

Specialization, Complexity & Resilience in Supply Chains

Alessandro Ferrari

University of Zurich & CEPR

Lorenzo Pesaresi

University of Zurich

Introduction

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Research Question

How do specialization choices and network complexity shape resilience in supply chains? Should governments promote resilience? If so, how?

1. Static model of sourcing with **endogenous product specialization**

- Input design problem: specialization $\uparrow \implies$ price \uparrow but share of compatible buyers \downarrow
- Complex network: Multiple key inputs needed for final production

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2. **Dynamic model of supply chain formation**

- Introduce long-term relationships and (stochastic) disruptions of final producers
- **Welfare-relevant notion of resilience** = avg time it takes for a final producer to restore production following a disruption
- Decompose resilience into search efficiency, avg specialization, and complexity

3. Normative analysis

- **Novel network externality in specialization:** intermediate producers do not internalize the cascading effect of halting final production on complementary input producers
- **If network is complex enough,** equilibrium displays over- specialization
⇒ **resilience is inefficiently low**

4. Normative implications

- **Targeted transaction subsidy** decentralizes efficient allocation
- Planner would like intermediate producers to have more *skin in the* (final production) *game* ⇒ align private and social cost of specialization

- **Fragility in production networks**

Levine (2012), Elliott et al. (2022), Acemoglu and Tahbaz-Salehi (2023), Carvalho et al. (2023), Grossman et al. (2023a)

⇒ *Separately identify fragility of production ("robustness") from ability to recover quickly from shocks ("resilience").*

- **Product design**

Bar-Isaac et al. (2012, 2023), Menzio (2023), Albrecht et al. (2023)

⇒ *Study product design choices with complementary inputs.*

- **Optimal number of varieties**

Spence (1976), Dixit and Stiglitz (1977), Zhelobodko et al. (2012), Parenti et al. (2017), Dhingra and Morrow (2019), Grossman et al. (2023b)

⇒ *Endogenous specialization and price posting make appropriability and business-stealing effects perfectly offset each other.*

Static Model

- **Specialization**
 - Characteristic of intermediate products
 - Determines the degree of **compatibility with final good production functions**
 - \neq general quality
- **Complexity**
 - Characteristic of final good production function
 - Equal the **number of key inputs** needed to produce
- **Resilience**
 - Equilibrium sourcing capacity of final producers
 - Equal the **probability that a final producer sources all key inputs**

- Rep household, measure 1 of final producers, measure m of intermediate producers
- Ex ante identical final producers, heterogeneous intermediate producers
- Perfectly competitive market for consumption good, **frictional markets for intermediate goods**
- Consumption good (of unit quality) is the numeraire

- Rep household's problem:

$$\max_{C_i, \ell} \mathcal{U} = C + \psi \log(1 - \ell)$$

$$\text{s.t } C = w\ell + \bar{\Pi}$$

$$C = \int_0^1 Q_i C_i di$$

- Q_i and C_i are quality and quantity of the consumption good produced by final producer i
- $\bar{\Pi}$ are profits rebated to the rep household

Final producers

- Each final producer needs to source N **key inputs** to produce $Y_i = 1$ unit of output (consumption good):

$$Y_i = \mathbb{1}\{\min\{y_1, \dots, y_N\} > 0\}$$

- Output quality Q_i depends on the value of inputs sourced A_j :

$$Q_i = \sum_{j=1}^N A_j$$

- Each final producer makes profits:

$$\pi_i = \left(Q_i Y_i - \sum_{j=1}^N p_j \right) \mathbb{1}\{Y_i = 1\}$$

Sourcing frictions in the real world ...

“At most organizations [...], hunting for new suppliers is a daunting, manual process. On average, it takes about three months to complete a single supplier search, with a sourcing professional logging more than 40 hours of work—and yet able to consider only a few dozen suppliers from a total population of thousands.”

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⇒ Intermediate market with **search and compatibility frictions**

- **Search and compatibility frictions:** each final producer meets a finite number n^c of *compatible* intermediate producers

$$n^c \sim \text{Poisson}(\lambda\bar{\phi})$$

- $\lambda = \text{exogenous}$ expected number of sellers met $\implies (\text{search frictions})^{-1}$
- $\bar{\phi} = \text{endogenous}$ average compatibility probability $\implies (\text{compatibility frictions})^{-1}$

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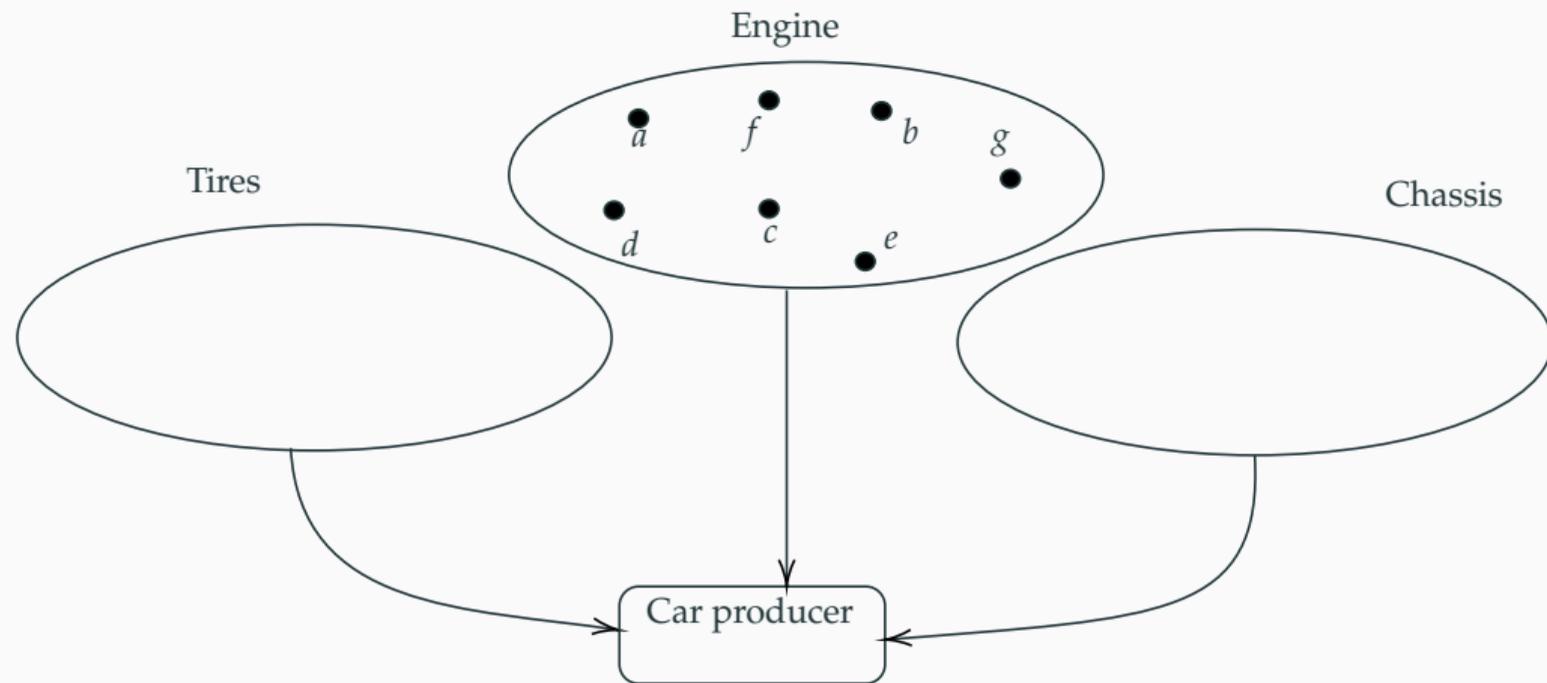
- **Finding probability** = probability that a final producer finds a compatible input:

$$f = 1 - \exp\{-\lambda\bar{\phi}\}$$

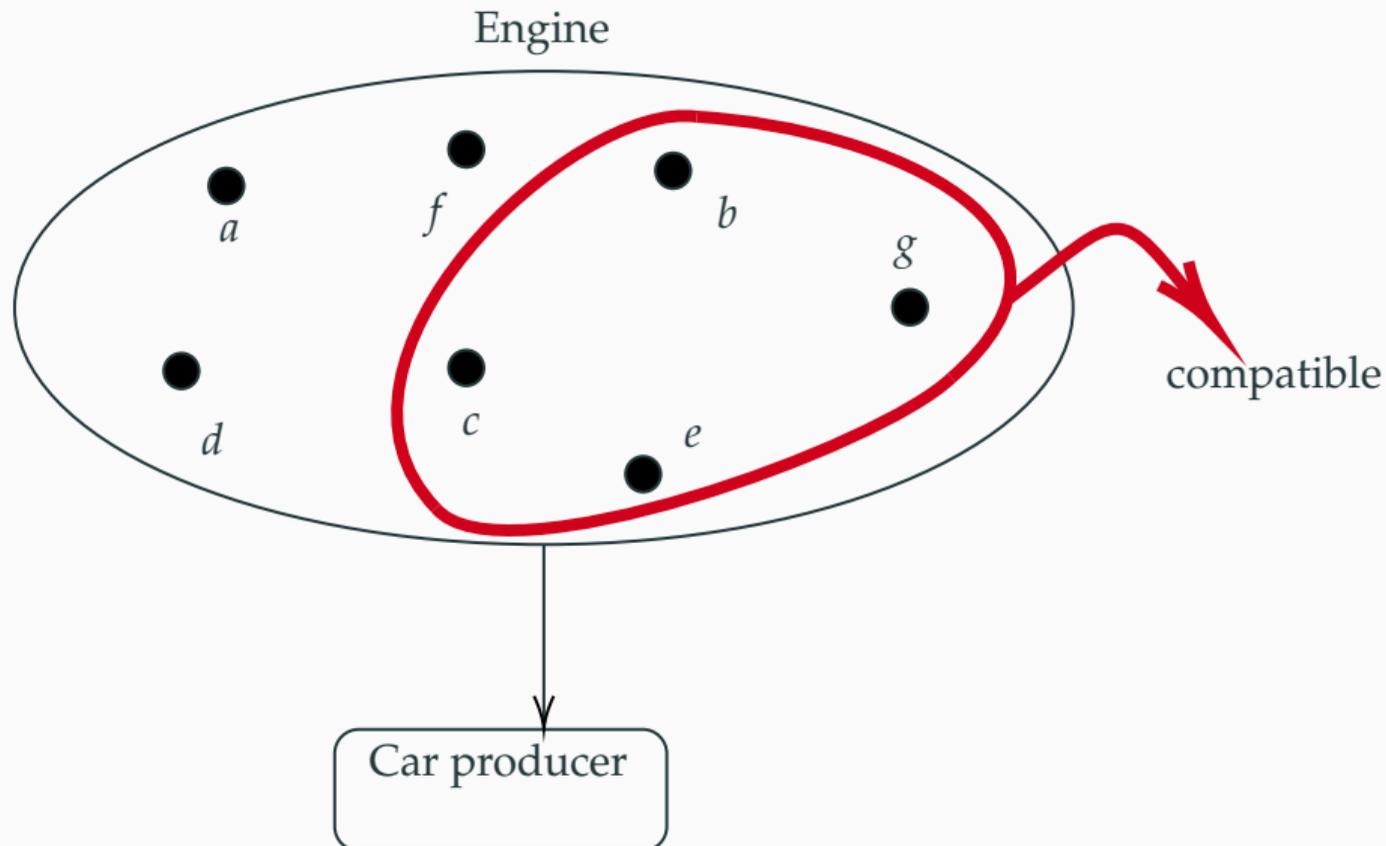
- Final producer **trades w/ best compatible seller** (= offering highest surplus), if any

Details

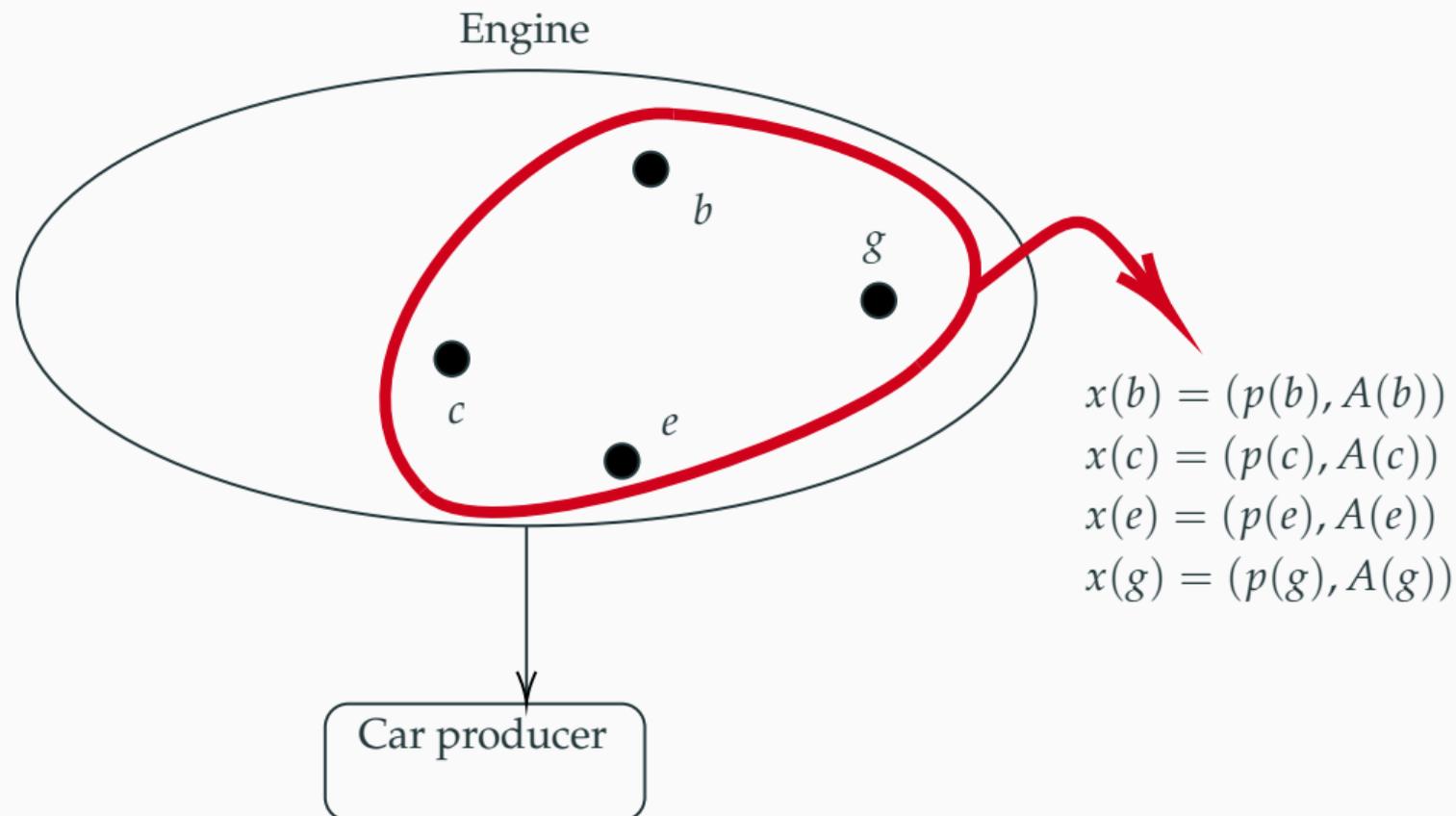
Market Structure

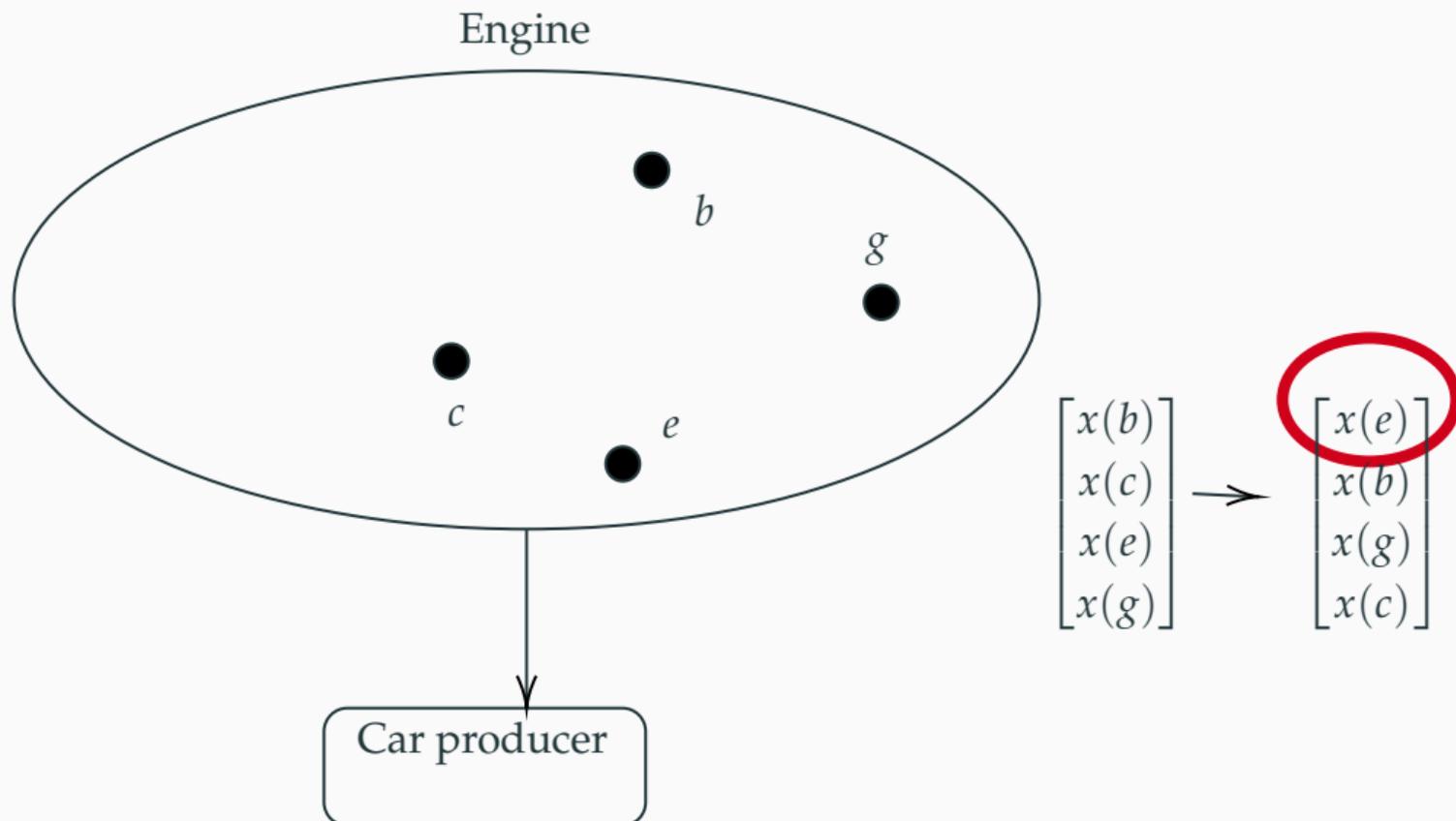


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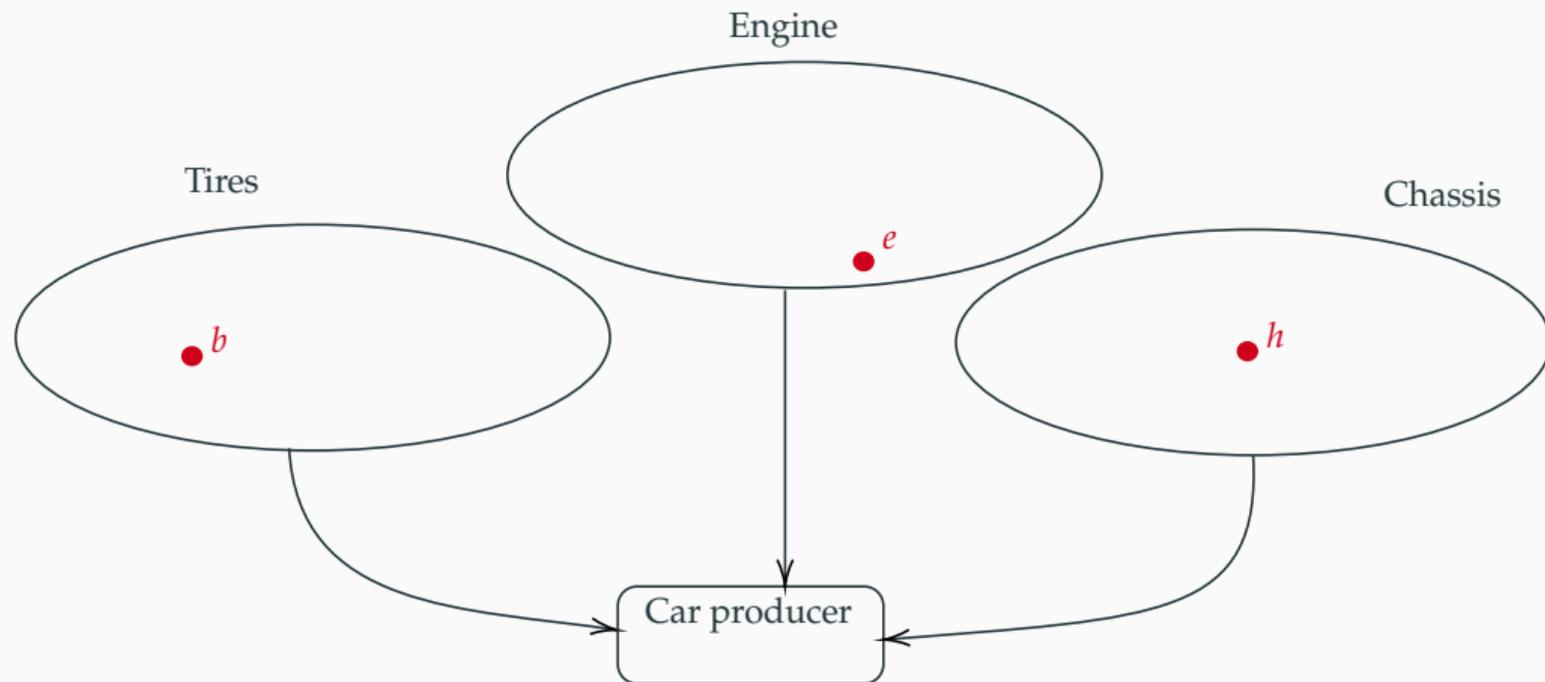


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Intermediate producers

- Intermediate producers differ in marginal cost $c(z), z \sim \Gamma(z)$
- **They choose specialization s** and offered surplus x (\iff post price p)
- Higher specialization increases final good quality $A'(s) > 0 \dots$
- \dots but reduces the share of compatible final producers $\phi'(s) < 0$

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- Higher specialization increases final good quality $A'(s) > 0 \dots$
- \dots but reduces the share of compatible final producers $\phi'(s) < 0$
- Key trade-off: $s \uparrow \implies \Pr(\text{match}) \downarrow \mathbb{E}[\text{Profits}|\text{match}] \uparrow$

Intermediate producer's profits

- Expected operating profits:

$$\Pi(s, x; z) = \underbrace{\theta\lambda}_{\substack{\text{exp \#} \\ \text{meetings}}} \underbrace{\mathcal{P}(s, x; N)}_{\substack{\text{trading prob.} \\ \text{meeting}}} \underbrace{[A(s) - c(z) - x]}_{\substack{\text{unit profit} \\ \equiv p - c(z)}}$$

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Derivation

Intermediate producer's problem – Offered Surplus

- Profit maximization problem:

$$V(s, x; z) = \max_{s, x} \Pi(s, x; z) - \underbrace{wq(s)}_{\text{specialization cost}}$$

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Lemma (Optimal Offered Surplus)

Optimal offered surplus equals the expected outside option of a final producer trading with the intermediate producer:

$$x^*(z) = \mathbb{E}_{\tilde{z}, \tilde{z} < z} [A(s(\tilde{z})) - c(\tilde{z})]$$

- **First-price auction w/ unknown # bidders** $\implies x^*(z)$ makes buyer indifferent b/w z and the second-best compatible seller in expectation Derivation Intuition

Intermediate producer's problem – Specialization

- Optimal specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(s(z)) + \frac{\phi'(s(z))}{\phi(s(z))} (A(s(z)) - c(z) - x(z)) \right] - wq'(s(z)) = 0$$

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1. $\theta \lambda \mathcal{P}(z; N) A'(s(z)) > 0$: marginal increase in profits conditional on trading

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2. $\theta \lambda \mathcal{P}(z; N) \frac{\phi'(s(z))}{\phi(s(z))} (A(s(z)) - c(z) - x(z)) < 0$: marginal reduction in trading probability

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3. $-wq'(s(z)) < 0$: marginal increase in specialization cost

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Lemma (Optimal Specialization)

If $\lambda \bar{\phi} < 1$, optimal specialization is increasing in search efficiency λ , and decreasing in complexity N .

- Labor market clearing:

$$1 - \frac{\psi}{w} = Nm\bar{q}$$

where $\bar{q} = \int q(s(z))d\Gamma(z)$

- Final good market clearing:

$$C = Y$$

where $Y = \underbrace{f^N}_{\text{prob. active}} \underbrace{N\mathbb{E}[A - c]/f}_{\text{expected surplus} \mid \text{active}}$

$$f^N = [1 - \exp\{-\lambda\bar{\phi}\}]^N$$

1. **Search efficiency** $\lambda \uparrow$ (ICT, AI, ...) increases resilience
2. **Avg product specialization** $\bar{s} \uparrow$ $\bar{\phi} \downarrow$ reduces resilience
3. **Production complexity** $N \uparrow$ reduces resilience

- Social planner problem

$$\max_{s_i(z)} \mathcal{W} = C + \psi \log(1 - \ell)$$

$$\text{s.t. } \ell = Nm\bar{q}$$

$$C = f^N \sum_{i=1}^N \mathbb{E}[A(s_i(z)) - c(z)]/f$$

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Proposition (Efficiency of Static Equilibrium)

The equilibrium is constrained efficient if and only if production is not complex, *i.e.* $N = 1$. If the production process is complex, *i.e.* $N > 1$, the equilibrium features over-specialization.

- Efficient specialization $\mathcal{S}(z)$:

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\implies **equilibrium over-specialization**

Network Externalities

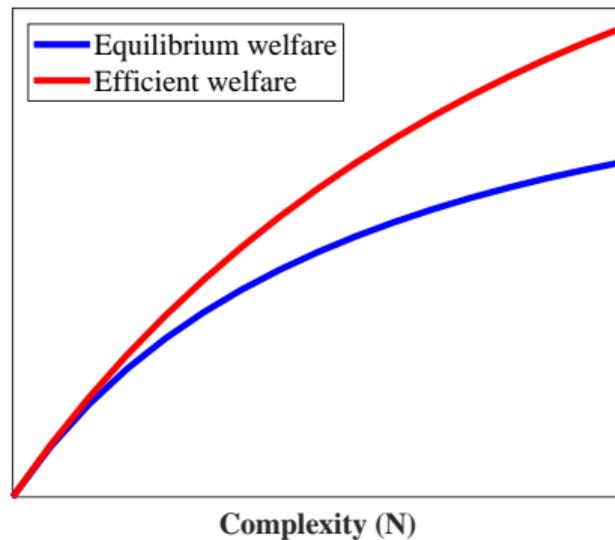
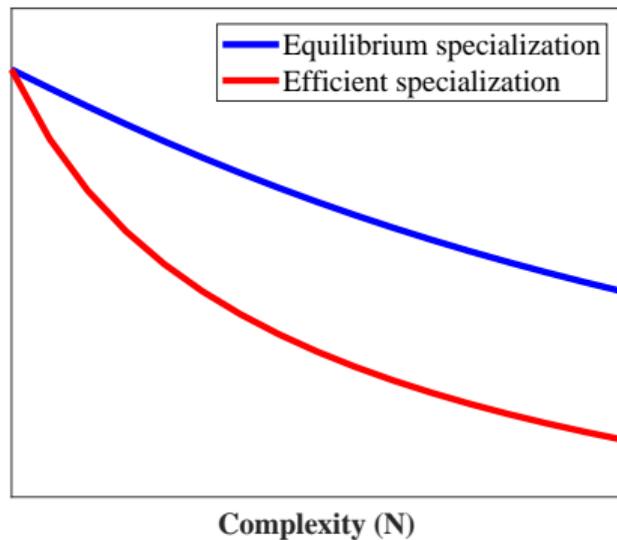
1. Compatibility frictions $\iff f < 1$
2. Endogenous specialization $\iff f'(s) < 0$
3. Complex production $\iff N > 1$

\implies Network externality in specialization

- Intermediate producers do not internalize the **cascading effect of halting final production on complementary input producers**
 - Example: higher specialization of engine makers hurts tire makers because cars are less likely to be produced

Example

Comparative Statics



- Network externality exacerbates as production becomes more complex

Remark (CES)

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The equilibrium allocation of our baseline model is the same as that of a general bargaining model where intermediate producers hold all bargaining power. The general bargaining model is also not efficient (network ext. + hold-up problem).

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Remark (Non-Contingent Contracts)

Economies with non-contingent contracts feature more equilibrium over-specialization than economies with contingent contracts.

Dynamic Model

- **So far:** Complex network \implies over-specialization
- **Now:** Over-specialization \implies under-resilience
- **How:** Extend static model to a dynamic setting with long-term relationships
 - Final producers face a disruption each period with probability δ
 - **Resilience** \equiv avg time it takes for a final producer to restore production following a disruption
 - **Robustness** $= 1/\delta$

Intermediate producer's problem

- Profit maximization problem:

$$\max_{x, \{s_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} [\mathcal{D}_t(\mathbf{s}_{1,t}, x) (a(s_t) - x - c(z)) - w_t q(s_t)]$$

$$\text{s.t. } \mathcal{D}_t(\mathbf{s}_{1,t}, x) = (1 - \delta)\mathcal{D}_{t-1}(\mathbf{s}_{1,t-1}, x) + \theta_t \lambda \mathcal{P}(s_t, x; N),$$

$$\mathcal{D}_0 = 0$$

Remark (Customization)

The level of specialization within a match increases over time.

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- Intuition: as intermediate producers “age” they build a customer base
 - ⇒ they are less subject to search frictions
 - ⇒ they optimally increase specialization.

Resilience – Output effects

$$Y = \underbrace{\mu(f, N)}_{\text{market size}} \underbrace{f^N}_{\text{resilience}} \underbrace{\frac{1}{\delta}}_{\text{robustness}} \underbrace{N\mathbb{E}[A - c]/f}_{\text{expected surplus | active}}$$

- **Separately identify** output effects of **resilience** (recovery from disruptions) and **robustness** (avoiding disruptions)
- Market size \equiv stationary share of searching final producers

- Marginal welfare effect of equilibrium specialization:

$$\begin{aligned}
 \frac{\partial \mathcal{W}}{\partial s(z)} \Big|_{\mathcal{S}(z)=s^*(z)} &\propto \underbrace{\mathbb{E}_{\tilde{z}, \tilde{z} < z} [A(s^*(\tilde{z})) - c(\tilde{z})]}_{\text{business-stealing externality}} \underbrace{- x^*(z)}_{\text{appropriability externality}} && = 0 \\
 &\underbrace{- (N-1) \exp\{-\lambda \hat{\phi}(z, z)\} \mathbb{E}[A - c] / f}_{\text{network externality}} && < 0 \\
 &\underbrace{+ \frac{\beta(1-\delta)}{1+\beta(1-\delta)} f^N N e^{-\lambda \hat{\phi}(z, z)} \hat{\mathbb{E}}[a(s^*(\tilde{z})) - c(\tilde{z})]}_{\text{search externality}} && > 0
 \end{aligned}$$

- If network is complex enough, equilibrium over-specialization \implies resilience is inefficiently low**

Proposition (Efficiency and Complexity)

If search efficiency is low enough, the equilibrium allocation features more over-specialization and under-resilience as complexity increases.

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As the frequency of disruption increases, the equilibrium allocation features more over-specialization and under-resilience.

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Proposition (Efficiency and Search Frictions)

If the elasticity of the average compatibility probability to search efficiency exceeds one, the equilibrium allocation features more over-specialization and under-resilience as search frictions decline.

Conclusions

- New model of **supply chain formation with endogenous compatibility frictions**
 - Heterogeneous intermediate producers solve input design problem: specialization
↑ \implies price ↑ but share of compatible buyers ↓
 - Complex network: Multiple key inputs needed for final production
- **Welfare-relevant notion of resilience** = avg time it takes for a final producer to restore production following a disruption
- **If network is complex enough**, equilibrium displays over- specialization \implies **resilience is inefficiently low**
 - **Network externality**: intermediate producers do not internalize the cascading effect of halting final production on complementary input producers

- **Dynamic model with link-specific destruction**
- **Optimal Policy**
 - Targeted transaction subsidy decentralizes efficient allocation
 - Planner would like intermediate producers to have more *skin in the* (final production) *game* \implies align private and social cost of specialization
- **Extensions**
 - Endogenize complexity (choose N) and robustness (invest to reduce δ)

Appendix

- **Search frictions:** each final producer meets a finite number n of intermediate producers

$$n \sim \text{Poisson}(\lambda)$$

where $n \in \mathbb{N}$ and $\mathbb{E}[n] = \lambda$

- **Compatibility frictions:** each intermediate producer is compatible with final producer's technology with probability

$$\phi \sim \mathcal{F}$$

where the distribution \mathcal{F} is endogenous and $\mathbb{E}[\phi] = \bar{\phi}$

- Conditional trading probability:

$$\mathcal{P}(s, x; N) \equiv \underbrace{\phi(s)}_{\text{prob. compatible}} \underbrace{\exp \{-\lambda \bar{\phi} [1 - G(x)]\}}_{\text{prob. best among compatible contacted}} \underbrace{f^{N-1}}_{\text{prob. other key inputs sourced}}$$

- $x \equiv A(s) - p$ is the **offered surplus** (profits granted to final producer)
- $G(x)$ denotes the **distribution of offered surplus**

- First-order condition (DE)

$$x'(z) = \lambda \phi(s(z)) \gamma(z) [A(s(z)) - c(z) - x(z)]$$

$$x(\underline{z}) = 0$$

- Optimal offered surplus:

$$\begin{aligned} x^*(z) &= \int_{\underline{z}}^z [A(s(\tilde{z})) - c(\tilde{z})] \gamma^*(\tilde{z}, z) d\tilde{z} \\ &= \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} \leq z} [A(s(\tilde{z})) - c(\tilde{z})] \end{aligned}$$

where $\gamma^*(\tilde{z}, z) \equiv \lambda \phi(s(z)) \exp\{-\lambda \hat{\phi}(\tilde{z}, z)\} \gamma(\tilde{z})$ is the final producer's **productivity density of outside option** when trading w/ intermediate of prod. z

Intermediate producer's problem – Offered Surplus

Lemma (Optimal Offered Surplus)

Optimal offered surplus equals the expected outside option of a final producer trading with the intermediate producer:

$$x^*(z) = \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} \leq z} [A(s(\tilde{z})) - c(\tilde{z})]$$

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Consider a seller with productivity z

- loses against anybody with productivity $z' > z$

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- buyer's outside option is given by the highest possible surplus by the best alternative

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$$x^*(z) = \underbrace{[A(s(\tilde{z})) - c(\tilde{z})]}_{\text{highest possible offered surplus by } \tilde{z}}$$

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$$x^*(z) = \underbrace{[A(s(\tilde{z})) - c(\tilde{z})]}_{\text{highest possible offered surplus by } \tilde{z}} \underbrace{\lambda \phi(s(\tilde{z})) \exp\{-\lambda \hat{\phi}(\tilde{z}, z)\}}_{\text{Pr } \tilde{z} \text{ is best alternative} | \tilde{z} < z}$$

Lemma (Optimal Offered Surplus)

Optimal offered surplus equals the expected outside option of a final producer trading with the intermediate producer:

$$x^*(z) = \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} \leq z} [A(s(\tilde{z})) - c(\tilde{z})]$$

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- buyer's outside option is given by the highest possible surplus by the best alternative

$$x^*(z) = \int_{\underline{z}}^z \underbrace{[A(s(\tilde{z})) - c(\tilde{z})]}_{\text{highest possible offered surplus by } \tilde{z}} \underbrace{\lambda \phi(s(\tilde{z})) \exp\{-\lambda \hat{\phi}(\tilde{z}, z)\}}_{\text{Pr } \tilde{z} \text{ is best alternative} | \tilde{z} < z} \underbrace{\gamma(\tilde{z}) d\tilde{z}}_{\text{weighted across all possible } \tilde{z}}$$

Intermediate producer's problem – Specialization

- Optimal specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(s(z)) + \frac{\phi'(s(z))}{\phi(s(z))} (A(s(z)) - c(z) - x(z)) \right] - wq'(s(z)) = 0$$

Lemma (Optimal Specialization)

Optimal specialization is decreasing in complexity N and increasing in search efficiency λ (if $\lambda \bar{\phi} < 1$).

- In a given product line, product specialization gives rise to two externalities:
 1. $x^*(z) > 0 \implies$ **appropriability externality**
 - Intermediate producers bear specialization cost but do not appropriate its whole return
 2. $\mathcal{P}(\hat{z}; N) \propto \exp \left\{ -\lambda \int_{\hat{z}}^{\bar{z}} \phi(s^*(\tilde{z})) \gamma(\tilde{z}) d\tilde{z} \right\} \implies$ **business-stealing externality**
 - Less productive intermediate producers ($\hat{z} \leq z$) increase their trading prob. if intermediate z specializes more ($\phi(s^*(z)) \downarrow$)
- Price posting + compatibility frictions \implies **the two externalities cancel out**

- Price posting protocol \iff **first-price sealed-bid auction among unknown number of compatible intermediate producers** contacted by a final producer
- Optimal offered surplus = **expected outside option of a final producer** when intermediate z is the best compatible seller contacted:

$$x^*(z) = \Pr(m(z) = 0) \cdot 0 + \Pr(m(z) = 1) \cdot \mathbb{E} \left[\max \{A(s(\tilde{z})) - c(\tilde{z})\} \mid \max\{\tilde{z}\} \leq z, m(z) = 1 \right]$$

where $m(z) = \mathbb{1}\{\text{final producer contacts at least one firm w/ productivity } \leq z\}$

- **Innovation** wrt std auction theory: **endogenous distribution of bidders** pinned down by search and compatibility frictions

Efficiency

- Efficient specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(\mathcal{S}(z)) + \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} \left([A(\mathcal{S}(z)) - c(z)] - \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} < z} [A(\mathcal{S}(\tilde{z})) - c(\tilde{z})] \right. \right. \\ \left. \left. + (N - 1) \exp\{-\lambda \hat{\phi}(\underline{z}, z)\} \mathbb{E}[A - c] / f \right) \right] - \frac{\psi}{1 - Nm\bar{q}} q'(\mathcal{S}(z)) = 0$$

1. $\theta \lambda \mathcal{P}(z; N) A'(\mathcal{S}(z)) > 0$: marginal increase in surplus conditional on trading

Efficiency

- Efficient specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(\mathcal{S}(z)) + \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} \left([A(\mathcal{S}(z)) - c(z)] - \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} < z} [A(\mathcal{S}(\tilde{z})) - c(\tilde{z})] \right. \right. \\ \left. \left. + (N-1) \exp\{-\lambda \hat{\phi}(\underline{z}, z)\} \mathbb{E}[A - c] / f \right) \right] - \frac{\psi}{1 - Nm\bar{q}} q'(\mathcal{S}(z)) = 0$$

2. $\theta \lambda \mathcal{P}(z; N) \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} [A(\mathcal{S}(z)) - c(z)] < 0$: marginal reduction in trading probability

Efficiency

- Efficient specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(\mathcal{S}(z)) + \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} \left([A(\mathcal{S}(z)) - c(z)] - \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} < z} [A(\mathcal{S}(\tilde{z})) - c(\tilde{z})] \right. \right. \\ \left. \left. + (N-1) \exp\{-\lambda \hat{\phi}(\underline{z}, z)\} \mathbb{E}[A - c] / f \right) \right] - \frac{\psi}{1 - Nm\bar{q}} q'(\mathcal{S}(z)) = 0$$

3. $-\frac{\psi}{1 - Nm\bar{q}} q'(\mathcal{S}(z)) < 0$: marginal increase in specialization cost

Efficiency

- Efficient specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(\mathcal{S}(z)) + \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} \left([A(\mathcal{S}(z)) - c(z)] - \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} < z} [A(\mathcal{S}(\tilde{z})) - c(\tilde{z})] \right) \right. \\ \left. + (N - 1) \exp\{-\lambda \hat{\phi}(\underline{z}, z)\} \mathbb{E}[A - c] / f \right) \Big] - \frac{\psi}{1 - Nm\bar{q}} q'(\mathcal{S}(z)) = 0$$

4. $-\theta \lambda \mathcal{P}(z; N) \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} < z} [A(\mathcal{S}(\tilde{z})) - c(\tilde{z})] > 0$: marginal increase in trading prob. of lower-productivity intermediates

- Efficient specialization (implicit):

$$\theta \lambda \mathcal{P}(z; N) \left[A'(\mathcal{S}(z)) + \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} \left([A(\mathcal{S}(z)) - c(z)] - \mathbb{E}_{\max\{\tilde{z}\} | \max\{\tilde{z}\} < z} [A(\mathcal{S}(\tilde{z})) - c(\tilde{z})] \right. \right. \\ \left. \left. + (N-1) \exp\{-\lambda \hat{\phi}(\underline{z}, z)\} \mathbb{E}[A - c]/f \right) \right] - \frac{\psi}{1 - Nm\bar{q}} q'(\mathcal{S}(z)) = 0$$

5. $\theta \lambda \mathcal{P}(z; N) \frac{\phi'(\mathcal{S}(z))}{\phi(\mathcal{S}(z))} (N-1) \exp\{-\lambda \bar{\phi}\} \mathbb{E}[A - c]/f < 0$: marginal reduction in trading prob. of other inputs' providers

Example: specialization of engine makers affecting tire makers because cars are less likely to be produced

- $N = 2$, final producers only meet one intermediate at a time ($x = 0$)
- Intermediate 1 profits: $\Pi_1 = f(s_1)f(s_2)A(s_1)$
- Social welfare: $\mathcal{W} = f(s_1)f(s_2)[A(s_1) + A(s_2)]$
- Equilibrium specialization:

$$\underbrace{f(s_1^*)A'(s_1^*)}_{\text{private MB}} = \underbrace{(-f'(s_1^*))A(s_1^*)}_{\text{private MC}}$$

- Efficient specialization:

$$\underbrace{f(\mathcal{S}_1)A'(\mathcal{S}_1)}_{\text{social MB}} = \underbrace{(-f'(\mathcal{S}_1))[A(\mathcal{S}_1) + A(\mathcal{S}_2)]}_{\text{social MC}}$$

Decentralization

Proposition (Decentralization)

The efficient allocation can be decentralized via a **targeted transaction tax schedule** $\tau(z)$ such that the price of the transactions is given by $\rho(z) = p(z) + \tau(z)$, where:

$$\tau(z) = \exp\{-\lambda\hat{\phi}(z, z)\}T$$

$$T \equiv \left(\mu(f; N)N - 1\right)\mathbb{E}[A - c]/f \lesseqgtr 0 \text{ (> 0 if over-specialized eq'm)}$$

\implies price of lower-productivity firms is more distorted