

The Bullwhip: Time to Build and Sectoral Fluctuations

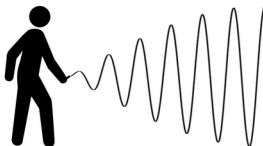
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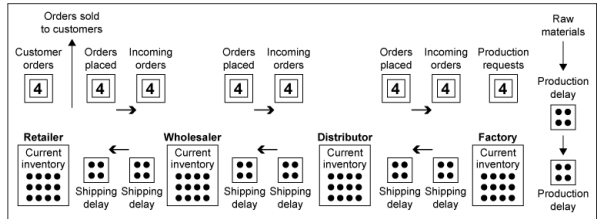
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The bullwhip effect: a supply-chain phenomenon where small demand variability from a downstream site (e.g., retail) create amplified volatility in an upstream site (e.g., manufacturer).



- Enormous literature in industrial engineering & operations management ([Forrester 1958](#), [Lee et al. 1997](#))
 - many case studies and anecdotal accounts; often attributed to delays between supply chain links

The Beer Game



Students at MIT Sloan School of Management playing the beer game, a role-playing simulation designed to teach principles of management science

Photo Credit: Courtesy of MIT Sloan School of Management

A theory of sectoral fluctuations in supply chains

- A framework to analyze dynamics interactions over supply chains, with two main ingredients:
 - sectoral demand shocks and heterogeneous time-to-build of inputs
- Challenge: dynamic fixed point problem over endogenous actions at all time horizons
 - each producer input choices depends on future demand all time horizons, arising from endogenous actions of producers at different positions in the supply chain who themselves may face different horizon delays

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 1. characterize the solution in closed-form over primitives, both cross-section & impulse response
 - full decomposition of equilibrium sectoral response to expected demand at different horizons across the network
 - the bullwhip effect arises if current demand shock implies greater expected demand in future
 - introduce exponential-decay & hump-shaped demand shocks; solve under complete and incomplete information

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 - the bullwhip effect arises if current demand shock implies greater expected demand in future
 - introduce exponential-decay & hump-shaped demand shocks; solve under complete and incomplete information
 2. show the bullwhip effect is significant across all supply chains within US production network
 - in the cross-section, upstream sectors have amplified volatility
 - impulse-response from downstream demand shocks: hump-shaped time profile & evidence of upstream learning

(preferences)
$$V_t \equiv \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\sum_{i=1}^N \theta_{is} \ln c_{is} - v(\ell_s) \right) \right]$$

(technology)
$$y_{it} = z_{it} \ell_{it}^{\alpha_i} \prod_{j=1}^N m_{ij,t-d_{ij}}^{\omega_{ij}}, \quad y_{jt} = c_{jt} + \sum_{i=1}^N m_{ijt}, \quad \ell_t = \sum_i \ell_{it}.$$

- $d_{ij} \in \mathbb{Z}_{>0}$: time-to-build for input j sent to producer i
- θ_{it} : demand shocks at time t . Steady-state levels: $\bar{\theta}_i$. Information set to be specified

The planner's problem:

$$\begin{aligned}
 V_t \left(\{m_{ij,t-q}\}_{q=1,\dots,d_{ij}} \right) = & \max_{\{\ell_{it}, c_{it}, m_{ijt}, \bar{\ell}_t\}} \sum_i \theta_{it} \ln c_{it} - v(\bar{\ell}_t) + \beta \mathbb{E}_t \left[V_{t+1} \left(\{m_{ij,t+1-q}\}_{q=1,\dots,d_{ij}} \right) \right] \\
 & + \sum_j p_{jt} \left[z_j \ell_{jt}^{\alpha_j} \prod_k m_{jk,t-d_{jk}}^{\omega_{jk}} - c_{jt} - \sum_i m_{ijt} \right] + w_t \left[\bar{\ell}_t - \sum_j \ell_{jt} \right],
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- closed-form solution despite the large state space; maps intuitively to a competitive equilibrium

Lemma. Consider a competitive equilibrium with the marginal util. of income normalized to one in each period. Let p_{jt} denote the price of good j when it is produced. Then:

$$\text{(consumer expenditure)} \quad p_{it} c_{it} = \theta_{it}, \quad \text{(revenue)} \quad \gamma_{jt} \equiv p_{jt} y_{jt} = \theta_{jt} + \sum_i \beta^{d_{ij}} \omega_{ij} \mathbb{E}_t [\gamma_{i,t+d_{ij}}].$$

Sectoral revenue expressed in future demand

- Let $\Omega \equiv [\omega_{ij}]'$ denote the matrix of input cost shares
- Let $\Omega_d \equiv [\omega_{ij} \mathbf{1}_{d_{ij}=d}]'$ the matrix of input cost shares with delay d
- Sectoral revenue follows

$$\gamma_t = \theta_t + \beta \Omega_1 \mathbb{E}_t [\theta_{t+1}] + \beta^2 (\Omega_1^2 + \Omega_2) \mathbb{E}_t [\theta_{t+2}] + \beta^3 (\Omega_1^3 + \Omega_2 \Omega_1 + \Omega_1 \Omega_2 + \Omega_3) \mathbb{E}_t [\theta_{t+3}] \dots$$

Proposition. Let Φ_s be the set of positive, finite sequences $\phi \equiv \{\phi_1, \dots, \phi_n\}$ that sum to s . Then

$$\gamma_t = \theta_t + \sum_{s=1}^{\infty} G_s \mathbb{E}_t [\theta_{t+s}], \quad \text{where } G_s \equiv \beta^s \sum_{\phi \in \Phi_s} \prod_{\phi_j \in \phi} \Omega_{\phi_j}.$$

- Corollary: steady-state revenue follows $\bar{\gamma} = (\mathbf{I} - \sum_{d=1}^{\infty} \beta^d \Omega_d)^{-1} \bar{\theta}$. Contrast with:
 - Long and Plosser (1983) with homogeneous, one-period time-to-build: $\bar{\gamma} = (\mathbf{I} - \beta \Omega)^{-1} \bar{\theta}$
 - Acemoglu et al. (2012)'s static model without time-to-build: $\bar{\gamma} = (\mathbf{I} - \Omega)^{-1} \bar{\theta}$

When demand follows $ARMA(p, q)$:

$$\theta_{it} = \sum_{s=1}^{\infty} \delta_s \theta_{it-s} + e_{it}. \quad (1)$$

Proposition. When demand follows (1), the revenue response to a demand shock is

$$\frac{\partial \gamma_t}{\partial \mathbf{e}_t} = \mathbf{I} + \sum_{s=1}^{\infty} \beta^s \left(\sum_{\phi \in \Phi_s} \prod_{\phi_j \in \phi} \delta_{\phi_j} \right) \left(\sum_{\phi \in \Phi_s} \prod_{\phi_j \in \phi} \Omega_{\phi_j} \right),$$

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Demand shocks: mean-reverting with transitory (ϵ_{it}) and persistent (u_{it}) components

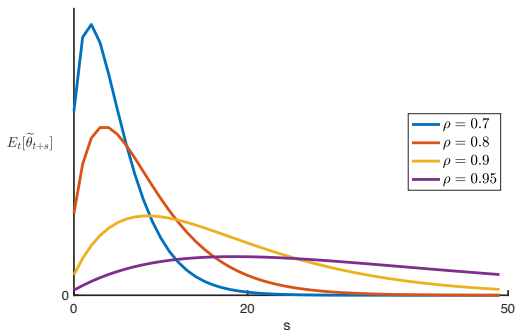
(demand shocks) $\theta_{it} - \bar{\theta}_i = \rho (\theta_{it-1} - \bar{\theta}_i) + x_{it} + \epsilon_{it},$ $x_{it} = \rho x_{i,t-1} + u_{it}.$

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- Starting from steady-state at $t - 1$, impulse-response of demand following a shock (ϵ_{it}, u_{it}):

$$\mathbb{E}_t [\theta_{i,t+s} - \bar{\theta}_i] = \underbrace{\rho^s \epsilon_{it}}_{\substack{\text{demand response to a} \\ \text{transitory shock;} \\ \text{exponential decay}}} + \underbrace{(s+1)\rho^s u_{it}}_{\substack{\text{demand response to a} \\ \text{persistent shock;} \\ \text{hump-shaped}}} \quad \text{for all } s \geq 0.$$



- First study complete information:** observe both θ and x ; later: incomplete info, observe θ only

$$\gamma_t = \theta_t + \beta \Omega_1 \mathbb{E}_t [\theta_{t+1}] + \beta^2 (\Omega_1^2 + \Omega_2) \mathbb{E}_t [\theta_{t+2}] + \beta^3 (\Omega_1^3 + \Omega_2 \Omega_1 + \Omega_1 \Omega_2 + \Omega_3) \mathbb{E}_t [\theta_{t+3}] \dots$$

- Concurrent revenue response to a transitory or persistent shock ($\tilde{\cdot}$ is deviation from steady-state):

$$\partial \gamma_t / \partial \epsilon_t = \mathbf{I} + \rho \beta \Omega_1 + (\rho \beta)^2 (\Omega_1^2 + \Omega_2) + (\rho \beta)^3 (\Omega_1^3 + \Omega_1 \Omega_2 + \Omega_2 \Omega_1 + \Omega_3) \dots$$

$$\partial \gamma_t / \partial \mathbf{u}_t = 1 \times \mathbf{I} + 2 \times \rho \beta \Omega_1 + 3 \times (\rho \beta)^2 (\Omega_1^2 + \Omega_2) + 4 \times (\rho \beta)^3 (\Omega_1^3 + \Omega_1 \Omega_2 + \Omega_2 \Omega_1 + \Omega_3) \dots$$

- if time-to-build is always one period, $\partial \gamma_t / \partial \epsilon_t = (\mathbf{I} - \rho \beta \Omega)^{-1}$, $\partial \gamma_t / \partial \mathbf{u}_t = (\mathbf{I} - \rho \beta \Omega)^{-2}$

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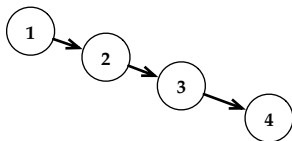
Proposition. Under complete information, sectoral revenue at time t follows

$$\tilde{\gamma}_t = \mathbf{G}_\infty^\epsilon \tilde{\theta}_t + \mathbf{G}_\infty^\epsilon \left(\sum_{d=1}^{\infty} d (\rho \beta)^d \Omega_d \right) \mathbf{G}_\infty^\epsilon \tilde{\mathbf{x}}_t, \quad \text{where } \mathbf{G}_\infty^\epsilon \equiv \left(\mathbf{I} - \sum_{d=1}^{\infty} (\rho \beta)^d \Omega_d \right)^{-1}.$$

The impulse-response functions are:

$$\frac{\partial \mathbb{E}_t [\gamma_{t+s}]}{\partial \epsilon_t} = \rho^s \mathbf{G}_\infty^\epsilon, \quad \frac{\partial \mathbb{E}_t [\gamma_{t+s}]}{\partial \mathbf{u}_t} = (s+1) \rho^s \mathbf{G}_\infty^\epsilon + \rho^s \mathbf{G}_\infty^\epsilon \left(d \sum_{d=1}^{\infty} (\rho \beta)^d \Omega_d \right) \mathbf{G}_\infty^\epsilon.$$

Example: a vertical supply chain



$$\left[\frac{d(\gamma_{it}/\bar{\gamma}_i)}{d\epsilon_{j,t}} \right] = \begin{bmatrix} 1 & \rho^{d_1} & \rho^{d_1+d_2} & \rho^{d_1+d_2+d_3} \\ 0 & 1 & \rho^{d_2} & \rho^{d_2+d_3} \\ 0 & 0 & 1 & \rho^{d_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\frac{d(\gamma_{it}/\bar{\gamma}_i)}{du_{j,t}} \right] = \begin{bmatrix} 1 & (1+d_1)\rho^{d_1} & (1+d_1+d_2)\rho^{d_1+d_2} & (1+d_1+d_2+d_3)\rho^{d_1+d_2+d_3} \\ 0 & 1 & (1+d_2)\rho^{d_2} & (1+d_2+d_3)\rho^{d_2+d_3} \\ 0 & 0 & 1 & (1+d_3)\rho^{d_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A measure of bilateral delays

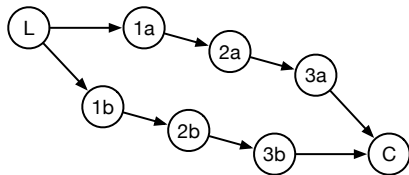
Definition. The delay from input i to producer j is $\xi_{ij} \equiv \begin{cases} \left[\frac{\partial \gamma_t}{\partial u_t} \right]_{ij} / \left[\frac{\partial \gamma_t}{\partial \epsilon_t} \right]_{ij} & \text{if } \left[\frac{\partial \gamma_t}{\partial \epsilon_t} \right]_{ij} \neq 0, \\ 0 & \text{otherwise.} \end{cases}$

- Interpretation: ξ_{ij} is the average delay for i 's output to reach producer j across all possible network routes
 - weighted by output response along each route following a transitory demand shock to j given ρ

$$\begin{aligned} \xi_{ij} = & 1 \times \frac{[I]_{ij}}{\left[\frac{\partial \tilde{\gamma}_t}{\partial \epsilon_t} \right]_{ij}} + 2 \times \frac{[\rho \beta \Omega_1]_{ij}}{\left[\frac{\partial \tilde{\gamma}_t}{\partial \epsilon_t} \right]_{ij}} + 3 \times \frac{[(\rho \beta)^2 (\Omega_1^2 + \Omega_2)]_{ij}}{\left[\frac{\partial \tilde{\gamma}_t}{\partial \epsilon_t} \right]_{ij}} \\ & + 4 \times \frac{[(\rho \beta)^3 (\Omega_1^3 + \Omega_1 \Omega_2 + \Omega_2 \Omega_1 + \Omega_3)]_{ij}}{\left[\frac{\partial \tilde{\gamma}_t}{\partial \epsilon_t} \right]_{ij}} + \dots \end{aligned}$$

- as $\rho \rightarrow 1$, weights become steady-state output response to permanent demand changes
- when delays are always one period, ξ_{ij} measures the stages of production from i to j

- ξ_{ij} : a bilateral measure of delays from any input supplier i to any input user j
 - contrast with the standard *Leontief inverse*, which captures a notion of network *dependence*
 - or Antras et al. (2012)'s *upstreamness*, which captures the distance of a sector relative to the consumer
- Illustration: ξ distinguishes supply chains A and B



Key: ξ enables us to empirically construct supply chains for every sector of the economy.

Persistent shocks lead to amplified upstream volatility

Proposition. Consider the volatility of revenue $Var_t(\Delta\gamma_{it+1})$ in a vertical network.

1. When all shocks are transitory, upstream sectors always have lower volatilities.
2. When shocks are persistent, for sufficiently high ρ , upstream sectors have higher volatility.

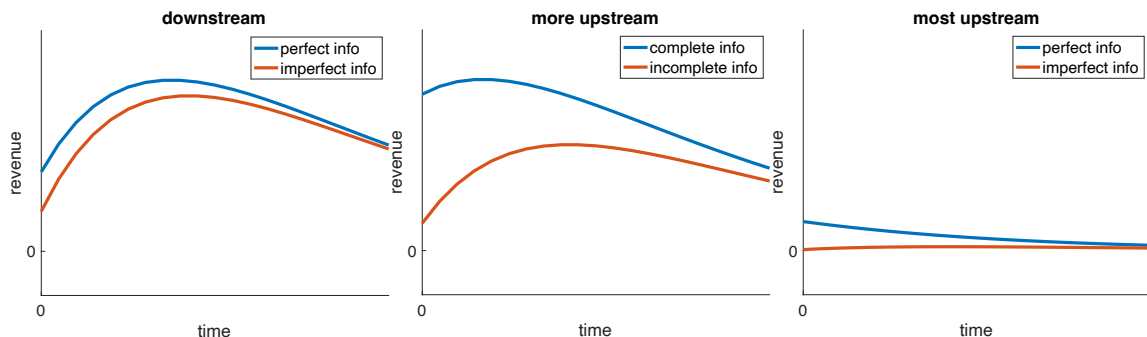
Generalization: learning under incomplete information

(recall demand shocks)
$$\tilde{\theta}_{it} = \rho\tilde{\theta}_{it-1} + x_{it} + \epsilon_{it}, \quad x_{it} = \rho x_{i,t-1} + u_{it}.$$

- Incomplete information: only θ is observable; x is not. Nowcast formed by Kalman filter

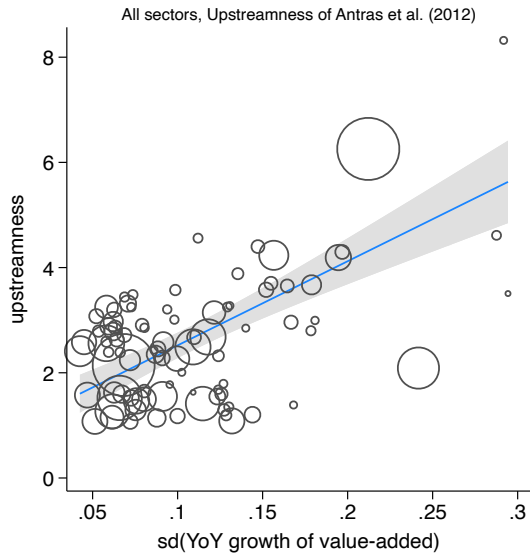
$$\hat{x}_{t+1} = \rho\hat{x}_t + \kappa_{t+1} \left(\tilde{\theta}_{t+1} - \mathbb{E}_t \left[\tilde{\theta}_{t+1} \right] \right), \quad \text{Kalman gain } \kappa \text{ is increasing in } \sigma_u/\sigma_\epsilon.$$

- delayed response to demand shocks
- momentum (correlated past shocks) affects nowcast \hat{x}_t and impulse response esp. for upstream sectors



- We provide evidence for the bullwhip effect across supply chains of the US production network
 - cross-section: within each supply chain, more upstream sectors tend to have higher volatility
 - impulse response: delayed and amplified response to downstream demand shocks
- Data
 - Industrial Production ([Foerster, Sarte, and Watson 2011](#)): value-added share for 114 sectors, 1972–2019
 - BEA IO table for 2007, 389 sectors
 - US Census M3 survey of manufacturers' shipments, inventories, and orders
 - measure delay with backlog ratio (between the stock value of unfilled orders & flow value of goods delivered)
 - imputation by durable/non-durable; impose one quarter minimum & 4 quarters maximum

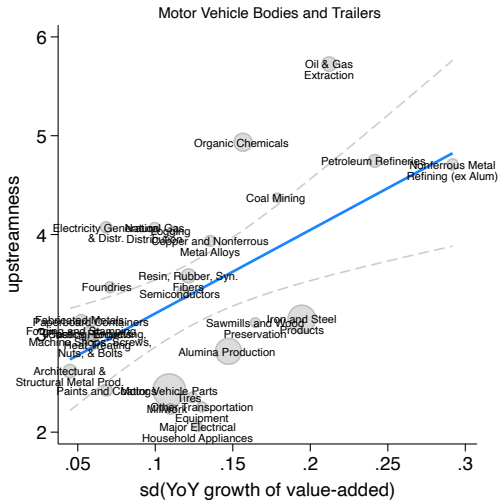
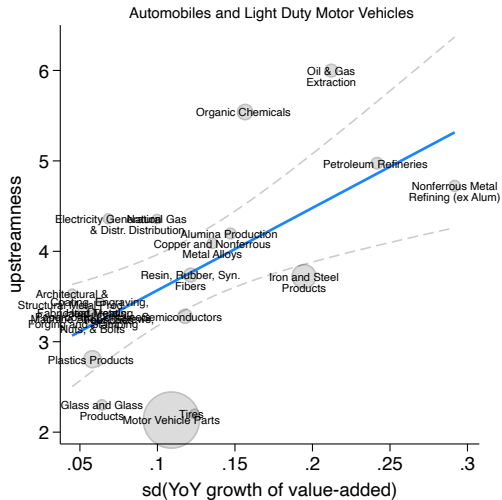
Upstream sectors have higher volatility



Identify supply chains

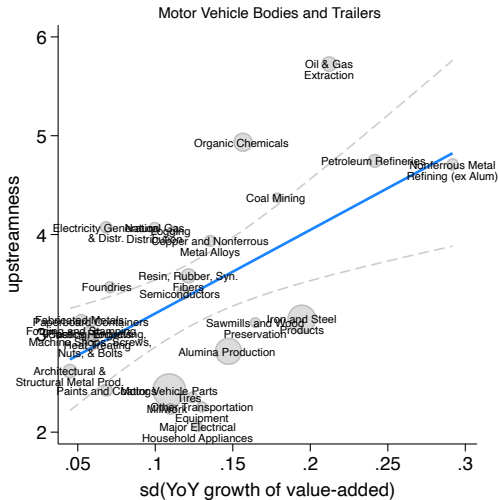
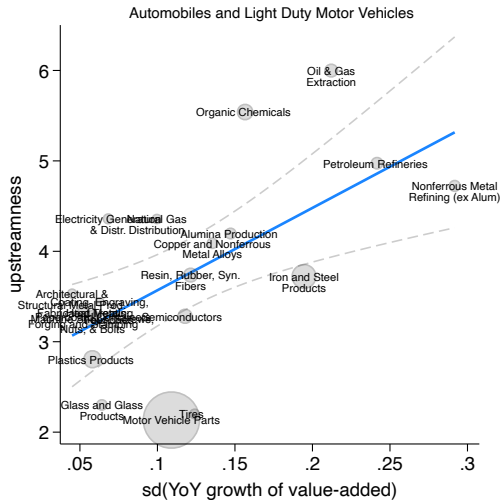
- For each downstream sector j ,
 - compute ξ_{ij} as the upstreamness of i relative to j
 - keep sectors that are “sufficiently connected” to j based on the Leontief inverse ($[(I - \beta\Omega)^{-1}]_{ij} > 0.01$)

Supply chains: automobiles; trailers



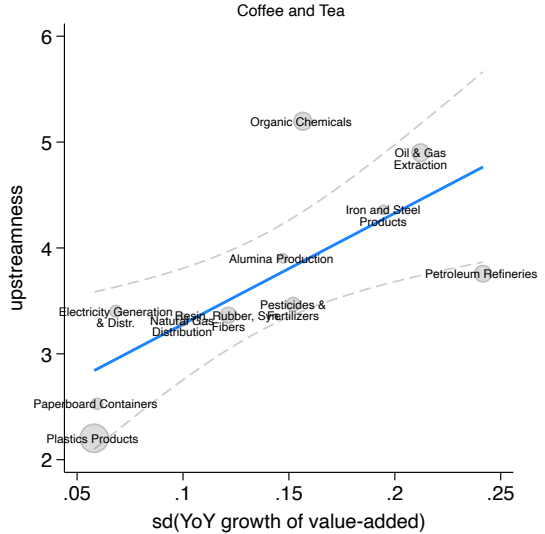
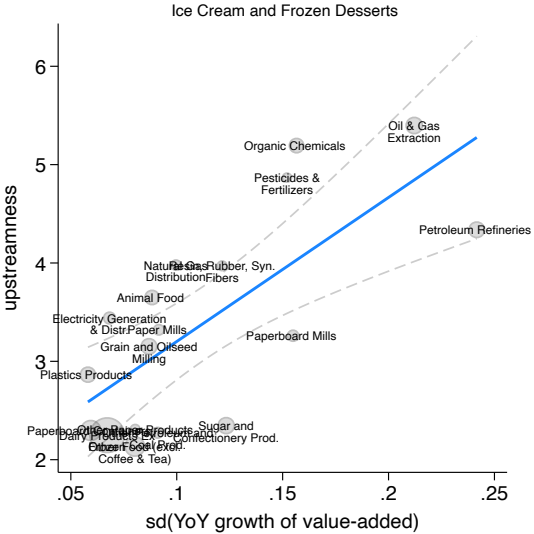
- What's immediately upstream to automobiles?
- What's immediately upstream to trailers?

Supply chains: automobiles; trailers

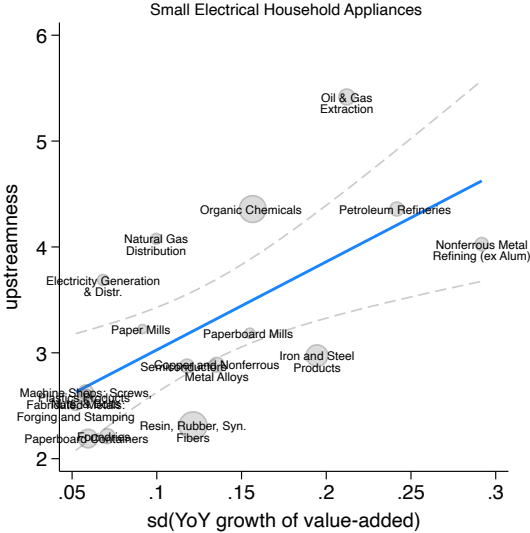
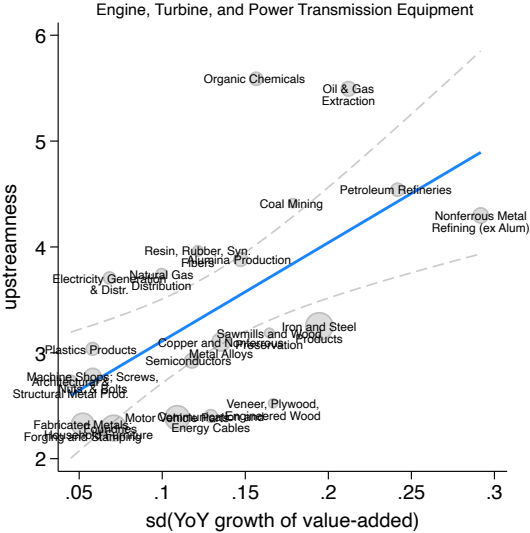


- What's immediately upstream to automobiles? Motor vehicle parts
- What's immediately upstream to trailers? Major electrical household appliances

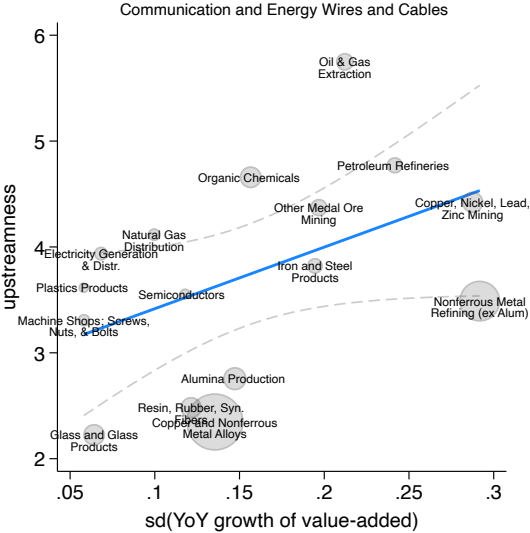
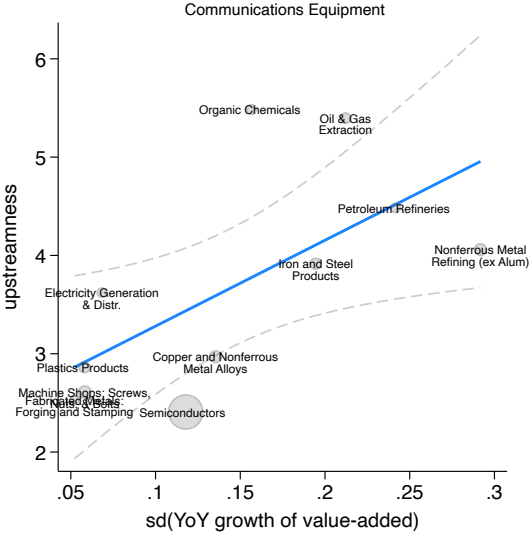
Supply chains: ice cream; coffee & tea



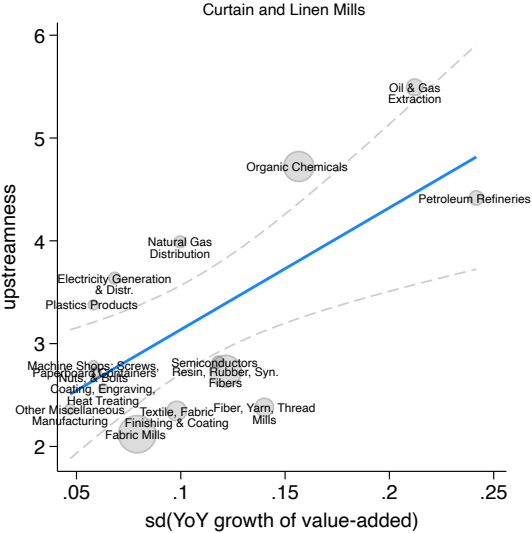
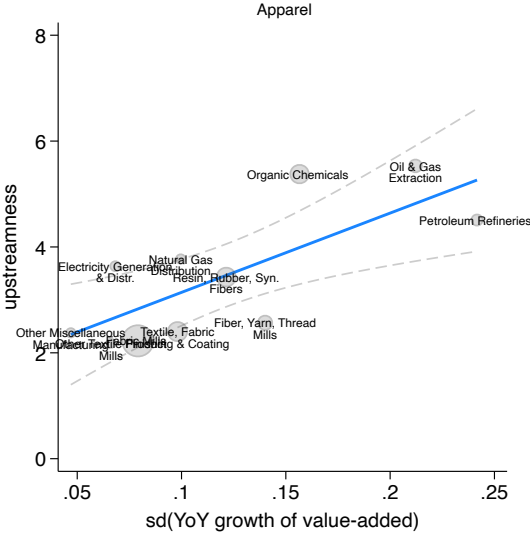
Supply chains: engines; small electrical household appliances



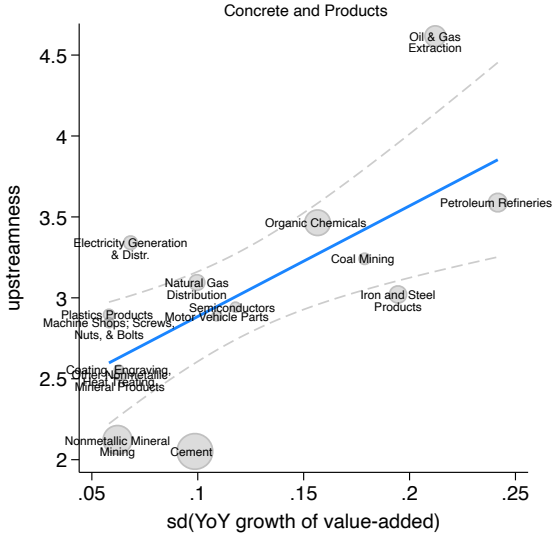
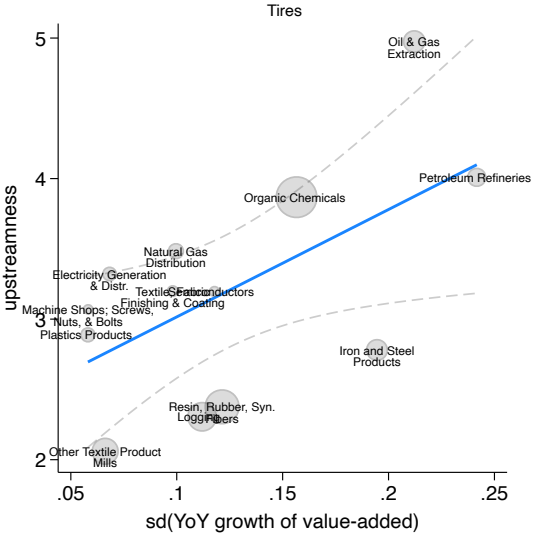
Supply chains: communications equipments;



Supply chains: apparel; curtain & linen mills



Supply chains: tires; concrete



Sectoral impulse response to demand shocks

- Identify 25 downstream sectors by the Antras et al. (2012) “upstreamness” measure

Automobiles and Light Duty Motor Vehicles	Ship and Boat Building	Breweries
Animal Slaughtering and Processing	Other Transportation Equipment	Heavy Duty Trucks
Soap, Cleaning Compounds, and Toilet Preparation	Medical Equipment and Supplies	Carpet and Rug Mills
Periodical, Book, and Other Publishers	Office And Other Furniture	Soft Drinks and Ice
Other Food Except Coffee and Tea	Aerospace Products and Parts	Tobacco
Sugar and Confectionery Products	Newspaper Publishers	Apparel
Fruit and Vegetable Preserving and Specialty Foods	Motor Vehicle Bodies and Trailers	Coffee and Tea
Major Electrical Household Appliances	Pharmaceuticals and Medicines	Bakeries and Tortilla
Navigational/Measuring/Electromedical/Control Instruments		

- Identify demand shock of 25 downstream sectors as innovations r_{it} in the linear regression

$$VA_{jt}^{down} = \sum_{s=1}^p \gamma_s VA_{j,t-s}^{down} + \mu_j + \mu_t + r_{jt}$$

- use normalized quarterly data on value-added; robust to monthly or HP filtered data

Sectoral impulse response to demand shocks

- The two sets of local projections are specified as follows:

$$VA_{j,t+h}^{down} = \alpha^h r_{jt} + X_{jt} + \zeta_{jt},$$

$$VA_{i,t+h}^{up,g} = \beta^{h,g} \sum_{w_{ij} \geq 0.01} w_{ij} r_{jt} + \delta^{h,g} \sum_{w_{ij} \geq 0.01} w_{ij} r_{jt} \times IC_{jt} + \sum_{w_{ij} \geq 0.01} w_{ij} IC_{jt} + Z_{it}^g + \epsilon_{it},$$

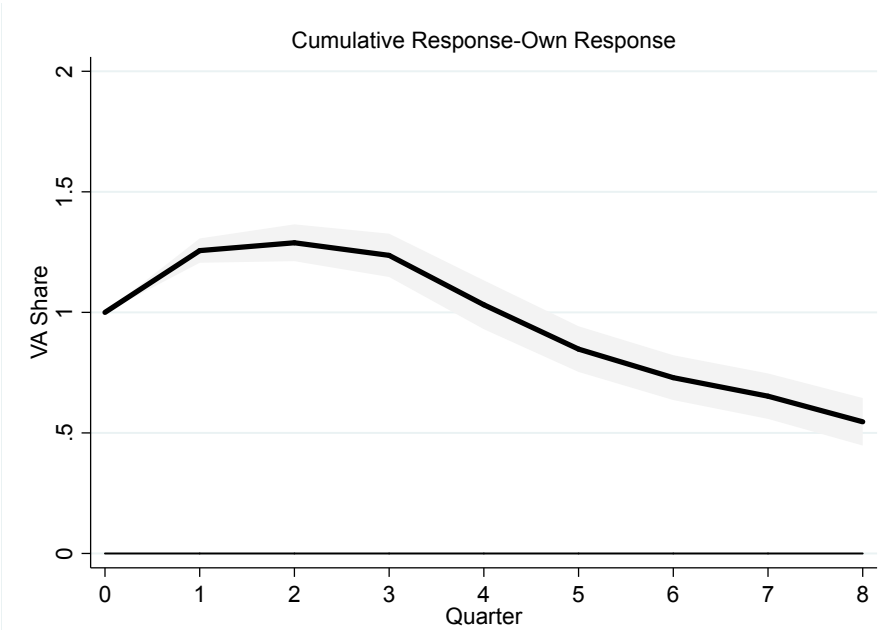
with controls $X_{jt} = \left\{ \left\{ VA_{j,t-s}^{down} \right\}_{s=1}^p, \mu_j, \mu_t \right\}$, $Z_{it}^g = \left\{ \left\{ \sum_{w_{ij} \geq 0.01} w_{ij} VA_{j,t-s}^{down}, VA_{i,t-s}^{up,g} \right\}_{s=1}^p, \mu_i, \mu_t \right\}$.

- $w_{ij} \equiv L_{ij} \times \frac{\bar{\theta}_j}{\bar{\gamma}_i}$ is the fraction of i 's revenue sold through supply chain to j in steady-state
- $IC_{j,t}$: "information continuity"; captures momentum (Da, Guren, Warachka 2014)

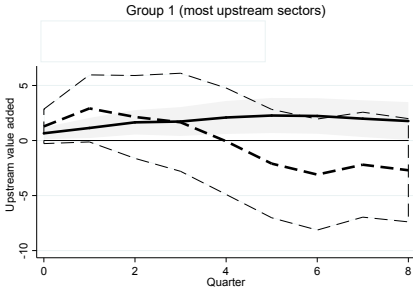
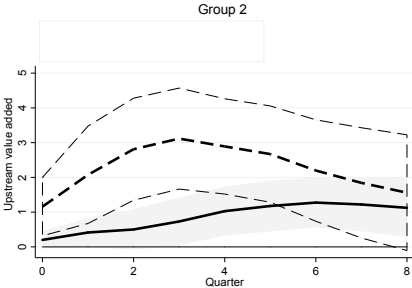
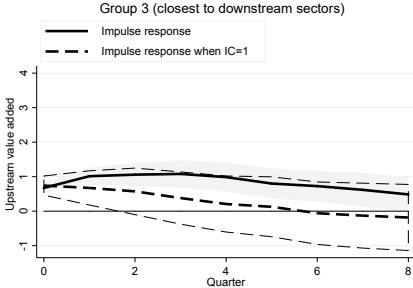
$$IC_{j,t} = \text{sign} \left(VA_{jt}^{down} - VA_{j,t-p}^{down} \right) \times [\% \text{ pos} - \% \text{ neg}]$$

- We estimate the response by grouping sectors together according to upstreamness

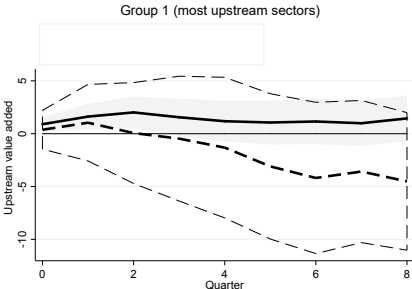
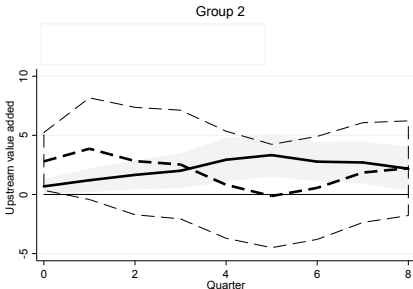
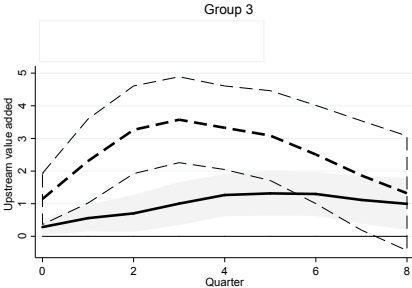
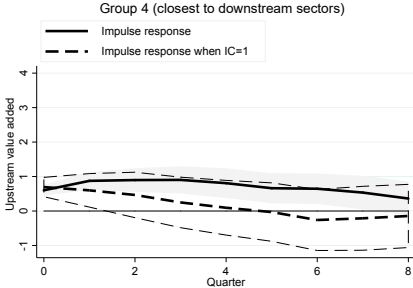
Downstream's own revenue response to demand shocks



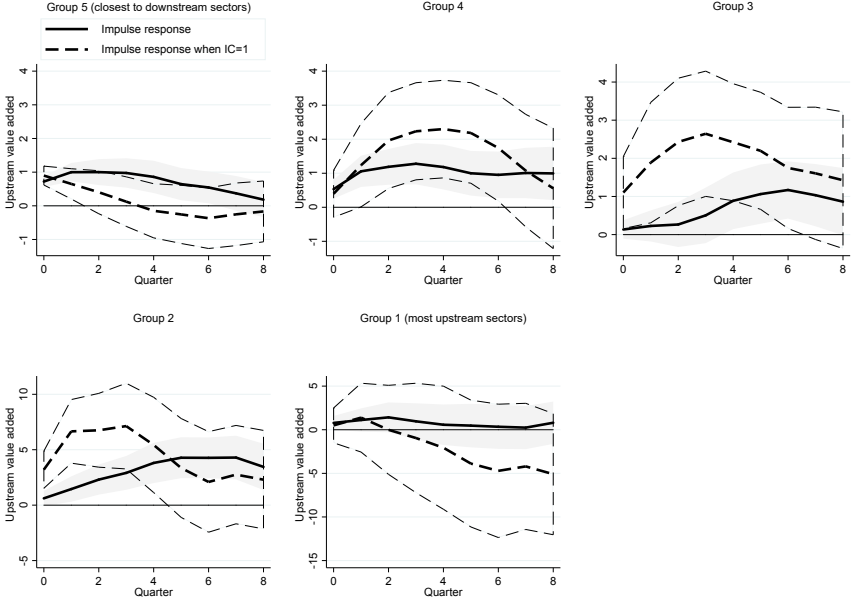
Response to downstream demand shock along the supply chain



Response to downstream demand shock along the supply chain



Response to downstream demand shock along the supply chain



Conclusion: a theory of sectoral fluctuations in supply chains

- A framework to analyze dynamics interactions over supply chains, with two main ingredients:
 - sectoral demand shocks and heterogeneous time-to-build of inputs
 - each producer input choices depends on future demand all time horizons, arising from endogenous actions of producers at different positions in the supply chain who themselves may face different horizon delays
- Our contributions:
 1. characterize the solution in closed-form over primitives, both cross-section & impulse response
 - full decomposition of equilibrium sectoral response to expected demand at different horizons across the network
 - the bullwhip effect arises if current demand shock implies greater expected demand in future
 - introduce exponential-decay & hump-shaped demand shocks; solve under complete and incomplete information
 2. show the bullwhip effect is significant across all supply chains within US production network
 - in the cross-section, upstream sectors have amplified volatility
 - impulse-response from downstream demand shocks: hump-shaped time profile & evidence of upstream learning