Global Value Chains, International Risk Sharing and the Transmission of Productivity Shocks

*Views are personal and should not be reported as representing the views of the Bank of England or its committees.

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- Welfare-relevant inefficiencies within GVCs have been elusive in the literature
- We focus on the interaction of (frictionless) GVCs with an important welfare-relevant friction: incomplete financial markets

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⇒ GVC integration can affect international risk sharing even when trade itself is frictionless

- The size of this effect depends on the degree of GVC integration and the trade elasticity

Plan for Today

1. Baseline NOEM model with GVCs
   - Dynamics of Relative Prices and Relative Consumption
   - International risk sharing
2. Preliminary empirical evidence

An Analytically Tractable Model of GVCs

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- Price of each good given by $P_{H,t}$ and $P_{F,t}$ (assume LOOP)
- Define the Terms of Trade $TOT_t = P_{F,t}/P_{H,t}$: an increase is a deterioration

Standard Model: Trade in Final Goods

- Households in each country consume a CES bundle of both goods:

$$C_t = \left({a_H}^{\frac{1}{\phi}} {C_{H,t}}^{\frac{\phi-1}{\phi}} + a_F^{\frac{1}{\phi}} {C_{F,t}}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}$$

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- So long as $a_H > 0.5$, RER and TOT comove

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- Note $b_H = 1$ and $\alpha = 1$ have similar impact

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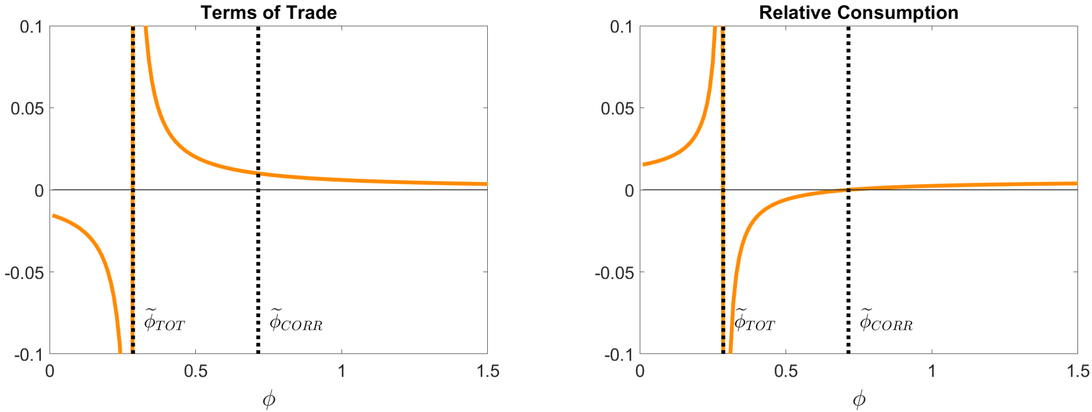
- Must understand how the TOT and Relative Consumption $\left(\widehat{C}_t - \widehat{C}_t^*\right)$ respond to shocks

Impact Responses: No GVC Baseline ($\alpha = 1$)



Note: Positive shock to $A_{H,t}$ with $a_H = 0.7$, under FA.

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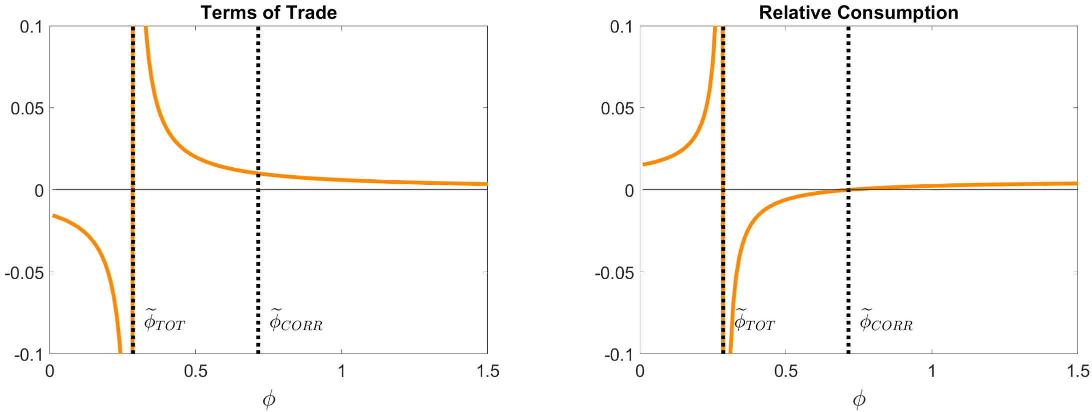

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- For $\widetilde{\phi}_{TOT} < \phi < \widetilde{\phi}_{CORR}$, there is "immiserising growth"

Trade Elasticity Thresholds: No GVCs

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- For $\phi < \widetilde{\phi}_{TOT}$, the TOT appreciate in response to a positive relative supply shock
- For $\phi < \widetilde{\phi}_{CORR}$, C/C^* is negatively correlated with the TOT following supply shocks

Trade Elasticity Thresholds with GVCs: Special Case I

Same home bias and trade elasticity in consumption goods and intermediates

- Assume $b_H = a_H$ and $\psi = \phi$

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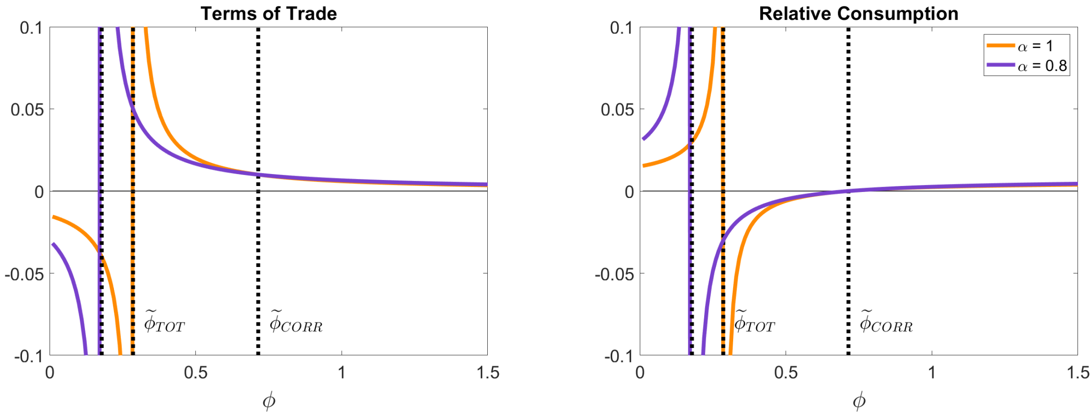
- The threshold for a TOT reversal is now an increasing function of α:

$$\widetilde{\phi}_{TOT}(\alpha) = 1 - \frac{1}{2a_H} - \frac{a_F}{a_H}\frac{1-\alpha}{\alpha}$$

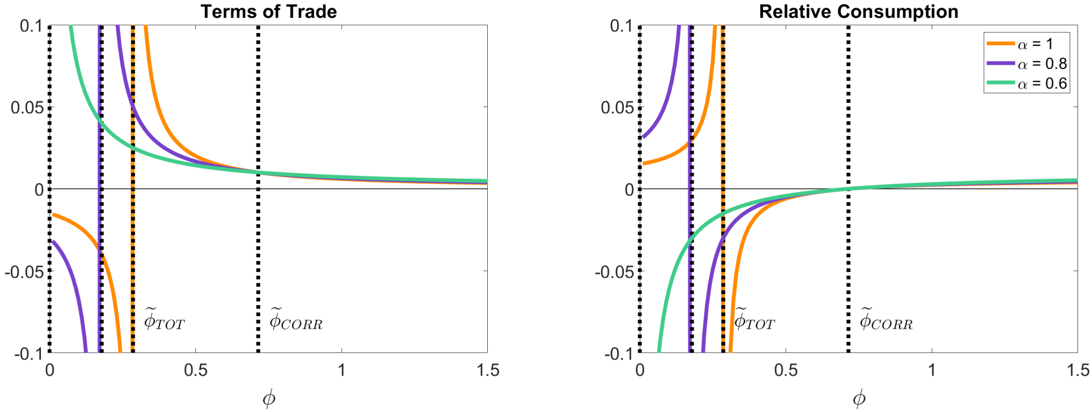
Impact Responses with and without GVCs



Note: Positive shock to $A_{H,t}$ with $a_H = 0.7$, under FA.

- Raising the intermediates share of output (i.e. lowering α) shifts $\widetilde{\phi}_{TOT}$ leftwards

GVCs can rule out TOT appreciation



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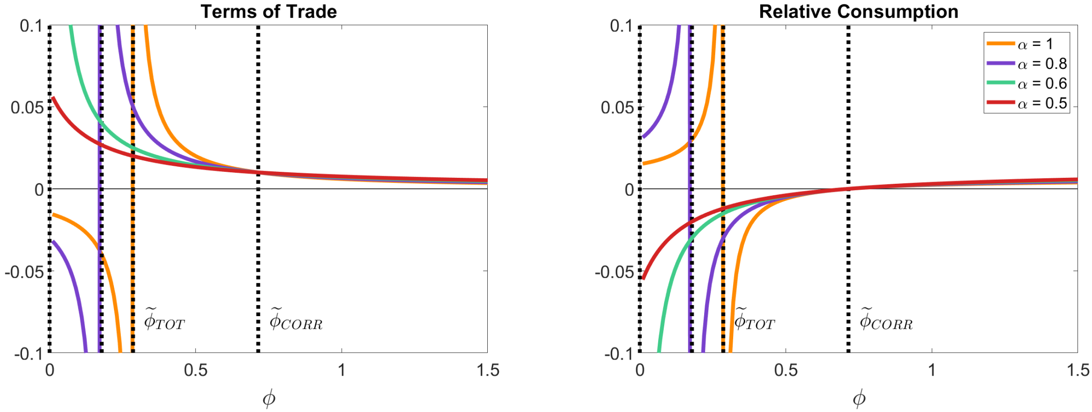

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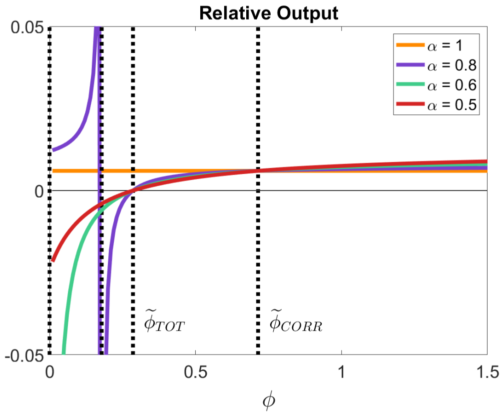
Key Mechanism: Supply-Side Effects



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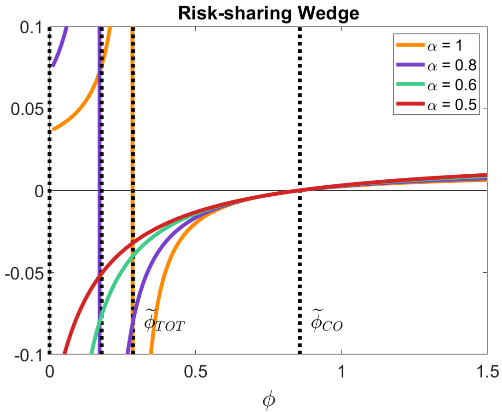
- Output adjusts endogenously due to marginal productivity effect

International Risk Sharing



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- At low elasticities, GVCs reduce the risk-sharing wedge

Back to Reality: Preliminary Empirical Evidence

Model has shown:

- **Intermediate-input linkages can affect countries' ability to share risks under incomplete financial markets, even when GVCs are frictionless**
- Sign and size of this depends on trade elasticity—notoriously difficult to estimate

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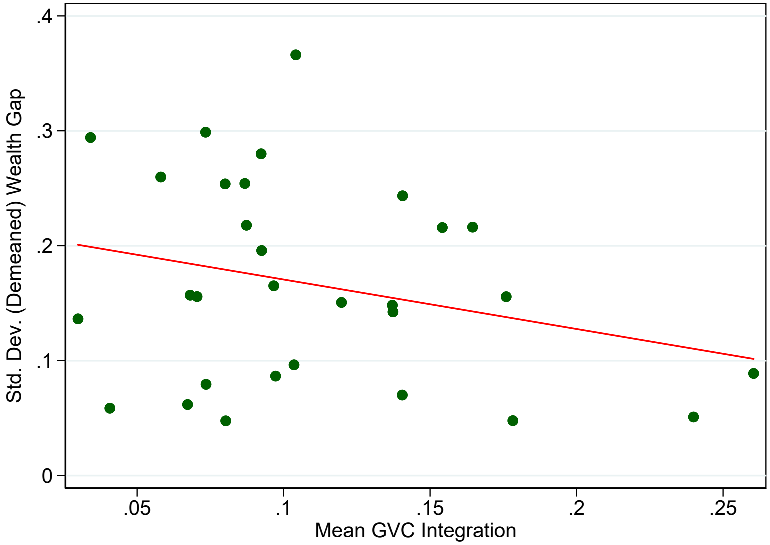
To take this to the data, we:

- Construct a model-consistent measure of risk sharing Var$(\mathcal{W})$, using $C^{(*)}$ and RER data (à la Corsetti, Dedola, Viani, 2012a,b), period 1960:Q1-2019:Q4
- Combine with measures of GVC integration (Backward $+$ Forward Linkages) (à la D'Aguanno et al., 2021) – data from WIOT, 2000-2014
- For now: $N = 43$ countries

Preliminary Empirical Evidence



Conclusions and Next Steps

- Summary:
  - The presence of GVCs has both demand-side and supply-side implications
  - The supply-side implications alone affect the threshold elasticity below which the terms of trade appreciate in response to relative supply shocks
  - Sufficient levels of GVC integration can rule out terms-of-trade appreciations altogether
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  - In this sense, **GVC integration affects risk sharing even when trade itself is frictionless**
- Next steps:
  - Further supporting empirical evidence, incl. assessment of $\text{Corr}(\hat{C} - \hat{C}^*, \hat{Y} - \hat{Y}^*)$
  - More general model (e.g., move away from financial autarky, calibrate parameters)

Appendix: Substitution and Income Effects

- Log-linearise the demand functions around the symmetric steady state:

$$\widehat{C}_{H,t} = \widehat{Y}_{H,t} + a_F(\phi - 1)\widehat{TOT}_t \qquad \widehat{C}_{F,t} = \widehat{Y}_{H,t} - [a_H\phi + a_F]\,\widehat{TOT}_t$$

- Derivative of $\widehat{C}_{F,t}$ wrt $\widehat{TOT}_t$ is unambiguously negative
- Sign of derivative of $\widehat{C}_{H,t}$ wrt $\widehat{TOT}_t$ depends on sign of $(\phi - 1)$
- As Home's terms of trade deteriorate (TOT rises):
  - Substitution effect: switch towards the relatively cheaper Home good
  - Income effect: reduce demand for all goods as the value of Home's endowment falls

Appendix: Equilibrium with GVCs

- In Special Case I, Relative Demand is unchanged:

$$\widehat{TOT}_t = \frac{1}{1 - 2a_H(1-\phi)} \left(\widehat{Y}_{H,t} - \widehat{Y}^*_{F,t} \right)$$

- Relative Gross Output is now a function of TOT:

$$\widehat{Y}_{H,t} - \widehat{Y}^*_{F,t} = \frac{1}{\alpha} \left(\widehat{A}_{H,t} - \widehat{A}^*_{F,t} \right) - 2a_F \frac{1-\alpha}{\alpha} \widehat{TOT}_t$$

- Equating this to relative demand and rearranging:

$$\widehat{TOT}_t = \frac{1}{\left(1 - 2a_H(1-\phi) + 2a_F \frac{1-\alpha}{\alpha} \right)} \frac{1}{\alpha} \left(\widehat{A}_{H,t} - \widehat{A}^*_{F,t} \right)$$

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$$\widehat{TOT}_t = \frac{1}{\left(1 - 2a_H(1-\Phi(\alpha)) + 2a_F\frac{1-\alpha}{\alpha}\right)}\frac{1}{\alpha}\left(\widehat{A}_{H,t} - \widehat{A}^*_{F,t}\right)$$

$$\widehat{C}_t - \widehat{C}^*_t = (2a_H\Phi(\alpha) - 1)\,\widehat{TOT}_t$$

- All results go through in the same way, with thresholds applying to the weighted elasticity, and no feasible TOT appreciation if $\alpha < \tilde{\alpha}$.

Appendix: Model with GVCs: General Case

Different home bias ($b_H \neq a_H$) and trade elasticities ($\psi \neq \phi$)

- In this case the equations are functions of all five parameters:

$$\widehat{TOT}_t = \frac{1}{\left(\Theta(\alpha) + 2b_F \frac{1-\alpha}{\alpha}\right)} \frac{1}{\alpha} \left(\widehat{A}_{H,t} - \widehat{A}^*_{F,t}\right)$$

$$\widehat{C}_t - \widehat{C}^*_t = \left(\Theta(\alpha) - 2a_F\right) \widehat{TOT}_t$$

where:

$$\Theta(\alpha) \equiv \frac{a_F \alpha \left[1 - 2a_H(1-\phi)\right] + b_F(1-\alpha)\left[1 - 2b_H(1-\varphi)\right]}{a_F \alpha + b_F(1-\alpha)}$$

- Sign of the responses will depend on the configuration of all parameters