Hiring Difficulties and Firms' Growth*

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Abstract

This paper studies the role of recruitment difficulties on firms' growth by combining unique vacancy-level data from France with an identification strategy based on a shift-share design. Specifically, we exploit cross-firm variation in exposure to recruiting difficulties stemming from initial differences in firms' occupational mix and we leverage recruiting difficulties shifts using market-level changes in the time it takes to fill a vacancy in a given occupation, with a leave-one-out correction at the industry-level. We find that higher hiring difficulties translate into fewer vacancies posted by firms employing workers in hard-to-recruit occupations. This hampers their employment, with a one-standard deviation increase in predicted recruiting time decreasing firms' employment by 5 to 9%. These effects are especially large when firms are labor-intensive and when they employ a higher share of workers in highly specialized occupations. Complementing the results on employment, we find evidence of negative effects also on firms' investment, profits, and sales. Finally, we show that firms partially adjust to hiring difficulties by increasing wages, retaining incumbent workers, and promoting them higher up into high-pay occupations.

Keywords: recruiting difficulties, firm growth, firm performance.

JEL Codes: J21, J63, G32, M51.

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1 Introduction

There is ample anecdotal evidence that firms face hiring difficulties. For example, in the recent survey of the U.S. National Federation of Independent Business run in 2021, a record-high 40% of small businesses reported they had jobs they could not fill.[1] Labor market frictions interfering with firms' hiring needs can affect firms' performance to different extents. On the one hand, hiring difficulties might lead firms to be short of essential inputs in their operations, and prevent them from growing. On the other hand, firms might be flexible enough to adapt to labor shortages, for instance by automating some tasks, or training their incumbent workers, in which case the impact of hiring difficulties on firms' performance might be limited.[2] Thus, investigating how firms' adjust to recruiting difficulties and quantifying their impact on corporate performance is key to understand the aggregate consequences of labor shortages for the economy.[3]

Even though a number of empirical studies have explored the reasons for why some firms have a hard time finding suitable workers for their jobs (see e.g. Haskel and Martin, 1993a, 2001; Weaver, 2021; Kerr et al., 2016)[4], we know surprisingly little about the causal impact of hiring difficulties on firms' outcomes. This is probably due to both identification and data challenges. First, the effect of hiring difficulties on firm performance may be confounded by unobserved market-level demand shocks, or rather reflects reverse causality where firms adapt their recruiting strategies to their growth opportunities. Second, large-scale datasets containing linked information on firms' balance sheets and hiring difficulty have not been

[1]Firms' hiring difficulties have increasingly attracted attention as the ratio of job openings to hires has hit an all-time high following the Covid-19 pandemic; however this problem is not unique to the post-Covid 19 period. Indeed in 2017, few years before the start of the pandemic, around two-thirds of firms reported that they experienced hiring difficulties according to the Federal Reserve Banks' Small Business Credit Survey. The three most frequently reported reasons were *"Lack of job-specific skills, education, or experience"*, *"Too few applicants"*, and *"Lack of soft skills"*, see Terry and de Zeeuw (2020) for more details.

[2]Labor shortages might also be an opportunity for the economy as a whole in that they could lead to an improvement in the quality of jobs, see Autor (2021).

[3]Acemoglu and Zilibotti (2001) use country-level data and show that technology-skill mismatch (between the skills' requirements of technologies imported from developed countries and the skills of workers in less developed countries) is associated with lower country productivity.

[4]More generally, labor supply has been shown to respond to several factors, including unemployment insurance benefits (Carrillo-Tudela et al., 2020b), training frictions (Acemoglu and Pischke, 1998), child care costs (Heckman, 1974), the quality of the education system (Katz and Murphy, 1992), or the introduction of new leisure technologies (Aguiar et al., 2021).

available until recently, as labor matching switches towards online platforms.

In this paper we overcome both these challenges and provide the first causal evidence on how hiring difficulties affect firms' outcomes. Our empirical setting exploits a large-scale micro dataset from the French Public Employment Services that contains detailed information on job vacancies over the sample period 2010-2017, which we can link to matched employer-employee data and balance sheets information for the universe of French firms. In order to purge from firm- and market-level shocks to demand or productivity that are likely to affect both corporate performance and hiring effort, we predict recruiting difficulties at the firm level by using a shift-share design combined with a battery of fixed-effects. Specifically, we combine occupation-specific changes in the time to fill job vacancies within a local labor market (the *shifts*) with variation in firms' exposure given by their pre-sampled occupation mix (the *shares*).[5]

Taking into consideration the recent papers on shift-share instruments (Borusyak et al., 2021; Goldsmith-Pinkham et al., 2020), there are three important clarifications to make when thinking about our empirical design. First, when computing the shares, we use baseline information about the occupation mix of a firm workforce. In this way, we make sure that our estimates are unaffected by contemporaneous shocks to firms' technologies that affect both the types of workers required to produce and employment decisions. Secondly, to account for the concern that firms' realized vacancy filling rates are likely to correlate with unobserved shocks to demand and productivity at the firm and product market level, we employ an instrumental-variable strategy with a leave-one-out correction at the industry-level. Specifically, for each firm we instrument the realized time it took to fill a vacancy in a given occupation by using the average time it took to other firms in the same local labor market but in different product markets to fill their vacancies in the same occupation. Finally, we complement our empirical strategy by including market-level (i.e. industry $\times$ commuting zones $\times$ year) fixed-effects in our most stringent specification. Indeed, while our occupation-specific shifts are plausibly not affected by endogenous changes in a firm own hiring effort, the inclusion of market-level fixed effects allows us to simultaneously control for unobserved labor demand shocks in the firm own market (industry x commuting zones).[6] Overall, as firms differ in

[5]See Rothwell (2014) for prior work using time-to-fill vacancies as a measure of hiring difficulties across occupations and geographical areas.

[6]While we argue that our empirical approach allows us to isolate quasi-exogenous supply-driven

their baseline occupation mix even within an industry and local labor market, our approach allows us to fully control for contemporaneous market-level shocks and to build a supply-driven measure of exposure to recruiting difficulties that varies at the *firm-level*.

Exploiting the richness of our data, we first document new stylized facts about the dispersion in time-to-fill job vacancies (or share of unsuccessful recruitment) by showing that time-to-fill features large variations across occupations, industries, and local labor markets. We also validate our vacancy-based measures by documenting that higher time-to-fill aggregated at the occupation and geography level correlates positively with survey-based measures of hiring difficulties (either the French Business Tendency Survey or in the Workforce Firm Survey).

Second, we show that our Bartik-like shift-share measure of recruiting difficulty significantly predicts the actual recruiting time of firms and thus we employ our predicted measure to estimate the causal impact of hiring difficulties on firms' employment. Our estimates imply that a one-standard deviation increase in recruiting time (around 70 days) is associated with a significant decrease in firms' employment by 5 to 9%. This semi-elasticity estimate is in line with significant vacancy posting cost in search and matching models: yearly vacancy posting cost equals between 6% and 12% of the yearly wages. Recruiting frictions have a first-order effect on dynamic labor demand.

Next, we turn to the effect of recruiting difficulties on other corporate outcomes, in particular firm investment, sales, and firm profitability. Our estimates imply that a one-standard deviation increase in predicted recruiting time is associated with a decrease in firms' investment, sales, and ROA of respectively 1, 3.5, and 0.25 percentage points. Taken together, these results indicate that in response to hiring difficulties firms scale is pushed downwards, and that any substitution effects towards capital are not large enough to generate an increase in investment. Down the road, firms' profitability is significantly affected: one standard deviation increase in recruiting time decrease their ROA by 3.5%.

shocks to an individual firm hiring-difficulties (see Borusyak et al. (2021)), we do not take a stance on the reasons for why firms in the same local labor market but different product markets might face a change in occupation-specific recruiting time. These may be supply-driven reasons (e.g. the current number of applicants is low or there is a mismatch between the skills of applicants and the skills requirement of vacancies) as well as demand-driven reasons (e.g. automation or innovation trends). Either way, in this paper, we take these hiring frictions as given, and estimate their impact on firms' outcomes.

We then exploit the richness of our micro data to investigate the adjustment margins of firms facing higher recruiting difficulties. First, we show that an increase in the expected time-to-fill a vacancy is associated with a decrease in vacancy posting, confirming that the negative scale effect decreases recruitment effort. This translates into a significant decrease in the number of new hires. Interestingly, we also find that firms partially adjust to hiring difficulties by increasing retention of incumbent workers, but, as mentioned above, the overall effect on employment remains negative.

Second, we investigate whether firms react to hiring difficulties by adjusting the hours worked and wages of their workers. We do not find that, when facing higher hiring difficulties, firms increase yearly hours per worker, not even for incumbent workers. Thus, firms do not seem to compensate for hiring constraints at the extensive-margin by adjusting hours worked at the intensive margin. In contrast, we find that yearly wages per worker increase, and that this results is stronger for incumbent workers. This result could be consistent with hourly wages being bid upward by workers when firms face greater competitive pressure to retain their workers. Alternatively, firms may have trained and increased their incumbent workers human capital to compensate for the lost hires due to hiring frictions. To investigate this further, we study to what extent incumbents switch occupation in firms that face hiring difficulties. We find that incumbent workers are pushed up in the occupation ladder towards high-wage occupation. To the extent that occupations define specific skills acquired by training, this result provides support for the human-capital channel described above. However, if occupation rather reflect ranks in the wage schedule of collective agreements, this evidence could also be consistent with the alternative promotion channel.

In line with the task definition of occupation, the effects of recruiting difficulties should be stronger when they relate to tasks that require more occupation-specific experience, in which case it is harder for the firm to find a way to circumvent them. We construct a new measure of the occupation-specificity of the workforce of a given firm to test whether this is the case. In particular, focusing on the sample of all workers switching employers, we compute the number of transitions from occupation O (”origin”) to occupation D (”destination”). Then, for each occupation D we compute the share of transitions in which the worker used to be employed in the same occupation ($O = D$), and interpret this ratio as a measure of the ”specificity" of a given occupation. For every firm, we finally compute the average of the occupation specificity of its employees at baseline (weighted by the firm occupation mix). We hypothesize that firms with high occupation-specific workforce should be less able to redirect their hiring to other types of workers when facing current recruiting difficulties on their occupation-mix. Consistent with this hypothesis, we find that the negative effects of recruiting frictions on employment are concentrated in the sample of firms characterized by high occupation-specificity, indicating that occupation-specificity is a key driver of the sensitivity of firms' employment to hiring difficulties.

In a similar spirit, we also look at firms' heterogeneity in the cross-section, and check whether hiring difficulties have a stronger effect on firms' outcomes in labor-intensive firms (as measured with their employment-to-asset ratio in the baseline year). We find that firms' employment sensitivity to hiring difficulties is twice as large in firms characterized by high labor intensity. Interestingly, when we compare the effects depending on firm size, we find that corporate outcomes of large firms are also significantly adversely affected by hiring difficulties, suggesting that hiring difficulties have aggregate effects for the whole economy.

We conduct a battery of tests to ensure that these results are not driven by alternative mechanisms. First, a possible threat to our strategy is that labor demand shocks may be correlated across connected industries. In this case, our leave-one-out correction at the industry-level may not be enough to isolate supply-driven shocks to hiring difficulties. In a robustness exercise, we show that our estimates are robust to removing information on time-to-fill from upstream and downstream industries when computing our occupation-specific shifts, which largely addresses the concern that our results could spuriously reflect demand or productivity shocks hitting connected sectors in the supply chain, rather than the causal impact of recruiting frictions on firms' outcomes.

Second, we address concerns over an upward bias in our estimates resulting from local business stealing effects from firms of the same local market. Namely, local business stealing would be a concern in the non-tradable industries (e.g. restaurants). When we remove non-tradable industries for our sample, we find comparable magnitudes of hiring difficulties effects.

Finally, another potential concern with our analysis is the selected nature of our vacancy level data. We obtain job vacancies from the French public employment agency, which is arguably less likely to be used for vacancies with high skill requirements. Hence, our sample measuring hiring difficulty is not representative of the full set of vacancies posted to attract job seekers, which might bias our estimates. However, to the extent that firms' outcomes are more sensitive to recruiting frictions on skilled labor, this selection effect should lead us to underestimate the true causal impact of recruiting difficulties on firms' outcomes.

Overall, our paper is the first to show that local hiring difficulties have a significant causal impact on firm employment and firms' overall performance. Our findings echo numerous press articles indicating that labor shortages hamper firm growth and harm the economy.[7] We also provide new evidence about the different adjustment margins used by firms in order to adapt to local hiring difficulties. Our work has important implications for the design of policies aiming at reducing the mismatch between firms' needs and the skills of the local workforce (such as e.g. targeted education and training, relocation assistance, immigration policy), and more generally for the design of location-based policies to foster growth (see e.g. Glaeser and Gottlieb, 2008; Kline, 2010).

In filling this knowledge gap, our paper relates to several strands of literature. At a broad level, our work contributes to existing empirical studies which provide evidence on the impact of operational constraints on firms' outcomes (Chava et al., 2020; Jagannathan et al., 2016). More specifically, we contribute to the scant body of work studying the implications of skill shortages for firms' behavior. While prior work uses aggregate data from surveys (Haskel and Martin, 1993b), or exploits variations in the supply of specific sets of skilled workers (D'Acunto et al., 2020; Sauvagnat and Schivardi, 2020; Beerli et al., 2021), we construct firm-level measure of recruiting difficulties for all types of workers using micro-level data on job vacancies. In doing so, our empirical setting allows us to interpret the magnitude of our estimates in light of key parameters of the search-and-matching literature.

Our empirical analysis also builds on prior work trying to understand skill and spatial mismatch between labor demand and labor supply (Şahin et al., 2014; Marinescu and Rathelot, 2018). Moretti (2011) highlights the role of thick labor markets in improving match quality, and in reducing the risk that firms cannot fill vacancies. Our work also speak to a series of recent papers using vacancy-level data in

[7]See for instance https://www.bloomberg.com/news/articles/2021-04-05/businesses-can-t-fill-jobs-despite-high-u-s-unemployment.

order to study firms' behavior on the labor market. While earlier work uses aggregate data on vacancies (Hall, 2005; Shimer, 2005, 2007) or micro data covering small samples of firms (Holzer, 1994), more recent studies exploit large-scale micro data on job postings in order to deliver novel insights on firms' hiring decisions (Davis et al., 2013; Bagger et al., 2021; Mueller et al., 2018), and skills' requirements (Hershbein and Kahn, 2018; Modestino et al., 2020).

Finally, we contribute to the literature on the causes and consequences of firm-level and aggregate variations in recruitment intensity (Davis et al., 2013; Kaas and Kircher, 2015; Gavazza et al., 2018; Forsythe and Weinstein, 2021; Carrillo-Tudela et al., 2020a).

The remainder of this paper proceeds as follows. Section 2 describes our data and provide stylized facts on recruiting time and success. Section 3 describes our empirical strategy. Section 4 presents our main results on firms' growth, while Section 5 discusses firms' margins in adjusting their workforce. Section 6 concludes.

2 Data

To overcome standard measurement issues of small-scale surveys, we measure recruitment difficulties in administrative micro data on job vacancies. Our main data source is the file of job postings and recruitment outcomes from the French Public Employment Services (Pole Emploi). We use two other complementary sources of administrative micro data from the French Statistical Office (INSEE): the exhaustive firm registry (balance-sheet data), and the exhaustive employment registers covering the totality of the French workforce. All files can be matched thanks to the individual firm identifier. The sample consists of all non-financial firms that were active in France in 2009. We follow them until 2017.

2.1 Vacancy-level data

Our main data on recruiting difficulties are from the French Public Employment Service (PES). The PES provides intermediation services on the French labor market. Namely, the PES maintains an online job board *pole-emploi.fr*, where firms post their job ads, and workers search for employment opportunities. Any firm may post on the website (private, public firms) and the service is free of charge. The

French PES provides the largest online job board of the French labor market. To assess its importance, we analyze data from a firm survey conducted by the French Ministry of Labor (the OFER survey). Around 50% of recruiters posting a job online declare posting on pole-emploi.fr.

For every job ad, we namely observe the occupation, the workplace location, the number of position offered, the firm identifier and industry. We observe the posting date and the delisting date and recruitment success. The information on recruitment success is collected by the PES employees in charge of the posting. When firms post job ads, they are assigned to a local public employment agency. Local employees are then in charge of monitoring the job ad, and checking its status.We have access to vacancy-level data from 2010 to 2017. We drop observations related to temporary work agencies. We keep only vacancies advertising jobs with at least 20 working hours per week (this excludes XX% of the sample of vacancies). We use two outcomes to measure hiring difficulties. We compute the share of unfilled vacancies (or failed recruitment) and the time-to-fill a vacancy as the duration between the initial posting date and the delisting date. We winsorize the time-to-fill variable at 365 days, and when stated, we also impute 365 days for failed recruitment.

2.2 Stylized Facts about Hiring Failures and Time-to-fill

First, we provide a series of stylized facts about the dispersion in hiring success and time to fill job vacancies across occupations, industries and geography.

Figure 1 illustrates the large variation in recruitment success across occupations and industries. We report in each panel the top 10 and bottom 10 hard-to-recruit occupation/industry. Nine out of ten recruitment of cashiers are successful, while for electrical technicians less than eight out of ten are. The range of variations across 2-digit industries is of the same order of magnitude: from 80% to 90% of success rate. We report in the appendix the descriptive statistics across all 85 2-digit occupation groups and ?? 2-digit industry code.

Figure 3 maps average failure rates across the 350 commuting zone in metropolitan France. Again we find substantial variations where the average probability of not filling a vacancy ranges from 7% to 30% across French CZs.

Figure 2 reports similar plots for our second measure of recruiting difficulties: time-to-fill where we have imputed unfilled vacancies at 365 days. Again we find large variations across occupations and industries. For cashiers, the average time-to-fill is just above 60 days, while it is almost the double for electrical technicians.

Second, we merge our data with two surveys of stated recruiting difficulties by firms in order to validate our vacancy-based measure. As discussed in more details in Appendix B, we find a robust correlation between our vacancy-based measures of hiring difficulties - time-to-fill and probabilities of failed recruitment - and the survey-based measures - the share of establishments reporting hiring difficulties at the industry $\times$ CZ level in the Business Tendency Survey of the French Statistical Institute, and the fraction of difficult recruiting searches aggregated at the occupation $\times$ department level in the PES manpower survey from the French Public Employment Service.

2.3 Firm-level Tax Filings

The second key administrative micro data we use is extracted from tax files. The data includes balance sheets as well as profit and loss statements for the universe of French firms. The data is not publicly available, but is available for academic research through a procedure similar to accessing Census data in the U.S. We track firms through time using their unique identifying number ascribed by INSEE. We retrieve industry classification using a historical four-digit industry classification code ascribed to each firm by INSEE itself, which is similar to the SIC coding system in the U.S, that we aggregate at the three-digit level for our purposes.

Our empirical analysis focuses on the following main firms' outcome variables: the logarithm of firms' employment, defined as the number of full-time employees at the end of the fiscal year, Return on assets (ROA), defined as earnings before interest, depreciation, and taxes (EBITDA) over assets; Investment, scaled by assets; and the logarithm of firms' sales. Table 1 reports summary statistics for our sample of non-financial firms from the private sector. Firms have on average 23 employees.

2.4 Employment registers

We also rely on matched employer-employee data (the *déclarations administratives de données sociales*, DADS) built by INSEE from social security contribution declarations of firms. Each year, firms declare the employment spells, the occupation code, the number of hours worked, and the associated wages for each worker. From the employment registers, we compute firm-level outcomes such as end-of-year employment counts, yearly hires and separations, average yearly wages and average hourly wages for new hires, for incumbents or both.

2.5 External Validity

One may wonder whether our results are informative for the impact of hiring difficulties on firms' outcomes beyond the case of France. Is France an outlier in terms of the recruitment frictions faced by firms on the labor market? Surveys about stated hiring difficulties are available in other countries. In the 2017 wave of the U.S. National Federation of Independent Business survey, around 30% of small businesses reported that they had jobs they could not fill. This compares well with the 30% of enterprises declaring that they encountered recruitment difficulties in the business tendency surveys run by the French Statistical Office in 2017. Similarly, Eurostat provides information on the fraction of firms that report having hard-to-fill vacancies for jobs requiring relevant ICT skills [8]: in France, over half (54%) of all enterprises that recruited or tried to recruit ICT specialists had difficulties in filling these vacancies, a number very close to the EU average (54%). Even though the survey covers only ICT occupations, the evidence suggests that France is similar to other developed countries in terms of the degree of recruitment difficulties faced by firms.

3 Empirical Strategy

We want to study the role of recruiting difficulties for corporate performance. However, because firm-level shocks to demand or productivity might affect both corporate performance and hiring effort establishing a causal link between these two variables is challenging. To address this problem, we predict the level of recruiting difficulties firms face using a shift-share instrument, also called Bartik instrument, which, in general terms, can be seen as a weighted average of a common set of shock (shift) with weights reflecting heterogeneity in shock exposure (shares).

[8]For more details, see https://ec.europa.eu/eurostat/en/web/products-eurostat-news/-/ddn-20190327-1

In practice, we follow this empirical strategy by interacting time-varying shocks to recruiting frictions that are specific to each occupation $\times$ local labor market, with the occupation-mix of a given firm. The basic intuition behind this methodology is that while aggregate variation in recruiting frictions are arguably exogenous to any given firm, their impact may vary significantly across companies precisely because each of them - even within the same industry and local labor market - has a different occupational structure. More specifically, we measure *shocks* to recruiting frictions as time variation in either the number of days it takes to fill a vacancy or the share of unfilled vacancies averaged at a 2-digit occupation $\times$ commuting zone level. To make sure that the recruiting shocks are indeed "exogenous" to the firm, we follow a leave-one's-industry-out approach and exclude from our measure observations on recruiting frictions from the firm of interest and any other firms operating in the same 3-digit industry and local market. [9]

The *shares* instead are specific to each firm and consist in the proportion of a company total workforce that is employed in each 2-digit occupation. To avoid that contemporaneous shocks affecting both a firm occupational structure and firm productivity bias our estimates, we pre-sample information on the occupation-mix and construct time-invariant shares using 2009 information on firm-level employment by occupation.[10]

Finally, to obtain our firm-level measure of recruiting difficulties we first multiply for each firm the shift component with the corresponding occupation share, and then we aggregate these occupation-specific products at the firm-level.[11] Formally, denoting by $\overline{RecTime}_{k,cz,-j,t}$, the average recruiting time for all posted vacancies for occupation k by firms in all industries except j, in year t, and by $s_{i,k,2009}$ the share of a firm i workforce employed in occupation k in the pre-sample year 2009 (with $\sum_k s_{i,k,2009} = 1$), we obtain our baseline measure of firm-level exposure to recruiting difficulties, which reads as follows:

[9] There are 84 distinct 2-digit occupations, 270 distinct 3-digit industries, and 322 distinct commuting zones. In robustness tests, we further exclude observations from local firms in related industries, namely operating in upstream and downstream sectors.

[10] Unfortunately, we cannot information from before 2009, as the occupation codes of firms' workforce is not available in earlier years. Our results are robust to using shares in 2010 or 2011.

[11] When the shift for a given occupation $\times$ local labor market $\times$ year cell is missing, we adjust firms' employment by re-calculating the total number of employees over the cells with non-missing shifts and by consequentially re-calculating occupation shares over the adjusted total employment.

$$\widehat{RecTime}_{i,cz,j,t} = \sum_k s_{i,k,2009}\overline{RecTime}_{k,cz,-j,t} \tag{1}$$

The intuition behind this approach is the following. Each firm i operating in industry j and located in the local labor market cz is characterized, at baseline, by a specific production function, which is reflected by a particular occupation-mix. Over time, there may be a strong increase in the time it takes to hire workers for a given occupation k in a given local labor market cz – for instance driven by a decline in the labor supply for that particular occupation. While these "shocks" to hiring difficulties, which vary across narrowly defined occupations × commuting zone, are plausibly exogenous to any given firm i (once we remove from their computations information from job vacancies posted by firm i and all other firms operating in the same industry as firm i), their impact still significantly varies across firms because each of them - even within the same local labor market and industry - has a different occupational structure. In particular, firms relying on occupation k more than others will be disproportionately more affected by the increase in hiring difficulties for occupation k.

Our identification strategy closely approximates the following example. Take two otherwise identical firms, A and B, located in the same Commuting Zone cz and operating in the same industry j (say producing elevators and escalators), with two types of occupations, IT engineers (k="IT"), and Mechanical engineers (k="MECHA"), with however different pre-determined occupation shares (s^A_{IT}, s^A_{MECHA}) and (s^B_{IT}, s^B_{MECHA}) (with $s^i_{IT} + s^i_{MECHA} = 1$ for $i = A, B$). We will compute the average time-to-fill job vacancies for both occupations "IT" and "MECHA" across all firms operating in all industries different than j "elevators and escalators" (leave-one's-industry-out), but in the same labor market cz as firms A and B, in order to construct our Bartik-like instrument for local hiring difficulties faced by firm A and firm B, defined as:

$$\widehat{RecTime}_{A,cz,j,t} = s^A_{IT} \times \overline{RecTime}_{IT,cz,-j,t} + s^A_{MECHA} \times \overline{RecTime}_{MECHA,cz,-j,t}$$

$$\widehat{RecTime}_{B,cz,j,t} = s^B_{IT} \times \overline{RecTime}_{IT,cz,-j,t} + s^B_{MECHA} \times \overline{RecTime}_{MECHA,cz,-j,t}$$

Suppose that firm A relies more on occupation IT than firm B, $s^A_{IT} > s^B_{IT}$ in the pre-sample period, and there is a negative local shock to recruiting time for occupation IT in commuting zone cz over the sample period, we will estimate whether this shock had a larger impact on the employment of firm A than firm B in a specification in which we control for any other confounding shocks that could occur at the narrowly defined market level, by including industry × cz × year fixed effects.

Specifically, to implement our identification strategy, we leverage our vacancy-level data, which provides us with granular information on the filling rate and recruiting time of each job posting, and run the following OLS specification at the firm-year level:

$$Y_{i,cz,j,t} = \alpha_i + \beta \widehat{RecTime}_{i,cz,j,t} + \mu_{cz,j,t} + \epsilon_{i,cz,j,t} \tag{2}$$

where $Y_{i,cz,j,t}$ is a given outcome variable of firm i (which operates in commuting zone cz and industry j) in year t, and $\widehat{RecTime}$ is the shift-share prediction of recruiting time defined in equation 1 above. Standard errors are clustered at the commuting zone level. We first present below the results of firm-stage specifications in which we check that the bartik instrument indeed predicts average recruiting time on all job postings of firm i in year t ($\overline{RecTime}_{i,t}$). In that case, by construction, the sample is restricted to the subset of firms posting at least one vacancy in a given year. We then consider the effect of the bartik variable $\widehat{RecTime}_{i,cz,j,t}$ on firms' vacancy rate and the offered jobs rate, as well as on a series of corporate outcomes, such as employment, investment, profits, or sales growth.

Formally, identifications rests on the assumption that shocks to recruiting filling rates and time for firms in other industries of the same commuting zone is orthogonal to the error term $E(\epsilon_{i,cz,j,t}|\widehat{RecTime}_{i,cz,j,t}) = 0$. Next, we discuss potential threats to this assumption and how to address it. First, there might be negative local or industry shocks that simultaneously affect firm performance and the average time-to-fill vacancies that they face. Importantly, our most saturated specifications include industry × commuting zone × year fixed effects (the $\mu_{cz,j,t}$'s in equation 2) so that we absorb any potentially confounding market-level shocks that could drive both changes in time-to-fill vacancies and say firms' employment (our main variable of interest). In other words, in equation 2, identification comes from comparing performance of two firms within the same market and year, based only on differences in their pre-determined occupation mix. One could still argue that the negative effect of higher recruiting time for the same occupations in other industries of the same local labor market on firms' employment is biased by the presence of inter-industry linkages between local firms.[12] To address this concern, in robustness tests, we remove all information on the time to fill vacancies of any firm located in both upstream and downstream industries with respect to firm i when constructing the bartik variable (using a 1% cutoff on input-output linkages at the sectoral level), and find virtually identical results.

Finally, one might worry that firms endogenously select their location by taking into account that local labor shortages in their most important occupations might have a negative impact on their performance. This is not a threat to the identification strategy: if anything, this should bias the results against finding any effect of recruiting difficulties on firm performance, given that the most vulnerable firms to hiring frictions are likely to endogenously select their location where there is a large supply of trained workers in the occupations for which they have a high demand.

3.1 First-Stage Regressions

We start by establishing the internal validity of our empirical setting, and check whether there is a strong relationship between the shift-share prediction of recruiting time, $\widehat{RecTime}_{i,cz,j,t}$, and the actual average recruiting time faced by firms on their posted vacancies, $RecTime_{i,cz,j,t}$. By construction, the sample is restricted to firms posting at least one vacancy in year t. Columns 1 and 2 of Table 7 present the results for the share of vacancies unfilled whereas columns 3 and 4 present the estimates for the time-to-fill. All specifications include firm fixed effects. Columns 1 and 3 add industry $\times$ year and CZ $\times$ year fixed effects whereas Columns 2 and 4 saturate the specification with industry $\times$ CZ $\times$ year fixed effects.

In each specification, the coefficient on *Share Not Filled Predicted* and *Time to Fill Predicted* is positive and highly statistically significant, indicating that our instrument has a strong predictive power for firms' recruiting difficulties.

[12]Consider for instance a positive productivity shock in upstream sectors driving both an increase in recruiting intensity per vacancy in upstream sectors and an increase in employment in downstream sectors. This could lead to a spurious association between our bartik variable and employment, even in the absence of any causal effect of recruiting difficulties and employment.

4 The Effect of Hiring Difficulties on firms' growth

This section shows the effects of recruiting difficulty on the firms' employment and other corporate outcomes.

4.1 The Effect of Recruiting Time on Firms' Employment

Reduced-form. We now present in Table 3 the results of our main specifications. For this, we relate changes in firms' employment to exogenous changes in local expected recruiting time to the occupations that firms' production activities require, after controlling finely for separately CZ $\times$ Year fixed effects and Industry $\times$ Year fixed effects in columns (1) and (3), and for CZ$\times$ Industry $\times$ Year fixed effects in columns (2) and (4). In columns (1) and (2), we find a negative relationship between the predicted share of unfilled vacancies and log employment, statistically significant at the one percent level where in columns (3) and (4), we find a negative relationship between the predicted time-to-fill and log employment, statistically significant at the one percent level. This is consistent with the view that recruiting difficulties have a significant adverse impact on firms' employment.

2-SLS specifications. In order to interpret the magnitude of the effect of recruiting difficulties on firms' employment, we perform a formal instrumental variable (IV) analysis, where the average share of unfilled vacancies (respectively time-to-fill of vacancies) at the firm level is instrumented with the shift-share variable. To be valid, this instrument needs to satisfy a relevance condition and an exclusion restriction. The former requires that the bartik variable is a strong enough predictor of firms' recruiting difficulties. We have shown in Table 7 that this is the case, and one can check that the F-tests are above conventional levels. The latter requires that the correlation between the instrument and the error term is zero. In other terms, the bartik variable should only affect firms' employment through its effect on the recruiting difficulties faced by firms when they post vacancies. This is the same assumption we need for our reduced form analysis to properly identify variations in employment caused by recruiting frictions. In Table 4, we report the IV estimate. To maximize statistical, we directly compute the Wald estimator, i.e. the ratio of the reduced-form estimate to the first stage coefficient (see Online Appendix C below). This allows to use the whole sample for the reduced form, even if we can compute the first stage on the subsample of posting firms only. The results presented in columns (1) and (2) indicate that a one standard-deviation increase (around 25 percentage points) in the expected probability of not filling a vacancy leads to a drop of around 5 to 9 percentage point in firms' employment.

Interpreting the magnitude of the estimates. Importantly, to check whether our estimates fall within a reasonable range, we compare them to the prediction of a baseline calibration of the simple search and matching model presented in Section C. As shown in the Appendix Equation 11, using an annual cost of a vacant job equal to 5.8% of the annual wage (Cahuc et al., 2018) and a labor share of 66%, we obtain a semi-elasticity of the logarithm of firm employment to τ, the expected vacancy duration expressed in fraction of the year (between 0 and 1, as in our data), equal to -0.17. Our empirical estimates are comparable (-0.24 and - 0.4, see columns 3 and 4 of Table 4), yet slightly higher. There are therefore consistent with the predictions of a search and matching model with relatively high levels of the flow vacancy cost.

Heterogeneous effects by labor-intensity. The negative effects of recruiting frictions on firms' employment should be stronger for labor-intensive firms. To see this formally, observe that Appendix Equation (11) indicates that the sensitivity of firm employment to τ, the expected vacancy duration, is increasing with the labor share. We sort firms into those with low and high labor-intensity, based on their ratio of employees over assets measured in 2009. The results are presented in Appendix Table A1. The effect of recruiting difficulties is indeed much stronger for labor-intensive firms (columns 1 and 2) than for not labor-intensive firms. By showing that recruiting difficulties have a larger effects on firms' employment precisely for those firms relying more on labor in their production function, these results make us confident that our baseline estimates indeed reflect the true causal impact of labor shortages on firms' outcomes.

4.2 Robustness checks

Local spillovers. A concern is that hiring difficulties by disrupting some firms might benefit other less-affected firms in the same industry and area if they are competitors in local product markets, leading us to overestimate the causal impact of recruiting frictions on firms' outcomes in particular in the specifications with industry $\times$ CZ $\times$ year fixed effects. To gauge the severity of this concern, we run our baseline specification after removing non-tradable industries from our sample (e.g. restaurants), where local demand spillovers could bias our estimates upward, and present the results in Appendix Table A3 and A2 with various definitions of tradable industries. Reassuringly, the estimates are quantitatively similar in non-tradable and tradable industries, indicating that business-stealing effects have only a negligible impact on our findings.

Input-output linkages. One may be concerned that our results could spuriously reflect demand or productivity shocks hitting connected sectors in the supply chain, rather than the causal impact of recruiting frictions on firms' outcomes. To address this concern, we check whether our estimates are robust to removing information on time-to-fill from upstream and downstream industries when computing our bartik instrument. Namely, we use international trade data to compute for each industry the share of inputs that come from other industries (upstream shares) and the share of output bought by other industries (downstream shares). We tag as connected any industry that represents 1% of either the upstream or downstream flows. We recompute the occupation-specific shifts, excluding not only the firms' industry but also all other industries tagged as connected. This results in shifts for which the exogeneity assumption is even more credible. Appendix Tables A4 and A5 present the results of our main reduced-form specification with this modified shift-share predicted hiring difficulty. The coefficient on employment is of the same order of magnitude as in the main specification, and remains highly statistically significant. This alleviates the concern that our result is driven by demand or productivity shocks fueling trhough the input and output linkage.

Sample selection on vacancy data. A potential concern with our analysis is the selected nature of our vacancy level data, which comes from the French job center. According to a recent survey run in 2016, around 50% of hires with online advertising use pole-emploi.fr. Even though we observe a large fraction of all the vacancies posted by French firms, pole-emploi.fr is arguably less likely to be used for job openings with high skill requirements. However, to the extent that firms' outcomes are more sensitive to recruiting frictions on skilled labor, this selection effect should if anything lead us to underestimate the true causal impact of recruiting difficulties on firms' outcomes.

4.3 Other corporate outcomes

We turn to the effect of recruiting difficulties on other corporate outcomes. For this, we run the specification in Equation 2 where the dependent variable is respectively firm investment, profitability, and sales growth. Table 5 presents the results. As shown in columns (1) to (8), recruiting difficulties have a negative and statistically significant effect on investment, profits, and sales growth.

Firms' scale Quantitatively, the estimates imply that a one-standard deviation increase in recruiting time is associated with a decrease of 0.3 to 0.4 percent in sales. As hiring costs increase in hard-to-recruit times, firms drive down their scale.

Investment A one-standard deviation increase in recruiting time is associated with a decrease of 0.075 basis point in investment rate. As the average investment rate in our sample is 3.8%, this represents a 2 percent decrease. The effect sign suggests two interpretations. First, if labor and capital are substitutes, the positive substitution effect triggered by the increase in the relative cost of labor wrt capital is not large enough to compensate for the negative scale effect, which pushes investment downwards. Second, capital and hard-to-recruit labor are indeed complements, which magnifies the effect of recruiting time on overall firms scale, as firms cannot smooth the cost shock by intensifying their production in labor.

Profits In our sample, the profit rate over assets is 6.9% on average. A one-standard deviation increase in recruiting time is associated with a decrease of 0.8 basis point in investment rate. In relative terms, this amounts to a 12% decrease in profit rates.

5 Mechanisms and adjustment margins

We now exploit the richness of our micro data to investigate the adjustment margins of firms facing high recruiting difficulties.

5.1 Recruitment Intensity and Labor turnover

Effect on job postings. We investigate in Table 6 whether firms open less or more vacancies following an increase in recruiting difficulties. In search and matching models, the effect of recruiting time on vacancy posting comes though two channels. On the one hand, the firm post less vacancies as targeted employment decreases (similar to a scale effect). On the other hand, it takes more time to replace workers who separated, so that firms need to post more vacancies to reach a given employment level (vacancy yield effects). Ultimately, the sign of the overall effect depends on the relative strength of the scale effect and the yield effect. To investigate the effect on vacancy posting, we consider in turn a vacancy dummy that equals one if the firm opens at least one vacancy in year t, and the vacancy (resp. jobs) rate defined as in Davis et al. (2013), namely as the number of vacancies (jobs) reported in year t divided by a measure of total jobs equal the sum of vacancies (jobs) and the simple average of employment in t-1 and t. The results consistently indicate that recruiting difficulties are associated with a decline in the number of vacancies posted by firms on the pole-emploi.fr website. Quantitatively, the estimate in Column (1) implies that a one-standard deviation increase in recruiting time is associated with a decrease of 3 percentage points in the probability of opening a vacancy.

Turnover In Table 7, we report the effects of predicted time-to-fill on firms labor turnover, yearly hirings in Column (1) and yearly separations in Column (2). We find significant negative effects on hirings. One standard-deviation increase in recruiting time translates into 0.08 less hirings, a percent decrease of 2% wrt average hirings in our sample. The employment decrease documented in the previous section is partly explained by a decrease in hirings. Our finding on hiring is an extra sanity check of internal consistency. It shows that even when we do no restrict on recruitment intensity within the pole-emploi.fr website, we do find negative effects on hirings. Our measure of hiring difficulties observed in pole-emploi.fr data is valid for recruitments outside of the platform.
Column (2) of Table 7 highlights another important margin that firms use to adjust their workforce when hirings become difficult. In difficult times for hirings, their separation rates decrease. In other words, firms seem to hoard on their incumbents, anticipating they would be difficult to replace.

5.2 Wages

We now investigate how firms adjust their wages to hiring difficulties. When hiring becomes difficult, firms may increase their wages to attract the few workers available on the market. In Table 8, we consider the effect of predicted recruiting time on the firms overall payroll wages. We find a negative and significant effect, which is smaller than the effect on employment (see column 4 in Table 3). Consequently, we find a positive effect on yearly wages per worker in Column (2). The effect is statistically and economically significant. One standard deviation in predicted recruiting time is associated to 0.4% higher wages. In Columns (3) and (4), we decompose the yearly wages into its two factors: yearly hours and hourly wages. We do not find that firms compensate for their lower number of employees by increasing the hours intensity of each worker. Workers in firms facing hiring difficulties do not work longer hours to substitute along the intensive margin for the lost hires. On the contrary, it seems that hiring difficulties are associated with increases in hourly wages. Such an effect is consistent with marginal workers bidding up their wages when firms face hiring difficulties. It could also be consistent with workers becoming more productive when firms lose hires because of recruiting difficulties. We shed light on the importance of these two explanations by analyzing separately incumbents employees and new hires.

In Table 9, we report the effects of predicted time-to-fill on yearly hours of incumbents and of new hires respectively in Columns (1) and (2), while Columns (3) and (4) consider effects on hourly wages for the same split of workers. To define incumbency status of workers, we use the short-panel structure of the matched employer-employee data. We define incumbents workers who are employed in the same firm on the last day of the previous calendar year. New hires are the complement group in the firms' workforce. In Columns (1) and (2), we do not find any significant effects on yearly hours for both workers' types. The overall absence of hours effects previously estimated in Table 8 does not mask heterogeneous and opposite effects by workers tenure, where incumbents workers would work longer hours and new hires would have shorter yearly hours, as it takes more time to recruit them. We find stronger heterogeneity in the effects on hourly wages. The effect on hourly wages is concentrated among incumbent workers. It is four times higher than the effect for new hires which is not statistically significant. In Column (5), we indeed verify that hiring difficulties widen significantly the hourly wage gap between incumbents and new hires. The empirical evidence is not consistent with a competition mechanism, where firms increase wages to attract new hires. The competition channel may rather operate on incumbents in an effort to retain them in the firm. This is in line with the above evidence of a decrease in separation rates. Alternatively, the hourly wages effects could also be related to an increase in incumbents productivity through training. While we do not observe training period in our data, we investigate this channel leveraging our detail information on the firm occupation structure.

5.3 Within-firm occupation ladder

We investigate whether firms make incumbents workers move up the occupation ladder in response to hiring difficulties. We do observe changes in the occupation codes of workers from one year to the other within the same firm. To characterize these occupational switches, we use occupational wages, defined as the average yearly wage for all workers employed in a given occupation in the baseline year (2009). Formally, we obtain $W_{occ,2009}$ where occupation is the ??-digit occupation code. For all workers employed during our post-2009 analysis period, we impute the occupational wages corresponding to their current occupation. In Column (2) of Table 10, we report the effect of predicted time to fill on occupational wages: it is statistically significant and large. One standard deviation in recruiting difficulty implies a % increase in occupational wages. In other words, firms facing hiring difficulties upgrade their occupational structure. Their occupational mix is tilted towards high-wage occupation. Interestingly, the occupational upgrading happens for both incumbent workers and new hires, but it is significantly stronger for incumbents (column 3 vs. Column 4). One interpretation of this result is that firms promote incumbents workers to high-wage occupation in an effort to retain them. In settings with rigid wage schedules tied to occupation level, as in typical collective agreements, firms fulfill wage increases by promoting workers to occupation higher up in the firm hierarchy. Another interpretation would be that firms actually train incumbent workers to perform new highly productive tasks. This may be to compensate for the difficulty to hire workers able to perform such tasks. Trained incumbents workers then switch to the occupation level corresponding to their task-ability. Their occupational wage increases. Interestingly, their actual yearly wages do not increase as much (see Column 1). This makes sense, as newly trained workers have probably a lower experience in their new occupation than the average worker employed in that occupation and over which we have computed the occupational wage. Overall, we find that firms make incumbents workers move up the occupation ladder in response to hiring difficulties. This is still uncertain whether this is driven by competition effect or by increase in workers' productivity through training.

5.4 Occupation specificity

Occupation-specificity of the workforce. The effects of recruiting difficulties should be stronger when they relate to tasks that require more occupation-specific experience, in which case it is harder for the firm to find a way to circumvent them. We construct a firm-level measure of workforce occupation-specificity to test whether this is the case. For this, we compute in the sample of all workers switching employers the number of transitions from occupation O ("origin") to occupation D ("destination"). Then, for each occupation D, we compute the share of firm-to-firm transitions in which the worker was employed in their previous firm in the same occupation ($O = D$), and interpret this ratio as a measure of occupation-specificity. Finally, for every firm, we compute the average of the occupation specificity of its employees in 2009 (using its occupation mix as weights).

We present the results in Figure 4. The effect of recruiting difficulties is indeed much stronger for firms with a high occupation-specific workforce. This indicates that these firms are indeed less able on average to redirect their hiring to other types of workers when facing current recruiting difficulties on their occupation-mix.

6 Conclusion

This paper studies the role of recruitment difficulties on firm growth. We build granular measure of vacancies filling rates and time-to-fill by commuting zone X occupation X industry in France. Within a shift-share design, we then show that recruitment difficulties translate into fewer vacancies posted by firms employing workers in hard-to-fill occupations. We then show that recruiting difficulties have real effects on firms' outcomes: their employment, investment, profits and sales are negatively affected. We find that the sensitivity of firm employment to recruiting difficulties is significantly stronger for labor-intensive firms, and for firms with high occupation-specificity. Taken together, our findings indicate that local labor shortages are an important determinant of the performance and growth of firms across time and space. Our work have therefore important implications for the design of policies aiming at reducing the mismatch between firms' needs and the skills of the local workforce, and more generally for the design of location-based policies to foster growth.

References

Acemoglu, D. and J.-S. Pischke (1998): "Why Do Firms Train? Theory and Evidence*," *The Quarterly Journal of Economics*, 113, 79–119.

Acemoglu, D. and F. Zilibotti (2001): "Productivity differences," *The Quarterly Journal of Economics*, 116, 563–606.

Aguiar, M., M. Bils, K. K. Charles, and E. Hurst (2021): "Leisure Luxuries and the Labor Supply of Young Men," *Journal of Political Economy*, 129, 337–382.

Autor, D. (2021): "Good News: There's a Labor Shortage," *New York Times*.

Bagger, J., F. Fontaine, M. Galenianos, and I. Trapeznikova (2021): "Vacancies, Employment Outcomes and Firm Growth: Evidence from Denmark," *Working Paper*.

Beerli, A., J. Ruffner, M. Siegenthaler, and G. Peri (2021): "The Abolition of Immigration Restrictions and the Performance of Firms and Workers: Evidence from Switzerland," *American Economic Review*, 111, 976–1012.

Besley, T., N. Fontana, and N. Limodio (2021): "Antitrust Policies and Profitability in Nontradable Sectors," *American Economic Review: Insights*, 3, 251–65.

Borusyak, K., P. Hull, and X. Jaravel (2021): "Quasi-Experimental Shift-Share Research Designs," *The Review of Economic Studies*, rdab030.

Cahuc, P., S. Carcillo, and T. Le Barbanchon (2018): "The Effectiveness of Hiring Credits," *The Review of Economic Studies*, 86, 593–626.

Carrillo-Tudela, C., H. Gartner, and L. Kaas (2020a): "Recruiting Policies, Job-Filling Rates and Matching Efficiency," *Working Paper*.

Carrillo-Tudela, C., A. Launov, and J.-M. Robin (2020b): "The Fall in German Unemployment: A Flow Analysis," *Working Paper*.

Chava, S., A. Danis, and A. Hsu (2020): "The economic impact of right-to-work laws: Evidence from collective bargaining agreements and corporate policies," *Journal of Financial Economics*, 137, 451–469.

D'Acunto, F., M. Weber, and S. Yang (2020): "Manpower Constraints and Corporate Policies," *Working Paper*.

Davis, S. J., R. J. Faberman, and J. C. Haltiwanger (2013): "The establishment-level behavior of vacancies and hiring," *The Quarterly Journal of Economics*, 128, 581–622.

Forsythe, E. and R. Weinstein (2021): "Recruiting Intensity, Hires, and Vacancies: Evidence from Firm-Level Data," *Working Paper*.

Gavazza, A., S. Mongey, and G. L. Violante (2018): "Aggregate Recruiting Intensity," *American Economic Review*, 108, 2088–2127.

Glaeser, E. L. and J. D. Gottlieb (2008): "The Economics of Place-Making Policies," *Brookings Papers on Economic Activity*, 1, 155–253.

Goldsmith-Pinkham, P., I. Sorkin, and H. Swift (2020): "Bartik Instruments: What, When, Why, and How," *American Economic Review*, 110, 2586–2624.

Hall, R. E. (2005): "Job Loss, Job Finding, and Unemployment in the US Economy over the Past Fifty Years," *NBER Macroeconomics Annual, ed. Mark Gertler and Kenneth Rogoff*.

Haskel, J. and C. Martin (1993a): "The Causes of Skill Shortages in Britain," *Oxford Economic Papers*, 45, 573–588.

——— (1993b): "Do Skill Shortages Reduce Productivity? Theory and Evidence from the United Kingdom," *The Economic Journal*, 103, 386–394.

——— (2001): "Technology, Wages, and Skill Shortages: Evidence from UK Micro Data," *Oxford Economic Papers*, 53, 642–658.

Heckman, J. J. (1974): "Effects of Child-Care Programs on Women's Work Effort," *Journal of Political Economy*, 82, S136–S163.

Hershbein, B. and L. B. Kahn (2018): "Do Recessions Accelerate Routine-Biased Technological Change? Evidence from Vacancy Postings," *American Economic Review*, 108, 1737–72.

Holzer, H. J. (1994): "Job Vacancy Rates in the Firm: An Empirical Analysis," *Economica*, 61, 17–36.

Jagannathan, R., D. A. Matsa, I. Meier, and V. Tarhan (2016): "Why do firms use high discount rates?" *Journal of Financial Economics*, 120, 445–463.

Kaas, L. and P. Kircher (2015): "Efficient Firm Dynamics in a Frictional Labor Market," *American Economic Review*, 105, 3030–60.

Katz, L. F. and K. M. Murphy (1992): "Changes in relative wages, 1963–1987: supply and demand factors," *The quarterly journal of economics*, 107, 35–78.

Kerr, S. P., W. Kerr, Ç. Özden, and C. Parsons (2016): "Global talent flows," *Journal of Economic Perspectives*, 30, 83–106.

Kline, P. (2010): "Place Based Policies, Heterogeneity, and Agglomeration," *American Economic Review*, 100, 383–87.

Marinescu, I. and R. Rathelot (2018): "Mismatch Unemployment and the Geography of Job Search," *American Economic Journal: Macroeconomics*, 10, 42–70.

Mian, A. and A. Sufi (2014): "What explains the 2007–2009 drop in employment?" *Econometrica*, 82, 2197–2223.

Modestino, A. S., D. Shoag, and J. Ballance (2020): "Upskilling: Do Employers Demand Greater Skill When Workers Are Plentiful?" *The Review of Economics and Statistics*, 102, 793–805.

Moretti, E. (2011): "Local Labor Markets," *Handbook of Labor Economics*, 1237–1313.

Mueller, A. I., D. Osterwalder, J. Zweimüller, and A. Kettemann (2018): "Vacancy Durations and Entry Wages: Evidence from Linked Vacancy-Employer-Employee Data," Working Paper 25118, National Bureau of Economic Research.

Rothwell, J. (2014): "Still searching: Job vacancies and STEM skills," *Metropolitan Policy Program at Brookings Institution*.

Sauvagnat, J. and F. Schivardi (2020): "Are Executives in Short Supply? Evidence from Death Events," *Working Paper*.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25–49.

——— (2007): "Mismatch," *American Economic Review*, 97, 1074–1101.

Terry, E. and M. de Zeeuw (2020): "How Do Firms Respond to Hiring Difficulties? Evidence from the Federal Reserve Banks' Small Business Credit Survey," *Federal Reserve Bank of Atlanta Discussion Paper*.

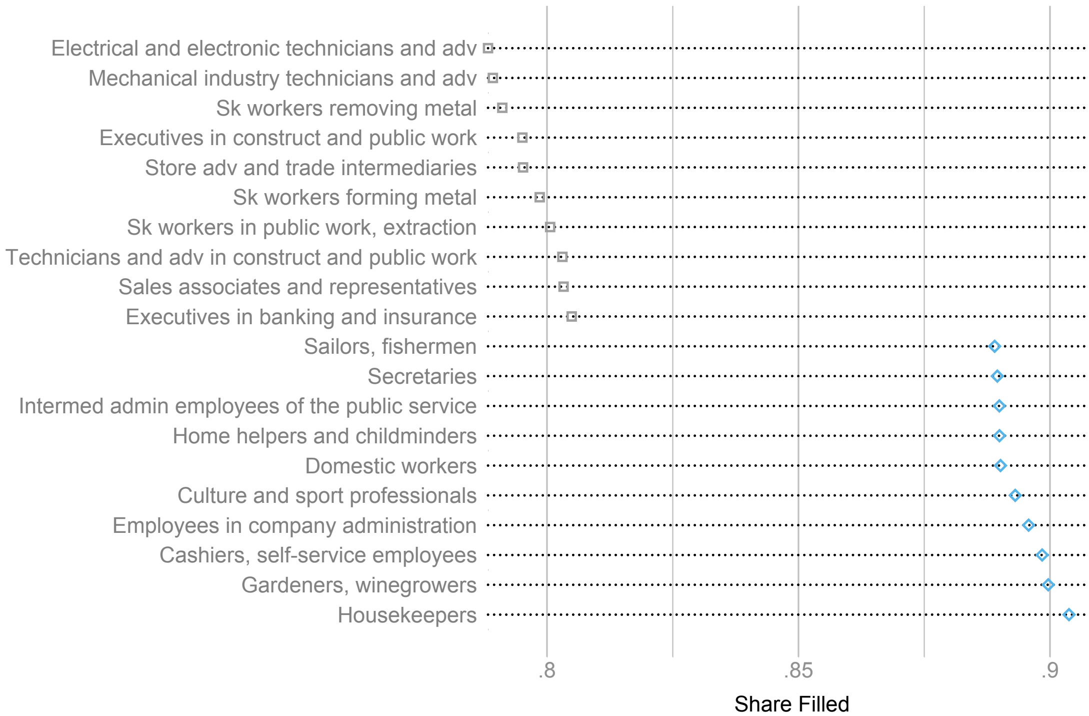
Weaver, A. (2021): "Who Has Trouble Hiring? Evidence from a National IT Survey," *ILR Review*.

Şahin, A., J. Song, G. Topa, and G. L. Violante (2014): "Mismatch Unemployment," *American Economic Review*, 104, 3529–64.

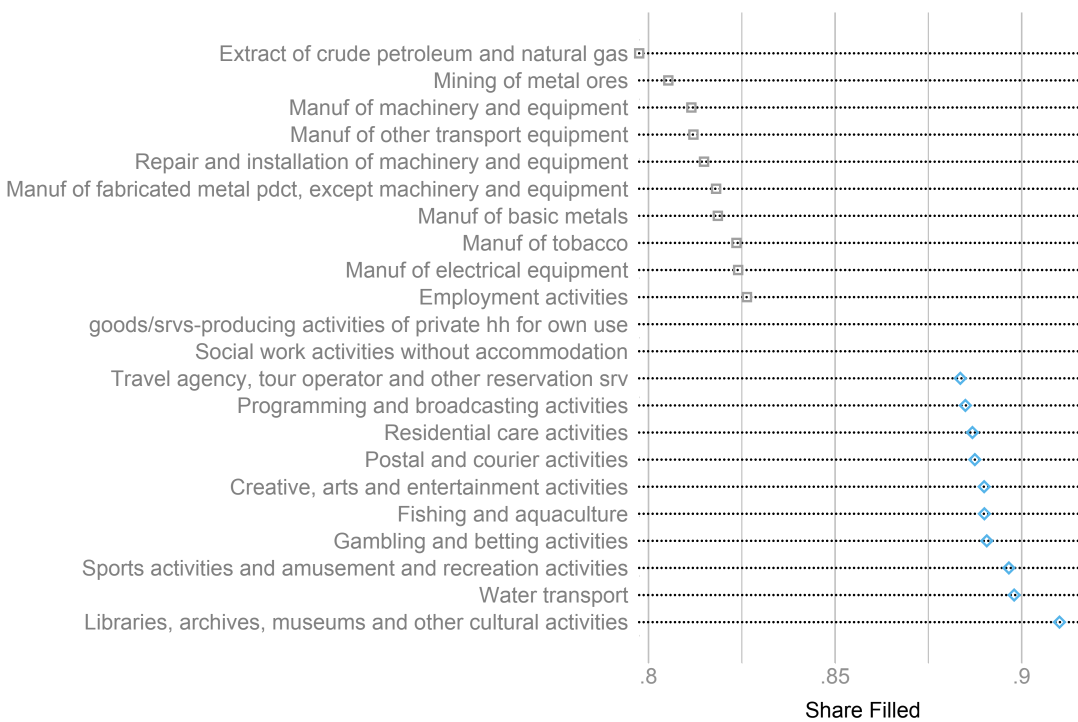
FIGURES

Figure 1: Share of filled vacancies

(a) 2-digit occupations



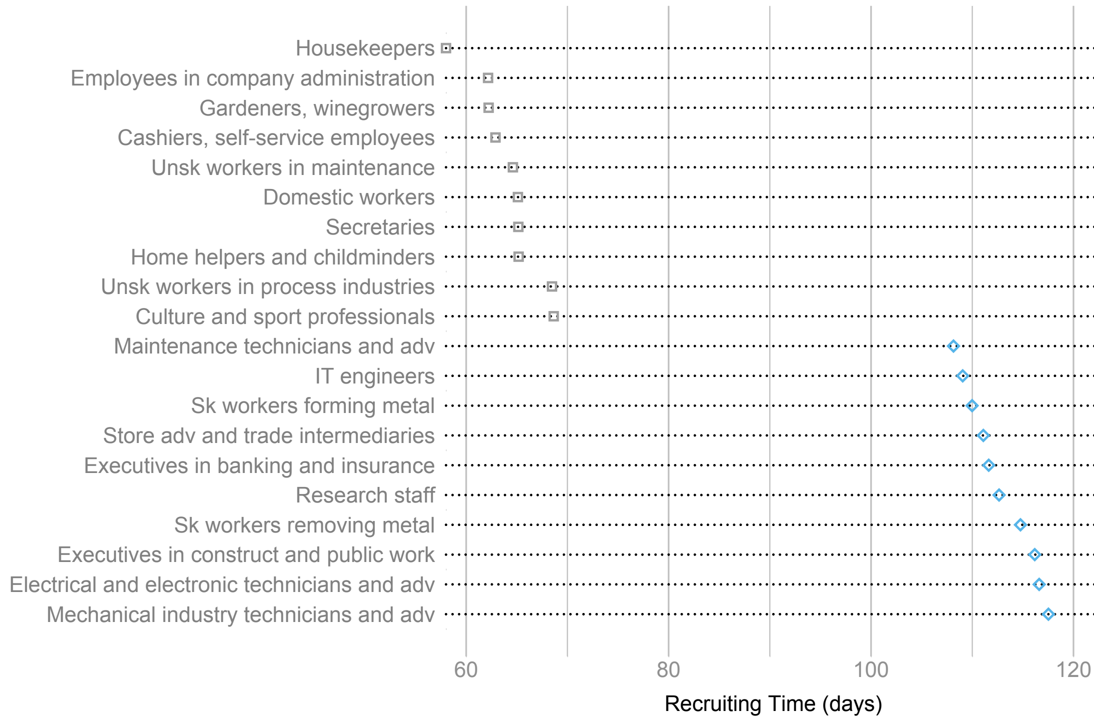
(b) 2-digit Industries



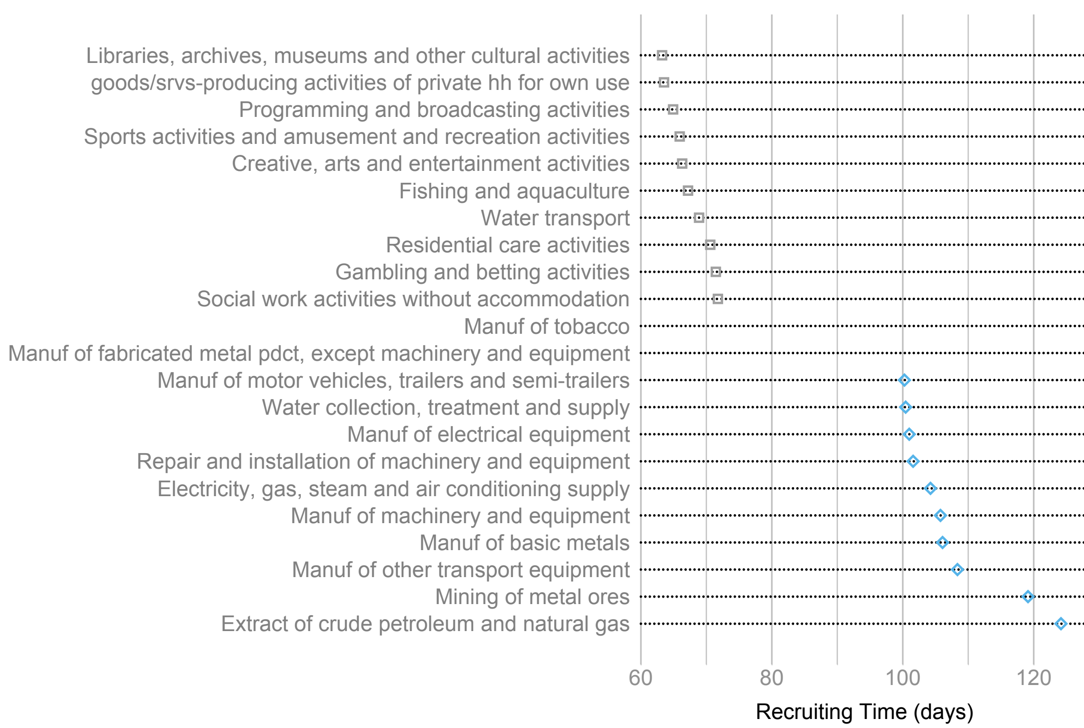
Source: vacancies posted on pole-emploi.fr from 2010 to 2017.
Note: This Figure shows average share of filled vacancies by 2-digit occupation group in panel 2a and by 2-digit industry in panel 2b.

Figure 2: Average recruiting time

(a) 2-digit occupations

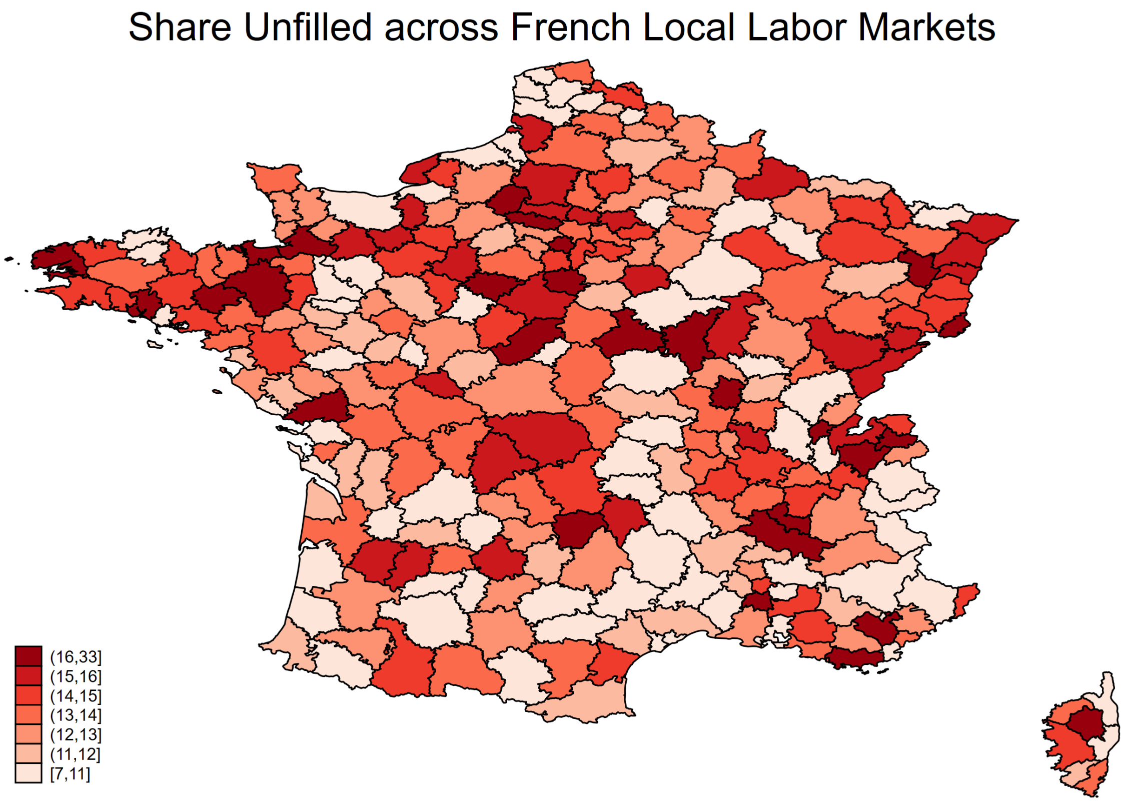


(b) 2-digit Industries



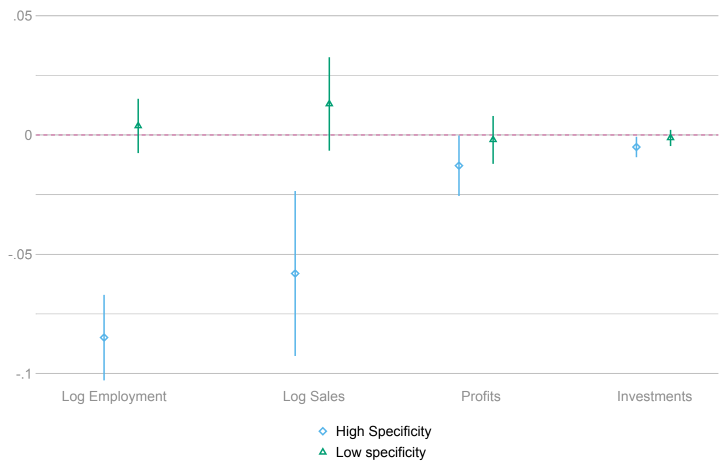
Source: vacancies posted on pole-emploi.fr from 2010 to 2017.
Note: This Figure shows average time-to-fill vacancies (in days) by 2-digit occupation group in panel **??** *and by 2-digit industry in panel* **??**. *Time-to-fill is capped at 365 days, and also set at 365 days for failed recruitments.*

Figure 3: Share of filled vacancies by commuting zones



This figure plots the share of filled vacancies by commuting zones.

Figure 4: Effects on Corporate Outcomes - Low vs High occupation specificity



To derive the specificity of a given occupation D (”destination”), we count in the sample of all workers switching employers the number of transitions from occupation O (”origin”) to occupation D, and compute the share of transitions in which the worker was employed in their previous firm in the same occupation ($O = D$). For every firm, we then compute the average of the occupation specificity of its employees in 2009 (using its occupation mix as weights). The workforce of a given firm is considered as being occupation-specific if its occupation-specificity ratio lies above the sample median in 2009.

TABLES

Table 1: **Descriptive Statistics**

	Mean	Sd	Min	Max	N
Hiring Difficulties					
Share Filled Predicted	0.849	0.075	0.000	1.000	2722997
Time to Fill Predicted	0.237	0.075	0.000	1.000	2722997
Share Filled	0.865	0.266	0.000	1.000	830716
Time to Fill	0.220	0.252	0.000	1.000	830716
Employment variables					
Employment	23.134	288.640	1	102860	3029009
Log Employment	2.044	1.093	0.693	11.541	3029009
Open Vacancy	0.274	0.446	0.000	1.000	3029009
Vac Rate	0.057	0.129	0.000	0.999	2987528
Jobs Rate	0.060	0.134	0.000	0.999	2987528
Other variables					
Investment	0.039	0.077	0.000	0.724	2963041
Profits (ROA)	0.066	0.259	-3.507	1.287	2962950
Log Sales	6.659	1.487	0.000	10.189	3028696

This table presents summary statistics for our sample, which consists of 3,029,009 firm-year observations between 2009 and 2017. There are XXX firms in this sample for which we observe the occupation-mix in 2009. Share Filled Predicted and Time to Fill Predicted are respectively the shift-share prediction of share filled and recruiting time computed as in Equation 1. Firms' employment is defined as the number of full-time employees at the end of the fiscal year. The vacancy (jobs) rate is computed as in Davis et al. (2013), namely as the number of vacancies (jobs) reported in year *t* divided by a measure of total jobs, defined as the sum of vacancies (jobs) and the simple average of employment in *t-1* and *t*. Return on assets (ROA) is defined as earnings before interest, depreciation, and taxes over assets. Investment is scaled by assets.

Table 2: **First Stage**

	(1)	(2)	(3)	(4)
	Share Not Filled		Time to Fill	
Share Not Filled Predicted	0.088***	0.069***		
	(0.010)	(0.013)		
Time to Fill Predicted			0.091***	0.075***
			(0.009)	(0.012)
Firm FE	Yes	Yes	Yes	Yes
Ind*Year	Yes	No	Yes	No
Cz*Year	Yes	No	Yes	No
Ind*Cz*Year	No	Yes	No	Yes
Observations	563474	563474	563474	563474
R-Sq	0.346	0.447	0.365	0.464
Dep Var Mean	0.133	0.133	0.218	0.218

This table show the results obtained from estimating equation 2 on the sub-sample of firms that in any given year are observed opening at least one vacancy. Both outcome variables take values between zero and one. Standard errors are clustered at the commuting zone level.

Table 3: **Effects on Employment**

	(1)	(2)	(3)	(4)
	Log Employment			
Share Not Filled Predicted	-0.017***	-0.022***		
	(0.004)	(0.005)		
Time to Fill Predicted			-0.022***	-0.029***
			(0.004)	(0.005)
Firm FE	Yes	Yes	Yes	Yes
Ind*Year	Yes	No	Yes	No
Cz*Year	Yes	No	Yes	No
Ind*Cz*Year	No	Yes	No	Yes
Observations	2616644	2616644	2616644	2616644

This table show the results obtained from estimating equation 2 on the entire sample of firms where the dependent variable is the logarithm of the number of full-time employees at the end of the fiscal year. Standard errors are clustered at the commuting zone level.

Table 4: **Two-Stage Least Square Effects on Employment**

	(1)	(2)	(3)	(4)
	Log Employment - 2SLS			
Share Not Filled	-0.193***	-0.314***		
	(0.049)	(0.092)		
Time to Fill			-0.24***	-0.391***
			(0.049)	(0.092)
Firm FE	Yes	Yes	Yes	Yes
Ind*Year	Yes	No	Yes	No
Cz*Year	Yes	No	Yes	No
Ind*Cz*Year	No	Yes	No	Yes
Obs. (red. form)	2616644	2616644	2616644	2616644
Obs. (1st stage)	563474	563474	563474	563474

This table show the results obtained from estimating the 2SLS equation 2, where the average share of unfilled vacancies (respectively time-to-fill of vacancies) at the firm level is instrumented with the shift-share variable. The Wald estimator is adjusted in order to derive the 2SLS estimates for the whole sample of firms (that is, for both those posting and not posting vacancies on Pôle Emploi). Standard errors are clustered at the commuting zone level.

Table 5: **Effects on Firm Outcomes**

	(1)	(2)	(3)	(4)	(5)	(6)
	Investment (basis points)		Profits (ROA) (basis points)		Log Sales	
Share Not Filled Predicted	-0.3**		-0.8**		-0.013*	
	(0.1)		(0.3)		(0.007)	
Time to Fill Predicted		-0.3**		-1.0***		-0.015**
		(0.1)		(0.3)		(0.007)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2558587	2558587	2558493	2558493	2616344	2616344
Dep Var Mean	3.8	3.8	6.9	6.9		
1 sd increase effects	0.075	0.075	0.20	0.25	0.3%	0.37%

This table show the results obtained from estimating equation 2 in specifications in which the dependent variable is respectively Investment, ROA, and the logarithm of firms' sales. Standard errors are clustered at the commuting zone level.

Table 6: **Effects on Vacancies**

	(1)	(2)	(3)	(4)	(5)	(6)
	Vacancy Dummy		Vacancy Rate		Offered Jobs Rate	
Share Not Filled Predicted	-0.012**		-0.004**		-0.004***	
	(0.005)		(0.002)		(0.002)	
Time to Fill Predicted		-0.015***		-0.005***		-0.005***
		(0.005)		(0.002)		(0.002)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2616644	2616644	2579014	2579014	2579014	2579014
Dep Var Mean	0.260	0.260	0.058	0.058	0.061	0.061

This table show the results obtained from estimating equation 2 on the entire sample of firms. The first two columns show the effects on the probability of opening at least one vacancy. Because each vacancy opened by a firm may refer to one or more jobs posting, columns (3)-(4) (5)-(6) show the effects on the vacancy rate and jobs rate respectively. We measure the vacancy (jobs) rate at t as in Davis et al. (2013), namely as the number of vacancies (jobs) reported in year t divided by a measure of total jobs, defined as the sum of vacancies (jobs) and the simple average of employment in t-1 and t. Standard errors are clustered at the commuting zone level.

Table 7: **Effects on Hirings and Separations**

	(1)	(2)
	Yearly hirings	yearly separations
Time to Fill Predicted	-0.308**	-0.263*
	(0.125)	(0.151)
Firm FE	Yes	Yes
Ind*Cz*Year	Yes	Yes
Observations	2616644	2191350
Dep Var Mean	4.693	4.488

Note: This table shows the results obtained from estimating equation 2 in specifications where the dependent variable is respectively the number of hirings and of separations. Standard errors are clustered at the commuting zone level.

Table 8: **Effects on Wages**

	(1) Payroll wages (log)	(2) Yearly wages per worker (log)	(3) Yearly Hours per worker (log)	(4) Hourly wages per worker (log)
Time to Fill Predicted	-0.020***	0.017***	0.006	0.035***
	(0.006)	(0.005)	(0.005)	(0.006)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	2616644	2616644	2616644	2615559

Note: This table shows the results obtained from estimating equation 2 in specifications where the dependent variable is respectively the overall payroll wages (in log), the yearly wage per worker (in log), the yearly hours per worker (in log) and the hourly wage per worker (in log). Standard errors are clustered at the commuting zone level.

Table 9: **Effects on Wages and on Hours of Incumbents and new Hires**

	(1)	(2)	(3)	(4)	(5)
	Yearly hours per worker (log)		Hourly wage per worker (log)		Wage gap
	Incumbent	Hires	Incumbent	Hires	Hires-Inc.
Time to Fill Predicted	-0.003	0.017	0.038***	0.010	-0.018**
	(0.005)	(0.016)	(0.007)	(0.008)	(0.009)
Firm FE	Yes	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes	Yes
Observations	2525568	1641530	2524385	1639580	1551397
Dep Var Mean	7.302	6.476	2.711	2.460	-0.257

Note: This table shows the results obtained from estimating equation 2 in specifications where the dependent variable is respectively the yearly hours of incumbents and of new hires (in log), and the hourly wage of incumbents and of new hires (in log). Standard errors are clustered at the commuting zone level.

Table 10: **Effects on Occupational Wages**

	(1)	(2)	(3)	(4)
	Yearly wages per worker (log)	Occupational wages yearly per worker (log)		
		All	Incumbents	Hires
Time to Fill Predicted	0.017***	0.062***	0.071***	0.025***
	(0.005)	(0.010)	(0.011)	(0.007)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	2616633	2616633	2525552	1641480

Note: This table shows the results obtained from estimating equation 2 in specifications where the dependent variable is respectively the yearly wage per worker (in log), the yearly occupational wages per worker (in log) eventually split by incumbency. For every worker we impute their occupational wage, defined as the average wages of workers in that occupation in the baseline year. Standard errors are clustered at the commuting zone level.

Online Appendix

TITLE

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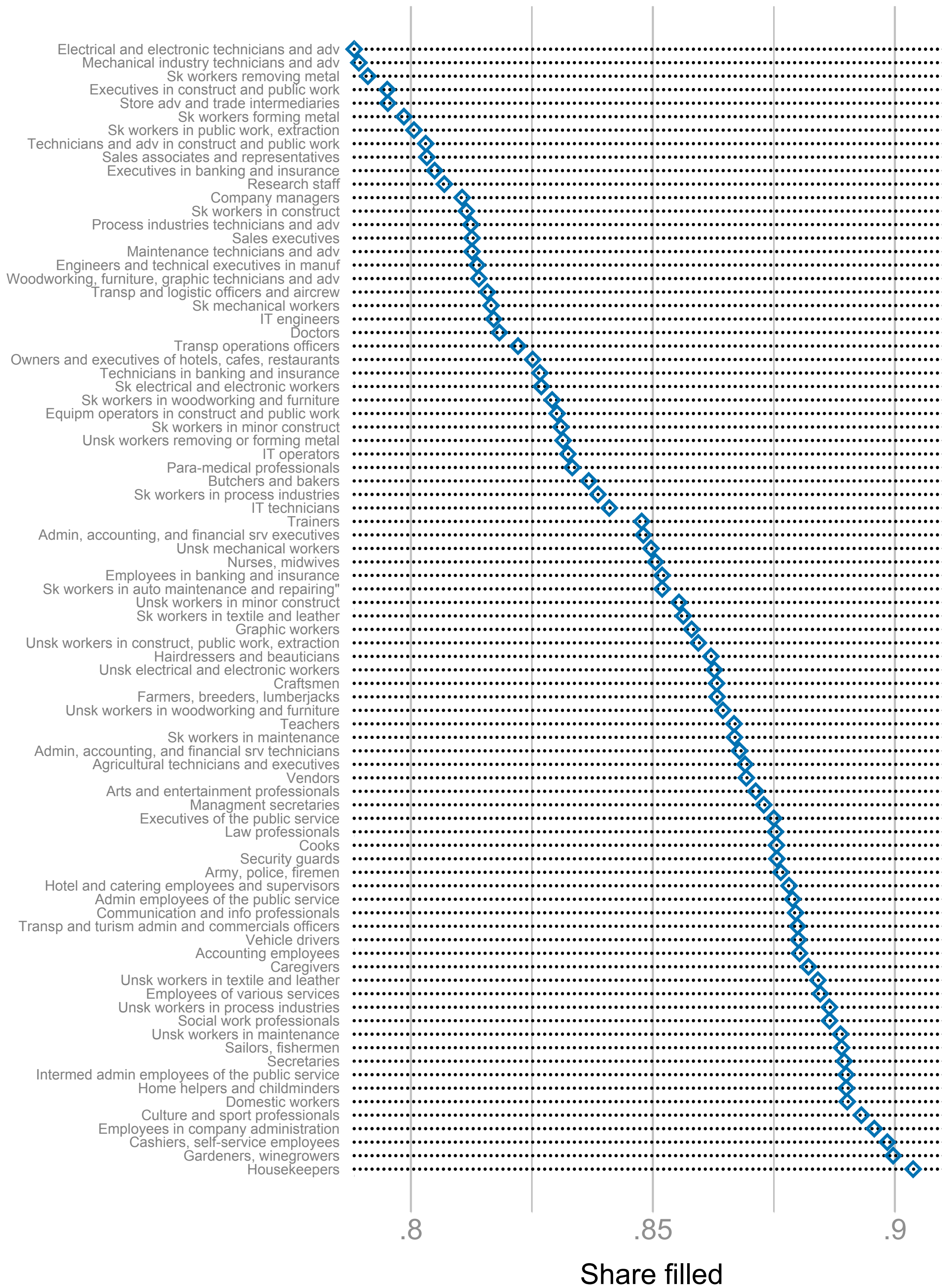
The online appendix has several sections. Appendix A includes extra figures and tables. In Appendix B, we compare our main measure of hiring difficulties from vacancy data to survey answers by firms.

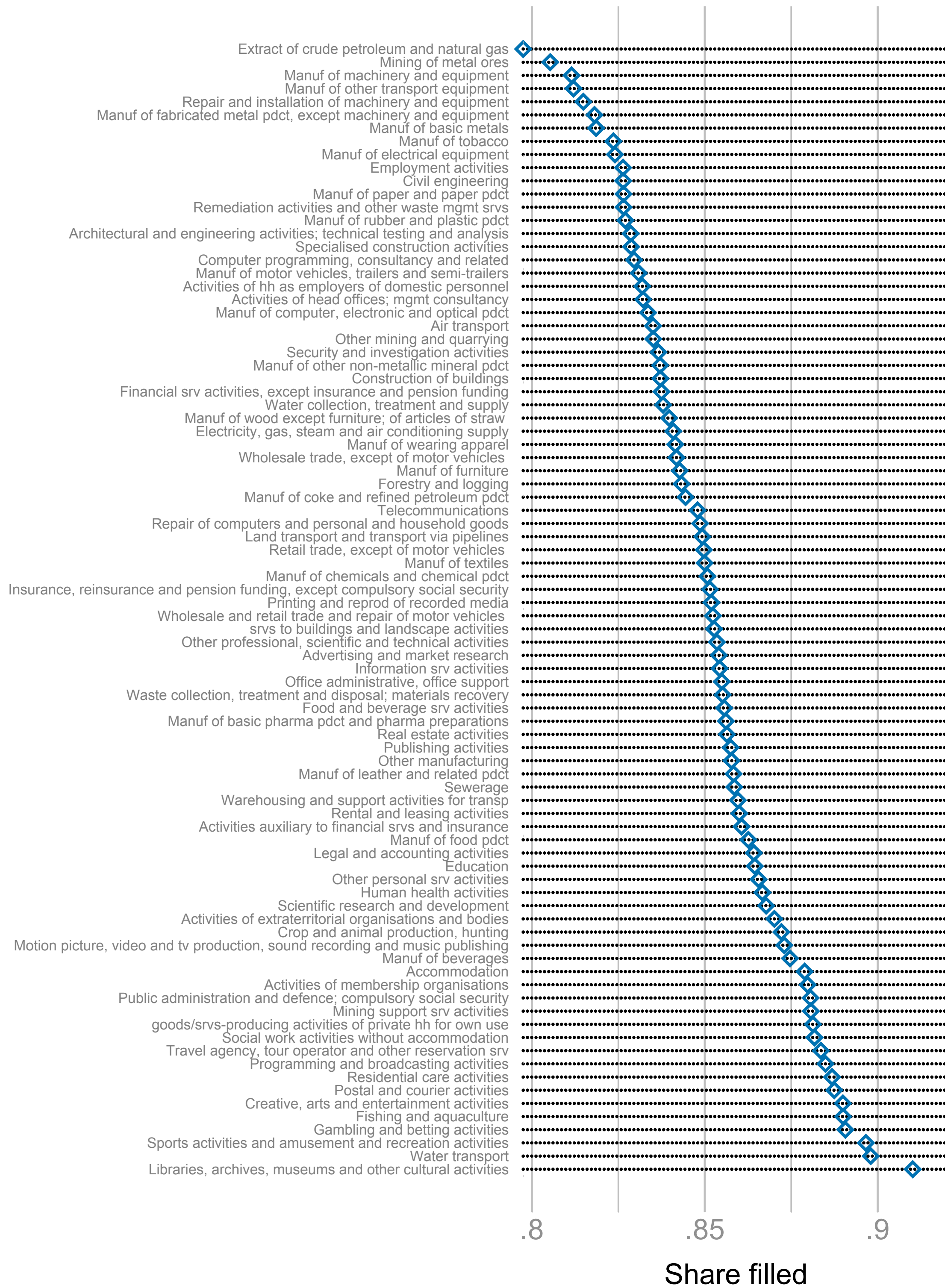
A Appendix Figures and Tables

Figure A1: Share of filled vacancies by 2-digit Occupations



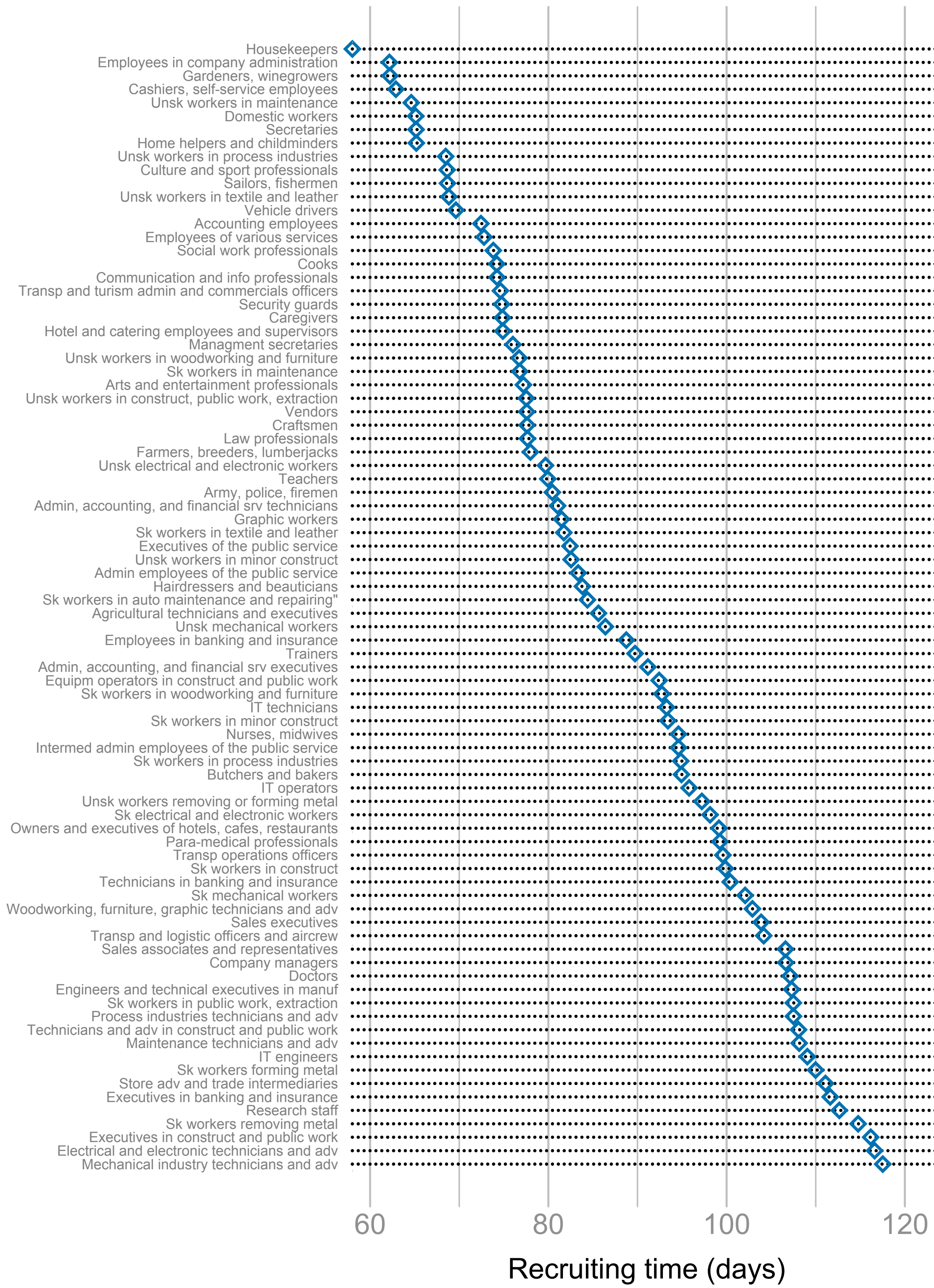
This figure plots the share of filled vacancies by 2-digit occupation group.

Figure A2: Share of filled vacancies by 2-digit Industries



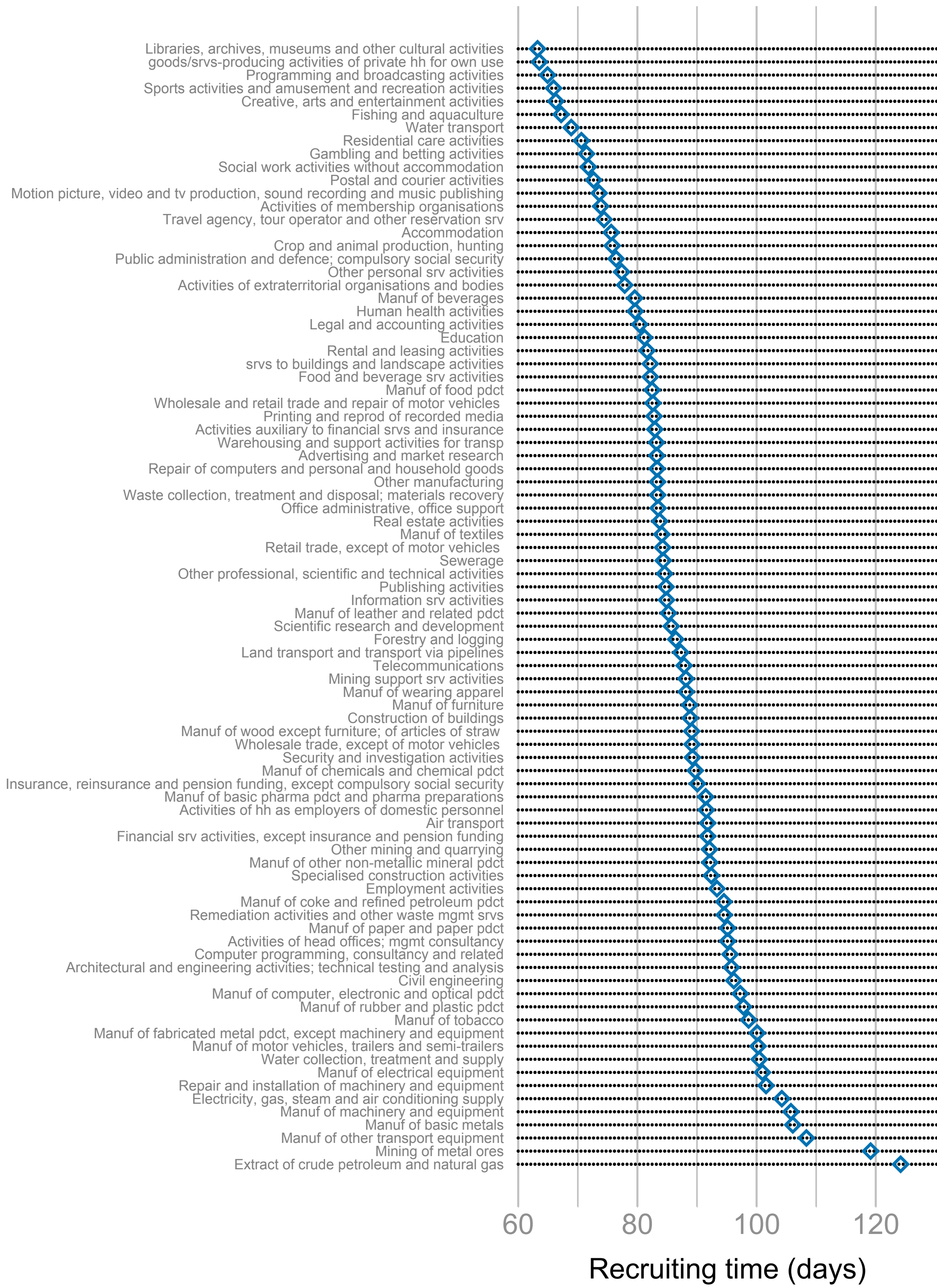
This figure plots the share of filled vacancies by 2-digit industry group.

Figure A3: Average Recruiting Time by 2-digit Occupations



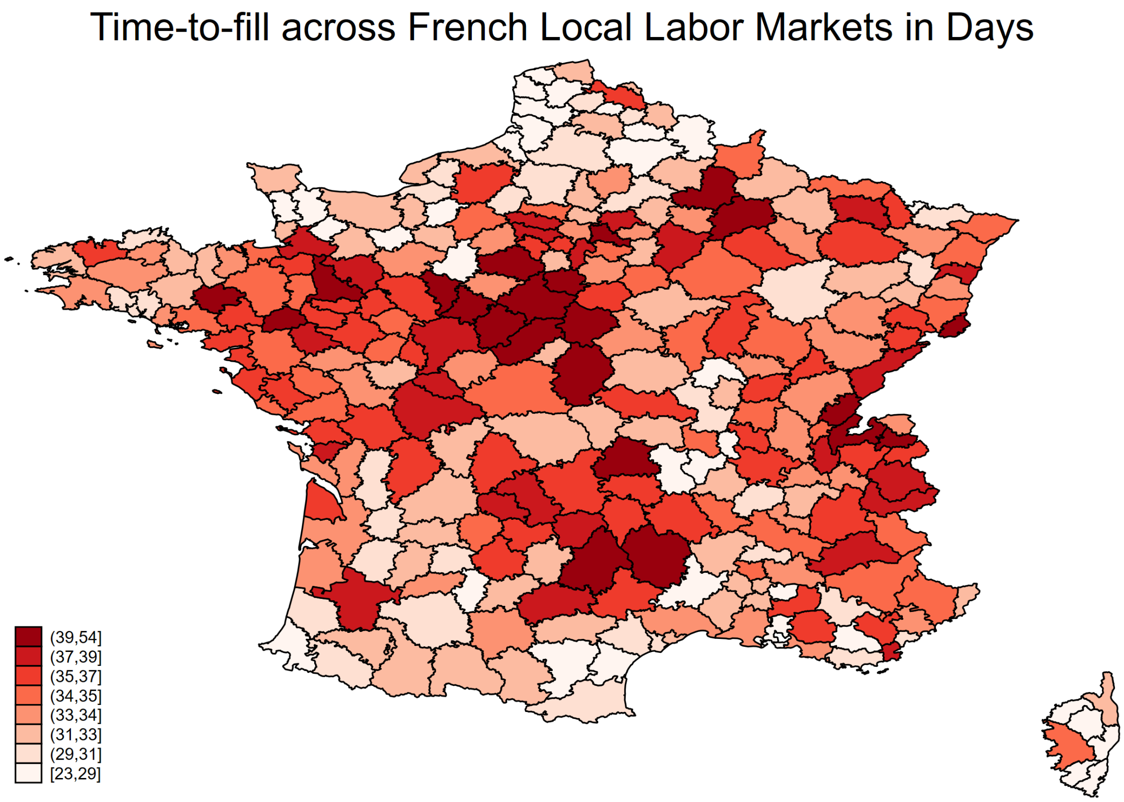
This figure plots average recruiting time over our sample period, measured in days for each two-digit occupation. Recruiting time is set to be equal to 365 if the vacancy remains unfilled.

Figure A4: Average Recruiting Time by 2-digit Sector



This figure plots average recruiting time measured in days by two-digit industry. Recruiting time is set to be equal to 365 if the vacancy is unfilled.

Figure A5: Average Recruiting Time by commuting zones



This figure plots average recruiting time measured in days by commuting zones. Recruiting time is set to be equal to 365 if the vacancy is unfilled.

Table A1: **Effects on Employment - Labor Intensive vs Not Labor Intensive**

	(1)	(2)	(3)	(4)
	Labor Intensive		Not Labor Intensive	
Share Not Filled Predicted	-0.028***		-0.010	
	(0.006)		(0.007)	
Time to Fill Predicted		-0.037***		-0.016**
		(0.007)		(0.007)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	1217001	1217001	1254940	1254940

This table show the results obtained from estimating our most stringent specification for labor intensive firms and not labor intensive firms separately. A firm is defined as (not) labor intensive if in 2009, i.e. our baseline year, it had an employment to asset ratio (below) above the median value in our sample (which is 68.5). Standard errors are clustered at the commuting zone level.

Table A2: **Robustness to business stealing effects: Effects on Employment - Tradable vs Nontradable (Besley et al. (2021) definition)**

	(1)	(2)	(3)	(4)
	Tradable		Nontradable	
Share Not Filled Predicted	-0.021*		-0.021***	
	(0.011)		(0.005)	
Time to Fill Predicted		-0.024*		-0.029***
		(0.013)		(0.006)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	276008	276008	2338327	2338327

This table show the results obtained from estimating our most stringent specification for firms operating in tradable and nontradable sectors separately. In **this version** of the analysis we follow the categorization of sectors by Besley et al. (2021), namely tradable sectors are agriculture, forestry, and fishing (A); mining and quarrying (B); and manufacturing (C). Standard errors are clustered at the commuting zone level.

Table A3: **Robustness to business stealing effects: Effects on Employment - Tradable vs Nontradable (Mian and Sufi (2014) definition)**

	(1)	(2)	(3)	(4)
	Tradable		Nontradable	
Share Not Filled Predicted	-0.029**		-0.020***	
	(0.013)		(0.005)	
Time to Fill Predicted		-0.034**		-0.028***
		(0.015)		(0.005)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	346972	346972	2266773	2266773

This table show the results obtained from estimating our most stringent specification for firms operating in tradable and nontradable sectors separately. We follow the categorization of sectors by Mian and Sufi (2014), namely tradable sectors are agriculture, forestry, and fishing (A); mining and quarrying (B); manufacturing (C); and information and communication (J). Standard errors are clustered at the commuting zone level.

Table A4: **Robustness to Input-Output Linkages**

	(1) Employment Log	(2) Investment	(3) Profits (ROA)	(4) Log Sales Log
Time to Fill Predicted	-0.020***	-0.003***	-0.008***	-0.006
	(0.005)	(0.001)	(0.003)	(0.006)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	2554492	2554492	2554492	2554492
Dep Var Mean		0.038	0.069	

We exclude from time-to-fill computation the industries linked to the firms industry through input-output matrix (more than 1% market share)

Table A5: **Robustness to Input-Output Linkages**

	(1) Employment Log	(2) Investment	(3) Profits (ROA)	(4) Log Sales Log
Share Not Filled Predicted	-0.013***	-0.002***	-0.006***	-0.006
	(0.004)	(0.001)	(0.002)	(0.005)
Firm FE	Yes	Yes	Yes	Yes
Ind*Cz*Year	Yes	Yes	Yes	Yes
Observations	2613634	2554492	2554492	2613634
Dep Var Mean		0.038	0.069	

We exclude from unfilled rate computation the industries linked to the firms industry through input-output matrix (more than 1% market share)

B Comparison of hiring difficulties measured in vacancy data vs. in firm surveys

In this section, we compare our main main measures of hiring difficulties from vacancy data to survey answers by firms. We use two surveys: the Business Tendency Survey (BTS) from the French Statistical Institute (Insee) and the Workforce Firm Survey from the French Public Employment Service (Pole Emploi).

The BTS surveys a panel of French establishments every month in order to forecast economic growth (*Enquête de conjoncture*). It includes questions on labor bottlenecks. The Workforce survey also surveys firms to assess manpower needs in the French labor market (*Besoin de Main d'oeuvre*).

In the BTS, firms are asked whether they currently encounter recruiting difficulties (yes/no question). The question is ventilated across three types of labor: executives, skilled workers, and unskilled workers. We have access to the BTS data covering manufacturing firms. We aggregate their answers at the year X industry level, where industries are within the fine 5-digit classification (NAF-5d). We restrict the period to 2010-2017 over which we have the vacancy data. Similarly, we collapse recruiting time and share of unfilled vacancies at the year X industry level, both across all vacancies, and separately for the subsamples of vacancies for executives, for skilled workers and for unskilled workers. Figure A6 (resp. A7) plots binscatters of recruiting time (resp. unfilled rate) against the average share of establishments reporting hiring difficulties. Each panel corresponds to one survey question. Year X Industry cells are weighted by the number of firms surveyed.

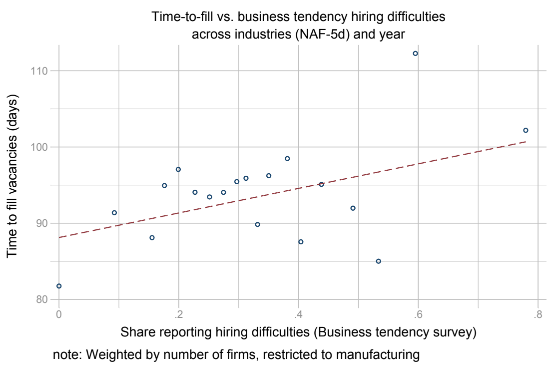
We find a positive and significant correlation between the survey measures and our measures of recruiting time / share of unfilled vacancies. Table A6 reports the slopes of the binscatter plots, which are all highly statistically significant. The across-cell standard deviation in reported hiring difficulties is 0.18, which would increase the time to recruit by 3 days (almost 4% of the average time to recruit, or 15% of its standard deviation). Such an increase in reported recruiting difficulties is associated with a 0.6 percentage points increase in the share of unfilled vacancies (around 4% of average rate, or of its standard deviation).

The PES manpower survey is instead available at the occupation level. It asks every firm in which occupation they intend to recruit, and for every such occupation the number of workers to be recruited and the number of difficult recruiting searches. Beyond the detailed occupation questions, the manpower survey has the key advantage of covering all industries. We have access to aggregate counts by occupation (fine 5-digit level, fap), year and department for the period 2015-2017. The French metropolitan territory is partitioned in 100 departments. This geographical unit is larger than the more than 300 commuting zones of the main analysis. We collapse the vacancy data at the same level and over the same period. Figure A8 reports binscatters of recruiting time / unfilled vacancies against the reported share of difficult recruiting processes. We weight cells by the overall number of recruiting processes. Table A7 reports the slope coefficients with or without controlling for occupation and departments fixed effects. Again, we find a significant and positive correlation between the survey-based measures and the vacancy-based measures of hiring difficulties. Quantitatively, one standard deviation increase in reported hiring difficulties is related to increase in recruiting time and unfilled vacancy shares of 8%-10%, compared to their averages.

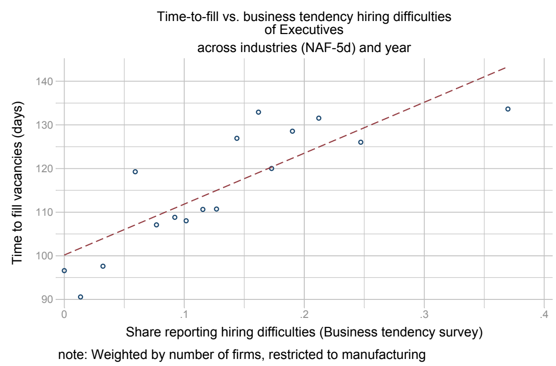
We conclude that our proxies for recruiting difficulties based on the expected probability of filling a vacancy and the average time it takes to recruit a worker indeed relate strongly with firms' own-assessment in surveys of the difficulty they face for findings workers on the labor market.

Figure A6: Recruiting Time vs. hiring difficulties in Business Tendency Survey

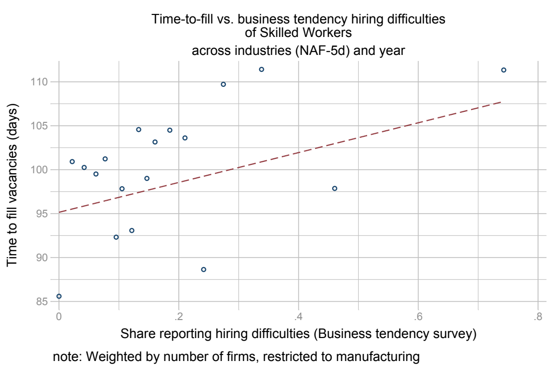
(a) All occupations



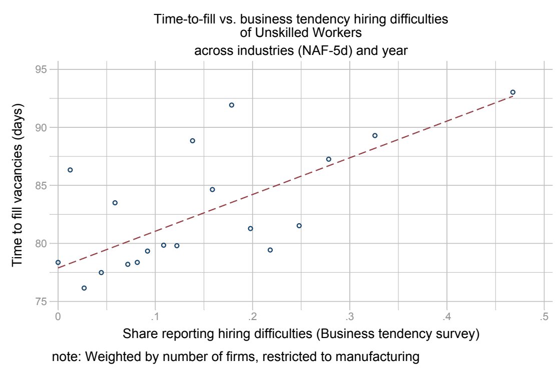
(b) Executives



(c) Skilled workers



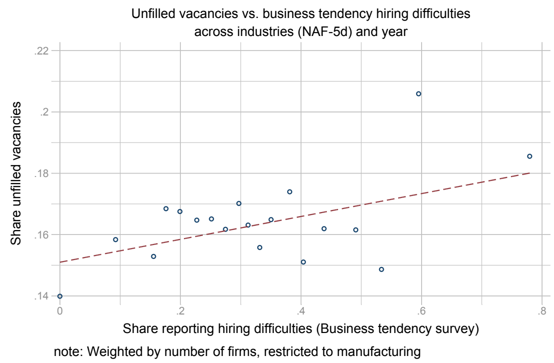
(d) Low-skill Workers



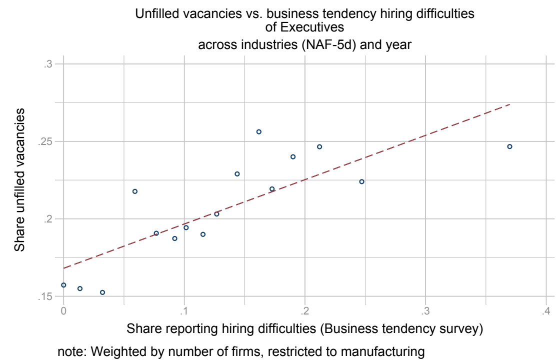
Source:
Note:

Figure A7: Unfilled vacancies vs. hiring difficulties in Business Tendency Survey

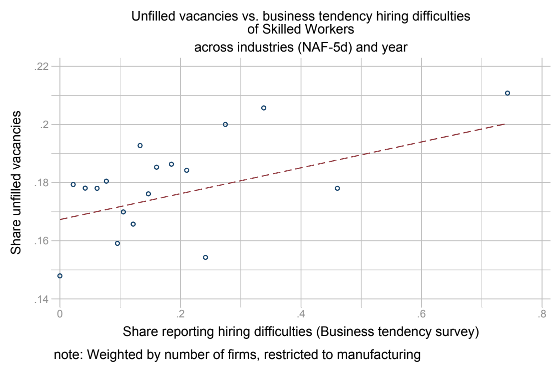
(a) All occupations



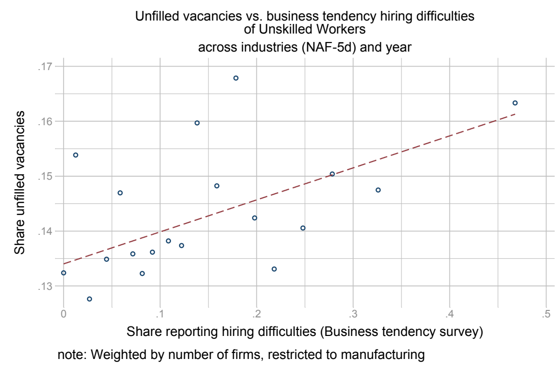
(b) Executives



(c) Skilled workers



(d) Low-skill Workers



Source:
Note:

Table A6: **Hiring difficulties in vacancy data vs. business tendency survey**

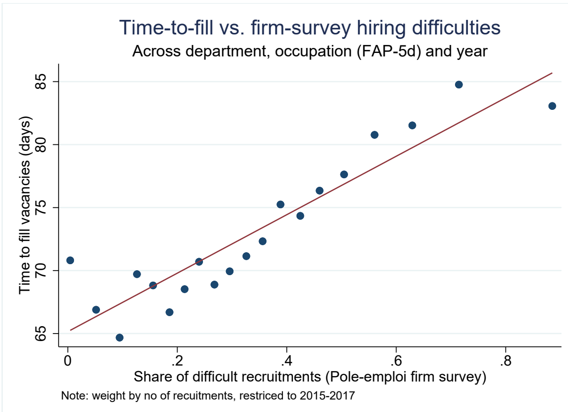
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	timetofill_un	timetofill_un_exec	timetofill_un_skilled	timetofill_un_unskill	unfilled	unfilled_exec	unfilled_skilled	unfilled_unskill
current_hiring_prob	16.1***				0.037***			
	(2.50)				(0.0064)			
hiringexec_prob		117***				0.29***		
		(10.3)				(0.030)		
hiringskilled_prob			17.0***				0.044***	
			(3.26)				(0.0083)	
hiring_prob				31.6***				0.058***
				(4.67)				(0.012)
Constant	88.1***	100***	95.1***	77.9***	0.15***	0.17***	0.17***	0.13***
	(0.95)	(1.50)	(0.74)	(0.86)	(0.0024)	(0.0044)	(0.0019)	(0.0023)
Observations	1,972	1,725	1,731	1,679	1,972	1,725	1,731	1,679
R-squared	0.021	0.069	0.015	0.027	0.017	0.050	0.016	0.013

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

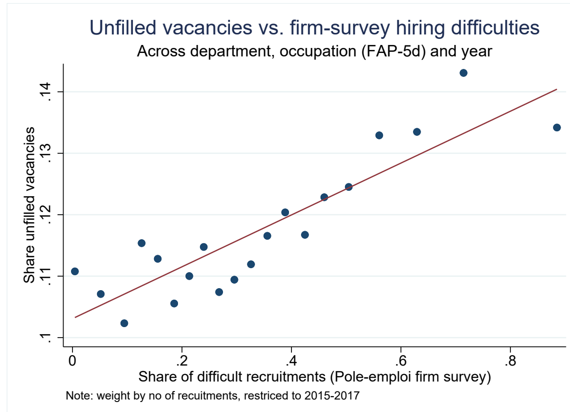
Note: This table show

Figure A8: Recruiting Time/Unfilled vacancies vs. hiring difficulties in *Pole Emploi* Firm Survey

(a) Recruiting Time



(b) Unfilled Vacancies



Source:
Note:

Table A7: **Hiring difficulties in vacancy data vs. Pole Emploi firm survey**

VARIABLES	(1) timetofill3	(2) timetofill3	(3) timetofill3	(4) unfilled	(5) unfilled	(6) unfilled
sh_xmet	23.2***	5.08***	22.3***	0.042***	0.0085***	0.043***
	(0.64)	(0.76)	(0.63)	(0.0018)	(0.0022)	(0.0018)
Observations	42,691	42,691	42,691	42,691	42,691	42,691
R-squared	0.030	0.174	0.097	0.013	0.118	0.093
Dep. mean	73.1	73.1	73.1	0.12	0.12	0.12
1-sd effect	5.25	1.15	5.05	0.0095	0.0019	0.0096
Year FE		Y	Y		Y	Y
Occ. FE		Y			Y	
Dept FE			Y			Y

Note: This table show

C Theoretical framework

In this section, we relate theoretically the notion of hiring difficulties to the time-to-fill of job vacancies in standard search and matching models (Cahuc et al., 2018). We consider firms of employment size L_t at the end of period t. Firms post vacancies to hire workers. Let's denote V_t the number of vacancies posted in period t and H_t the number of hires. There is an exogenous separation rate q_t generating S_t separations in period t. First we have an accounting relation where

$$L_t - L_{t-1} = H_t - S_t \tag{3}$$

Vacancies are filled at rate m_t, so that $H_t = V_t \times m_t$. The previous accounting relation becomes:

$$L_t - L_{t-1} = V_t \times m_t - L_{t-1} \times q_t \tag{4}$$

Firms have a revenue $A_t R(L_t)$ when employing L_t workers, where A_t is labor productivity. We denote R_L the marginal return to labor. Firms pay wages w_t and a flow vacancy cost c_v. They maximize their profits subject to the employment law of motion above.

$$\Pi(L_{t-1}) = \max_{L_t, V_t} A_t R(L_t) - w_t L_t - c_v V_t + \beta \mathbb{E}\left[\Pi(L_t)\right] \tag{5}$$

Taking the foc wrt V_t, we obtain:

$$A_t R_L(L_t) = w_t + \frac{c_v}{m_t} - \beta \mathbb{E}\left[\Pi_L(L_t)\right] \tag{6}$$

Using the envelope theorem, we obtain:

$$\Pi_L(L_t) = \frac{(1 - q_{t+1})c_v}{m_{t+1}} \tag{7}$$

We can then write the dynamic labor demand equation:

$$A_t R_L(L_t) = w_t + \frac{c_v}{m_t} - \beta E\left[\frac{(1 - q_{t+1})c_v}{m_{t+1}}\right] \tag{8}$$

Note that $\tau_t = 1/m_t$ represents the average recruiting time. Let's further assume that the revenue function has decreasing marginal returns. With $R = L^\alpha/\alpha$ and

$\alpha \in (0,1)$, the marginal return is $R_L = L^{\alpha-1}$.
Let's take the log of the labor demand equation:

$$a_t + (\alpha - 1)l_t = \log(w_t) + \log\left(1 + \frac{c_v \tau_t}{w_t} - \frac{\beta}{w_t} E\left[(1 - q_{t+1})c_v \tau_{t+1}\right]\right) \tag{9}$$

Let's consider a deviation $d\tau_t$, the change in employment then writes:

$$(\alpha - 1)dl_t = \frac{c_v}{w_t} \frac{d\tau_t}{\left(1 + \frac{c_v \tau_t}{w_t} - \frac{\beta}{w_t} E\left[(1 - q_{t+1})c_v \tau_{t+1}\right]\right)} \tag{10}$$

where we have taken wages as exogenous.
Let's first consider the denominator. We assume stationary recruiting time and separations, so that $E\left[(1 - q_{t+1})c_v \tau_{t+1}\right] = (1 - q_t)c_v \tau_t$. We can then factorize write the second and third terms of the denominator as: $\frac{c_v \tau_t}{w_t}(1 - \beta(1 - q_t))$. We note the annual discount factor can be set to 0.95, and the separation rate around 0.5, so that $0 < \beta(1 - q_t) < 1$. In Cahuc et al. (2018), "the hiring cost amounts to 1.2% of the annual wage": $\frac{c_v \tau_t}{w_t} = 0.012$, which can then be neglected wrt 1. We obtain the following approximated expression:

$$dl_t = \frac{c_v}{w_t} \frac{1}{(\alpha - 1)} d\tau_t = \frac{0.058}{-0.33} d\tau_t = -0.168 d\tau \tag{11}$$

where the ratio of flow cost of vacancy posting wrt wage is from Cahuc et al. (2018) and we chose 2/3 as the elasticity of revenue to employment. The calibrated semi-elasticity in Equation (11) is comparable to our empirical estimate, where the regression specification has τ, the vacancy duration in fraction of year (between 0 to 1).

This expression shows that when α increases, then the semi-elasticity of employment wrt recruiting time remains negative and increases in absolute value. The parameter α is related to the employment intensity of the firm production function. The current model abstracts from capital and assets. If we were to include the capital input K_t, then we would assume a standard Cobb-Douglas production function: $A_t K^{1-\alpha} L^{\alpha}$. At equilibrium, we would obtain that the share of labor costs in firms output would be equal to α. So that more capital intensive firms have lower α and more labor intensive firms higher α.

D Wald estimator with two different samples

The IV estimator can be computed as the Wald ratio of the reduced-form estimate ($\hat{r}$) and of the first-stage estimate ($\hat{f}$). Let us denote $se(r)$ (resp. $se(f)$) the standard errors of $\hat{r}$ (resp. $\hat{f}$). Then using the delta method, we obtain the standard errors of the Wald ratio ($\hat{w} = \hat{r}/\hat{f}$) as:

$$se(w) = \left(\frac{se(r)^2}{\left(\hat{f}\right)^2} + \frac{se(f)^2 \, (\hat{r})^2}{\left(\hat{f}\right)^4} \right)^{-1/2}$$

Recall the delta method, where Σ is the variance-covariance matrix of the real vector X:

$$Var(h(X)) = \nabla h^t \Sigma \nabla h$$