

# **A Sufficient Statistics Approach for Endogenous Production Networks: Theory and Evidence from Ukraine's War**

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Vasily Korovkin, CERGE-EI

Alexey Makarin, MIT Sloan

Yuhei Miyauchi, Boston University

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## Motivation: Disruption & Reorganization of Production Networks

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- Countries and regions are interconnected through production networks
- These networks propagate localized shocks to surrounding countries and regions
  - Transient shocks: e.g., natural disasters, trade shocks
  - Intense & prolonged shocks: e.g., war & conflict
- Firms endogenously reorganize production networks as a response to shocks, e.g.,
  - Mitigation through substitution
  - Amplification through cascading failures
  - Changes in local factor prices and economic activity
- Need framework to capture many different channels simultaneously

## This Paper: Theory of Suff. Stat. and Evidence from 2014 Ukraine Conflict

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- Theory: welfare changes in many multi-location endogenous network models follow:

$$\widehat{W}_i = \widehat{\Lambda}_{ii}^{-\frac{1-\beta}{\beta} \frac{1}{\varepsilon}} \widehat{M}_{ii}^{\frac{1-\beta}{\beta} \eta}$$

- $\beta$ : labor share,  $\varepsilon$ : input substitution (trade) elasticity
- $\widehat{\Lambda}_{ii}$ : change in within-region sourcing share (Arkolakis, Costinot, Rodriguez-Clare; ACR '12)
- $\widehat{M}_{ii}$ : change in measures of suppliers per buyer within a region;  $\eta$ : “supplier link elasticity”

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- Reduced-form evidence using firm-to-firm railroad shipments in 2012–16 within Ukraine
    - Disruption of firm sales depending on supplier & buyer conflict exposure
    - Increase of supplier & buyer linkages strictly outside conflict areas

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- Reduced-form evidence using firm-to-firm railroad shipments in 2012–16 within Ukraine
    - Disruption of firm sales depending on supplier & buyer conflict exposure
    - Increase of supplier & buyer linkages strictly outside conflict areas
  - Sufficient-statistics results:
    - Estimate supplier link elasticity ( $\eta$ ) using variation in exposures to conflict
    - $\downarrow$  17% for an average region (relative to no conflict exposure regions)
    - Overestimation without  $\widehat{M}_{ii}^{\frac{1-\beta}{\beta} \eta}$  (31% instead of 17%)

# Contributions to the Literature

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- Economic Costs of Conflict: Guidolin & La Ferrara '07; Hjort '14; Amodio & Di Maio '18; Rohner & Thoenig '21; Ksoll, Macchiavello, Morjaria '22; Couttenier, Monnet, Piemontese '22; Korovkin & Makarin '23

⇒ Show large propagation of localized conflict through disruption & reorganization of production networks

- Endogenous Production Networks:

- Relationship-specific fixed cost: Bernard, Moxnes, Ulltveit-Moe '18; Lim '18; Huneus '18; Bernard, Moxnes, Saito '19; Zou '20; Bernard, Dhyne, Magerman, Manova, Moxnes '22; Dhyne, Kikkawa, Kong, Mogstad, Tintelnot '22
- Optimal supplier choice: Oberfield '18; Boehm & Oberfield '20; Acemoglu & Azar '20; Taschereau-Dumouchel '20; Eaton, Kortum, Kramarz '22; Antras & de Gortari '20; Miyauchi '23; Panigrahi '21; Lenoir, Martin, Mejean '22
- Endogenous search intensity: Demir, Fieler, Xu, Yang '21; Arkolakis, Huneus, Miyauchi '23
- Sufficient Statistics in Trade and Production Networks: Arkolakis, Costinot, Rodriguez-Clare '12; Blaum, Lelarge, Peters '18; Donaldson '18; Baqaee, Burstein, Duperez, Farhi '23

⇒ Develop common (ex-post) welfare sufficient statistics and use it to study causal effects of conflicts

# Outline

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Theory

Background and Data of Ukrainian Conflict

Reduced-Form Evidence

Sufficient Statistics Analysis

Conclusion

**Theory**



## Model Set-up

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- “Locations”  $i, u, d \in \mathcal{L}$
- Intermediate goods produced by “firms”; final goods produced by “retailers”
- $\Omega_i$ : set of firms in location  $i$ 
  - Use local labor and intermediate inputs for production
- Intermediate goods are traded among connected firms across different locations
  - $\mathcal{S}_{ui}(\omega) \subset \Omega_u$ : set of suppliers in location  $u$  that firm  $\omega \in \Omega_i$  in  $i$  is connected to
  - Endogenous, but do not model how it is determined

# Equilibrium

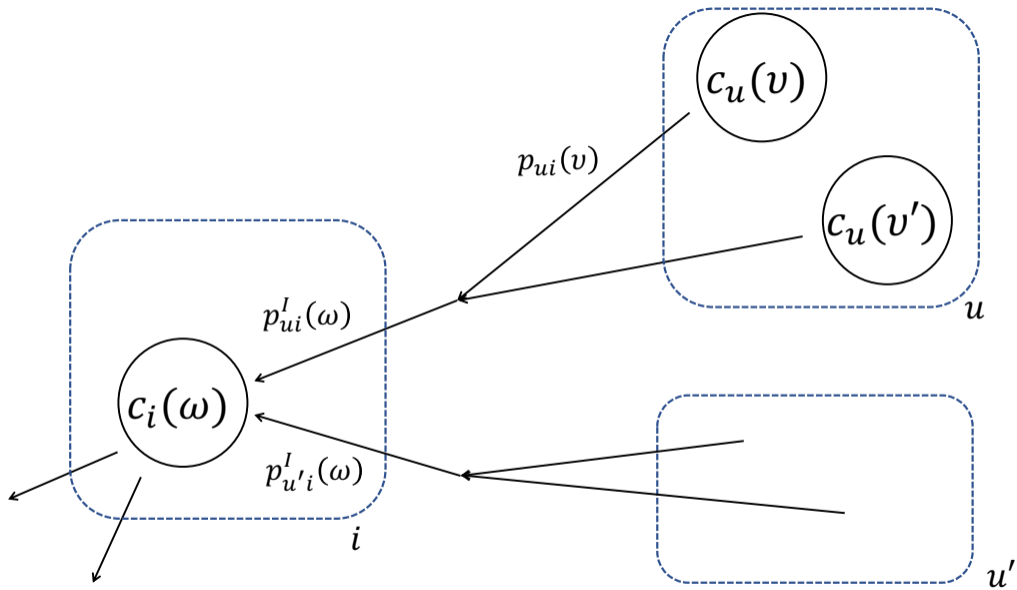
- Unit cost of firm  $\omega$  in location  $i$ :

$$c_i(\omega) = \frac{1}{z_i(\omega)} w_i^\beta \left( \sum_{u \in \mathcal{L}} (p_{ui}^l(\omega))^{-\varepsilon} \right)^{\frac{1-\beta}{-\varepsilon}}, \quad p_{ui}^l(\omega) = f_{ui,\omega} \left( \{p_{ui}(v)\}_{v \in \mathcal{S}_{ui}(\omega)} \right)$$

- $z_i(\omega)$ : productivity;  $w_i$ : wage
  - $\beta$ : labor share;  $\varepsilon$ : input substitution (trade) elasticity
- $p_{ui}(v)$ : unit price of supplier  $v$  to sell firms in location  $i$

$$p_{ui}(v) = c_u(v) \underbrace{\tau_{ui}(v)}_{\text{iceberg trade cost}} \underbrace{\rho_{ui}(v)}_{\text{(exogenous) markups}}$$

- Final goods produced using local intermediate inputs:  $P_i^F = h_i \left( \{c_i(v)\}_{v \in \Omega_i} \right)$



## Assumption (1. Aggregation)

Price index of input bundle can be expressed as:

$$p_{ui}^I(\omega) = P_{ui}^I g_i(\omega),$$

where  $g_i(\omega)$  only depends on the exogenous variable and parameters.

- Implies  $c_i(\omega) = C_i g_i^C(\omega)$ ,  $p_{ui}(\omega) = P_{ui} g_{ui}^P(\omega)$
- Only need to keep track of  $\{P_{ui}^I, P_{ui}, C_i\}$
- High-level assumption satisfied in many parametric production network models

multiple firm types

multiple sectors

## Lemma

Under Assumption 1, the changes in real wages from external shocks are given by

$$\frac{\widehat{w}_i}{\widehat{P}_i^F} = \left( \underbrace{\widehat{\Lambda}_{ij}}_{\text{within-region source share}} \right)^{-\frac{1-\beta}{\beta} \frac{1}{\varepsilon}} \left( \underbrace{\widehat{P}_{ij}^I / \widehat{C}_i}_{\text{input bundle price / average supplier's cost}} \right)^{-\frac{1-\beta}{\beta}}$$

- Proof: Shephard's Lemma + CES input demand + ( $\widehat{P}_i^F = \widehat{C}_i$ )

$$\left( \widehat{C}_i \right)^{-\varepsilon} = \widehat{w}_i^{-\beta\varepsilon} \left( \underbrace{\left( \widehat{C}_i \right)^{-\varepsilon}}_{\text{"value of supplier bundles" within a region}} \underbrace{\left( \frac{\widehat{P}_{ij}^I}{\widehat{C}_i} \right)^{-\varepsilon}}_{\text{terms of trade}} \underbrace{\widehat{\Lambda}_{ij}^{-1}}_{\text{terms of trade}} \right)^{1-\beta}$$

- Without changes in production networks,  $\widehat{P}_{ij}^I / \widehat{C}_i = 1$  (ACR '12)

- $\hat{P}_{ii}^I / \hat{C}_i$  hard to observe / estimate
- In many existing parametric production network models (Assumption 2),

$$\hat{P}_{ii}^I / \hat{C}_i = \hat{M}_{ii}^{-\eta},$$

- $\hat{M}_{ii}$ : a common change in the measure of suppliers within a region ( $\hat{m}_{ii}(\omega) = \hat{M}_{ii}$ )
- $\eta$ : **supplier link elasticity** (elas' of marginal cost w.r.t. measure of supplier linkages)

## Proposition

*Under Assumption 1 and 2,*

$$\frac{\widehat{w}_i}{\widehat{P}_i^F} = \widehat{\Lambda}_{ii}^{-\frac{1-\beta}{\beta} \frac{1}{\varepsilon}} \widehat{M}_{ii}^{\frac{1-\beta}{\beta} \eta}$$

- Endogenous search intensity (e.g., Arkolakis, Huneus, Miyauchi '23)
  - CES production function
  - $\varepsilon = \sigma - 1$ ,  $\eta = \frac{1}{\sigma-1} (= 1/\varepsilon)$
- Relationship-specific fixed cost (e.g., Bernard, Moxnes, Ulltveit-Moe '18)
  - CES + selection with Pareto productivity dispersion  $\theta$
  - $\varepsilon = \sigma - 1$ ,  $\eta = \frac{1}{\sigma-1} - \frac{1}{\theta} (< 1/\varepsilon)$
- Optimal supplier choice (e.g., Eaton, Kortum, Kramarz '22)
  - Homogeneous inputs, Pareto productivity dispersion  $\theta$ , biased matching  $\gamma$
  - $\varepsilon = \theta(1 - \gamma)$ ,  $\eta = \frac{1}{\theta(1-\gamma)} (= 1/\varepsilon)$
- Other examples
  - Separate variety gains from substitution (Benassy '98; Acemoglu, Antras, Helpman '07)
  - Entry into input market (Antras, Fort, Tintelnot '17)
  - Diversifying idiosyncratic supplier risks (Anderson, de Palma, Thisse '92)
  - Network formation under adjustment frictions (Lim '18, Huneus '19)

## Discussion and Extensions

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- Firm profit
  - Wage  $\propto$  total firm profit under trade balance & constant markup  $\rho_{id}(\omega)$   
(Assumption 1 & 2 of ACR)
- Firm entry
  - Additional effect arises only from the change in final prices  $N_i \uparrow \Rightarrow \hat{P}_i^F / \hat{C}_i \downarrow$
  - Same argument for labor shocks and mobility
- Final goods trade [detail](#)
- Multiple sector (i.e., Caliendo & Parro '15) [detail](#)
- Multiple firm types [detail](#)
- Nonparametric production function [detail](#)
- Alternative sufficient statistics using Domar weights [detail](#)



## **Background and Data of Ukrainian Conflict**

## Background: 2014 Ukraine War

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- In February 2014, right after Ukrainian revolution, Russia annexed Crimea and started supporting Donbas separatists
- Intense but localized conflict in Donbas regions (until February 2022)
- Donbas (and Crimea) were economic centers of Ukraine before the war
  - Donbas: extractive industry (coal), metallurgy, manufacturing
  - Crimea: agriculture, tourism, some industry
  - Jointly covered 17.5% of Ukraine's 2013 GDP
- Sudden and large drop in production in Donbas (and Crimea) regions event study
  - Production disruption, disconnected from transportation networks

Q. How did the conflict affect economic activity & welfare outside direct conflict areas?

# Background: 2014 Ukraine War

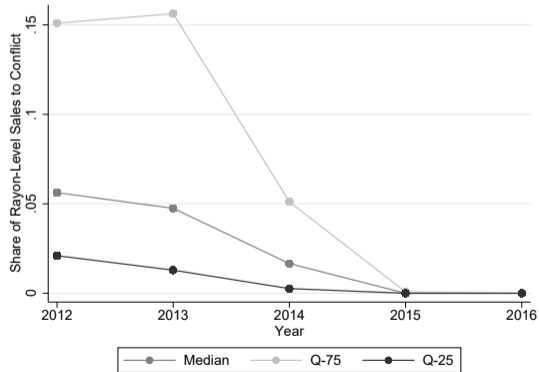
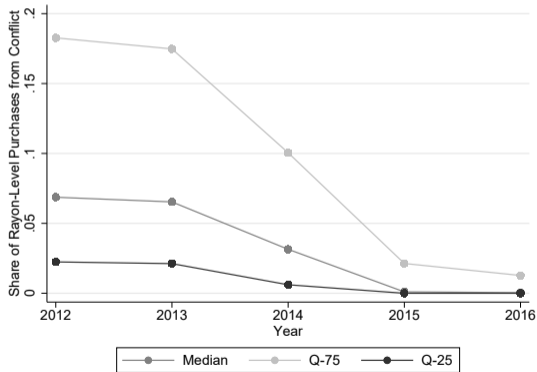


- Universe of firm-to-firm railroad shipments in Ukraine, 2012–2016 [map](#)
  - >41 mln transactions between >7 k firms
  - Sender and receiver firm IDs, dates, weights (kg), freight charges, product codes, origin & destination station codes
  - 80% of all freight in ton-km within Ukraine is through railways (Ukr Stat, 2018)
- Accounting data for Ukrainian firms, 2010–2017
  - Sources: Spark-Interfax database; ORBIS

## **Reduced-Form Evidence**

# Sudden and Large Drop of Trade from & to Conflict Areas

- Weighted fraction of suppliers (left) and buyers (right) from/to conflict areas
- Samples: rayons (regions) outside direct conflict areas ( $\approx 400$ )



## Firm-Level Impacts of Conflict Exposure

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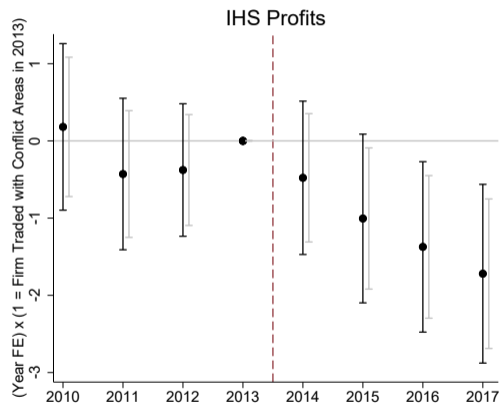
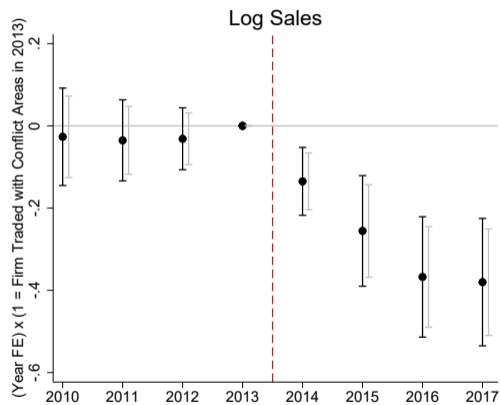
Difference-in-differences specification:

$$Y_{ft} = \alpha_f + \delta_t + \beta_t \times \text{ConflictTradeExposure}_{f,2013} + \varepsilon_{ft}$$

- $Y_{ft}$  — sales of firm  $f$  (in non-conflict area of Ukraine) at year  $t$
- $\text{ConflictTradeExposure}_{f,2013}$  — whether firm  $f$  traded with Crimea, DPR, or LPR before the start of the conflict

**Identifying assumption:** Absent the conflict, firms with varying pre-war ties to Donbas & Crimea would have evolved along parallel trends

# Firm-Level Impacts of Conflict Exposure: Results





## Firm-Level Impacts of Conflict Exposure: By Supplier and Buyer Exposures

	(1)	(2)	(3)	(4)
	Log Sales	IHS Profits	Log Sales	IHS Profits
Post x High buyer conflict exposure, 2013	-0.196*** (0.074)	-0.942* (0.542)		
Post x High seller conflict exposure, 2013	-0.216*** (0.074)	0.192 (0.519)		
Post x Buyer conflict exposure, 2013			-0.338* (0.187)	-0.697 (1.733)
Post x Seller conflict exposure, 2013			-0.301*** (0.101)	-0.017 (0.727)
Firm FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Mean	17.079	6.765	17.079	6.765
SD	2.407	13.124	2.407	13.124
R <sup>2</sup>	0.83	0.48	0.83	0.48
Observations	25,491	24,751	25,491	24,751
Number of Firms	3,713	3,677	3,713	3,677

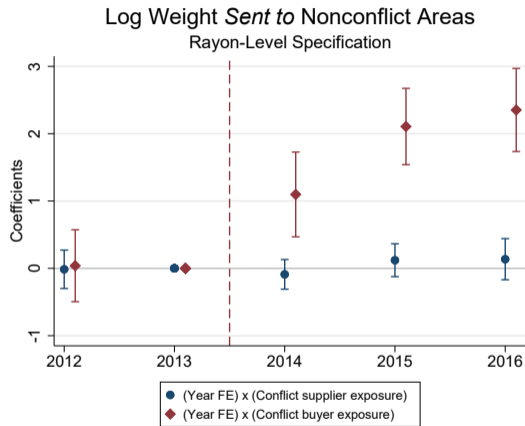
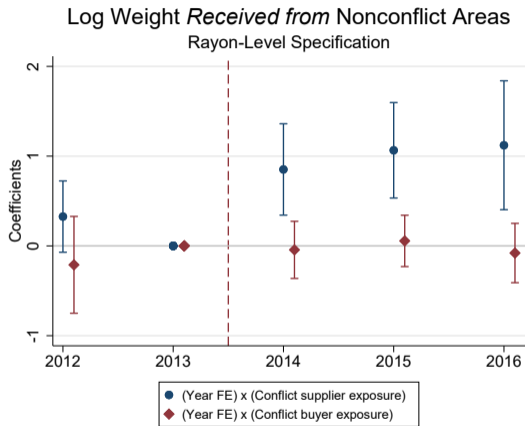
## Impacts of Conflict Exposure on Trade and Linkages in Nonconflict Areas

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$$Y_{it} = \gamma \times Post_i \times SupplierExposure_i + \beta \times Post_t \times BuyerExposure_i + \alpha_i + \delta_t + \varepsilon_{it}$$

- $i$ : rayons (excluding conflict areas)
- $SupplierExposure_i$ : Weighted fraction of shipment *from* conflict areas in 2013 in  $i$
- $BuyerExposure_i$ : Weighted fraction of shipment *to* conflict areas in 2013 in  $i$
- $Y_{it}$ : Sales or purchases (weight) of rayon  $i$  to or from **nonconflict areas**

# Impacts of Conflict Exposure on Trade and Linkages in Nonconflict Areas



- Left: Supplier exposure  $\uparrow$  purchases in non-conflict areas: substitution
- Right: Buyer exposure  $\uparrow$  sales in non-conflict areas: capacity constraint or GE effect

# **Sufficient Statistics Analysis**

# Quantify Welfare Losses from Propagation Effects outside Conflict Areas

$$\frac{\widehat{w}_i}{P_i^F} = \widehat{\Lambda}_{ij}^{-\frac{1-\beta}{\beta} \frac{1}{\varepsilon}} \widehat{M}_{ij}^{\frac{1-\beta}{\beta} \eta}$$

1. Measure time changes in  $\Lambda_{ij}$  and  $M_{ij}$  before and after conflict
  - Convert shipment weight to value using product code (in progress)
  - Project on empirical gravity equations for data sparseness (Dingel & Tintelnot '21)
2. Calibrate / estimate  $\{\beta, \varepsilon, \eta\}$ 
  - Labor share  $\beta = 0.2$ ; input substitution  $\varepsilon = 4$  (Oberfield & Raval '21)
  - Supplier link elasticity  $\eta = 1.23/\varepsilon$ : estimate using conflict exposure variations detail
3. Same diff-in-diff design with the sufficient statistics as outcome variables

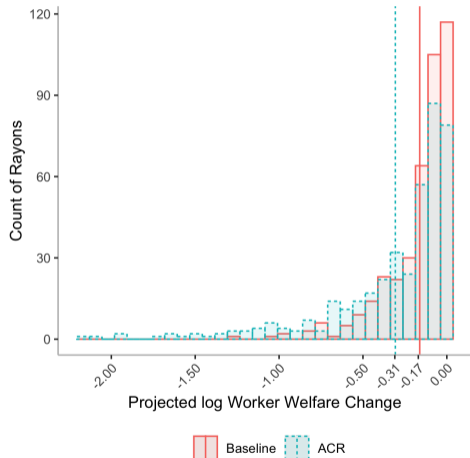
# More Reduction of Welfare in Higher Conflict Exposure Rayons

	Dependent Variables: Sufficient Statistics for Worker Welfare					
	Baseline ( $\frac{1-\beta}{\varepsilon}\tilde{\Lambda}_{ii} + \frac{1-\beta}{\beta}\eta\tilde{M}_{ii}$ )		ACR ( $\frac{1-\beta}{\varepsilon}\tilde{\Lambda}_{ii}$ )		Supplier Link Margin ( $\frac{1-\beta}{\beta}\eta\tilde{M}_{ii}$ )	
	(1)	(2)	(3)	(4)	(5)	(6)
Conflict Supplier Exposure (Value)	-1.000*** (0.205)		-0.883*** (0.208)	-0.968*** (0.257)	-0.996*** (0.211)	0.112 (0.204)
Conflict Buyer Exposure (Value)		-0.730*** (0.206)	-0.542*** (0.207)	-0.569*** (0.212)	-1.781*** (0.209)	1.238*** (0.202)
$\sum$ Conflict $\times$ Forward Domar Weights				0.170 (0.302)		
Constant	0.891*** (0.043)	0.851*** (0.042)	0.935*** (0.045)	0.890*** (0.092)	0.681*** (0.046)	0.254*** (0.045)
Observations	403	403	403	403	403	403
Adjusted R <sup>2</sup>	0.054	0.028	0.067	0.066	0.222	0.088

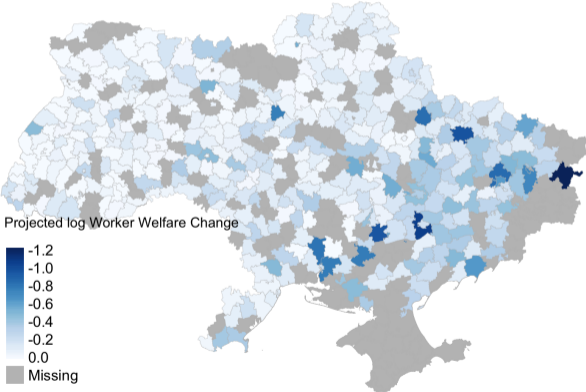
- Ignoring “supplier link margin” overestimate the relationships (Column 5 and 6)

# Projected Welfare Loss outside Conflict Areas

- Predict welfare loss using supplier & buyer conflict exposures using the previous regression
- Welfare ↓ 17% for an average region (relative to regions with zero exposures)
- Substantial overestimation of welfare loss (↓ 31%) if we ignore supplier link margin
- Large regional heterogeneity



# Regional Heterogeneity in Welfare Loss outside Conflict Areas





## **Conclusion**

## Conclusion

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- Develop common welfare sufficient statistics under endogenous production networks
- Show large propagation effects of 2014 Ukraine War, beyond Donbas and Crimea
- Highlights a key mechanism in which localized conflict often have far-reaching detrimental consequences for the broader economy (Rohner & Thoenig '21)



# Appendix

## Different Endogenous Network Models, Same Welfare Changes

- Endogenous search intensity: Demir, Fieler, Xu, Yang '21; **Arkolakis, Huneus, Miyauchi '23**
- Relationship-specific fixed cost: **Bernard, Moxnes, Ulltveit-Moe '18**; Lim '18; Huneus '18; Bernard, Moxnes, Saito '19; Bernard, Dhyne, Magerman, Manova, Moxnes '22; Dhyne, Kikkawa, Kong, Mogstad, Tintelnot '22
- Optimal supplier choice: Oberfield '18; Boehm & Oberfield '20; Acemoglu & Azar '20; Taschereau-Dumouchel '20; **Eaton, Kortum, Kramarz '22**; Antras & de Gortari '20; Miyauchi '23; Panigrahi '21; Lenoir, Martin, Mejean '22

## Example: Endogenous Search Intensity

- Single-sector version of [Arkolakis, Huneus, Miyauchi '23](#)
- [CES](#) production function, common  $\sigma$  within and across regions ( $\varepsilon = \sigma - 1$ )

$$p_{ui}^l(\omega) = \left( \int_{v \in \mathcal{S}_{ui}(\omega)} c_u(v)^{1-\sigma} dv \right)^{\frac{1}{1-\sigma}}$$

- Suppliers and buyers choose endogenous intensity of search, match realizes based on matching technology
- $\varepsilon, \eta$  are given by

$$\varepsilon = \sigma - 1, \quad \eta = \frac{1}{\sigma - 1} (= \frac{1}{\varepsilon}),$$

- Do not depend on matching technology and search decisions (summarized by  $\hat{M}_{ij}$ )

## Example: Relationship-Specific Fixed Cost

- A version of Bernard, Moxnes, Ulltveit-Moe '19 with input-output loops
- CES production function as Arkolakis, Huneus, Miyauchi '23
- Relationship forms if supplier  $v$  is willing to pay fixed cost  $f_{ui}$
- Productivity follows Pareto distribution with dispersion parameter  $\theta$
- $\varepsilon, \eta$  are given by

$$\varepsilon = \sigma - 1, \quad \eta = \frac{1}{\sigma - 1} - \frac{1}{\theta} (< \frac{1}{\varepsilon})$$

- $1/\theta$  comes from negative assortative matching

## Example: Optimal Supplier Choice

- A version of Eaton, Kortum, Kramarz '22 without in-house production
- Suppliers and buyers randomly match, and buyers choose the best supplier

$$p_{ui}^l(\omega) = \min_{v \in \mathcal{S}_{ui}(\omega)} p_{ui}(v)$$

- Pareto Productivity with dispersion  $\theta$ ; matching technology is biased toward lower-cost suppliers with weight  $\gamma$
- $\varepsilon, \eta$  are given by

$$\varepsilon = \theta(1 - \gamma), \quad \eta = \frac{1}{\theta(1 - \gamma)} (= \frac{1}{\varepsilon})$$

- Note:  $\mathcal{S}_{ui}(\omega)$  is potential ( $\neq$  realized) set of suppliers
  - With exogenous matching rates, formula still holds with  $\eta = 0$  (Oberfield '20)
  - Otherwise, can use gravity to back out measure of potential suppliers



## Examples: Additional Remarks [go back](#)

- Substantially general than existing models
  - Allow more flexible firm heterogeneity in productivity  $z_i(\cdot)$ , trade costs  $\tau_{id}(\cdot)$ , (exogenous) markups  $\rho_{id}(\cdot)$ , depending on models
  - Different elasticity of substitution within and across locations
- Other examples
  - Separate variety gains from substitution (Benassy '98; Acemoglu, Antras, Helpman '07)
  - Entry into input market (Antras, Fort, Tintelnot '17)
    - Expression unchanged if firms always enter own region
  - Diversifying idiosyncratic supplier risks (Anderson, de Palma, Thisse '92)
  - Network formation under adjustment frictions (Lim '18, Huneeus '19)
- Some models imply non-iso-elastic function of  $\hat{M}_{ij}$  in welfare sufficient statistics e.g., Miyauchi '21; EKK '22 with in-house production

## Final Goods Trade [go back](#)

- CES preference for final goods

$$P_i^F = \left( \sum_{\ell} \left( \tau_{\ell i}^F C_{\ell} \right)^{\nu} \right)^{\frac{1}{\nu}}$$

- Real Wages:

$$\hat{W}_i = \hat{\Lambda}_{ii}^{-\frac{1-\beta}{\beta} \frac{1}{\varepsilon}} \hat{M}_{ii}^{\frac{1-\beta}{\beta} \eta} \left( \hat{\Lambda}_{ii}^F \right)^{-\frac{1}{\nu}}$$

where  $\hat{\Lambda}_{ii}^F$  is the within-region expenditure share in final goods

## Multiple Sectors [go back](#)

- $k, m \in K$ : sectors (Caliendo & Parro '15; Costinot & Rodriguez-Clare '14)
- Unit cost

$$c_{i,k}(\omega) = z_{i,k}(\omega) w_i^{\beta_{i,Lk}} \prod_{m \in K} \left( \sum_u \left( p_{ui,mk}^l(\omega) \right)^{-\varepsilon_m} \right)^{\frac{\beta_{i,mk}}{-\varepsilon_m}}$$

- Cobb-Douglas preference:

$$\hat{P}_i^F = \prod_k \hat{C}_{i,k}^{\alpha_{i,k}}$$

- Real Wages:

$$\log \frac{\hat{w}_i}{\hat{P}_i^F} = \sum_k \alpha_k \sum_{m,h \in K} \tilde{\beta}_{i,hk} \beta_{i,mh} \left( -\frac{1}{\varepsilon_m} \log \hat{\Lambda}_{ii,mk} + \log \frac{\hat{P}_{ii,mk}^l}{\hat{C}_{i,m}} \right)$$

where  $\tilde{\beta}_{i,mk}$  is  $(m, k)$ -th element of Leontief inverse:  $(I - B_i)^{-1}$  with  $B_{i,mk} = \beta_{i,mk}$

## Multiple Firm Types [go back](#)

- Unit cost of type  $\vartheta$  firm

$$c_{i,\vartheta}(\omega) = z_{i,\vartheta}(\omega) w_i^{\beta_{i,\vartheta}} \left( \sum_u (p_{ui}^l(\omega))^{-\varepsilon} \right)^{\frac{1-\beta_{i,\vartheta}}{-\varepsilon}}$$

- First-order approximation of external shocks on real wages:

$$d \log \frac{w_i}{P_i^F} = - \sum_{\vartheta} \Lambda_{i,\vartheta}^F \frac{1-\beta_{i,\vartheta}}{\beta_{i,\vartheta}} \left( \frac{1}{\varepsilon} d \log \Lambda_{ii,\vartheta'\vartheta} + d \log \frac{P_{ii,\vartheta'\vartheta}^l}{C_{i,\vartheta'}} \right)$$

- $\Lambda_{i,\vartheta}^F$ : share of final goods expenditure for  $\vartheta$
- $\tilde{\Lambda}_{ii,\vartheta'\vartheta}$ : type  $\vartheta$  and location  $i$  firms' share of intermediate inputs within same type and location

## Nonparametric Production Function [go back](#)

- Nonparametric production function

$$c_i(\omega) = f_i\left(w_i, \left\{p_{ui}^l(\omega)\right\}_u\right),$$

- Define elasticity of substitution for inputs sourced within a region:

$$\mathcal{E} \equiv \frac{d \log \Lambda_{ii}}{(1 - \Lambda_i^L)^{-1} \sum_u \Lambda_{ui} d \log p_{ui}^l - d \log p_{ii}^l}$$

- First-order changes in real wages:

$$d \log \frac{w_i}{c_i} = - \left( \frac{1 - \Lambda_i^L}{\Lambda_i^L} \right) d \log \frac{p_{ii}^l}{c_i} - \left( \frac{1 - \Lambda_i^L}{\Lambda_i^L} \right) \frac{1}{\mathcal{E}} d \log \Lambda_{ii}$$

## Alternative Decomposition using Domar Weights [go back](#)

- For simplicity, consider a change in variable trade costs  $\{\tau_{ij}\}$
- Change in production cost is also rewritten as

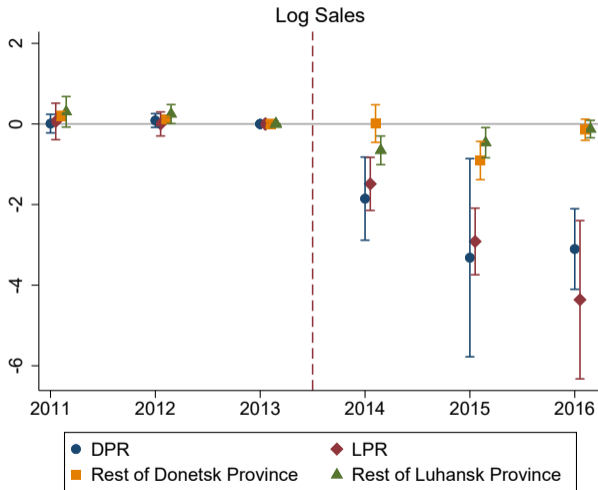
$$\log C_i = \sum_u \psi_{ui}^L \log w_u + \sum_u \psi_{uj} \left( d \log \tau_{ij} + d \log \left( \hat{P}_{ui}^I / \hat{P}_{ui} \right) \right)$$

- $\psi_{ui}^L, \psi_{ui}$ : forward Domar weights
- To obtain real wage changes, need to keep track of the changes in the wage vector in all locations  $\{\log w_u\}_u$

# Sudden and Large Drop of Total Firm Sales in Conflict Areas [go back](#)

$$Y_{rt} = \beta_t^{LPR} \times LPR_r \times Post_t + \beta_t^{DPR} \times DPR_r \times Post_t + \beta_t^{DON} \times Donetsk_r \times Post_t + \beta_t^{LUH} \times Luhansk_r \times Post_t + \alpha_r + \kappa_t + \varepsilon_{rt}$$

- $r$ : rayon (district)
- Exclude Crimea due to data quality after the annexation



# Ukrainian Railroads with Stations

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## Estimation Strategy: $\eta$

- Input expenditure share of firms in  $d$  from  $i$ :

$$\tilde{\Lambda}_{id} = -\varepsilon \tilde{C}_i + \eta \varepsilon \tilde{M}_{id} - \varepsilon \tilde{\tau}_{id} + \tilde{\xi}_d$$

- Shepard's Lemma + CES input demand

$$\tilde{C}_i = \beta \tilde{w}_i + (1 - \beta) \left( \tilde{C}_d + \eta \tilde{M}_{di} + \tilde{\tau}_{di} - \frac{1}{\varepsilon} \tilde{\Lambda}_{di} \right)$$

- Combining, our estimating equation:

$$\tilde{\Lambda}_{id} + (1 - \beta) \tilde{\Lambda}_{di} + \beta \varepsilon \tilde{w}_i = \eta \varepsilon \left( \tilde{M}_{id} + (1 - \beta) \tilde{M}_{di} \right) + \tilde{\xi}_d^* + \tilde{\tau}_{id}^*$$

- $\tilde{\xi}_d^*$ : destination FE;  $\tilde{\tau}_{id}^*$ : residuals
- Samples: region pairs excluding if  $i$  or  $d$  are in direct conflict areas
- **IV: supplier and buyer conflict exposures of region  $i$**

# Estimation Results of $\eta \times \varepsilon$ [go back](#)

	Dependent variable:			
	$\tilde{M}_{id} + (1 - \beta)\tilde{M}_{di}$ OLS (1)	$\tilde{M}_{id}$ OLS (2)	$\tilde{M}_{di}$ OLS (3)	$\tilde{\Lambda}_{id} + (1 - \beta)\tilde{\Lambda}_{di} + \beta\varepsilon\tilde{w}_i$ IV (4)
Conflict Supplier Exposure <sub>i</sub>	0.729** (0.313)	0.101 (0.276)	0.785*** (0.124)	
Conflict Buyer Exposure <sub>i</sub>	1.137*** (0.418)	1.177*** (0.362)	-0.050 (0.138)	
$\tilde{M}_{id} + (1 - \beta)\tilde{M}_{di}$				1.231*** (0.296)
IV				Supplier and Buyer Exposures
First-Stage F-stat				6.56
d FE	X	X	X	X
Observations	155,555	155,555	155,555	155,555
Adjusted R <sup>2</sup>	0.480	0.250	0.820	0.357

Existing models imply  $\eta\varepsilon = 1$  (Arkolakis et al '23; Eaton et al '22) or  $\eta\varepsilon < 1$  (Bernard et al '18)