The Size-Centrality Relationship in Production Networks

Nikola Dacic[1] Marko Melolinna[2]

[1]Goldman Sachs & Co.

[2]Bank of England

November 24, 2020

4th Annual Workshop of the ESCB Research Cluster 2

Disclaimer

Any views expressed in this presentation are solely those of the authors and cannot be taken to represent those of the Bank of England or to state Bank of England policy. This presentation should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee. This paper was written in Nikola Dacic's individual capacity and is not related to his role at Goldman Sachs. The analysis, content and conclusions set forth in this paper are those of the authors alone and not of Goldman Sachs & Co. or any of its affiliate companies.

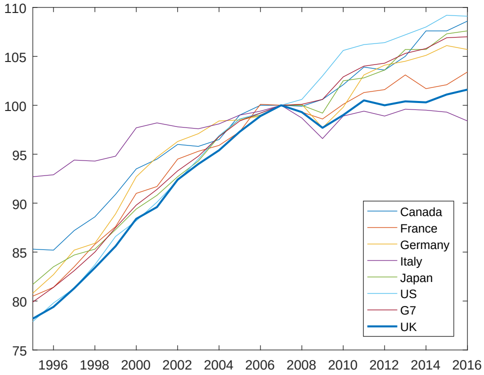
Outline

Outline

1. Introduction
2. Size and Centrality Measures
3. Model
4. Empirical Results
5. Summary

Aggregate productivity

Figure 1: *Real GDP per hour worked (2007=100)*



Sources: OECD, ONS

Motivation

- Production of goods and services typically involves inputs that firms source from other producers, in addition to the value-added they generate themselves. The existence of such input linkages gives rise to production networks.
- In this paper, we focus on two characteristics of producers that determine their importance in a production network: their *size* and *centrality*.
- Our main interest is in the relationship between size and centrality, and how this relationship changes over time.
- Use a structural model calibrated on UK data to shed light on UK productivity growth slowdown since the GFC.

Our Contributions

- We analyse characteristics of the UK's input-output network, showing there are significant asymmetries in the degree of importance of industries as input suppliers.
- We show that there is significant time-variation in the input-output network over time, which is inconsistent with a Cobb-Douglas aggregation of intermediate inputs in the production function.
- We set up a structural model with a production network calibrated using UK data, which we use to filter out technology and demand shocks.
- We show that technology shocks imply a negative relationship between size and centrality, which is inconsistent with the empirical relationship. We show that demand shocks can help to reconcile the model's predictions with the data.
- We use this model to analyse the UK's productivity growth puzzle and show that shocks to the manufacturing and finance sectors as well as common shocks have played a key role in driving the slowdown.

Related Literature

- Importance of microeconomic shocks for aggregate macroeconomic dynamics (Hulten (1978), Baqaee and Farhi (2019)) and importance of production networks (Carvalho and Tahbaz-Salehi (2019)).
- Established literature on the role of production networks in the transmission of common and idiosyncratic shocks (e.g. Long and Plosser (1983), Dupor (1999), Acemoglu et al. (2012))
- In production networks, complementarities and substitutability among production inputs play a key role (Jones (2011)).
- Atalay (2017) shows that elasticities of substitution (e.g. among intermediate inputs) have a significant effect on the importance of idiosyncratic (as opposed to common) shocks in driving aggregate fluctuations.
- Sectoral location of UK productivity puzzle (e.g. Riley et al. (2018), Tenreyro (2018).

Outline

Key definitions

Key variables of interest:

- *size* - output (level of real output of producer j) (Q_{jt}),
- *size* - Domar weight (level of nominal output of a producer divided by nominal GDP);

$$\lambda_{jt} = \frac{P_{jt} Q_{jt}}{\mathrm{P}_t Q_t} \tag{1}$$

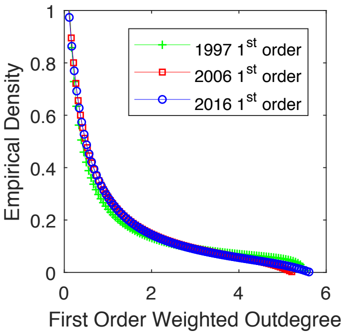
- *centrality* - first–order weighted outdegree of sector j;

$$D_{jt}^{out} = \sum_{i=1}^{N} \omega_{ijt}, \tag{2}$$

where ω_{ijt} is the share of industry i's inputs that are produced by industry j at time t, for total of N industries, P_{jt} is the price of the output of sector j, P_t is the aggregate price level and Q_t is the aggregate real output.

Weighted outdegrees - UK data

Figure 2: *Empirical density and counter-cumulative distribution of weighted outdegrees*



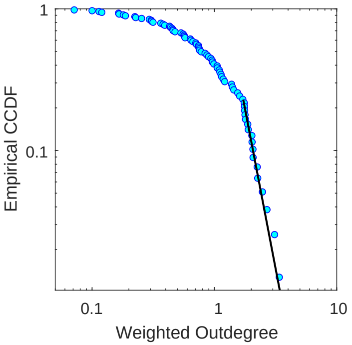
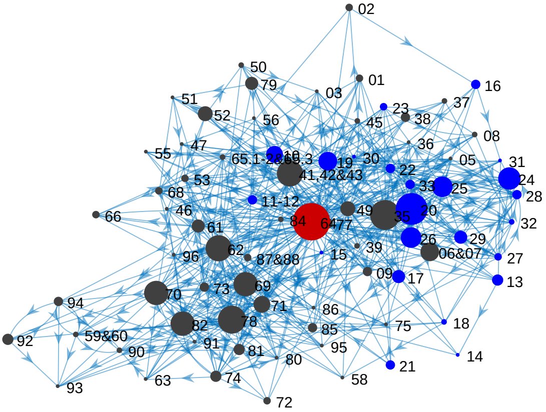


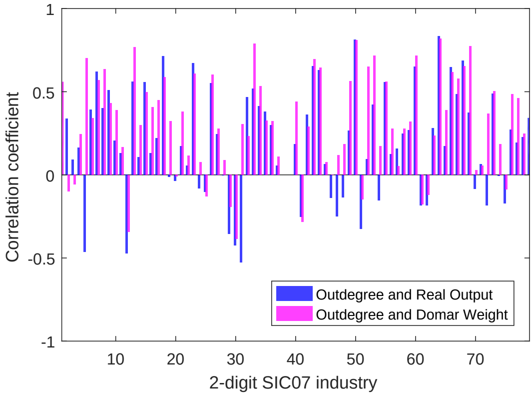
Figure 3: *A graphical representation of the UK input-output network in 2015*



Notes: Finance (64) is the red node, manufacturing industries (10-33) are blue nodes.

Outdegrees and size - data

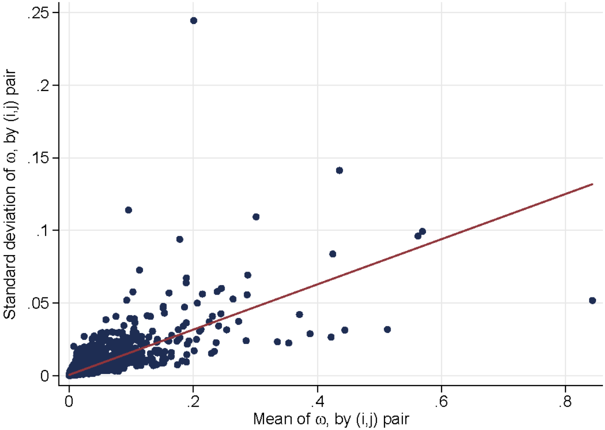
Figure 4: *Correlation between the growth rates of outdegrees and real output and Domar weights, by industry*



Notes: Correlations are calculated between year-on-year growth rates. All 2-digit industries included. Sample covers 1997-2016.

Stability of input–output linkages

Figure 5: *Scatterplot of mean and standard deviation of intermediate input shares*



Outline

Key Features of the Model

- A multisector model where agents (households and firms) face a dynamic optimisation problem (similar to Atalay (2017)).
- Representative household derives utility from the N different consumption goods (produced by the N industries) and disutility from supplying labour.
- Each industry produces a quantity of output using capital, labour and intermediate inputs according to a CRS production function.
- Capital stock is accumulated via an industry-specific bundle of investment goods, and intermediate inputs can be sourced from all industries.
  - Parameter ε_M determines how easily substitutable the goods in the intermediate bundle are. We estimate this with UK data and calibrate $\varepsilon_M = 0.4$.
- The market-clearing condition for each industry states that output can be used for consumption, as an intermediate input, or for investment.
- We solve for the competitive equilibrium of a log–linearised version of the model, and calibrate parameters from UK data and Atalay (2017).

Selected Definitions and Derivations

Intermediate input bundle of industry i is defined as follows (CES):

$$M_{it} = \left(\sum_{l=1}^{N} \left(\Gamma_{ij}^{M} \right)^{\frac{1}{\varepsilon_M}} (M_{ijt})^{\frac{\varepsilon_M - 1}{\varepsilon_M}} \right)^{\frac{\varepsilon_M}{\varepsilon_M - 1}} . \tag{3}$$

where Γ_{ij}^{M} is weight of industry j as an intermediate–good supplier to industry i.
The partial derivative of a industry j outdegree with respect to its price is given by:

$$\frac{\partial D_{jt}^{out}}{\partial P_{jt}} = \sum_{i=1}^{N} (1 - \varepsilon_M)\omega_{ijt}(1 - \omega_{ijt})P_{jt}^{-1} \tag{4}$$

where

$$\omega_{ijt} = \frac{P_{jt} M_{ijt}}{P_{it}^{M} M_{it}} \tag{5}$$

In other words, if $\varepsilon_M < 1$, the price and the outdegree move in the same direction (as $0 < \omega_{ijt} < 1$).

Processes for Key Endogenous and Exogenous Variables

Exogenous Processes (A_t is vector of technology shocks in N industries, $(A_{t1}, ..., A_{tN})'$, and D_t is vector of preference shocks in N industries, $(D_{t1}, ..., D_{tN})'$);

$$\log A_t = \log A_{t-1} + \omega_t^A, \tag{6}$$

$$\log D_t = \log D_{t-1} + \omega_t^D. \tag{7}$$

For endogenous processes of size, denote the vector of log-deviations of real gross output and Domar weights from their steady-state values as $\hat{q}_t$ and $\hat{\lambda}_t$, respectively. They can be expressed as functions of capital ($\hat{k}_t$) and the shocks:

$$\begin{bmatrix} \Delta\hat{q}_t \\ \Delta\hat{\lambda}_t \end{bmatrix} = \begin{bmatrix} \mathbf{F}_k \\ \mathbf{L}_k \end{bmatrix} \Delta\hat{k}_t + \begin{bmatrix} \mathbf{F}_a \\ \mathbf{L}_a \end{bmatrix} \omega_t^A + \begin{bmatrix} \mathbf{F}_d \\ \mathbf{L}_d \end{bmatrix} \omega_t^D, \tag{8}$$

Log-deviation of weighted outdegrees from their steady state values:

$$\Delta\hat{d}_t^{out} = \mathbf{D}_k \Delta\hat{k}_t + \mathbf{D}_a \omega_t^A + \mathbf{D}_d \omega_t^D. \tag{9}$$

where all the matrices in square brackets are functions of model parameters only.

Model Filter

We use quarterly data on UK industries' value-added and labour input ($\hat{l}_t$) to filter out the technology and preference shocks in the model. Atalay (2017) shows that the model filter follows a VARMA(1,1) process:

$$\begin{bmatrix} \Delta \hat{v}_{t+1} \\ \Delta \hat{l}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_v & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_l \end{bmatrix} \begin{bmatrix} \Delta \hat{v}_t \\ \Delta \hat{l}_t \end{bmatrix} + \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_d \\ \mathbf{L}_a & \mathbf{L}_d \end{bmatrix} \begin{bmatrix} \omega^A_{t+1} \\ \omega^D_{t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{W}_a & \mathbf{W}_d \\ \mathbf{LL}_a & \mathbf{LL}_d \end{bmatrix} \begin{bmatrix} \omega^A_t \\ \omega^D_t \end{bmatrix} \tag{10}$$

where matrices in square brackets are functions of model parameters only.

- By expressing the shocks as a function of data and the lagged shock itself and assuming that the initial shocks are zero, we can solve equation (10) forward to filter out both technology and preference shocks.
- Once we have filtered out the shocks, we can back out the implied size and centrality variables, using equations (8) and (9).

Productivity contributions of shocks vs sectors

The growth of aggregate labour productivity can be decomposed as follows:

$$\Delta\hat{Y}_t - \Delta\hat{L}_t = S^Y \Delta\hat{v}_t - L\Delta\hat{l}_t$$
$$= S^Y \left(\mathbf{V}_k \Delta\hat{k}_t + \mathbf{V}_a \omega_t^A + \mathbf{V}_d \omega_t^D\right) - L\left(\mathbf{L}_k \Delta\hat{k}_t + \mathbf{L}_a \omega_t^A + \mathbf{L}_d \omega_t^D\right) \quad (11)$$

where S^Y corresponds to the vector of steady-state shares of industry i's value-added in aggregate value-added and L denotes a vector of industry i's steady-state share of aggregate labour.

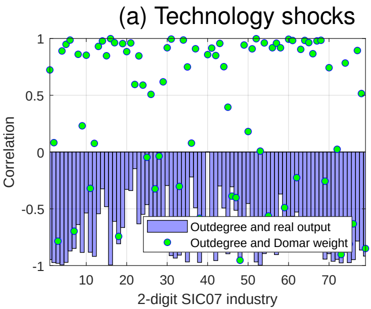
- Hence, we can decompose aggregate labour productivity growth into contributions from industries, and contributions from shocks.

Shocks and their effects

- Positive **technology shock** in industry j leads to...
  - ...lower price of output; $P_{jt} \downarrow$
  - ...higher output of sector j; $Q_{jt} \uparrow$
  - ...lower outdegree; $D_{jt}^{out} \downarrow$ (as $\varepsilon_M < 1$ (0.4), and the negative price effect dominates the positive quantity effect)
  - ...lower Domar weight; $\lambda_{it} \downarrow$ (as $\varepsilon_M < 1$ (0.4), and the negative price effect dominates the positive quantity effect)
  - ...positive correlation between effects on outdegrees vs Domar weights, and negative correlation between effects on outdegrees vs real outputs.
- Positive **preference shock** in industry j leads to...
  - ...higher price of output; $P_{jt} \uparrow$.
  - ...higher output of sector j; $Q_{jt} \uparrow$
  - ...higher outdegree; D_{jt}^{out} (as prices and quantities move in the same direction)
  - ...higher Domar weight; $\lambda_{it} \uparrow$ (as prices and quantities move in the same direction)
  - ...positive correlation between effects on outdegrees vs Domar weights, and between effects on outdegrees vs real outputs.

Outdegrees and size - shocks in the model

Figure 6: *Model-implied relationship between size and centrality*



Outline

Data

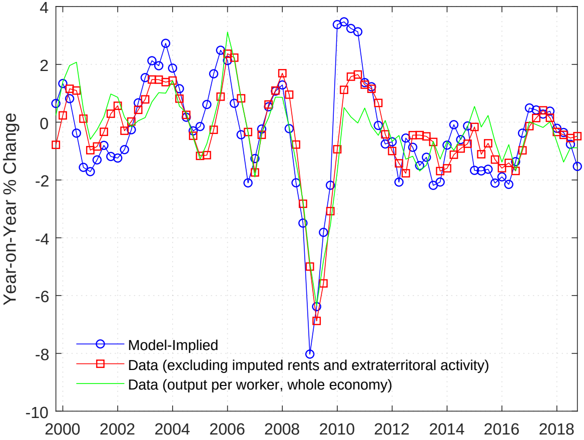
All data used in this paper is provided by the UK Office for National Statistics (ONS) and is publicly available:

- **supply and use tables** are published annually (1997 to 2016).
- data on **industries' value-added** can be obtained from the ONS's GDP(O) low-level aggregates dataset (1990Q1 to 2019Q1).
- data on **jobs per industry** (1978Q2 to 2018Q4).
- data on **industries' price deflators** (1997Q1 to 2018Q2). We use the implied GVA deflators to impute the deflators over the last two quarters of 2018

The full dataset we use to filter out the shocks has $T = 87$ (1997Q2–2018Q4) and $N = 79$. The burn-in period is set to equal 7 quarters. To seasonally adjust the data, we use the Census X-13 method in EViews.

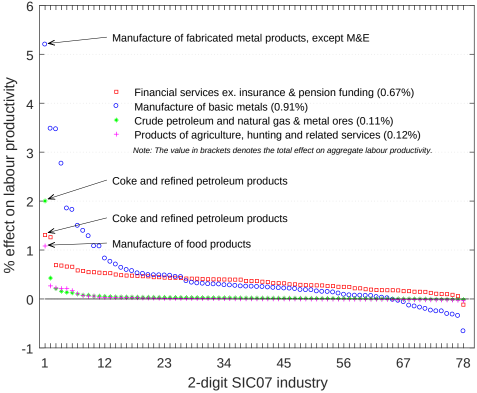
Aggregate productivity - data vs model

Figure 7: *Aggregate labour productivity de-meaned growth: data vs. model*



Notes: The model-implied series is based on de-meaned data on industries' value-added and jobs growth. The two series based on ONS data have been de-meaned so as to ensure that all three series have the same mean over the period shown in the figure.

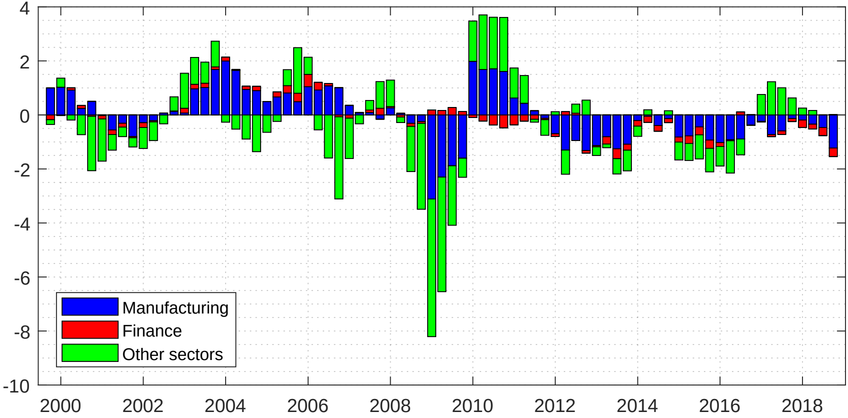
Figure 8: *IRFs of labour productivity to a 10% technology shock in selected industries*



Notes:The *x*-axis has 78 ticks, corresponding to 78 industries other than the industry which is the source of the technology shock. In each of the four cases, the first tick corresponds to the most affected industry other than the industry in which the shock has originated, the second tick to the second most affected industry other than the industry in which the shock has originated, and so on.

Historical decomposition - by sector

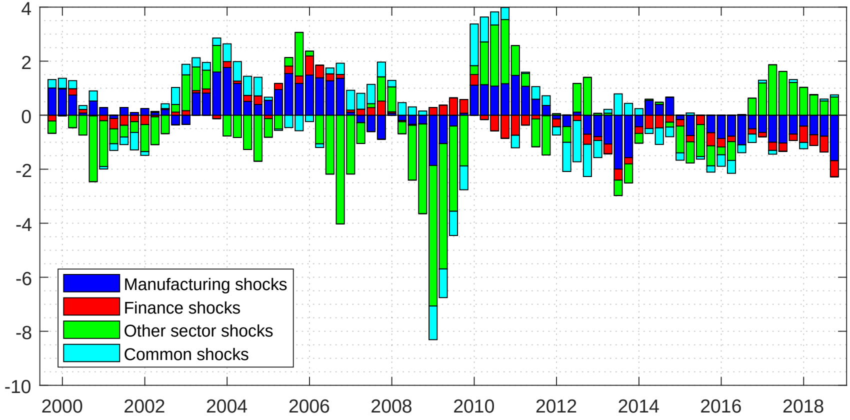
Figure 9: *Historical decomposition of contributions to aggregate labour productivity fluctuations, by sector* ($\varepsilon_M = 0.4$)



Notes: All 2-digit industries included.

Historical decomposition - by shock

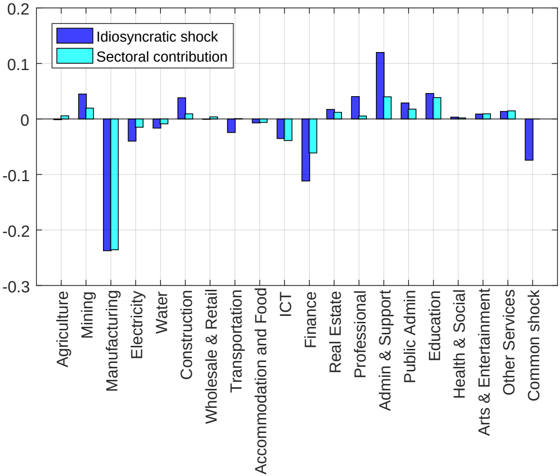
Figure 10: *Historical decomposition of contributions to aggregate labour productivity fluctuations, by shock* ($\varepsilon_M = 0.4$)



Notes: All 2-digit industries included.

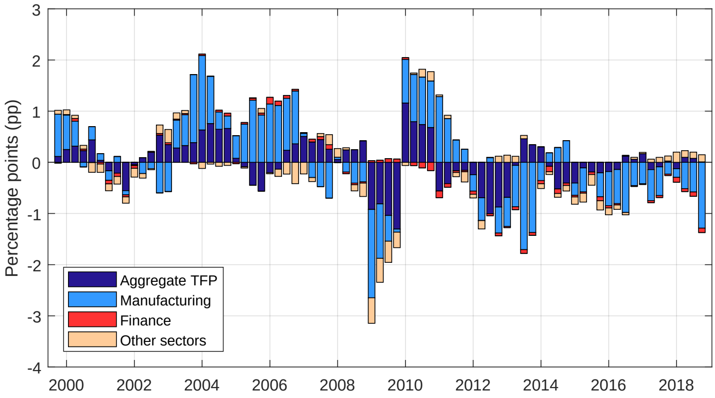
UK productivity growth puzzle - sectors vs shocks

Figure 11: *Contributions to the growth puzzle: sectors vs. shocks*



Historical decomposition - manufacturing sector

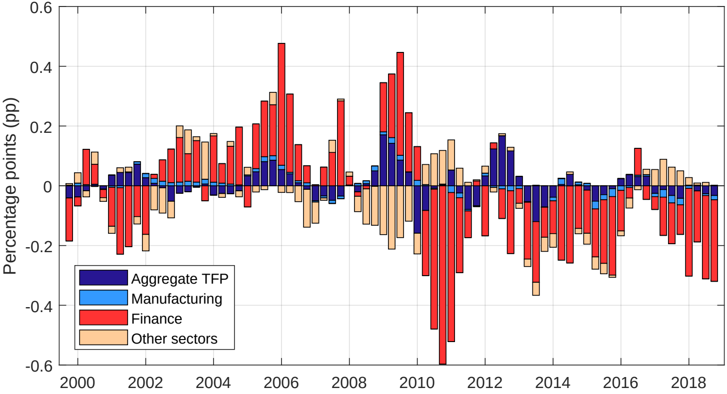
Figure 12: *Sectoral contributions to aggregate labour productivity growth, decomposed by source of shock* ($\varepsilon_M = 0.4$)



Notes: Aggregate TFP denotes the common shock.

Historical decomposition - finance sector

Figure 13: *Sectoral contributions to aggregate labour productivity growth, decomposed by source of shock* ($\varepsilon_M = 0.4$)



Notes: Aggregate TFP denotes the common shock.

Outline

Summary of main results

- We show that there is significant time-variation in the input-output network over time, which is inconsistent with a Cobb-Douglas aggregation of intermediate inputs in the production function.
- In our structural model, we show that technology shocks imply a negative relationship between size and centrality, which is inconsistent with the empirical relationship. We show that demand shocks can help to reconcile the model's predictions with the data.
- We use this model to analyse the UK's productivity growth puzzle and show that shocks to the manufacturing and finance sectors as well as common shocks have played a key role in driving the slowdown.
- Could the model be used to say something about the Covid-19 shock? Maybe, though not in a self–contained way...

References

Acemoglu, D. , Carvalho, V. M., Ozdaglar, A. and Tahbaz-Salehi, A., (2012), The Network Origins of Aggregate Fluctuations, *Econometrica*, 80, p. 1977-2016.

Atalay, E. (2017). How Important Are Sectoral Shocks?. *American Economic Journal: Macroeconomics*, 4, p. 254-80.

Baqaee, D. and Farhi, E. (2019), The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem', *Econometrica* (forthcoming).

Carvalho, V. M., and Tahbaz-Salehi, A., (2019), Production Networks: A Primer, *Annual Review of Economics*, 11:1, p. 635-663.

Dupor, B., (1999), Aggregation and irrelevance in multi–sector models, *Journal of Monetary Economics*, 43, p. 391-409.

Hulten, C. R., (1978), Growth Accounting with Intermediate Inputs, *Review of Economic Studies*, 45, issue 3, p. 511-518.

Jones, C. I., (2011), Intermediate Goods and Weak Links in the Theory of Economic Development, *American Economic Journal: Macroeconomisc*, Vol. 3, No. 2, pp. 1-28.

Long, J. B., and Plosser, C. I., (1983), Real Business Cycles, *Journal of Political Economy* Vol. 91, No. 1 (Feb., 1983), p. 39-69.

Riley, R., Rosazza-Bondibene, C., and Young, G. (2015), The UK productivity puzzle 2008-13: evidence from British businesses, *Bank of England Working Paper*, No. 531.

Tenreyro, S. ,(2018), The fall in productivity growth: causes and implications, Speech at Peston Lecture Theatre, Queen Mary University of London, 15 January 2018.