Appendix 7 showed that $\frac{\partial u_0}{\partial D_0^d} < 0$. Putting these together implies that $\frac{\partial u_0}{\partial \ell_1(s_L)} < 0$. Moreover, we have $\frac{\partial i_0}{\partial D_0}, \frac{\partial i_0}{\partial r_0^k k_0} = 1$, and in the demand-determined regime at date 0, we have $r_0^k k_0 = y_0$, so $\frac{dr_0^k k_0}{dy_0} = 1$.

Thus we have

$$\frac{di_0}{dz_1(s_L)}=\underbrace{\frac{\partial i_0}{\partial D_0}}_{=1}\left[\frac{\partial D_0^*}{\partial z_1(s_L)}+\frac{\partial D_0^*}{\partial u_0^*}\underbrace{\frac{du_0^*}{dz_1(s_L)}}_{>0}\right]+\underbrace{\frac{\partial i_0}{\partial r_0^k k_0}}_{=1}\underbrace{\frac{dr_0^k k_0}{dy_0}}_{=1}\underbrace{\frac{\partial y_0}{\partial u_0}}_{>0}\underbrace{\frac{du_0}{dz_1(s_L)}}_{<0}$$

i.e.

$$\frac{di_0}{dz_1(s_L)}=\left[\underbrace{\frac{\partial i_0}{\partial D_0}\frac{\partial D_0^*}{\partial u_0^*}}_{recession\ on\ inv\ via\ saving}+\underbrace{\frac{\partial i_0}{\partial r_0^k k_0}\frac{dr_0^k k_0}{dy_0}\frac{\partial y_0}{\partial u_0}}_{recession\ on\ inv\ via\ rental\ income}\right]\underbrace{\frac{du_0}{dz_1(s_L)}}_{shock\ on\ recession}+\underbrace{\frac{\partial i_0}{\partial D_0}\frac{\partial D_0^*}{\partial z_1(s_L)}}_{shock\ on\ inv\ via\ saving\ directly}$$

$$\frac{di_0}{dz_1(s_L)}=\left[\underbrace{\frac{\partial i_0}{\partial D_0}}_{=1}\frac{\partial D_0^*}{\partial u_0^*}+\underbrace{\frac{\partial i_0}{\partial r_0^k k_0}}_{=1}\underbrace{\frac{dr_0^k k_0}{dy_0}}_{=1}\underbrace{\frac{\partial y_0}{\partial u_0}}_{>0}\right]\underbrace{\frac{du_0}{dz_1(s_L)}}_{>0}+\underbrace{\frac{\partial i_0}{\partial D_0}}_{=1}\frac{\partial D_0^*}{\partial z_1(s_L)}$$

Simplifying

$$\frac{di_0}{dz_1(s_L)}=\left[\frac{\partial D_0^*}{\partial u_0^*}+\underbrace{\frac{\partial y_0}{\partial u_0}}_{>0}\right]\underbrace{\frac{du_0}{dz_1(s_L)}}_{>0}+\frac{\partial D_0^*}{\partial z_1(s_L)}$$

where $\frac{\partial y_0}{\partial u_0}=\alpha\frac{y_0}{u_0}$.

$$\frac{di_0}{dz_1(s_L)}=\left[\frac{\partial D_0^*}{\partial u_0^*}+\alpha\frac{y_0}{u_0}\right]\underbrace{\frac{du_0}{dz_1(s_L)}}_{>0}+\frac{\partial D_0^*}{\partial z_1(s_L)}$$

$$\frac{di_0}{dz_1(s_L)}=\underbrace{\overbrace{\frac{\partial D_0^*}{\partial u_0^*}\underbrace{\frac{du_0}{dz_1(s_L)}}_{>0}}^{GE\ effect\ on\ saving}+\overbrace{\frac{\partial D_0^*}{\partial z_1(s_L)}}^{direct\ effect\ on\ saving}}_{effect\ via\ equilibriu\ saving}+\underbrace{\overbrace{\alpha\frac{y_0}{u_0}\underbrace{\frac{du_0}{dz_1(s_L)}}_{>0}}^{GE\ effect\ on\ income}}_{effect\ via\ bank\ income}$$

Thus, the response of date 0 investment to the shock consists broadly of two effects. The first is the effect on equilibrium saving D_0, which comprises a direct effect of the shock on equilibrium saving, and a general equilibrium effect via the response in utilization. The net effect on the equilibrium quantity saved is ambiguous. The second effect is the effect of the shock on the bank's rental income via the recession at date 0. The sign of this term is unambiguously positive: The recession erodes the bank's date 0 income which reduces investment at date 0. Thus, effect of the shock on investment at date 0 depends on the net of these two effects. If the effect of the date 0 recession on date 0 investment i_0 via the bank's income is sufficiently severe (and the recession erodes the bank's income sufficiently), then date 0 investment will decline in response tot he shock. Thus, date 0 investment falls in response to the MIT shock if and only if the effect of the recession on investment (via rental income and the resources available for saving and investment) dominates the consumption smoothing motive whereby the lower future productivity and the more severe future crises lead to more saving and investment.

Effect of shock on date 1 output The effects of the shock on future variables (output and capital accumulation) depend on the response pf date 0 investment, described above. First begin with date 1 output.

$$y_1(s) = z_1(s)\,(k_1)^{\alpha}\,\bar{n}^{1-\alpha} \tag{55}$$

$$k_1 = i_0 + k_0 \tag{56}$$

The effect of the shock on date 1 output is comprised of the direct effect of the fall in productivity on date 1 output, and an indirect effect through capital accumulation.

$$\frac{dy_1(s)}{dz_1(s_L)} = \frac{\partial y_1(s)}{\partial z_1(s_L)} + \frac{\partial y_1(s)}{\partial k_1}\frac{\partial k_1}{\partial i_0}\frac{di_0}{dz_1(s_L)}$$

First consider the good state. We have $\frac{\partial y_1(s_H)}{\partial z_1(s_L)} = 0$, so

$$\frac{dy_1(s_H)}{dz_1(s_L)} = \frac{\partial y_1(s_H)}{\partial k_1}\frac{\partial k_1}{\partial i_0}\frac{di_0}{dz_1(s_L)}$$

$$= \alpha\frac{y_1(s_H)}{k_1}\frac{di_0}{dz_1(s_L)}$$

where $\frac{di_0}{dz_1(s_L)}$, the effect of the shock on date 0 investment, is given above. Thus, $\frac{dy_1(s_H)}{dz_1(s_L)} > 0$ as long as $\frac{di_0}{dz_1(s_L)} > 0$ – that is, date 1 output in the bad state falls if and only if date 0 investment falls (i.e. if the effect of the recession on investment at date 0 is sufficiently severe).

In the bad state, we have $\frac{\partial y_1(s_L)}{\partial z_1(s_L)} = \frac{y_1(s_L)}{z_1(s_L)}$, so

$$\frac{dy_1(s)}{dz_1(s_L)} = \frac{\partial y_1(s_L)}{\partial z_1(s_L)} + \frac{\partial y_1(s_L)}{\partial k_1}\frac{\partial k_1}{\partial i_0}\frac{di_0}{dz_1(s_L)}$$

$$= \frac{y_1(s_L)}{z_1(s_L)} + \alpha \frac{y_1(s_L)}{k_1} \frac{di_0}{dz_1(s_L)}$$

Thus, if $\frac{di_0}{dz_1(s_L)} > 0$, then $\frac{dy_1(s_L)}{dz_1(s_L)} > 0$. That is, date 1 output in the bad state falls if and only if date 0 investment falls (i.e. if the effect of the recession at date 0 on investment is sufficiently severe). However, output in the bad state may fall $\frac{dy_1(s)}{dz_1(s_L)} > 0$ even if investment rises $\frac{di_0}{dz_1(s_L)}$, due to the direct effect $\frac{\partial y_1(s_L)}{\partial z_1(s_L)}$. (Note also the effect on expected date 1 output: If $\frac{di_0}{dz_1(s_L)} > 0$ (i.e. if the recession's effect on investment is sufficiently severe), then $\frac{dE[y_1(s)]}{dz_1(s_L)} = \pi^H \frac{dy_1(s_H)}{dz_1(s_L)} + \pi^L \frac{dy_1(s_L)}{dz_1(s_L)} > 0$.)

Effect of shock on date 1 investment Recall that date 1 investment is given by

$$i_1(s) = r_1^k(s)k_1 + \ell_1(s)k_1 + T_1^E - D_0R_0 \tag{57}$$

Note that $r_1^k(s)k_1 = \alpha y_1(s)$. In the good state, this implies (and imposing $R_0 = 1$)

$$i_1(s_H) = \alpha y_1(s_H) + T_1^E - D_0 \tag{58}$$

and so

$$\frac{di_1(s_H)}{dz_1(s_L)} = \alpha \frac{dy_1(s_H)}{dz_1(s_L)} - \frac{\partial i_1(s_H)}{\partial D_0} \frac{dD_0}{dz_1(s_L)}$$
$$= \alpha \frac{dy_1(s_H)}{dz_1(s_L)} - \frac{dD_0}{dz_1(s_L)}$$

where $\frac{dy_1(s_H)}{dz_1(s_L)}$ and $\frac{dD_0}{dz_1(s_L)}$ are given above. Recall that $\frac{dy_1(s_H)}{dz_1(s_L)}$ depends on $\frac{di_0}{dz_1(s_L)}$, which itself depends on $\frac{dD_0}{dz_1(s_L)}$. Therefore, if the overall effect of the shock and recession on date 0 investment is sufficiently large, then date 1 investment falls in the good state. Intuitively, this can occur because the fall in date 0 investment reduces capital and output at date 1, which lowers the resources available at date 1 for further investment.

In the bad state,

$$i_1(s) = \alpha y_1(s_L) + \ell_1(s_L)k_1 + T_1^E - D_0R_0 \tag{59}$$

$$k_1 = i_0 + k_0 \tag{60}$$

so

$$\frac{di_1(s_L)}{dz_1(s_L)} = \alpha \frac{dy_1(s_L)}{dz_1(s_L)} - \frac{\partial i_1(s_L)}{\partial D_0} \frac{dD_0}{dz_1(s_L)} + k_1 \frac{d\ell_1(s_L)}{dz_1(s_L)} + \ell_1(s_L) \frac{dk_1}{dz_1(s_L)}$$

$$= \alpha \frac{dy_1(s_L)}{dz_1(s_L)} - \frac{dD_0}{dz_1(s_L)} + k_1 \underbrace{\frac{d\ell_1(s_L)}{dz_1(s_L)}}_{<0} + \ell_1(s_L) \frac{di_0}{dz_1(s_L)}$$

where $\frac{dy_1(s_L)}{dz_1(s_L)}$, $\frac{di_0}{dz_1(s_L)}$, and $\frac{dD_0}{dz_1(s_L)}$ are given above, and I showed above that $\frac{d\ell_1(s_L)}{dz_1(s_L)} = -\alpha\frac{y_1(s_L)}{z_1(s_L)k_1} < 0$. Therefore, if the overall effect of the shock and recession on date 0 investment is sufficiently large, then date 1 investment falls in both states. Intuitively, this can occur because the fall in date 0 investment reduces capital and output at date 1, which lowers the resources available at date 1 for further investment.

Effect of shock on date 2 capital and output The effect of the shock on date 2 capital and output reflects the effect of the shock on date 1 investment and liquidation.

$$k_2(s) = i_1(s) + (1 - \ell_1(s) - \phi(\ell_1(s)))\, k_1 \tag{61}$$

Since $\frac{dk_1}{dz_1(s_L)} = \frac{di_0}{dz_1(s_L)}$

$$\frac{dk_2(s)}{dz_1(s_L)} = \underbrace{\frac{\partial k_2(s)}{\partial i_1(s)}}_{=1}\frac{di_1(s)}{dz_1(s_L)} + (1 - \ell_1(s) - \phi(\ell_1(s)))\frac{di_0}{dz_1(s_L)} - \left(1+\phi'\right)k_1 \underbrace{\frac{d\ell_1(s)}{dz_1(s_L)}}_{<0}$$

$$\frac{dk_2(s)}{dz_1(s_L)} = \frac{di_1(s)}{dz_1(s_L)} + (1 - \ell_1(s) - \phi(\ell_1(s)))\frac{di_0}{dz_1(s_L)} - \left(1+\phi'\right)k_1 \underbrace{\frac{d\ell_1(s)}{dz_1(s_L)}}_{<0}$$

where $\frac{di_1(s)}{dz_1(s_L)}$, $\frac{di_0}{dz_1(s_L)}$, and $\frac{d\ell_1(s)}{dz_1(s_L)} < 0$ are given above. Also, we can see from the date 2 production function $y_2(s) = z_2\,(k_2(s))^{\alpha}\,\overline{n}^{1-\alpha}$ that

$$\frac{dy_2(s)}{dz_1(s_L)} = \alpha\frac{dk_2(s)}{dz_1(s_L)}$$

Thus, if the shock sufficiently reduces investment at dates 0 and 1, then the capital stock and output at date 2 will be lower at the margin as well.

APPENDIX 7: Risk-absorption channel of quantitative easing

To trace out the risk-absorption channel of QE, I first characterize the effect of government purchases of capital k_1^G on date 1 and 2 variables, and then given these ex post effects, characterize the effect on date 0.

Recall that I assumed that $T_1(s_H) = 0$ and $T_1(s_L) = R_0B_0$. The government's date 1 budget constraint then implies that

$$T_1^B(s_H) = r_1^k(s_H)k_1^G - R_0^BB_0 \tag{62}$$

$$T_1^B(s_L) = r_1^k(s_L)k_1^G \tag{63}$$

To focus on the risk-absorption channel, we can first leave out general equilibrium effects of asset purchases on date 0 utilization u_0 and output y_0 fixed. (This also implies we keep c_0, w_0 fixed). Then higher asset purchases k_1^G lowers k_1 but leaves the aggregate capital stock $\tilde{k}_1 := k_1 + k_1^G$.

$$\frac{dk_1}{dk_1^G} = -1 \ , \quad \frac{d\tilde{k}_1}{dk_1^G} = 0$$

Note from the Saving Demand curve that, holding c_0, w_0 constant (partial equilibrium), government asset purchases k_1^G crowd out private debt D_0 out one for one at the effective lower bound (recall that, here, $\tau_0, T_0^B = 0$)

$$D_0^d = e_0 + \tau_0 - T_0^B - k_1^G - c_0 + w_0\bar{n} \tag{64}$$

Consider the bad state at date 1.

$$\ell_1(s_L) = Lev_0 - r_1^k(s_L) \tag{65}$$

where $Lev_0 := \frac{\tau_0^R D_0 R_0^D - T_1^B}{k_1}$, and $\tau_0^R = 1$ and $R_0^D = 1$ in the demand-determined regime. First note that as k_1^G rises, $\ell_1(s_L)$ falls in partial equilibrium (keeping u_0 fixed).

$$\frac{d\ell_1(s)}{dk_1^G} = -\frac{\tau_0^R D_0 R_0^D - T_1^B}{(k_1)^2}\frac{dk_1}{dk_1^G} + \frac{\tau_0^R R_0^D}{k_1}\frac{dD_0}{dk_1^G} - \frac{dr_1^k(s)}{dk_1^G}$$

Since $\frac{dr_1^k(s)}{dk_1^G} = 0$ in partial equilibrium, we have

$$\frac{d\ell_1(s)}{dk_1^G} = \frac{\tau_0^R D_0 R_0^D - T_1^B}{(k_1)^2} - \frac{\tau_0^R R_0^D}{k_1} <?0$$

$$\frac{\tau_0^R D_0 R_0^D - T_1^B}{(k_1)} <?\tau_0^R R_0^D$$

$$D_0 <?k_1 + T_1^B$$

Recall the bank's date 0 budget constraint and law of motion for k_1: $k_1 = D_0 + r_0^k k_0 + T_0^B + k_0$. So

$$D_0 <?D_0 + r_0^k k_0 + T_0^B + k_0 + T_1^B$$

$$0 <? \left(1 + r_0^k\right) k_0 + T_1^B$$

This holds, since $T_1^B = r_1^k(s_L)k_1^G > 0$. Thus, $\frac{d\ell_1(s)}{dk_1^G} < 0$. Moreover, $\frac{d\ell_1(s)}{dk_1^G} < 0$ and $\frac{dk_1}{dk_1^G} < 0$ together imply that deadweight losses fall in partial equilibrium as k_1^G rises, $\frac{d}{dk_1^G}\left(\phi\left(\ell_1(s_L)\right)k_1\right) < 0$.

Effects at date 2 What is the effect of a rise in k_1^G on the date 2 stock of capital held by the bank, $k_2(s)$? Recall that $k_2(s)$ is given by

$$k_2(s) = (k_1 - D_0) + \left(T_1^B + r_1^k(s)k_1\right) - \phi\left(\ell_1(s)\right)k_1 \tag{66}$$

Note that

$$(k_1 - D_0) = k_1 - \left(e_0 + \tau_0 - T_0^B - k_1^G - c_0 + w_0\bar{n}\right)$$

$$= k_1 + k_1^G - e_0 + c_0 - w_0\bar{n}$$

Note also that

$$\underbrace{R_0^B B_0}_{debt\ repayment} = \underbrace{r_1^k(s)k_1^G}_{rental\ income} + \underbrace{T_1(s) - T_1^B(s)}_{lump-sum\ taxes\ and\ transfers}$$

$$B_0 = r_1^k(s)k_1^G + T_1(s) - T_1^B(s)$$

So $T_1^B + r_1^k(s)k_1 = r_1^k(s)k_1^G + T_1(s) - k_1^G + r_1^k(s)k_1$.

So returning to the equation for $k_2(s)$, in the bad state we have

$$k_2(s) = (k_1 - D_0) + \left(T_1^B + r_1^k(s)k_1\right) - \phi\left(\ell_1(s)\right)k_1 \tag{67}$$

$$= k_1 + T_1(s) - e_0 + c_0 - w_0\bar{n} + r_1^k(s)\left(k_1 + k_1^G\right) - \phi\left(\ell_1(s)\right)k_1 \tag{68}$$

$$= k_1 + T_1(s) - e_0 + c_0 - w_0\bar{n} + r_1^k(s)\tilde{k}_1 - \phi\left(\ell_1(s)\right)k_1 \tag{69}$$

In the good state, this is

$$k_2(s_H) = k_1 + T_1(s_H) - e_0 + c_0 - w_0\bar{n} + r_1^k(s_H)\tilde{k}_1 \tag{70}$$

$$= k_1 - e_0 + c_0 - w_0\bar{n} + r_1^k(s_H)\tilde{k}_1 \tag{71}$$

Recall from above that $\frac{d\tilde{k}_1}{dk_1^G} = 0$, in partial equilibrium. Moreover, since $r_1^k(s)$ depends on $y_1(s)$, which depends on the total capital stock $\tilde{k}_1$, it follows that $\frac{dr_1^k(s)}{dk_1^G} = 0$ in partial equilibrium. Therefore, we have

$$\frac{dk_2(s_H)}{dk_1^G} = -1$$

In the bad state, we have

$$k_2(s_L) = k_1 + T_1(s) - e_0 + c_0 - w_0\bar{n} + r_1^k(s)\tilde{k}_1 - \phi\left(\ell_1(s)\right)k_1 \tag{72}$$

$$= k_1 + R_0B_0 - e_0 + c_0 - w_0\bar{n} + r_1^k(s)\tilde{k}_1 - \phi\left(\ell_1(s)\right)k_1 \tag{73}$$

$$= k_1 + k_1^G - e_0 + c_0 - w_0\bar{n} + r_1^k(s)\tilde{k}_1 - \phi\left(\ell_1(s)\right)k_1 \tag{74}$$

$$= \tilde{k}_1 - e_0 + c_0 - w_0\bar{n} + r_1^k(s)\tilde{k}_1 - \phi\left(\ell_1(s)\right)k_1 \tag{75}$$

Again, recall from above that $\frac{d\tilde{k}_1}{dk_1^G}, \frac{dr_1^k(s)}{dk_1^G} = 0$ in partial equilibrium. Therefore, we have

$$\frac{dk_2(s_L)}{dk_1^G} = -\frac{d}{dk_1^G}\left[\phi\left(\ell_1(s)\right)k_1\right] > 0$$

Recall also that

$$k_2^G = k_1^G$$

$$\tilde{k}_2(s) = k_2(s) + k_2^G \tag{76}$$

In partial equilibrium (holding u_0 constant), we have

$$\frac{d\tilde{k}_2(s_H)}{dk_1^G} = \frac{dk_2(s_H)}{dk_1^G} + 1 = 0$$

We also have

$$\frac{d\tilde{k}_2(s_L)}{dk_1^G} = \frac{dk_2(s_L)}{dk_1^G} + \frac{dk_2^G}{dk_1^G}$$

$$= \frac{dk_2(s_L)}{dk_1^G} + 1 > 0$$

Thus, leaving aside general equilibrium effects of QE on u_0 (and therefore c_0, y_0, w_0), we have $\frac{dk_2(s_H)}{dk_1^G} = -1$ and $\frac{dk_2(s_L)}{dk_1^G} = -\frac{d}{dk_1^G}\left[\phi\left(\ell_1(s)\right)k_1\right] > 0$. Since $\tilde{k}_2 = k_2 + k_2^G$ this implies $\frac{dk_2(s_H)}{dk_1^G} = -1$ so that $\frac{d\tilde{k}_2(s_H)}{dk_1^G} = \frac{dk_2(s_H)}{dk_1^G} + 1 = 0$ and $\frac{d\tilde{k}_2(s_L)}{dk_1^G} = \frac{dk_2(s_L)}{dk_1^G} + 1 > 0$.

Effect on date 2 labor income Recall that we have

$$y_2 = z_2\left(\tilde{k}_2\right)^{\alpha}\bar{n}^{1-\alpha} \tag{77}$$

$$w_2 = (1-\alpha)\frac{y_2}{\bar{n}} \tag{78}$$

Thus, labor income at date 2 goes up in response to risky asset purchases iff aggregate capital stock at date 2 goes up. Therefore, $\frac{d}{dk_1^G}\left(w_2(s_H)\bar{n}\right) = 0$ and $\frac{d}{dk_1^G}\left(w_2(s_L)\bar{n}\right) > 0$. Then QE at the margin (in PE) boosts labor income in bad state, so reduces precautionary saving at date 0, reduces excess demand for safe assets, and hence leads to a rise in u_0, mitigating the severity of the date 0 recession.

Effects of QE on the severity of the recession at date 0 I now show that k_1^G increases u_0 in the demand-determined regime. The response of u_0 depends on the response of the excess demand for saving at the ELB ($D_0^d - D_0^s$) to k_1^G.

First consider the effect of QE on D_0^d. Above I showed that if the necessary and sufficient condition for QE to boost $k_2(s_L)$ holds, then QE boosts the household's date 2 labor income in the bad state $w_2(s_L)\bar{n}$. In turn, Lemma 2 implies that this reduces the household's date 0 precautionary saving D_0^d, from the Saving Demand curve. Hence, QE increases D_0^d to the extent that it boosts the household's date 2 labor income in the bad state.

I already showed that if the model's mechanism stronger (that is, if the macroeconomic spillover and effect on precautionary saving are larger), then D_0^d will respond more to $w_2(s_L)\bar{n}$. Thus, if the model's mechanism is stronger, then the effect of QE in reducing D_0^d will be stronger.

Now consider the effect on D_0^s. I showed above that QE reduces D_0^d. If QE also weakly increases D_0^s, then it unambiguously increases $D_0^d - D_0^s$ at the ELB, and therefore increases the level of u_0 required to reach equilibrium. The bank's supply of safe assets D_0^s is determined implicitly to equate the bank's date 0 marginal cost and marginal benefit of debt, as discussed in the optimality conditions $MB\left(D_0^s, z_1(s_L)\right) = MC\left(D_0^s, z_1(s_L)\right)$, i.e.

$$\underbrace{E\left[r_2^k\left(r_1^k(s)+1\right)\right] + \lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]}_{marginal\ benefit} = \underbrace{E\left[r_2^k\left(\phi\left(\ell_1(s)\right)+\tau_0^R R_0^D\right)\right] + \lambda_1(s_L)\tau_0^R R_0^D}_{marginal\ cost} \tag{79}$$

We can implicitly derive this equation with respect to k_1^G

$$\frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial k_1^G} + \frac{\partial MB\left(D_0^s, z_1(s_L)\right)}{\partial D_0^s}\frac{dD_0^s}{dk_1^G} = \frac{\partial MC\left(D_0^s, z_1(s_L)\right)}{\partial k_1^G} + \frac{\partial MC\left(D_0^s, z_1(s_L)\right)}{\partial D_0^s}\frac{dD_0^s}{\partial k_1^G}$$

Solving for $\frac{dD_0^s}{\partial k_1^G}$ yields the first-order effect of the shock on the supply of bank debt.

$$\frac{dD_0^s}{dk_1^G} = \frac{\frac{\partial MB}{\partial k_1^G} - \frac{\partial MC}{\partial k_1^G}}{\frac{\partial MC}{\partial D_0^s} - \frac{\partial MB}{\partial D_0^s}}$$

Note that k_1^G increases the supply of safe assets $\frac{dD_0^s}{dk_1^G} > 0$ if the numerator is positive, since the denominator is positive. This occurs if a rise in k_1^G increases the expected marginal benefit of debt to the bank more than it increases the expected marginal cost.

We already showed in Appendix 1 that, under Condition 1, we have $\frac{\partial MC}{\partial D_0^s} - \frac{\partial MB}{\partial D_0^s} > 0$. What about $\frac{\partial MB}{\partial k_1^G} - \frac{\partial MC}{\partial k_1^G}$? We can express

$$\frac{\partial MC}{\partial k_1^G} = \pi_L\left(\phi\left(\ell_1\right)+1\right)\frac{\partial r_2^k(s_L)}{\partial k_1^G} + \pi_L r_2^k(s_L)\phi'\frac{d\ell_1(s_L)}{dk_1^G} + \frac{d\lambda_1(s_L)}{dk_1^G}$$

Recall that $\lambda_1(s_L) = r_2^k(s_L)\phi'\left(\ell_1(s_L)\right)$. Also recall that $\frac{d\ell_1(s)}{dk_1^G} < 0$ and, since $\frac{\partial k_2(s_L)}{\partial k_1^G} > 0$, we have $\frac{\partial r_2^k(s_L)}{\partial k_1^G} < 0$. Guess and verify that $\frac{\partial r_2^k(s_H)}{\partial k_1^G} = 0$. So

$$\frac{d\lambda_1(s_L)}{dk_1^G} = \underbrace{\phi'\left(\ell_1(s_L)\right)}_{>0}\underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + r_2^k(s_L)\underbrace{\phi''\left(\ell_1(s_L)\right)}_{>0}\underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} < 0$$

Also, since $\frac{\partial r_1^k(s)}{\partial k_1^G} = 0$, we have

$$\frac{\partial MB}{\partial k_1^G} = \pi_L\left(r_1^k(s)+1\right)\frac{\partial r_2^k(s_L)}{\partial k_1^G} + \lambda_1(s_L)\frac{d\ell_1(s)}{dk_1^G} + \left(r_1^k(s_L)+\ell_1(s_L)\right)\frac{d\lambda_1(s_L)}{dk_1^G}$$

$$= \pi_L\left(r_1^k(s)+1\right)\underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + \lambda_1(s_L)\underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} + \left(r_1^k(s_L)+\ell_1(s_L)\right)\underbrace{\frac{d\lambda_1(s_L)}{dk_1^G}}_{<0}$$

So

$$\frac{\partial MB}{\partial k_1^G} - \frac{\partial MC}{\partial k_1^G} = \pi_L\left(r_1^k(s)+1\right)\underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + \lambda_1(s_L)\underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} + \left(r_1^k(s_L)+\ell_1(s_L)\right)\underbrace{\frac{d\lambda_1(s_L)}{dk_1^G}}_{<0} \ldots$$

$$\ldots - \left[\pi_L\left(\phi\left(\ell_1\right)+1\right)\frac{\partial r_2^k(s_L)}{\partial k_1^G} + \pi_L r_2^k(s_L)\phi'\frac{d\ell_1(s_L)}{dk_1^G} + \frac{d\lambda_1(s_L)}{dk_1^G}\right]$$

$$= \left[\pi_L \left(r_1^k(s)+1\right) - \pi_L \left(\phi\left(\ell_1\right)+1\right)\right] \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + \left[\lambda_1(s_L) - \pi_L r_2^k(s_L)\phi'\right] \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} + \left(r_1^k(s_L)+\ell_1(s_L)-1\right) \underbrace{\frac{d\lambda_1(s_L)}{dk_1^G}}_{<0}$$

$$= \pi_L \left[r_1^k(s) - \phi\left(\ell_1\right)\right] \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + \lambda_1(s_L)\left[1-\pi_L\right] \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} + \left(r_1^k(s_L)+\ell_1(s_L)-1\right) \underbrace{\frac{d\lambda_1(s_L)}{dk_1^G}}_{<0}$$

$$= \pi_L \left[r_1^k(s) - \phi\left(\ell_1\right)\right] \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + \lambda_1(s_L)\left[1-\pi_L\right] \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} ...$$

$$... + \left(r_1^k(s_L)+\ell_1(s_L)-1\right) \left[\underbrace{\phi'\left(\ell_1(s_L)\right)}_{>0} \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} + r_2^k(s_L) \underbrace{\phi''\left(\ell_1(s_L)\right)}_{>0} \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0}\right]$$

$$= \left(\pi_L \left[r_1^k(s) - \phi\left(\ell_1\right)\right] + \left[r_1^k(s_L)+\ell_1(s_L)-1\right] \underbrace{\phi'\left(\ell_1(s_L)\right)}_{>0}\right) \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} ...$$

$$... + \left(\underbrace{\lambda_1(s_L)\left[1-\pi_L\right]}_{>0} + \left(r_1^k(s_L)+\ell_1(s_L)-1\right) r_2^k(s_L) \underbrace{\phi''\left(\ell_1(s_L)\right)}_{>0}\right) \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0}$$

Note that $r_1^k(s_L)+\ell_1(s_L)-1 = \frac{\tau_0^R D_0 R_0 - T_1^B(s)}{k_1} - 1$. At the ELB, and when $\tau_0^R = 1$ and $T_1^B(s) = 0$, this is $r_1^k(s_L)+\ell_1(s_L)-1 = \frac{D_0}{k_1} - 1 < 0$. So we have

$$= \left(\pi_L \left[r_1^k(s) - \phi\left(\ell_1\right)\right] + \underbrace{\left[r_1^k(s_L)+\ell_1(s_L)-1\right]}_{<0} \underbrace{\phi'\left(\ell_1(s_L)\right)}_{>0}\right) \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} ...$$

$$... + \left(\underbrace{\lambda_1(s_L)\left[1-\pi_L\right]}_{>0} + \underbrace{\left(r_1^k(s_L)+\ell_1(s_L)-1\right)}_{<0} r_2^k(s_L) \underbrace{\phi''\left(\ell_1(s_L)\right)}_{>0}\right) \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0}$$

Then $\frac{\partial MB}{\partial k_1^G} - \frac{\partial MC}{\partial k_1^G} > 0$ iff

$$\left(\underbrace{\lambda_1(s_L)\left[1-\pi_L\right]}_{>0} + \underbrace{\left(r_1^k(s_L)+\ell_1(s_L)-1\right)}_{<0} r_2^k(s_L) \underbrace{\phi''\left(\ell_1(s_L)\right)}_{>0} \right) \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} \dots$$

$$\dots + \underbrace{\left[r_1^k(s_L)+\ell_1(s_L)-1\right]}_{<0} \underbrace{\phi'\left(\ell_1(s_L)\right)}_{>0} \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} > -\pi_L \left[r_1^k(s_L)-\phi\left(\ell_1\right)\right] \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0}$$

i.e.

$$\left(\underbrace{\lambda_1(s_L)\left[1-\pi_L\right]}_{>0} + \underbrace{\left(r_1^k(s_L)+\ell_1(s_L)-1\right)}_{<0} r_2^k(s_L) \underbrace{\phi''\left(\ell_1(s_L)\right)}_{>0} \right) \underbrace{\frac{d\ell_1(s)}{dk_1^G}}_{<0} \dots$$

$$\dots + \underbrace{\left[r_1^k(s_L)+\ell_1(s_L)-1\right]}_{<0} \underbrace{\phi'\left(\ell_1(s_L)\right)}_{>0} \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0} > \pi_L \left[\phi\left(\ell_1\right)-r_1^k(s_L)\right] \underbrace{\frac{dr_2^k(s_L)}{dk_1^G}}_{<0}$$

where the left-hand side is strictly positive, and the right-hand side is negative iff $\phi\left(\ell_1\right) > r_1^k(s)$. Thus, a sufficient condition for $\frac{\partial MB}{\partial k_1^G} - \frac{\partial MC}{\partial k_1^G} > 0$, and therefore for $\frac{dD_0^s}{dk_1^G} > 0$ and hence for $\frac{du_0}{dk_1^G} > 0$, is that liquidation costs are sufficiently large relative to the date 1 rental rate of capital in the bad state.

$$\phi\left(\ell_1(s_L)\right) > r_1^k(s_L)$$

This is a sufficient condition for QE to lead to a rise in D_0^s the supply of private safe assets, which is a sufficient condition for the excess demand for safe assets at $R_0 = 1$ to fall, and hence for u_0 and y_0 to rise. Intuitively, the asset purchases have two effects on the supply of safe assets: QE reduces liquidation in the bad state, and hence reduces the expected marginal cost of issuing debt. At the same time, by doing so, it increases the date 2 capital stock in the bad state, which lowers the expected date 2 rental rate of capital and hence the expected marginal benefit of investment. As long as this latter effect is not too large, D_0^s will not fall as QE rises at the margin. And hence the excess demand for safe assets at the ELB will fall.

I already showed that if the model's mechanism stronger (that is, if the macroeconomic spillover and effect on precautionary saving are larger), then D_0^d will respond more to $w_2(s_L)\bar{n}$. Suppose that the above condition holds so that QE increases date 0 output. Then it follows that, if the model's mechanism is stronger, then the risk-absorption channel will be stronger: QE will have a larger stimulative effect on date 0 output through the risk borne by the household.

APPENDIX 8: Macroprudential policy

Recall the bank's first-order condition for debt.

$$\underbrace{\left(1-\tau_0^D\right)\left\{E\left[r_2^k\left(r_1^k(s)+1\right)\right]+\lambda_1(s_L)\left[r_1^k(s_L)+\ell_1(s_L)\right]\right\}}_{marginal\ benefit}=\underbrace{E\left[r_2^k\left(\tau_0^D R_0+\phi\left(\ell_1(s)\right)\left(1-\tau_0^D\right)\right)\right]+\lambda_1(s_L)\tau_0^D R_0}_{marginal\ cost} \tag{80}$$

Effect of τ_0^R on D_0^s Note that $\frac{\partial MC}{\partial \tau_0^R}>0$ and $\frac{\partial MB}{\partial \tau_0^R}=0$. Therefore $\frac{\partial D_0^s}{\partial \tau_0^R}<0$.

Differentiating the FOC for D_0 with respect to τ_0^R yields

$$\frac{\partial MB\left(D_0^s,\tau_0^R\right)}{\partial \tau_0^R}+\frac{\partial MB\left(D_0^s,\tau_0^R\right)}{\partial D_0^s}\frac{dD_0^s}{d\tau_0^R}=\frac{\partial MC\left(D_0^s,\tau_0^R\right)}{\partial \tau_0^R}+\frac{\partial MC\left(D_0^s,\tau_0^R\right)}{\partial D_0^s}\frac{dD_0^s}{\partial \tau_0^R}$$

Solving for $\frac{dD_0^s}{\partial k_1^G}$ yields the first-order effect of the shock on the supply of bank debt.

$$\frac{dD_0^s}{d\tau_0^R}=\frac{\frac{\partial MB}{\partial \tau_0^R}-\frac{\partial MC}{\partial \tau_0^R}}{\frac{\partial MC}{\partial D_0^s}-\frac{\partial MB}{\partial D_0^s}}$$

We already showed in Appendix 1 that

$$\frac{\partial MC}{\partial D_0^s}-\frac{\partial MB}{\partial D_0^s}>0$$

Consider now $\frac{\partial MB}{\partial \tau_0^R}$:

$$\frac{\partial MB}{\partial \tau_0^D}=0$$

And

$$\underbrace{E\left[r_2^k\left(\tau_0^D R_0+\phi\left(\ell_1(s)\right)\left(1-\tau_0^D\right)\right)\right]+\lambda_1(s_L)\tau_0^D R_0}_{marginal\ cost}$$

$$\frac{\partial MC}{\partial \tau_0^D}=R_0 E\left[r_2^k\right]+\lambda_1(s_L)R_0>0$$

Therefore $\frac{dD_0^s}{d\tau_0^R}<0$.

Effect of τ_0^R on $\ell_1(s_L)$ Note also that

$$\ell_1(s_L) = \frac{\tau_0^R D_0 R_0 - T_1^B(s)}{k_1} - r_1^k(s_L) \tag{81}$$

Therefore

$$\frac{d\ell_1(s_L)}{d\tau_0^R} = \frac{\partial \ell_1(s_L)}{\partial \tau_0^R} + \frac{\partial \ell_1(s_L)}{\partial D_0^s}\frac{dD_0^s}{d\tau_0^R}$$

$$= \frac{\tau_0^R R_0}{k_1}\frac{\partial D_0}{\partial \tau_0^R} + \frac{D_0 R_0}{k_1}$$

Thus, there are two opposing effects of an increase in τ_0^R on $\ell_1(s_L)$. On the one hand, it reduces D_0, which reduces leverage and the amount of liquidation. On the other hand, it increase interest expenses (at any level of D_0), which increase liquidation. For ease of exposition, I restrict analysis to the case in which the former effect dominates. I have $\frac{d\ell_1(s_L)}{d\tau_0^R} < 0$ iff

$$\frac{D_0 R_0}{k_1} < -\frac{\tau_0^R R_0}{k_1}\underbrace{\frac{\partial D_0}{\partial \tau_0^R}}_{<0}$$

i.e.

$$D_0 < -\tau_0^R \underbrace{\frac{\partial D_0}{\partial \tau_0^R}}_{<0}$$

I restrict analysis to the case in which this holds, so that, at the margin, an increase in τ_0^R reduces $\ell_1(s_L)$.

Effect of macroprudential policy on the excess demand for saving and output The condition for macroprudential taxes to reduces the severity of the date 0 recession is that it reduces the excess demand for safe assets at the effective lower bound. The effect of the tax on this is given by $\frac{d}{d\tau_0^R}\left(D_0^d - D_0^s\right) = \frac{dD_0^d}{d\tau_0^R} - \frac{dD_0^s}{d\tau_0^R}$. (I have suppressed the notation that these derivatives are evaluated at the equilibrium, including $u_0 = u_0^*$.) Therefore, we have that an increase in τ_0^R reduces the severity of the date 0 recession if and only if

$$\frac{dD_0^d}{d\tau_0^R} - \frac{dD_0^s}{d\tau_0^R} < 0$$

Note from the Saving Demand curve that

$$\frac{dD_0^d}{d\tau_0^R} = \frac{\partial D_0^d}{\partial d_0^F}\frac{dd_0^F}{d\tau_0^R} + \frac{\partial D_0^d}{\partial w_0}\frac{dw_0}{d\tau_0^R} + \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{d\tau_0^R}$$

Again, I have in the demand-determined regime that $d_0^F = 0$, $w_0 = 0$. Then $\frac{dd_0^F}{d\tau_0^R}, \frac{dw_0}{d\tau_0^R} = 0$ and $\frac{dD_0^d}{d\tau_0^R}$ in the demand-determined regime, simplifies to

$$\frac{dD_0^d}{d\tau_0^R} = \frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{d\tau_0^R} = \frac{1}{R_0}\left(E_0\left[\frac{1}{c_1(s)}\right]\right)^{-2}\frac{dE_0\left[\frac{1}{c_1(s)}\right]}{d\tau_0^R}$$

This is the effect of the tax on the household's consumption risk (through the macroeconomic spillover, and the effect of consumption risk on D_0^d (precautionary saving demand). Moreover, recall these elasticities can be decomposed into a channel reflecting the effect of systemic risk. Moreover, these elasticities can be decomposed into a channel reflecting the effect of systemic risk.

Therefore, the condition $\frac{dD_0^d}{d\tau_0^R} - \frac{dD_0^s}{d\tau_0^R} < 0$ is

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \tau_0^R} + \underbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}_{>0}\frac{d\ell_1(s_L)}{d\tau_0^R} < \frac{dD_0^s}{d\tau_0^R}$$

where the second term on the left-hand side reflects captures the effect of the tax on safe asset demand via systemic risk and expected future marginal utility of consumption, while the first term captures other effects such as through the household's interest income on holdings of bonds in both states. I showed in Appendix 2 that $\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)} > 0$.

I showed above that $\frac{dD_0^s}{d\tau_0^R} < 0$. Also, as discussed above, I restrict analysis to the case in which the tax reduces liquidation, $\frac{d\ell_1(s_L)}{d\tau_0^R} = \frac{\partial \ell_1(s_L)}{\partial \tau_0^R} + \frac{\partial \ell_1(s_L)}{\partial D_0^s}\frac{dD_0^s}{d\tau_0^R} < 0$. So the condition is

$$\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \tau_0^R} + \underbrace{\overbrace{\frac{\partial D_0^d}{\partial E_0\left[\frac{1}{c_1(s)}\right]}}^{\textit{precautionary saving}}\overbrace{\frac{\partial E_0\left[\frac{1}{c_1(s)}\right]}{\partial \ell_1(s_L)}}^{\textit{macro spillover}}}_{>0}\underbrace{\frac{d\ell_1(s_L)}{d\tau_0^R}}_{<0} < \underbrace{\frac{dD_0^s}{d\tau_0^R}}_{<0}$$

So we can see that, the stronger is the macroeconomic spillover and precautionary saving effect (that is, the stronger the dynamic interaction between systemic risk and aggregate demand), the more likely this condition is to hold.